# Hydrodynamic electron fluid

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Based on works with
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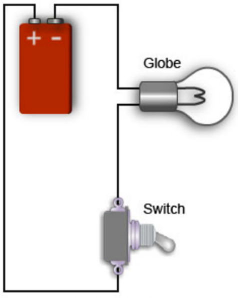
# Hydrodynamic electron fluid

Can this eventually be true?

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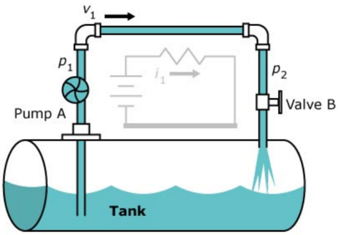
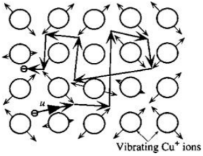


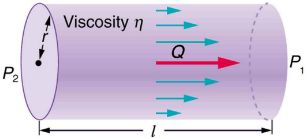
# Electric conduction versus water flow

**Metal**



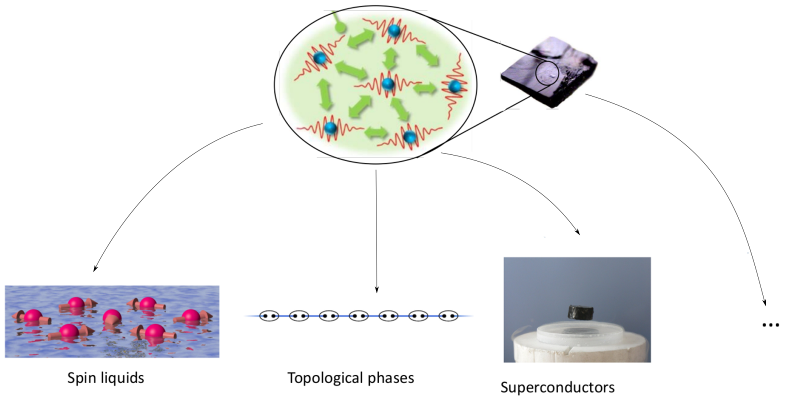

- Resistance arises through external scattering due to the lattice (impurities, phonons,...)

**Water**




- Resistance arises through internal scattering (viscosity)

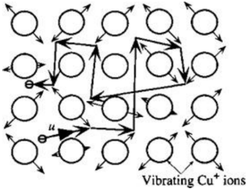
# “More is different” [Anderson ‘72]

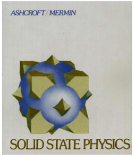


# Conventional metallic transport [Drude 1900]

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

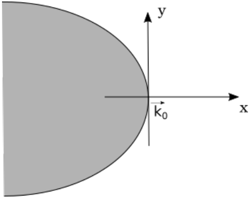
Mean free time of electrons, set by defects and vibrations of the lattice




Electron-electron scattering[5] plays a relatively minor role in the theory of conduction in solids, for reasons to be described in Chapter 17.

# Landau's Fermi liquid theory [Landau '56]

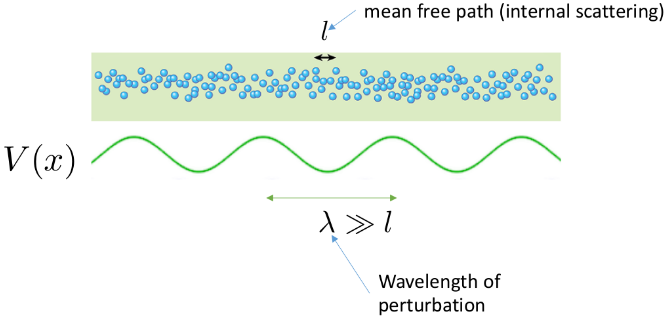



- Electrons in solids organize themselves into a ground state characterized by a Fermi surface
- Excitations around the Fermi surface are weakly interacting collective modes called *quasi-particles*
- Quasi-particles are long-lived and determine the transport properties in normal metals

There are systems in which this is not true.... Hydrodynamics might help....

# Hydrodynamics

- Hydridynamics is an EFT in which the fundamental DOFs are conserved quantities: momentum, energy, charge,...
- Works at length/time scales much larger than the microscopic ones




- At strong coupling with no well defined quasi-particles is the only suitable EFT

# Hydrodynamics as an EFT

- At large length and time scales, only a small number of DOFs survive to become hydrodynamic modes
  - If no spontaneously broken symmetries: (almost)-conserved currents.
- EOMS are determined by symmetries. Eg in a the relativistic charged fluid there are two conserved currents:

$$\partial_\mu J^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0$$

- Local thermal equilibrium: everything is function of $\mu(x)$, $T(x)$ and $u^\mu(x) \Rightarrow$ gradients expansion:

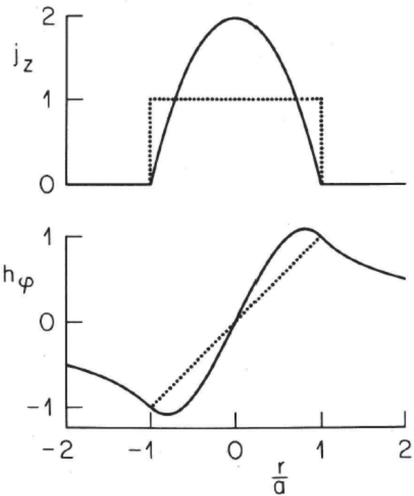
$$J^\mu = n u^\mu + \mathcal{O}(\partial), \qquad T^{\mu\nu} = (n+p)u^\mu u^\nu - p g^{\mu\nu} + \mathcal{O}(\partial)$$

Eventually one solves the EOMs order by order to find the relevant observables

# The prehistory of electron hydrodynamics

- **Hydrodynamic effects in solids at low temperature**, R. N. Gurzhi, Usp. Fiz. Nauk. 94, 689 [Sov Phys Usp (1968)]
- **Two-fluid hydrodynamic description of ordered systems**, C. P. Enz, RMP (1974)
- **Electronfluid model for dc size effect**, R. Jaggi, J. Appl. Phys. (1991), Helvetica Physica Acta (1989); ibid (1980)
- **Hydrodynamic electron flow in high-mobility wires**, M. J. M. de Jong and L. W. Molenkamp, PRB (1995)

# Electronfluid model for dc size effect [Jaggi]




Dotted: classical (fluid) behaviour
Solid: viscous flow

$$\mathbf{J} = Ne\mathbf{v} \tag{2}$$

and the Navier-Stokes equation of motion is expressed in terms of drift velocity **v**, electron concentration *N*, electron mass *m*, and electron charge *e*,

$$Nm\frac{D\mathbf{v}}{Dt} + \mathbf{F} + \nabla p = Ne(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - N\frac{m}{\tau}\mathbf{v}. \tag{3}$$

The total or substantial acceleration $D\mathbf{v}/Dt$ is the sum of local acceleration and convection:

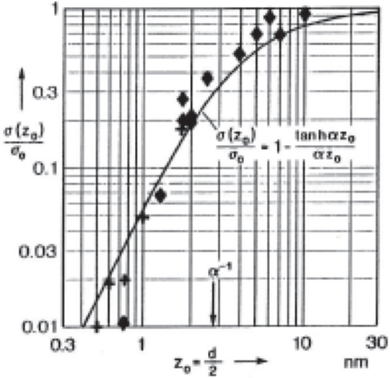
$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}. \tag{4}$$

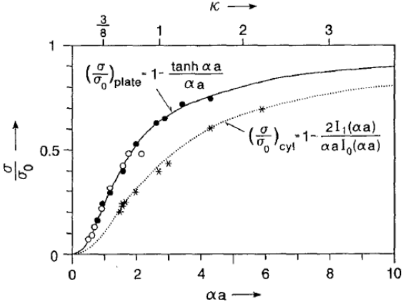
The second term in Eq. (3) represents internal friction:

$$\mathbf{F} = -\eta[\Delta\mathbf{v} + \tfrac{1}{3}\nabla(\nabla\mathbf{v})], \tag{5}$$

characterized by a shear viscosity coefficient $\eta$. The pressure

# Electronfluid model for dc size effect [Jaggi]



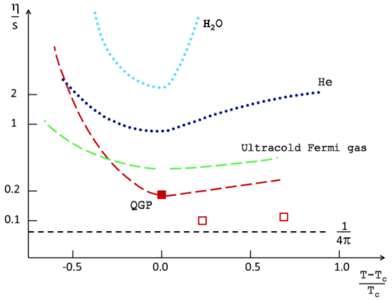
# Holography and hydrodynamic revival

[Kovtun, Son, Starinets, PRL 2005]

- Shear viscosity bound

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

- Class of strongly interacting QFTs with gravity dual saturate this bound
- Empirical evidence: first in QGP and then in other materials (Dirac Materials, Graphene, Cuprates, Kagome...)
- The more the system is strongly coupled the more the bound is saturated



## Intense theoretical effort in the last decade...

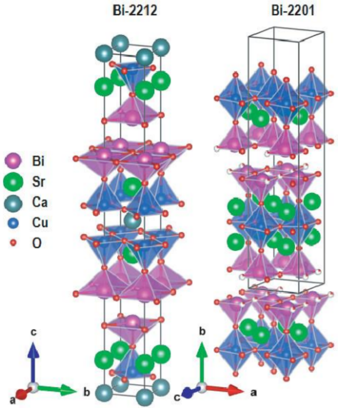
- Magneto-transport with momentum dissipation in holography and hydrodynamics [Davison,Hartnoll, Sachdev, Kiritsis, Goutéraux, Baggioli, Donos Gauntlett, Pantelidou, Musso Arean, Lucas, Andrade, Krikun, Schalm, Zaanen, Amoretti, Musso, Pujolas, Kovtun] and many mores....
- Hydrodynamics in Dirac Materials (Graphene, Kagome) [Lucas, Sachdev, Erdmenger, Meyers...]
- Anomalous hydrodynamics, holography and Weyl semi-metals [Landsteiner, Abbasi....]
- Holography and hydrodynamics with spontaneously broken symmetries Baggioli, Pujolas, Krikun, Andrade, Poovuttikul, Amoretti, Musso, Arean, Goutéraux, Delacretaz, Karlson, Hartnoll, Armas, Jain...]
- Holography in turbulent systems [Pantelidou, Andrade, Krikun, Baggioli....]

# Followed by experiments...

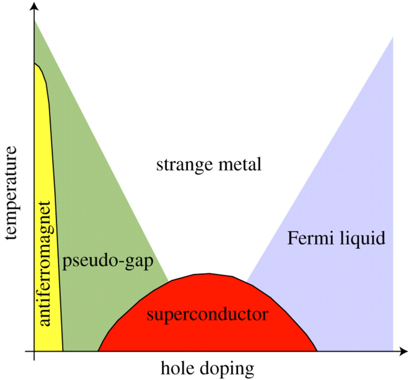
- **Evidence for hydrodynamic electron flow in $PdCoO_2$**, Moll et al. Science (2016)
- **Negative local resistance due to viscous electron backflow in graphene**, Baudurin et al., Science (2016)
- **Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene**, Crossno et al., Science (2016)
- **Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide**, Gooth et al., Nature Comm. (2018)
- **Scanning gate microscopy in a viscous electron fluid**, Braem et al., PRB (2018)
- **Ballistic and hydrodynamic magnetotransport in narrow channels**, Holder et al., PRB (2019)

# Cuprates

- Layers of $CuO_2$ planes bounded by rare earths
- Superconductivity and the most part of exotic properties happen on the $CuO_2$ plane $\rightarrow$ 2D materials
- Universal properties despite many different compounds
- Among High-$T_c$ superconductors Bi-2201 has a relatively low critical temperature even at optimal doping $\Rightarrow$ ideal to test low $T$ properties of the normal phase

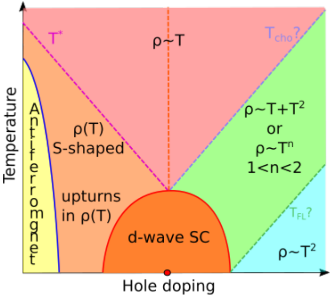


# Cuprates phase diagram




- Cuprates have almost the same Temperature vs doping (concentration of rare earth) phase diagram, characterized by many intertwined phases appearing at the same time.

# Phase diagram, QCP and scaling laws




- QCP is supposed to affect the properties of the strange metal phase:
  - transport coefficients should assume simple scaling laws
  - *Strong coupling:* no well defined quasi-particles.

# The Resistivity and Hall angle issue

- In normal Fermi liquid (magnetic field perpendicular to $CuO_2$ planes)

$$\rho_{xx} \sim T^2 \,, \qquad \cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2$$

- In most of the cuprates

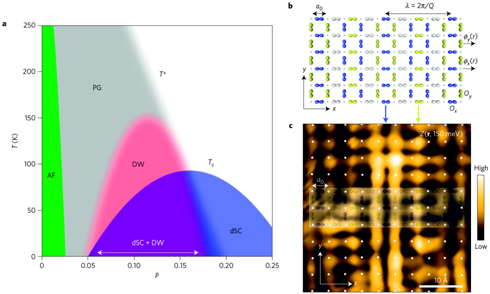
$$\rho_{xx} \sim T \,, \qquad \cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}} \sim T^2$$

- Actually in Bi-2201 is known that $\cot\theta_H \sim T^{1.5}$

# Other transport coefficients are less known

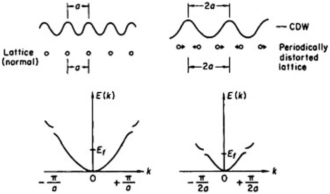
- Some of them are just dominated by lattice vibration
  - $\kappa_{xx}$ has an 80 % of lattice phonon contribution
- Transverse transport coefficients are independent of phonons contribution (typically very small signal)
  - The Nernst coefficient $N$ ([Wang, 2006] for a review)
  - The thermal Hall conductivity $\kappa_{xy}$ (measured in LSCO [Grissonnanche, 2019] and in YBCO [Zhang, 2000][Matusiak, 2009])
  - Magnetoresistance typically $B^2$ suppressed

# More orderings discovered recently




- Charge-density wave (CDW) order appears to be a ubiquitous feature of cuprate superconductors.
- Our material, $Bi_2Sr_2CuO_6$:
  - 2D CDW confirmed (by X-ray diffraction) to extend to optimal and over-doped region [Peng 2018],
  - low critical temperature ( $T_c \sim 10 - 33$ K).

# Charge density wave order




- **What are charge density waves?**
  - Peierls (1955) suggested periodic distortion of 1D lattice can lower total energy.
  - Start with first Brillouin zone $k = \pm\pi/a$ half filled.
  - CDW distortion $\rightarrow$ new superlattice of spacing $2a$. New first Brillouin zone band gap at $k = \pm\pi/2a$.
  - Gain in creating energy gaps can overcome loss of lattice distortion.
- Incommensurate CDW $\rightarrow$ broken translation invariance.

# CDW and pinning

As soon as the translation SB is pseudo-spontaneous (Goldstone Bosons have a small mass) the AC conductivity can have an off-axes peak [Fukuyama-Lee-Rice '78,Delacretaz 2017]

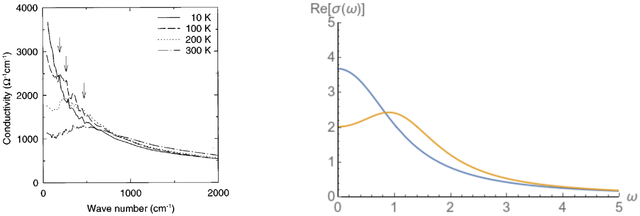


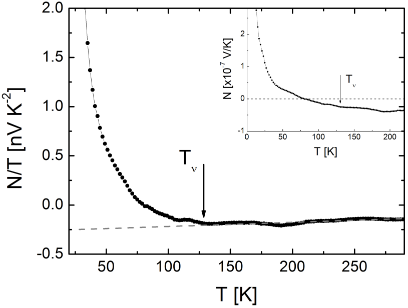

Figure: Experimental BiSCO conductivity from [Tsvetkov 1997]

$$\sigma(\omega) = \sigma_0 + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}$$

- for $\omega_0^2 > \Omega^3/(\Gamma + 2\Omega)$ there is an off-axes peak
- can the Drude to off axes peak originate from the same mechanism?

# CDW not only affects the conductivity

- Usually the enhancement in the Nernst effect at low $T$ was attributed to fluctuating superconductivity
- [Cyr-Choinière 2009] found a relation between $T_{CDW}$ and the enhancement temperature



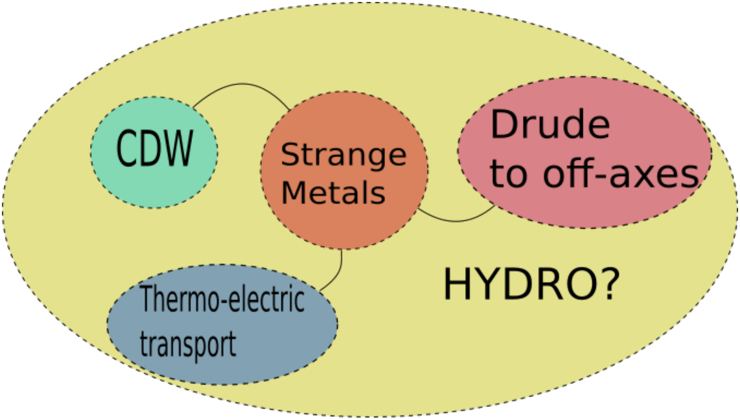

- $T_\nu$ is the temperature at which one recovers a Fermi Liquid expectation ($T_\nu \sim 2T_{CDW}$)
- CDW affects the Nernst signal also at fluctuating level

# Where do we stand?

- Can one mechanism takes into account consistently all the thermo-electric transport coefficients?
- Many intertwined phases $\Rightarrow$ difficult to uncover
- We need a metallic behavior
- Strange metals are strongly coupled by nature

Hydrodynamics might come to help

# A unified hydrodynamic picture?




Let us play simple and start with DC transport coefficients

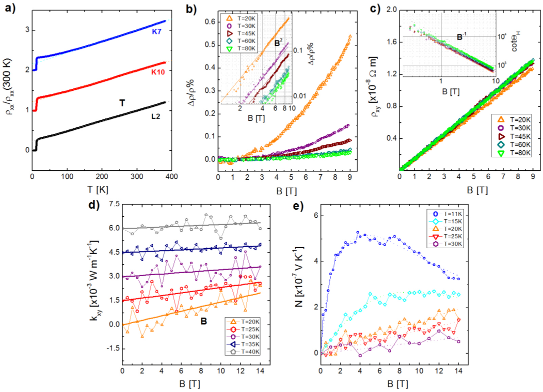
# Experiment

- We want to measure the temperature $T$ and magnetic field $B$ dependence of all the thermo-electric transport coefficients

- We will restrict to transverse or electric transport coefficients to avoid phonons contribution (no $\kappa_{xx}$)
  - The electric conductivity $\rho_{xx}$
  - The Hall angle $\cot\theta_H = \frac{\rho_{xy}}{\rho_{xx}}$
  - The magnetoresistence $\frac{\rho_{xx}(B)-\rho_{xx}(0)}{\rho_{xx}(0)}$
  - The thermal Hall conductivity $\kappa_{xy}$
  - The Nernst signal $N$

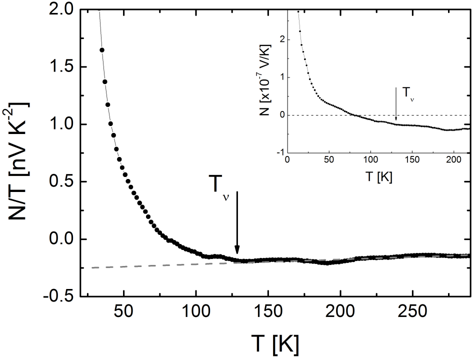
- Many coexisting phases $\Rightarrow$ we need to properly define the temperature range where the picture is supposed to be valid

# $B$ dependence of the DC transport coefficients



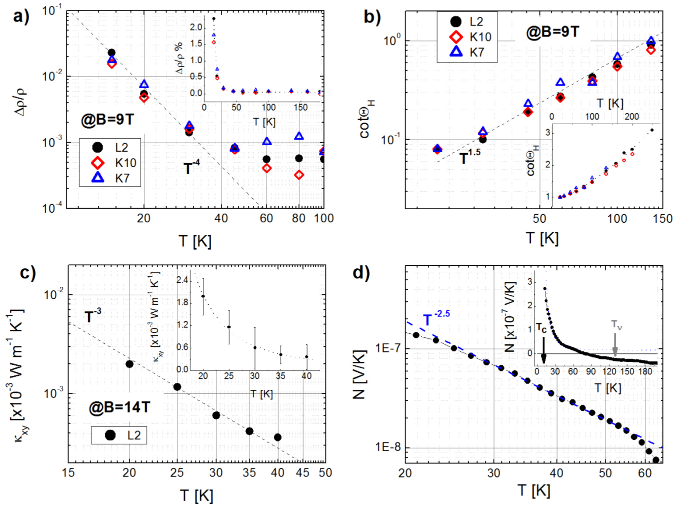

- For $T < 20$ K the Nernst starts to deviate from linearity $\Rightarrow$ Vortex effect [Wang 2006]
- For $T > 20$ K the $B$ dependence is the one expected for a parity invariant system

# $T$ dependence of the DC transport coefficients upper bound




- **Estimation of $T_\nu$:** the point where $N/T$ deviates from linearity at high temperature : $T_{CDW} \sim T_\nu/2 = 65$ K [Cyr-Choinière 2009]
- In accordance with [Peng, 2018]

# $T$ dependence of the DC transport coefficients




- Relevant temperature interval 20 K $< T <$ 65 K

# Summary of experimental results

- **How do experimental parameters depend on $T$ and $B$?**
  - $\rho_{xx} \sim B^0 T$ as expected for strange metals
  - $\Delta\rho/\rho \sim B^2 T^{-4}$
  - $\cot\theta_{\mathrm{H}} \sim B^{-1} T^{1.5}$ as expected in Bi-2201 but different from other materials (YBCO $\cot\theta_{\mathrm{H}} \sim B^{-1} T^2$)
  - $\kappa_{xy} \sim B T^{-3}$
  - $N \sim B T^{-2.5}$

# Hydrodynamics with broken continuous symmetries

When continuous symmetries are spontaneously broken, in addition to currents there are also Goldstone Bosons which are long lived and need to be included in the Hydro description.

Poisson brackets method [Son 2000]

- Define the Poisson brackets between the GBs $\phi_i$ and the other DOFs $A_i$:

$$[A_i, \phi_j] = ...$$

- Define the correction to the Hamiltonian due to $\phi_i$:

$$\Delta H_\phi = \int d^d x \, F(\phi_i, \partial_i \phi_j)$$

- Compute the corrections to the EOMs

$$\dot{A} = i[\Delta H_\phi, A]$$

# When translations are broken

- The phonon $\phi_i$ is the conjugate vriable of momentum density $\pi_j$:

$$[\phi_i, \pi_j] = \delta_{ij}$$

- the most general phonon Hamiltonian is [Chaikin&Lubensky]:

$$\Delta H_\phi = \int d^2x \left[ \frac{K}{2}(\partial_i \phi^i)^2 + \frac{G}{2}\left(\partial_i \phi_j \partial^i \phi^j + k_0^2 \phi_i \phi^i\right)\right]$$

- The EOMs for $\phi_i$ and $\pi_i$ are modified due to the non-trivial commutation relations:

$$[\pi_i, H] = -k_0^2 G \phi_i$$
$$[\phi_i, H] = -v_i$$

# Hydrodynamics with broken continuous symmetries and dissipation

The breaking of translations can be pseudo-spontaneous

- Momentum dissipation rate $\Gamma$: coupling to external lattice
- phase relaxation $\Omega_1$ of the GBs: present as soon as translations are explicitly broken [Amoretti 2018]
- The magnetic fields $F^{xy} = B$ enters only as an external field via the Lorentz term

The total EOMs:

$$\begin{aligned} \partial_t (n, s) + \partial_i \left( J^i, Q^i/T \right) &= 0 \,, \\ \partial_t \pi^i + \partial_j T^{ji} &= F^{ij} J_j - \Gamma \pi^i - k_0^2 G \phi^i \,, \\ \partial_t \phi_a + \partial_i J^i_{\phi_a} &= -\Omega_1 \phi_a \,. \end{aligned}$$

# Constitutive relations

The only missing step is to provide constitutive relations for the currents $J_i$, $Q_i/T$, $T^{ij}$ and $J^i_{\phi_a}$ to first order in the gradients expansion around the equilibrium configuration $T+\delta T$, $\mu+\delta\mu$:

$$
\begin{aligned}
\frac{Q^i}{T} &= s v^i - \alpha_0\left(\partial^i\delta\mu - F^{ij}v_j\right) - \frac{\bar{\kappa}_0}{T}\partial^i\delta T - \gamma_2\partial^i\theta_1 \,, \\
J^i &= n v^i - \sigma_0\left(\partial^i\delta\mu - F^{ij}v_j\right) - \alpha_0\partial^i\delta T - \gamma_1\partial^i\theta_1 \,, \\
T^{ij} &= \left(n\delta\mu + s\delta T - (G+K)\,\chi_1\theta_1\right)\delta^{ij} - G\chi_2\theta_2\epsilon^{ij} \\
&\quad -\eta\left(\partial^i v^j + \partial^j v^i - \partial_k v^k\delta^{ij}\right) - \zeta\partial_k v^k\delta^{ij} + \gamma_1 B\theta_2\delta^{ij} \,, \\
J_1^i &= -v^i - \gamma_1\left(\partial^i\delta\mu - F^{ij}v_j\right) - \gamma_2\partial^i\delta T - \xi_1\chi_1\partial^i\theta_1 + \xi_2\chi_2\epsilon^{ij}\partial_j\theta_2 \,, \\
J_2^i &= \epsilon^{ij}J_1^j \,,
\end{aligned}
$$

- Transport coefficients
- Susceptibilities

# Constraints

- Typical constraints for charged fluid:

$$\sigma_0,\ \bar{\kappa}_0,\ \eta\ ,\Gamma\ ,\Omega_1 \geq 0\ , \qquad \bar{\kappa}_0\sigma_0 - T\alpha_0^2 \geq 0\ .$$

- Special to CDW: $\xi_1 > 0$.
- This subsequently leads to bounds on $\gamma_1$ and $\gamma_2$:

$$(\gamma_1^2, \gamma_2^2) \leq \left(\sigma_0, \frac{\bar{\kappa}_0}{T}\right) \min\left[\frac{\xi_1}{K+G}, \frac{\Omega_1}{\chi_{\pi\pi}\omega_0^2}\right]\ .$$

- We will assume $\gamma_{1,2}$ are small enough to be treated as vanishing.
- If we assume a relativistic covariant fixed point then

$$\alpha_0 = -\frac{\mu\sigma_0}{T}\ , \qquad \bar{\kappa}_0 = \frac{\mu^2\sigma_0}{T}$$

# The Martin-Kadanoff method

Having the modified EOMs and the constitutive relations one can apply the Martin-Kadanoff procedure

- One can cast the EOMs in the following way ($q_A$ are the relevant fields, $s_A^0$ are the sources):

$$\partial_t q_A(t,\vec{k}) + M_A^C(\vec{k},B) s_C(t,\vec{k}) \;=\; \chi_A^B s_B^0(\vec{k}) \,.$$

- The retarded Green's function can eventually be computed

$$-\left(I_6 + i\omega\left(-i\omega I_6 + M\chi^{-1}\right)^{-1}\right)\chi \,.$$

# Conductivities at low $B$

- Taking the DC transport coefficients to lowest order in $B$:
  - Charge resistivity: $\rho_{xx} = \frac{1}{\sigma_0+\tilde{\sigma}} + \mathcal{O}(B^2)$.
  - Magnetoresistance: $\frac{\Delta\rho}{\rho} = B^2 \frac{\sigma_0^3\ \tilde{\sigma}}{n^2} \frac{1}{(\sigma_0+\tilde{\sigma})^2} + \mathcal{O}(B^4)$.
  - Thermal Hall conductivity: $\kappa_{xy} = -BT \frac{\tilde{\sigma}^2 s}{n^4} \left( ns - 2\frac{\mu\sigma_0 n^2}{T\tilde{\sigma}} \right) + \mathcal{O}(B^3)$.
  - Hall angle: $\cot\Theta_H = \frac{n}{B\tilde{\sigma}} \frac{1+\frac{\sigma_0}{\tilde{\sigma}}}{1+2\frac{\sigma_0}{\tilde{\sigma}}} + \mathcal{O}(B)$.
  - Nernst coefficient: $N = \frac{B\ \sigma_0\ \tilde{\sigma}}{n^2(\sigma_0+\tilde{\sigma})^2}\sigma_0(s+\frac{\mu}{T}) + \mathcal{O}(B^3)$.
- DC conductivities are a sum of incoherent and relaxation conductivities

$$\sigma_{\mathrm{DC}} = \sigma_0 + \tilde{\sigma} \qquad \text{with} \qquad \tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_1}{\Omega_1\Gamma + \omega_0^2}\ .$$

- Only four variables $\sigma_0$, $\tilde{\sigma}$, $n$ and $s$. But we measure five observables - system overconstrained.

# Determining the hydrodynamic variables

- **What does experiment imply for our hydrodynamic variables?**
  - Consistency requires $\rho_{xx}$ dominated by $\sigma_0$ at low $T$ i.e.

$$\rho_{xx} \sim \frac{1}{\sigma_0} \sim T \ ,$$

  - and

$$\cot\Theta_{\mathrm{H}} \sim \frac{n}{B\tilde{\sigma}} \sim T^{1.5} \ .$$

  - Using $\Delta\rho/\rho \sim T^{-4}$ fixes

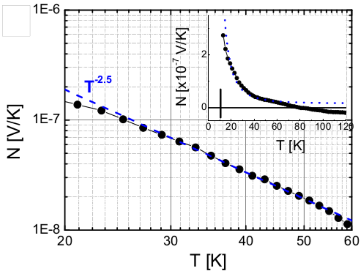
$$n \sim T^{1.5} \qquad \text{and} \qquad \tilde{\sigma} \sim T^0 \ .$$

  - Finally $s$ is given through $\kappa_{xy}$

$$\kappa_{xy} \sim \mu B \frac{\sigma_0 \ \tilde{\sigma}}{n^2} s \sim T^{-3} \qquad \Rightarrow \qquad s \sim T \ .$$

    - $s$ is in accordance with specific heat measurement on our sample and on YBCO [Loram 1991]

# Recovering the Nernst behavior




- The Nernst coefficient behaves as

$$N \sim \frac{\mu B \tilde{\sigma}}{nT} \sim \frac{\mu}{T \cot \Theta_H} \sim T^{-2.5} \ .$$

- The temperature range where the scaling agrees is exactly the one predicted from other principles (vortices at low $T$ and $T_\nu/2$ at high $T$)

# Relevance of CDW order

- The Nernst coefficient is dominated solely by $\tilde{\sigma}$:

$$\tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_1}{\Omega_1 \Gamma + \omega_0^2} \sim T^0$$

  - If $\tilde{\sigma}$ is dominated by CDW order (in accordance with [Cyr-Choinière 2009]) then $\Omega \sim T$, $\omega_0 \sim T^2$ (compatible with a QCP behavior with a massive operator (remember the mass of the phonon)
  - If external momentum dissipation is dominating istead ($\Omega_1 \to 0$ and $\omega_0 \to 0$) then $\Gamma \sim T^3$ (compatible with electron-phonon scattering in a multiband metal, not our case...)

# Outlook

- This is a consistency check of the validity of hydro
  - to say something conclusive on $\tilde{\sigma}$ we need precision spectral measurements
  - If hydro is valid down to low $T$ the Drude to off-axes peak should be explained within the same picture
- Other cuprates have different temperature scalings for the transport coefficients (eg Hall angle and $\kappa_{xy}$ in YBCO)
  - CDW order is measured almost in every cuprates $\Rightarrow$ try to find a consistent picture
  - Is hydro a valid description in different points of the phase diagram?

# Thank you