Hunting for new physics with leptonic g-2

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Plan of the talk

1. **Status of the muon $g-2$ as of early 2021**
2. **New Physics explanations of the muon $g-2$ anomaly**
   - Heavy NP: Effective Field Theory (EFT) approach
   - Light NP: the axion-like particle (ALP) solution
3. **Testing the muon $g-2$ anomaly at a Muon Collider**
4. **Testing the muon $g-2$ anomaly with the electron $g-2$**
5. **Conclusions**

- **Status of the muon $a_\mu \equiv \frac{g_\mu - 2}{2}$ as of early 2021** [T. Aoyama *et al.*, Phys. Rept. '20]

$$a_\mu^{\rm EXP} = 116592089(63) \times 10^{-11} \qquad a_\mu^{\rm SM} = 116591810(43) \times 10^{-11}$$

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = 279\,(76) \times 10^{-11} \qquad \textbf{(3.7}\sigma\textbf{ discrepancy!)}$$

$$\underbrace{(0.1)_{\rm QED}\,, \quad (1)_{\rm EW}\,, \quad (18)_{\rm HLbL}\,, \quad (40)_{\rm HVP}\,,}_{(43)_{\rm TH}} \quad (63)_{\delta a_\mu^{\rm EXP}}\,.$$

  - Hadronic uncertainties (HLbL & HVP) are very hard to improve. [see Colangelo's talk]
  - The E989 Muon $g-2$ experiment will deliver a measure of $a_\mu^{\rm EXP}$ by this spring.
  - We expect $\delta a_\mu^{\rm EXP} \lesssim 2 \times 10^{-10}$ by the E989 Muon $g-2$ experiment in a few years.

- **Low-energy determinations of Δa_μ assume that systematic and hadronic uncertainties are under control at the outstanding level of $\Delta a_\mu \lesssim 10^{-9}$!**

An independent test of Δa_μ would be very desirable!

The NP scale of the muon g-2 anomaly

- Δa_μ **discrepancy at** $\sim 3.7\,\sigma$ **level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.79 \pm 0.76) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - A weakly interacting NP at $\Lambda \approx v$ can naturally explain $\Delta a_\mu \approx 2 \times 10^{-9}$.
  - $\Lambda \approx v$ favoured by the *hierarchy problem* and by a WIMP DM candidate.

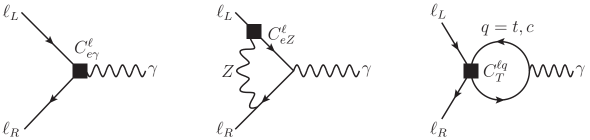
- **LEP and LHC bounds disfavour $\Lambda \approx v$ and two possibilities emerge:**
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.

$$\mathcal{L}_{\rm EFT}(\Lambda \gg v) = \frac{C^{\ell}_{e\gamma}}{\Lambda^2}\left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right) H F_{\mu\nu} + h.c. \quad \Longrightarrow \quad \Delta a_\mu = \frac{4 m_\mu v}{e\Lambda^2} C^{\mu}_{e\gamma}$$

- **What is the NP scale Λ probed by $\Delta a_\mu \equiv a_\mu^{\rm NP} = (2.79 \pm 0.76) \times 10^{-9}$?**

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^{\ell}_{eV}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\bar{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



$$\Delta a_\ell \simeq \frac{4 m_\ell v}{e\Lambda^2} \left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^\ell_{eZ} \log\frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell m_q}{\pi^2} \frac{C^{\ell q}_T}{\Lambda^2} \log\frac{\Lambda}{m_q},$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{250\ \mathrm{TeV}}{\Lambda}\right)^2 |C^\mu_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{50\ \mathrm{TeV}}{\Lambda}\right)^2 |C^\mu_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{100\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu t}_T| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{10\ \mathrm{TeV}}{\Lambda}\right)^2 |C^{\mu c}_T|$$

- **Strongly coupled NP:** $C^\mu_{e\gamma}, C^{\mu t}_T \sim g^2_{\rm NP}/16\pi^2 \lesssim 1$ implying $\Lambda \lesssim$ *few* x 100 TeV, beyond the direct production reach of any foreseen collider.
- **Weakly coupled NP:** $C^\mu_{e\gamma}, C^{\mu t}_T \lesssim 1/16\pi^2$ implying $\Lambda \lesssim 20$ TeV maybe within the direct production reach of a very high-energy Muon Collider

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} \left(\bar\ell_L \sigma^{\mu\nu} e_R\right) H V_{\mu\nu} + \sum_{q=c,t} \frac{C^{\ell q}_T}{\Lambda^2} (\bar\ell_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$

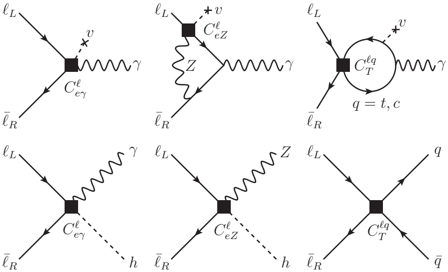


Figure: Connection between the Feynman diagrams for leptonic g-2 (upper row) and high-energy scattering processes (lower row) within the SMEFT: $\boldsymbol{H = v + h/\sqrt{2}}$

$$\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T} \qquad \Longleftrightarrow \qquad \sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4} |C_{eV,T}|^2 \qquad (f = e\gamma, eZ, q\bar q)$$

High-energy probes of $(g-2)_\mu$ [Buttazzo and P.P., '20]

- **Connecting $\mu^+\mu^- \to h\gamma$ with Δa_μ**

$$\sigma_{\mu\mu\to h\gamma} = \frac{s}{48\pi}\frac{|C^\mu_{e\gamma}|^2}{\Lambda^4} \approx 0.7\,\mathrm{ab}\left(\frac{\sqrt{s}}{30\,\mathrm{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$

- **SM irreducible background:**
  - $\sigma^{\rm SM}_{\mu\mu\to h\gamma} \approx (\alpha y_\mu^2/4s)\times \ln(s/m_\mu^2)|_{\sqrt{s}=30\,\mathrm{TeV}} \sim 4\times 10^{-3}\,\mathrm{ab}$: negligible!

- **SM reducible background:**

$$\frac{d\sigma_{\mu\mu\to Z\gamma}}{d\cos\theta} \sim \frac{\pi\alpha^2}{4s}\frac{1+\cos^2\theta}{\sin^2\theta} \qquad \frac{d\sigma_{\mu\mu\to h\gamma}}{d\cos\theta} = \frac{|C^\mu_{e\gamma}|^2}{\Lambda^4}\frac{s}{64\pi}(1-\cos^2\theta)$$

  - The significance of the signal $S = N_S/\sqrt{N_B+N_S}$ maximal for $|\cos\theta| \lesssim 0.6$.

$$\sigma^{\rm cut}_{\mu\mu\to h\gamma} \approx 0.53\,\mathrm{ab}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2, \qquad \sigma^{\rm cut}_{\mu\mu\to Z\gamma} \approx 82\,\mathrm{ab} \qquad (\sqrt{s}=30\,\mathrm{TeV})$$

  - S/B isolation: i) angular distributions and ii) h/Z invariant mass reconstruction.
  - Cut-and-count exp. with $b\bar{b}$ final state, $\mathcal{B}(h/Z\to b\bar{b}) = 0.58/0.15$ and $\epsilon_b = 80\%$.
  - For a Z/h misident. prob. of 10%, $N_{S(B)} = 22(88)$ and $S = 2$ at $\sqrt{s} = 30$ TeV.

High-energy probes of $(g-2)_\mu$ [Buttazzo and P.P., '20]

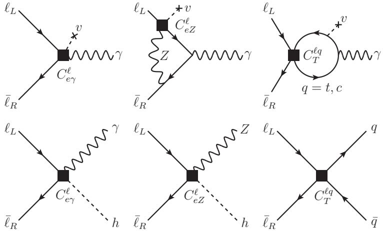
- **Connecting $\mu^+\mu^- \to (h\gamma, Zh, t\bar{t}, c\bar{c})$ with Δa_μ**

$$\sigma^{\rm cut}_{\mu\mu\to h\gamma} \approx 0.5\,{\rm ab} \left(\frac{\sqrt{s}}{30\,{\rm TeV}}\right)^2 \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to Zh} \approx 38\,{\rm ab} \left(\frac{\sqrt{s}}{10\,{\rm TeV}}\right)^2 \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to t\bar{t}} \approx 58\,{\rm ab} \left(\frac{\sqrt{s}}{10\,{\rm TeV}}\right)^2 \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$

$$\sigma_{\mu\mu\to c\bar{c}} \approx 100\,{\rm fb} \left(\frac{\sqrt{s}}{3\,{\rm TeV}}\right)^2 \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2$$



- **Δa_μ predictions in the SMEFT**

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{250\,{\rm TeV}}{\Lambda}\right)^2 |C^\mu_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{50\,{\rm TeV}}{\Lambda}\right)^2 |C^\mu_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{100\,{\rm TeV}}{\Lambda}\right)^2 |C^{\mu t}_{T}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{10\,{\rm TeV}}{\Lambda}\right)^2 |C^{\mu c}_{T}|$$

- **SM irreducible background**

$$\sigma^{\rm SM,cut}_{\mu\mu\to Z\gamma} \approx 82\,{\rm ab} \left(\frac{30\,{\rm TeV}}{\sqrt{s}}\right)^2 \qquad \sigma^{\rm SM}_{\mu\mu\to Zh} \approx 122\,{\rm ab} \left(\frac{10\,{\rm TeV}}{\sqrt{s}}\right)^2$$

$$\sigma^{\rm SM}_{\mu\mu\to t\bar{t}} \approx 1.7\,{\rm fb} \left(\frac{10\,{\rm TeV}}{\sqrt{s}}\right)^2 \qquad \sigma^{\rm SM}_{\mu\mu\to c\bar{c}} \approx 19\,{\rm fb} \left(\frac{3\,{\rm TeV}}{\sqrt{s}}\right)^2$$

The muon g-2 at a Muon Collider [Buttazzo and P.P., '20]

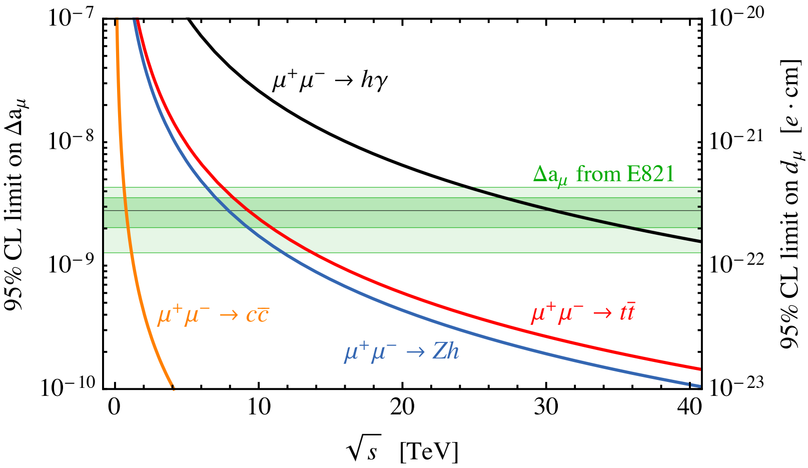


Figure: 95% C.L. reach on Δa_μ, as well as on the muon EDM d_μ, as a function of $\sqrt{s}$ from various processes for the reference integrated luminosity $\mathcal{L} = \left(\sqrt{s}/10\,\mathrm{TeV}\right)^2 \times 10\,\mathrm{ab}^{-1}$.

$$d_\mu = \frac{\Delta a_\mu \tan\phi_\mu}{2m_\mu}\, e \simeq 3\times 10^{-22}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)\tan\phi_\mu \; e\,\mathrm{cm}$$

Axion-like Particle effective Lagrangian

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2}\frac{\partial^\nu a}{\Lambda}\bar{\mu}\gamma_\nu\gamma_5\mu$$

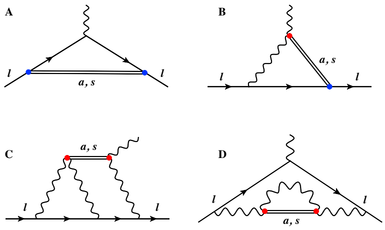


Figure: Contributions of a scalar 's' and a pseudoscalar 'a' ALP to the $(g-2)_\ell$.

[Marciano, Masiero, Paradisi, Passera '16]

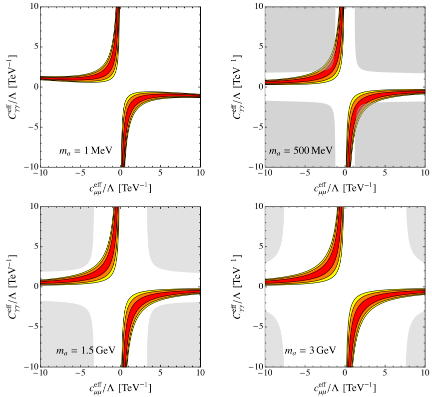


Figure: Regions where the $(g-2)_\mu$ exp. value is reproduced at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by $e^+e^- \to \mu^+\mu^- + \mu^+\mu^-$ searches at BaBar.

[Bauer, Neubert, Thamm, '17]

$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2}\left[\frac{12\alpha^3}{\pi} C_{\gamma\gamma}^2 \ln^2\frac{\Lambda^2}{m_\mu^2} - \frac{(c_{\mu\mu})^2}{16\pi^2} h_1\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi} c_{\mu\mu} C_{\gamma\gamma} \ln\frac{\Lambda^2}{m_\mu^2}\right]$$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau\,,$$

  - **Branching ratios of $\ell \to \ell'\gamma$**

$$\frac{\mathrm{BR}(\ell\to\ell'\gamma)}{\mathrm{BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

  - **"Naive scaling":** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}\,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j \gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}} \right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}} \right)^2$$

- **EDMs vs.** $(g-2)_\mu$

$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-28} \left(\frac{\phi_e^{CPV}}{10^{-4}} \right) e\,\mathrm{cm}\,,$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22}\, \phi_\mu^{CPV}\ e\,\mathrm{cm}\,.$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\ e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \approx (3 \pm 1) \times 10^{-9}$$

$$\Delta a_\mu \approx a_\mu^{\rm EW} = \frac{m_\mu^2}{(4\pi v)^2}\left(1 - \frac{4}{3}s_{\rm W}^2 + \frac{8}{3}s_{\rm W}^4\right) \approx 2 \times 10^{-9}.$$

- **Testing the muon $g-2$ anomaly through the electron $g-2$**

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \qquad \Longleftrightarrow \qquad \Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}$$

  - a_e has never played a role in testing NP effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which was is the most precise value of α up to 2018!
  - The situation has now changed thanks to th. and exp. progresses.
  - α can be extracted from atomic physics and a_e used to perform NP tests!

[Giudice, P.P, & Passera, '12]

The Standard Model prediction of the electron $g-2$

- **Status of Δa_e as of 2012**

$$\Delta a_e = a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -9.2\,(8.1) \times 10^{-13},$$

$$\delta a_e \times 10^{13}: \quad (0.6)_{\mathrm{QED4}},\; (0.4)_{\mathrm{QED5}},\; (0.2)_{\mathrm{HAD}},\; (7.6)_{\delta\alpha},\; (2.8)_{\delta a_e^{\mathrm{EXP}}}.$$

  - The errors from QED4 and QED5 will be reduced soon to 0.1×10^{-13} [Kinoshita]
  - We expect a reduction of $\delta a_e^{\mathrm{EXP}}$ to a part in 10^{-13} (or better). [Gabrielse]
  - Work is also in progress for a significant reduction of $\delta\alpha$. [Nez]

- **Status of Δa_e as of 2018: 2.4σ discrepancy** [Parker et al., Science, '18]

$$\Delta a_e = a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}}(\alpha_{\mathrm{Berkeley}}) = -8.8\,(3.6) \times 10^{-13}$$

$$\delta a_e \times 10^{13}: \quad (0.1)_{\mathrm{QED5}},\; (0.1)_{\mathrm{HAD}},\; (2.3)_{\delta\alpha},\; (2.8)_{\delta a_e^{\mathrm{EXP}}}.$$

- **Status of Δa_e as of 2020: 1.6σ discrepancy** [Morel et al., Nature, '20]

$$\Delta a_e = a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}}(\alpha_{\mathrm{LKB2020}}) = 4.8\,(3.0) \times 10^{-13}$$

$$\delta a_e \times 10^{13}: \quad (0.1)_{\mathrm{QED5}},\; (0.1)_{\mathrm{HAD}},\; (0.9)_{\delta\alpha},\; (2.8)_{\delta a_e^{\mathrm{EXP}}}.$$

- **$\Delta a_e \lesssim 10^{-13}$ is not too far! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.** [Giudice, P.P, & Passera, '12]

Conclusions

- The E989 Muon $g-2$ experiment will deliver a measure of $a_\mu^{\rm EXP}$ by this spring with an uncertainty $\delta a_\mu^{\rm EXP}$ comparable to the current one by BNL.
- If the current muon $g-2$ anomaly will be confirmed, then its interpretation in terms of New Physics will become more and more plausible.
- Both heavy New Physics ($\Lambda \gg 1\,{\rm TeV}$) and ligh New Physics ($\Lambda \lesssim$ *few* $\times\,{\rm GeV}$) scenarios have the potential to account for the muon $g-2$ anomaly.
- A Muon Collider running at $\sqrt{s} \gg 1\,$TeV would provide a unique opportunity to probe heavy New Physics effects in the muon g-2 in a model-independent way:
  - Direct determination of NP, not hampered by the hadronic uncertainties of $a_\mu^{\rm SM}$.
  - A high-energy measurement with $\mathcal{O}(1)$ precision is sufficient to probe $\Delta a_\mu \sim 10^{-9}$.
- Testing New Physics effects in the electron $g-2$ at the 10^{-13} is not too far! This will bring a_e to play a pivotal role in probing New Physics in the leptonic sector as well as in confirming the muon $g-2$ anomaly.

Message: an exciting Physics program is in progress at the Intensity Frontier!