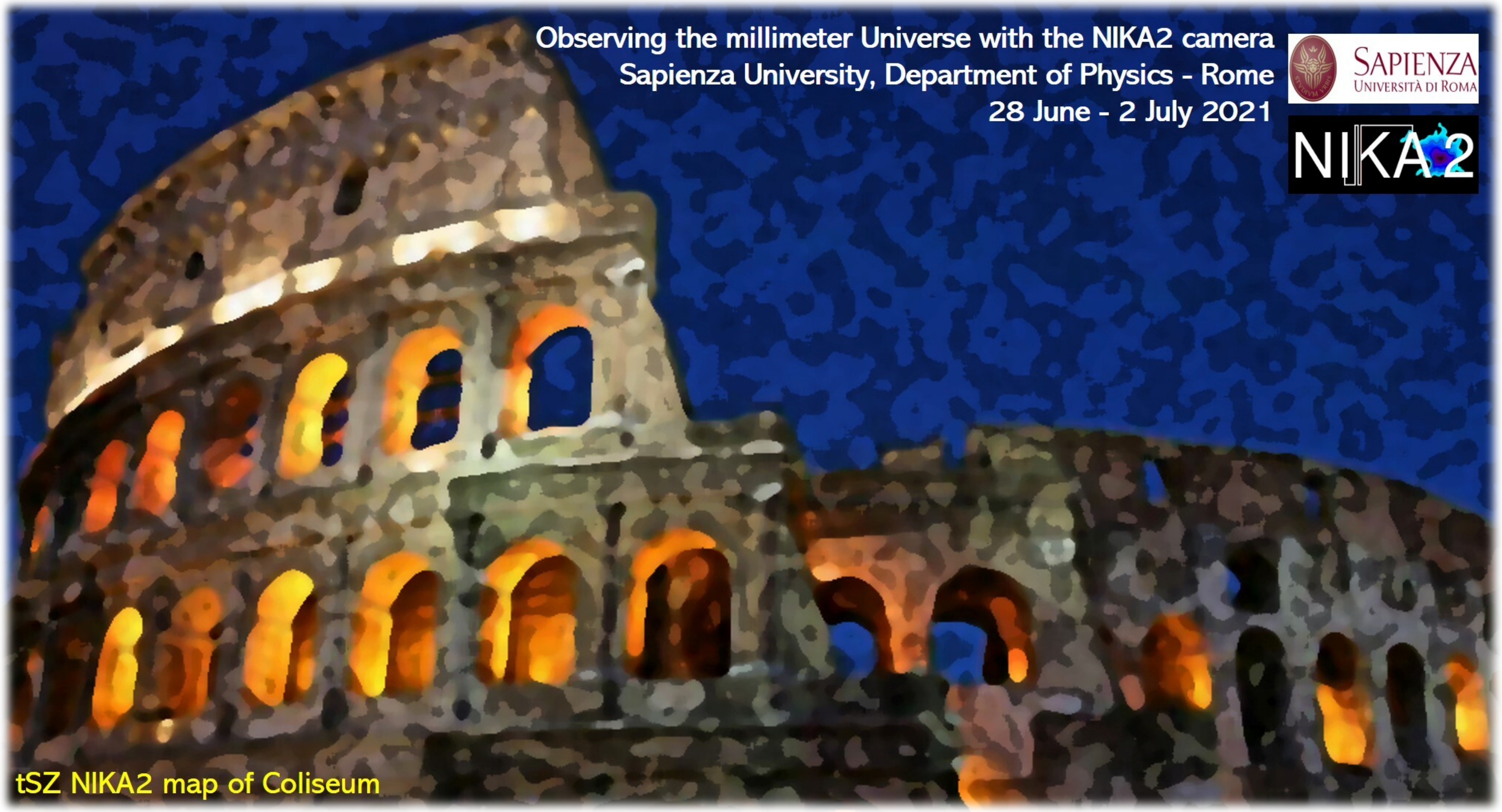


Morphological analysis of SZ and X-ray maps of galaxy clusters with Zernike polynomials

V. Capalbo, M. De Petris, G. Yepes, F. De Luca, W. Cui, A. Knebe, E. Rasia, F. Ruppin, A. Ferragamo

Introduction

- **Purpose:** study the morphology of galaxy clusters from multi-wavelength maps to infer, as possible, their dynamical state

- **Method:** analytic approach using ***Zernike polynomials***

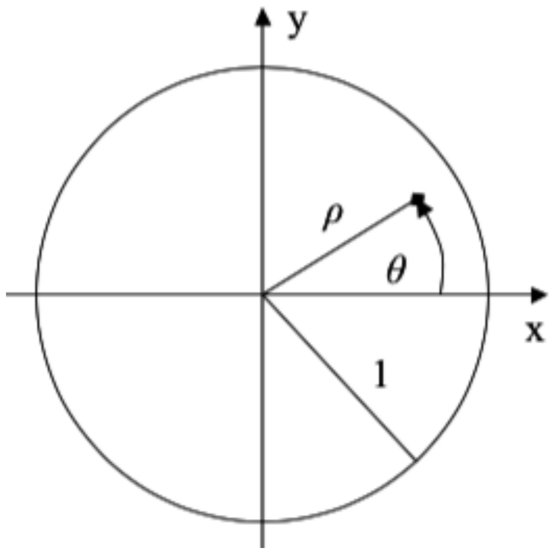
Why?

ZPs are a ***complete*** and ***orthogonal*** set of functions defined over a unit circle, useful for modelling functions in circular domains

Common applications of ZPs in several fields:

- adaptive optics *(see e.g. Noll R. G., 1976, J. Opt. Soc. Am.,66,207; Rigaut F. et al.,1991, A&A, 250, 280)*
- image analysis and pattern recognition *(see e.g. Teague M. R., 1980, J. Opt. Soc. Am., 70, 920)*
- ophthalmology, optometry, medicine *(see e.g. Liang J., Williams D. R.,1997,J. Opt. Soc. Am. A,14,2873; Tahmasbi A., et al., 201, Comput. Biol. Med., 41,726; Alizadeh E., et al., 2016, Integr. Biol., 8, 1183)*

Zernike polynomials: definition



(Noll R. G., 1976, J. Opt. Soc. Am., 66, 207)

$$\mathbf{Z_n^m}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\cos(\mathbf{m}\theta)$$

$$\mathbf{Z_n^{-m}}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\sin(\mathbf{m}\theta)$$

order ***n*** and frequency ***m***:

$\in \mathbb{N},\ m \leqslant n,\ n-m = even$

normalization factor:

$$\mathbf{N_n^m} = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}}$$

radial term:

$$\mathbf{R_n^m}(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s!\left(\frac{n+m}{2}-s\right)!\left(\frac{n-m}{2}-s\right)!}\rho^{n-2s}$$

➡ orthogonality: $\int_0^{2\pi}\int_0^1 Z_n^m(\rho,\theta) Z_{n'}^{m'}(\rho,\theta)\rho\, d\rho\, d\theta = \pi\delta_{nn'}\delta_{mm'}$

➡ <u>linear expansion</u>: an arbitrary function $\phi(\rho,\theta)$ over a unit circle aperture can be expressed as a weighted sum of ZPs

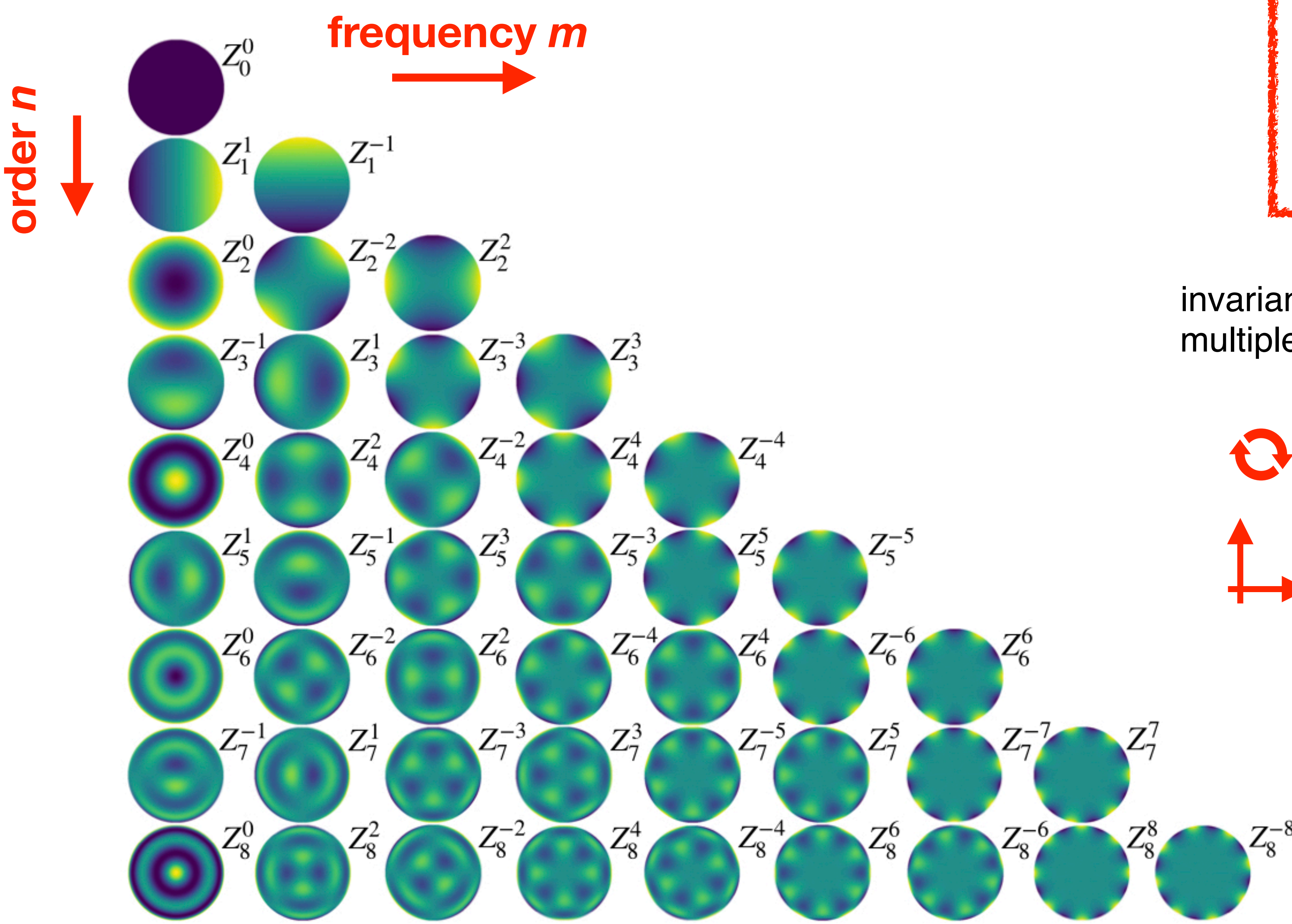
$$\phi(\rho,\theta) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} c_{nm} Z_n^m(\rho,\theta)$$

➡ expansion coefficients: $c_{nm} = \frac{1}{\pi}\int_0^1\int_0^{2\pi}\phi(\rho,\theta) Z_n^m(\rho,\theta)\rho\, d\rho\, d\theta$

*note that $c_{00} = <\phi>$

Zernike polynomials: definition

(Noll R. G., 1976, J. Opt. Soc. Am., 66, 207)



$$\mathbf{Z_n^m}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\cos(\mathbf{m}\theta)$$

$$\mathbf{Z_n^{-m}}(\rho,\theta) = \mathbf{N_n^m}\,\mathbf{R_n^m}(\rho)\sin(\mathbf{m}\theta)$$

invariance of form with respect to rotations of multiple of $2\pi/m$ around the centre of the aperture

m=0: no angular dependence, continuous circular symmetry

m≠0: axial symmetry/antisymmetry

spatial frequency resolution k for a combination of ZPs up to order n: $k \approx (n+1)/2\pi$ in unit of $1/R_{ap}$, from the Fourier transform of ZPs
(Svechnikov M. et al., 2015, Opt. Express 23,14677)

First application of the Zernike fitting to study galaxy clusters morphology from Compton parameter maps

(Capalbo V. et al., 2021, MNRAS, 503, 6155)

Zernike fitting: validation of the method on mock *y*-maps

Data set: 324 mock *y*-maps of galaxy clusters from *The Three Hundred Project* at 3 redshifts (*z*=0, 0.45, 1.03) and different angular resolution, up to 5 arcmin. *(Cui W. et al., 2018, MNRAS, 480, 2898)*

Each *y*-map is modelled with 45 ZPs up to the order $n = 8$, within a circular aperture with radius R_{500} centred on the *y*-centroid

$$y = \sum_{n=0}^{8} \sum_{m=0}^{n} c_{nm} Z_n^m \longrightarrow c_{nm} = \frac{\sum y \times Z_n^m}{\pi (R_{500})^2}$$

Common morphological/dynamical parameter used as references:

(De Luca F. et al., 2021, MNRAS, 504, 5383 ⟶ talk on Thursday)

➡ *M* - a combination of some morphological parameters

- *Asymmetry parameter (A)*
- *Light concentration parameter (c)*
- *Power ratio parameter (P)*
- *Centroid shift parameter (w)*
- *Gaussian fit parameter (G)*
- *Strip parameter (S)*

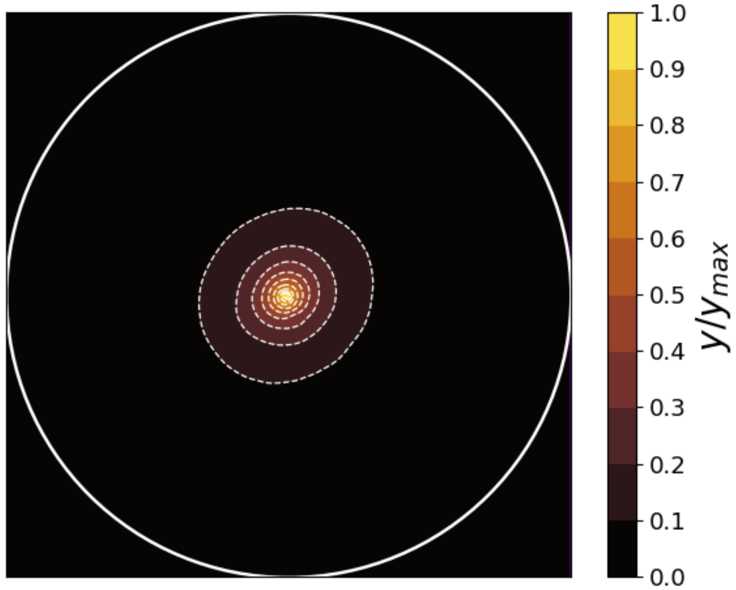
➡ χ - a combination of some 3D dynamical indicators χ_i

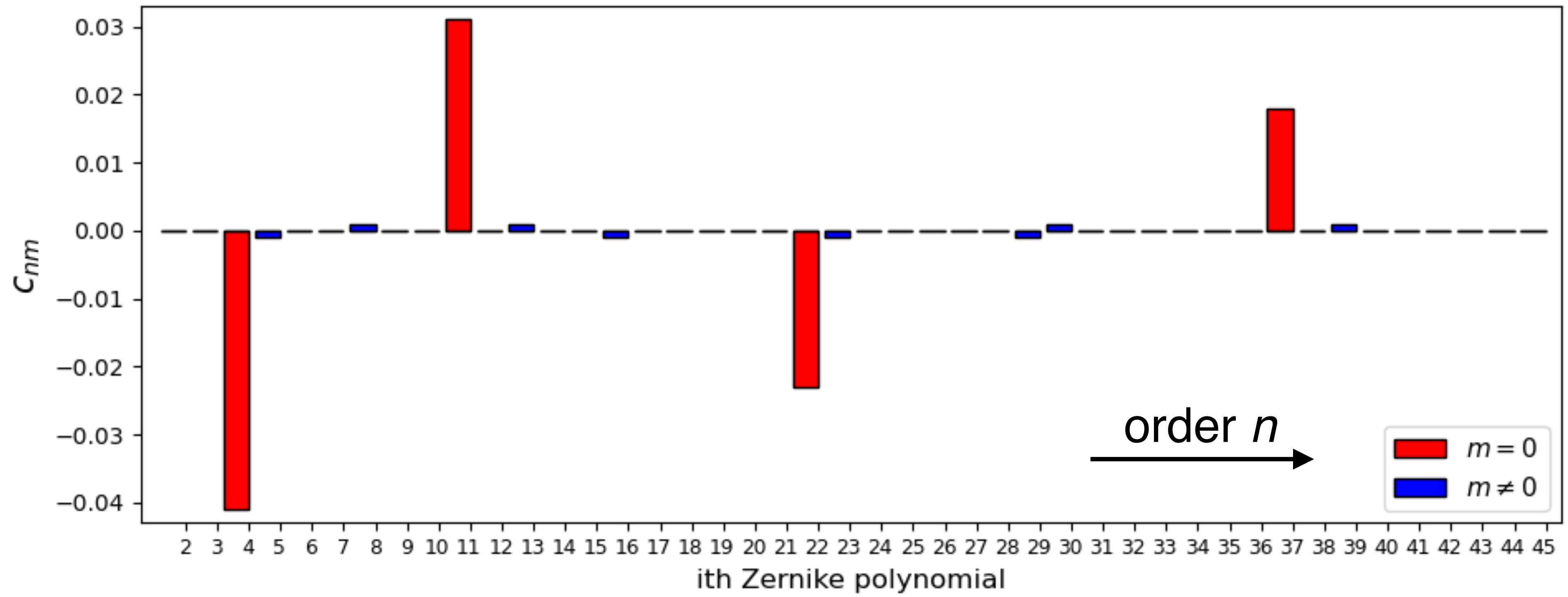
- *Centre-of-mass offset* (f_s)
- *Fraction of mass in subhalos* (Δ_r)

How the Zernike fitting works? Two examples

1) Relaxed cluster (classified with M and χ)

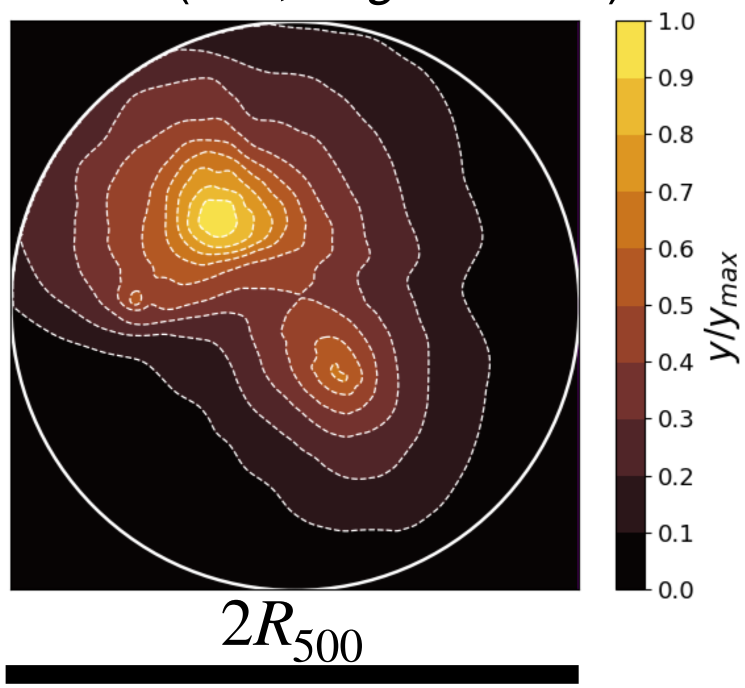
#299 (*z=0, ang.res=*$5.2''$)

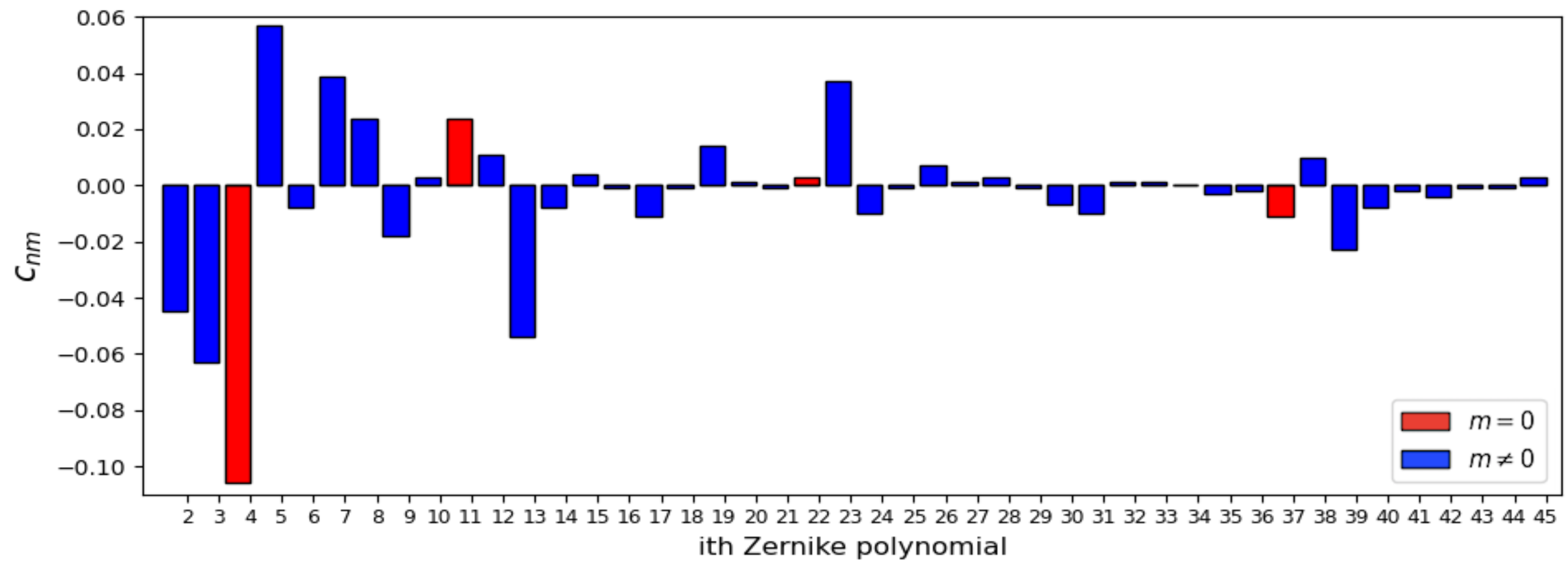




2) Disturbed cluster (classified with M and χ)

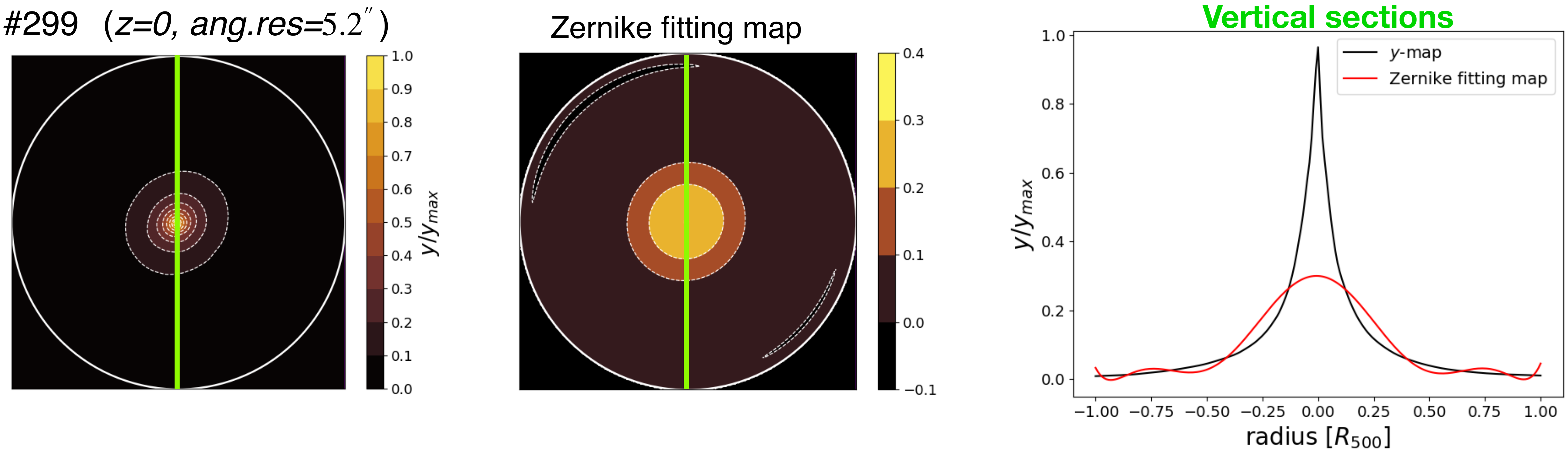
#244 (*z=0, ang.res=*$5.2''$)



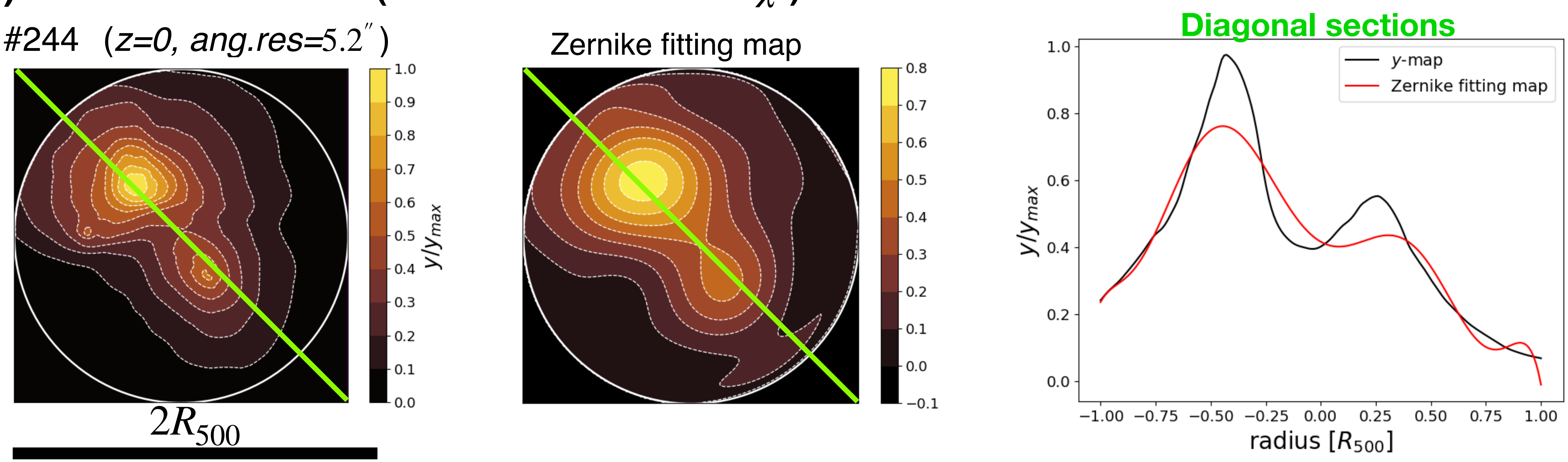


How the Zernike fitting works? Two examples

1) Relaxed cluster (classified with M and χ)



2) Disturbed cluster (classified with M and χ)



For the Zernike fitting we estimate a spatial resolution $\sim 0.5R_{500}$

Results *(Capalbo V. et al., 2021, MNRAS, 503, 6155)*

1) Correlate the results of the Zernike fitting with morphological parameters

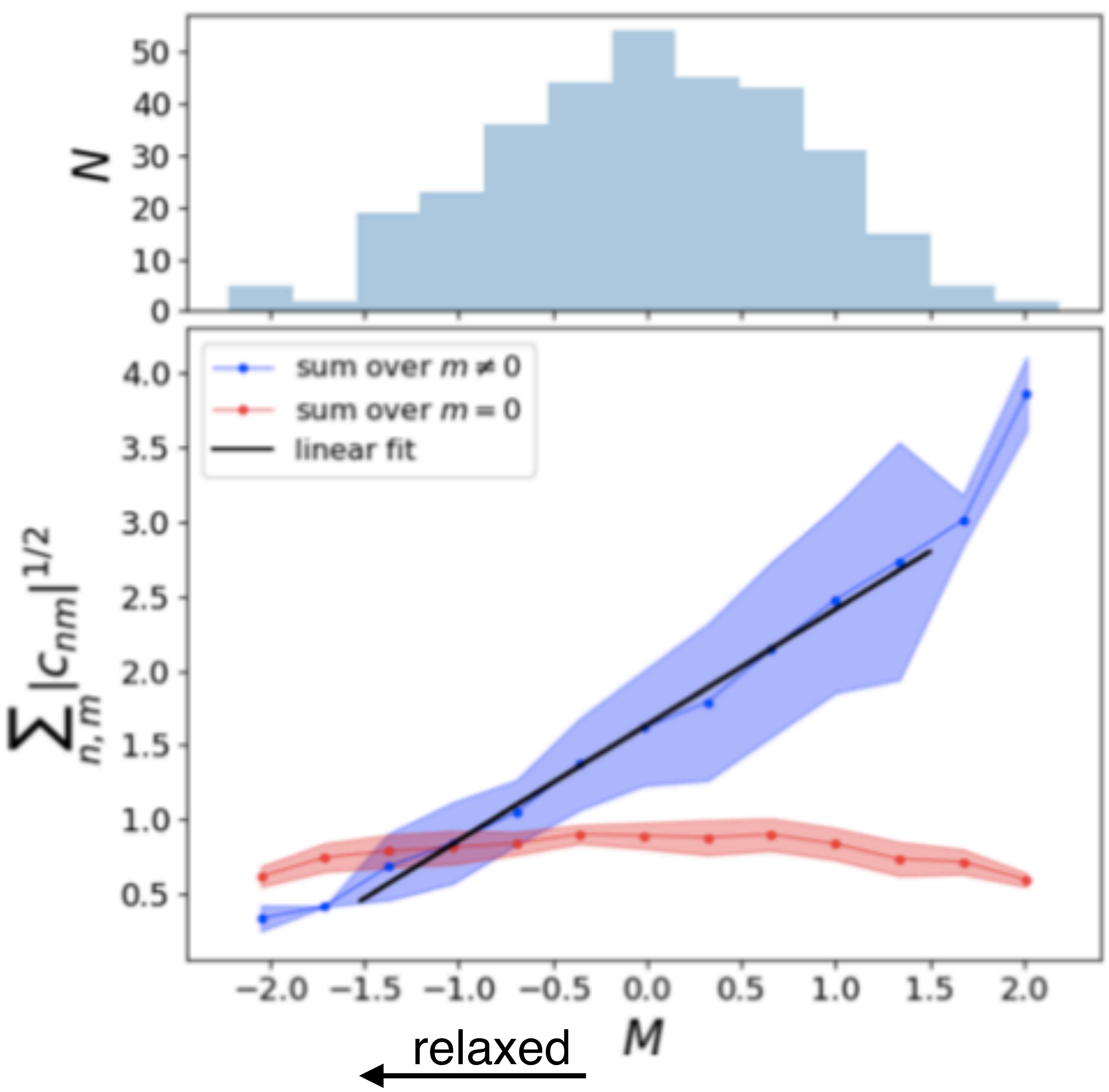


Figure 4. Top: Number clusters distribution along the parameter M with binning of 0.3. Bottom: Sum over all the fitting Zernike moments c_{nm} with $m \neq 0$ (in blue) and $m = 0$ (in red) *versus* the combined parameter M, for all clusters at $z = 0$. Each point represents the mean value of the sum in each M bin and the coloured regions are referring to $\pm 1\sigma$. The black line is the fitting line of equation $C = (0.78 \pm 0.04)M + (1.64 \pm 0.03)$

$$C = \sum_{n,m\neq 0} |c_{nm}|^{1/2}$$

Pearson correlation coefficient r between C and M

z	Angular resolution (arcsec)	r
0	5.25	0.78 (± 0.02)
	20	0.78
	60	0.79
	300	0.77
0.45	1.14	0.73 (± 0.03)
	5	0.73
	20	0.73
	60	0.68
	300	-
1.03	0.59	0.75 (± 0.02)
	5	0.75
	20	0.70
	60	-
	300	-

Spearman correlation coefficient r_s between C and the single parameters combined in M

Parameter	r_S
A	0.69
c	-0.85
P	0.56
w	0.61
G	-0.25
S	0.61

Results *(Capalbo V. et al., 2021, MNRAS, 503, 6155)*

2) Correlate the results of the Zernike fitting with 3D dynamical indicator

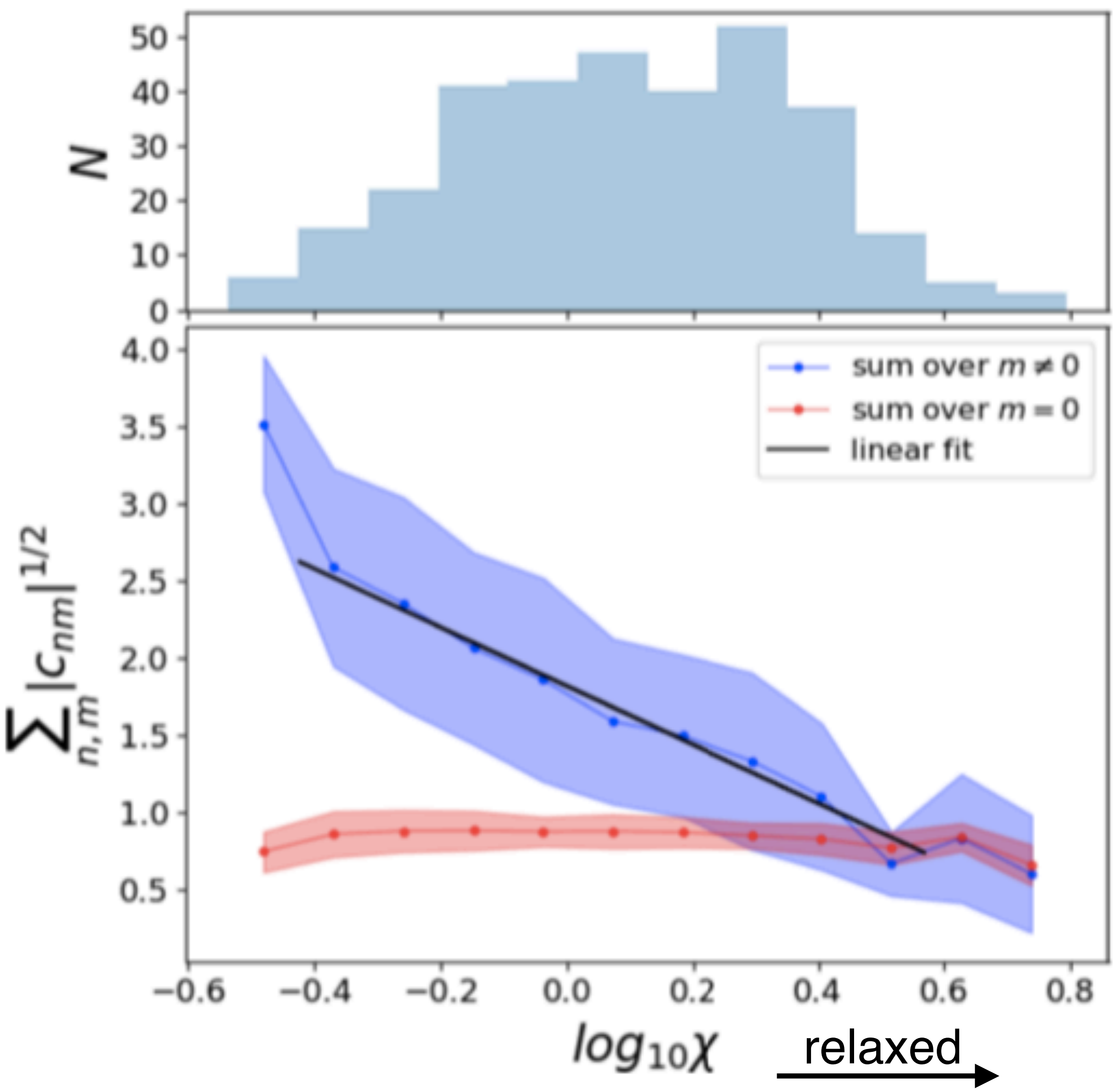


Figure 5. Top: Number clusters distribution along $\log_{10}\chi$ with binning of 0.1. Bottom: $\mathcal{C}$ *versus* $\log_{10}\chi$, for all clusters at $z = 0$. Each point represents the mean value of $\mathcal{C}$ in each bin and the coloured regions are referring to $\pm 1\sigma$. The black line is the fitting line of equation $\mathcal{C} = (-1.90 \pm 0.14)\log_{10}\chi + (1.82 \pm 0.04)$

$$C = \sum_{n,m\neq 0} |c_{nm}|^{1/2}$$

Pearson correlation coefficient r between C and χ

z	Angular resolution (arcsec)	r
0	5.25	-0.61 (± 0.04)
	20	-0.62
	60	-0.62
	300	-0.64
0.45	1.14	-0.50 (± 0.05)
	5	-0.50
	20	-0.53
	60	-0.54
	300	-
1.03	0.59	-0.45 (± 0.05)
	5	-0.47
	20	-0.50
	60	-
	300	-

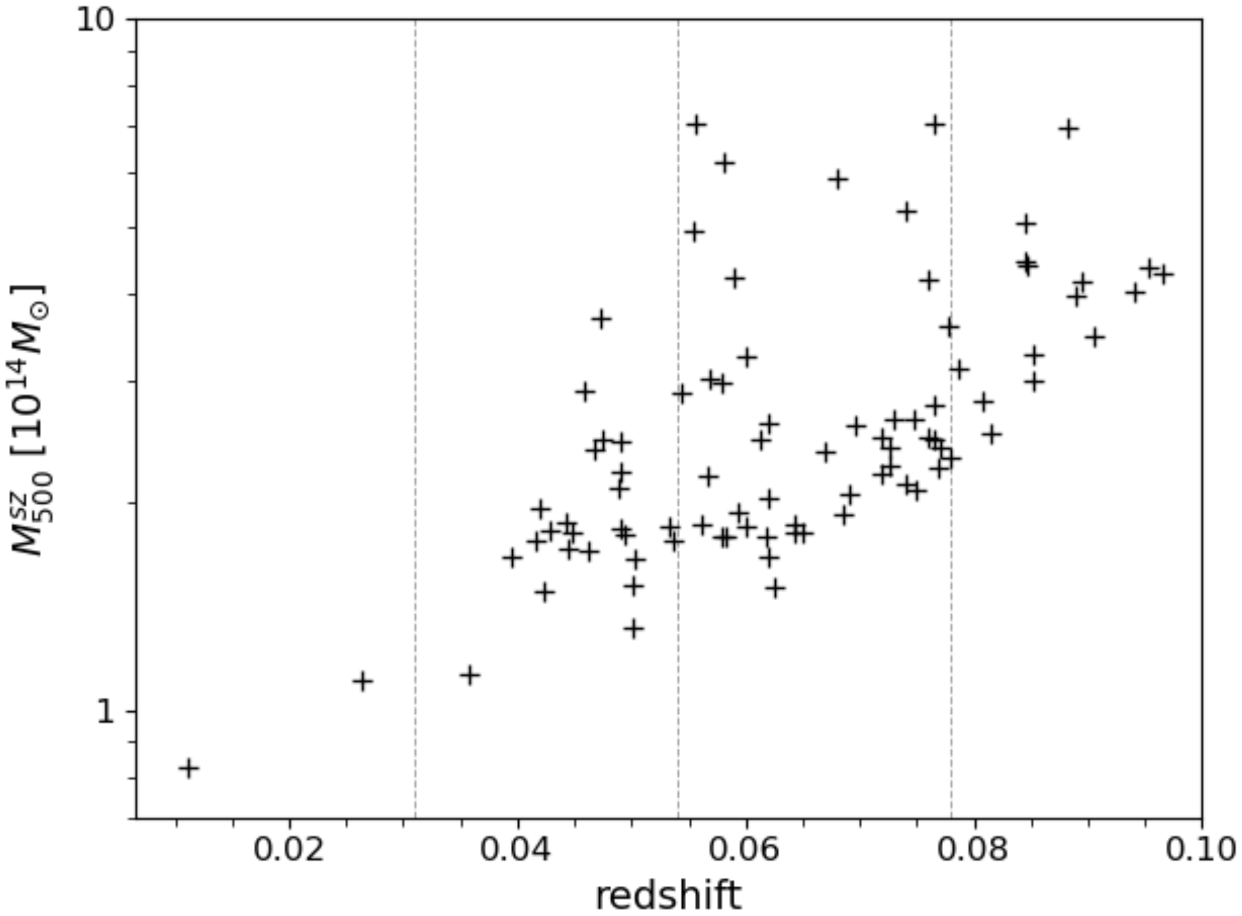
Correlation for 3 different directions along which the *y*-maps are generated, at z=0

Direction	r
x	−0.56
y	−0.59
z	−0.62

Zernike fitting for *Planck*-SZ clusters *(Capalbo V. et al., in preparation)*

Data set: 87 clusters at *z<0.1*, selected from PSZ2 Union catalogue *(Planck Collab. XXVII 2016)*

- only resolved clusters ($\theta_{500} \geqslant 10'$)
- *y*-maps with low residual contamination from radio and IR source (by using PSMASK) *(Planck Collab. XXII 2016)*



Distribution in the $M_{500}^{sz} - z$ plane of 87 PSZ2 clusters selected. The thin vertical lines indicate the boundaries of the redshift bins (width ~ 0.02) centred on the redshift of the 4 snapshots of The Three Hundred clusters.

Planck y-maps:

- gnomonic projections extracted from the public released all-sky *y*-maps (angular resolution=$10'$) *(Planck Collab. XXII 2016)*
- each map is centred on the clusters coordinates, with side-length=$2\theta_{500}$
- we use both MILCA and NILC *y*-maps

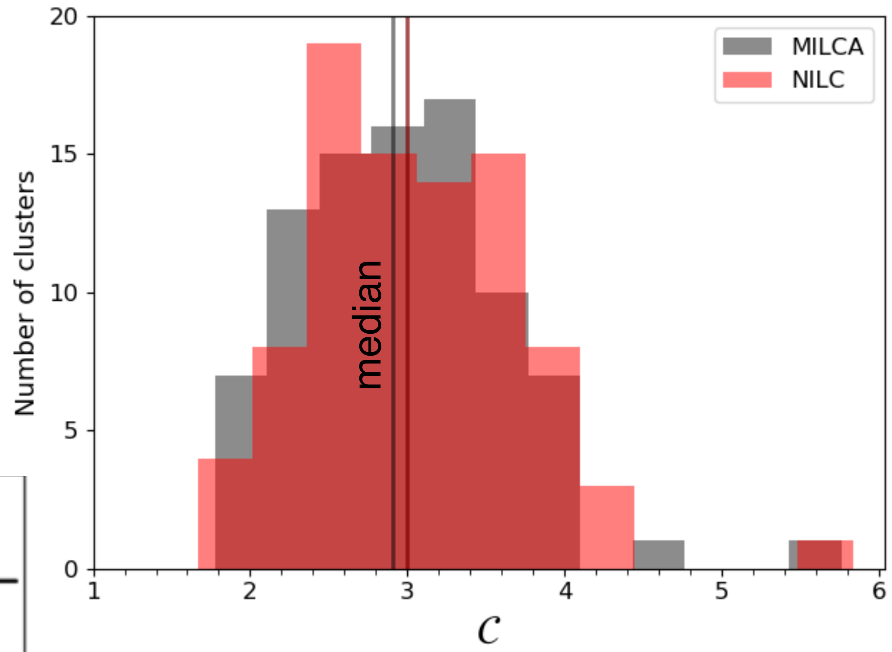
Synthetic data set:

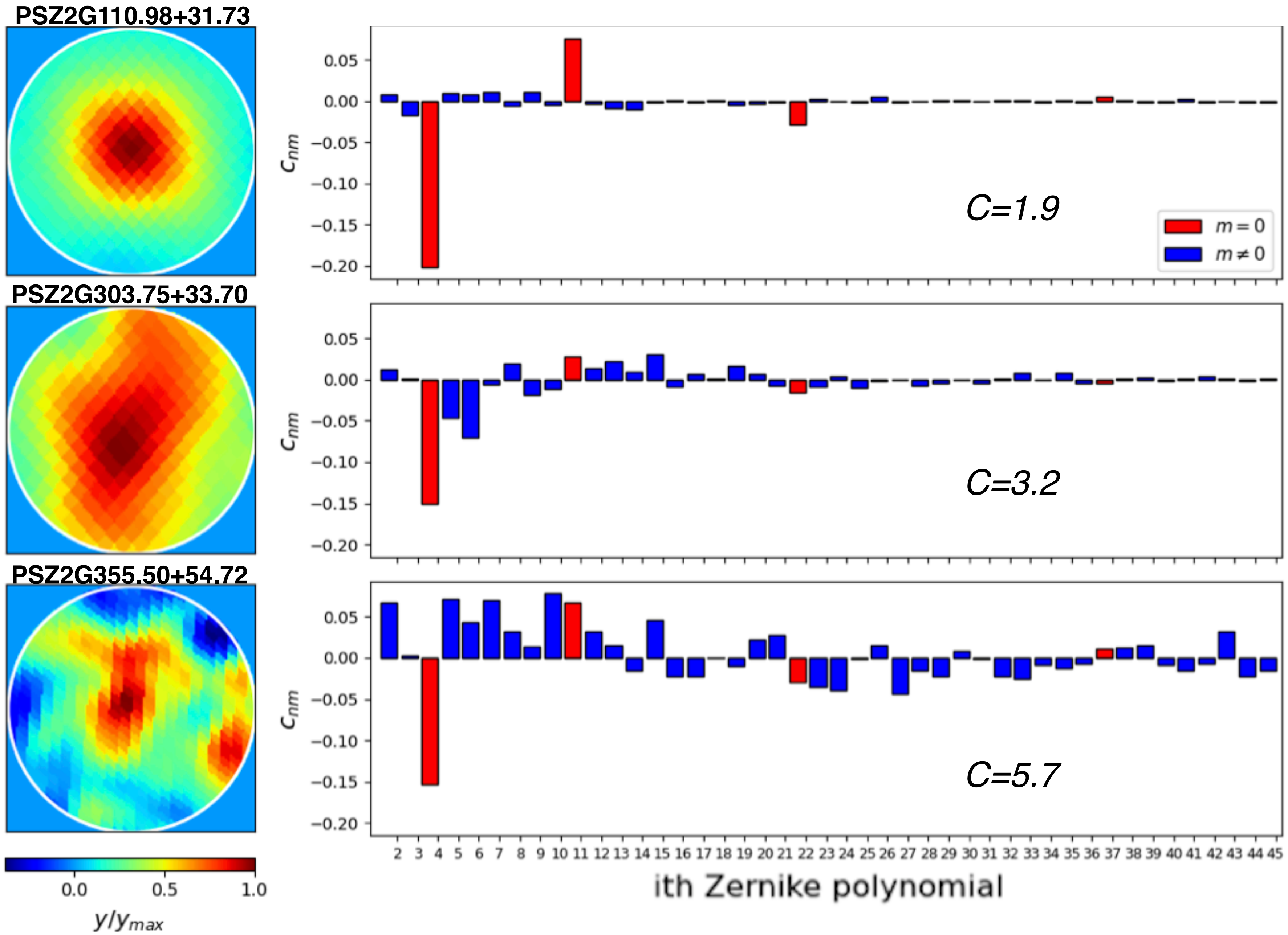
- mock *Planck y*-maps realized for The Three Hundred clusters at 4 redshift snapshots (*z=0.021, 0.044, 0.068, 0.092*)
- The Three Hundred clusters are classified for the dynamical state by using the 3D indicators (f_s , Δ_r , χ) computed within R_{500}

Preliminary results *(Capalbo V. et al., in preparation)*

All the *Planck* *y*-maps are modelled with 45 ZPs, up to *n=8*

$$y = \sum_{n=0}^{8} \sum_{m=0}^{n} c_{nm} Z_n^m \longrightarrow C = \sum_{n,m \neq 0} |c_{nm}|^{1/2} \longrightarrow$$





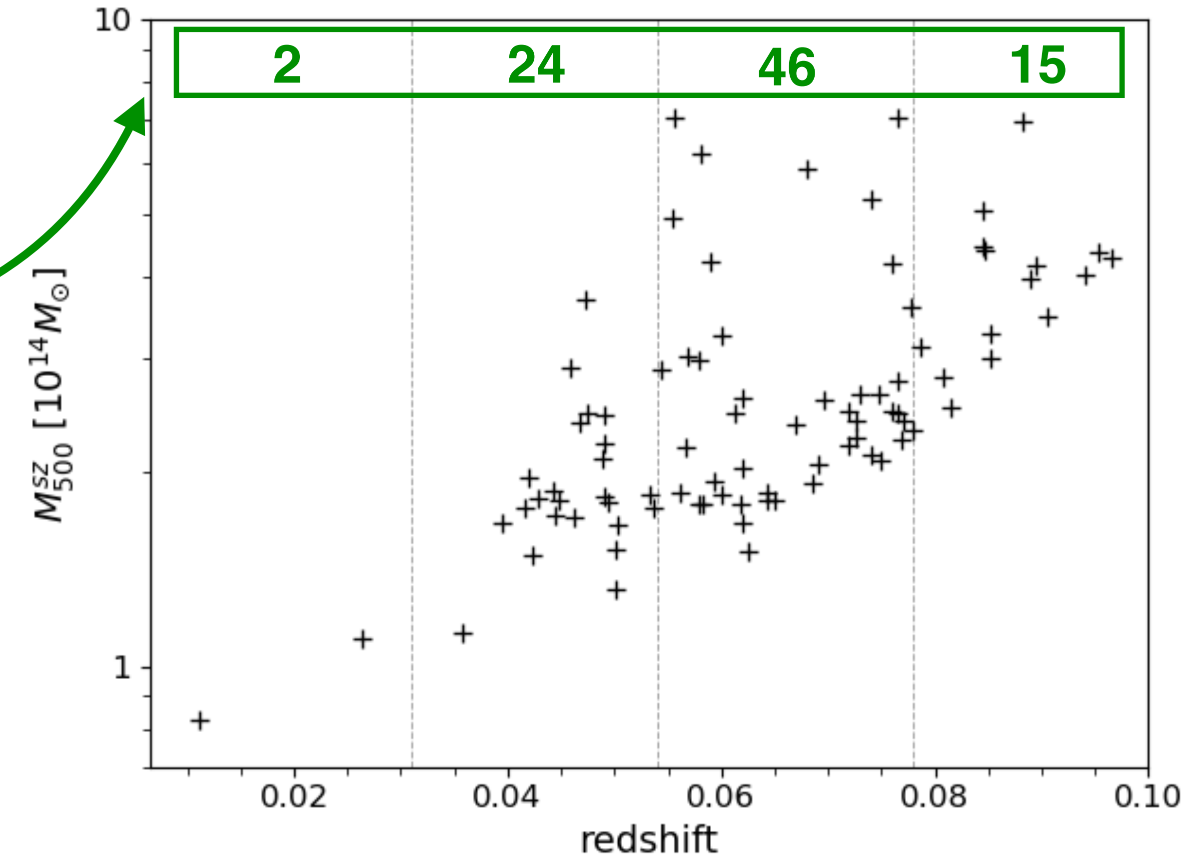
What about the dynamical states?

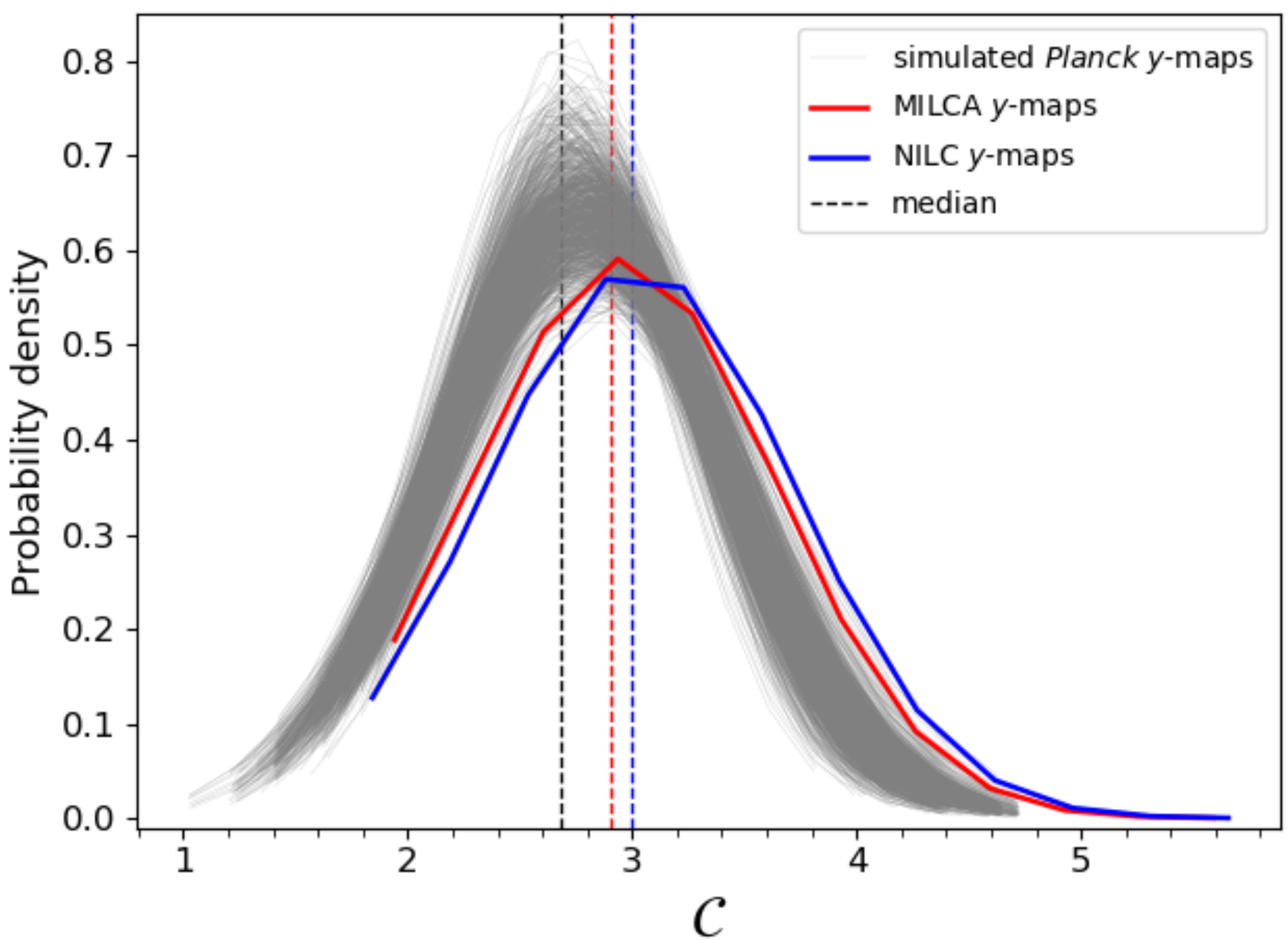
We search for a correlation between the morphological analysis with ZPs and the dynamical state, by using the synthetic Three Hundred clusters

Preliminary results *(Capalbo V. et al., in preparation)*

We perform 1000 random extractions of 87 clusters from the 4 redshift snapshots of The Three Hundred simulations, to mimic the PSZ2 sample

Each simulated sample is constructed by extracting the clusters from the 4 snapshots and following this partition (avoiding the extraction of the same cluster in more than one snapshot)





Good matching between the distributions of *C* parameter from mock *Planck* *y*-maps and from real *Planck* *y*-maps

Zernike fitting on X-ray maps *(Ferragamo A. et al., in preparation)*

Compton parameter *vs* X-ray surface brightness

$$y \propto \int n_e T_e \, dl$$

$$S_x \propto \int n_e^2 \Lambda_X \, dl$$

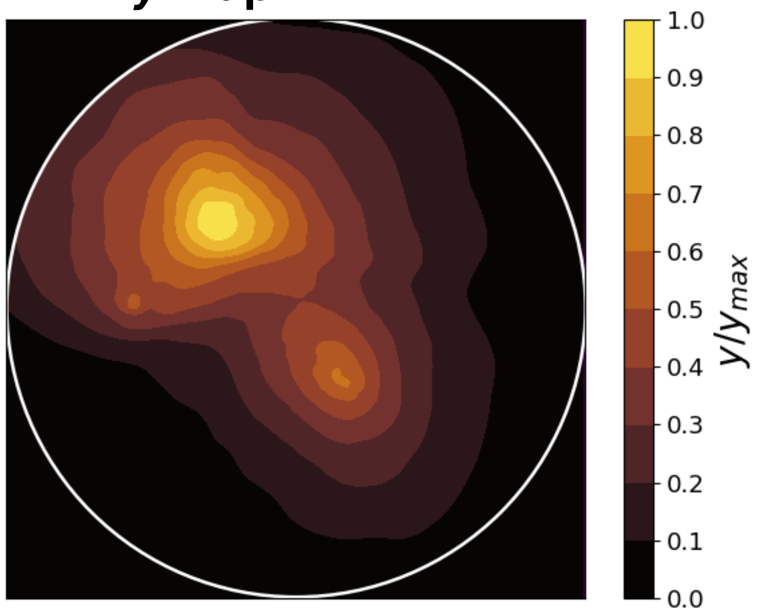
- *y*-maps are sensitive to the diffuse signal of ICM
- X-ray maps show larger spatial frequencies

↓

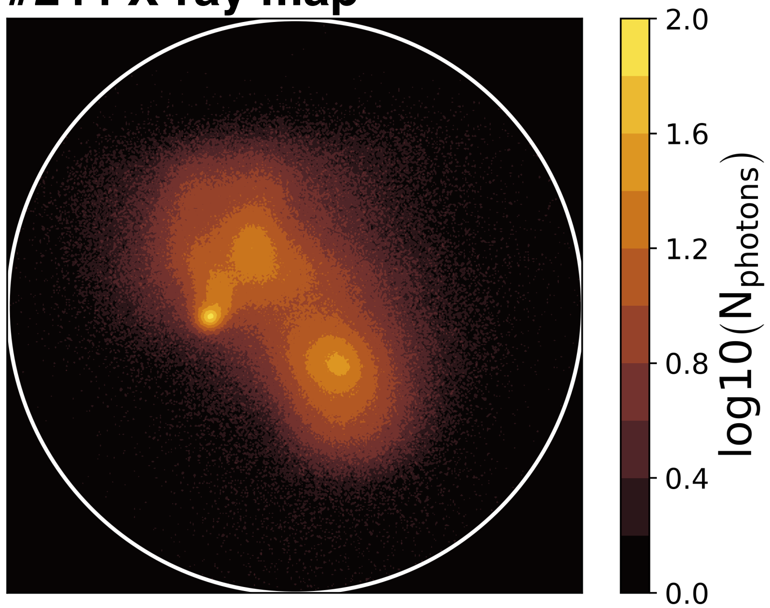
poor modelled with low-order ZPs ($n \leqslant 8$)

Data set: mock X-ray maps for *The Three Hundred* clusters, at 3 redshifts (*z*=0, 0.45, 1.03).
The maps are in terms of number counts of detected photons, realized in the spectral band 0.2-15 keV (as for the *WFI instrument* for the *Athena X-ray Observatory*), with fixed resolution of 10kpc/px.

#244 *y*-map



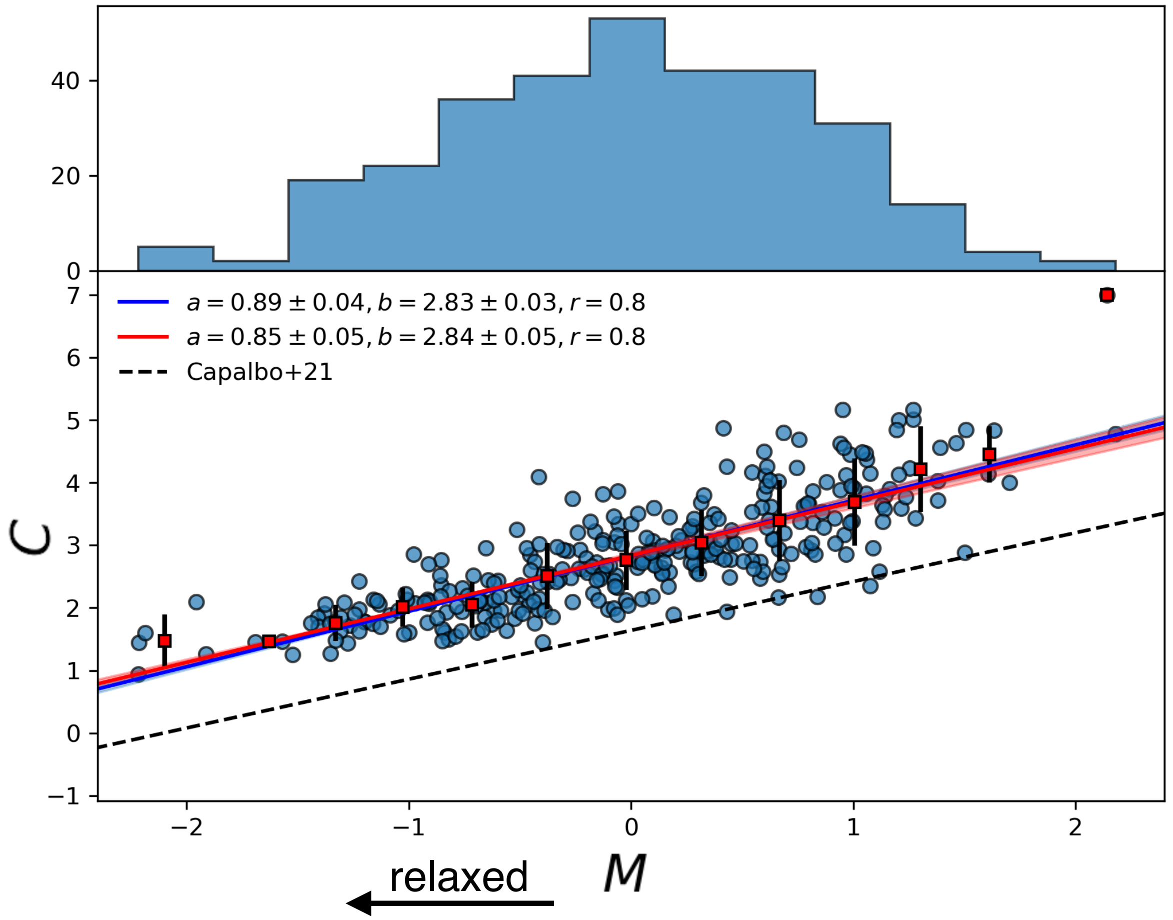
#244 X-ray map

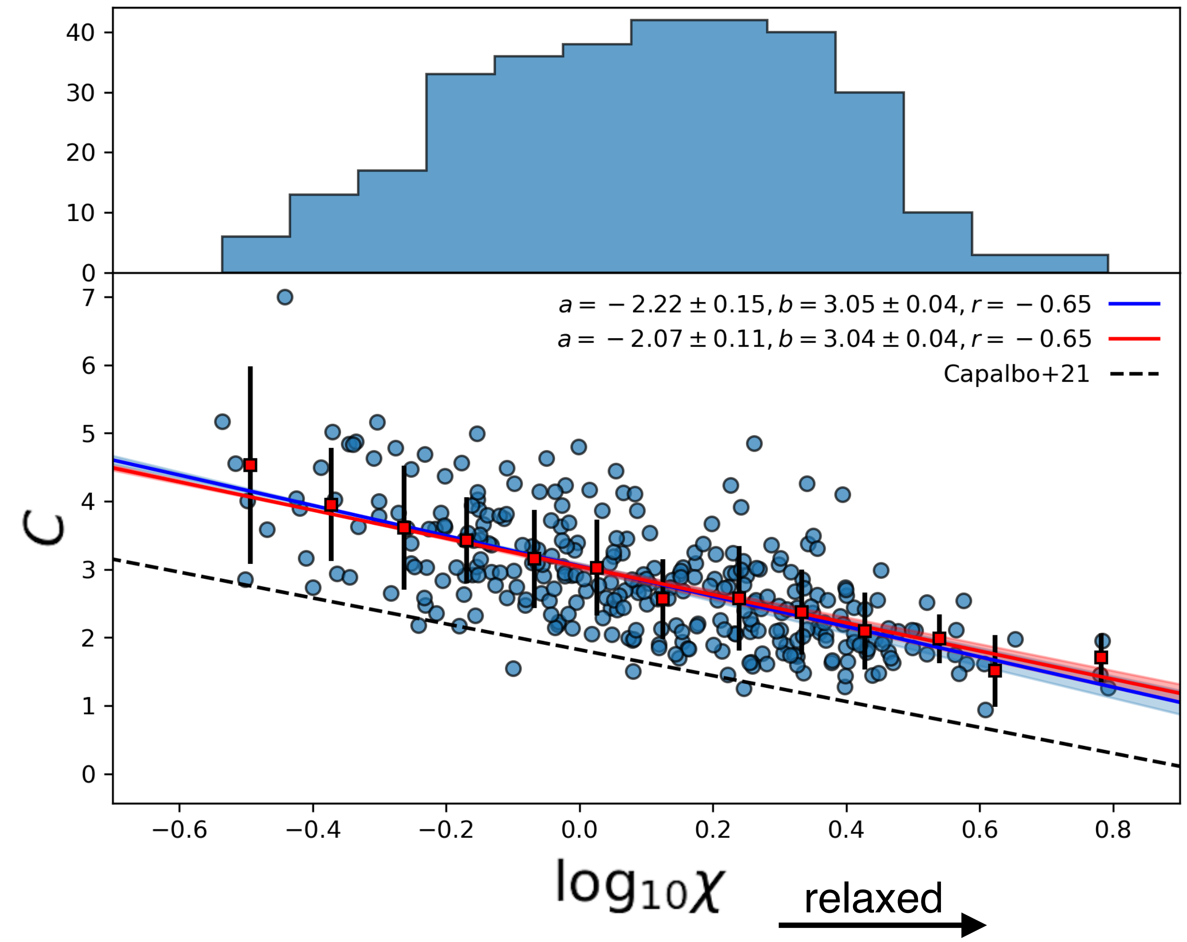


We analyse the maps in logarithmic scale, within a circular aperture with radius R_{500} centred on the X-ray centroid (normalization to the mean within $0.5R_{500}$)

Preliminary results *(Ferragamo A. et al., in preparation)*

M **and** χ **from** ***De Luca F. et al., 2021, MNRAS, 504, 5383***





Pearson correlation coefficient r between C and M

Redshift	r
0	0.80
0.45	0.76
1.03	0.72

Pearson correlation coefficient r between C and χ

Redshift	r
0	-0.65
0.45	-0.57
1.03	-0.47

Summary

- ✓ Zernike polynomials are used for the first time to model mock *y*-maps of galaxy clusters
- ✓ By defining a single parameter that includes the contribution of the different ZPs to the fit of the maps, it is possible to quantify their morphological differences
- ✓ The results are correlated with other (common) morphological and dynamical estimators
- ✓ This method is easily applicable to large surveys of clusters

- ➡ First application of ZPs on real *y*-maps of the *Planck*-SZ clusters *(Capalbo V. et al., in prep)*
- ➡ Validation of the method on X-ray maps *(Ferragamo A. et al., in prep)*

THANK YOU