Precision Measurement of ππ Scattering Lengths from Ke4 and K3π Decays

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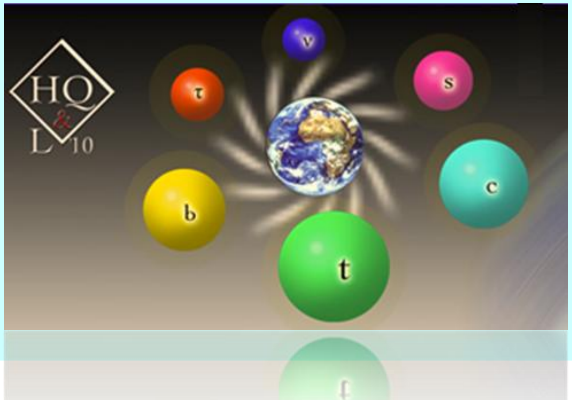
HQL 2010 – Heavy Quarks & Leptons



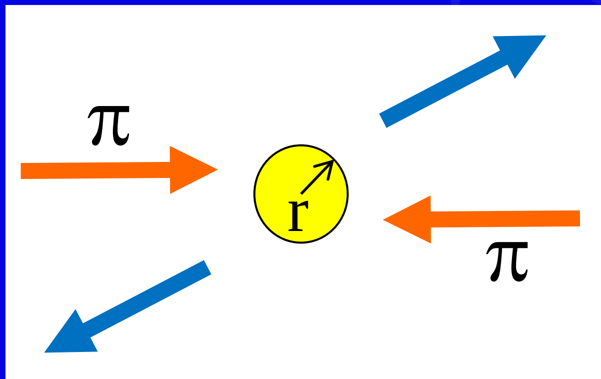
LNF-INFN, 11-15 October, 2010

*On behalf of the NA48 collaboration

Nicola Cabibbo
(1935-2010)

Collaborator of the NA48 and NA62 experiments since 2004

ππ scattering lengths



$$k = \frac{\sqrt{2mE}}{\hbar}$$

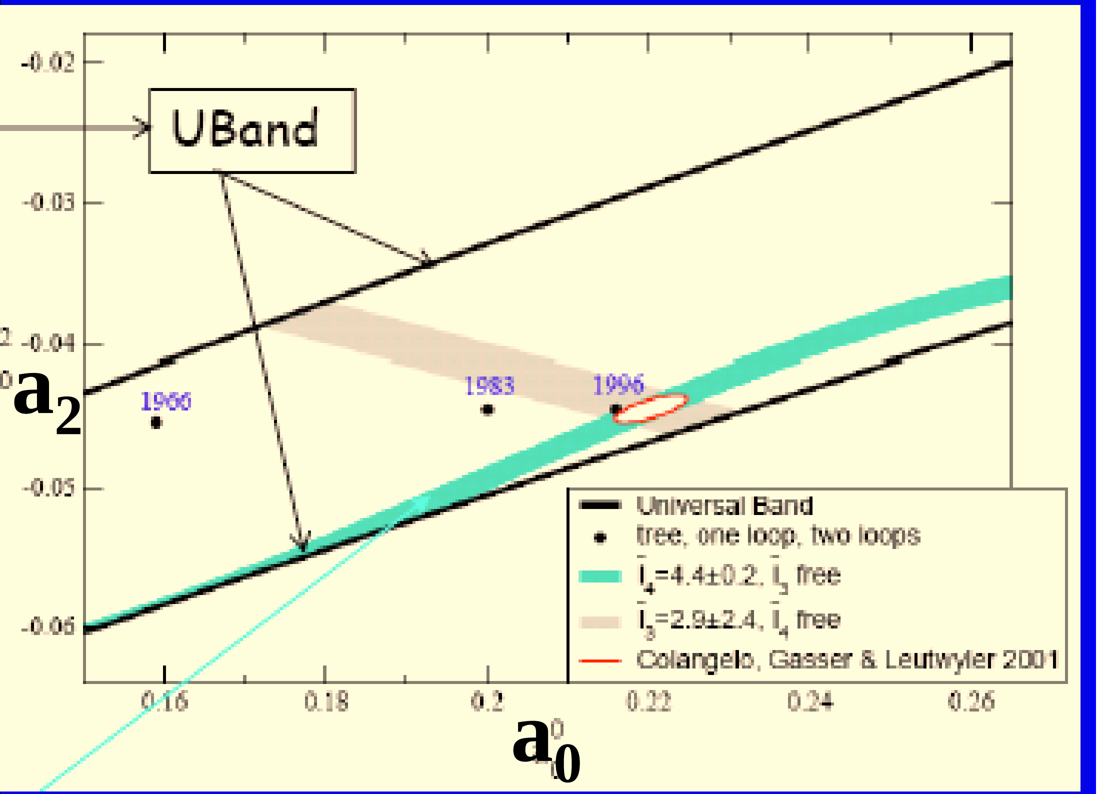
At low energy, **kr « 1**, **S-wave** dominates the scattering amplitudes, with **Isospin = 0, 2**

- Scattering matrix : $S|\pi\pi> = e^{2i\delta}|\pi\pi>$
  may be parametrized using 2 phases: $\delta_{0,2} = a_{0,2}k + O(k^2)$

- At low energy **S-wave scattering lengths** a_0, a_2 are essential parameters of **Chiral Perturbation Theory (ChPT).**

- Progress in experimental measurements allows a **stringent test of ChPT predictions.**

Theoretical predictions



ChPT constraint:

$$a_2 = -0.0444(8) + 0.236\,(a_0 - 0.22) - 0.61\,(a_0 - 0.22)^2 - 9.9\,(a_0 - 0.22)^3$$

- Scattering lengths $\mathbf{a_0, a_2}$ are directly connected to $\mathbf{m_\pi}$:

$$a_0 \sim \frac{7\, m_\pi^2}{32\, \pi\, F_\pi^2} = 0.16$$

$$a_2 \sim \frac{-\, m_\pi^2}{16\, \pi\, F_\pi^2} = -0.045$$

(Weinberg, PRL 17 (1978) 275)

- Precise prediction within ChPT:

$$a_0 = 0.220 \pm 0.005$$
$$a_2 = -0.0444 \pm 0.0010$$

(Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

Measuring ππ scattering lengths

Three different approaches to measure a_0 and a_2

Semileptonic decay mode Ke4

$K^{\pm} \rightarrow \pi^{+} \pi^{-} e^{\pm} \nu$

- **Measurement of a_0, a_2**

- small BR= 4.1 x 10^{-5}
- S118 (1977): 30,000 events
- BNL E685 (2003): 400,000 events
- NA48/2 (2009): 1.1 x 10^{6} events

Pionium lifetime $(\pi^{+} \pi^{-})_{atom}$

- **Measurement of $|a_2 - a_0|$**
- DIRAC CERN/PS ~40% data analyzed: 6,500 events *(PLB 619 (2005)*

Hadronic decay mode K3π

- **Measurement of $(a_2 - a_0)$, a_2**

Cusp in $K^{\pm} \rightarrow \pi^{0} \pi^{0} \pi^{\pm}$

- large BR = 1.7%
- NA48/2: 60 x 10^{6} events

Cusp in $K_L \rightarrow \pi^{0} \pi^{0} \pi^{0}$

- large BR = 19.6%
- KTEV, NA48/2: 70+100 x 10^{6} events

$\pi\pi$ scattering lengths in Ke4 decays

in Ke4 decays

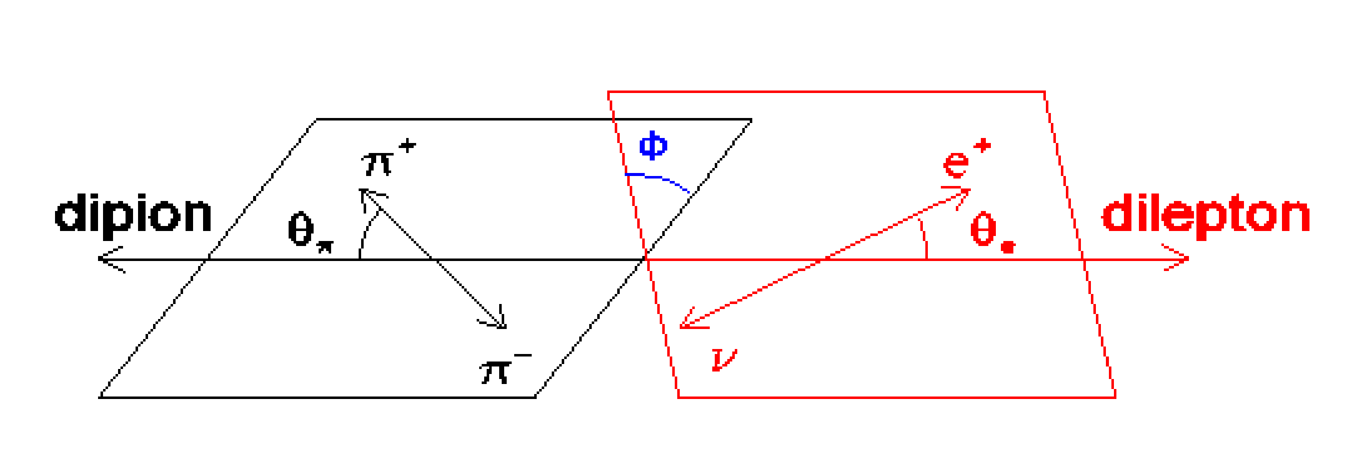
$\pi\pi$ scattering lengths from K_{e4} decays

- Ke4 decay amplitude depends on two complex phases:
  - δ_S ➡ $\pi\pi$ scattering phase shift for **I=0, l=0** (S-wave)
  - δ_P ➡ $\pi\pi$ scattering phase shift for **I=1, l=1** (P-wave)

  (I=2 is suppressed by the $\Delta I=1/2$ rule)

- Decay rate depends on difference $\delta = \delta_S - \delta_P$, with $\delta = \delta(m_{\pi\pi})$

- $\delta \neq 0$ implies **asymmetric distribution of lepton** w.r.t. $\pi\pi$ plane

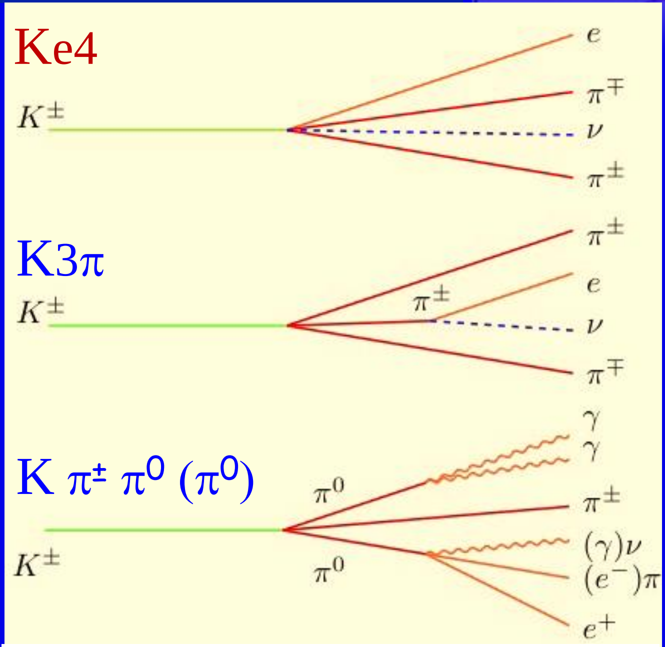


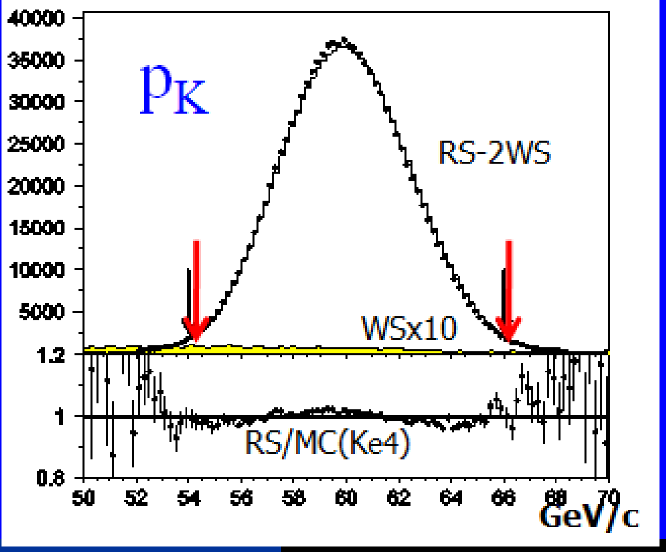
K_{e4} events selection

- **Ke4 decay event selection**:
  - 3 charged tracks and 1 good vertex
  - 2 opposite sign π, 1 electron (E/p ~1)
  - missing transverse momentum
  - kaon momentum close to 60 GeV/c
- **Background**:
  - $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ with $\pi \to e\nu$ or mis-identified pion
  - $K^{\pm} \to \pi^{\pm}\pi^{0}(\pi^{0})$ with $\pi^{0} \to e^{+}e^{-}\gamma$ and mis-identified electron
  - Background estimation from wrong sign (WS) i.e. $K^{+} \to \pi^{+}\pi^{+}e^{-}$ events

➡ **Background ~ 0.6%**

➡ **1.13×10^{6} events**



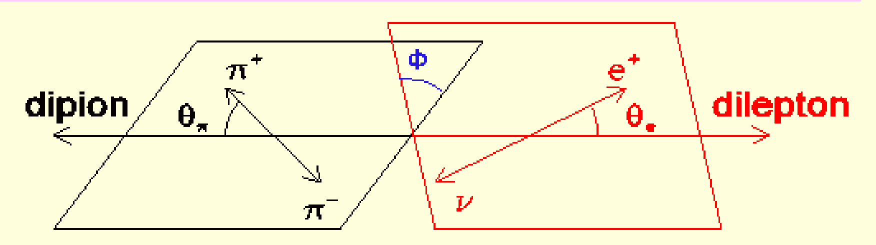


Formalism of K_{e4} decay

- K_{e4} is a 4-body decay ⟹ **5 independent kinematic variables**

(Cabibbo-Maksymowicz variables, 1965)

$$\mathbf{S_\pi = m^2_{\pi\pi}\ ;\ S_e = m^2_{e\nu}\ ;\ \cos\vartheta_\pi\ ;\ \cos\vartheta_e\ ;\ \Phi}$$



- **Partial wave expansion of the amplitude** *(Pais-Treiman 1968)*

F,G are 2 Axial Form Factors:

- $F = F_S\, e^{i\delta_s} + F_P\, e^{i\delta_p} \cos\vartheta_\pi$ + D-wave term
- $G = G_P\, e^{i\delta_p}$ + D-wave term

H= 1 Vector Form Factor:

- $H = H_P\, e^{i\delta_p}$ + D-wave term

$F(F_S, F_P)$, G, H, $\delta = \delta_P - \delta_S$ are the fit parameters

- **q^2 dependence** can be studied expanding fitted form factors:

(Amoros-Bijens 1999)

- $F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e\, (m^2_{e\nu}/4\, m^2_\pi) + \dots$
- $F_p = f_p + f_p' q^2 + \dots$
- $G_p = g_p + g_p' q^2 + \dots$
- $H_p = h_p + h_p' q^2 + \dots$

with $q^2 = (m^2_{\pi\pi}/4\, m^2_\pi) - 1$

(this Taylor expansions are valid in the Isospin symmetry limit)

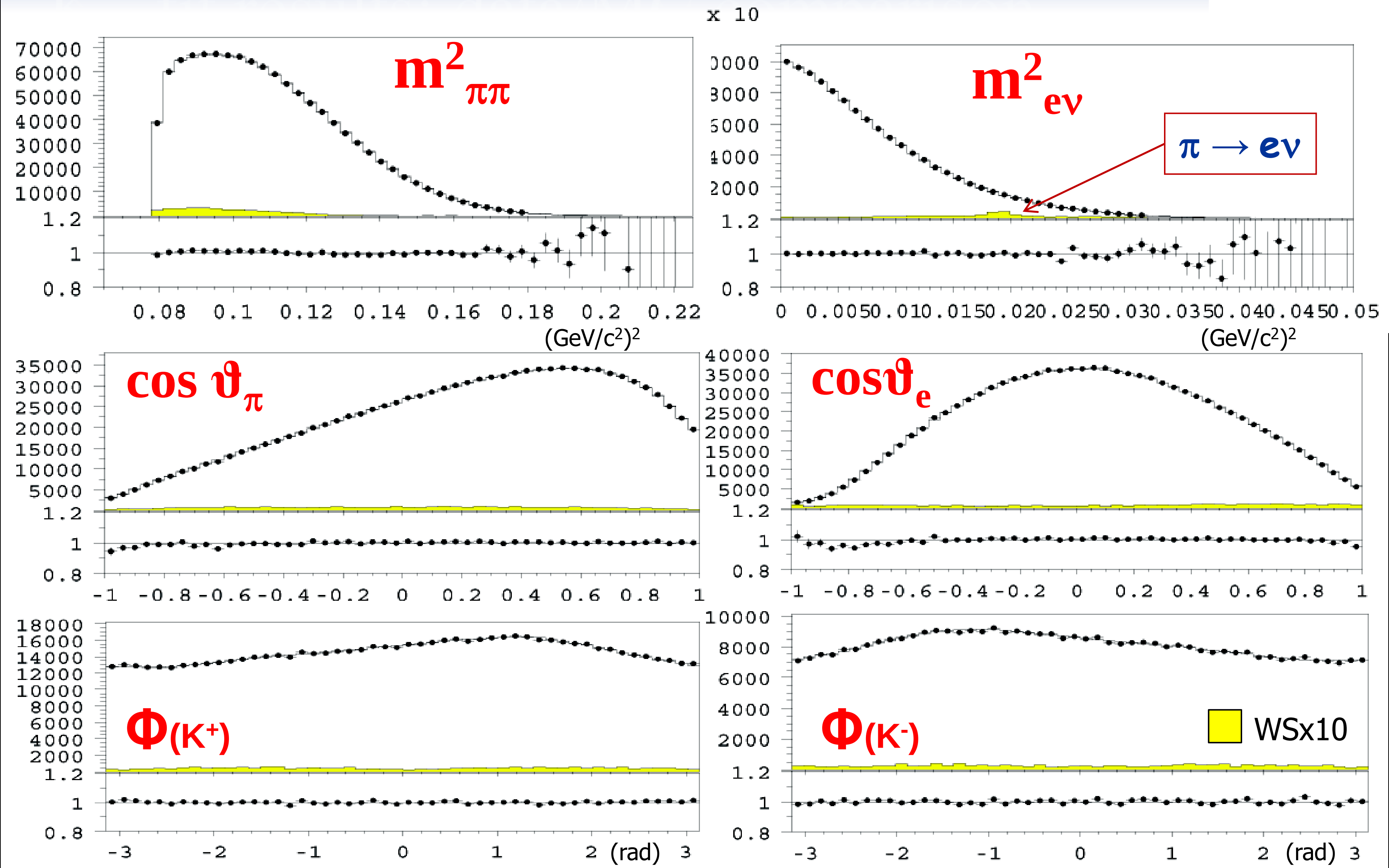
K_{e4} fitting procedure

- The fit parameters are F_S, F_P, G_P, H_P and $\delta=\delta_S-\delta_P$
- Full event sample (2003&2004): 1.13 million Ke4 decays
- Fit in iso-populated boxes in the 5-dim variables:

  10(**$m_{\pi\pi}$**) x 5(**$m_{e\nu}$**) x 5(cos **ϑ_π**) x 5(cos **ϑ_e**) x 12(**ϕ**) = **15,000 boxes**
- The form factors and phase shift are extracted by minimizing a log-likelihood estimator in **10 independent $m_{\pi\pi}$ bins**
- K^+ and K^- samples fitted separately and then combined in each $m_{\pi\pi}$ bin according to their statistical error.

Data:	K^+ sample: 726 400 events	→	48 events/box
	K^- sample: 404 400 events	→	27 events/box
MC:	K^+ sample: 17.4 10^6 events	→	1160 events/box
	K^- sample: 9.7 10^6 events	→	650 events/box

K_{e4} fit results: data/MC comparison



K_{e4} form factor results

- Only relative form factor (F_p/F_s, G_p/F_s, H_p/F_s) are measured (no overall normalization from BR)
- The form factor structure is studied in 10 bins of q^2

$$f_s' / f_s = 0.152 \pm 0.007_{stat} \pm 0.005_{syst}$$
$$f_s'' / f_s = -0.073 \pm 0.007_{stat} \pm 0.006_{syst}$$
$$f_e' / f_s = 0.068 \pm 0.006_{stat} \pm 0.007_{syst}$$
$$f_p / f_s = -0.048 \pm 0.003_{stat} \pm 0.004_{syst}$$
$$g_p / f_s = 0.868 \pm 0.010_{stat} \pm 0.010_{syst}$$
$$g_p' / f_s = 0.089 \pm 0.017_{stat} \pm 0.013_{syst}$$
$$h_p / f_s = -0.398 \pm 0.015_{stat} \pm 0.008_{syst}$$

Systematics from acceptance and background control

First evidence of non-zero f'_e and f_p

(Submitted to Eur. Phys. J. - 17 Sept 2010)

In agreement with our partial data sample publication: (R.Batley et al. EPJC 54-3, 411, 2008)

From phase shift to scattering length

<u>Corrections to be applied:</u>

- Radiative effects:

  included in the simulation (Coulomb attraction, IB)

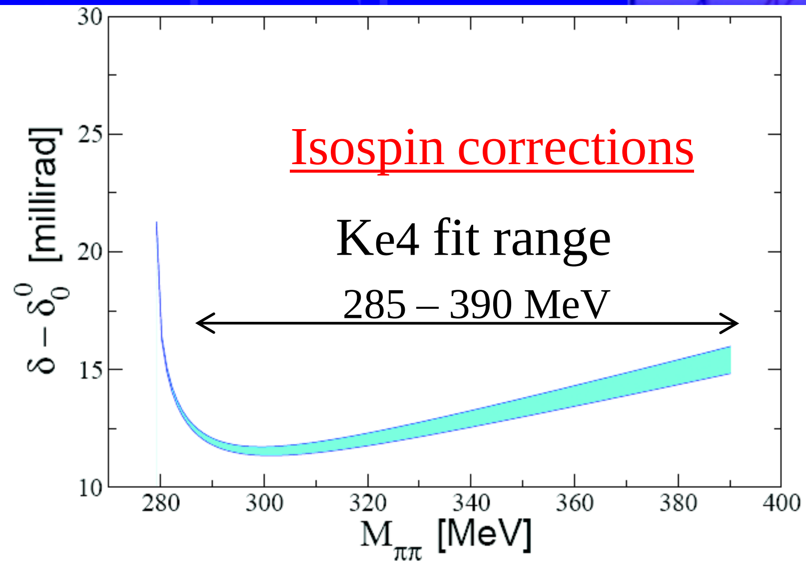
- Mass effects:

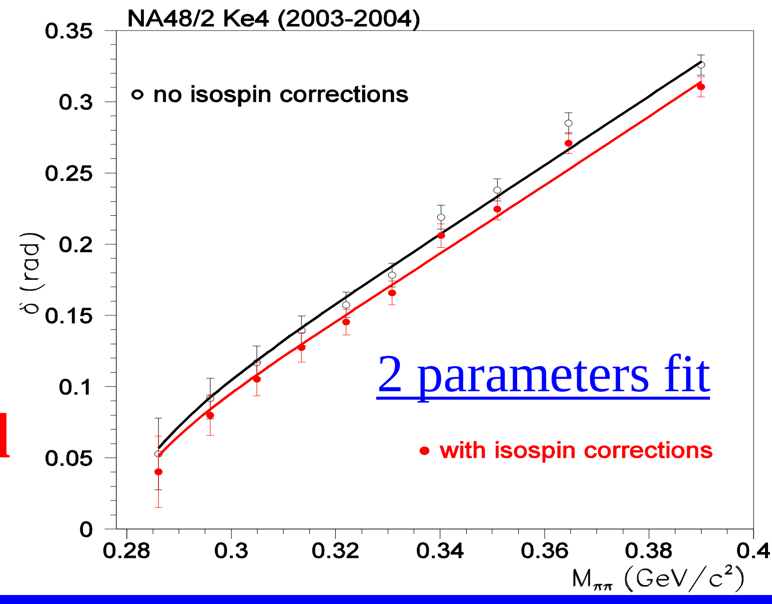
  Isospin corrections have to be applied to δ. Developed in close collaboration with NA48/2.

(Colangelo, Gasser, Rusetsky, EPJC 59 (2009) 777)

Size of correction on δ ~10-15 mrad

(Exp. stat. precision ~7-8 mrad)





Fit in the (a_0 , a_2) plane

1par. fit (ChPT)	
a_0	$0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{theor}$
2par. fit	
a_0	$0.2220 \pm 0.0128_{stat} \pm 0.0050_{syst} \pm 0.0037_{theor}$
a_2	$-0.0432 \pm 0.0086_{stat} \pm 0.0034_{syst} \pm 0.0028_{theor}$

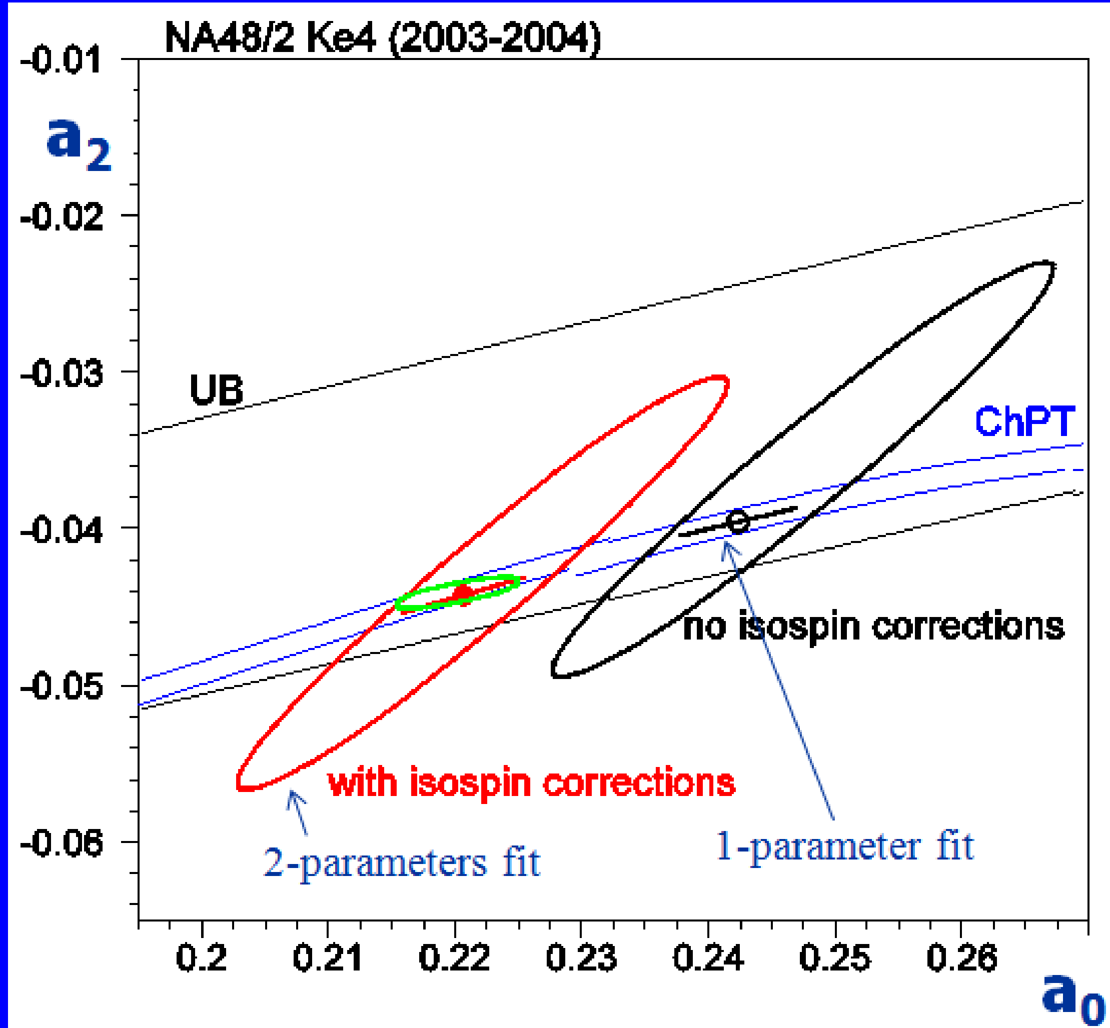
Precise ChPT prediction,

CGL NPB 603(2001), PRL86(2001)

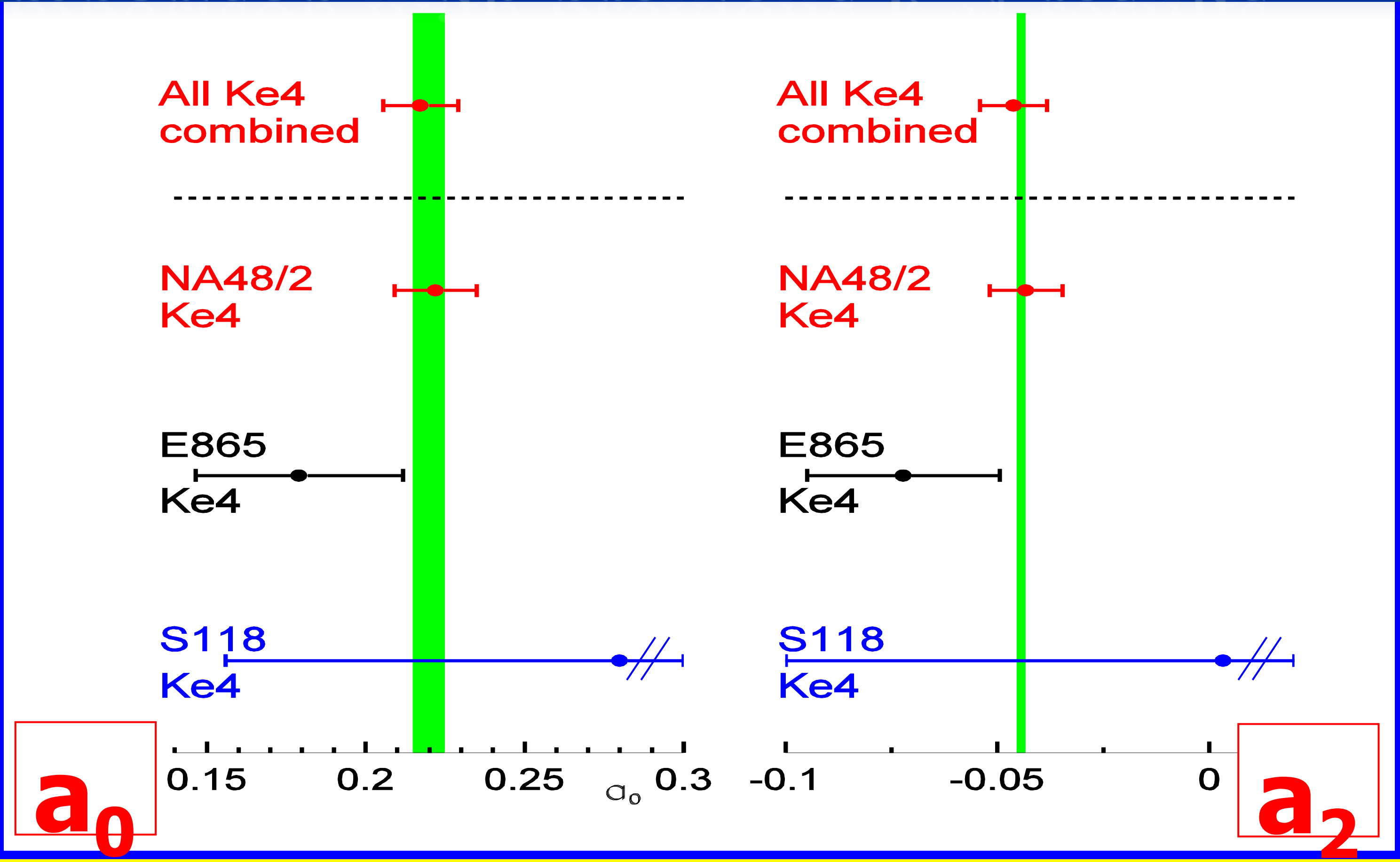
$a_0 = 0.220 \pm 0.005$ and

$a_2 = -0.0444 \pm 0.0008$

or $(a0 - a2) = 0.265 \pm 0.005$



Comparison with previous Ke4 results



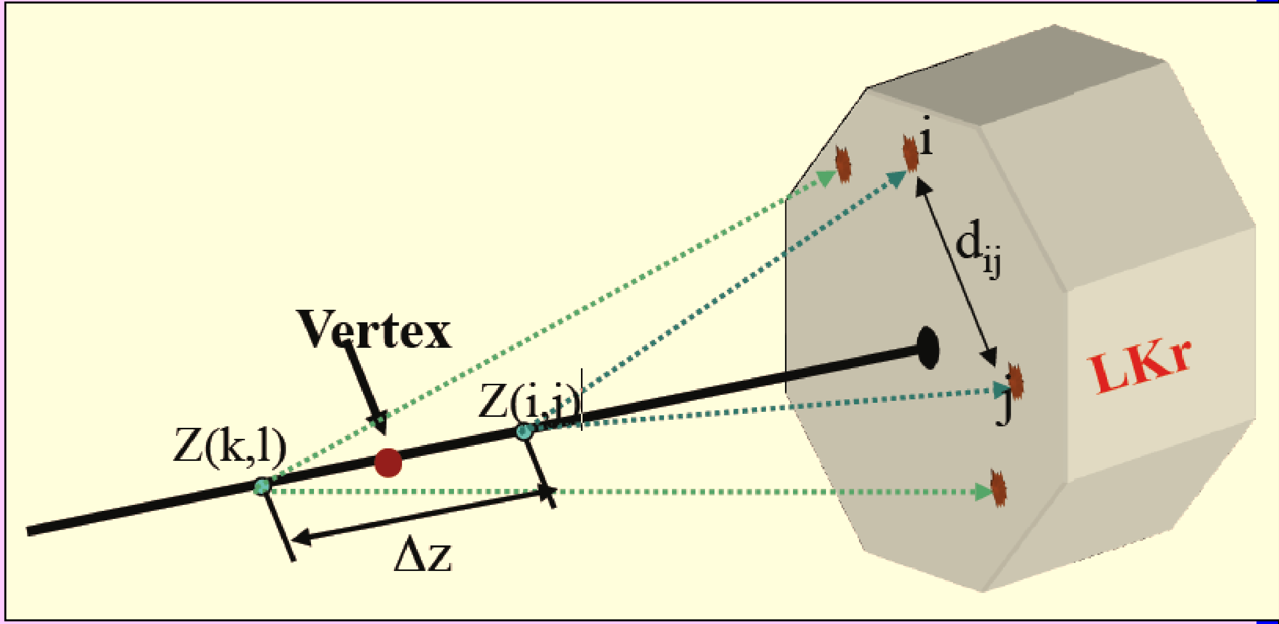
Two parameters best fit values for a_0 and a_2 from each Ke4 experiment and combined.

Vertical bands correspond to the best predictions from ChPT.

$\pi\pi$ scattering lengths in K3π decays

$K_{3\pi}$ events selection

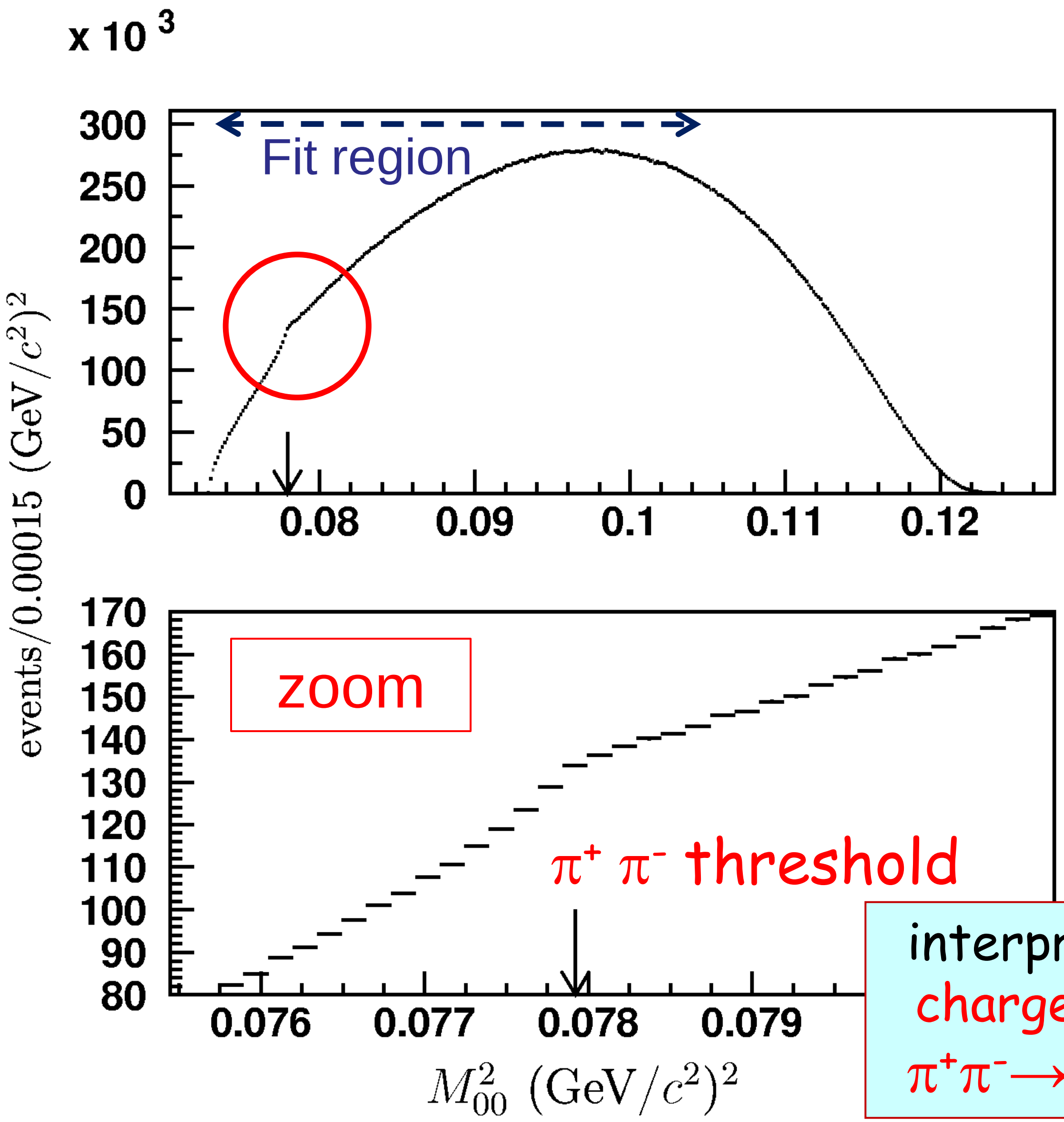
- Offline $K^{\pm} \rightarrow \pi^0 \pi^0 \pi^{\pm}$ selection:
  - Require 1 track and 4 e.m. LKr calorimeter clusters
  - $\pi^0 \rightarrow \gamma\gamma$ cluster pairing: consider all 3 combinations and choose 2-vtx combination with closest vertices (minimize vertex difference ΔZ)
  - Calculate $M_{00} = M(\pi^0 \pi^0)$ invariant mass using only e.m. calorimeter and vertex information



60 million events

with mass resolution of 1.3 MeV and negligible background

Cusp in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decays



Thanks to:
- ✓Very high statistics
- ✓Very good calorimeter resolution
- ✓Proper M_{00} reconstruction strategy

$M_{00} = M(\pi^0\pi^0)$ distribution shows a clear **CUSP** structure at **$M(\pi^0\pi^0) = 2\,M(\pi^{\pm})$**

interpreted as due to the final state charge exchange scattering process $\pi^+\pi^- \rightarrow \pi^0\pi^0$ in $K^{\pm} \rightarrow \pi^{\pm}\pi^+\pi^-$ decays

Cusp: Theoretical approach (CI)

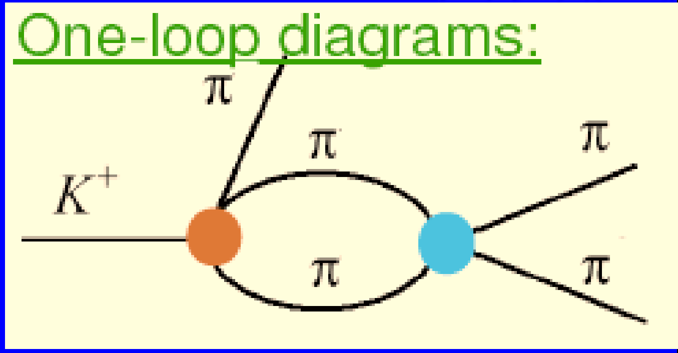
•$\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ phenomenological approach:

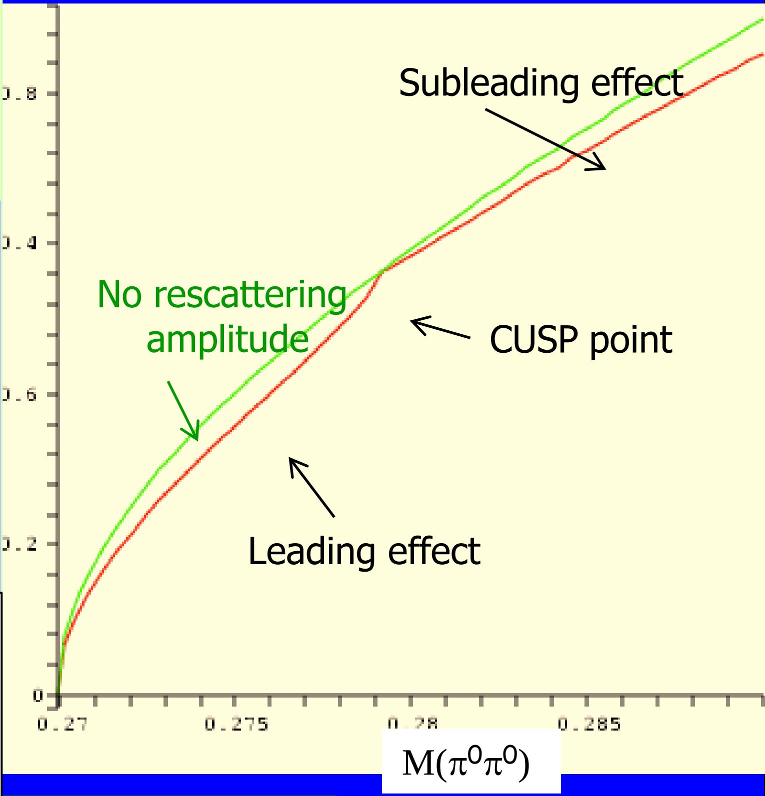
•The no-rescattering term is given by the standard PDG expansion

•The first-rescattering term is real below threshold and imaginary above. Negative interference below threshold arise.

•The amplitude depends on **a_0-a_2**

(Cabibbo, PRL 93 121801, 2004 but predicted earlier *Budini, Fonda, PRL6, 419, 1961)*





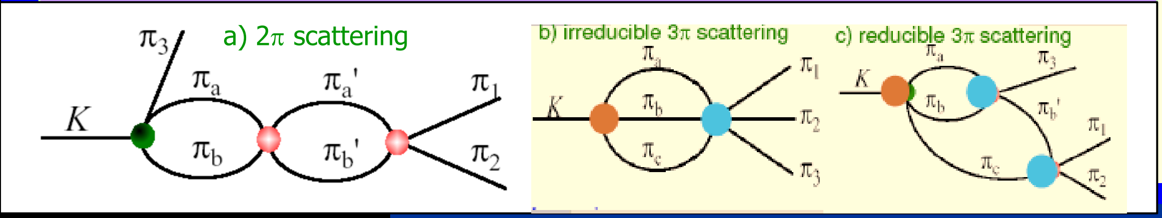
•More complete calculation

•rescattering corrections from $\pi^+\pi^- \rightarrow \pi^0\pi^0$, $\pi^+\pi^0 \rightarrow \pi^+\pi^0$,…………

•Two-loop level corrections, $O(a_i^2)$

•No $O(a_i^3)$, no radiative corrections

(Cabibbo, Isidori, JHEP03, 21, 2005)



Cusp: Theoretical approach (BB)

Cusp: Theoretical approach (BB)

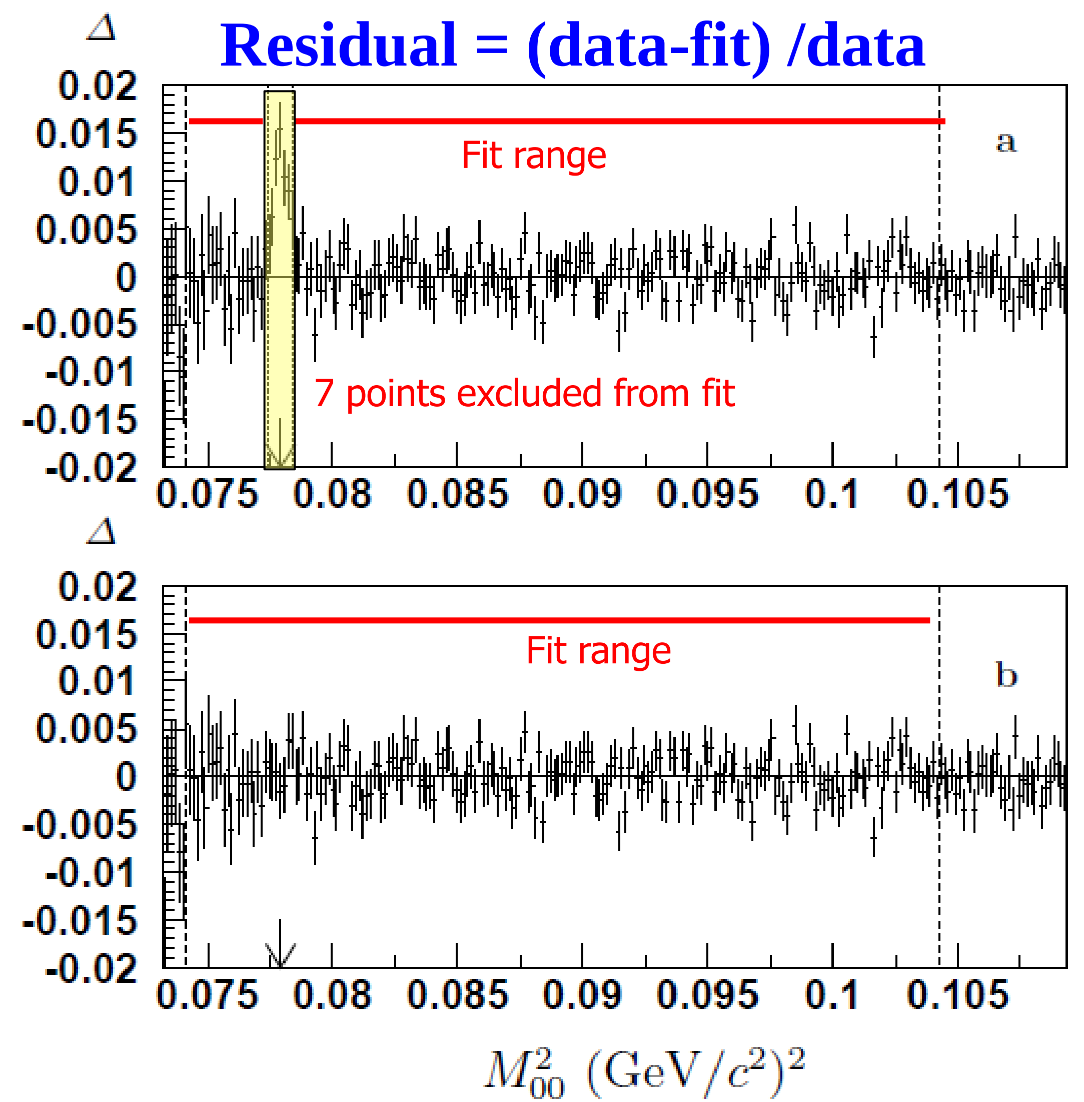
Approach by the Bern-Bonn group:

- Different approach based on an effective non-relativistic Lagrangian
- Different structure of the expansion with respect to CI (kinetic energy and threshold parameter).
- Simultaneous fitting of charged and neutral amplitudes to extract Dalitz plot slope parameters
- Electromagnetic effects naturally included
- Radiative corrections, outside the cusp point included.

(Colangelo, Gasser, Kubis, Rusetsky, Phys.Lett.B 638, 187, 2006; Bissenger, Fuhrer, Gasser, Kubis, Rusetsky, Phys.Lett.B 659,576, 2008; Bissenger, Fuhrer, Gasser, Kubis, Rusetsky, NPH B806, 178, 2009)

Provides so far the most complete description of rescattering effect

Fit to the M($\pi^0\pi^0$) spectrum (BB)



Free fit parameters:

- a_0-a_2, a_2,
- Dalitz plot parameters,
- normalizations (fit also includes $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ decays).

b) Pionium fraction F_{atom} left free in the fit

Cusp fitting results

fit	χ^2/ndf	a_0-a_2	a_2	f_{atom}
CI	206.3/195	0.2727(46)	-0.0392(80)	0.0533(91)
CI (a)	201.6/189	0.2689(50)	-0.0344(86)	0.0533
CI (c)	210.6/196	0.2749(21)	-0.0413	0.0441(76)
CI (a,c)	207.6/190	0.2741(21)	-0.0415	0.0441
BB	462.9/452	0.2815(43)	-0.0693(136)	0.0530(95)
BB (a)	458.5/446	0.2775(48)	-0.0593(142)	0.0542
BB (c)	467.3/453	0.2737(26)	-0.0417	0.0647(76)
BB (a,c)	459.8/447	0.2722(27)	-0.0421	0.0647
CI	205.6/195	0.2483(45)	-0.0092(91)	0.0625(92)
CI (a)	202.9/189	0.2461(49)	-0.0061(98)	0.0625
CI (c)	222.1/196	0.2646(21)	-0.0443	0.0420(77)
CI (a,c)	219.7/190	0.2645(22)	-0.0444	0.0420
BB	477.4/452	0.2571(48)	-0.0241(129)	0.0631(97)
BB (a)	474.4/446	0.2544(51)	-0.0194(132)	0.0631
BB (c)	479.8/453	0.2633(24)	-0.0447	0.0538(77)
BB (ac)	478.1/447	0.2627(25)	-0.0449	0.0538

(…) stat error
no(…) = fixed par.

Rad. Corr. Off

a) Pionium f_{atom} fixed

c) with ChPt constraint

a,c) both

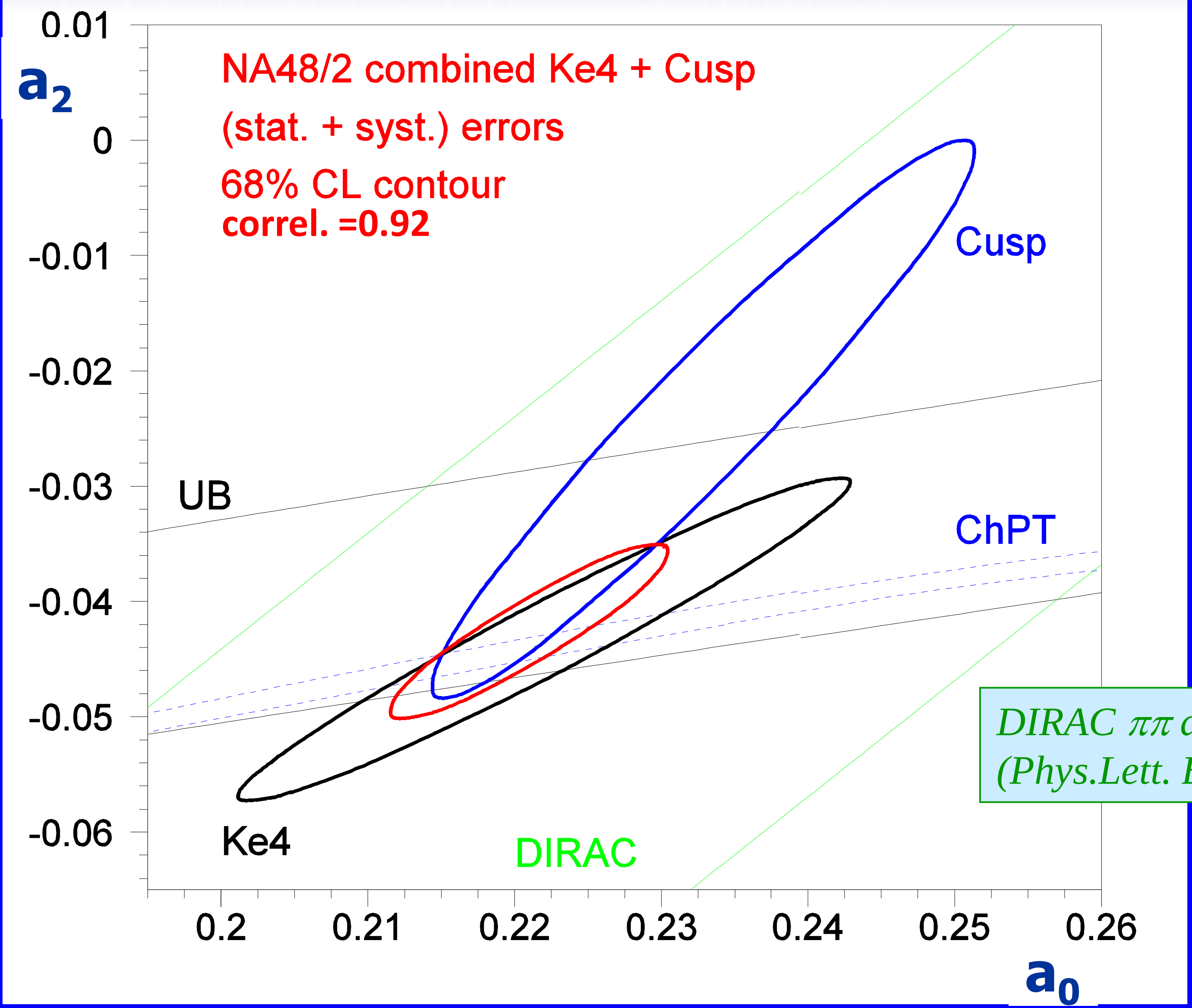
Rad. Corr. On

Final result (f_{atom}, a_0-a_2, a_2 free in the fit)

ππ scattering lengths in Ke4 and K3π decays

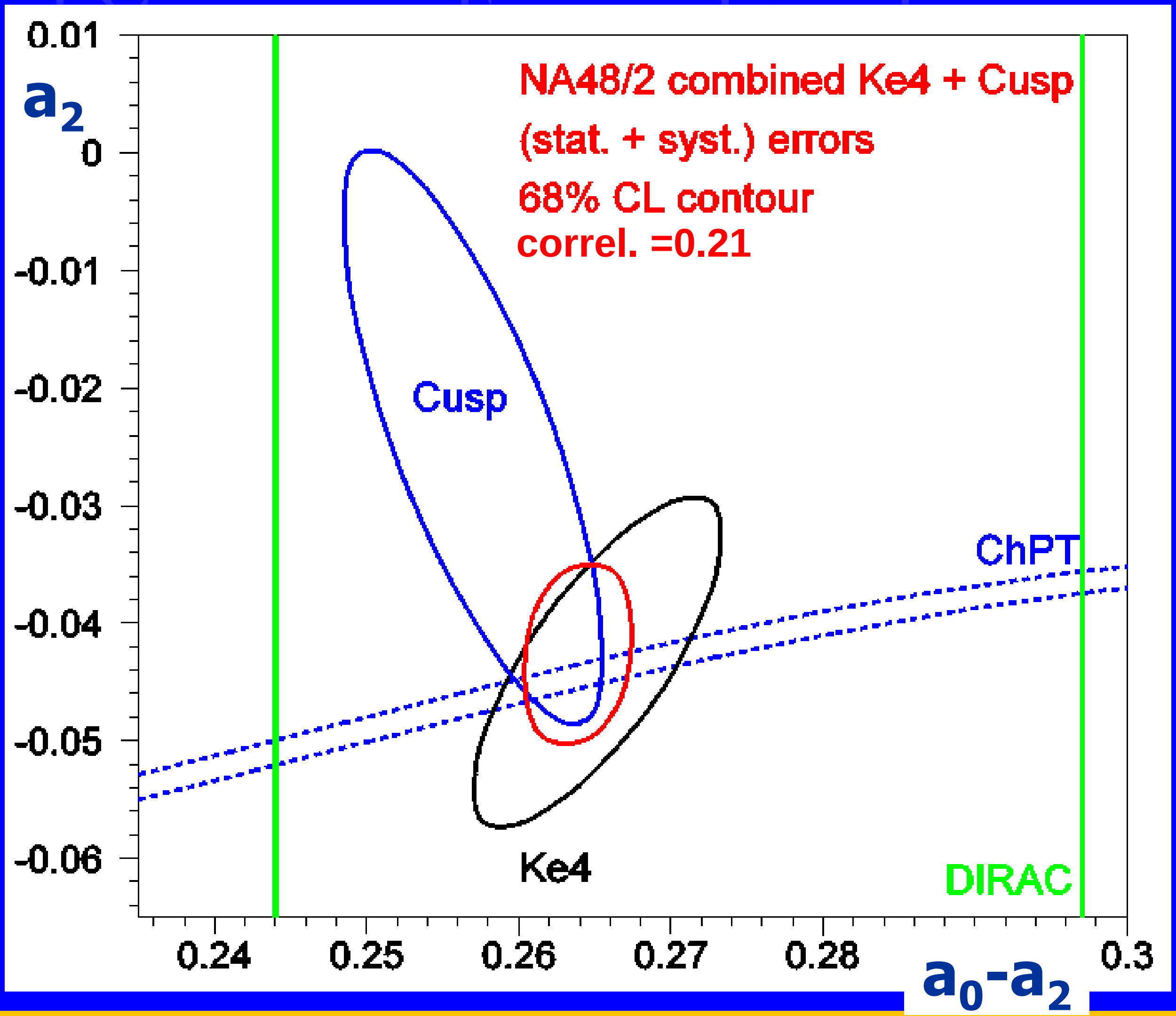
in Ke4 and K3π decays

Ke4 and Cusp results combined



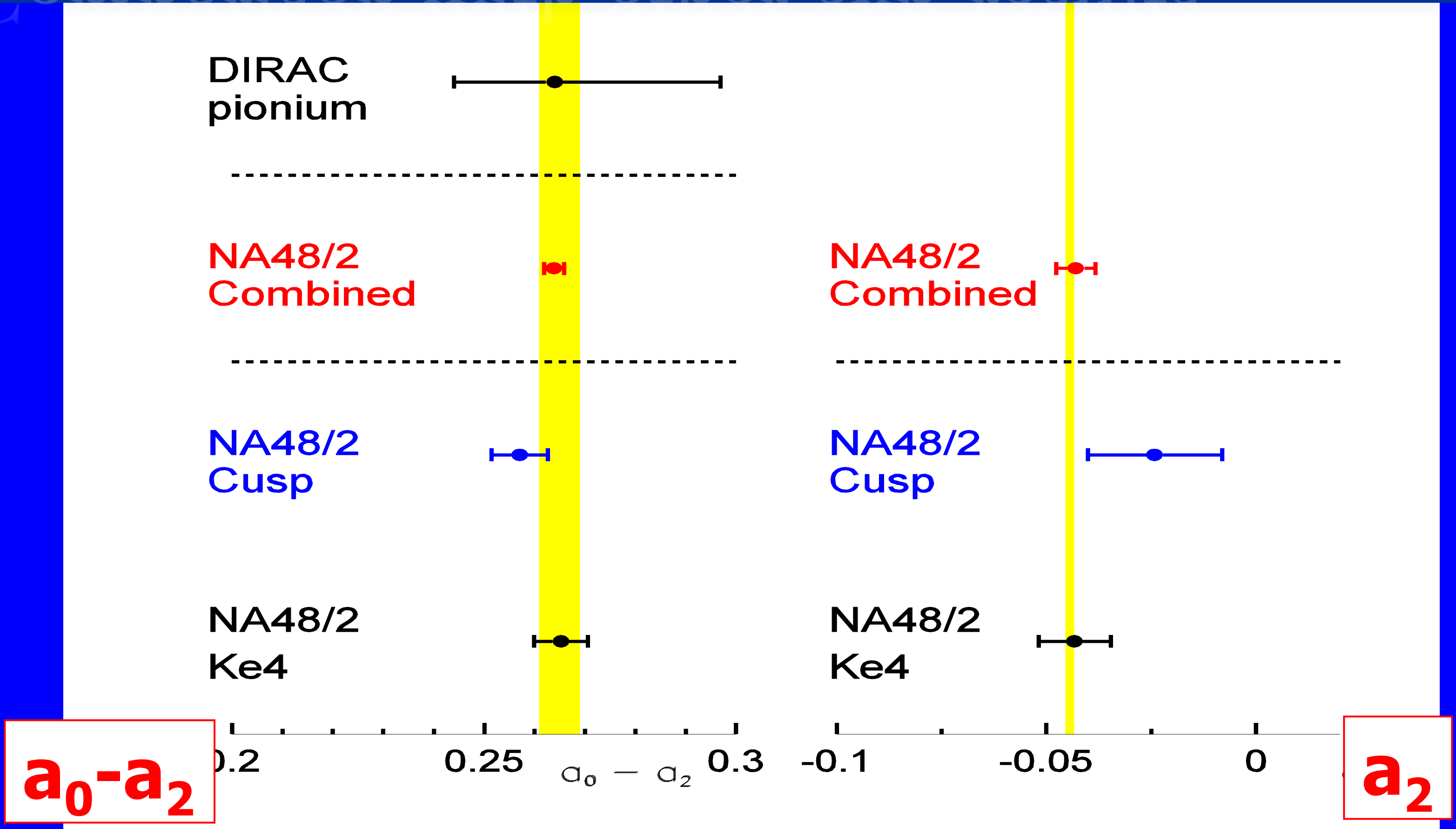
DIRAC ππ atom lifetime (Phys.Lett. B 619 (2005).

Ke4 and Cusp results combined



Very good agreement with ChPT prediction: $(a_0 - a_2)\, m_+ = 0.265 \pm 0.004$

Comparison with other exp-results



Two parameters best fit values for **a_0-a_2** and **a_2** from both NA48/2 channels and combined result.

Vertical bands correspond to the best predictions from ChPT.

Conclusions

Kaon decays give unique possibility to study low-energy hadronic interactions with high precision.

Two statistically independent measurements by NA48/2 of the
<u>ππ scattering lengths in Ke4 and K3π decays</u>:

$1.15 \cdot 10^6$ Ke4 and **$60 \cdot 10^6$ K3π (Cusp)** events

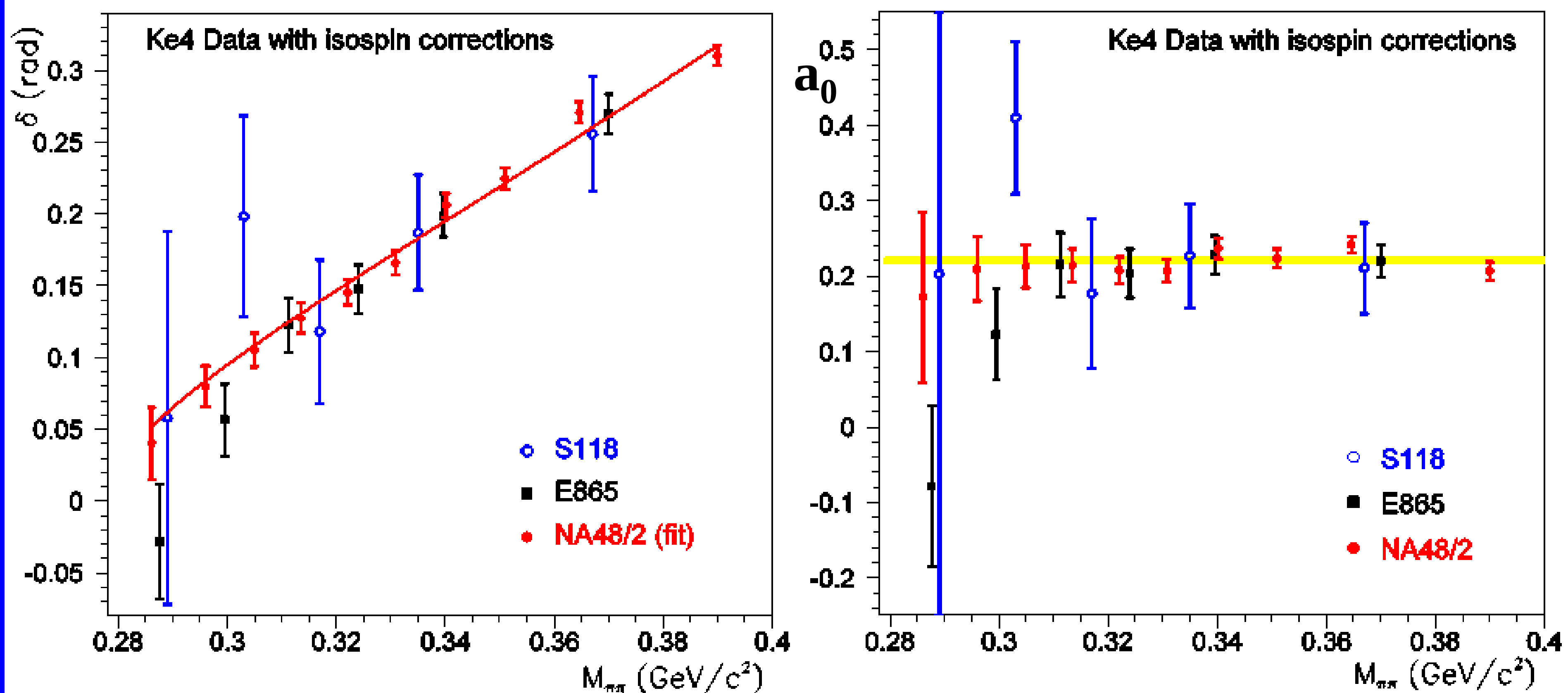
✓ Different systematics: **<u>K3π :</u>** calorimeter and trigger; **<u>Ke4:</u>** electron misID and background

✓ Different theoretical inputs: **<u>K3π :</u>** rescattering in final state and ChPT expansion; **<u>Ke4:</u>** Roy equation and isospin breaking connection

- Large overlap in the (**a_0-a_2, a_2**) plane
- ππ scattering lengths results from Ke4 and K3π are fully consistent
- the experimental results are in very good agreement with ChPT
- the achieved experimental precision on a_0 is now competitive with the theoretical precision (±0.005) in both decay modes

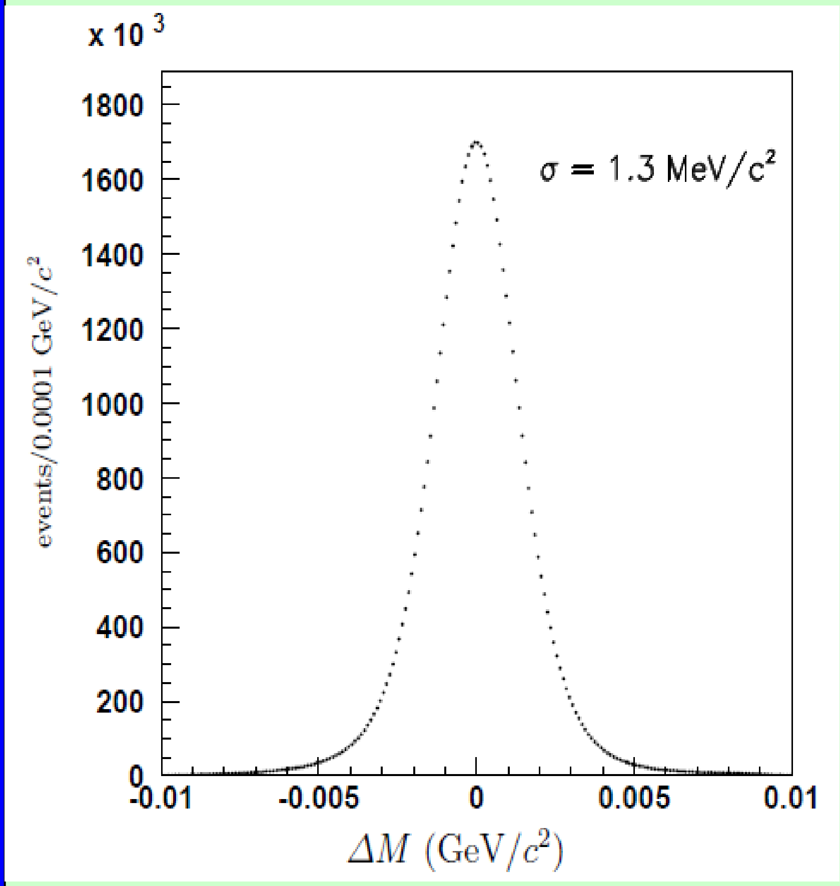
Spares

Comparison with previous Ke4 results



Phase shift measurements for all Ke4 available results. The line corresponds to the two-par. fit of NA48 data alone. On the right values obtained for each individual result from the inverted ChPT constraint. The yellow band is from the global fit of NA48 data.

$K_{3\pi}$ events invariant mass



Distribution of the difference between the $\pi^0\,\pi^0\,\pi^\pm$ invariant mass and the nominal $K^\pm$ mass for the selected $K^\pm \rightarrow \pi^0\,\pi^0\,\pi^\pm$ decays

Fit of phase shift $\delta=\delta_0-\delta_1$

2-parameters fit:

$$a_0 = \quad 0.2220 \pm 0.0128_{stat} \pm 0.0050_{syst} \pm 0.0037_{theo}$$

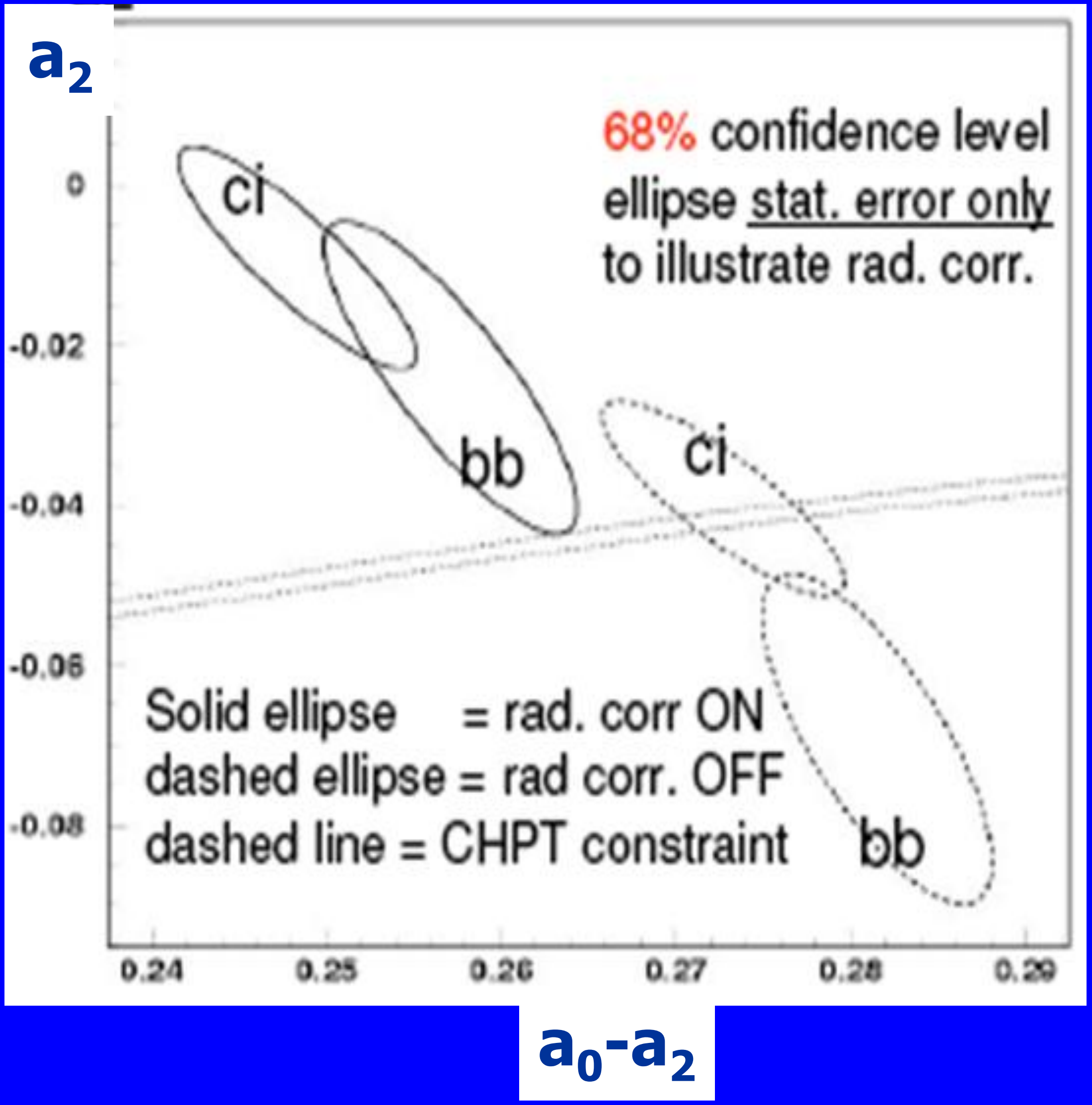
$$a_2 = - 0.0432 \pm 0.086_{stat} \pm 0.0034_{syst} \pm 0.0028_{theo}$$

1-parameter fit (with ChPT constraint):

$$a_0 = \quad 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{theo}$$

Theoretical error computed from isospin corrections and Roy equation inputs *(Gasser et al. Eur.Phys.J. C59:777, 209).*

Cusp results on a_0- a_2 and a_0



- **Final result:**

(EPJC 64 (2009) 589)

a0-a2 =
$0.257(5)_{stat}(3)_{sys}(1)_{ext}$

a2 =
$- 0.024(13)_{stat}(9)_{sys}(2)_{ext}$

- **with ChPT constraint:**

a0-a2 =
$0.2633(24)_{stat}(14)_{sys}(19)_{ext}$

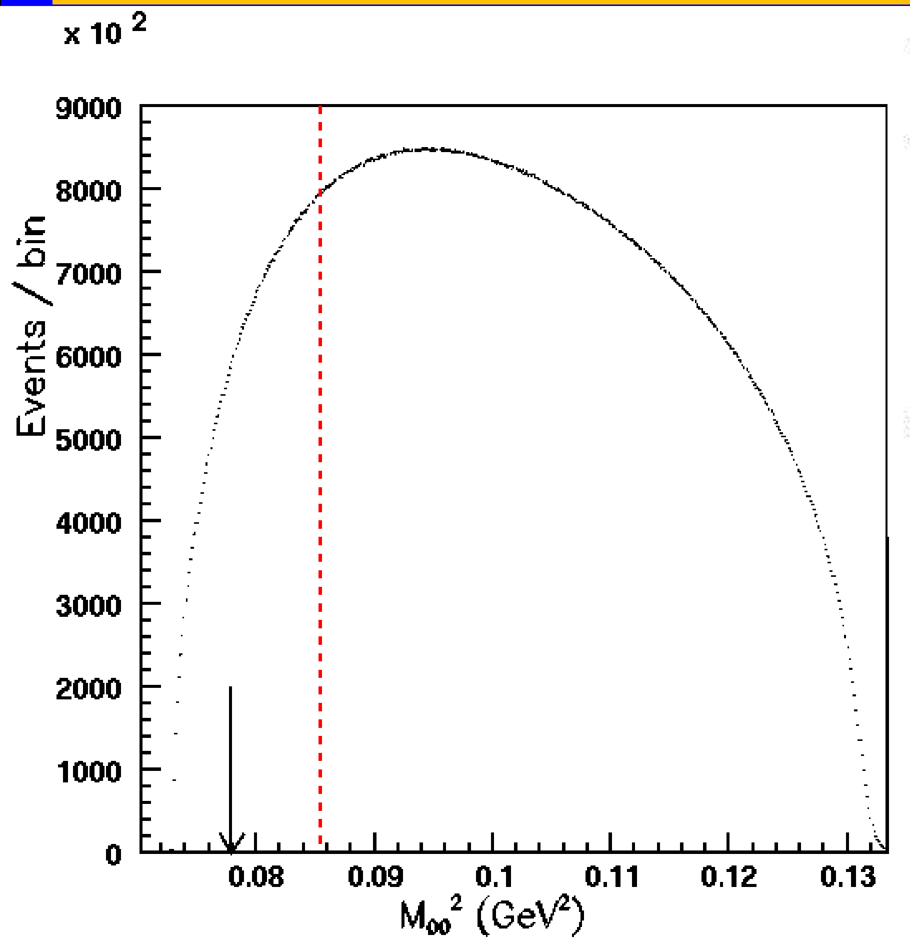
- **ChPT prediction:**

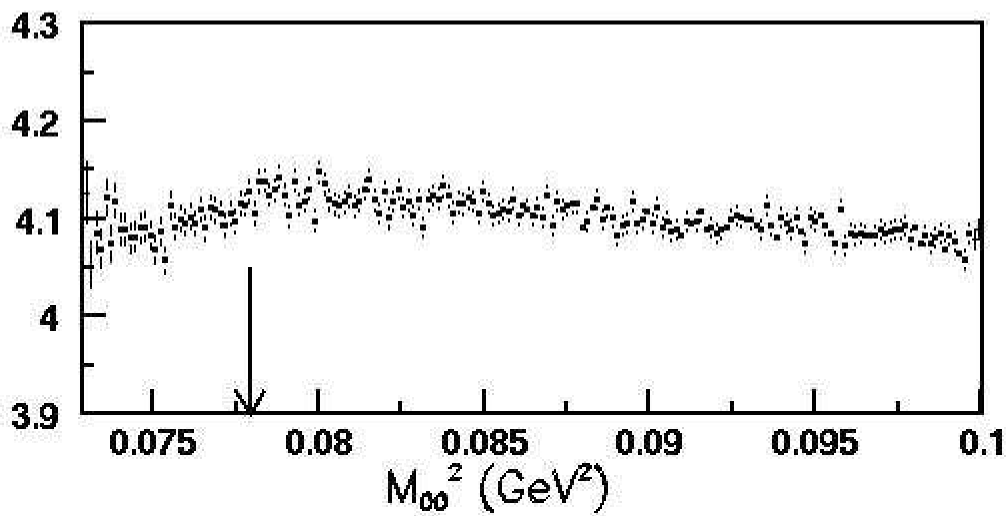
a0-a2 = 0.265(4)

$\pi\pi$ scattering lengths in $K_L \rightarrow \pi^0\pi^0\pi^0$ decays

$\pi\pi$ scattering lengths in $K_L \rightarrow \pi^0\pi^0\pi^0$

NA48/2 data taking in 2000:



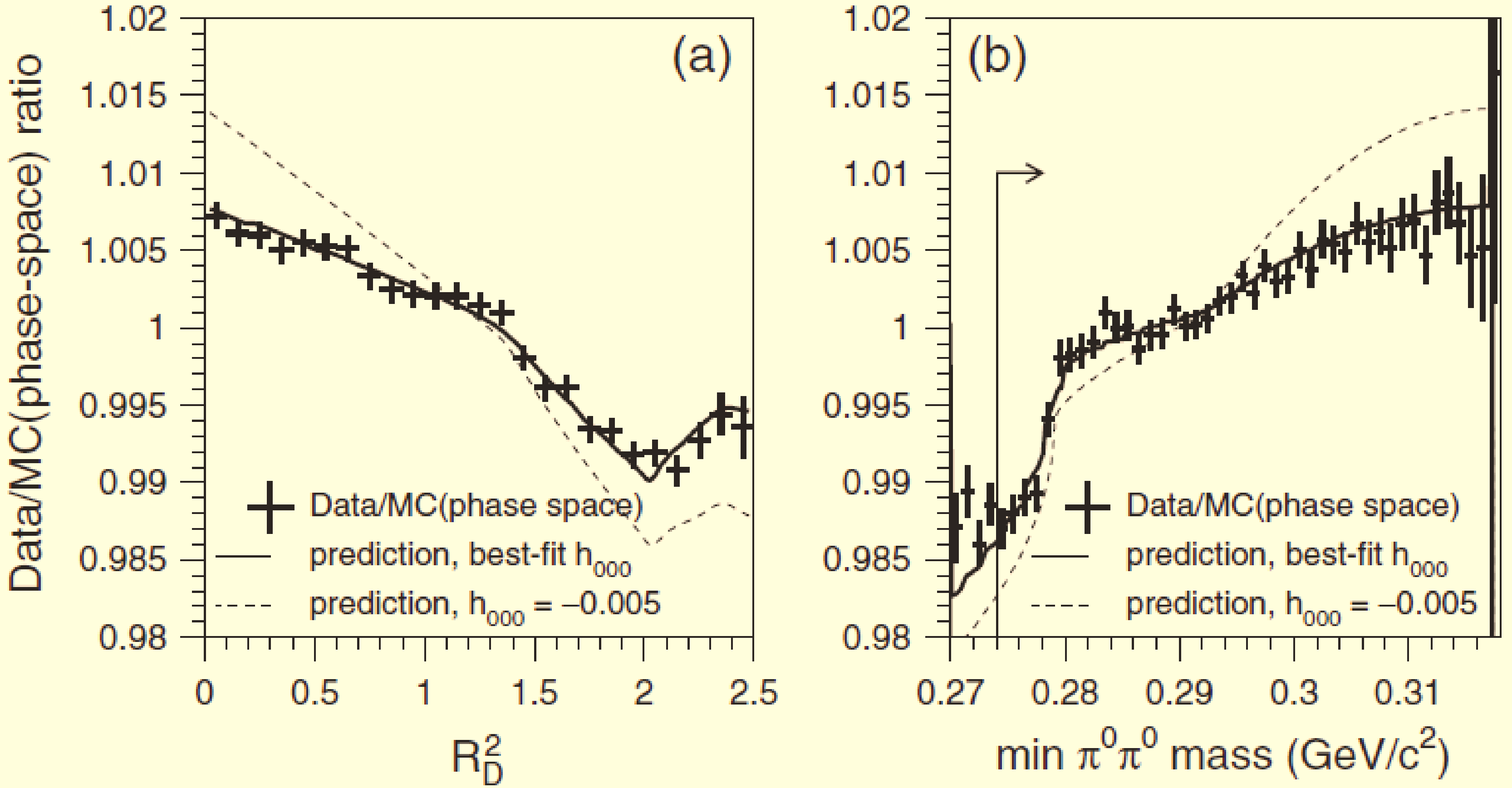


Evidence for a change of slope near the cusp point.

$\pi\pi$ scattering lengths in $K_L \rightarrow \pi^0\pi^0\pi^0$

(KTEV, Phys.Rev. D78, 032009, 2008)

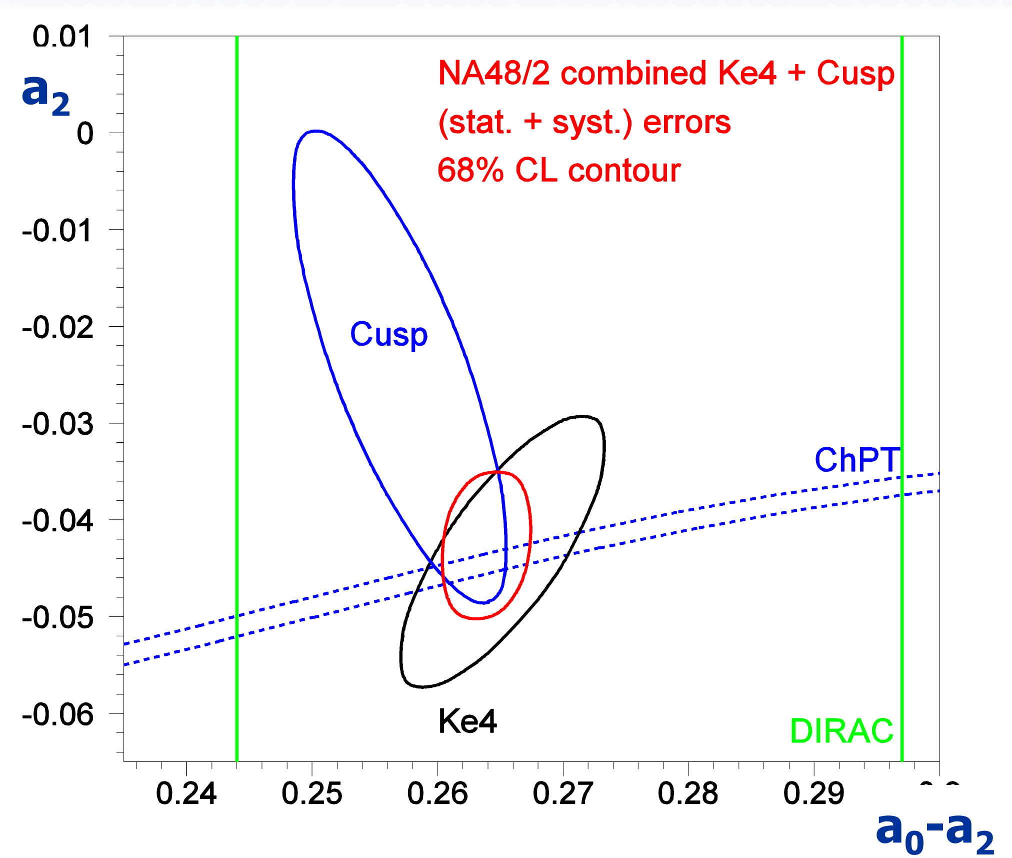
Fit for h_{000} and a_0-a_2



$$m_{\pi+}(a_0-a_2) = 0.215 \pm 0.014_{stat} \pm 0.025_{syst} \pm 0.006_{ext}$$

$$h_{000} = (-2.09 \pm 0.62_{stat} \pm 0.72_{syst} \pm 0.28_{ext})$$

Ke4 and Cusp results combined

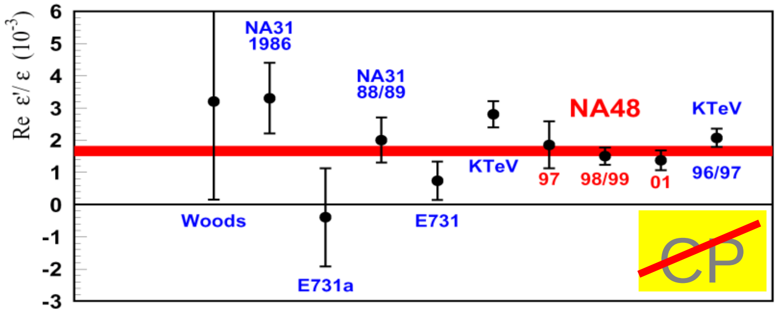


- ✓ Two statistically independent measurements by NA48/2:
  60 M K3π ; 1.13 M Ke4
- ✓ Different systematics:
  - Cusp: calorimeter and trigger
  - Ke4: electron misID and background
- ✓ Different theoretical inputs:
  - Cusp: rescattering in final state and ChPT expansion
  - Ke4: Roy equation and isospin breaking connection

- ✓ Large overlap in the $(a_0\text{-}a_2, a_2)$ plane
- ✓ Impressive agreement with ChPT predictions
- ✓ Also shown DIRAC results: $|a_0\text{-}a_2|$ extracted from pionium lifetime PLB619(2005)
- ✓ Cusp effect in $K_L \rightarrow \pi^0\,\pi^0\,\pi^0$ KTeV ($68\cdot10^6$) [PRD 78, 032009 (2008)]
  NA48 ($100\cdot10^6$)

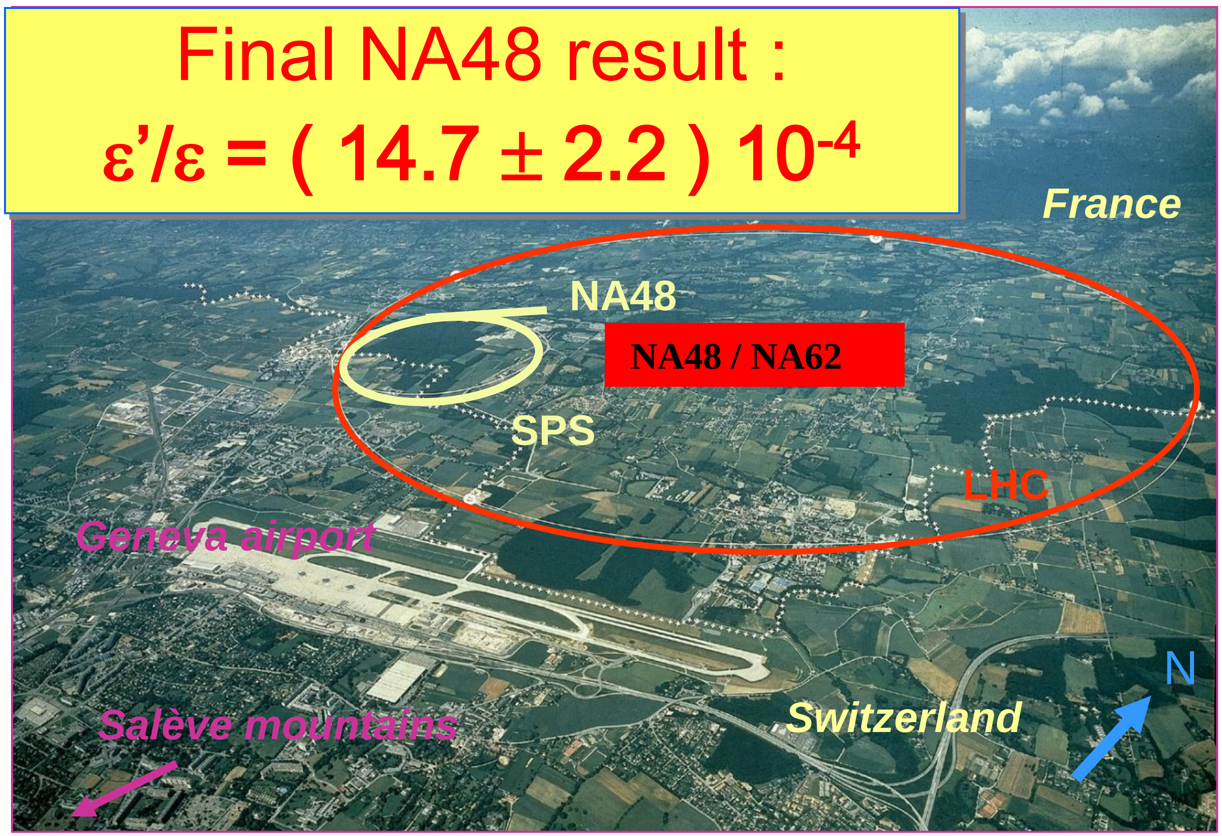
The NA48/NA62 experiment

A fixed target experiment at the CERN SPS dedicated the study of CP violation and rare decays in the kaon sector



Final NA48 result :

$\varepsilon'/\varepsilon = (14.7 \pm 2.2)\ 10^{-4}$



NA48
- 1997 ε'/ε run $K_L + K_S$
- 1998 ε'/ε run $K_L + K_S$
- 1999 ε'/ε run $K_L + K_S$ | K_S Hi. Int.
- 2000 K_L only | K_S High Intensity | *NO Spectrometer*
- 2001 ε'/ε run $K_L + K_S$ | K_S High Int.

NA48/1
- 2002 K_S High Intensity

NA48/2
- 2003 $K^{\pm}$ High Intensity
- 2004 $K^{\pm}$ High Intensity

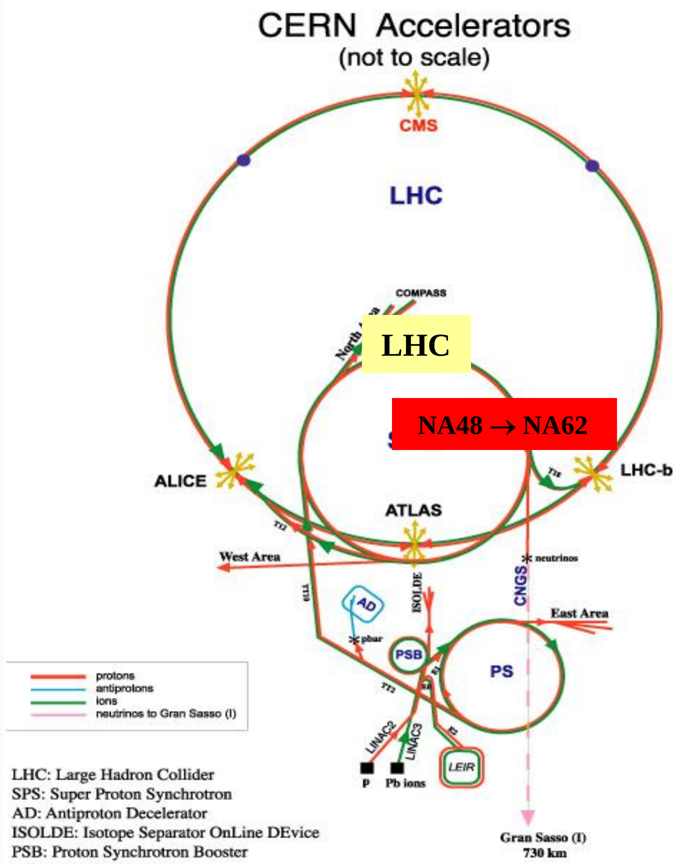
NA62
- 2007 $K^{\pm}_{e2}/K^{\pm}_{\mu 2}$ run

NA62 phase II
measurement of the decay

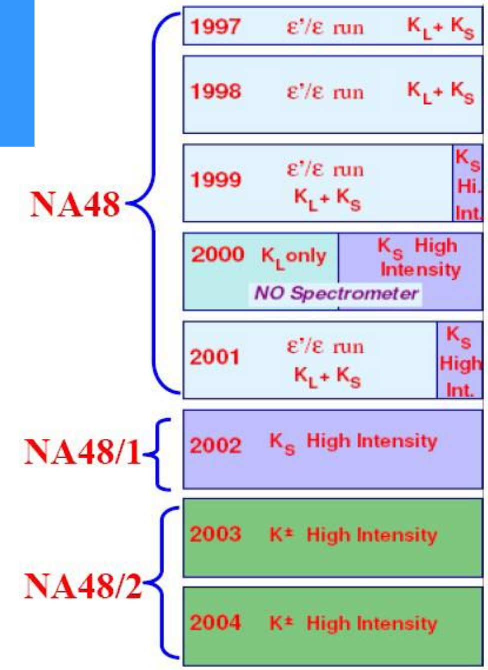
$$K^+ \to \pi^+ \nu \bar{\nu}$$

(2008-2010 R&D
& construction
2011 start of data taking)

The NA62 experiment



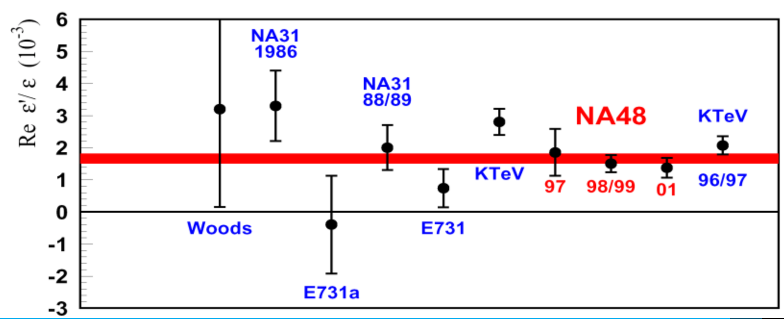
LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Separator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso



NA62 phase I

Dedicated 2007 run to measure:

$$R_K = \frac{\Gamma\left(K^\pm \to e^\pm \nu_e\right)}{\Gamma\left(K^\pm \to \mu^\pm \nu_\mu\right)}$$



NA62 phase II

measurement of the decay

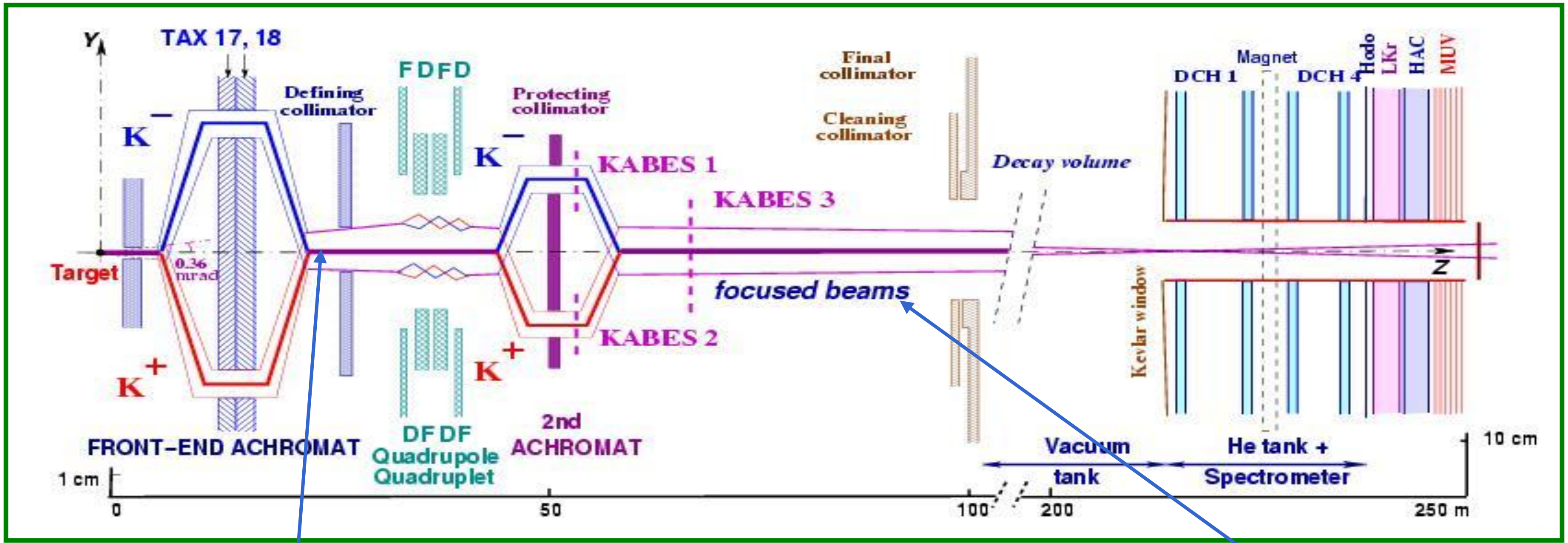
$$K^+ \to \pi^+ \nu \bar{\nu}$$

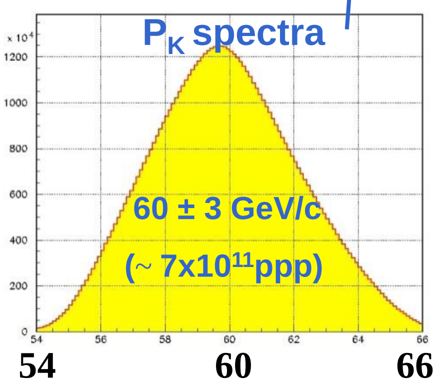
(2008-2010 R&D
& construction
2011 start of data taking)

NA48/2 simultaneous $K^{\pm}$ beam

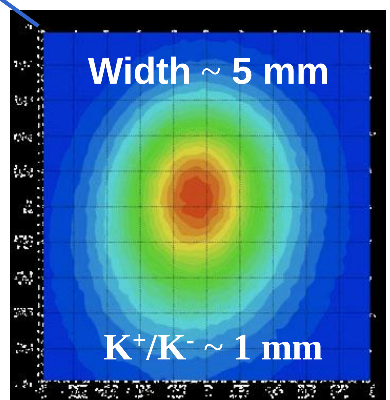
NA48-2 beams: simultaneous K^+/K^-, focused, high momentum, narrow band

designed to precisely measure $K^{\pm} \rightarrow \pi^+\pi^-\pi^{\pm}$ ($\pi^0 \pi^0 \pi^{\pm}$) Dalitz-plot density to search for direct CPV and **tuned for K_{e2} measurement.**





- **Simultaneous, unseparated, focused beams**
- **Flux ratio: $K^+/K^- \sim 1.8$**
- **Similar acceptance for K^+ and K^-** decays
- **Large charge symmetrization of experimental conditions**



NA48 detector

- **Magnetic spectrometer (4 DCHs)**
  - 4 views : redundancy ⇒ high efficiency;

$\sigma_p/p = (1.0 \oplus 0.044\ p)\%$ (p in GeV/c)

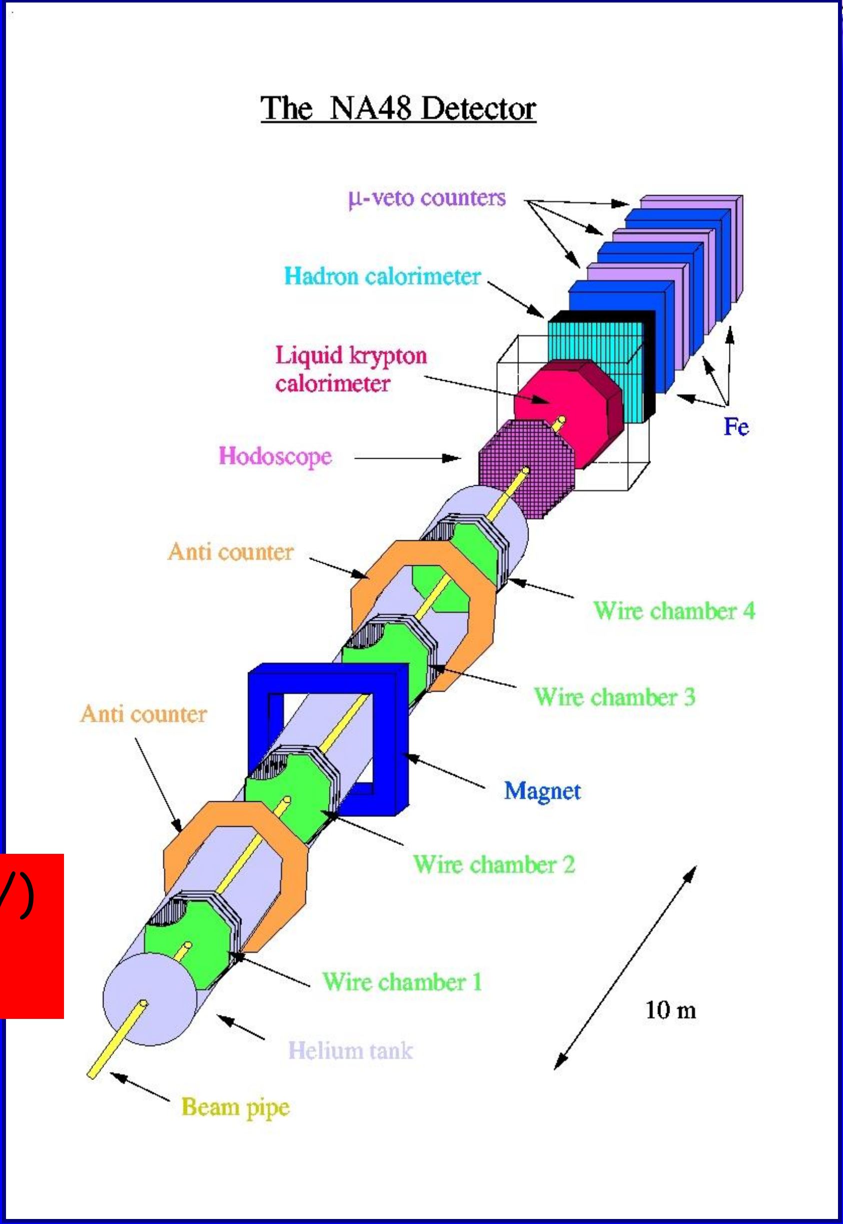
- **Hodoscope**
  - fast trigger;
  - precise time measurement (σ_t = 150 ps) .

- **Liquid Krypton EM calorimeter (LKr)**
  - Quasi-homogeneous ionization chamber
  - 27 electromagnetic radiation lengths long active volume
  - Segmented transversally 13248 cells, 2x2 cm2
  - Energy resolution (E in GeV):

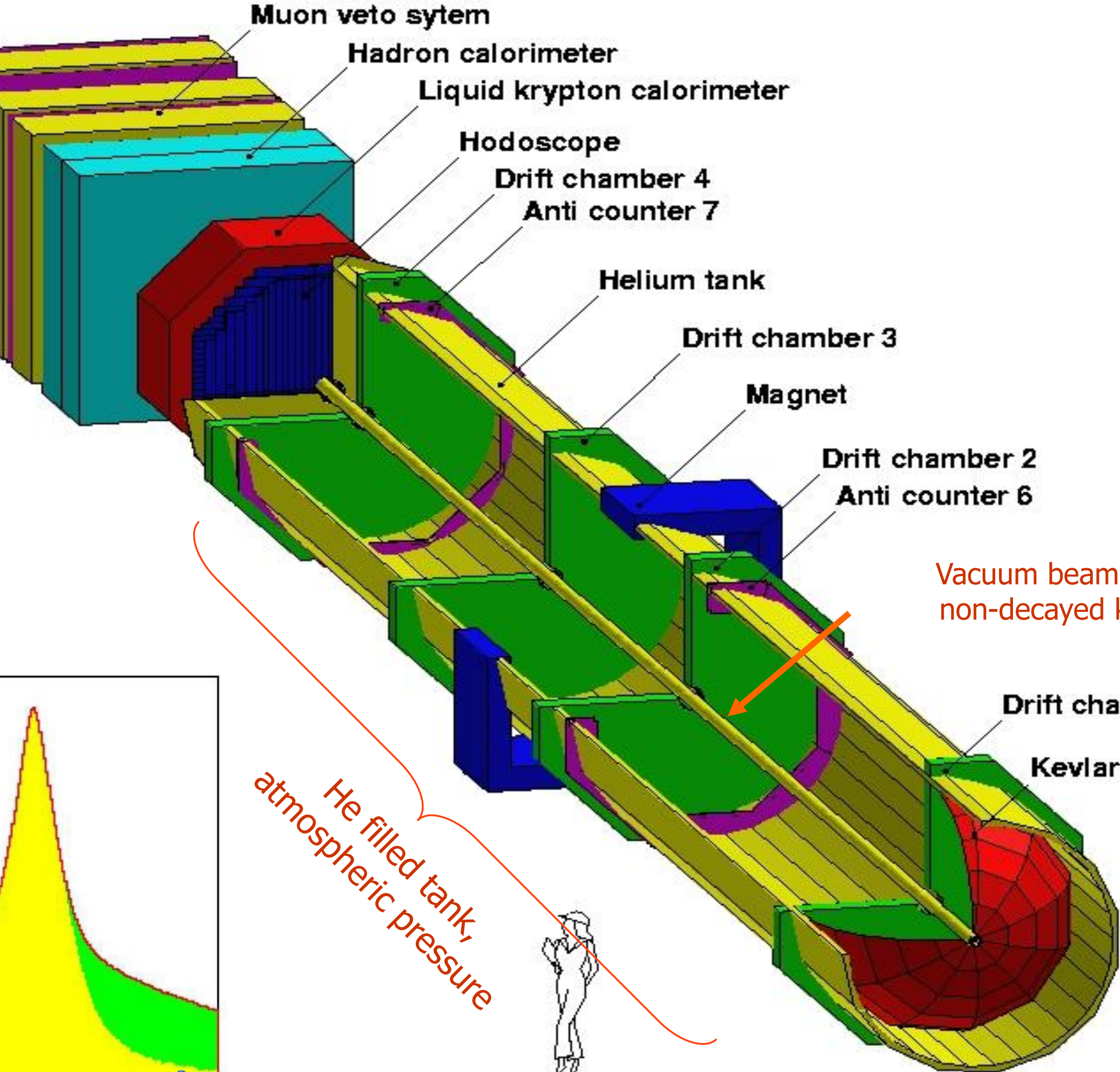
$\sigma_E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV/)

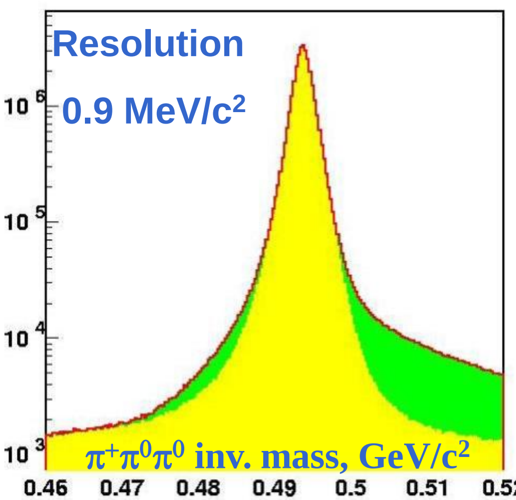
$\sigma_x = \sigma_y = 0.42/E^{\frac{1}{2}} + 0.6$ mm

Cambridge, CERN, Chicago, Dubna, Edimburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

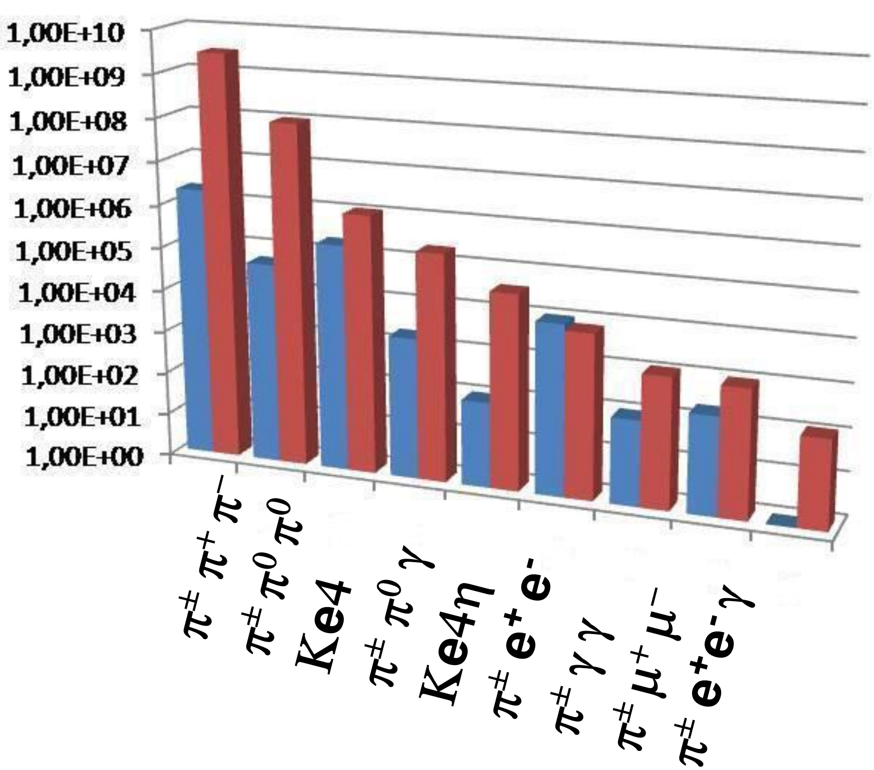


The NA48/NA62 experiment



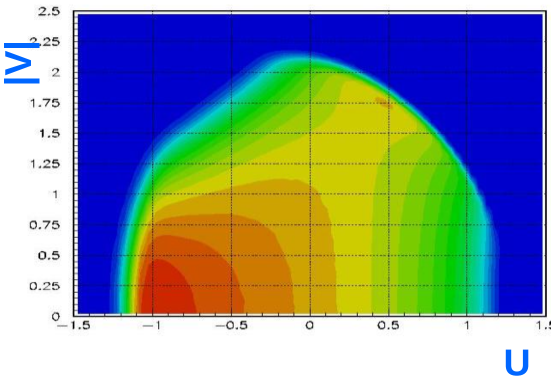


NA48/2 Data taking



NA48 Experimental hall

- Unprecedented statistics in many channels
- Two years of data taking (2003 and 2004)
- Main purpose was to measure direct CP violation in charged kaon decays, through asymmetry in Dalitz plot distribution
- New limits on CP violation in charged kaon decays



$A_g^{ch} = (-1.5 \pm 2.1) \times 10^{-4}$

$A_g^0 = (1.8 \pm 1.8) \times 10^{-4}$