Theoretical status of $(g-2)_\mu$ in the Standard Model

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Outline

Introduction: $(g-2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

Hadronic light-by-light contribution to $(g-2)_\mu$

Conclusions

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White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

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White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier
Simon Eidelman
Aida El-Khadra (co-chair)
Martin Hoferichter
Christoph Lehner (co-chair)
Tsutomu Mibe (J-PARC E34 experiment)
(*Andreas Nyffeler* until summer 2020)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

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Muon $g-2$ Theory Initiative
Workshops:

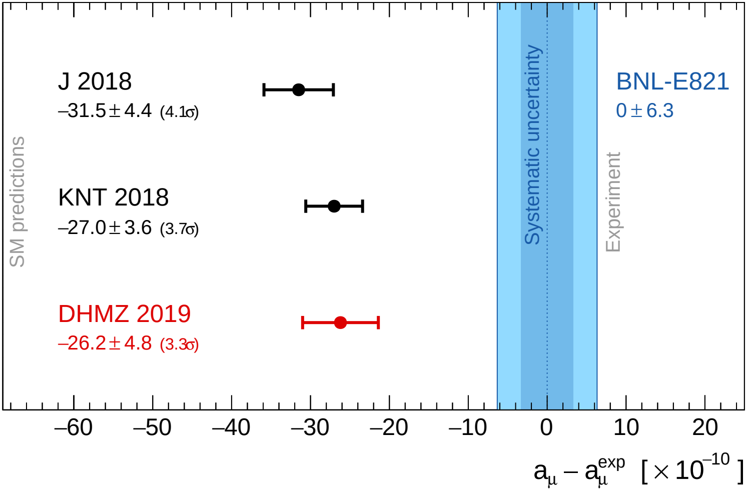
- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021

White Paper executive summary (my own)

- QED and EW known and stable, negligible uncertainties
- HVP dispersive: consensus number, conservative uncertainty (KNT20, DHMZ20, CHS19, HKS19)
- HVP lattice: consensus number, uncertainty $\sim 4.5\times$ larger than dispersive (Fermilab-HPQCD-MILC18,20, BMW18, RBC/UKQCD18, ETM19,SK19, Mainz19, ABTGJP20)
- HVP BMW20: central value $\rightarrow$ discrepancy $< 2\sigma$; uncertainty $\sim$ to dispersive; not yet published $\rightarrow$ not in WP
- HLbL dispersive: w/ recent improvements $\rightarrow$ uncertainty $\sim 1/2$ HVP uncertainty
- HLbL lattice: one number, agrees with dispersive: average reduces final uncertainty (RBC/UKQCD20)

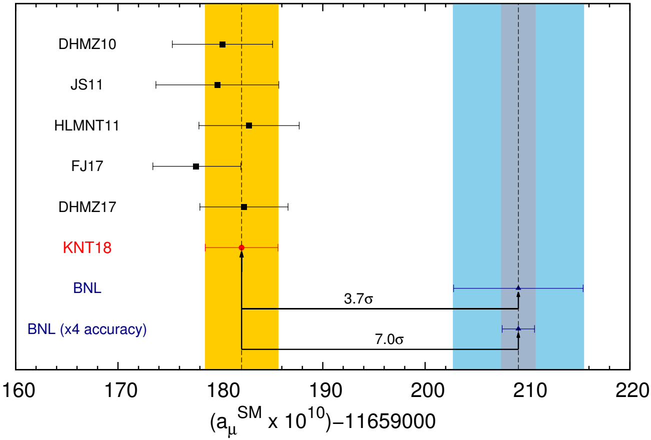
Status of $(g-2)_\mu$, experiment vs SM

Davier, Hoecker, Malaescu, Zhang 2019



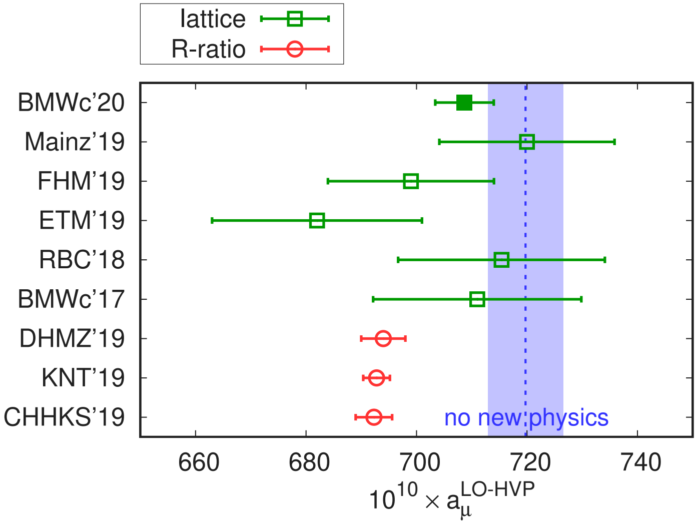
Status of $(g-2)_\mu$, experiment vs SM

Keshavarzi, Nomura, Teubner, 2018 (KNT18)



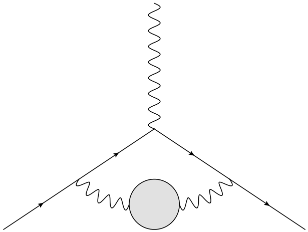
Status of $(g-2)_\mu$, experiment vs SM

BMW20



Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) can be systematically improved



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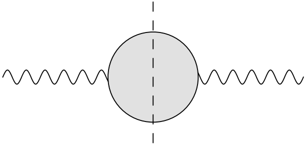
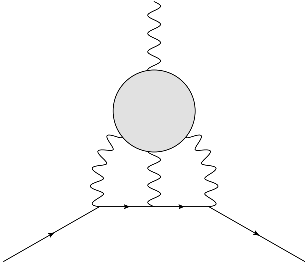


  - basic principles: unitarity and analyticity
  - direct relation to experiment: $\sigma_{\rm tot}(e^+e^- \to \gamma^* \to \text{hadrons})$
  - dedicated e^+e^- program: BaBar, Belle, BESIII, CMD3, KLOE2, SND
  - alternative approach: lattice (ETMC, Fermilab, Mainz, MILC, HPQCD, BMW, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) can be systematically improved
- Hadronic light-by-light (HLbL) is more problematic:



  - 4-point fct. of em currents in QCD
  - "*it cannot be expressed in terms of measurable quantities*"
  - recently, a dispersive approach has been successfully applied here too
  - lattice QCD is becoming competitive

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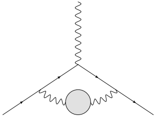
Calculating the HVP contribution

- HVP can be calculated with a data-driven approach



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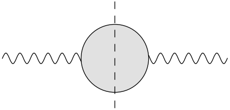
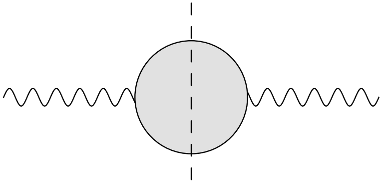


- direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \gamma^* \to \text{hadrons})$
- dedicated e^+e^- program: BaBar, Belle, BESIII, CMD3, KLOE2, SND

Unitarity relation for HVP

For HVP the unitarity relation is simple and looks the same for all possible intermediate states



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

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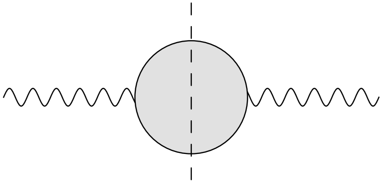


$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

$$a_\mu^{\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty} \frac{ds}{s} K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

WP(20)

2π: comparison with the dispersive approach

Energy range	ACD18	CHS18	DHMZ19	DHMZ19$'$	KNT19
$\leq$ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
$\leq$ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
$\leq$ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
$\leq$ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
$\leq$ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
$\leq$ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

WP(20)

Combination method and final result

The complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been combined with the following methodology:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$\begin{aligned} a_\mu^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\ &= 693.1(4.0) \times 10^{-10} \end{aligned}$$

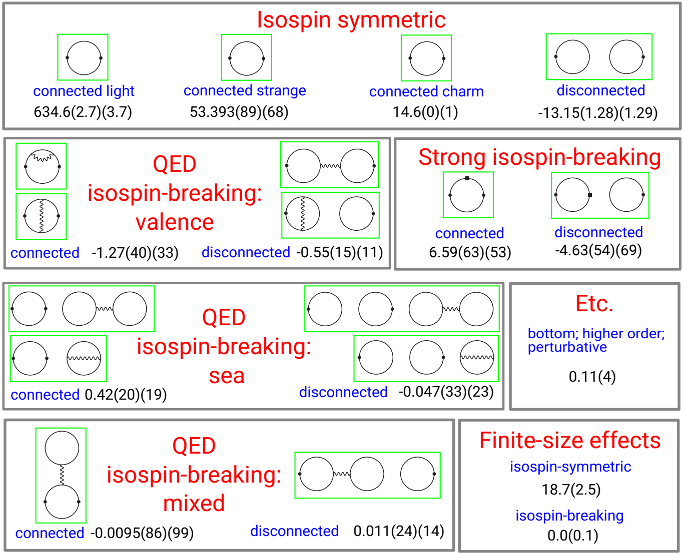
WP(20)

The BMW result

State-of-the-art lattice calculation of $a_\mu^{\text{HVP, LO}}$ based on

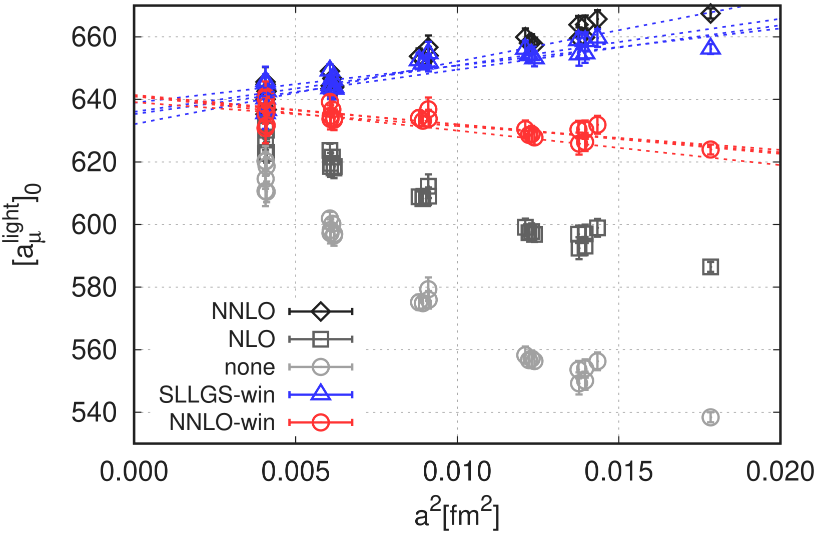
- current-current correlator, summed over all distances, integrated in time with appropriate kernel function
- using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin breaking effects

The BMW result



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 708.7(2.8)_{\text{stat}}(4.5)_{\text{sys}}[5.3]_{\text{tot}}$$

The BMW result



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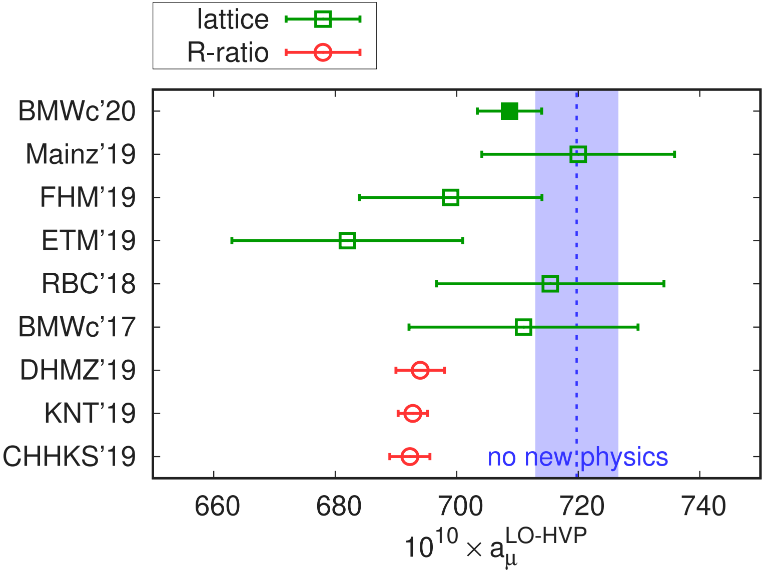

Consequences of the BMW result

A shift in the established value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (with a different weight function)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a change in $\Delta\alpha_{\text{had}}(M_Z^2)$: its size depends on the energy range of the shift in $\sigma(e^+e^- \to \text{hadrons})$
- a change in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- two independent analyses have shown: to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below ~ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20), Keshavarzi, Marciano, Passera, Sirlin (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?

- Below $1-2$ GeV there is only one significant channel: $\pi^+\pi^-$
- This is strongly constrained by analyticity and unitarity (for the pion vector form factor $F_\pi^V(s)$)
- A parametrization which respects these has been built and depends on a small number of parameters

  GC, Hoferichter, Stoffer (18)
- We have analyzed possible shifts in these parameters and the corresponding scenarios

  GC, Hoferichter, Stoffer (21)

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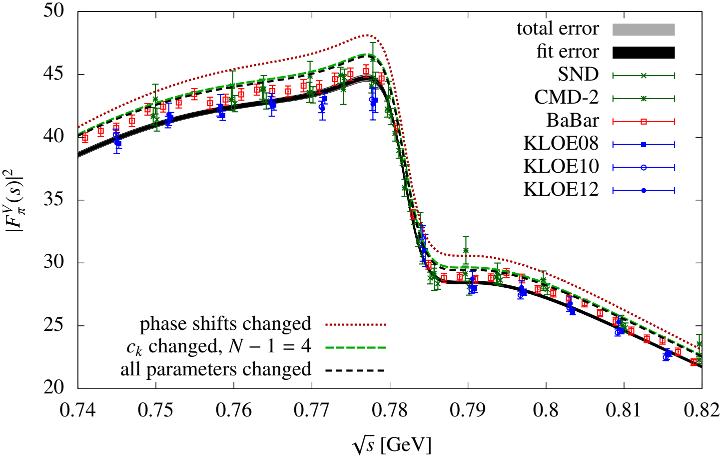

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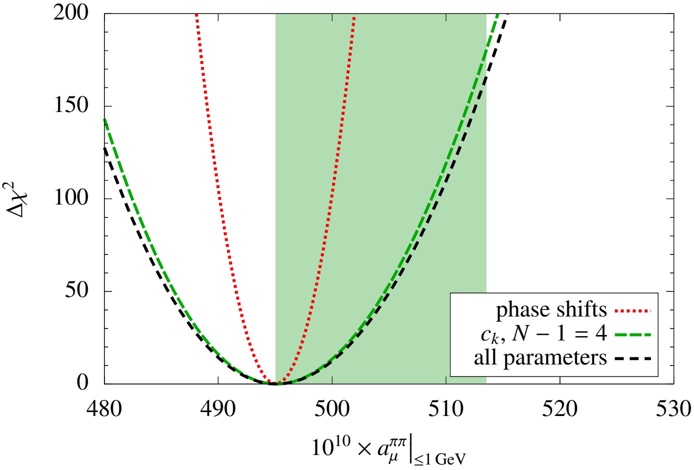

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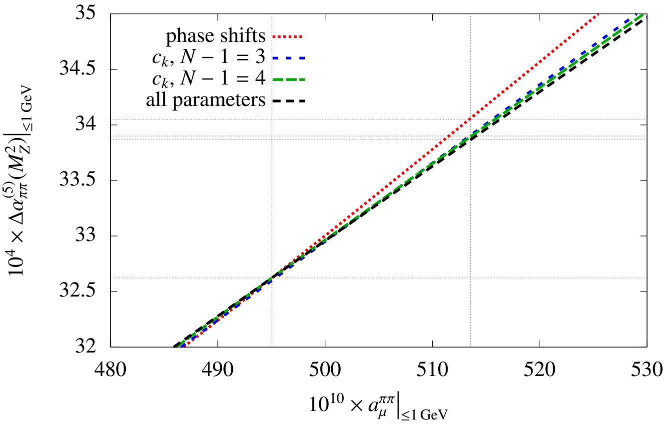
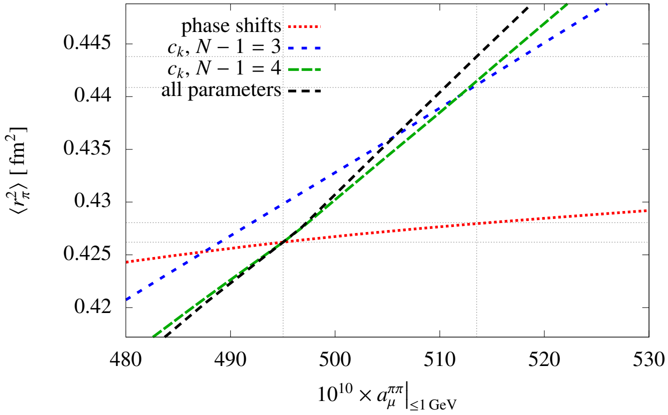


GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

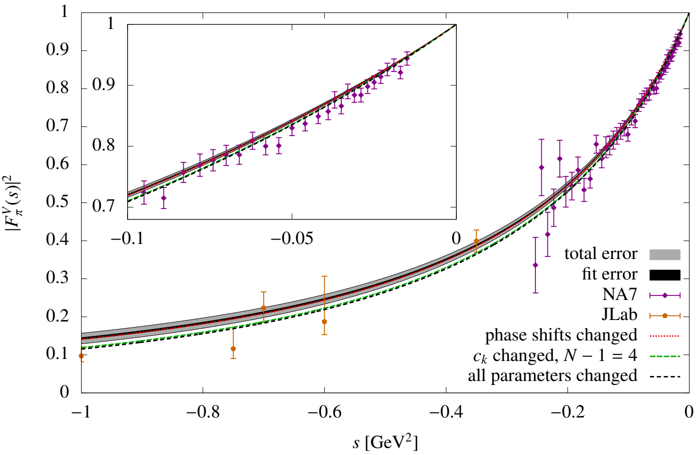
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

$$\langle r_\pi^2 \rangle = \begin{cases} 0.429(4)\text{fm}^2 & \text{CHS(18)} \\ 0.436(5)(12)\text{fm}^2 & \chi\text{QCD(20)} \end{cases}$$

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

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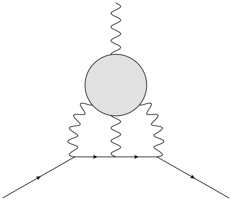
Calculating the HLbL contribution is complicated

- 4-point function of em currents in QCD

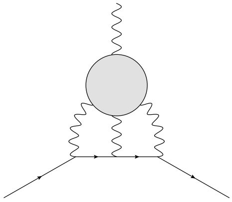


- until recently, only model calculations
- a data-driven approach like for HVP seemed hopeless
  "*it cannot be expressed in terms of measurable quantities*"
- lattice QCD is an alternative and is making fast progress

Master Formula

$$a_\mu^{\text{HLbL}} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p + q_1)^2 - m_\mu^2][(p - q_2)^2 - m_\mu^2]}$$

- $\hat{T}_i$: known kernel functions
- $\hat{\Pi}_i$: linear combinations of the Π_i
- the Π_i are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses
- 5 integrals can be performed with Gegenbauer polynomial techniques



GC, Hoferichter, Procura, Stoffer (2015)

Master Formula

After performing the 5 integrations:

$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1^4 \int_0^\infty dQ_2^4 \int_{-1}^1 d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

where Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$.

GC, Hoferichter, Procura, Stoffer (2015)

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	$-19(19)$	$-19(13)$	$-20(5)$	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-7(7)$	$-7(2)$	$-5.98(1.20)$	$-8(1)$
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	$-1(3)$ (scalars + tensors)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

- significant reduction of uncertainties in the first three rows: low-energy region well constrained by a dispersive approach
- $1-2$ GeV and asymptotic region (short distance constraints) have been improved, but still work in progress

Bijnens, Hermansson-Truedsson, Laub, Rodriguez-Sànchez (20,21)

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- The White Paper has provided the current status of the SM evaluation of $(g-2)_\mu$: 3.7σ discrepancy with experiment
- The evaluation of the HVP contribution relies only on the dispersive approach. With a 0.6% error it gives the largest contribution to the theory uncertainty
- A recent lattice calculation, BMW(20) has reached a similar precision but differs from the dispersive evaluation. If confirmed, it would bring the discrepancy with experiment to below 2σ
- The evaluation of the HLbL contribution based on the dispersive approach has reached 20% accuracy. A lattice calculation, RBC/UKQCD(20), agrees with it, though with larger uncertainty
- The Fermilab Muon $g-2$ experiment aims to reduce the experimental uncertainty by a factor four, potentially leading to a 7σ discrepancy. First results are expected soon.