4th UniverseNET School Lecce, 17/09/10

Coupled DBI inflation

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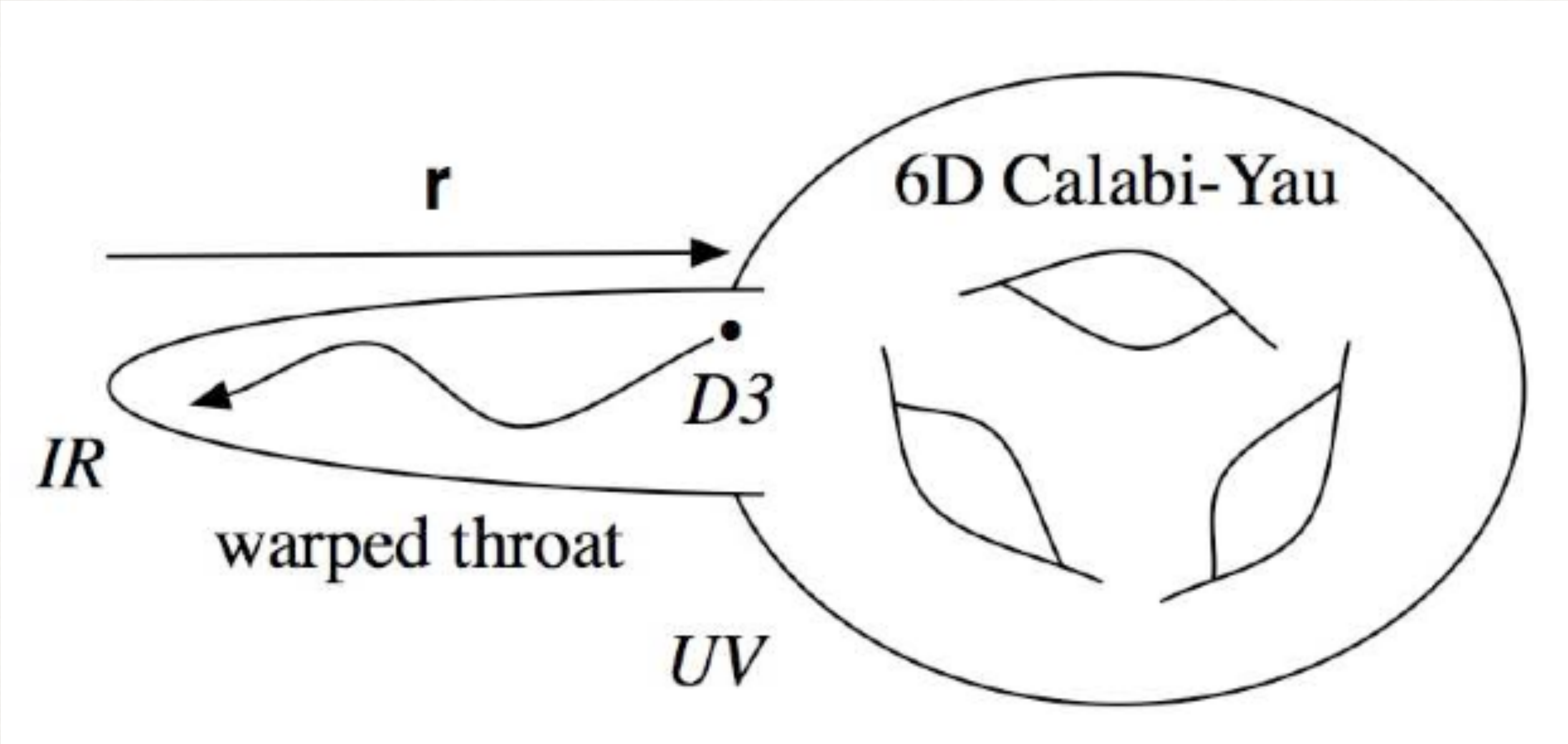
Based on ongoing work with C. van de Bruck & D. F. Mota, supported by
The Research Council of Norway

DBI Inflation



Picture Credit: Easson et al., 2008

- The inflaton is the radial coordinate of a D3 brane, moving down a warped throat.
- A speed limit is imposed upon the motion of the brane which is dependent on the warping (Silverstein & Tong, 2004; Alishahiha et al., 2004).

DBI Inflation

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DBI Inflation

Boost Factor

$$\gamma = \frac{1}{\sqrt{1 - f\dot{\chi}^2}}$$

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Potential

$$V(\chi) \sim m^2\chi^2$$

DBI Inflation

$$\ddot{\chi} + 3H\gamma^{-2}\dot{\chi} + \tfrac{1}{2}\frac{f_\chi}{f^2}(1 - 3\gamma^{-2} + 2\gamma^{-3}) + \gamma^{-3}V_\chi = 0$$

- As the DBI field starts to roll down its potential, the boost factor becomes large.
- In the simplest models with an AdS throat, the warp factor is $f(\chi) \approx \lambda\chi^{-4}$
- So the late-time solution is, $\ddot{\chi} - \frac{2}{\lambda}\chi^3 \approx 0 \quad \Rightarrow \quad \chi \rightarrow \frac{\sqrt{\lambda}}{t}$

$$\epsilon = \frac{2}{\gamma}\left(\frac{H'}{H}\right)^2 \approx \sqrt{\frac{3}{\lambda}}\frac{1}{m}$$

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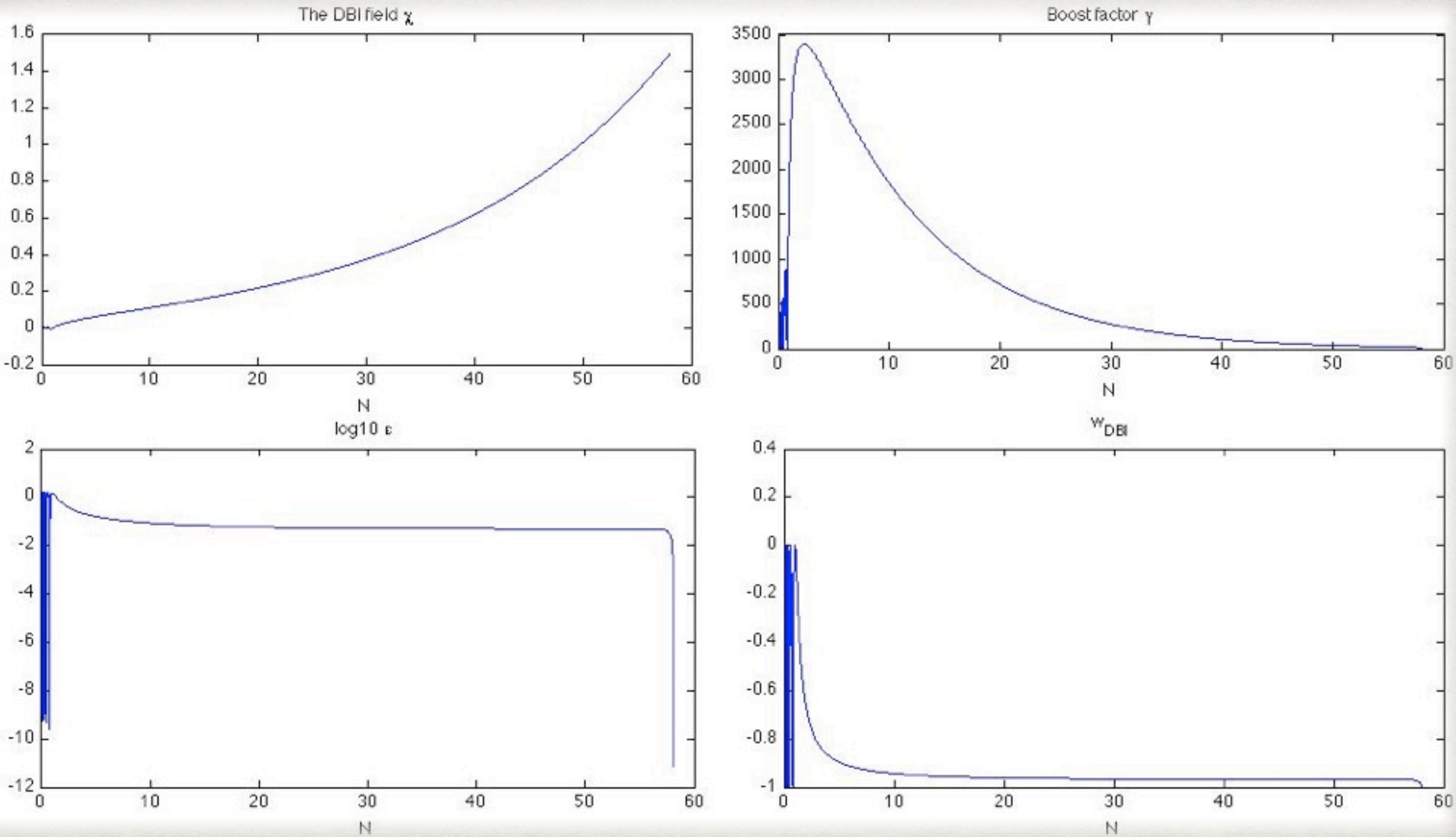
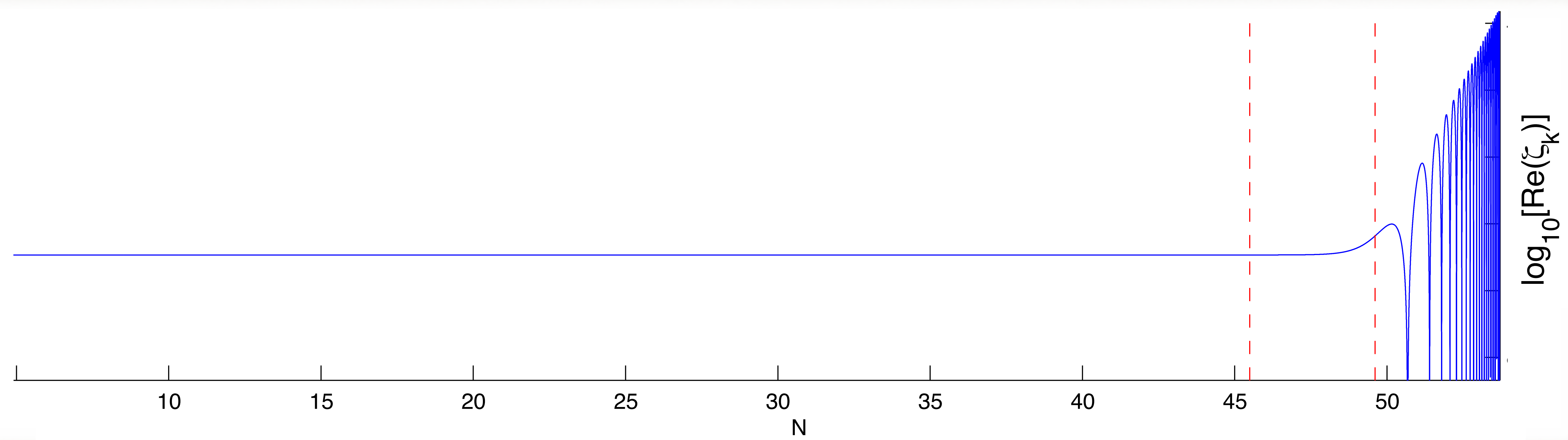


DBI Inflation

To understand the evolution of the curvature perturbation ζ we use a result from k-inflation.

$$\frac{d^2 v_k}{d\tau^2} + \left(\frac{k^2}{\gamma^2} - \frac{1}{z}\frac{d^2 z}{d\tau^2}\right) v_k = 0 \qquad \text{where } z = \frac{a\gamma^{3/2}\dot{\chi}}{H} \text{ and } \nu = z\zeta$$

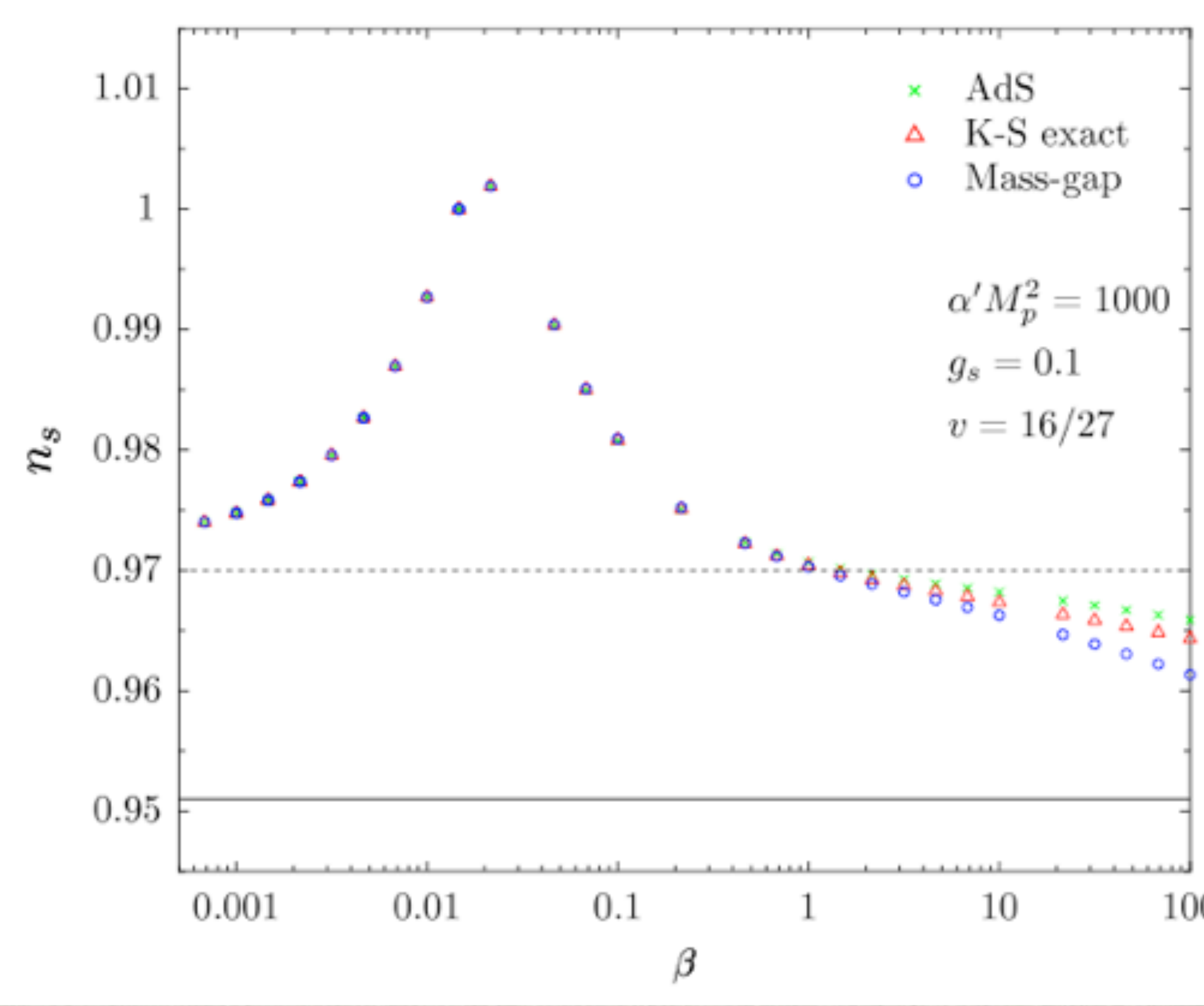
The speed of sound of the perturbations is: $c_s^2 = \gamma^{-2}$



DBI Inflation

$$\mathcal{P}_\zeta = \frac{1}{8\pi^2}\left(\frac{H^2}{c_s\epsilon}\right)_{|c_s k = aH} \approx \frac{1}{36\pi^2} m^4 \lambda$$

- One can use the power spectrum amplitude to fix the parameters, but this can lead to a relatively small number of efolds of inflation.
- Modes freeze-in at smaller scales as inflation progresses, cancelling the red-tilt due to the evolution of H.
- The spectral index is dependent on the warped geometry and the background dynamics.



Picture Credit: Bean et al., 2007

Spectral index in DBI models ($m^2 \sim \beta H^2$)

DBI Inflation

- The non-linearity parameter is a typical measure of the level of non-Gaussianity in the perturbation.

$$\zeta = \zeta_L - \frac{3}{5} f_{NL} \zeta_L^2$$

- Perturbations in DBI inflation are characterised by high levels of non-Gaussianities.

$$f_{NL} \approx 0.32\gamma^2 \qquad \Rightarrow \qquad \gamma \lesssim 20$$

- This can be a stringent constraint on DBI inflation.

Motivations

- In a realistic model of inflation, we can expect more than one field to be present.
- We can also consider the possibility of couplings between the fields.
- Multi-field inflationary models with multiple sound speeds have not received much attention in the literature.
- Can the problems with DBI inflation be ameliorated by interactions with a second field?

Coupled DBI Inflation

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[\tfrac{1}{2}R - \tfrac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\mu} - U(\varphi)\right] \\
&+ \int d^4x\sqrt{-g}A^4(\varphi)\left\{f^{-1}(\chi)\left(1-\gamma^{-1}\right) - V(\chi)\right\}
\end{aligned}
$$

The coupling term enters into the boost factor

$$A(\varphi) = \exp(\alpha\varphi) \qquad \gamma = \frac{1}{\sqrt{1 + A^{-2} f g^{\mu\nu}\chi_{,\mu}\chi_{,\nu}}}$$

To study inflation, we can take a flat FRW metric and get the Friedmann equations,

$$
\begin{aligned}
3H^2 &= \tfrac{1}{2}\dot{\varphi}^2 + U + A^4\left[f^{-1}(\gamma - 1) + V\right] \\
-2\dot{H} &= \dot{\varphi}^2 + A^2\gamma\dot{\chi}^2
\end{aligned}
$$

Background Dynamics

The standard scalar field and DBI equations of motion are modified by the coupling.

$$\ddot{\chi} + 3H\gamma^{-2}\dot{\chi} + \tfrac{1}{2}A^2\frac{f_\chi}{f^2}(1 - 3\gamma^{-2} + 2\gamma^{-3}) + A^2\gamma^{-3}V_\chi = -\alpha\dot{\chi}\dot{\varphi}(3\gamma^{-2} - 1)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + U_\varphi = \alpha T^{\mathrm{DBI}}$$

where,

$$T^{\mathrm{DBI}} = A^4 f^{-1}(4 - \gamma - 3\gamma^{-1}) - 4A^4 V$$

and the boost factor depends on A:

$$\gamma = (1 - A^{-2} f\dot{\chi}^2)^{-1/2}$$

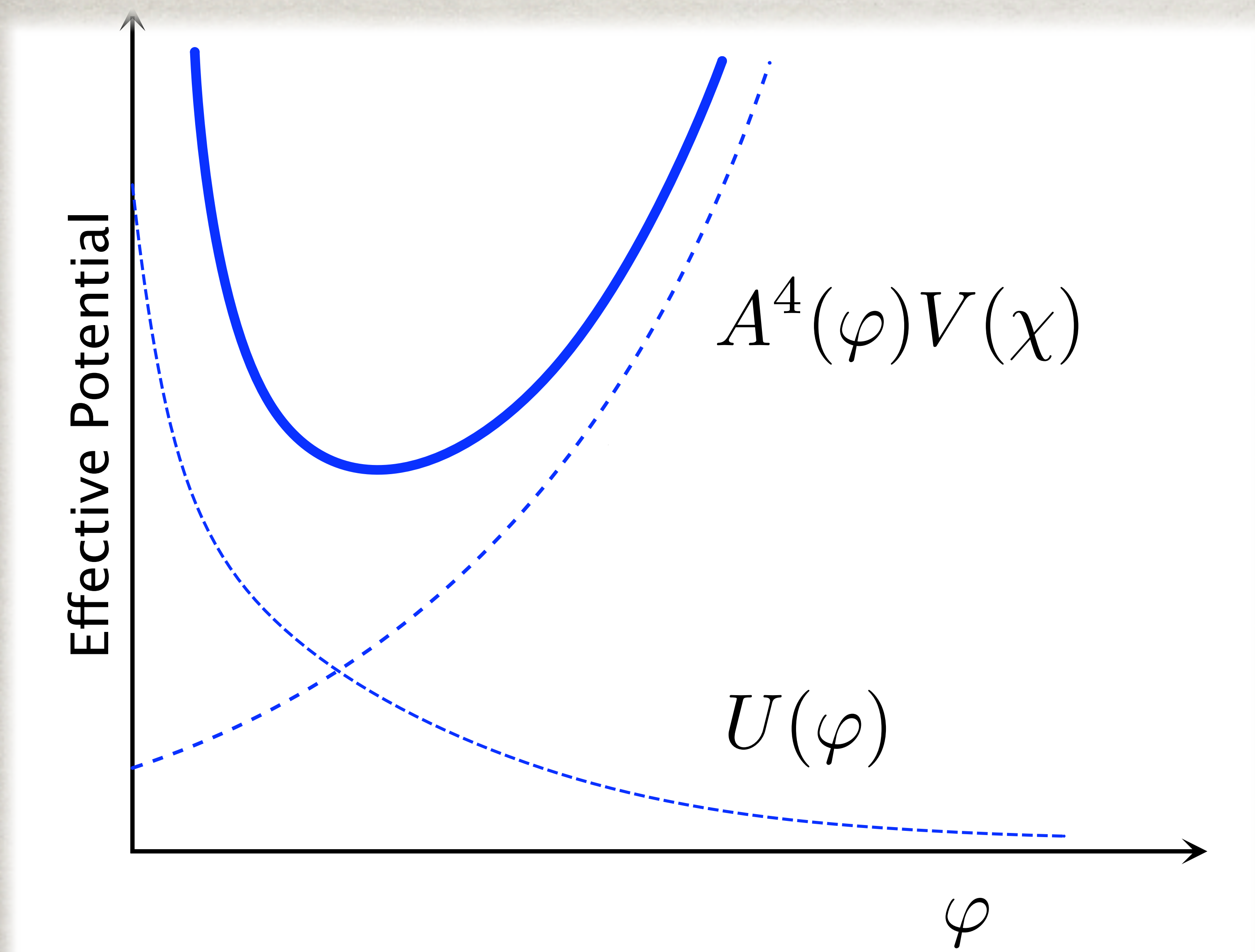
For numerical work, we choose standard forms for the DBI warp factor & potential.

$$V(\chi) = \tfrac{1}{2}m^2\chi^2 \qquad f(\chi) = \frac{\lambda}{(\chi^2 + \mu^2)^2}$$

Background Dynamics

- The scalar field moves in an effective potential determined by the coupling to the DBI field.

$$
\begin{aligned}
U_{\mathrm{eff}} &= U(\varphi) - \tfrac{1}{4} T^{DBI} \\
&\approx U(\varphi) + A^4(\varphi) V(\chi)
\end{aligned}
$$



- We can work with potentials for the scalar field that lead to a minimum at positive values.

- If the potential is steep, $\varphi = \varphi_{min}$ and the field tracks the minimum and the coupling A >1

Background Dynamics

- Offset quadratic potential

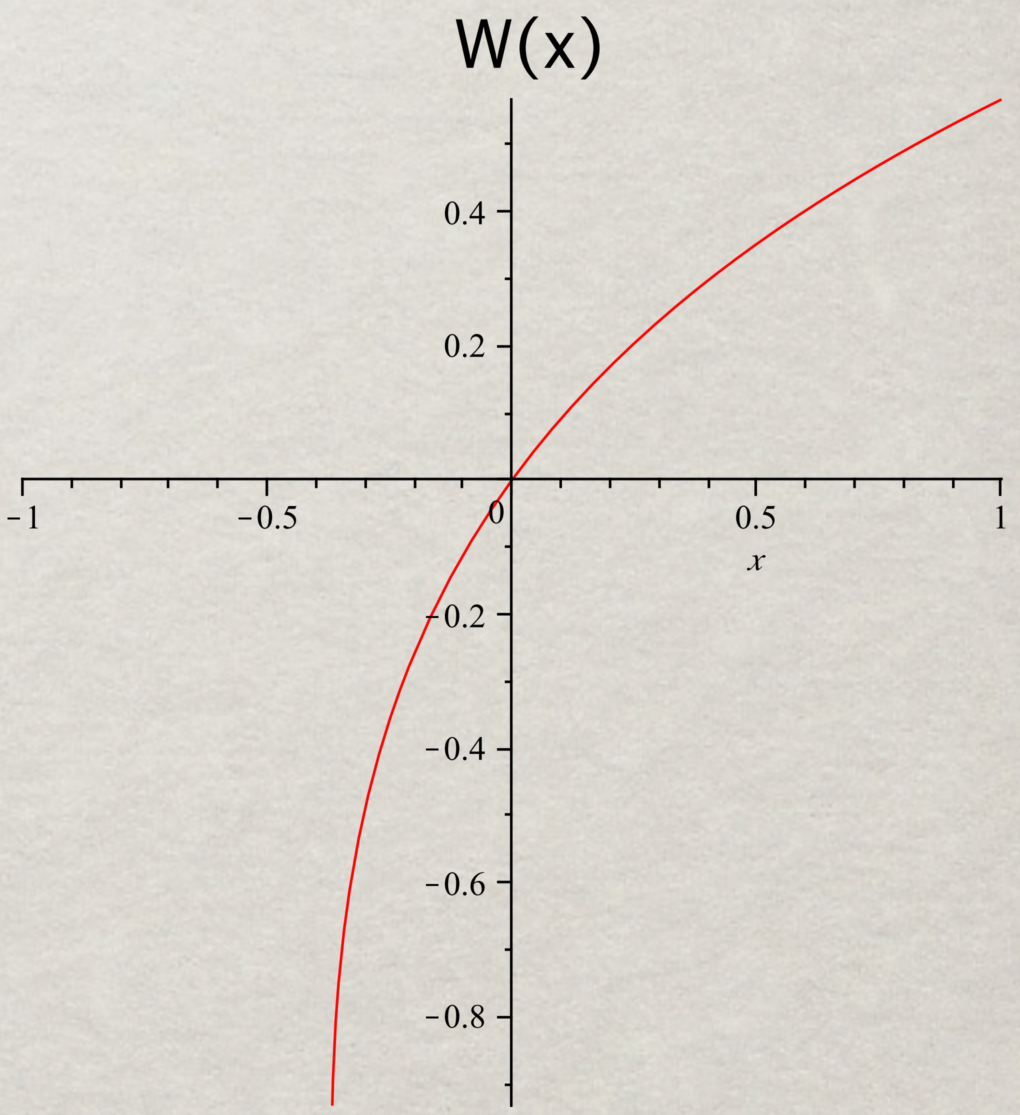
$$U(\varphi) = U_0(\varphi - \eta)^2$$

$$\varphi_{\min} = \eta - \frac{1}{4\alpha} W\left(\frac{8\alpha^2 V(\chi) e^{4\alpha\varphi}}{U_0}\right)$$

- Exponential Potential

$$U(\varphi) = U_0 \exp(-p\varphi)$$

$$\varphi_{\min} = \frac{1}{4\alpha + p} \log\left(\frac{pU_0}{2\alpha m^2 \chi^2}\right)$$



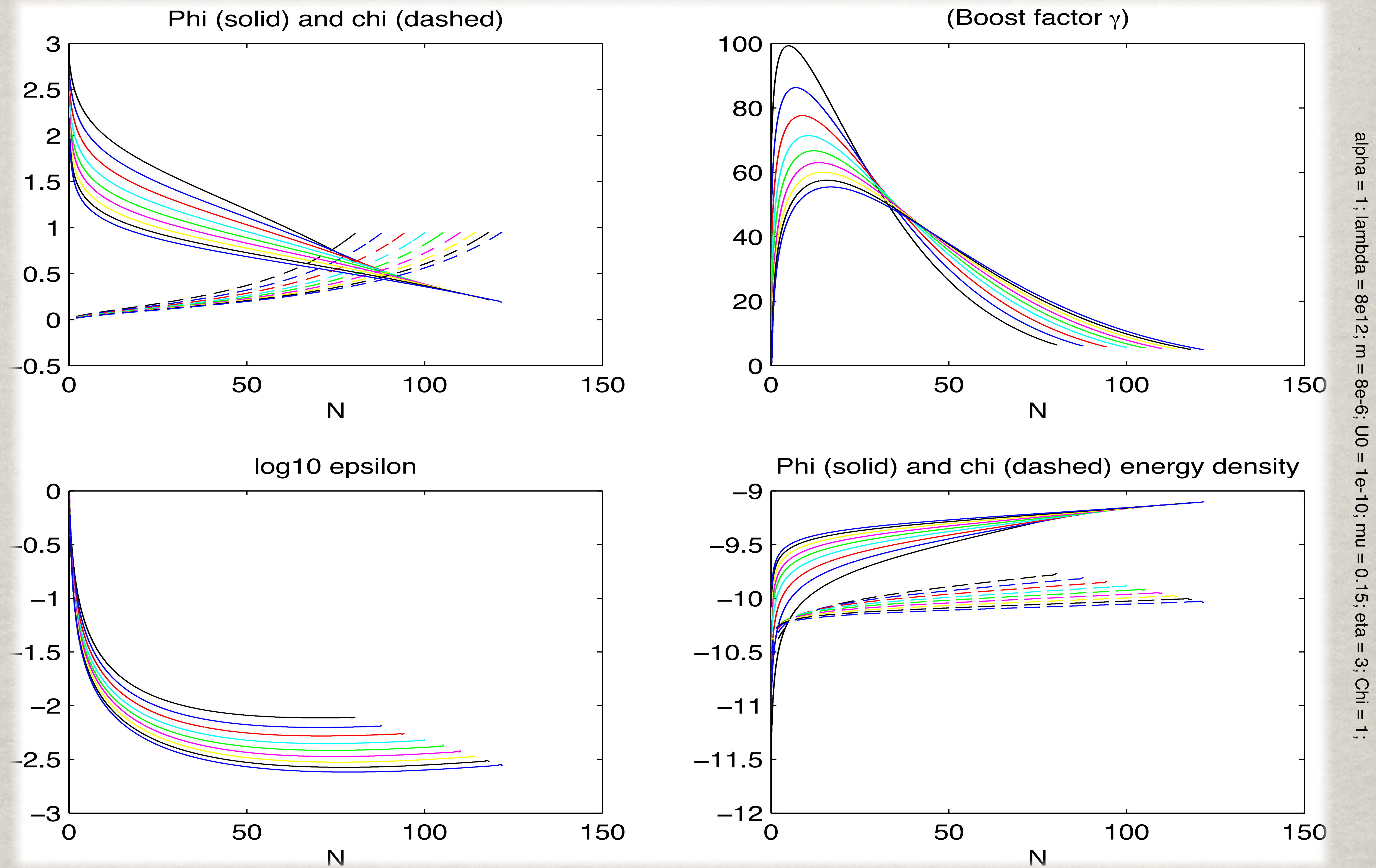
Background Dynamics

- When the scalar field sits in its minimum, A>1.
- This increases the speed limit and leads to smaller values of γ, whilst retaining the interesting DBI effects.

$$\gamma = (1 - A^{-2} f \dot{\chi}^2)^{-1/2}$$

- The coupling increases the effective mass of the DBI field, so the slow-roll parameter takes smaller values and the number of efolds of inflation is increased.

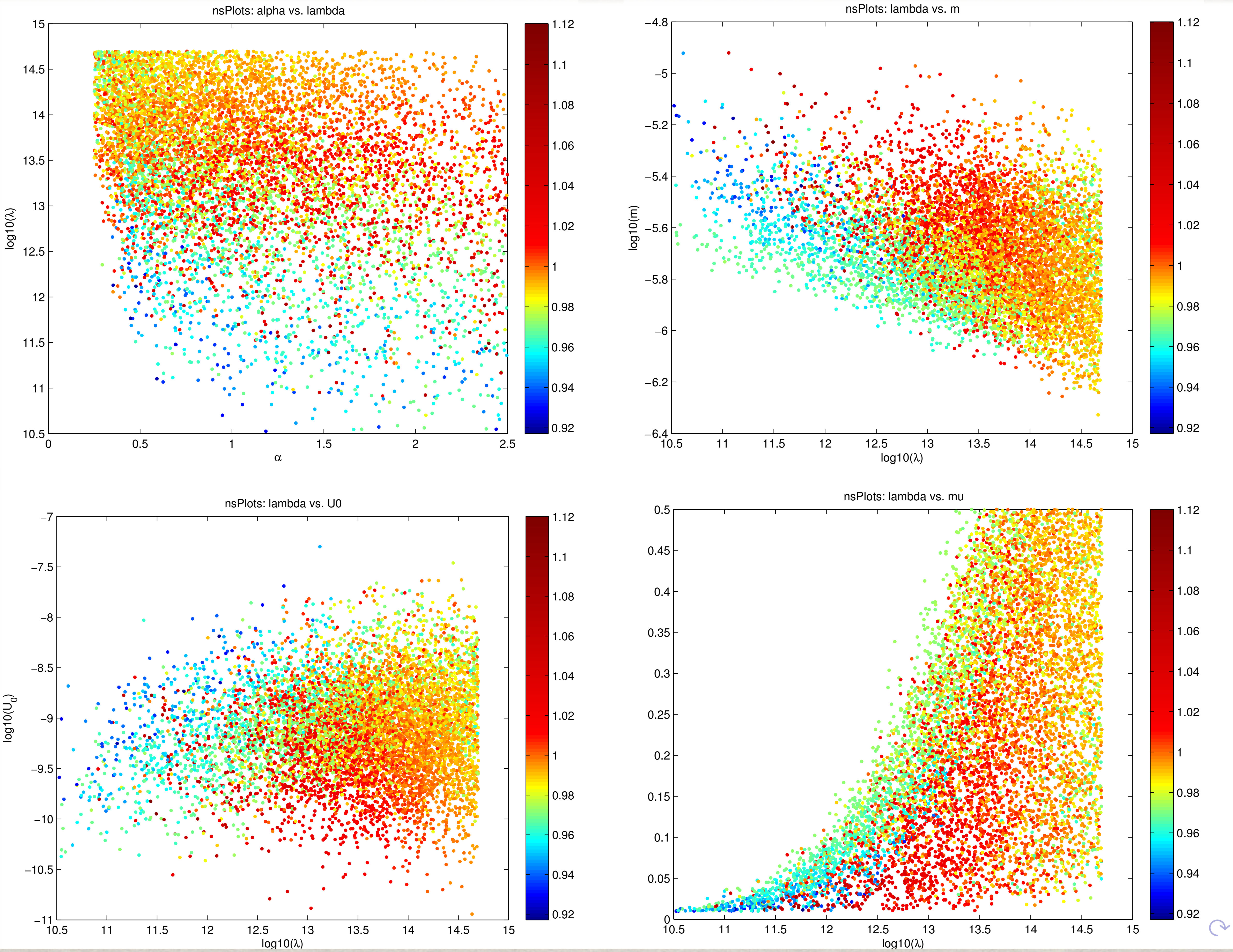
Background Dynamics



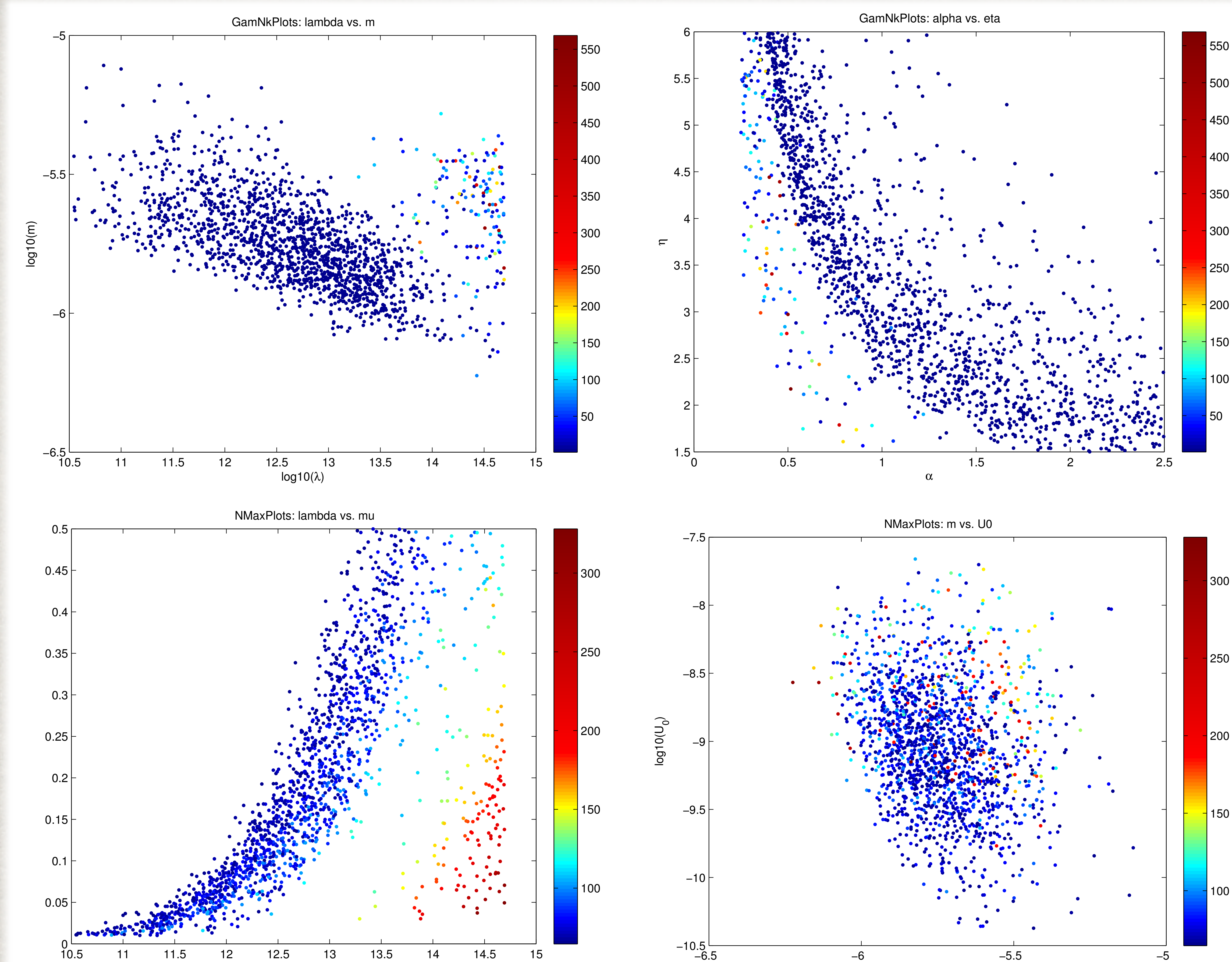
`Alpha = [0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5];`

Parameters & Predictions

- There are six parameters in the model that can affect the behaviour of the perturbations:
  - The coupling α
  - DBI warp factor parameters λ and μ
  - Parameters in the field potentials U_0, m, η
- We can constrain the parameter space by considering the effect on:
  - background quantities e.g. no. of efolds, boost factor
  - perturbation quantities e.g. the power spectrum amplitude, spectral index



Spectral Index predictions -- Quadratic potential



The runs that have n_s within the current limits have similar properties: low Boost factor and ~100 efolds of DBI inflation.

Conclusions

- DBI provides an interesting example of k-inflation with 'stringy' motivations. However, a large boost factor and short duration of inflation can be a problem.
- Realistic inflationary models are likely to involve multiple scalar fields and couplings. Coupled DBI inflation combines these elements in a two-field model.
- The coupling forces the scalar field into the minimum of its effective potential, extending the number of efolds of DBI inflation and decreasing the boost factor. The effect of this on the non-Gaussianity predictions of the DBI model will be the subject of future work.
- A range of parameters affect the prediction for the spectral index (for both the exponential and the quadratic potential) including a considerable set compatible with current observational limits.