Bimetric structure formation: Non-Gaussian predictions

Based on:
J. Magueijo, JN, F. Piazza; Bimetric structure formation: Non-Gaussian predictions;
Phys.Rev.D82:043521,2010 (arXiv: 1006.3216)

Johannes Noller, UniverseNet Lecce, 17/09/10

The bottom line

- We derive the ***non-Gaussian amplitudes for a set of bimetric models***…

- …establishing a ***consistency relationship*** between the non-Gaussian amplitudes/shapes and the spectral index n_s

Physical and gravitational geometry

Status quo - a one-geometry description of physics (e.g. GR):

$$S \propto \int d^4x \sqrt{-g} \left(\frac{R[g_{\mu\nu}]}{2} + \mathcal{L}_m[g_{\mu\nu}, \Phi_{Matt}] \right)$$

What if nature has two geometries? E.g. gravity and ordinary matter being governed by two different metrics

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An example: Brans-Dicke theory 1961 (in the Jordan frame)

$$S \propto \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right] + \int d^4x \sqrt{-g}\, \mathcal{L}_m$$

Bekenstein 1992: If there are two metrics at play, what is the most general relation between them that respects weak equivalence and causality?

$$\hat{g}_{\mu\nu} = e^{2\phi} \left[A(x, \phi) g_{\mu\nu} - B(x, \phi) \partial_\mu \phi \partial_\nu \phi \right]$$

The model

A bimetric theory with action:

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R[g_{\mu\nu}] + \int d^4x\sqrt{-\hat{g}}\,\mathcal{L}_m[\hat{g}_{\mu\nu},\Phi_{Matt}]$$

$$-\int d^4x\sqrt{-g}\left(\frac{1}{B(\phi)}+V(\phi)\right)+\int d^4x\sqrt{-\hat{g}}\frac{1}{B(\phi)}$$

The "matter" and "gravity" metrics are related by

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - B(\phi)\partial_\mu\phi\partial_\nu\phi$$

The Einstein frame

Using

$$\hat{g} = g(1 + 2B(\phi)X) \qquad X \equiv -\tfrac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

we map bimetric action into the Einstein frame:

$$S = \int d^4x\sqrt{-g}\left(\tfrac{R}{2} + \sqrt{1 + 2B(\phi)X}\,\tfrac{1}{B(\phi)} - \tfrac{1}{B(\phi)} + V(\phi)\right)$$

compare with

K-essence: $$S = \int \mathrm{d}^4x\sqrt{-g}\left[\tfrac{R}{2} + P(X,\phi)\right]$$

DBI models: $$P(X,\phi) = -f^{-1}(\phi)\sqrt{1 - 2f(\phi)X} + f^{-1}(\phi) - V(\phi)$$

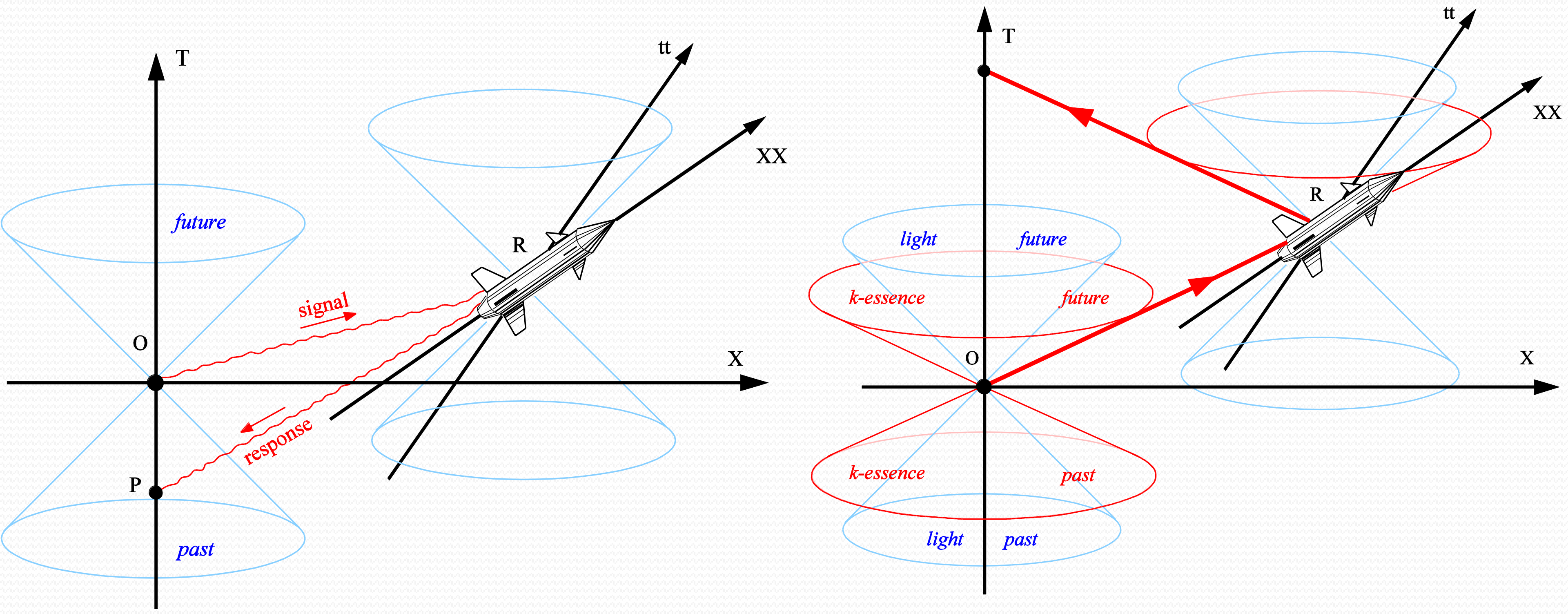
“Superluminality” – Feature or Bug?

Two light cones at any point, with different propagation speeds corresponding to each metric

$$B(\phi) > 0 \quad \Rightarrow \quad c_{light} > c_{grav} \quad \Rightarrow \quad c_s > 1$$
$$B(\phi) < 0 \quad \Rightarrow \quad c_{light} < c_{grav} \quad \Rightarrow \quad c_s < 1$$

- K-essence models are known to give rise to an effective bimetric description (see e.g. Bruneton ‘06). “Superluminal” behaviour a problem?
- VSL models with dynamically varying sound speed known to resolve horizon & flatness problems and produce (near)scale-invariant power spectra (see e.g. Magueijo ‘03).
- Causality concerns?

The (anti-)telephone?



Babichev, Mukhanov and Vikman ‘08

The 2-point function

Taking the k-essence type action ansatz

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(X,\phi)\right]$$

Consider subset of solutions with

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \approx 0, \quad \eta_s \equiv \frac{\dot{\epsilon_s}}{\epsilon_s H} \approx 0$$

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We find

$$B(\phi) = -f(\phi) = \frac{3\epsilon_s^2}{16\epsilon^2 V_0}\left(\frac{\phi}{M_{\rm Pl}}\right)^{2+\frac{4\epsilon}{\epsilon_s}}\left[1 + \mathcal{O}\left(\frac{\phi}{M_{\rm Pl}}\right)^{-4}\right]$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - B(\phi)\partial_\mu\phi\partial_\nu\phi$$

$$n_s - 1 = \frac{2\epsilon+\epsilon_s}{\epsilon_s+\epsilon-1}$$

The non-Gaussian amplitude

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3){P_\zeta}^2\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

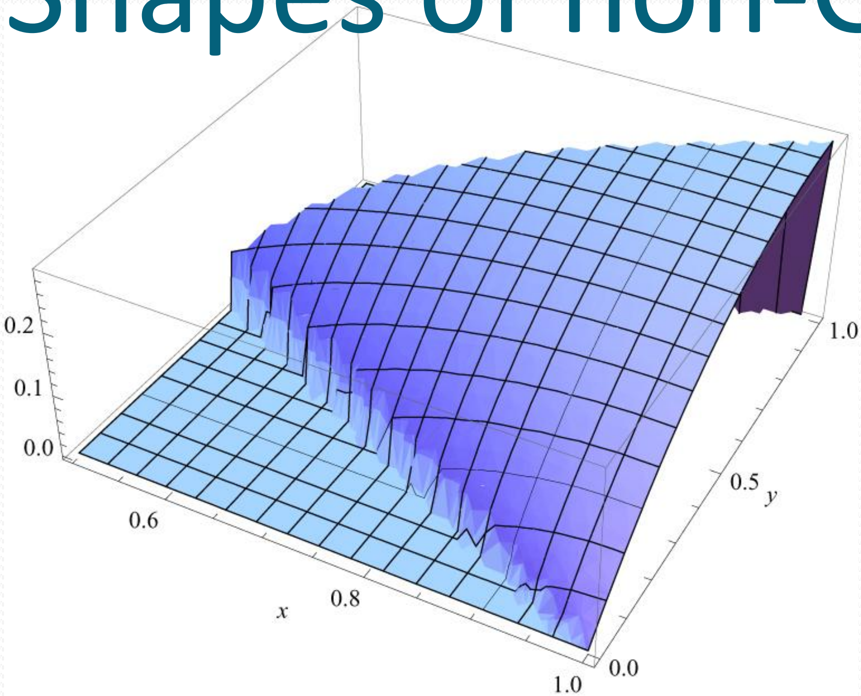
In the large "speed of sound" limit we obtain the following amplitude

$$\mathcal{A} = \left(\frac{k_1k_2k_3}{2K^3}\right)^{n_s-1}\left[-\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j}k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right.$$

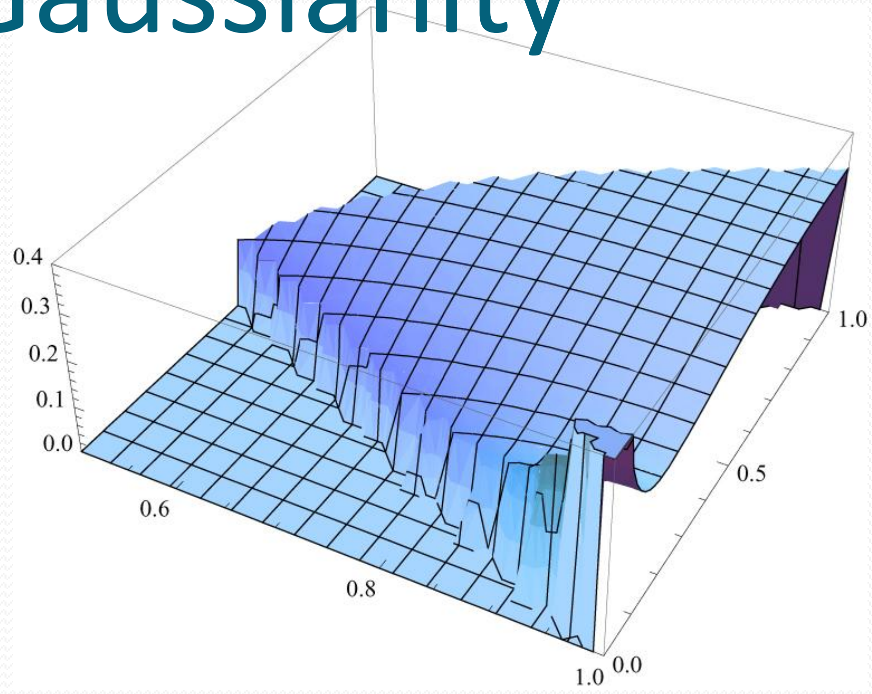
$$+(n_s-1)\left(-\frac{1}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j}k_ik_j^2 + \frac{1}{8}k_1k_2k_3 + \frac{1}{2K}\sum_{i<j}k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right)$$

$$\left.+\mathcal{O}\left(\frac{1}{c_s^2}\right)\right],$$

Shapes of non-Gaussianity



$\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 1$



$\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 0.96$

In equilateral limit: $\mathcal{A}(n_s = 0.96) \approx \mathcal{A}(n_s = 1) + \mathcal{O}(0.1\mathcal{A})$

f_{NL}

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta{}^2\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

f_{NL} as a single parameter measure of non-Gaussianity

$$\zeta = \zeta_g(x) + \frac{3}{5}f_{\rm NL}\zeta_g^2$$

$$f_{\rm NL} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}$$

How does f_{NL} here compare to other models of structure formation?

"Standard" inflation: $f_{\rm NL} \approx \mathcal{O}(\epsilon)$

DBI inflation (with ε -> 0): $f_{\rm NL} \approx 0.31 - 0.324\bar{c}_s^{-2}$ (Khoury & Piazza 08)

Our **bimetric model**: $f_{\rm NL} \approx \mathcal{O}(1)$

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- We have derived the ***non-Gaussian amplitudes for a set of bimetric models***…

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- Work in Progress: ***Extend results to other types of k-essence models***.

Thank you!