Statistical considerations for the next data taking at CNAO

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Introduction

- Next data taking at CNAO is not a physics run, but we might as well try to do tests while taking useful data
- One carbon energy (200 MeV/u) and two targets
- There are several questions to answer before going to CNAO:
  - How many primaries do we expect to use for the next physics run?
  - How to divide them over the 2 targets? Should we collect the same amount of statistics for both targets? Not a priori clear, since **targets have different densities and cross sections, and cross section on H is obtained through subtraction**
- Today's presentation: make some statistical considerations (no efficiencies, no systematic errors, no background, etc) in order **to optimize data taking at CNAO with 2 targets**, keeping in mind:
  - The cross section subtraction technique
  - The limited amount of time available at CNAO
- Outline:
  - Statistical considerations (analytical)
  - Check their correctness with MC
  - Some plots of what's we can expect at CNAO
  - Conclusion

Reminder: cross section formulas

- Goal of FOOT: measure (single and double differential) cross sections of heavy ion beams (C, O) on tissue like targets (H, C, O)
- Reminder: cross section for production of fragments ***i*** on target (neglecting efficiency factors)

$$\sigma_{i,t} = \frac{Y_{i,t}}{N_p}\frac{A_t}{N_A\,\rho_t\,\delta_t} \quad \textbf{(1)}$$

With:
$\sigma_{i,t}$ = cross section to produce fragment i on target t [cm^2]
$Y_{i,t}$ = Number of fragments of type i [ ]
A_t = molecular mass of target [g mol^{-1}]
N_p = number of primary particles [ ]
N_A = Avogado's number [mol^{-1}]
ρ_t = density of target [g cm^{-3}]
δ_t = thickness of target [cm^{-1}]

- This CNAO data taking:
  - C beam on C target
  - C beam on C_2H_4 target

$$\sigma_{i,C} = \frac{Y_{i,C}}{N_p}\frac{A_C}{N_A\,\rho_C\,\delta_C} \quad \textbf{(1a)}$$

$$\sigma_{i,C_2H_4} = \frac{Y_{i,C_2H_4}}{N_p}\frac{A_{C_2H_4}}{N_A\,\rho_{C_2H_4}\,\delta_{C_2H_4}} \quad \textbf{(1b)}$$

$$\sigma_{i,H} = \frac{1}{4}\left(\sigma_{i,C_2H_4} - 2\sigma_{i,C}\right) \quad \textbf{(2)}$$

- For the targets inherited from GSI:
  - $\delta_C = \delta_{C_2H_4}$= 5 mm, ρ_C = 1.83 g/cm^3, $\rho_{C_2H_4}$= 0.94 g/cm^3, A_C ~12 g mol^{-1}, $A_{C_2H_4}$~28 g mol^{-1}

Reminder: cross section formulas

- Reminder: **statistical errors** on cross section for production of fragment ***i*** on target (neglecting efficiency factors). Essentially they are only determined by the yield of the detected fragments

$$\Delta\sigma_{i,t} = \frac{\sqrt{Y_{i,t}}}{N_p} \frac{A_t}{N_A\, \rho_t\, \delta_t} \quad \textbf{(3)}$$

With:
$\sigma_{i,t}$ = cross section to produce fragment i on target t [cm^2]
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- This CNAO data taking:
  - C beam on C target
  - C beam on C_2H_4 target

$$\Delta\sigma_{i,C} = \frac{\sqrt{Y_{i,C}}}{N_p} \frac{A_C}{N_A\, \rho_C\, \delta_C} \quad \textbf{(3a)}$$

$$\Delta\sigma_{i,C_2H_4} = \frac{\sqrt{Y_{i,C_2H_4}}}{N_p} \frac{A_{C_2H_4}}{N_A\, \rho_{C_2H_4}\, \delta_{C_2H_4}} \quad \textbf{(3b)}$$

$$\Delta\sigma_{i,H} = \frac{1}{4}\sqrt{(\Delta\sigma_{i,C_2H_4})^2 + 4(\Delta\sigma_{i,C})^2} \quad \textbf{(4)}$$

- For the targets inherited from GSI:
  - $\delta_C = \delta_{C_2H_4}$ = 5 mm, ρ_C = 1.83 g/cm^3, . $\rho_{C_2H_4}$ = 0.94 g/cm^3, A_C ~12 g mol^{-1}, $A_{C_2H_4}$ ~28 g mol^{-1}

Note

Note that targets have the same thickness → for the same nr. of primaries, the measurement with the C_2H_4 target, having a density smaller by a factor of ~2 w.r.t. the carbon target, will have a larger relative statistical error

What errors do we expect?

- What can we expect for $\Delta\boldsymbol{\sigma}_{i,H}$, $\Delta\boldsymbol{\sigma}_{i,C}$ and $\Delta\boldsymbol{\sigma}_{i,C_2H_4}$ if the same number of primaries is used on both targets? (efficiencies same)
- Using 200 MeV/u carbon ions, assuming similar cross sections , we estimate for fragment type ***i*** for our targets:

$$\frac{Y_{i,C}}{Y_{i,C_2H_4}} = \frac{\boldsymbol{\sigma}_{i,C}}{\boldsymbol{\sigma}_{i,C_2H_4}} \frac{\rho_C}{\rho_{C_2H_4}} \frac{A_{C_2H_4}}{A_C} \quad \textbf{(5)}$$

$$\frac{Y_{i,C}}{Y_{i,C_2H_4}} \approx 4.54 \frac{\boldsymbol{\sigma}_{i,C}}{\boldsymbol{\sigma}_{i,C_2H_4}} \approx \mathbf{1.4} \quad \textbf{(7)}$$

From previous publications and simulations:

$$\frac{\boldsymbol{\sigma}_{tot,C}}{\boldsymbol{\sigma}_{tot,C_2H_4}} \approx 0.3 \quad \textbf{(6)}$$

This is for $\boldsymbol{\sigma}_{tot,C}$ but may depend on fragment type i

$$\Delta\sigma_{i,H} = \frac{1}{4}\sqrt{(\Delta\sigma_{i,C_2H_4})^2 + 4(\Delta\sigma_{i,C})^2}$$
$$= \frac{1}{4}\sqrt{(3.8\Delta\sigma_{i,C})^2 + 4\Delta\sigma_{i,C}^2}$$
$$\approx \frac{1}{4}\sqrt{18.8}\Delta\sigma_{i,C}\,\Delta\sigma_{i,C} \approx \mathbf{1.08}\,\Delta\boldsymbol{\sigma}_{i,C} \quad \textbf{(9)}$$

$$\frac{\Delta\boldsymbol{\sigma}_{i,C_2H_4}}{\Delta\boldsymbol{\sigma}_{i,C}} = \sqrt{\frac{Y_{i,C_2H_4}}{Y_{i,C}} \frac{\rho_C\, A_{C_2H_4}}{\rho_{C_2H_4} A_C}} \approx \sqrt{\frac{1}{1.4}4.54} \approx \mathbf{3.84} \quad \textbf{(8)}$$

What errors do we expect?

- But actually, what matters are the relative errors… $\frac{\Delta\boldsymbol{\sigma}_{i,H}}{\boldsymbol{\sigma}_{i,H}}$ $\frac{\Delta\boldsymbol{\sigma}_{i,C}}{\boldsymbol{\sigma}_{i,C}}$

$$\sigma_{i,H} = \frac{1}{4}\left(\sigma_{i,C_2H_4} - 2\sigma_{i,C}\right) = \frac{1}{4}\sigma_{i,C}\left(\frac{\sigma_{i,C_2H_4}}{\sigma_{i,C}} - 2\right) \sim \frac{1}{4}\sigma_{i,C}\left(\frac{1}{0.3} - 2\right) \sim \mathbf{0.33}\ \boldsymbol{\sigma}_{i,C}$$

$$\Delta\sigma_{i,H} \approx \mathbf{1.08}\ \Delta\boldsymbol{\sigma}_{i,C} \qquad \textbf{(9)}$$

$$\frac{\Delta\boldsymbol{\sigma}_{i,H}}{\boldsymbol{\sigma}_{i,H}} \sim \frac{1.08}{0.33}\frac{\Delta\boldsymbol{\sigma}_{i,C}}{\boldsymbol{\sigma}_{i,C}} \sim \mathbf{3.3}\ \frac{\Delta\boldsymbol{\sigma}_{i,C}}{\boldsymbol{\sigma}_{i,C}} \qquad \textbf{(10)}$$

For the same nr of primaries in both target runs, relative cross section error on H is > 3 times larger than that on C (the most accurate case)…

- Does it depend on i? (type of fragment?) → see slide 9 and further (MC)

What if we double the statistics of the C_2H_4 run?

- If doubling N_p for the C_2H_4 target w.r.t. C target, we obtain:

$$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}} \sim 2.5 \frac{\Delta\sigma_{i,C}}{\sigma_{i,C}} \qquad (12)$$

- If 4 times N_p for the C_2H_4 target we obtain:

$$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}} \sim 2.1 \frac{\Delta\sigma_{i,C}}{\sigma_{i,C}} \qquad (13)$$

Decrease of statistical error is slow…

- In the case of dσ/dE and dσ/dΩ, the correct numerical factor of course depends on the actual value of $\frac{\sigma_{i,C_2H_4}}{\sigma_{i,C}}$ (or equivalently $\frac{Y_{i,C_2H_4}}{Y_{i,C}}$) in each ΔE, $\Delta\Omega$ bin for each secondary fragment type of interest, ***i***

Note

A factor 2 more for the C_2H_4 target than for C target the is the 'minimum' we should do (assuming same target thicknesses of 5 mm for now. We can also increase them if needed…)

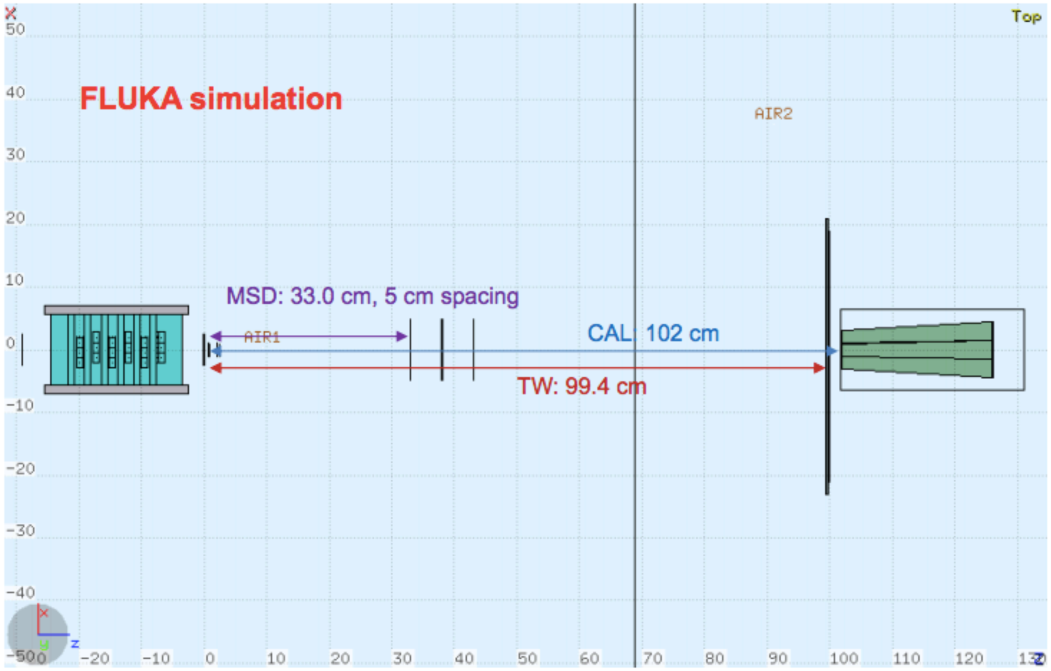
Let's now try to confirm some of these considerations with MC and check behaviour of different fragments

MC files and software used

MC files from: /gpfs_data/local/foot/Simulation/CNAO2020

- 12C at 200 MeV/u on C
  - filename:12C_C_200.root
  - 284246 events on file
  - 5 mm
  - rho=1.83 g/cm3)
  - 10^7 primaries
- 12C at 200 MeV/u on C2H4
  - Filename:
    - 12C_C2H4_200_1.root, 198215 events
    - 12C_C2H4_200_2.root, 197621 events
  - 5 mm
  - rho=0.94 g/cm3
  - 10^7 primaries

Geometry: campaign CNAO2020



Focus on what we can do only with SC and TOF-Wall (no mass discrimination)

Fragment production at CNAO

To cross check formulas 5 and 7, analyzed yield for fragments of 200 MeV/u ^{12}C produced in C and C_2H_4 target (both with $N_p=10^7$)

Z of fragment i	$Y_{i,C}$	Y_{i,C_2H_4}	$\frac{Y_{i,C}}{Y_{i,C_2H_4}}$
1	334288	207099	1.61
2	274852	197885	1.39
3	28158	22329	1.26
4	15405	13240	1.16
5	32617	26699	1.22
6	26183	26396	0.99

Starting with $N_p=10^7$, how many have inelastic interactions?

- Carbon: about 6%
- Ethylene: about 4%

$$\frac{Y_{i,C}}{Y_{i,C_2H_4}} \approx 4.54 \frac{\sigma_{i,C}}{\sigma_{i,C_2H_4}} \approx \mathbf{1.4} \qquad \textbf{(7)}$$

Note that mostly ^{11}C (see backup for overview of produced isotopes), may be hard to distinguish from ^{12}C primary

- Ratio between C yield and C_2H_4 yield varies with Z
- Goes down for heavier fragments

Fragment production at CNAO

To cross check formula 6, converted fragment (from target) yields to cross sections with formula 1a and 1b for C and C_2H_4, respectively

Z of fragment i	$\frac{\sigma_{i,C}}{\sigma_{i,C_2H_4}}$
1	0.36
2	0.31
3	0.27
4	0.26
5	0.27
6	0.21

$$\frac{\sigma_{i,C}}{\sigma_{i,C_2H_4}} \approx 0.3 \qquad \textbf{(6)}$$

- **Cross section ratio $\frac{\sigma_{i,C}}{\sigma_{i,C_2H_4}}$ not constant**
- **Decreases for heavier fragments**

Fragment production at CNAO

To cross check formula 8, evaluated fragment yields and factors:

Z of fragment i	$\frac{\Delta\sigma_{i,C_2H_4}}{\Delta\sigma_{i,C}}$
1	3.57
2	3.85
3	4.04
4	4.21
5	4.11
6	4.56

$$\frac{\Delta\sigma_{i,C_2H_4}}{\Delta\sigma_{i,C}} = \sqrt{\frac{Y_{i,C_2H_4}}{Y_{i,C}}}\,\frac{\rho_C\,A_{C_2H_4}}{\rho_{C_2H_4}A_C} \approx \sqrt{\frac{1}{1.4}}\,4.54 \approx \mathbf{3.8} \qquad (8)$$

- $\frac{\Delta\sigma_{i,C_2H_4}}{\Delta\sigma_{i,C}}$ **is not constant**
- **Increases for heavier fragments**

Fragment production at CNAO

To cross check formulas 9, 10 and 11, evaluated all statistical errors:

Z of fragment i	$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}}$	$\frac{\Delta\sigma_{i,C}}{\sigma_{i,C}}$	$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}} / \frac{\Delta\sigma_{i,C}}{\sigma_{i,C}}$
1	0.87	0.17	5.0
2	0.65	0.18	3.4
3	1.68	0.60	2.8
4	1.97	0.81	2.4
5	1.47	0.55	2.7
6	1.19	0.62	1.9

$$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}} \sim \frac{1.08}{0.33}\frac{\Delta\sigma_{i,C}}{\sigma_{i,C}} \sim 3.3\frac{\Delta\sigma_{i,C}}{\sigma_{i,C}}$$

Note

The relative error on cross section varies with Z

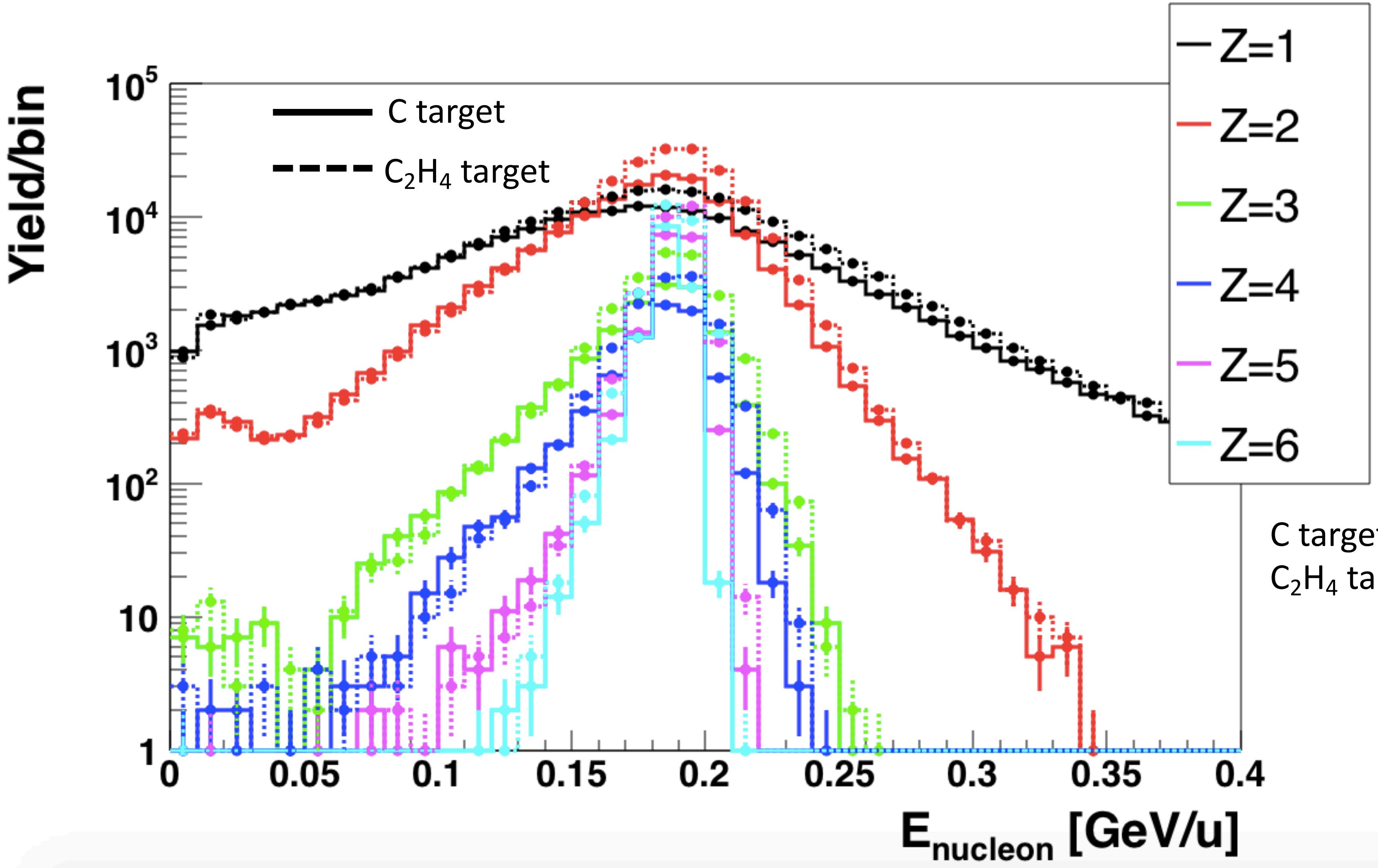
What do we expect at CNAO?

- Assume that we take data at low intensity: about 1000 primaries/s in the spill → given that the duty cycle is 50%, about 500 primaries/s
- Firing 10^7 primaries would take $10^7/500$ s, i.e., 5.5 hours… which is long… (shift is about 8 hours)
- As said before (slide 7), run with C_2H_4 target with double number of primaries

N_p for C target	N_p for C_2H_4 target	Total estimated run time
10^7	2 x 10^7	5.5+11=16.5 hours: **no**
$5x10^6$	10^7	2.7+5.5~8.2 $\gtrsim$ 8 hours: **maybe**
$4x10^6$	$8x10^6$	2.2+4.4~6.6 < 8 hours: **ok**

- What would be obtain with $5x10^6$ primaries for C target and 10^7 primaries for C_2H_4 target ?
  - dN/dE (per nucleon)
  - $d\sigma/dE$ (per nucleon)
- Distinguish the fragments only in Z for now, MC truth

dN/dE MC truth: fragments from target



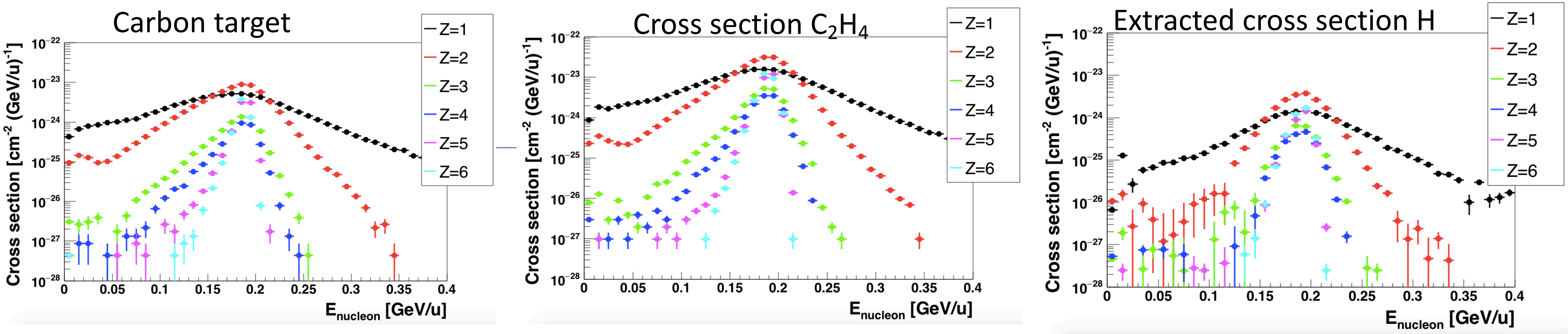
Bin size=10 MeV

C target: 5×10^6 primaries
C_2H_4 target: 10^7 primaries

d$\boldsymbol{\sigma}$/$E_{nucleon}$ for MC truth (fragments from target)

C target: $5\text{x}10^6$ primaries
C_2H_4 target: 10^7 primaries

Applying the appropriate factors to translate yields into cross sections:



- Would be at the limits of run time (~8 hours)
- Still acceptable result with $5\text{x}10^6$ primaries for C target, and 10^7 primaries for C_2H_4 target
- Errors: heavier fragments have large errors

What about the relative errors

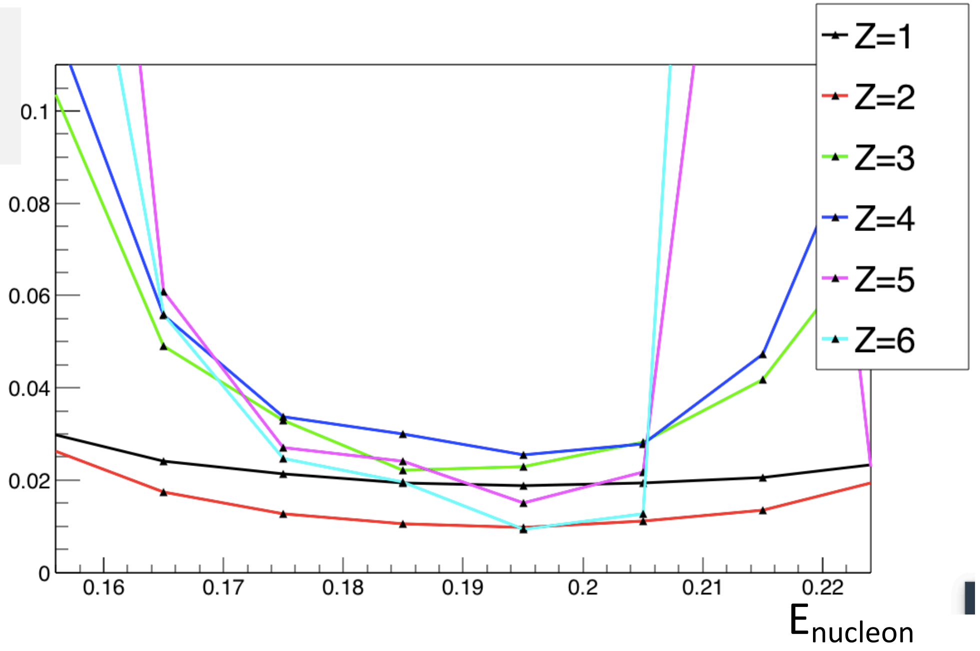
- What about the relative errors?

$$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}} \qquad \frac{\Delta\sigma_{i,C}}{\sigma_{i,C}} \qquad \frac{\Delta\sigma_{i,C_2H_4}}{\sigma_{i,C_2H_4}}$$

No details… (apologies for the ugly plot), but we saw that:

- Largest relative errors are expected at higher Z: Z≥3
- Most problematic in less populated energy bins

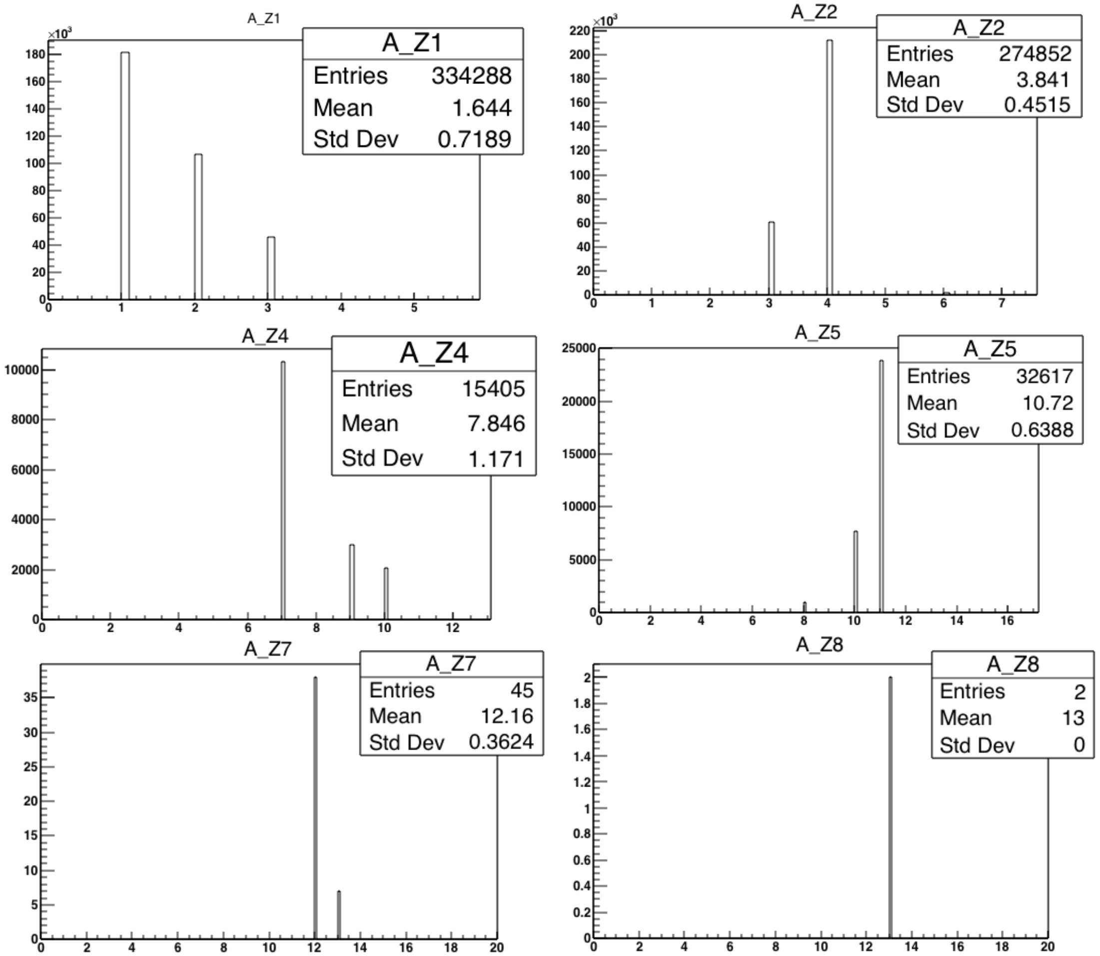
$$\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}}$$



Conclusions

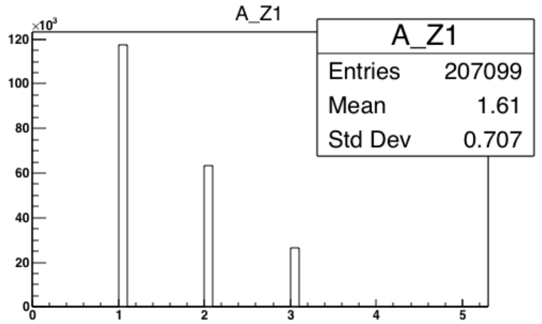
- Some statistical considerations were made about the CNAO run
- We need more primaries for the C_2H_4 target than for the C target
- Given the slow decrease of the error on $\frac{\Delta\sigma_{i,H}}{\sigma_{i,H}}$, probably for a given energy we can point at $n*10^6$ primaries of C (preferably with n not too far away from 5) and $2n*10^6$ for C_2H_4
- Showed some first plots of what can be expected at CNAO with n=5, which is at the limit of what we can get (~8 hours)
- Largest relative errors on cross sections for larger Z (say $Z\geq3$)
- The present analysis is preliminary and there are other aspects in the overall aspect of measurement errors which are connected to the size of statistical sample. An example can be
  - The evaluation of background
  - ….

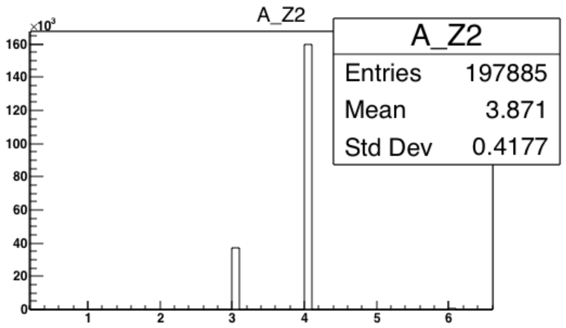
Mass isotopes for carbon target

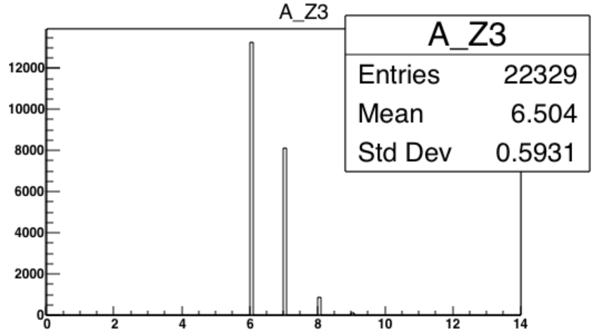


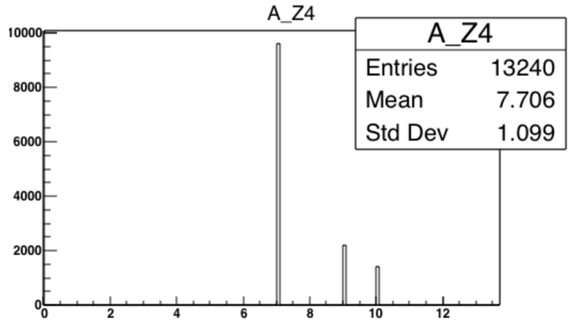
Use to extract average mass for carbon target

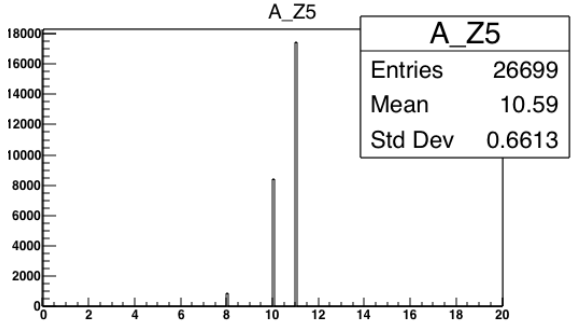
Mass isotopes for C_2H_4 target

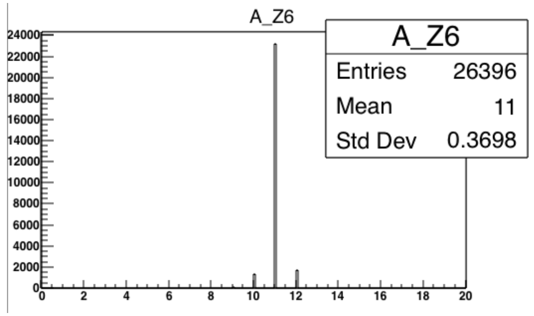


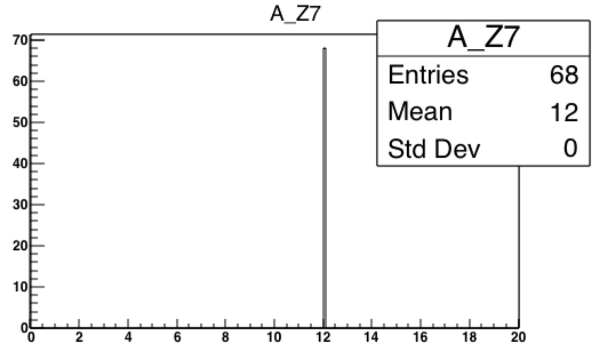


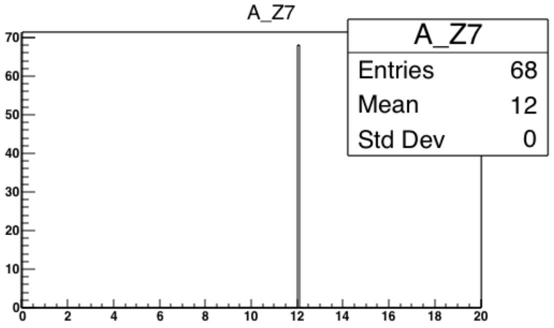












Use to extract average mass for C2H4 target