EMISSION OF MASSIVE SCALAR FIELDS BY A HIGHER-DIMENSIONAL ROTATING BLACK-HOLE

N. Pappas
University of Ioannina

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MOTIVATION

- Possibility of BH creation at LHC due to the existence of Large Extra Dimensions (ADD, RS models).
- Microscopic BH are higher-dimensional objects, so their decay pattern depends on the number and the nature (flat, warped) of the extra dimensions.
- The existence of both “bulk” and “brane” channel is a characteristic feature of Hawking radiation. The knowledge of the balance between them would be a guide to evaluate relevant detector signals (if any!).
- We built a more realistic model since massive scalars are predicted both by the Standard Model and supersymmetric theories.

$\tilde{\omega}$

Solving the eq. of motion

We start from Myers Perry line-element

$$ds^2 = -\left(1 - \frac{\mu}{\Sigma\, r^{n-1}}\right) dt^2 - \frac{2a\mu \sin^2\theta}{\Sigma\, r^{n-1}}\, dt\, d\varphi + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2$$
$$+ \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma\, r^{n-1}}\right) \sin^2\theta\, d\varphi^2 + r^2\cos^2\theta\, d\Omega_n^2$$

and then try to solve the eq. of motion for massive particles

$$\frac{1}{\sqrt{-G}}\,\partial_M\left(\sqrt{-G}\, G^{MN}\partial_N\Phi\right) - m_\Phi^2\Phi = 0$$

by using the ansatz $\Phi = e^{-i\omega t} e^{im\varphi} R(r)\, S(\theta)\, Y_{ln}(\theta_1, \ldots, \theta_{n-1}, \phi)$

where $Y_{ln}(\theta_1, \ldots, \theta_{n-1}, \phi)$ are the hyper-spherical harmonics, $R(r)$ the radial part and $S(\theta)$ the spheroidal harmonics, whose known eigenvalues $E_{j\ell m}(a\omega)$ are now taken to be $\tilde{E}_{j\ell m}(a\tilde{\omega})$ since, instead of ω, we encounter the shifted form $\tilde{\omega} \equiv \sqrt{\omega^2 - m_\Phi^2}$.
Finally we end up with decoupled equations for all three functions.

Defining Absorption Probability

Near the horizon:

- the eq. for the radial part R(r) takes the form of a hypergeometric DE.
- for r $\rightarrow$ r_h we find free plane waves (potential equals zero there) and demand only ingoing waves to exist.

Far from the horizon:

- the eq. for the radial part R(r) takes the form of a Bessel DE.
- for r $\rightarrow$ ∞ we retrieve again free waves both in- and outgoing.
- we define the absorption probability as $|\mathcal{A}_{j\ell m}|^2 = 1 - \left| \frac{A_{out}^{(\infty)}}{A_{in}^{(\infty)}} \right|^2$

In the intermediate region:

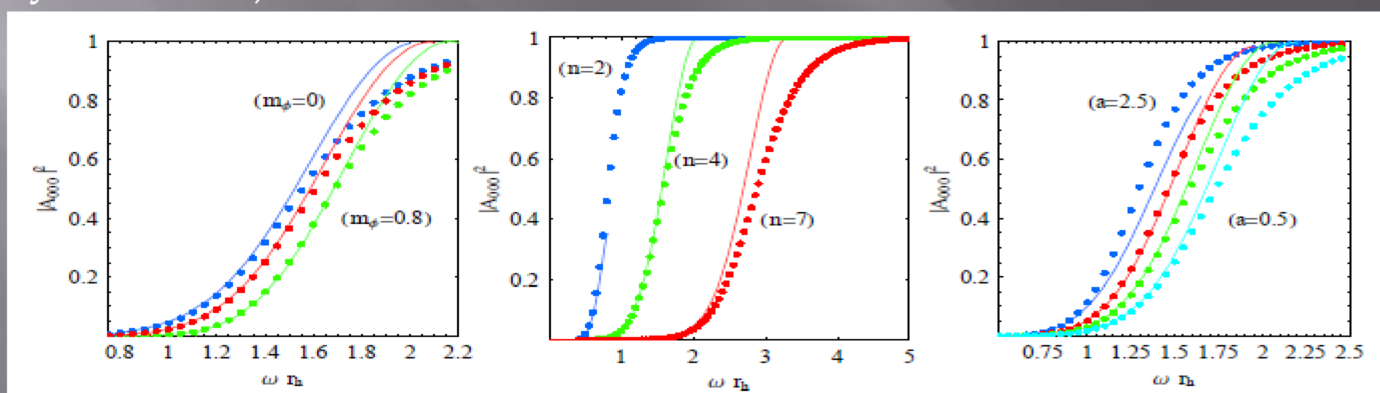
- we demand that the two solutions match perfectly (at least at the low-energy limit)
- this allows us to calculate the ratio $\left| \frac{A_{out}^{(\infty)}}{A_{in}^{(\infty)}} \right|$

The Bulk Case

For perfect match to occur m_Φ<1TeV (with M_* =1TeV, M_{BH} =5TeV, n=1-7). Then we find

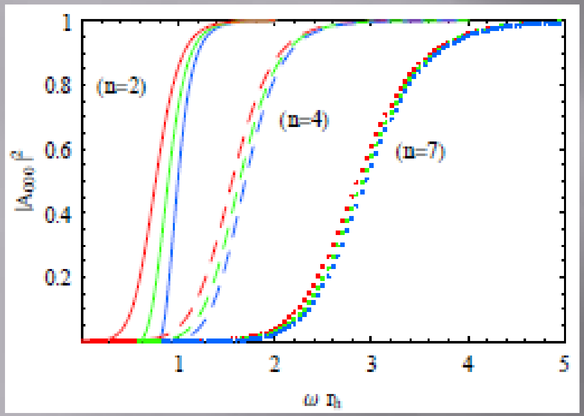
$$|\mathcal{A}_{j\ell m}|^2 = \frac{4\pi\,(\tilde{\omega} r_h/2)^{2j+n+1}\,K_*\,\sin^2\pi(2\beta+D_*)\,\Gamma^2(2\beta+D_*-2)\,\Gamma^2(1-\beta)\,(2-D_*-2\beta)}{A_*\,(1+a_*^2)^{-\frac{2j+n+1}{n+1}}\,(j+\frac{n+1}{2})\,\Gamma^2(j+\frac{n+1}{2})\,\Gamma^2(\beta+D_*-1)\,\sin^2\pi(\beta+D_*)}$$

where $K_* = (1+a_*^2)\omega r_h - a_* m$. For K_* <0 the absorption probability also becomes negative (superradiance). This happens for m_Φ <ω<m Ω_h (Ω_h the rotational velocity of the bh)

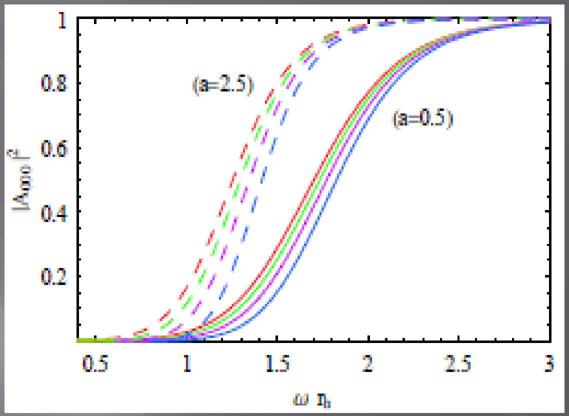


Comparison between numerical (dotted line) and analytical (solid line) results.

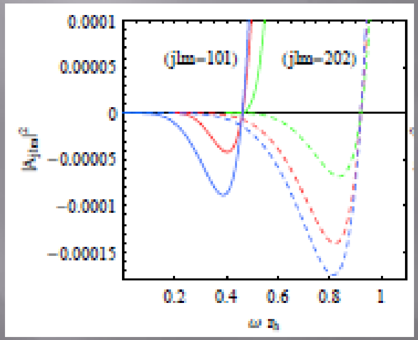
Absorption Probability in the Bulk



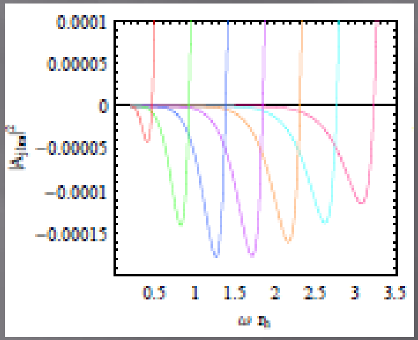
j = l = m = 0
$a_* = 1$
m_Φ = 0, 0.6, 0.8
(left to right)



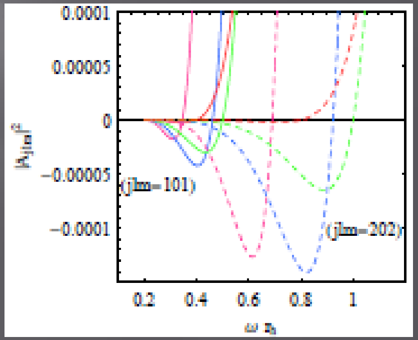
j = l = m = 0
n = 4
m_Φ = 0, 0.4, 0.6, 0.8
(left to right)



(1)



(2)



(3)

Superradiance for various values of the scalar mass (fig.1), different modes (fig.2) and various values of a_* (fig3).

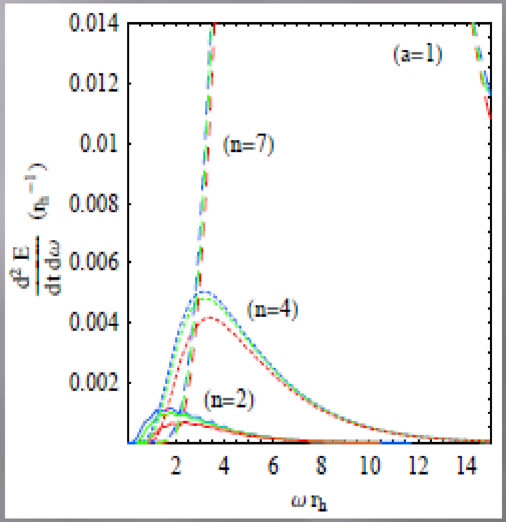
Energy Emission Rate in the Bulk

We had to perform an extensive summation according to the formula

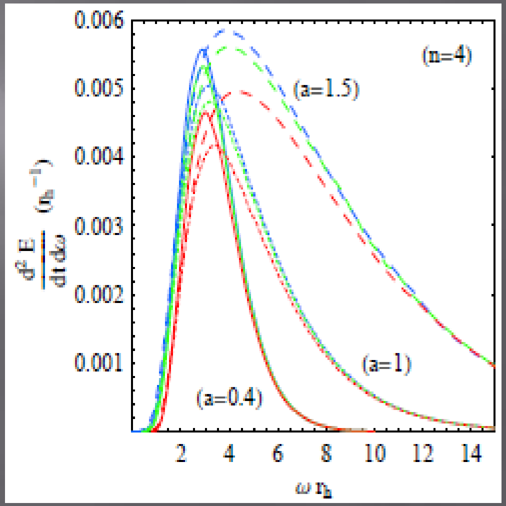
$$\frac{d^2E}{dtd\omega} = \frac{1}{2\pi}\sum_{j,\ell,m} \frac{\omega}{\exp\left[k/T_{\rm H}\right]-1} N_\ell \left|\mathcal{A}_{j\ell m}\right|^2$$

where

$$T_{\rm H} = \frac{(n+1)+(n-1)a_*^2}{4\pi(1+a_*^2)r_h}$$



$a_*=1$
$n=2, 4, 7$
$m_\Phi=0, 0.4, 0.8$



$n=4$
$a_*=0.4, 1, 1.5$
$m_\Phi=0, 0.4, 0.8$

Total number of modes summed: 5456

Contributions down to 10^{-6} were included.

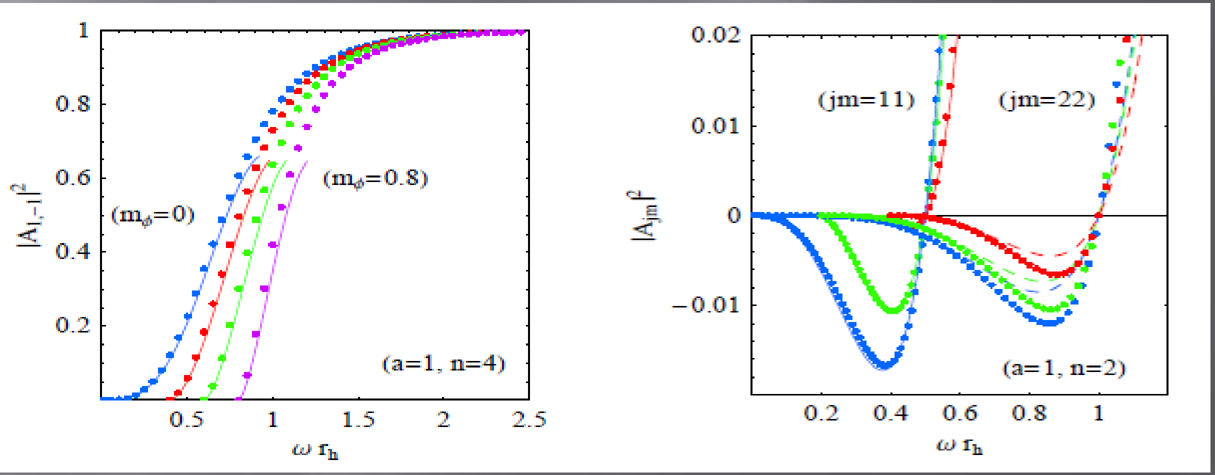
Estimated error: 0.001%(n=2, $a_*=0.5$, $m_\Phi=0$ case) - 5%(n=7, $a_*=1.5$, $m_\Phi=0.8$ case)

The Brane Case

For perfect match to occur m_Φ<0.5TeV (with M_* =1TeV, M_{BH} =5TeV, n=1-7). Then we find

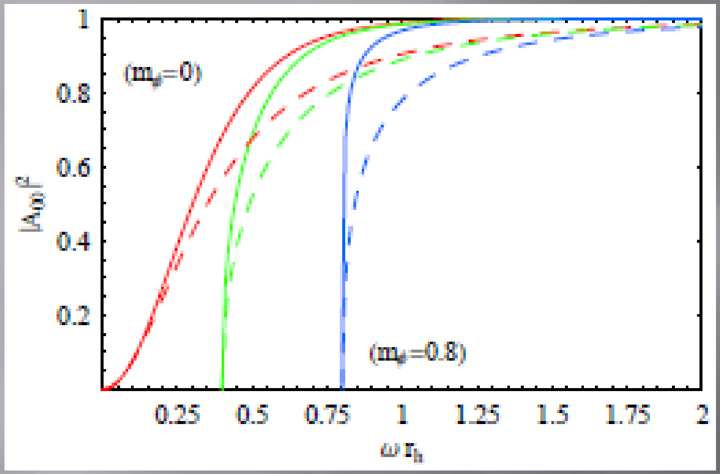
$$|\mathcal{A}_{jm}|^2 = \frac{4\pi\,(\tilde{\omega} r_h/2)^{2j+1}\,K_*\,\sin^2\pi(2\beta+D_*)\,\Gamma^2(2\beta+D_*-2)\,\Gamma^2(1-\beta)\,(2-D_*-2\beta)}{A_*\,(1+a_*^2)^{-\frac{2j+1}{n+1}}\,(j+\frac{1}{2})\,\Gamma^2(j+\frac{1}{2})\,\Gamma^2(\beta+D_*-1)\,\sin^2\pi(\beta+D_*)}$$

where $K_* = (1+a_*^2)\omega r_h - a_* m$. For K_* <0 superradiance occurs (negative absorption probability).

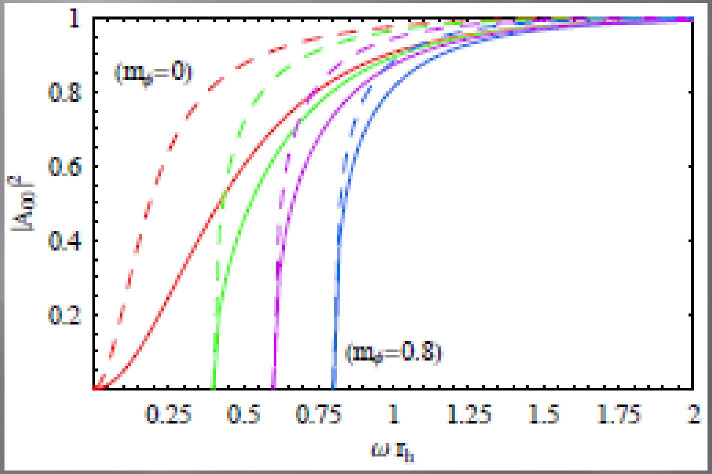


Comparison between numerical (dotted line) and analytical (solid line) results

Absorption Probability on the Brane



j = m = 0 mode
$a_* = 1$
n = 2 (solid lines)
n = 7 (dashed lines)
m_Φ=0, 0.4, 0.8



j = m = 0 mode
n = 4
$a_* = 0.5$ (solid lines)
$a_* = 2.5$ (dashed lines)
m_Φ=0, 0.4, 0.6, 0.8

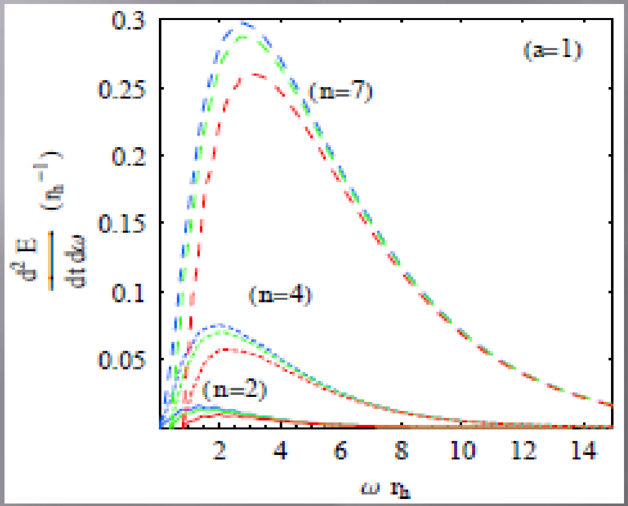
Energy Emission Rate on the Brane

We used a formula very similar to the one for the emission in the bulk:

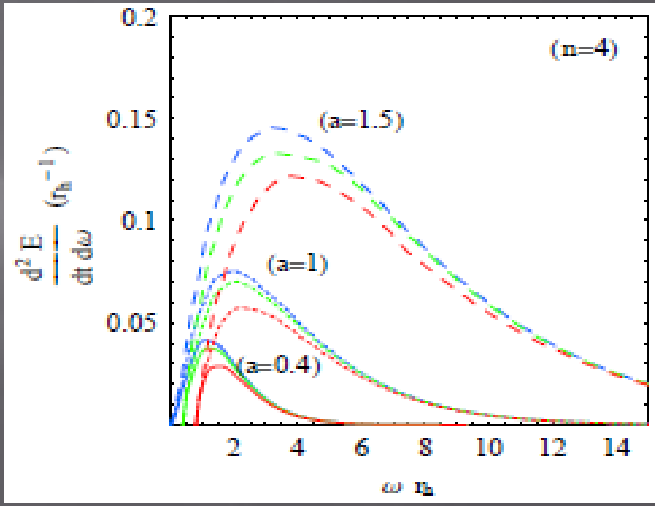
$$\frac{d^2E}{dtd\omega} = \frac{1}{2\pi}\sum_{j,m}\frac{\omega}{\exp\left[k/T_{\rm H}\right]-1}\,|\mathcal{A}_{jm}|^2$$

where

$$T_{\rm H} = \frac{(n+1)+(n-1)a_*^2}{4\pi(1+a_*^2)r_h}$$



$m_\Phi = 0$ (red)
$m_\Phi = 0.4$ (green)
$m_\Phi = 0.8$ (blue)



Total number of modes summed: 1681

Contributions down to 10^{-6} were included.

Estimated error: 0.001% ($n=2$, $a_*=0.5$, $m_\Phi=0$ case) - 5% ($n=7$, $a_*=1.5$, $m_\Phi=0.8$ case)

Bulk and Brane Total Energy Emission

BULK

		$a_*=0.4$	$a_*=1.0$	$a_*=1.5$
$n=2$	$m_\Phi=0$	1.00	1.54	3.46
	$m_\Phi=0.4$	0.84	1.34	3.05
	$m_\Phi=0.8$	0.52	0.95	2.46
$n=4$	$m_\Phi=0$	6.29	9.57	19.22
	$m_\Phi=0.4$	5.97	9.13	18.61
	$m_\Phi=0.8$	5.12	7.99	16.74
$n=7$	$m_\Phi=0$	131.47	202.48	327.37
	$m_\Phi=0.4$	128.56	197.27	322.87
	$m_\Phi=0.8$	121.57	188.58	310.18

BRANE

		$a_*=0.4$	$a_*=1.0$	$a_*=1.5$
$n=2$	$m_\Phi=0$	1.00	3.37	13.18
	$m_\Phi=0.4$	0.75	3.10	11.98
	$m_\Phi=0.8$	0.39	2.16	9.51
$n=4$	$m_\Phi=0$	6.56	25.73	89.89
	$m_\Phi=0.4$	5.73	23.75	84.18
	$m_\Phi=0.8$	4.14	19.51	83.39
$n=7$	$m_\Phi=0$	36.75	144.53	483.83
	$m_\Phi=0.4$	34.48	138.86	471.08
	$m_\Phi=0.8$	29.28	126.53	440.77

Results normalized with respect to the n=2, a_*=0.4, m_Φ=0 case (separately for bulk and brane).

- Total Energy Emission increases with the increase of n .
- Total Energy Emission increases with the increase of a_* .
- Total Energy Emission decreases with the increase of m_Φ . The magnitude of suppression (bulk max suppression 48% - brane max suppression 61%) diminishes as n and/or a_* take larger values.

Bulk-over-Brane Total Energy Emission

		$a_* = 0.4$	$a_* = 1.0$	$a_* = 1.5$
$n = 2$	$m_\Phi = 0$	0.180	0.076	0.0451
	$m_\Phi = 0.4$	0.202	0.078	0.0458
	$m_\Phi = 0.8$	0.24	0.079	0.0466
$n = 4$	$m_\Phi = 0$	0.173	0.067	0.038
	$m_\Phi = 0.4$	0.188	0.069	0.039
	$m_\Phi = 0.8$	0.223	0.074	0.040
$n = 7$	$m_\Phi = 0$	0.645	0.253	0.122
	$m_\Phi = 0.4$	0.673	0.256	0.124
	$m_\Phi = 0.8$	0.749	0.269	0.127

In all cases the brane channel remains the dominant one and predominates even more the faster the BH rotates. Interestingly enough, the presence of the scalar mass gives a considerable boost to the ratio even by 33% ($n=2$, $a_*=0.4$), a feature that should be taken under consideration in order to credibly determine the balance between the bulk and the brane channel and correctly evaluate any relevant detector signals.