



Paolo Creminelli, ICTP (Trieste)

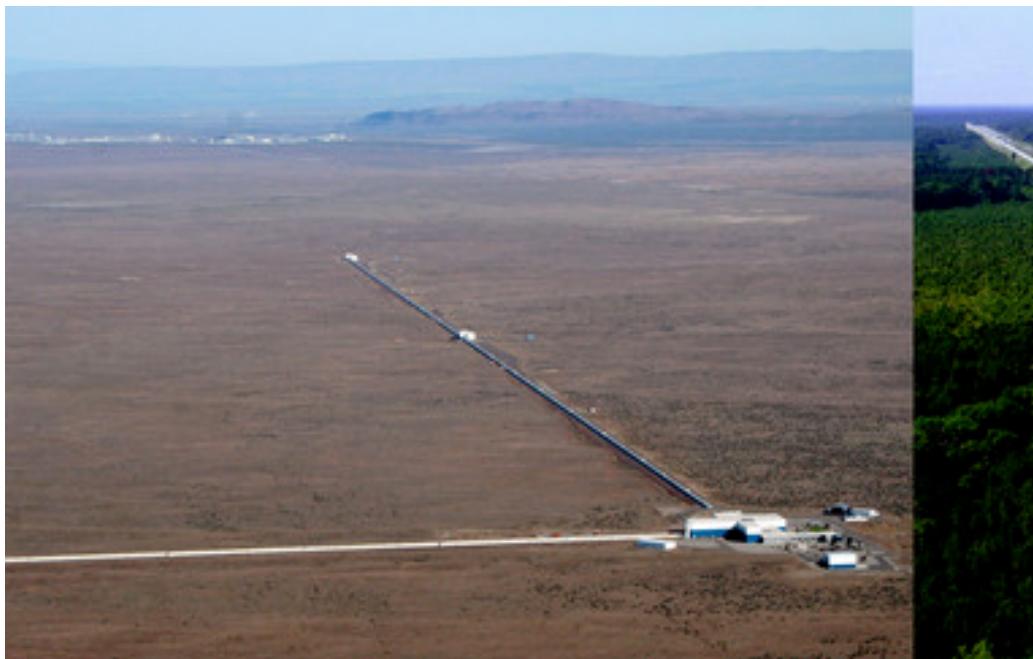
# Dark Energy and GW Observations

with F. Vernizzi 1710.05877, with M. Lewandowski, G.Tambalo and F. Vernizzi 1809.03484

with G.Tambalo, F. Vernizzi and V. Yingcharoenrat 1906.07015 + 1910.14035 + ...

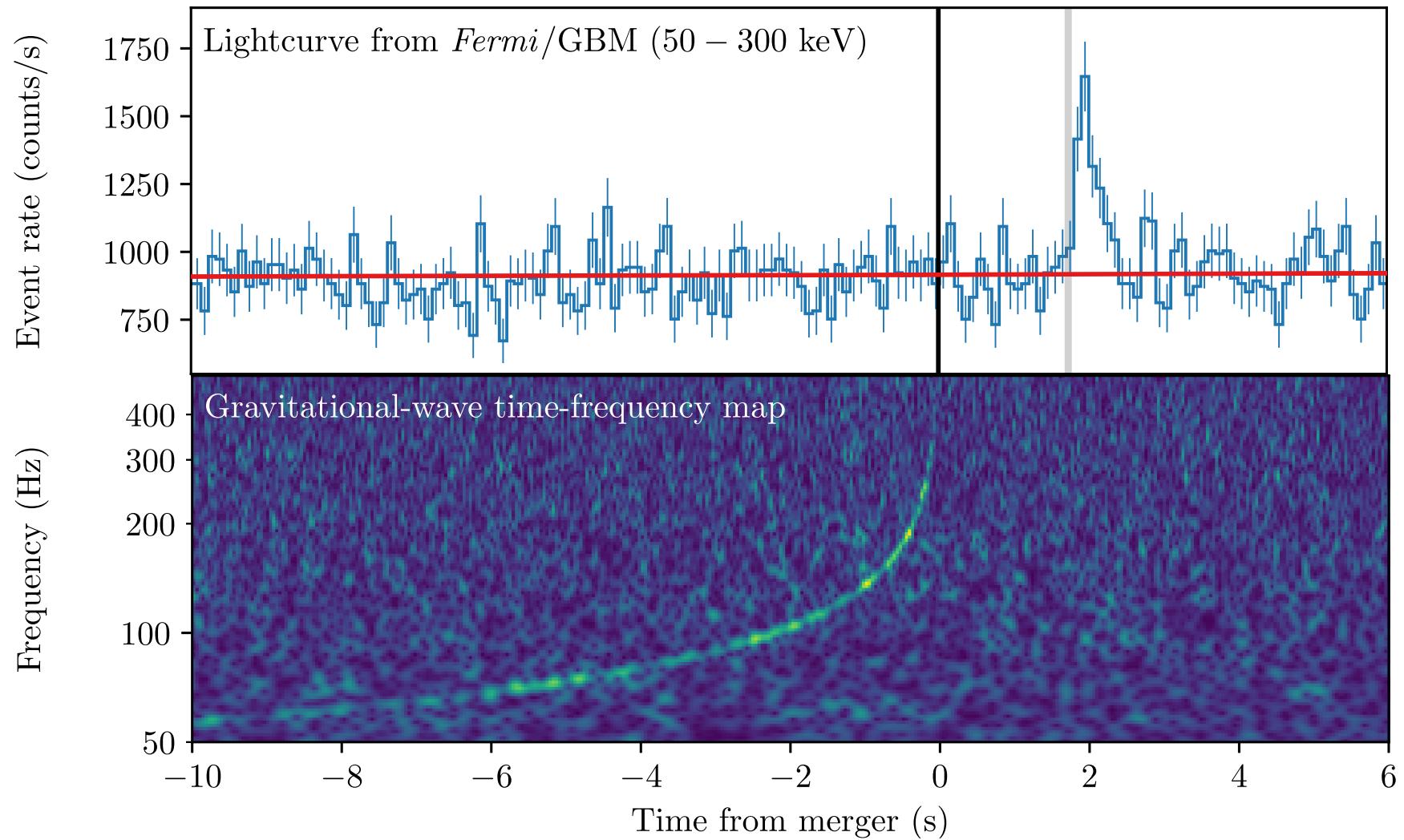
“La Thuile”, 9 March 2021

# First (direct) GW observation on 14.09.15



The two LIGO detectors (USA)  
and VIRGO near Pisa, Italy

# GW170817 = GRB170817A



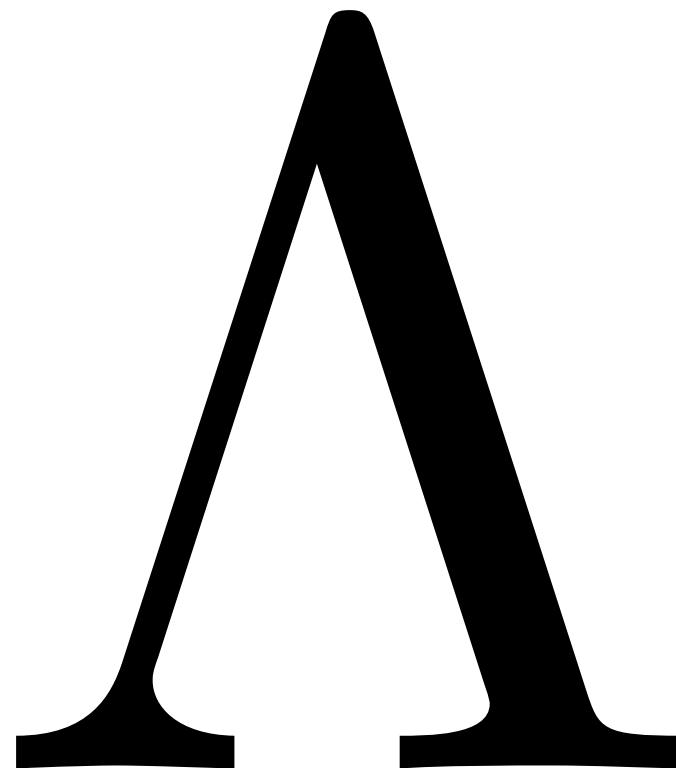
# Dark Energy = Lorentz-violating Medium



In general the speed of GWs is different from photons,  
GWs can be absorbed, have dispersion...

Use GWs to probe Dark Energy as light probes a material

The (small) elephant in the room



# GW170817 = GRB170817A

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$

- **Low energy:**  $\lambda \sim 10\,000$  km

Reasonable one can use the same EFT as for cosmo scales

- Over **~ cosmological distances:** 40 Mpc

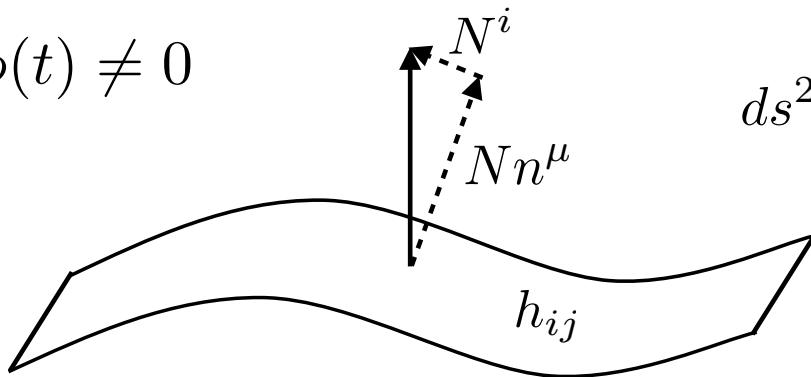
Screening can (probably) be neglected

# EFT of Dark Energy

PC, Luty, Nicolis, Senatore 03  
+ Vernizzi, Piazza + many others

Parametrization of possible deviations from CC

$$\dot{\phi}(t) \neq 0$$



$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S = \int d^4x \sqrt{-g} L[t; N, K_j^i, {}^{(3)}R_j^i, \dots] \quad g^{00} = -N^{-2}$$

Action contains all possible scalars under spatial diffs, ordered by number of perturbations and derivatives

Assume universal metric coupled to SM and DM

# EFT of Dark Energy

Focus on theories with **second order EOM**

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\ \left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \right. \\ \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu , \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2\delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho .$$

# EFT of Dark Energy

Quintessence and Brans-Dicke

Speed of sound  
of DE

DGP and braiding

$$\begin{aligned}
 S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)}R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} + m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].
 \end{aligned}$$

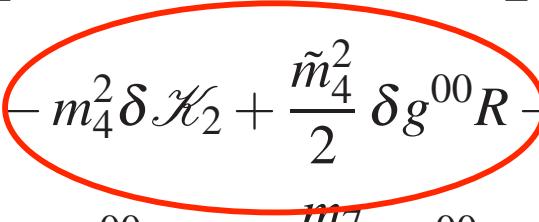
$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu , \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho .$$

# EFT of Dark Energy

Galileon, Horndeski and Beyond Horndeski

$$\begin{aligned}
 S = & \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
 & - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \\
 \text{Non-linear terms,} \\
 \text{screening} & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].
 \end{aligned}$$


$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu , \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho .$$

**Horndeski:** most generic theory with 2<sup>nd</sup> order EOM  $\tilde{m}_4^2 = m_4^2$      $\tilde{m}_6 = m_6$

**Beyond Horndeski:** more than 2 derivatives, but degenerate with no ghost

# EFT of Dark Energy

This term changes the speed of GWs

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$

$$\dot{\gamma}_{ij}^2 \subset \delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu, \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R / 2, \\ \delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2\delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.$$

Experiment also probes propagation in a perturbed universe

# EFT of Dark Energy

see also Sakstein, Jain 17,  
 Ezquiaga, Zumalacarregui 17,  
 Baker et al 17, Copeland et al 18

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right.$$

$$\left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \right.$$

$$\left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$

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$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho.$$

$$\alpha_B \equiv -\frac{m_3^3}{2M_{Pl}^2 H}$$

$$\alpha_H \equiv \frac{2\tilde{m}_4^2}{M_{Pl}^2}$$

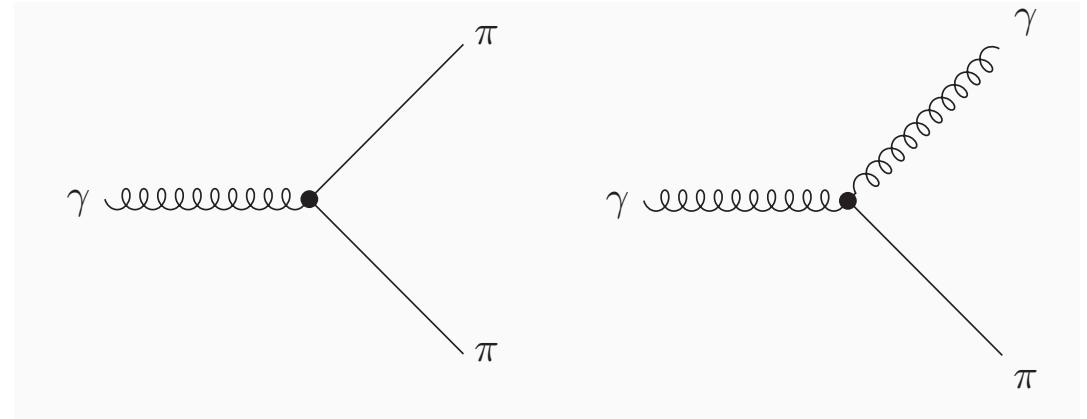
For LSS we are interested  
 in the regime  $\alpha \sim 0.1$

# Graviton decay into dark energy

Light is also absorbed as it travels in a material

The (spontaneous) **breaking of Lorentz invariance allows graviton decay**

$$\frac{\tilde{m}_4^2}{2} \delta g^{00} \left( {}^{(3)}R - \delta \mathcal{K}_2 \right)$$



Solving constraints: 
$$\frac{M_\star^2 \tilde{m}_4^2}{M_\star^2 + 2\tilde{m}_4^2} \int d^4x a \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

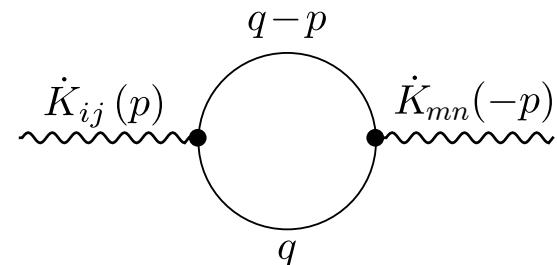
Non-relativistic kinematic: 
$$\Gamma_\gamma = \frac{p^7 (1 - c_s^2)^2}{480 \pi c_s^7 \Lambda^6}$$
       $\Lambda \sim (H_0^2 M_P)^{1/3}$

$$\alpha_H \equiv \frac{2\tilde{m}_4^2}{M_{Pl}^2} \lesssim 10^{-10}$$

irrelevant for LSS observations  
(unless  $c_s=1$  with great precision)

# Higher-derivative terms and dispersion

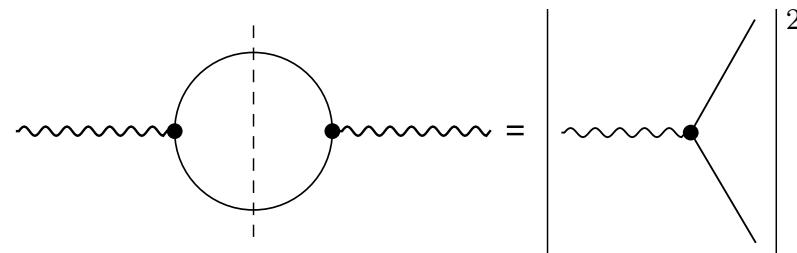
Loops generate higher-derivative corrections



$$S_{\text{eff}} = \frac{M_{\text{Pl}}^2}{480\pi^2\Lambda_*^6 c_s^7} \int \frac{d^4 p}{(2\pi)^4} \bar{p}^4 \dot{K}_{ij}(p) \dot{K}_{ij}(-p) \left[ \frac{1}{\varepsilon} + \frac{23}{15} - \frac{\gamma_E}{2} - \frac{1}{2} \log \left( \frac{\bar{p}^2}{4\pi c_s^2 \mu^2} - i\epsilon \right) \right]$$

$$\omega^2 = k^2 - \frac{k^8 (1 - c_s^2)^2}{480\pi^2 \Lambda_*^6 c_s^7} \log \left( -(1 - c_s^2) \frac{k^2}{\mu_0^2} - i\epsilon \right)$$

It applies to  $c_s > 1$  as well. For  $c_s = 1$  one still has power divergences



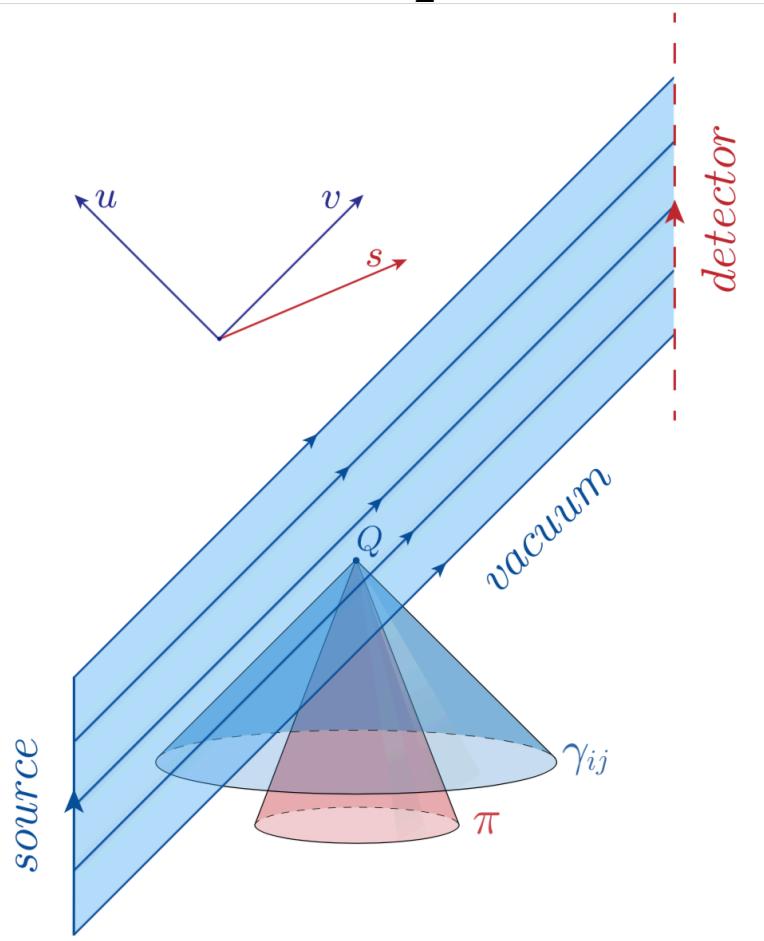
**It is inconsistent to look at Horndeski only**

# Coherent decay

The decay of  $\gamma$  enhanced by the **large occupation number** of GW  
 $\sim$  preheating

$$\mathcal{L}_{m_3} = -\frac{1}{2}\sqrt{-g} m_3^3 \delta K \delta g^{00}$$

$$\ddot{\pi} - c_s^2 \nabla^2 \pi + \frac{2}{\Lambda^2} \dot{\gamma}^{ij} \partial_i \partial_j \pi = 0$$



$$\ddot{\pi} - c_s^2 \nabla^2 \pi + c_s^2 \beta \cos[\omega(t-z)] (\partial_x^2 - \partial_y^2) \pi = 0$$

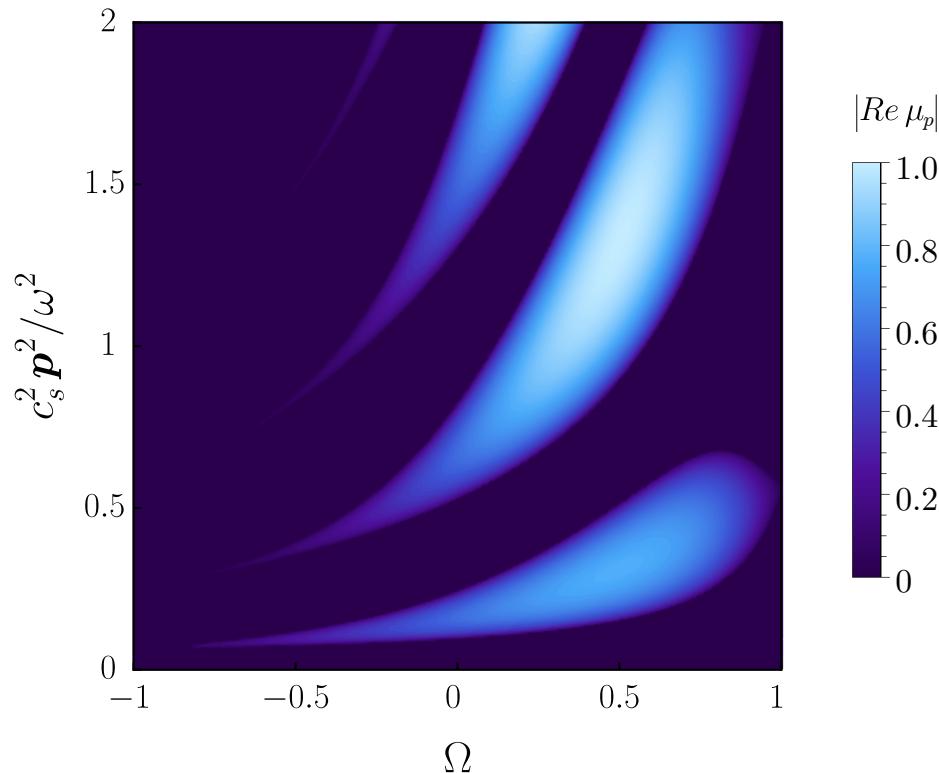
$$\beta \equiv \frac{2\omega M_{\text{Pl}} h_0^+}{c_s^2 |\Lambda^2|}$$

$$u = t - z, \quad s = -t + c_s^{-2} z, \quad v = t + z$$

Similar to a Mathieu equation:  
**parametric resonance**

# Narrow resonance ( $\beta \ll 1$ )

$$\frac{d^2}{d\tau^2}\varphi + [A - 2q \cos(2\tau)]\varphi = 0 \quad \tau \equiv \frac{\omega u}{2}$$



$$A = 4 \frac{c_s^2 \mathbf{p}^2}{\omega^2} \frac{(1 - c_s \Omega)^2}{(1 - c_s^2)^2} = \frac{4 p_u^2}{\omega^2} ,$$

$$q = 2\beta \frac{c_s^2 \mathbf{p}^2}{\omega^2} \frac{(1 - \Omega^2) \cos(2\phi)}{1 - c_s^2} .$$

**Backreaction on  $\gamma$**

$$\ddot{\gamma}_{ij} - \nabla^2 \gamma_{ij} + \frac{2}{\Lambda^2} \Lambda_{ij,kl} \partial_t (\partial_k \pi \partial_l \pi) = 0$$

# Saddle point

$$J(\tau) = \int g(X) e^{\tau f(X)} d^3X \quad J(\tau) \approx \frac{1}{\sqrt{\det(-\partial_i \partial_j f(X_0))}} \left( \frac{2\pi}{\tau} \right)^{3/2} g(X_0) e^{\tau f(X_0)}$$

$$\Delta \gamma_{ij}(u, v) \simeq -\frac{v}{4\Lambda^2} \frac{(1 - c_s^2)^2}{c_s^5 \sqrt{\beta}} \frac{\omega^{5/2}}{(8u\pi)^{3/2}} \sin\left(\omega u + \frac{\beta}{2}\right) \exp\left(\frac{\beta}{4}\omega u\right) \epsilon_{ij}^+$$

- Generalize the perturbative formula
- At higher order in  $\beta$  one gets harmonics.  
They enter in the obs window before: **precursors**

# $\pi$ interactions

**BUT** one expects the exp growth is quenched when  $\pi$  self-interactions are relevant

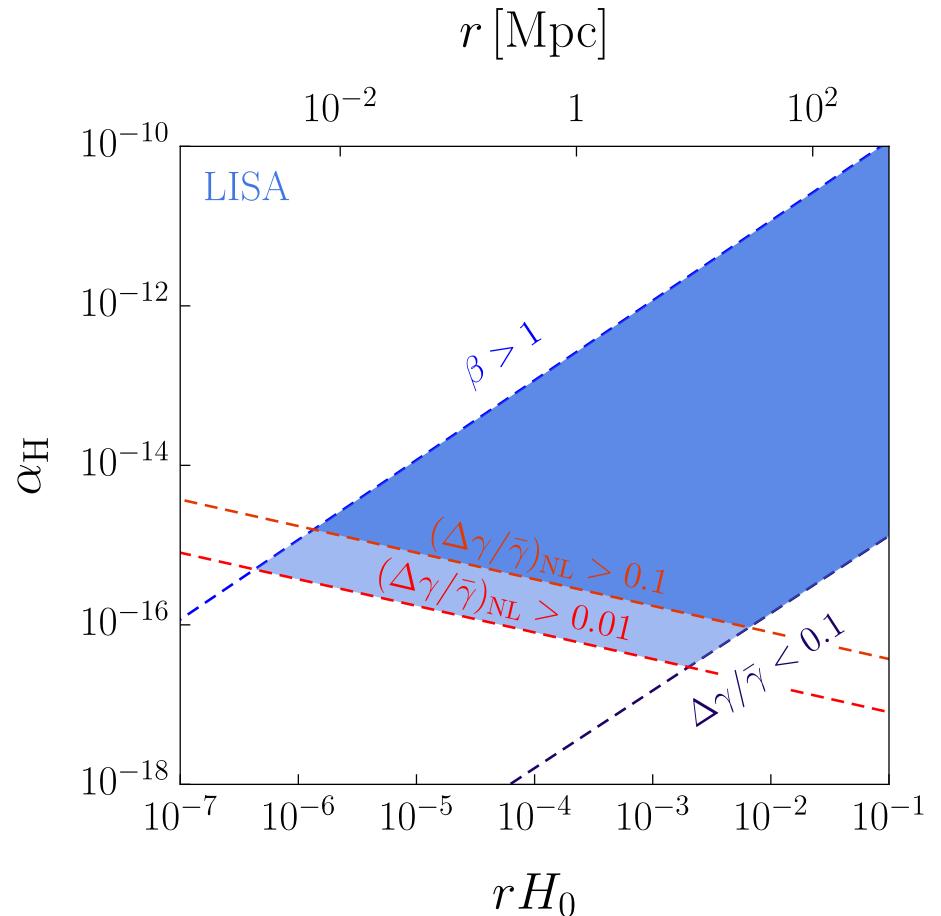
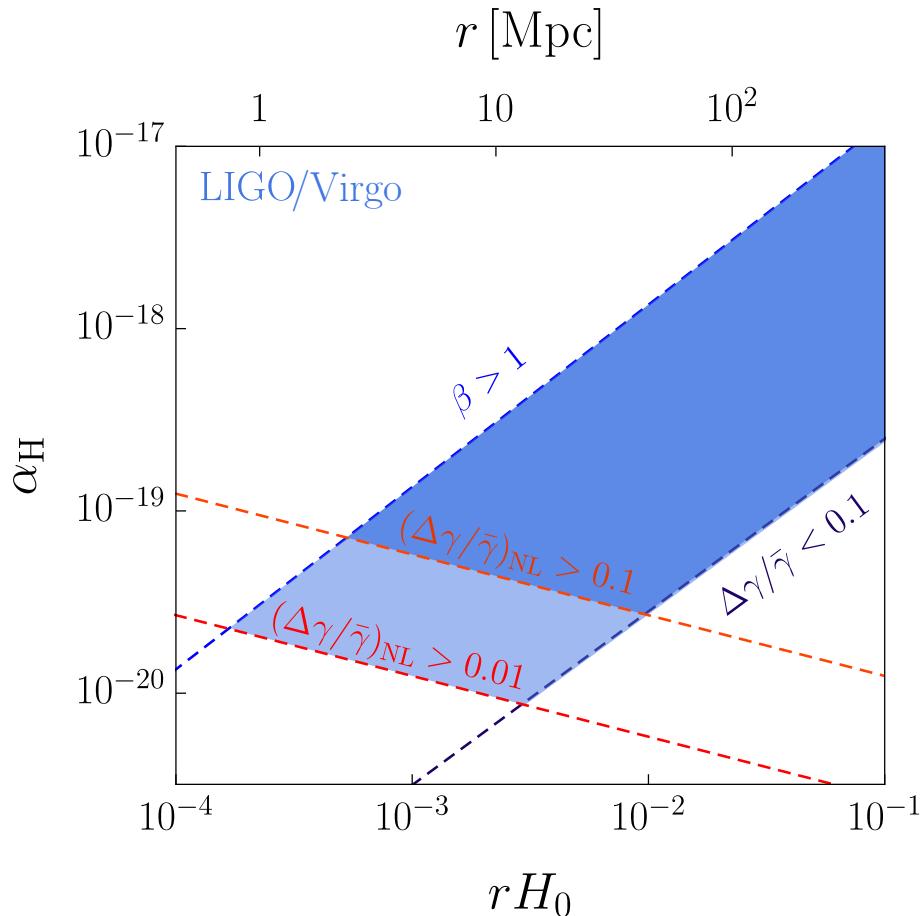
$$\frac{1}{\Lambda_3^3} \nabla^2 \pi (\partial \pi)^2 \quad \text{vs} \quad \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

- Kills the effect!? Simulations  $\sim$  preheating
- For  $\tilde{m}_4^2$  self-interactions are small: same scale  $\Lambda_* \gg \Lambda_3$

On saddle point, accidental cancellation due to Galileon structure

$$\frac{\Delta \gamma}{\bar{\gamma}} \sim \frac{v \omega (\partial_i \pi)^2}{\Lambda_*^3 M_{\text{Pl}} h_0^+} \lesssim \beta c_s^3 \alpha(v H_0) \frac{H_0}{\omega h_0^+} \sqrt{\beta \tau}$$

# GW modification



$f = 30$  Hz,  $M_c = 1.2 M_{\text{sun}}$

$f = 10^{-2}$  Hz,  $M_c = 30 M_{\text{sun}}$

Perturbative bound:  $\alpha_H < 10^{-10}$

# $\beta > 1$ : $\pi$ instability

$$\ddot{\pi} - c_s^2 \nabla^2 \pi + c_s^2 \beta \cos[\omega(t - z)] (\partial_x^2 - \partial_y^2) \pi = 0 \quad \text{Gradient instability}$$

- Generically present for  $m_3$  (but  $\pi$  non-linearities must be considered)
- To be contrasted with non-linear stability of cubic Galileon

Nicolis Rattazzi 04

$$\mathcal{L}_{(2)} = Z^{\mu\nu} \partial_\mu \pi \partial_\nu \pi$$

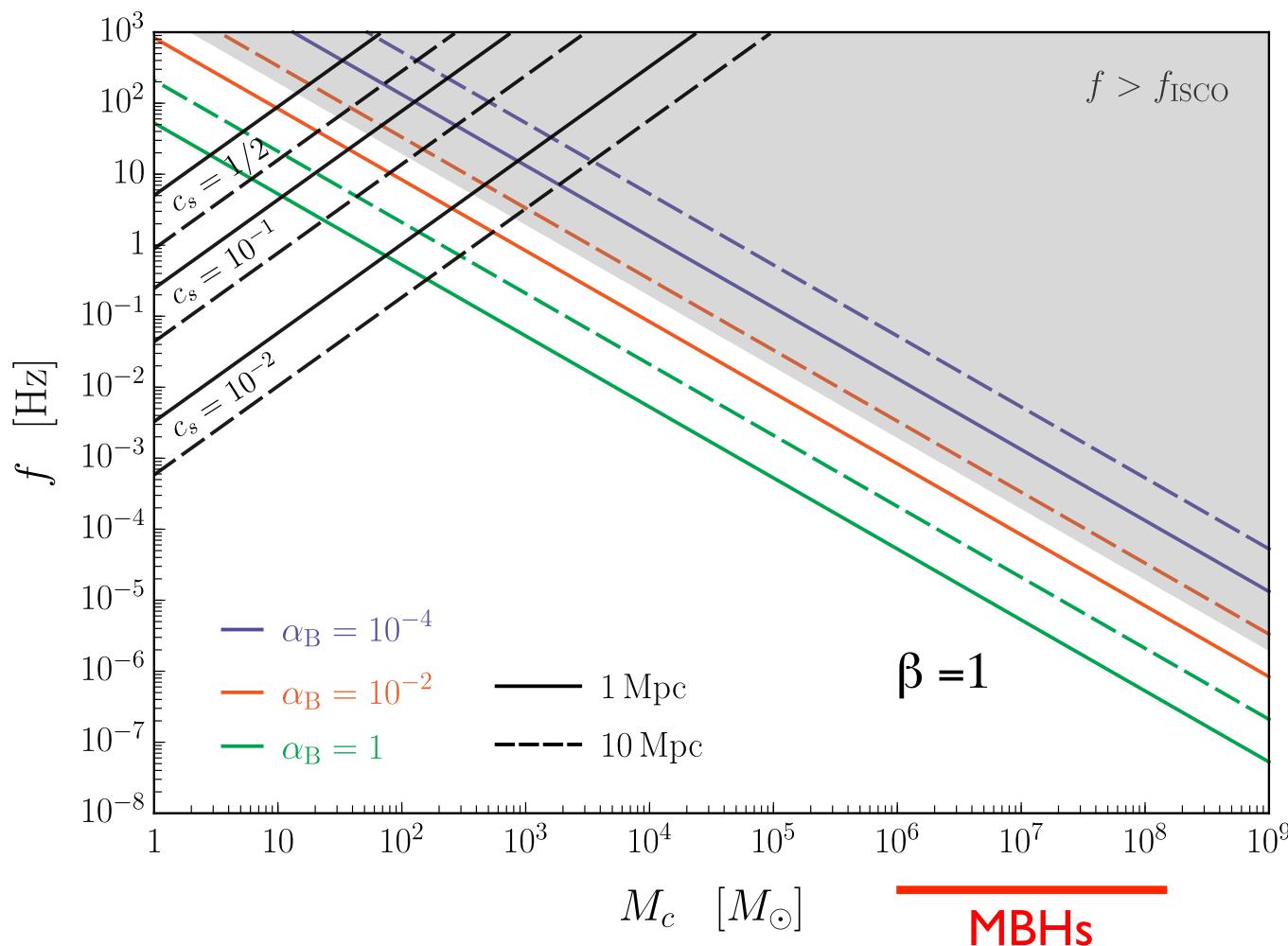
Eom imply eigenvalues of the matrix  $Z^{\mu\nu}$  do not flip sign

- Fate of instability is **UV sensitive**. Does it affect GWs?

But in general the system finds a new vacuum with a different EFT

# Instability triggered by binaries

$$\beta \equiv \frac{2\omega M_{\text{Pl}} h_0^+}{c_s^2 |\Lambda^2|} = \frac{2\sqrt{2}|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+ \sim 1 \quad \text{For typical LIGO-Virgo event}$$



Population of  
MBHs is enough  
to globally trigger  
the instability

Down to  $10^{10}$  km

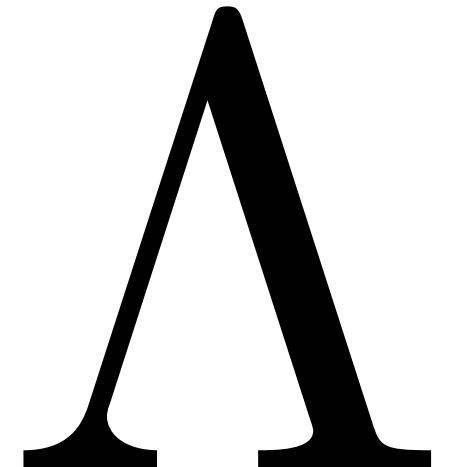
# Conclusions

GWs probe Dark Energy as light probes a material

In many cases better than what LSS can do

- Speed of GWs
- Perturbative graviton decay and dispersion
- Resonant graviton decay
- Instability due to GW

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right.$$
$$\left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \right.$$
$$\left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right].$$



# Backup slides

# Covariant theory

Horndeski 74

Gleyzes, Langlois, Piazza, Vernizzi 14

$$S = \int d^4x \sqrt{-g} \sum_I L_I$$

Horndeski

Beyond Horndeski

$$\begin{aligned} L_2 &\equiv G_2(\phi, X), \quad L_3 \equiv G_3(\phi, X) \square \phi, \\ L_4 &\equiv G_4(\phi, X)^{(4)}R - 2G_{4,X}(\phi, X)(\square \phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}) \\ &+ F_4(\phi, X)\underline{\varepsilon^{\mu\nu\rho}}\varepsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}, \\ L_5 &\equiv \underline{G_5(\phi, X)}^{(4)}G_{\mu\nu}\phi^{\mu\nu} \\ &+ \frac{1}{3}G_{5,X}(\phi, X)(\square \phi^3 - 3\square \phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\mu\sigma}\phi^\nu_\sigma) \\ &+ F_5(\phi, X)\underline{\varepsilon^{\mu\nu\rho\sigma}}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}, \end{aligned}$$

Degeneracy Constraint:  $XG_{5,X}F_4 = 3F_5 [G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}]$

$$c_T = 1 \quad m_4^2 = X^2 F_4 - 3H\dot{\phi}X^2 F_5 - [2XG_{4,X} + XG_{5,\phi} + (H\dot{\phi} - \ddot{\phi})XG_{5,X}] = 0$$



$$G_{5,X} = 0, \quad F_5 = 0, \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0$$

$$\begin{aligned} L_{c_T=1} &= G_2(\phi, X) + G_3(\phi, X) \square \phi + B_4(\phi, X)^{(4)}R \\ &- \frac{4}{X}B_{4,X}(\phi, X)(\phi^\mu\phi^\nu\phi_{\mu\nu}\square\phi - \phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}), \end{aligned}$$

# Radiative stability

Some operators must be set to zero: is this choice stable?

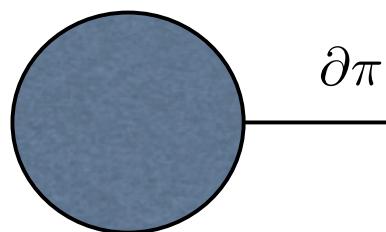
- Approximate Galilean invariance  $\phi \rightarrow \phi + b_\mu x^\mu$

$$\mathcal{L}_3 = (\partial\phi)^2 [\Phi] ,$$

$$\mathcal{L}_4 = (\partial\phi)^2 ([\Phi]^2 - [\Phi^2]) ,$$

$$\mathcal{L}_5 = (\partial\phi)^2 ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

- Non renormalization of Galileons Luty, Porrati, Rattazzi 03

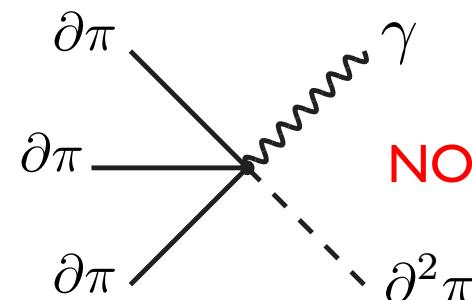


$$\partial^\mu \pi_{\text{ext}} \partial_\mu \pi_{\text{int}} \square_4 \pi_{\text{int}} = \partial^\mu \pi_{\text{ext}} \partial_\nu \left[ \partial_\mu \pi_{\text{int}} \partial_\nu \pi_{\text{int}} - \frac{1}{2} \eta_{\mu\nu} \partial^\rho \pi_{\text{int}} \partial_\rho \pi_{\text{int}} \right]$$

- Broken by gravity

Pirstkhalava, Santoni, Trincherini, Vernizzi 15

The particular coupling giving 2<sup>nd</sup> order EOM  
keeps approximate Galilean invariance



# Radiative stability

$$\Lambda_3 \sim (M_P H_0^2)^{1/3} \sim 1000 \text{ km}$$

$$\mathcal{L}_2^{\text{WBG}} = \Lambda_2^4 G_2(X) , \quad \Lambda_2 \sim (M_P H_0)^{1/2} \sim 0.1 \text{ mm}$$

$$\mathcal{L}_3^{\text{WBG}} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X)[\Phi] ,$$

$$\mathcal{L}_4^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X)R + 2\frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X) ([\Phi]^2 - [\Phi^2]) ,$$

$$\mathcal{L}_5^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^9} G_5(X)G_{\mu\nu}\Phi^{\mu\nu} - \frac{\Lambda_2^4}{3\Lambda_3^9} G_{5X}(X) ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

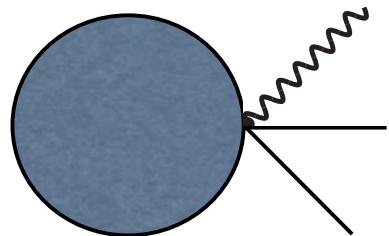
$$\delta c_n \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40} \ll 10^{-15}$$

The tuning is stable

Same holds for Beyond Horndeski theories

# Generic loops

External graviton lines



$$\Lambda_3^4 F \left( \frac{(\partial\phi)^2}{\Lambda_2^4}, \frac{\partial^2\phi}{\Lambda_3^3}, \frac{\gamma^{(c)}}{\Lambda_3}, \frac{\partial}{\Lambda_3} \right)$$

Quartic/quintic Horndeski give large  
higher derivative terms: not exp viable

Cubic case scales in a **different** way

$$H M_{\text{Pl}}^2 \delta g^{00} \delta K \sim H M_{\text{Pl}}^2 \dot{\pi} \partial_i \partial_j \pi \gamma_{ij} \quad \Lambda_2 \equiv (H_0 M_{\text{Pl}})^{1/2}$$

A leg with one derivative must go inside

# Caveat

De Rham, Melville 18

We do not really know whether the EFT of cosmological scales applies to LIGO/Virgo

The theory may break down (new states appear) at energy scales **parametrically** lower than the cut-off ( $\sim 1000$  km)

The speed of GWs may go back to 1 at “short” scale

Analogous to :  $n(\omega) \simeq 1 + \frac{2\pi Ne^2}{m} \frac{f}{\bar{\omega}^2 - \omega^2 - i\bar{\nu}\omega}$

- Naively requires new physics at scales of order  $10^8 \times 1000$  km to satisfy constraints
- How to reconcile **with local tests of gravity?**
- Can we say something general about the UV completion?  $\sim$  Kramers-Kronig?

# Some examples

$$-\frac{m_3^3}{2} \delta K \delta g^{00}$$

Braiding, the scalar mixes with gravity

Deffayet, Pujolas, Sawicki, Vikman 10

$$\delta g^{00} \rightarrow -2(\dot{\pi} - \Phi) , \quad \delta K \rightarrow -(3\dot{\Psi} + a^{-2}\nabla^2\pi)$$

Different from usual Brans-Dicke, e.g.  $\Phi = \Psi$

$$+ \frac{\tilde{m}_4^2}{2} \delta g^{00} R$$

Modifications **inside** matter:  
violation of Vainshtein screening

Kobayashi, Watanabe, Yamauchi 14

$$\frac{d\Phi}{dr} = G_N \left( \frac{\mathcal{M}}{r^2} - \epsilon \frac{d^2\mathcal{M}}{dr^2} \right)$$

D'amico, Huang, Mancarella, Vernizzi 14

$$\mathcal{L} = \frac{1}{2} \left\{ \left( 1 + \frac{c_s^2}{c_m^2} \lambda^2 \right) \dot{\pi}_c^2 - c_s^2 (\nabla \pi_c)^2 + \dot{v}_c^2 - c_m^2 (\nabla v_c)^2 + 2 \frac{c_s}{c_m} \lambda \dot{v}_c \dot{\pi}_c \right\}$$

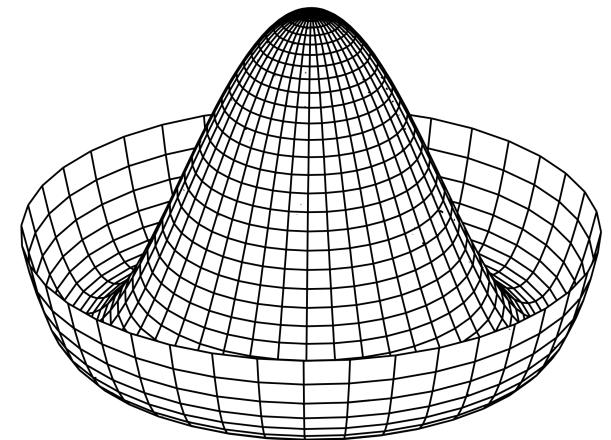
**Kinetic matter  
mixing**

# Instabilities in a UV complete model

Spontaneously broken global U(1)

$$\mathcal{L}_{\text{UV}} = -|\partial h|^2 - V(|h|), \quad V(|h|) = \lambda(|h|^2 - v^2)^2$$

Integrate out radial direction  $P(X) \simeq -\frac{1}{4\lambda} X(\mu^2 - X)$



Gradient instabilities for  $\frac{1}{6}\mu^2 < \hat{X} < \frac{1}{2}\mu^2$

In UV theory, instability is saturated at  $\mu$ : stable for  $k \gg \mu$

Ghost instabilities for  $\hat{X} > \frac{1}{2}\mu^2$

EFT has no applicability: radial mode becomes massless

# Why consider so complicated theories ??

To modify gravity one has to introduce extra dof

Scalars will play with the graviton through  $\partial_\mu \partial_\nu \pi$

- Massive gravity. Longitudinal mode  $g_{\mu\nu} \supset \partial_\mu \partial_\nu \pi$       E.g. De Rham, Gabadadze, Tolley 10
- DGP model



5D Minkowski bulk

Dvali, Gabadadze, Porrati 00

Brane bending mode

$$g_{5\mu} \sim \partial_\mu \pi$$



Actions for scalars with many derivatives  
(but 2<sup>nd</sup> order equations)

# Another caveat

Copeland, Kopp, Padilla,  
Saffin, Skordis 18

We imposed that  $c_T=1$  is robust to changes of  $H(t)$  and  $\phi(t)$ ,  
but there are possible cancellations that we missed

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\ \xrightarrow{\quad} & \left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{K}_2 \right. \\ & \left. - \frac{m_6}{3} \delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{K}_3 \right]. \end{aligned}$$

Red lines indicate terms that are crossed out, while blue arrows indicate terms that are being compared.

The cancellation does not work in the presence of curvature  
and thus in the perturbed universe

# Beyond Beyond Horndeski: DHOST

Even more general theories propagating a single dof

A combination of:  $\int d^4x \sqrt{-g} \frac{M^2}{2} \left( -\frac{2}{3}\alpha_L \delta K^2 + 4\beta_1 \delta KV + \beta_2 V^2 + \beta_3 a_i a^i \right)$

These do not affect GWs on any background

Can be obtained by:  $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$

$$\begin{aligned} L_{c_T=1} = & \tilde{B}_2 + \tilde{B}_3 \square \phi + C B_4 {}^{(4)}R - \frac{4CB_{4,X}}{X} \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi \\ & + \left( \frac{4CB_{4,X}}{X} + \frac{6B_4 C_{,X}{}^2}{C} + 8C_{,X} B_{4,X} \right) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ & - \frac{8C_{,X} B_{4,X}}{X} (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2. \end{aligned}$$