UNIVERSITY OF OXFORD

Non-Fierz-Pauli Massive Gravities

Giulio Gambuti

Theoretical Physics Seminar - Università degli Studi di Genova - 9nd Dec 2020

Outline

- Gravity as a Field Theory
- The Fierz-Pauli theory
- What is wrong with Fierz-Pauli?
- How to fix it

The interpretations of Gravity

Geometry or Particle?

General Relativity	Quantum Field Theory
$R_{\mu\nu\rho\sigma}$	$h_{\mu\nu}$

Equivalent

The Graviton

~~Spin 0~~ ← Interaction with photons

~~Spin 1~~ ← Repulsive classical force

✓ Spin 2 → Attractive classical force

→ Interacts with EMT $T^{\mu\nu}$

Linearized Gravity

The Linearized Einstein-Hilbert Action

$$S_{EH}[g] = \int \mathrm{d}^4x \, \sqrt{-g} \, R$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$O(h^2)$$

$$S_{LG}[h] = \int \mathrm{d}^4\mathrm{x} \, \left[\frac{1}{2} h \partial^2 h \, - \, h_{\mu\nu} \partial^\mu \partial^\nu h \, - \, \frac{1}{2} h^{\mu\nu} \partial^2 h_{\mu\nu} \, + \, h^{\mu\nu} \partial_\nu \partial^\rho h_{\mu\rho} \right]$$

The Linearized Einstein-Hilbert Action

$$S_{LG}[h] = \int \mathrm{d}^4\mathrm{x} \ \left[\frac{1}{2} h \partial^2 h \ - \ h_{\mu\nu}\partial^\mu\partial^\nu h \ - \ \frac{1}{2} h^{\mu\nu}\partial^2 h_{\mu\nu} \ + \ h^{\mu\nu}\partial_\nu\partial^\rho h_{\mu\rho} \right]$$

- Gauge symmetry

$$\delta h_{\mu\nu}(x) = \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)$$

$$\delta A_\mu = \partial_\mu \theta$$

- Unique theory with this symmetry

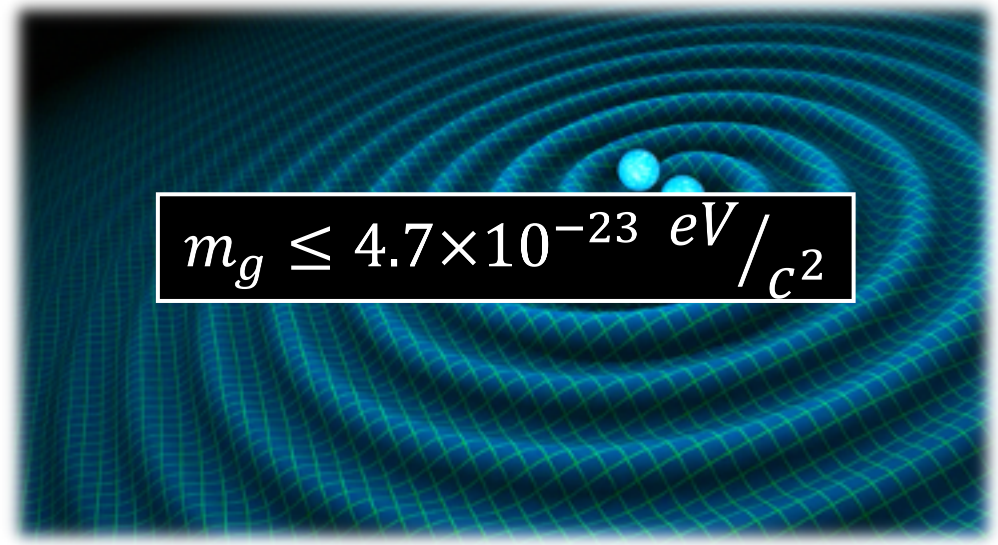
Uniqueness of GR $\neq$ **Equivalence Spin-2 and Covariance**

Massive Gravity

MG: Why?

Cons:

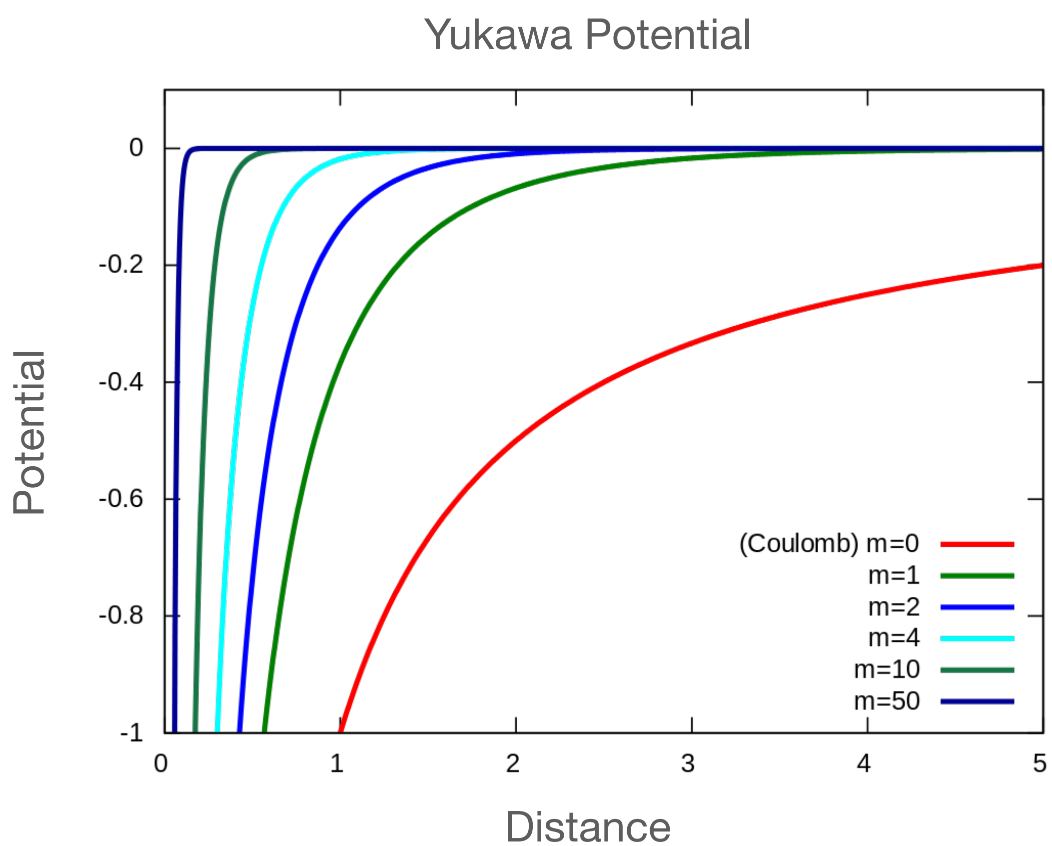
- Strong constraints on the mass



LIGO Scientific and Virgo Collaborations, Phys.Rev.D (2019)

Pros:

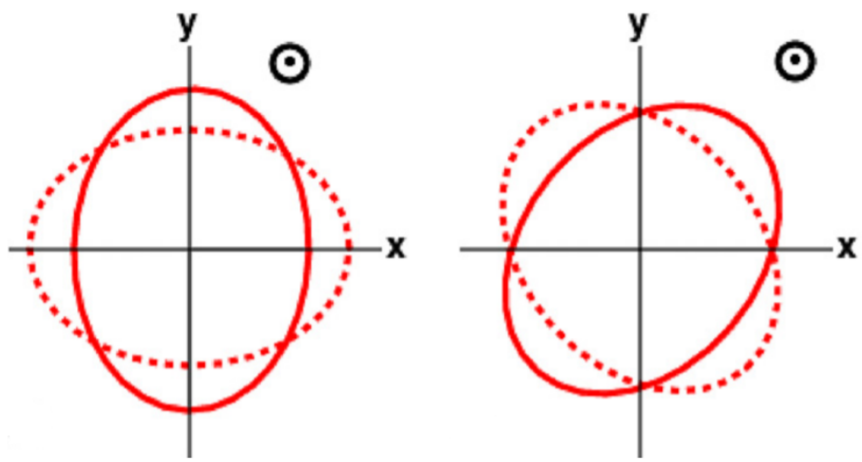
- Interesting modification
- Universe without Dark Energy



MG: How?

Massless graviton

2 degrees of freedom



General Relativity

Massive graviton

5 degrees of freedom

2S +1

Massive Gravity

$0 \leftarrow m_g$

Fierz-Pauli theory of MG

Fierz-Pauli Theory

- Define the Action
- Find the EOMs
- Find the Propagator

The Fierz-Pauli Action

$$S = S_{LG} + S_m$$

$$S_m[h; m_1^2, m_2^2] = \frac{1}{2}\int \mathrm{d}^4x \; (m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2)$$

Fierz-Pauli Tuning

$m_1^2 + m_2^2 = 0$ $\Longrightarrow$ 5 DOFs

D. Boulware and S. Deser, Phys. Rev. D 6 (1972)

The Equations of Motion

$$\frac{\delta S_{FP}}{\delta h^{\mu\nu}} = \partial^2 h_{\mu\nu} - \partial_\alpha \partial_\mu h^\alpha{}_\nu - \partial_\alpha \partial_\nu h^\alpha{}_\mu + \eta_{\mu\nu} \partial_\alpha \partial_\beta h^{\alpha\beta} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \partial^2 h - m_1^2 (h_{\mu\nu} - \eta_{\mu\nu} h) = 0$$

$$\downarrow$$

$$\boxed{h = 0 \qquad \partial^\mu h_{\mu\nu} = 0}$$

10 components of h
+
5 constraints
$\Downarrow$
5 DOFs ✓

The Equations of Motion

$$h = 0 \qquad\qquad \partial^\mu h_{\mu\nu} = 0$$

$$\frac{\delta S_{FP}}{\delta h^{\mu\nu}} = \partial^2 h_{\mu\nu} - \cancel{\partial_\alpha \partial_\mu h^\alpha{}_\nu} - \cancel{\partial_\alpha \partial_\nu h^\alpha{}_\mu} + \cancel{\eta_{\mu\nu}\partial_\alpha \partial_\beta h^{\alpha\beta}} + \cancel{\partial_\mu \partial_\nu h} - \cancel{\eta_{\mu\nu}\partial^2 h} - m_1^2(h_{\mu\nu} - \cancel{\eta_{\mu\nu} h}) = 0$$

$$(\partial^2 - m_1^2) h_{\mu\nu}(x) = 0$$ Wave Equation

$$\partial^\mu h_{\mu\nu}(x) = 0$$ Transversality

$$h(x) = 0\ .$$ Tracelessness

The Fierz-Pauli Propagator

$$G^{FP}_{\mu\nu,\alpha\beta}(p) = \frac{2}{p^2 + m_1^2}\left[\frac{1}{2}(P_{\mu\alpha}P_{\nu\beta} + P_{\nu\alpha}P_{\mu\beta}) - \frac{1}{3}P_{\mu\nu}P_{\alpha\beta}\right]$$

$$P_{\mu\nu} = \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m_1^2}$$

What is wrong with FP?

The Fierz-Pauli Propagator

$$G^{FP}_{\mu\nu,\alpha\beta}(p) = \frac{2}{p^2 + m_1^2}\left[\frac{1}{2}(P_{\mu\alpha}P_{\nu\beta} + P_{\nu\alpha}P_{\mu\beta}) - \frac{1}{3}P_{\mu\nu}P_{\alpha\beta}\right]$$

$$P_{\mu\nu} = \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m_1^2}$$

The Interactions

vDVZ discontinuity

$$S_{int} = \lambda \int \mathrm{d}^4 x \; h_{\mu\nu} T^{\mu\nu}$$

$$U(\mathbf{r}) = -\lambda^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \, \tilde{T}_1^{\mu\nu} \, G_{\mu\nu,\alpha\beta}(\mathbf{k}) \, \tilde{T}_2^{\alpha\beta} \, e^{i\mathbf{k}\cdot\mathbf{r}}$$

Case 1:

$$T_1^{\mu\nu} = M_1 \delta(\mathbf{r} - \mathbf{r}_1) \, \mathrm{diag}(-1,0,0,0)^{\mu\nu}$$

= massive body

$$T_2^{\alpha\beta} = M_2 \delta(\mathbf{r} - \mathbf{r}_2) \, \mathrm{diag}(-1,0,0,0)^{\alpha\beta}$$

= massive body

Case 2:

$$T_1^{\mu\nu} = M_1 \delta(\mathbf{r} - \mathbf{r}_1) \, \mathrm{diag}(-1,0,0,0)^{\mu\nu}$$

= massive body

$$T_2^{\alpha\beta} = -E \, \mathrm{diag}\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^{\alpha\beta}$$

= radiation (light)

The Interactions

vDVZ discontinuity

$$S_{int} = \lambda \int \mathrm{d}^4x \, h_{\mu\nu} T^{\mu\nu}$$

Case 1: mass-mass

$$m_1 \to 0 \qquad U(r) = -\frac{4}{3}\frac{\lambda^2}{4\pi}\frac{M_1 M_2}{r}$$

Case 2: mass-light

$$m_1 \to 0 \qquad U(r) = -2\frac{\lambda^2}{4\pi}\frac{M_1 M_2}{r}$$

What is the origin of these flaws?

$$S_{LG}[h] = \int \mathrm{d}^4\mathrm{x} \ \left[\frac{1}{2} h \partial^2 h \ - \ h_{\mu\nu} \partial^\mu \partial^\nu h \ - \ \frac{1}{2} h^{\mu\nu} \partial^2 h_{\mu\nu} \ + \ h^{\mu\nu} \partial_\nu \partial^\rho h_{\mu\rho} \right]$$

Gauge Invariance $\qquad \delta h_{\mu\nu}(x) = \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x)$

Mass term used as GF! $\qquad S = S_{LG} + S_m$

What is the origin of these flaws?

Mass as gauge-fixing

Propagator

$$\begin{tikzcd} S = S_{LG} + S_m \arrow[rr] \arrow[d, "m_1 \to 0"] & & G^{FP}_{\mu\nu,\alpha\beta} \arrow[d] \\ S_{LG} \arrow[rr, "\times" description] & & \text{Divergence \& Discontinuity} \end{tikzcd}$$

Fierz Pauli theory of MG*

*the real one

The original theory

$$L = \kappa^2 A_{ik} A_{ik} + \frac{\partial A_{ik}}{\partial x_l}\frac{\partial A_{ik}}{\partial x_l} + a_1 \frac{\partial A_{rk}}{\partial x_r}\frac{\partial A_{sk}}{\partial x_s} + a_2 \kappa^2 C^2 + a_3 \frac{\partial C}{\partial x_l}\frac{\partial C}{\partial x_l} + \frac{\partial A_{rk}}{\partial x_r}\frac{\partial C}{\partial x_k}.$$

trace condition $A_{ii} = 0$

C has the role of a Lagrange multiplier!

The modern theory

$$S = S_{LG} + S_m$$

No gauge fixing

Fierz-Pauli Tuning

$$m_1^2 + m_2^2 = 0$$

The Gauge Fixed Theory

Back to Massless

Gauge invariant action:

$$S_{LG}[h] = \int \mathrm{d}^4\mathrm{x} \ \left[\frac{1}{2} h\partial^2 h \ - \ h_{\mu\nu}\partial^\mu\partial^\nu h \ - \ \frac{1}{2} h^{\mu\nu}\partial^2 h_{\mu\nu} \ + \ h^{\mu\nu}\partial_\nu\partial^\rho h_{\mu\rho} \right]$$

Usual procedure in QFT: add gauge fixing

$$S[h; k, \kappa] = S_{inv}[h] + S_{gf}[h; k, \kappa]$$

Comparison with Maxwell

$$S_{LG}[h]$$

$$S_{Maxwell} = -\int \mathrm{d}^4x \, \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = \partial_\mu \theta$$

$$S_{gf} = -\frac{1}{2k} \int \mathrm{d}^4\mathrm{x} \, \left[\partial_\mu h^{\mu\nu} + \kappa \partial^\nu h\right]^2$$

$$S_{gf} = -\int \mathrm{d}^4x \, \frac{(\partial_\mu A^\mu)^2}{2k}$$

Breaking of symmetry: mass

Massless (well defined) action:

$$S[h;k,\kappa] = S_{inv}[h] + S_{gf}[h;k,\kappa]$$

Linearized massive gravity action:

$$S_{MG} = S_{inv}[h] + S_{gf}[h;k,\kappa] + S_m[h;m_1^2,m_2^2]$$

$$S_m[h;m_1^2,m_2^2] = \int \mathrm{d}^4x \left[\frac{1}{2}m_1^2 h_{\mu\nu}h^{\mu\nu} + \frac{1}{2}m_2^2 h^2\right]$$

NO TUNING!

The Equations of Motion

$$S_{MG} = S_{inv}[h] + S_{gf}[h; k, \kappa] + S_m[h; m_1^2, m_2^2]$$

$$\frac{\delta S}{\delta \tilde{h}_{\mu\nu}} = -\left(1 + \frac{\kappa^2}{k}\right) \eta^{\mu\nu} p^2 \tilde{h} \; + \; \left(1 - \frac{\kappa}{k}\right) p^\mu p^\nu \tilde{h} \; + \; \left(1 - \frac{\kappa}{k}\right) \eta^{\mu\nu} p^\alpha p^\beta \tilde{h}_{\alpha\beta}$$

$$+ p^2 \tilde{h}^{\mu\nu} - \left(1 + \frac{1}{2k}\right) \left(p^\mu p^\alpha \tilde{h}^\nu_\alpha + p^\nu p^\alpha \tilde{h}^\mu_\alpha\right) \; + \; m_1^2 \, \tilde{h}^{\mu\nu} + m_2^2 \, \eta^{\mu\nu} \tilde{h} = 0$$

$$(p^2 + m_1^2) \tilde{h}_{\mu\nu} = 0$$

$$\tilde{h} = 0$$

$$p^\mu \tilde{h}_{\mu\nu} = 0$$

5 DOFs

The massive photon propagator

With gauge fixing

$$S = S_{Maxwell}[A^\mu] + S_{gf}[A^\mu, k] + S_m[A^\mu, m]$$

$$G_{\mu\nu}(p) = \frac{2}{p^2 + m^2}\left[g_{\mu\nu} - \frac{1-k}{p^2 + k m^2} p^\mu p^\nu\right]$$

$$j_1^\mu G_{\mu\nu} j_2^\nu$$

$$p_\mu j^\mu = 0$$

$$k = 0$$

The massive graviton propagator

$$X_{\mu\nu,\alpha\beta} \equiv (A, B, C, D, E)_{\mu\nu,\alpha\beta}$$

$$\boxed{G_{\mu\nu,\alpha\beta} = \hat{t} A_{\mu\nu,\alpha\beta} + \hat{u} B_{\mu\nu,\alpha\beta} + \hat{v} C_{\mu\nu,\alpha\beta} + \hat{z} D_{\mu\nu,\alpha\beta} + \hat{w} E_{\mu\nu,\alpha\beta}}$$

The massive graviton propagator

$$\boxed{G_{\mu\nu,\alpha\beta} = \ \hat{t}A_{\mu\nu,\alpha\beta} + \hat{u}B_{\mu\nu,\alpha\beta} + \hat{v}C_{\mu\nu,\alpha\beta} + \hat{z}D_{\mu\nu,\alpha\beta} + \hat{w}E_{\mu\nu,\alpha\beta}}$$

$$\hat{t} = \frac{2(1+\kappa)(1+4\kappa)p^2 - 2k(m_1^2+4m_2^2)}{\mathrm{DN}(m_1,m_2,k,\kappa,p^2)}$$

$$\hat{u} = \frac{2\big[\kappa(1+4\kappa)+2k\big]p^2 - 2k(m_1^2+4m_2^2)}{\mathrm{DN}(m_1,m_2,k,\kappa,p^2)}$$

$$\hat{v} = \frac{2}{p^2+m_1^2}$$

$$\hat{z} = \frac{-4k}{p^2-2km_1^2}$$

$$\hat{w} = \frac{8km_2^2 - 8\kappa(1+\kappa)p^2}{\mathrm{DN}(m_1,m_2,k,\kappa,p^2)} \ ,$$

The massive graviton propagator

Hint of gauge independence

$$\tilde{T}^{(1)}_{\mu\nu}\left(\hat{t}\cancel{A}^{\mu\nu,\alpha\beta}+\hat{u}\cancel{B}^{\mu\nu,\alpha\beta}+\hat{v}C^{\mu\nu,\alpha\beta}+\hat{z}\cancel{D}^{\mu\nu,\alpha\beta}+\hat{w}\cancel{E}^{\mu\nu,\alpha\beta}\right)\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}\,G^{\mu\nu,\alpha\beta}\,\tilde{T}^{(2)}_{\alpha\beta}$$

The massive graviton propagator

Hint of gauge independence

$$\tilde{T}^{(1)}_{\mu\nu} G^{\mu\nu,\alpha\beta} \tilde{T}^{(2)}_{\alpha\beta} = \underline{\frac{2}{p^2 + m_1^2}} \tilde{T}^{(1)}_{\mu\nu} \left(\frac{1}{2} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} \eta^{\mu\beta} \eta^{\nu\alpha} \right) \tilde{T}^{(2)}_{\alpha\beta}$$

- **Correct pole**
- **Gauge independent**

The massless limit

$$\boxed{G_{\mu\nu,\alpha\beta} = \hat{t}A_{\mu\nu,\alpha\beta} + \hat{u}B_{\mu\nu,\alpha\beta} + \hat{v}C_{\mu\nu,\alpha\beta} + \hat{z}D_{\mu\nu,\alpha\beta} + \hat{w}E_{\mu\nu,\alpha\beta}}$$

$$m_1 \to 0$$

$$m_2 \to 0$$

$$G^{\mu\nu,\alpha\beta} = \frac{2}{p^2}\left(\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

Linearized gravity propagator!

NO vDVZ ✓

Conclusions

Conclusions

$$S_{MG} = S_{inv}[h] + \cancel{S_{gf}}[h; k, \kappa] + \cancel{S_m}[h; m_1^2, m_2^2]$$

$$(\partial^2 - m_1^2) h_{\mu\nu}(x) = 0$$

$$\partial^\mu h_{\mu\nu}(x) = 0$$

$$h(x) = 0$$

Still missing:

- A gauge independent mass-mass interaction
- BRS formulation of the theory
- Quantum extension

Thanks for your attention

The basis tensors

$$A_{\mu\nu,\alpha\beta} = \frac{d_{\mu\nu} d_{\alpha\beta}}{3}$$

$$B_{\mu\nu,\alpha\beta} = e_{\mu\nu} e_{\alpha\beta}$$

$$C_{\mu\nu,\alpha\beta} = \frac{1}{2}\left(d_{\mu\alpha} d_{\nu\beta} + d_{\mu\beta} d_{\nu\alpha} - \frac{2}{3} d_{\mu\nu} d_{\alpha\beta}\right)$$

$$D_{\mu\nu,\alpha\beta} = \frac{1}{2}\left(d_{\mu\alpha} e_{\nu\beta} + d_{\mu\beta} e_{\nu\alpha} + e_{\mu\alpha} d_{\nu\beta} + e_{\mu\beta} d_{\nu\alpha}\right)$$

$$E_{\mu\nu,\alpha\beta} = \frac{\eta_{\mu\nu} \eta_{\alpha\beta}}{4} \ ,$$

The massive propagator

$$S_{MG} = S_{inv}[h] + S_{gf}[h; k, \kappa] + S_m[h; m_1^2, m_2^2]$$

$$S_{MG} = \int d^4p \; \tilde{h}_{\mu\nu} \, \Omega_{MG}^{\mu\nu,\alpha\beta} \, \tilde{h}_{\alpha\beta}$$

$${\Omega_{MG}}_{\mu\nu}{}^{\alpha\beta} G_{\alpha\beta,\rho\sigma} = \mathcal{I}_{\mu\nu,\rho\sigma}$$

The massive graviton propagator

$$\tilde{T}^{(1)}_{\mu\nu}\left(\hat{t}A^{\mu\nu,\alpha\beta} + \cancel{\hat{u}B^{\mu\nu,\alpha\beta}} + \hat{v}C^{\mu\nu,\alpha\beta} + \cancel{\hat{z}D^{\mu\nu,\alpha\beta}} + \hat{w}E^{\mu\nu,\alpha\beta}\right)\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}A^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta} = \frac{1}{3}\tilde{T}^{(1)}_{\mu\nu}\eta^{\mu\nu}\eta^{\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}B^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta} = 0$$

$$\tilde{T}^{(1)}_{\mu\nu}C^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta} = \frac{1}{2}\tilde{T}^{(1)}_{\mu\nu}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{3}\eta^{\mu\nu}\eta^{\alpha\beta})\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}D^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta} = 0$$

$$\tilde{T}^{(1)}_{\mu\nu}E^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta} = \frac{1}{4}\tilde{T}^{(1)}_{\mu\nu}\eta^{\mu\nu}\eta^{\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}\ .$$

The massive graviton propagator

Hint of gauge independence

$$\tilde{T}^{(1)}_{\mu\nu}\left(\cancel{\hat{t}A^{\mu\nu,\alpha\beta}}+\cancel{\hat{u}B^{\mu\nu,\alpha\beta}}+\hat{v}C^{\mu\nu,\alpha\beta}+\cancel{\hat{z}D^{\mu\nu,\alpha\beta}}+\cancel{\hat{w}E^{\mu\nu,\alpha\beta}}\right)\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}A^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}=\frac{1}{3}\tilde{T}^{(1)}_{\mu\nu}\cancel{\eta^{\mu\nu}\eta^{\alpha\beta}}\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}B^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}=0$$

$$\tilde{T}^{(1)}_{\mu\nu}C^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}=\frac{1}{2}\tilde{T}^{(1)}_{\mu\nu}(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\frac{2}{3}\eta^{\mu\nu}\eta^{\alpha\beta})\tilde{T}^{(2)}_{\alpha\beta}$$

$$\tilde{T}^{(1)}_{\mu\nu}D^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}=0$$

$$\tilde{T}^{(1)}_{\mu\nu}E^{\mu\nu,\alpha\beta}\tilde{T}^{(2)}_{\alpha\beta}=\frac{1}{4}\tilde{T}^{(1)}_{\mu\nu}\cancel{\eta^{\mu\nu}\eta^{\alpha\beta}}\tilde{T}^{(2)}_{\alpha\beta}\ .$$