Threshold resummation of Drell-Yan rapidity distribution

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in collaboration with

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Outline

- Inclusive Cross Section
- Fixed Order Predictions and Associated Problems
- All Order Resummation
- Rapidity Spectrum
- Fixed Order Vs All Order
- Summary & Future Outlook

Drell-Yan: Inclusive Production

Process:

$$h_1(p_1^\mu) + h_2(p_2^\mu) \to l\bar{l}(Q^\mu) + X$$

Parton model factorization

$$Q^2 \frac{d\sigma}{dQ^2} = \sigma_0(Q^2,\tau) \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz\, \delta(\tau - x_1 x_2 z)$$
$$\times f_a^{h_1}(x_1,\mu_F^2) f_b^{h_2}(x_2,\mu_F^2) \Delta^{ab}\left(z, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$\tau = \frac{Q^2}{s(= 2p_1.p_2)}; z = \frac{Q^2}{x_1 x_2 s} = \frac{Q^2}{\hat{s}}$$

μ_R : renormalization scale

μ_F : mass factorization scale

Structure of Partonic Coefficient Function

At each perturbative order

$z \to 1$ (Soft/Threshold limit)

$$\Delta(z) = \Delta^{\text{sing.}}(z) + \Delta^{\text{reg.}}(z)$$

$$\Delta^{\text{sing.}}(z) = C_\delta(\alpha_s)\delta(1-z) + \sum_{i=0}^{\infty} C_{\mathcal{D}_i}(\alpha_s)\mathcal{D}_i(z)$$ + Coll. Logs

Sub-leading

$$D_i(z) \equiv \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

$$\int_0^1 dz f(z)\mathcal{D}_i(z) \equiv \int_0^1 dz \frac{f(z)-f(1)}{1-z}\ln^i(1-z)$$

One of the solutions: All order resummation

- Can be large enough to potentially spoil a perturbative series

- Solution: Threshold resummation, a technique to include/resum these logarithms to all orders in perturbation theory

  [Sterman (1987), Catani-Trentadue (1989)]

- Can be used to improve the calculation accuracy that needs high precision and has been very successful in perturbative QCD

Mellin Space: Phase-Space Factorization

- Resummation is most naturally performed in Mellin, N-space

- Underlying reason: factorization of phase-space happens in N-space not in z-space $\Rightarrow$ threshold logarithms are exponentiated

$$\sigma_N(Q^2) \equiv \int_0^1 d\tau \tau^{N-1} \sigma(Q^2, \tau)$$
$$= f_N * f_N * \Delta_N$$

$z \to 1$

$\downarrow$

$N \to \infty$

Under Mellin transformation

$$\left[\frac{\ln^i(1-z)}{1-z}\right]_+ \to (\ln N)^{i+1} + \ldots + \text{sub-leading terms}$$

Resummation

- Contribution to the threshold logarithms to all orders in perturbation theory is known to be an exponential of the form:

$$\Delta_N^{\text{sing.}} \to \Delta_N^{\text{res.}} = g_0(\alpha_s) e^{G(N,\alpha_s)}$$

Born kinematics

where

$$G(N,\alpha_s) = \ln N \, g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \ldots$$

$1 \approx \lambda = \beta_0 \alpha_s \ln N$

Consecutive terms are separated by a power of strong coupling

Resummation: Accuracy

- Accuracy: Power of the threshold logarithms included in $G(N, \alpha_s)$ relative to the strong coupling

- To have next(k-times)-leading logarithmic accuracy $(N^k LL)$, g_{k+1} must be included and g_0 must be computed up to α_s^k order

- Expansion of $\Delta_N^{\text{res.}}$ with accuracy $N^n LL$ in powers of $\alpha_s(Q^2)$ up to order n correctly predicts all the threshold enhanced logarithmic contributions in $\Delta_N^{\text{sing.}}$ up to the same order

Resummation: Matching

- Exclusively using resummation to calculate an observable loses information about kinematic regions away from threshold and hence, not recommended

- Adding a fixed order result to the resummed calculation retains the information that is not enhanced at threshold and it is done through a process called matching to avoid double counting

$$\sigma^{\text{res.}}(Q^2,\tau) = \sigma^{\text{f.o.}}(Q^2,\tau) + \sigma_0(Q^2,\tau)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN \tau^{-N}\,(f_N)^2 [\Delta_N^{\text{res.}} - \Delta_N^{\text{res.}}|_{\text{f.o.}}]$$

A parameter determined by resummation prescription

Matching

Resummation: Prescriptions

- The Mellin inversion involves integration of the running strong coupling at the Landau pole

$$N = N_L = \exp\left(\frac{1}{2\beta_0\alpha_s}\right)$$

- Therefore naively taking inverse Mellin transform of the resummation formula will include information from the Landau pole

- A prescription which makes a divergent series asymptotic must be adopted to remove these spurious effects from the resummation calculation

Prescriptions: Minimal Vs Borel

Minimal(MP)	Borel(BP)
[Catani, Mangano, Nason, Trantadue (1996)]	[Forte, Ridolfi, Rojo, Ubiali, Abbate (2006, 2007)]

- Differ in the way the high-order behaviour of the divergent series is handled
- In particular, they differ in sub-leading terms that do not increase as rapidly as threshold logarithms of the desired logarithmic accuracy
- To further pinpoint, in the MP all the 1/N suppressed terms in N-space are set to 0, but this leads to (1-z) power suppressed terms in z-space, while in the BP the opposite is true

State-of-the-art Calculations

- Threshold resummation method has been very successful for the inclusive cross section

- Using resummation the theoretical uncertainty from missing higher orders has been brought down to few % for the Higgs production

  [Catani-de Florian-Grazzini-Nason (2003), Bonvini-Marzani-Muselli-Rottoli (2016)]

- Similar high accuracy has also been achieved for the lepton pair production through Drell-Yan

  [Moch-Vogt (2005), Catani-Cieri-de Florian-Ferrera-Grazzini (2014)]

- Obvious extension: to more differential observables such as Rapidity

DY rapidity distribution

Rapidity distribution is given by

$$Q^2 \frac{d\sigma}{dQ^2 dY}(\tau, Y, Q^2) = \sigma_0(Q^2, \tau) \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz_1 \int_0^1 dz_2 \delta(x_1^0 - x_1 z_1) \delta(x_2^0 - x_2 z_2)$$

$$\times f_a^{h_1}(x_1, \mu_F^2) f_b^{h_2}(x_2, \mu_F^2) \Delta_y^{ab} \left(z_1, z_2, y, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right)$$

$$\tau = \frac{Q^2}{s(= 2p_1.p_2)}; x_{1,2}^0 = \sqrt{\tau} e^{\pm Y} \quad \Big| \quad y = Y - \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right); z = z_1 z_2 = \frac{\tau}{x_1 x_2}$$

Fixed order: Integrable Singularity

Similarly, to the inclusive cross section

$z_{1,2} \to 1$ (Soft / threshold limit)

$$\Delta(z_1, z_2) = \Delta^{\text{sing.}}(z_1, z_2) + \Delta^{\text{reg.}}(z_1, z_2)$$

$$\Delta^{\text{sing.}}(z_1, z_2) = C_\delta(\alpha_s)\delta(1-z_1)\delta(1-z_2) + \sum_{i,j=-1}^{\infty} C_{\mathcal{D}_{ij}} \mathcal{D}_i(z_1)\mathcal{D}_j(z_2)$$ + Coll. logs

Sub-leading

$$\mathcal{D}_{-1}(z_j) = \delta(1-z_j)$$

Rapidity Resummation: Comments

- It follows similar steps as for the inclusive case

- Technically it is more difficult as hadronic rapidity complicates the factorization of soft gluon phase space

$$\delta(x_1^0 - x_1 z_1) * \delta(x_2^0 - x_2 z_2)$$

- Unlike inclusive case, two integral transforms are required for the rapidity distribution to do the same

Rapidity Resummation: Existing Approaches

- In methods currently employed in phenomenological studies: a Fourier transform w.r.t. rapidity and Mellin transform w.r.t. z

$$\{z_1, z_2\} \leftrightarrow \{z, y\}$$

- By E. Laenen and G. F. Sterman: it was conjectured to provide an approximation to the threshold resummed rapidity spectrum at y=0

- Performing Mellin-Fourier (M-F) transform and neglecting some terms, one could express resummation in rapidity effectively in terms of rapidity integrated resummed exponent

[Mukherjee-Vogelsang (2006), Bolzoni (2006)]

Rapidity Resummation: Existing Results

- Following the approach, a phenomenologically complete study with higher resummation accuracy (NNLO+NNLL) was achieved both for neutral as well as charged DY

[Bonvini-Forte-Ridolfi (2010)]

- Similar studies are also performed from the SCET side

[Becher-Neubert-Xu (2007)]

- In all the above studies: $\left[\frac{\ln^i(y)}{y}\right]_+ \to 0$

Existing Approaches: A Different One

- There is a second approach originally applied to Feynman variable (x_F) which includes distributions in both partonic rapidity (y) and inclusive scaling variable, z

[Catani-Trentadue (1989)]

- Two Mellin (M-M) transforms were performed corresponding to two variables to have phase-space factorization and include all the threshold enhanced logarithms into an exponential

$$\int_0^1 dx_1^0 (x_1^0)^{N_1-1} \int_0^1 dx_2^0 (x_2^0)^{N_2-1} \delta(x_1^0 - x_1 z_1)\delta(x_2^0 - x_2 z_2)$$

Existing Result and Our Extension

- The latter approach was successfully applied to both Feynman variable and rapidity spectrum at NLO+NLL accuracy and shown to improve the existing result in high-x_F/Y region resulting in better agreement with the data

  [Westmark-Owens (2017)]

- We extended the above work to a higher resummation accuracy i.e. NNLO+NNLL in M-M space and presented the predictions for both Higgs and DY rapidity spectrum at the LHC energies

  [Banerjee-Das-Ravindran+PKD (2017, 2018)]

Comments

- One of the consequences of the resummation exponent existing in M-M space is that the Landau pole no longer corresponds to a single Mellin moment, instead the Landau pole occurs on the curve

$$N_1 N_2 = \exp\left(\frac{1}{\alpha_s \beta_0}\right)$$

- While using MP, the paths of the two inverse Mellin transforms must be to the right of all poles in both N_1 and N_2 space, but performed in such a way that the Landau pole lies to the right of the paths

Rapidity Resummation: Matching

Using MP

NNLO

$$\sigma^{\text{res.}}(Q^2,\tau,Y) = \sigma^{\text{f.o.}}(Q^2,\tau,Y) + \sigma_0(Q^2,\tau,Y)\int_{c_1-i\infty}^{c_1+i\infty}\frac{dN_1}{2\pi i}\int_{c_2-i\infty}^{c_2+i\infty}\frac{dN_2}{2\pi i}$$

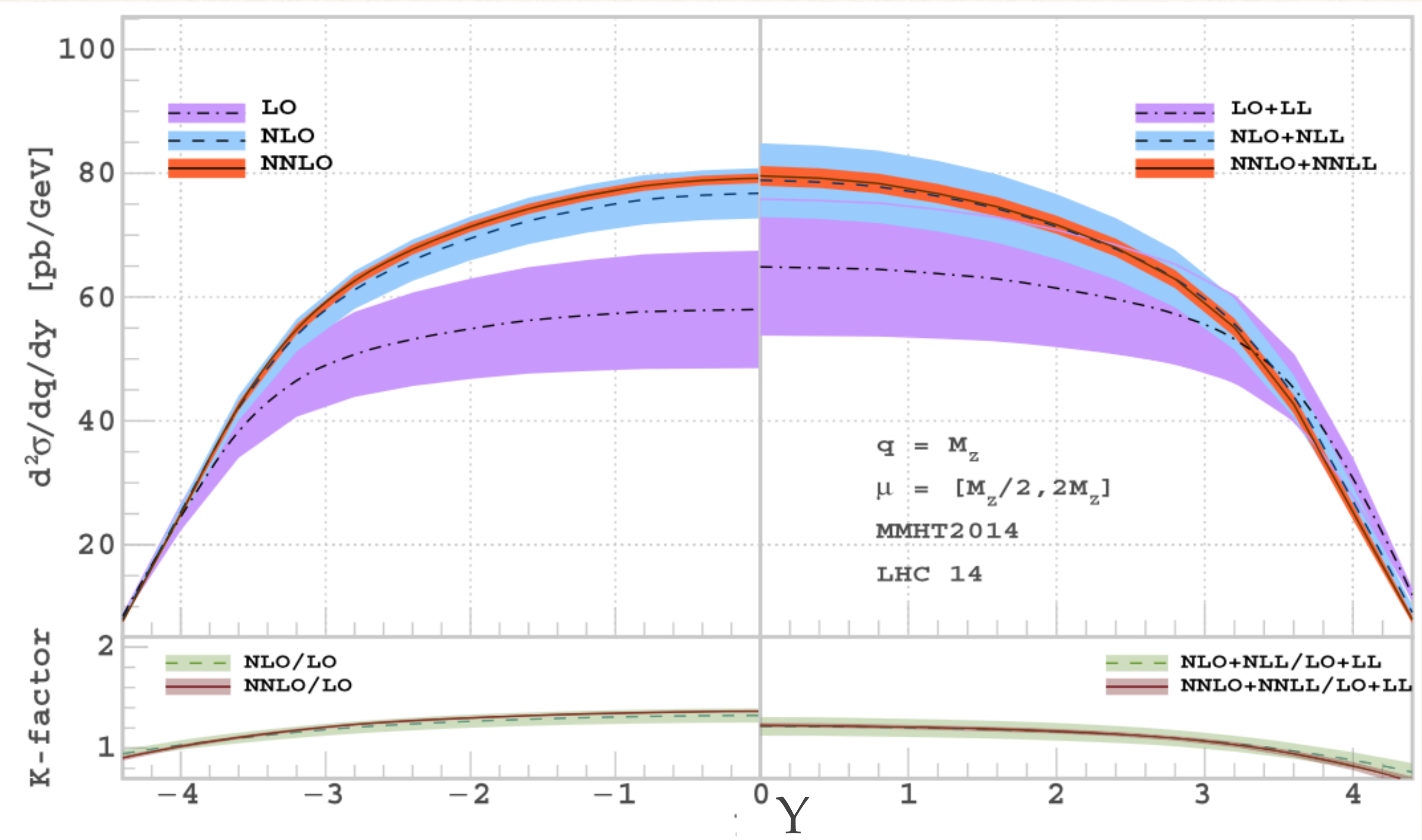
$$\times\tau^{-\frac{N_1+N_2}{2}}e^{Y(N_2-N_1)}f_{N_1}f_{N_2}\left[\Delta^{\text{res.}}_{N_1,N_2} - \Delta^{\text{res.}}_{N_1,N_2}\big|_{\text{f.o.}}\right]$$

NNLL

Phenomenology: $\sqrt{s} = 14\,\text{TeV}\,;\;\; Q = M_Z\,;$ PDF: MMHT14(68cl)

- Fixed order from Vrap [Anastasiou-Dixon-Melnikov-Petriello (2003)]

Fixed Order Vs All Order



Resummation increases the cross section across Y-values

Better overlap compared to F.O.

But large uncertainty band

Reason:

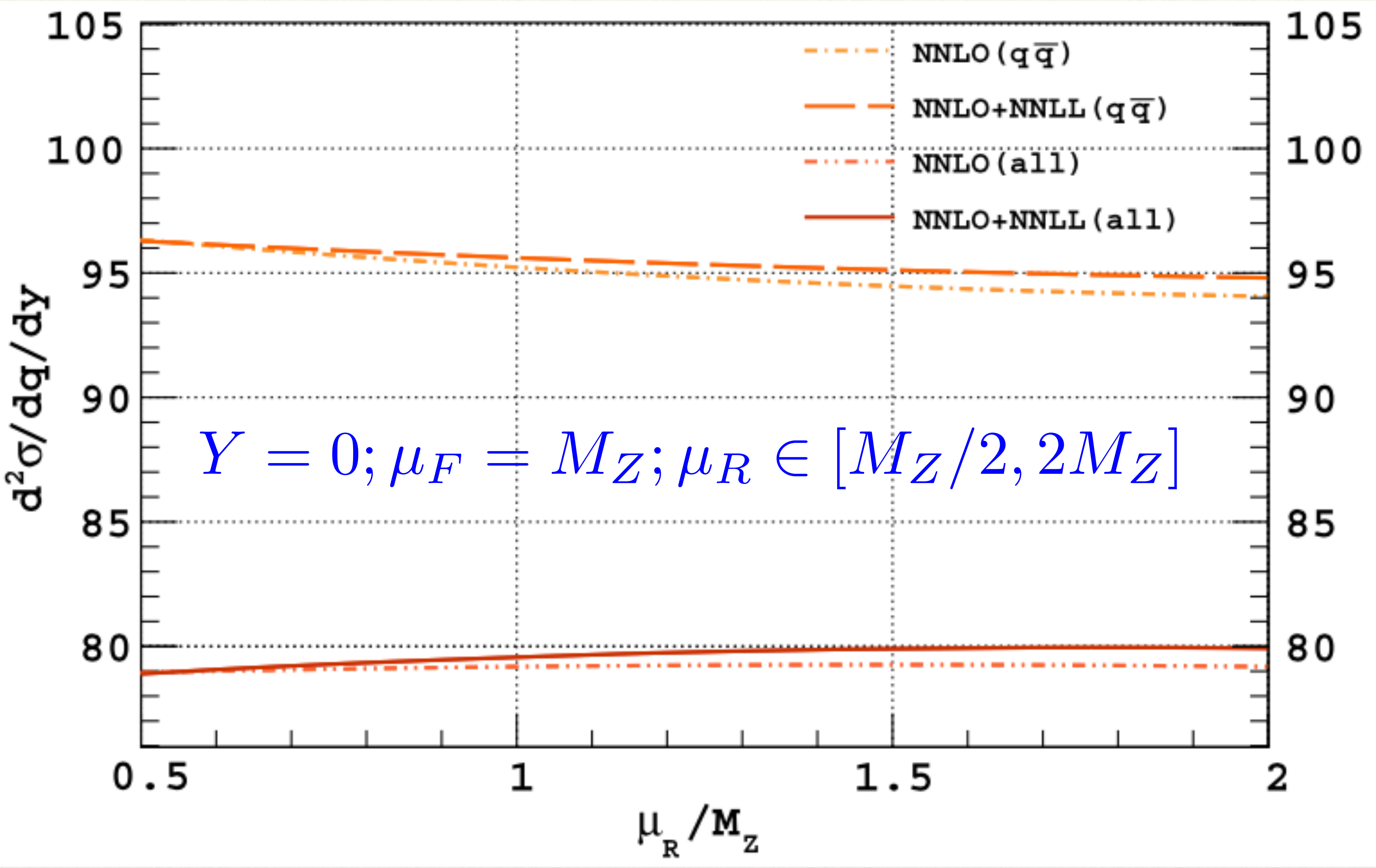
[Banerjee-Das-Ravindran+PKD (2018)]

Central scale choice: $\mu_R = \mu_F = M_Z$

Only qQ-channel contributes

Most PDFs are lack of resummation effect

qQ Vs All other channels



qQ: resummation brings more stability

From M_Z/2 to 2M_Z

F.O. decreases by 2.36 %

Corresponding number for the resummation one is 1.53%

[Banerjee-Das-Ravindran+PKD (2018)]

Fixed Order Vs All Order

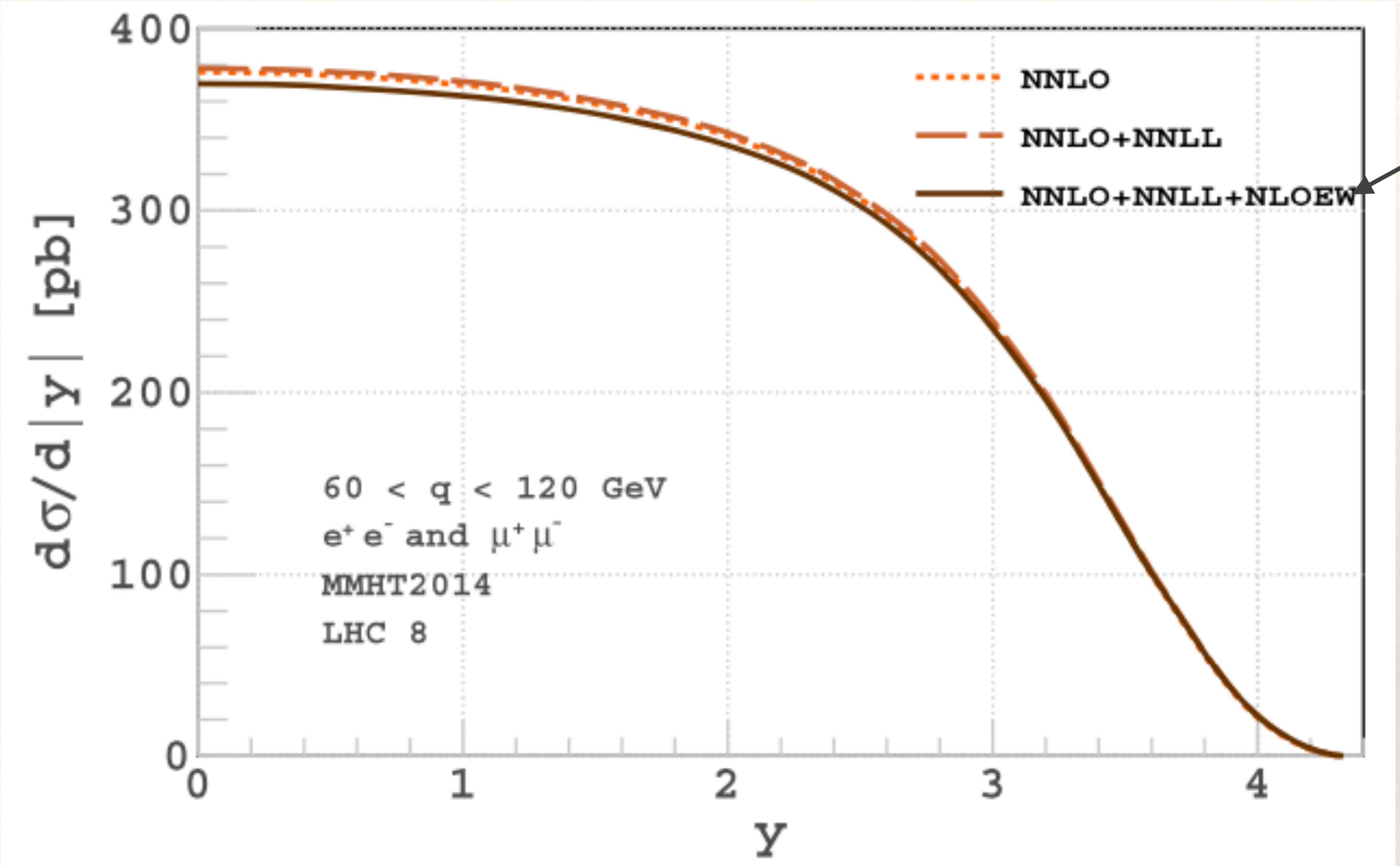
Y	NLO	NLO + NLL	NNLO	NNLO + NNLL
0.0	76.758±5.28%	78.867±7.56%	79.182±0.98%	79.568±2.02%
0.8	75.727±5.26%	77.797±7.53%	77.968±1.04%	78.340±2.03%
1.6	72.295±5.17%	74.274±7.45%	74.239±1.11%	74.588±2.08%
2.4	65.953±5.04%	67.772±7.33%	67.678±1.21%	67.985±2.11%

[Banerjee-Das-Ravindran+PKD (2018)]

$K_{\mathrm{NLO}} \in \{1.3, 1.2\}$ $\qquad$ $K_{\mathrm{NNLO}} \in \{1.37, 1.30\}$

$$K_{\mathrm{N(N)LO+N(N)LL}} \sim 1.2$$

Q-integrated Rapidity Distribution



Horace-3.2

NNLL: 0.5 %

NLO_EW is -ve

Overall 2.3% decrease

[Banerjee-Das-Ravindran+PKD (2018)]

M-M Vs M-F

y	$(\frac{\mu_R}{M_Z}, \frac{\mu_F}{M_Z})$	LO	$LL_{M\text{-}F}$	$LL_{M\text{-}M}$	NLO	$NLL_{M\text{-}F}$	$NLL_{M\text{-}M}$	NNLO	$NNLL_{M\text{-}F}$	$NNLL_{M\text{-}M}$
0.0	(2, 2)	72.626	+0.988	+3.219	73.450	+1.639	+1.796	70.894	+0.630	+0.646
0.0	(2, 1)	63.197	+0.768	+2.595	70.625	+0.761	+1.017	70.360	+0.292	+0.317
0.0	(1, 2)	72.626	+1.095	+3.577	73.535	+1.912	+1.760	70.509	+0.510	+0.395
0.0	(1, 1)	63.197	+0.851	+2.887	71.395	+0.858	+0.901	70.537	+0.248	+0.167
0.0	(1, 1/2)	53.241	+0.621	+2.216	67.581	+0.156	+0.140	69.834	−0.001	−0.094
0.0	(1/2, 1)	63.197	+0.953	+3.278	72.355	+0.945	+0.681	70.266	+0.091	−0.015
0.0	(1/2, 1/2)	53.241	+0.695	+2.504	69.259	+0.102	−0.154	70.283	−0.039	−0.146

[Banerjee-Das-Ravindran+PKD (2018)]

- For the M-F results, we use publicly available code ReDY [Bonvini-Forte-Ridolfi (2010)]

- We find that predictions from M-F and M-M are comparable at higher resummation accuracy in the central rapidity region.

Summary & Future Outlook

- Using threshold resummation technique, we have included large threshold logarithms in both partonic rapidity and scaling variable up to NNLO+NNLL accuracy for the lepton pair production

- Resummation has been performed in M-M space where kinematics gets factorized and matched to fixed order result to have better prediction for the rapidity spectrum

- Finally, we have presented our predictions for the LHC-14 and compared our results with the existing results in the literature.

Comments & Future Outlook

- Note that, I only talked about resummation of dominant contributions in the threshold limit i.e. distributions in the partonic variables.

- For some observables, next-to dominant terms such as $\ln(1 - z_i)$ are important and should be included to high orders

- Very recent work from SCET suggesting a formalism for generalized threshold limit i.e.

  $z_1 \to 1$ for arbitrary z_2 and vice versa

  [Lustermans-Michel-Tackmann (2019)]

- It would be interesting to see such a formalism from QCD side

Comments & Future Outlook

- Fixed order calculations also contain logarithms that are enhanced in the small-z limit suggesting their inclusion to high orders in perturbation theory. Such terms are of the form

$$\frac{\ln^i(z)}{z} \quad \text{enhanced in the limit } z \to 0$$

[Balitsky-Fadin-Kuraev-Lipatov]

- Recent work on inclusive Higgs production which includes towers of logarithms in both extremes

[Bonvini-Marzani (2018)]

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Thank You