Schwinger effect in superconductors

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Collaboration

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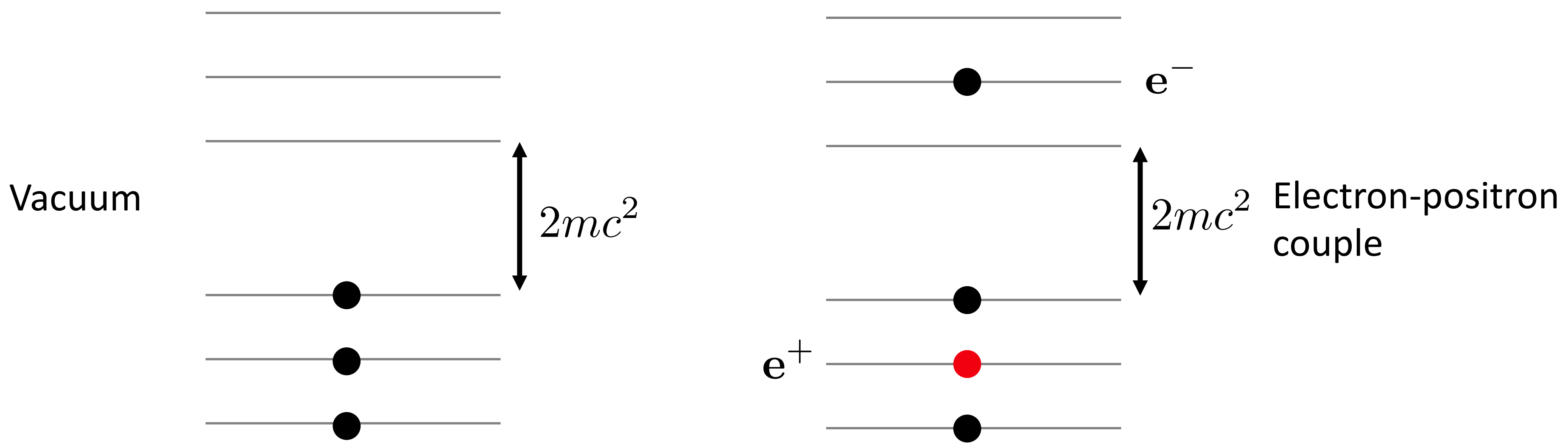
Details in arXiv:2007.08323

Outline

- From Dirac to Schwinger
- Connection between QED and superconductivity theory
- Schwinger effect in superconductor
- Experiments
- Future

Brief history

- Dirac equation (1929)
- It predicts the existence of negative energy solutions
- To be stable, we assume that these are occupied: Dirac sea



Brief history

- Sauter (1931): the electric field can accelerate an electron from the Dirac sea and excite it
- Tunneling through a potential barrier of energy $2mc^2$

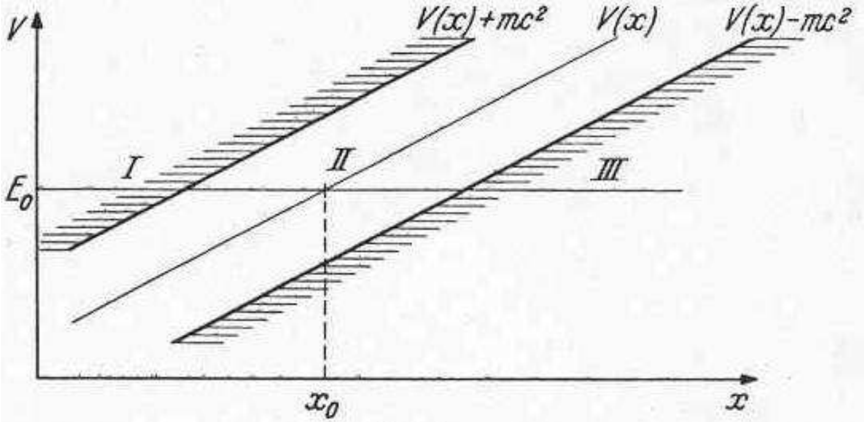


Fig. 1.

Brief history

- Heisenberg and Euler (1936) “Consequences of Dirac’s theory of positrons”
- The electromagnetic field creates particle pairs and polarizes the vacuum
- It modifies the Maxwell equations in vacuum

Brief history

- Schwinger (1951) full treatment using QED
- The electromagnetic vacuum is unstable in presence of an electric field

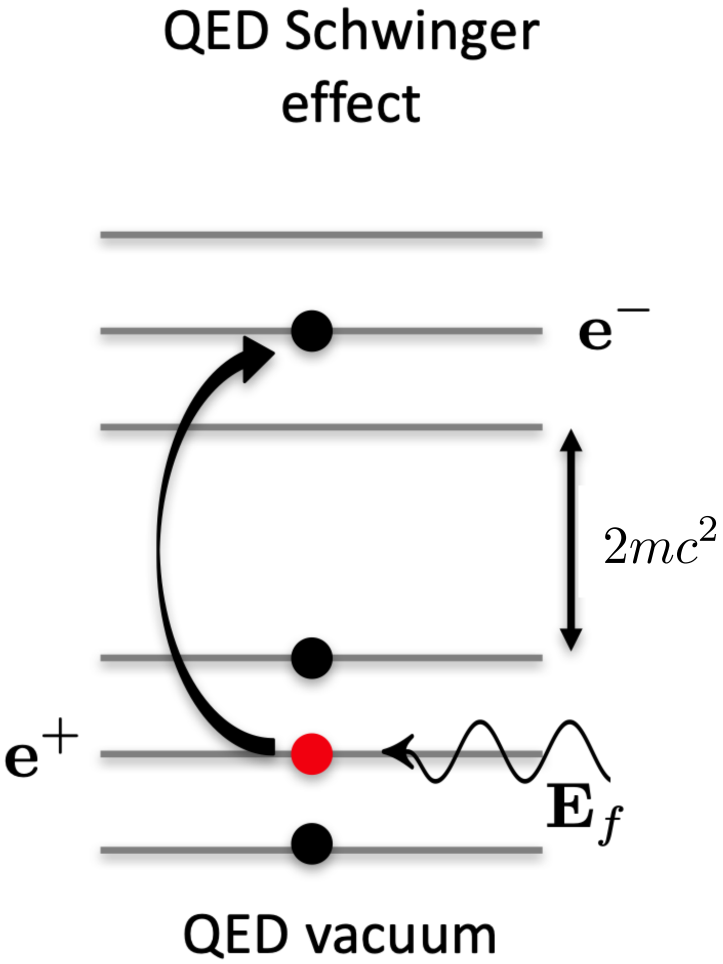
$$\Gamma = \eta \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\pi n \frac{m^2 c^3}{eE\hbar}}$$

Pair production rate

$$E_c = \frac{m^2 c^3}{e\hbar}$$

Critical electric field

Pictorial description



Absence of experiments

- The Schwinger effect in QED has not yet been observed

- The critical electric field needed cannot be generated in laboratory

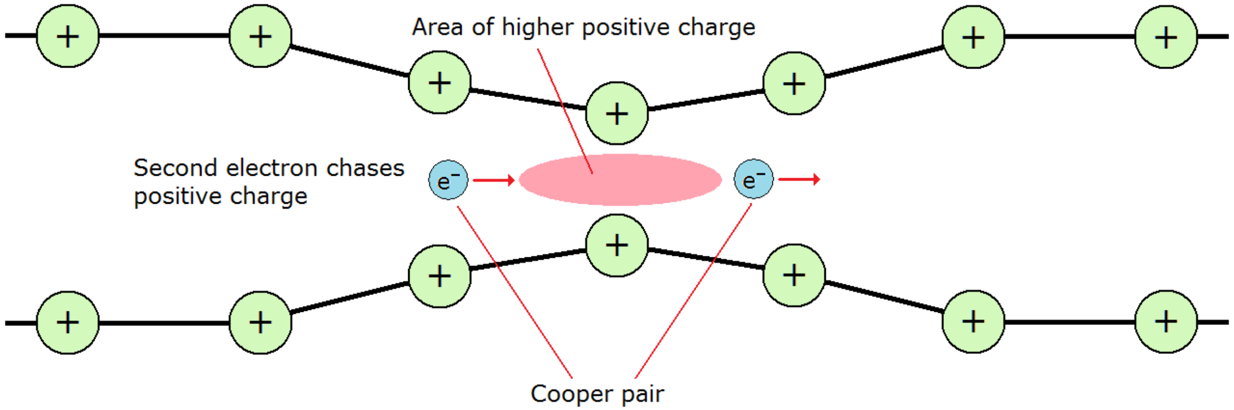
$$E_c = 10^{18}\mathrm{V\ m}^{-1}$$

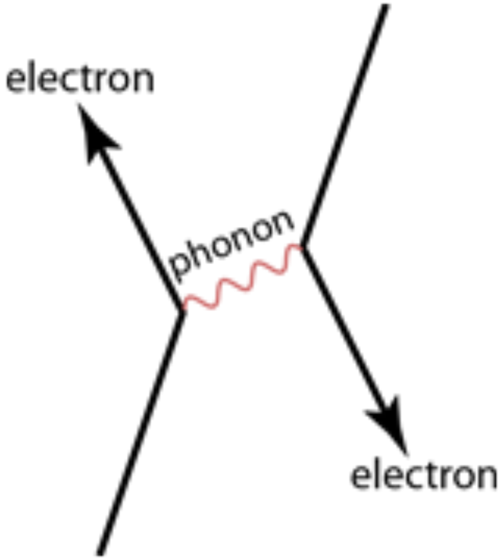
On the other side…

- Heike Kamerlingh Onnes (1911) experimental evidence of superconductivity
- Ginzburg–Landau theory (1950) phenomenological description
- Bardeen–Cooper–Schrieffer theory (1957) microscopic description

BCS theory

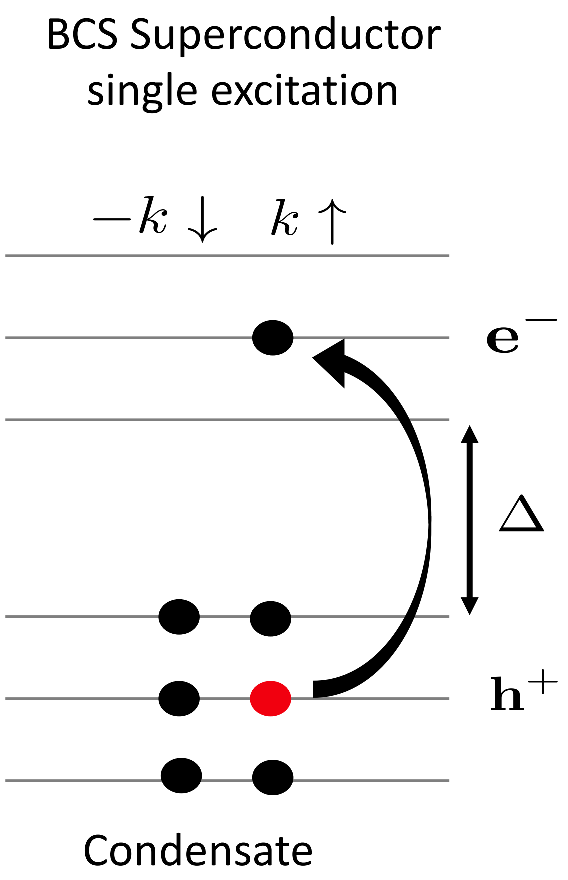
- The electrons moving in a lattice feel an effective attractive force
- They can condensate forming a “Cooper pair”
- This is the superconducting ground state





BCS theory

- The Cooper pair can be broken leading to excitation called "quasiparticle"
- These are quasi-electron (negative charge) and quasi-hole (positive charge)
- The energy needed is the superconducting energy gap Δ



BCS theory

- The "quasiparticle" excitation are described by the Bogoliubov-de Gennes equations

$$
\begin{aligned}
E\ u^*_{\mathbf{k}} &= (\epsilon_{\mathbf{k}} - \mu) u^*_{\mathbf{k}} + \Delta\ v^*_{\mathbf{k}} \\
E\ v^*_{\mathbf{k}} &= -(\epsilon_{\mathbf{k}} - \mu) v^*_{\mathbf{k}} + \Delta\ u^*_{\mathbf{k}}
\end{aligned}
$$

μ Chemical potential

$\epsilon_{\mathbf{k}}$ Single particle energy

$$
|\Psi_{gs}\rangle = \Pi_k \left(u_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} \right) |vac\rangle
$$

BdG-Dirac equation connection

- In 1961 Nambu and Jona-Lasinio noticed a formal similarity between the BdG and the Dirac equations

Dirac

$$
\begin{aligned}
E\,\psi_L &= \sigma\cdot\mathbf{k}\,\psi_L + m\psi_R \\
E\,\psi_R &= -\sigma\cdot\mathbf{k}\,\psi_L + m\psi_R
\end{aligned}
$$

Bogoliubov-de Gennes equations

$$
\begin{aligned}
E\,u^*_{\mathbf{k}} &= (\epsilon_{\mathbf{k}}-\mu)u^*_{\mathbf{k}} + \Delta\,v^*_{\mathbf{k}} \\
E\,v^*_{\mathbf{k}} &= -(\epsilon_{\mathbf{k}}-\mu)v^*_{\mathbf{k}} + \Delta\,u^*_{\mathbf{k}}
\end{aligned}
$$

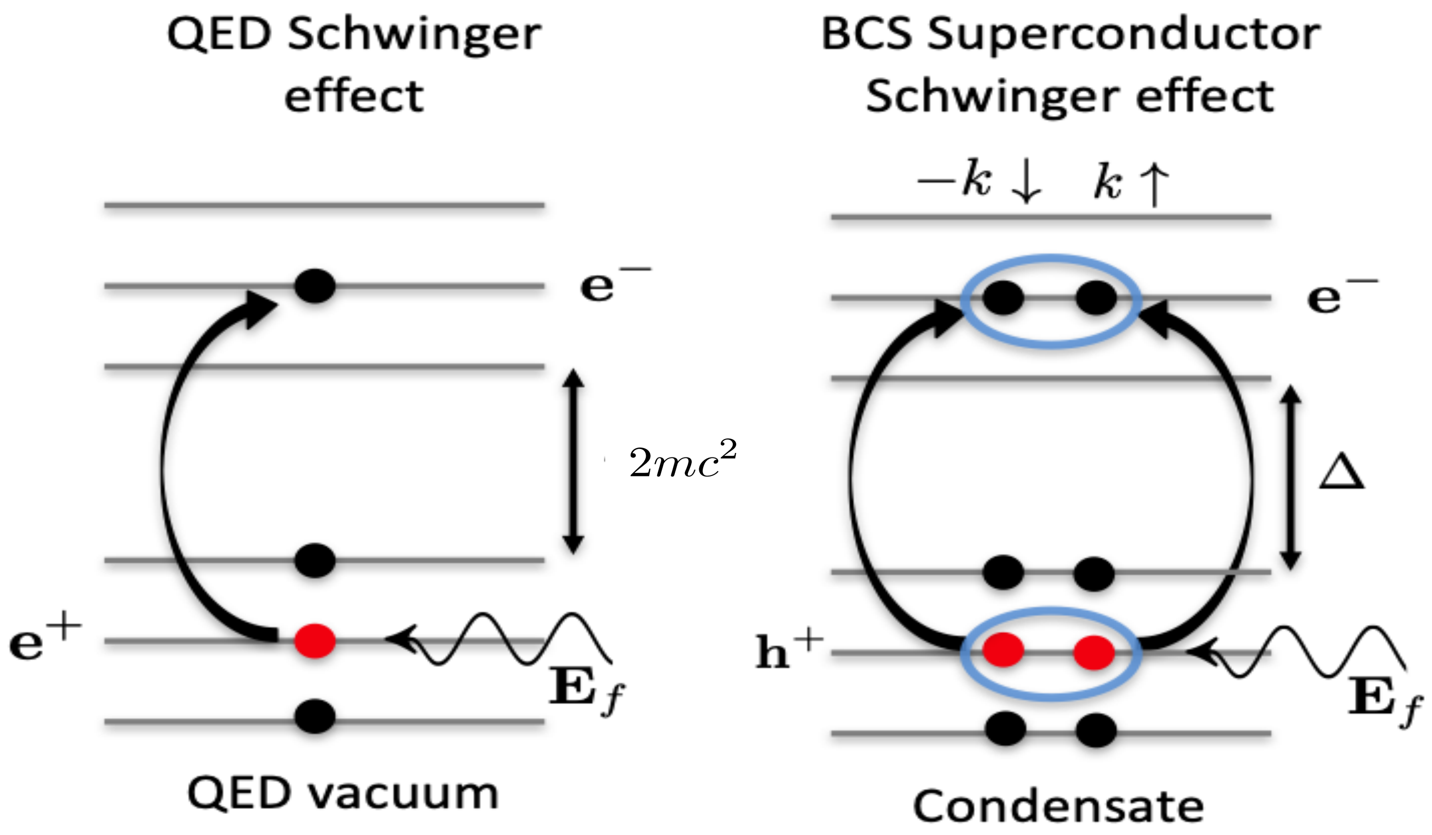
Mapping

$$
\begin{aligned}
\psi_L &\leftrightarrow u^*_{\mathbf{k}} \\
\psi_R &\leftrightarrow v^*_{\mathbf{k}} \\
\sigma\cdot\mathbf{k} &\leftrightarrow (\epsilon_{\mathbf{k}}-\mu) \\
m &\leftrightarrow \Delta
\end{aligned}
$$

vacuum $\leftrightarrow$ Ground state condensate

BdG-Dirac equation connection

- If the Dirac equation predicts the Schwinger effect, the BdG equations should predict a superconducting Schwinger effect

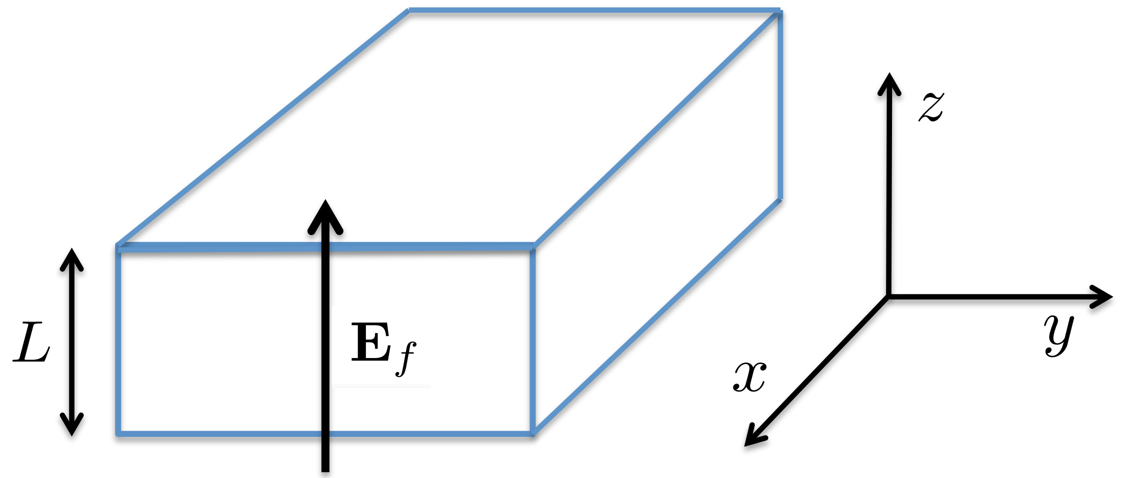


Differences and analogies

- Analogy: same equations – same effect
- Differences:
- The physical quantities can change in time, e.g., gap
- Screening of the electric field
- The energy gap is much smaller than the electron rest energy. It could be observed in the laboratory.

Model

- Electric field along z
- It completely penetrates the superconductors



Theoretical framework

BCS Hamiltonian

$$H_{eff} = \int d\mathbf{r}\Big\{\sum_{\alpha}\Big[\Psi^{\dagger}(\alpha\mathbf{r})H_e(\mathbf{r})\Psi(\alpha\mathbf{r}) + \Delta(\mathbf{r})\Psi^{\dagger}(\mathbf{r}\uparrow)\Psi^{\dagger}(\mathbf{r}\downarrow) + \Delta^{*}(\mathbf{r})\Psi(\mathbf{r}\downarrow)\Psi(\mathbf{r}\uparrow)\Big\}$$

Single particle Hamiltonian

$$H_e(\mathbf{r}) = \frac{1}{2m}\Big(-i\hbar\nabla - \frac{e}{c}\mathbf{A}\Big)^2 + U_0(\mathbf{r}) - \mu$$

Gauge choice

$$\mathbf{A} = \{0, 0, -cE_f t\}$$

Expansion of fermionic field

$$\Psi(\mathbf{r}\alpha) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k}\alpha}$$

Theoretical framework

Only opposite momentum particles are coupled

Hamiltonian

$$H_{eff} = 2\sum_{\mathbf{k}} \begin{pmatrix} \xi_k & -\Delta \\ -\Delta^* & -\xi_k \end{pmatrix}$$

Kinetic energy

$$\xi_k = \frac{\hbar^2 k^2}{2m} + \frac{e^2 E_f^2 t^2}{2m} - \mu,$$

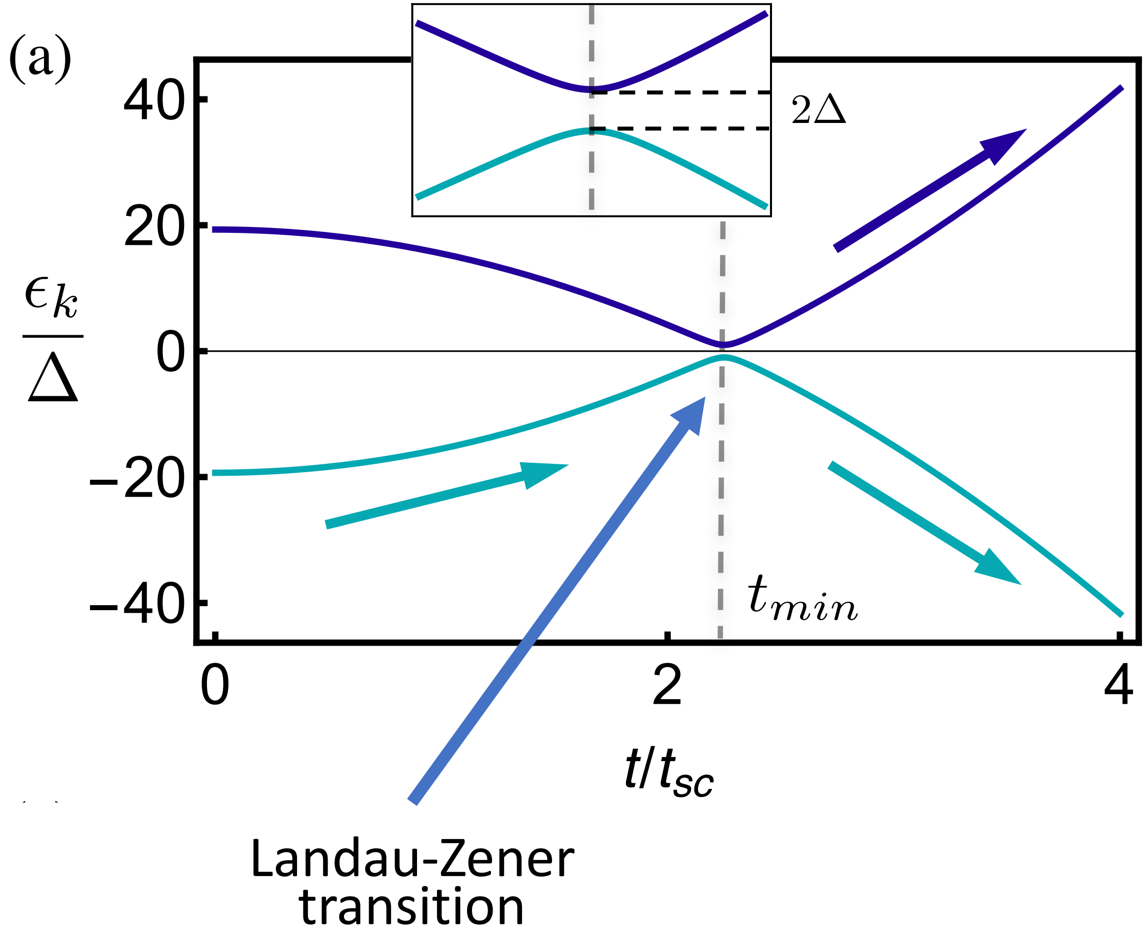
Energy spectrum

$$\pm\epsilon_k = \pm\sqrt{\xi_k^2 + |\Delta|^2}$$

We can solve the Schroedinger equation with the time-dependent Hamitonian

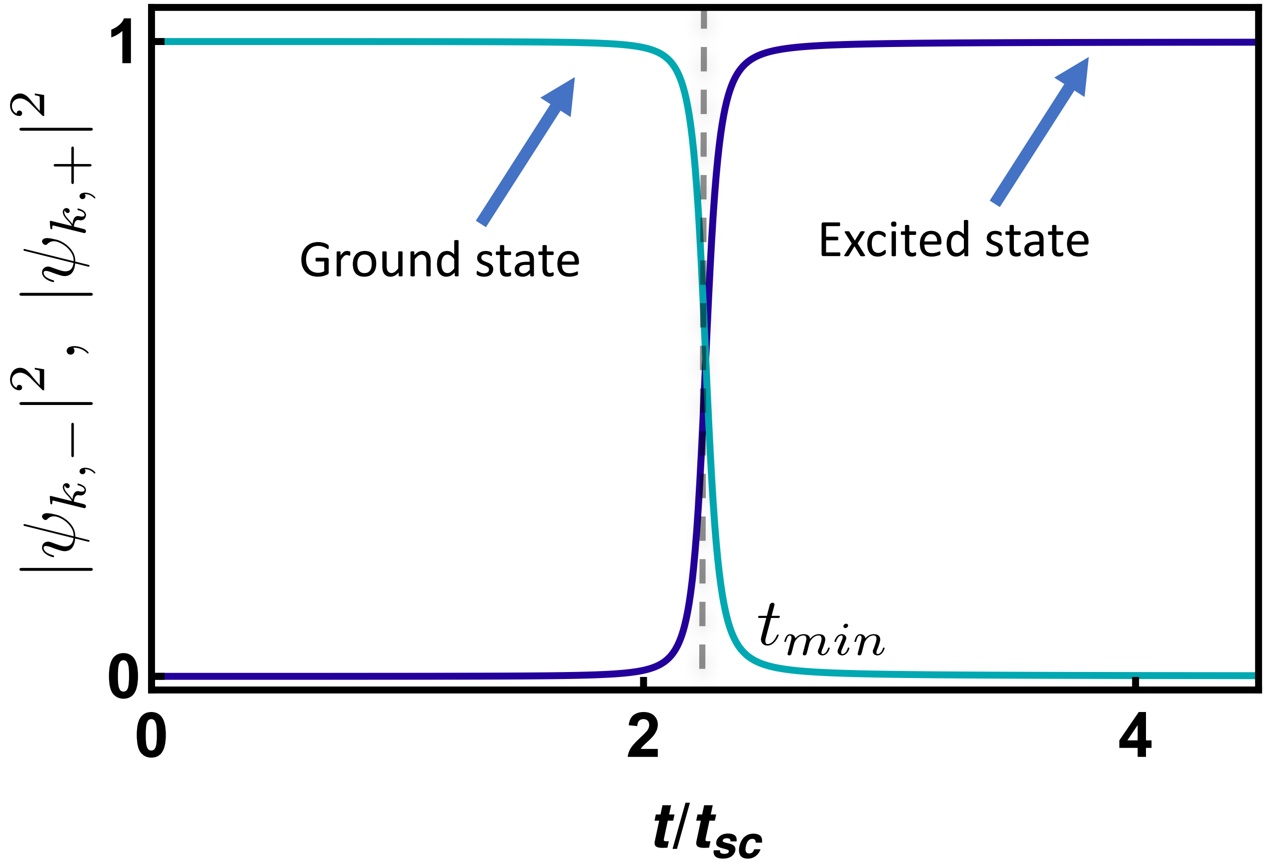
Results

- Time dependent energy spectrum (k mode)
- Landau-Zener transition at the minimum
- The ground state is excited



Results

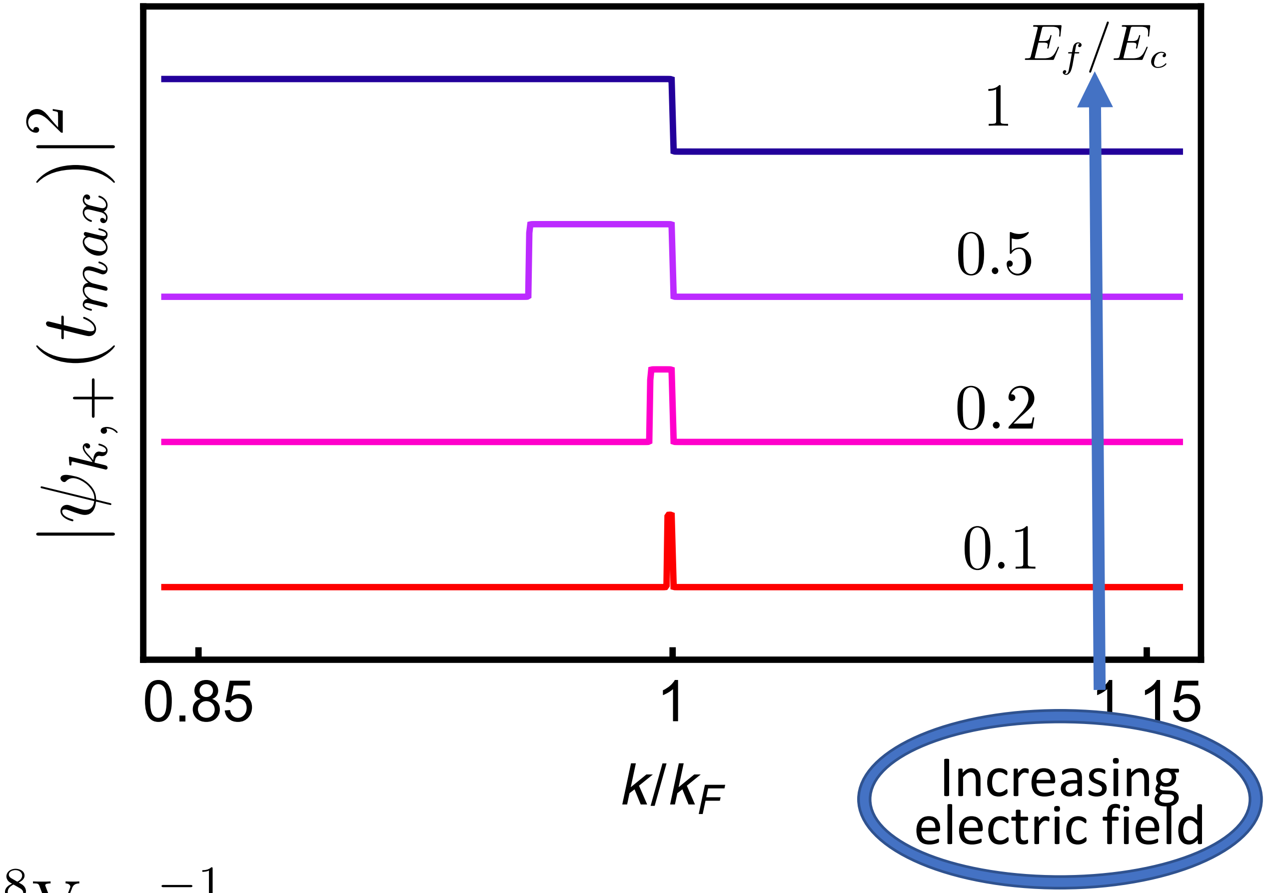
- Ground and excited population dynamics
- The ground state is excited at the minimum gap



Results

- Excited final population as a function of k

- Increasing the electric field leads to full excitation of the ground state



- Critical field $E_c = 5 \times 10^8 \text{V m}^{-1}$

Critical fields

QED (Schwinger)

$$E_c = 10^{18}\text{V m}^{-1}$$

Superconductor

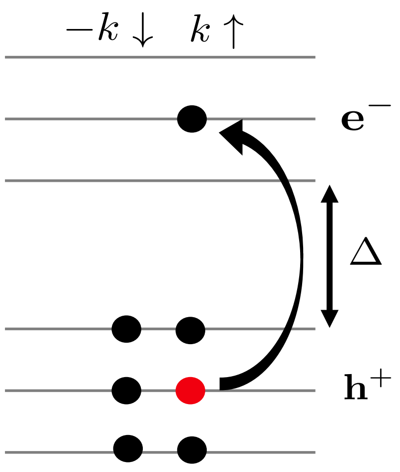
$$E_c = 5 \times 10^{8}\text{V m}^{-1}$$

The critical electric field is much smaller in a superconductor.

Notes on excitation

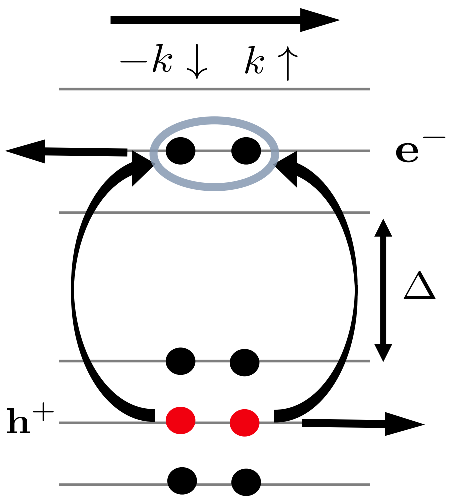
- The excitation induced by the electric field are different from the thermal ones
- Two couples are excited
- They are superconducting
- The energy gap is the same for a completely excited states
- They are coherent excitations

BCS single (thermal) excitation



Condensate

BCS Superconductor Schwinger effect

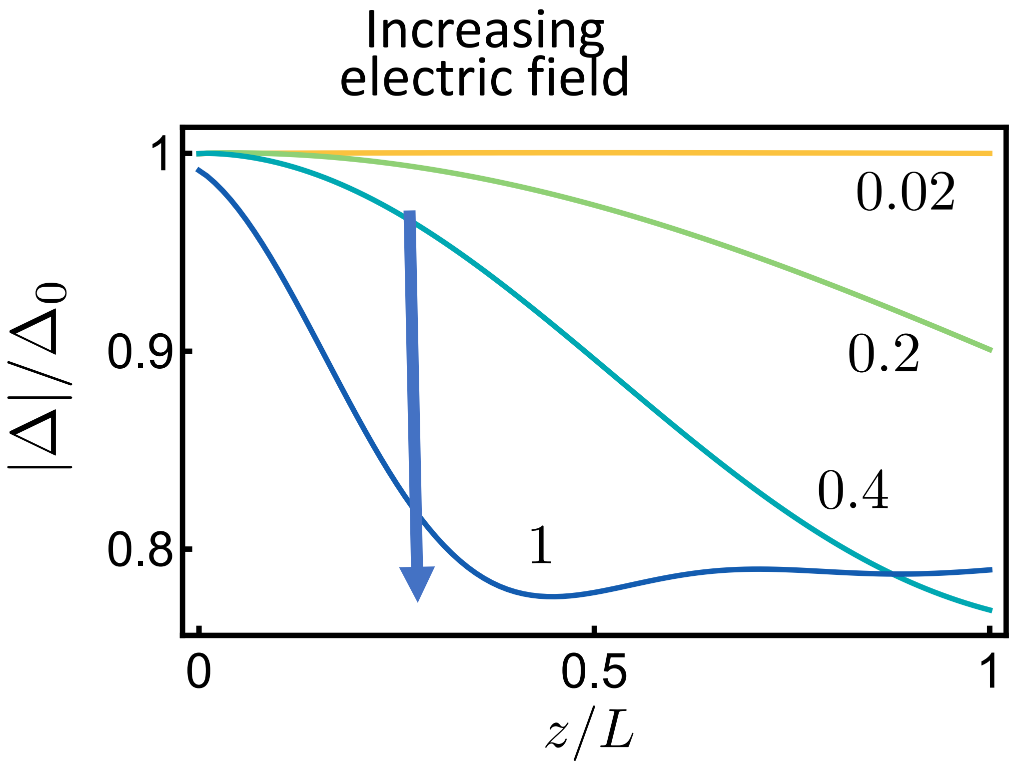


Condensate

Predictions

- The superconductivity should be weakened

- Spatial dependence of the superconducting gap



Predictions

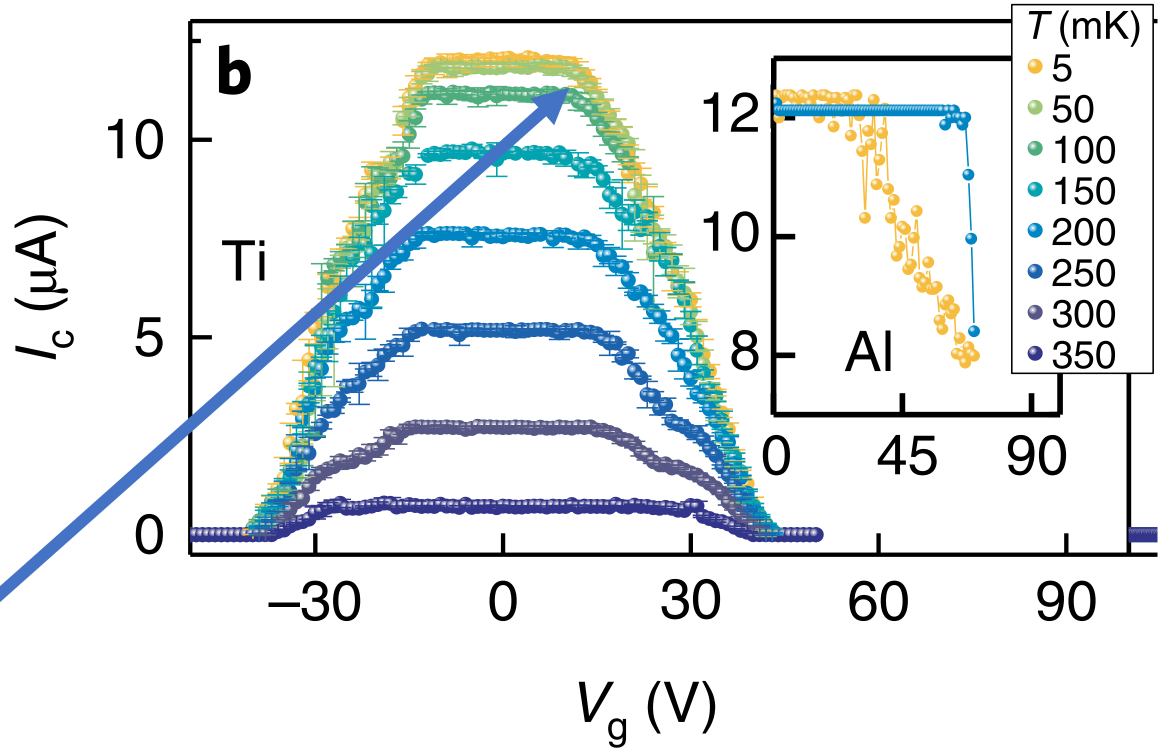
- The superconductivity should be weakened
- At low temperature there should be an excess of quasiparticles
- We expect the quasiparticles to have a non-thermal distribution

Recent experiments

- The decrease of the critical current is associated to the weakening of supercurrent

- Same critical electric field predicted by the theory

$$E_c \sim 10^8 \mathrm{V\ m^{-1}}$$

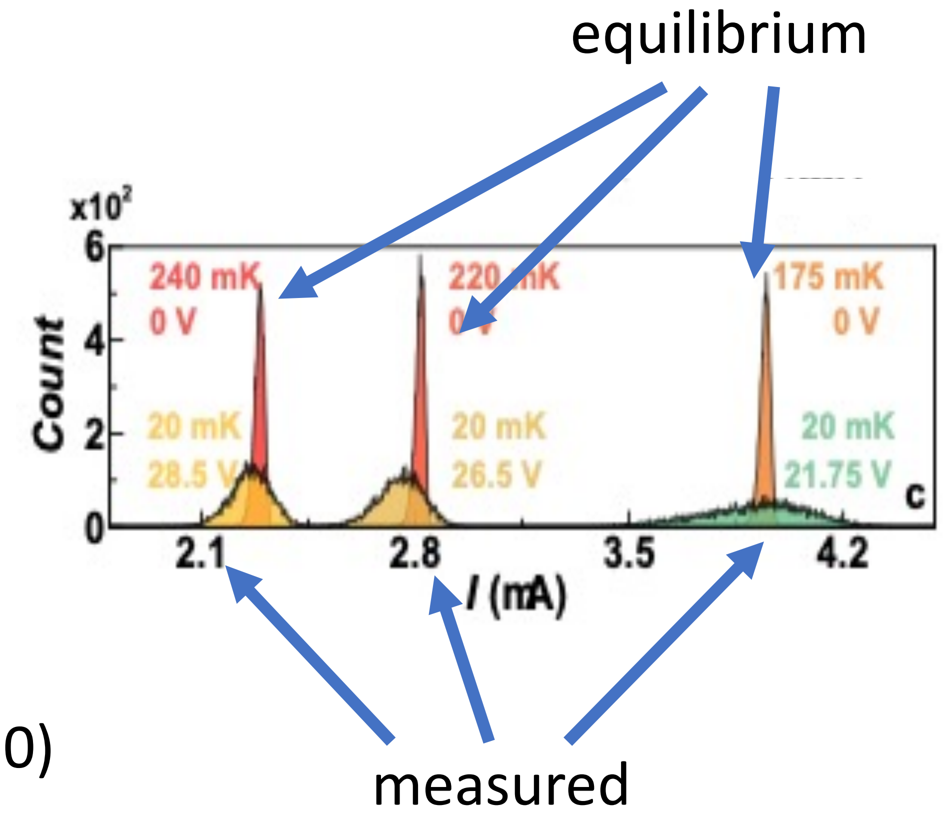


De Simoni et al. Nat. Nanotechnology, 13, 802 (2018)

Recent experiments

- Experiments point out an excess of quasi-particles at low temperature (non-thermal)

- They seem to suggest the presence of non-equilibrium



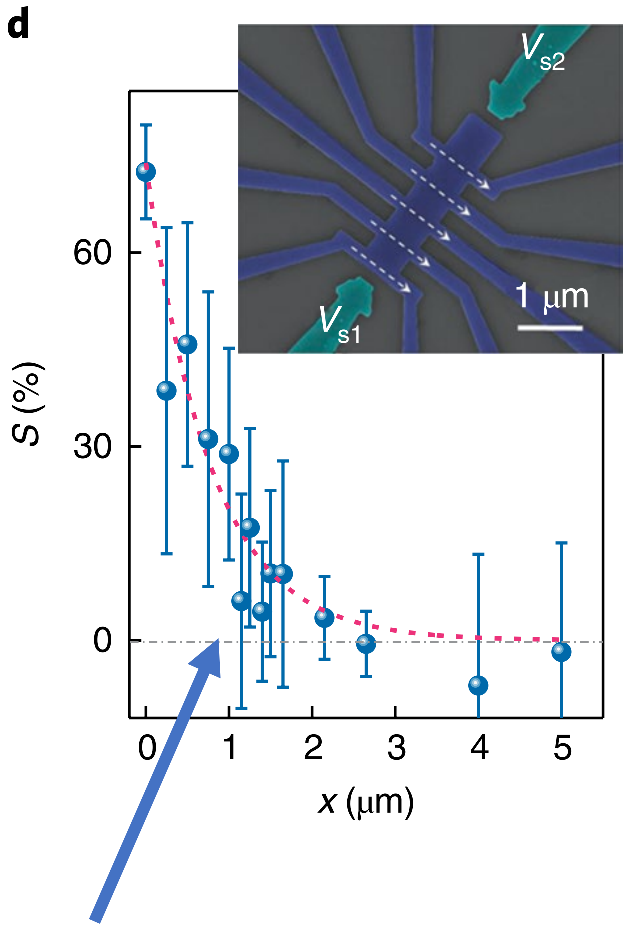
Puglia et al. Phys. Rev. Applied, 13, 054026 (2020)

Open problems 1

- To have a quantitative comparison we need a dissipative interaction
- The superconductor should be able to dissipate the energy excess
- Effective theory with energy dissipation

Open problems 2

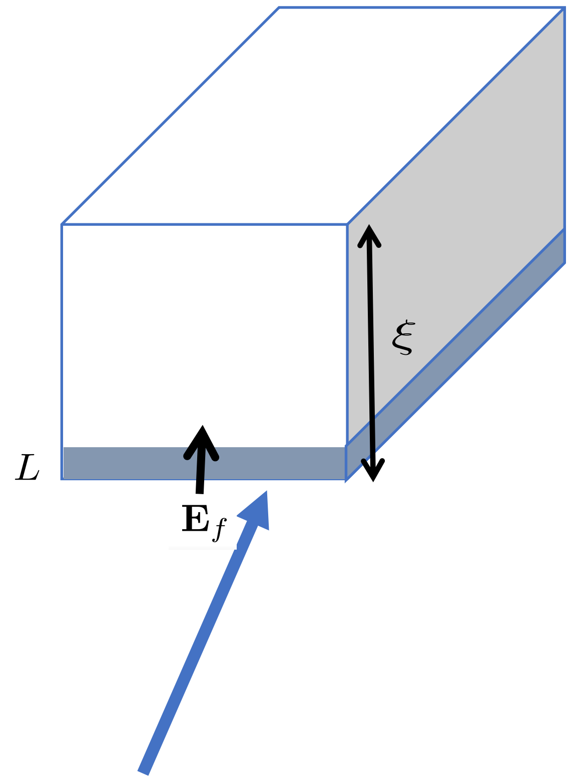
- The effects seem to a length scale of several times the coherence length ξ



Exponential decay with length scale of several ξ

Open problems 2

- The effects seem to a length scale of several times the coherence length ξ
- Assuming that the electric field penetrates only close to the edge (fraction of ξ), these are non-local effect
- It might be related to the extension of superconducting excitations



Penetration in
a small part of SC

Future

- As discovered by Heisenberg, Euler and Schwinger, the vacuum polarization changes the Maxwell equations
- There are additional non-linear corrections to the Lagrangian

$$\frac{2\alpha^2}{45}\frac{(h/mc)^3}{mc^2}\left[(\mathbf{E}^2-\mathbf{H}^2)^2+7(\mathbf{E}\cdot\mathbf{H})^2\right]$$

- Are similar non-linear corrections present in the superconductor?

Summary

- Starting from QED-superconductivity analogy we have predicted the presence of the superconducting Schwinger effect
- The e.f. excites two electron-hole pairs
- The model predicts an effect on the gap and an excess of (out-of-equilibrium) quasi-particles
- These are compatible with recent experiments

Thank you