# Studies of exclusive semileptonic $B$ decays and extraction of |V_\{ub\}| at BaBar 

Paul Taras

Université de Montréal
(on behalf of the BaBar collaboration)

## Exclusive charmless semileptonic B decays

$$
\begin{gathered}
B^{0} \rightarrow \pi^{-} \ell^{+} v \quad B^{+} \rightarrow \pi^{0} \ell^{+} v \quad B^{+} \rightarrow \eta \ell^{+} v \quad B^{+} \rightarrow \eta^{\prime} \ell^{+} v \\
B^{0} \rightarrow \rho^{-} \ell^{+} v \quad B^{+} \rightarrow \rho^{0} \ell^{+} v
\end{gathered}
$$

In Standard model, quark flavour changes occur through weak interactions via coupling of a W gauge boson

Such couplings are prop. to relevant CKM matrix elements
Probability of a b quark to decay into a u quark is prop. to $\left|V_{u b}\right|^{2}$
Unitarity of CKM matrix represented by triangle in the $\rho-\eta$ complex plane

$\left|V_{u b}\right|$ second poorest known element, its precise measurement would constrain the description of weak interactions and CP violation
$\left|V_{u b}\right|$ results presented at FPCP2010 (Bob Kowaleski)

$$
\begin{gathered}
B \rightarrow \pi \ell v:(2.95 \pm 0.31) \times 10^{-3} \quad \text { (exclusive) } \\
B \rightarrow u \ell v:(4.27 \pm 0.38) \times 10^{-3} \quad \text { (inclusive) }
\end{gathered}
$$

Should we take this difference of $2.7 \sigma$ seriously?

Measurement of $\left|V_{u b}\right|$ requires study of $b \rightarrow u$ transition
Semileptonic $b \rightarrow u \ell \nu$ decays are best :

- much easier to understand theoretically than hadronic decays
- much easier to study experimentally than purely leptonic decays because far more abundant

Exclusive semileptonic decays e.g. $B^{0} \rightarrow \pi^{-} \ell^{+} v \quad \& \quad B^{+} \rightarrow \eta^{( } \ell^{+} v$ involve a $b \rightarrow u$ transition


The value of $\left|V_{u b}\right|$ is extracted from the measured $\Delta B\left(q^{2}\right)$ as a function of $q^{2}$, the momentum transferred squared

$$
\Delta B\left(q^{2}\right)=\frac{\tau_{B^{0}}\left|V_{u b}\right|^{2} G_{F}^{2}}{24 \pi^{3}} \int_{q_{\text {min }}^{2}}^{q_{\text {max }}^{2}}\left|\vec{p}_{X u}\right|^{3} f_{+}\left(q^{2}\right)^{2} d q^{2}
$$

$f_{+}\left(q^{2}\right)$ : form factor provided by QCD calculations

Experimental data used to discriminate between various QCD calculations by measuring the $f_{+}\left(q^{2}\right)$ i.e. $\Delta B\left(q^{2}\right)$ shape precisely

$$
\left|V_{u b}\right|=\sqrt{\Delta B /\left(\tau_{B^{0}} \Delta \zeta\right)} \quad \Delta \zeta=\frac{G_{F}^{2}}{24 \pi^{3}} \int_{q_{\min }^{2}}^{q_{\max }^{2}}\left|\vec{p}_{X u}\right|^{3} f_{+}\left(q^{2}\right)^{2} d q^{2}
$$

Report on 2 recent analyses in BaBar :

- In the $\pi-\eta$ analysis, study 3 decay modes and measure $\boldsymbol{q}^{2}=\left(\boldsymbol{P}_{\boldsymbol{B}}-\boldsymbol{P}_{\text {meson }}\right)^{\mathbf{2}}$

$$
B^{0} \rightarrow \pi^{-} \ell^{+} v \quad B^{+} \rightarrow \eta \ell^{+} v \quad B^{+} \rightarrow \eta^{\prime} \ell^{+} v
$$

- In the $\pi-\rho$ analysis, study 4 decay modes and measure $\boldsymbol{q}^{2}=\left(\boldsymbol{P}_{\ell}+\boldsymbol{P}_{v}\right)^{\mathbf{2}}$
$B^{0} \rightarrow \pi^{-} \ell^{+} v \quad B^{+} \rightarrow \pi^{0} \ell^{+} v \quad B^{0} \rightarrow \rho^{-} \ell^{+} v \quad B^{+} \rightarrow \rho^{0} \ell^{+} v$

Will concentrate on $B \rightarrow \pi \ell \nu$ mode (mostly $\pi-\eta$ analysis)

- most precise measurement
- value of $\left|V_{u b}\right|$

Comparison of the 2 analyses
$\pi-\eta$ analysis
$\pi-\rho$ analysis

| Luminosity on peak $\Upsilon(4 S)$ | $422.6 \mathrm{fb}^{-1}$ | $349.0 \mathrm{fb}^{-1}$ |
| :--- | :---: | :---: |
| Number of BB pair events | 464 millions | 377 millions |
| $q^{2}$ evaluation | $\left(P_{\mathrm{B}}-P_{\text {meson }}\right)^{2}$ | $\left(P_{\ell}+P_{v}\right)^{2}$ |
| Cut strategy | $q^{2}$ dependent, cuts | $q^{2}$ dependent, NN |
| Cut selection | Loose $v$ cuts | Tighter $v$ cuts |
| Signal efficiency | $8 \%$ to $15 \%$ | $6 \%$ to $7 \%$ |
| Background/Signal | 11.5 | 6.3 |
| $B^{0} \rightarrow \pi^{-} \ell^{+} v$ yield | $11778 \pm 435$ | $7181 \pm 279$ |
| Number of $q^{2}$ bins in $\pi$ mode | 12 | 6 |
| Fit strategy | 1 -mode $\left(\pi^{-}, \eta, \eta^{\prime}\right) \ell v$ | 4 -modes $\left(\pi^{-}, \pi^{0}, \rho^{-}, \rho^{0}\right) \ell v$ |
| Systematic uncertainties | Full gaussian | $\pm 1 \sigma$ |

Estimated overlap < 20\%

## $\Delta B\left(q_{i}^{2}\right)=N_{i} / 2 \varepsilon_{i} N_{B}$ $N_{i}$ : number of observed signal events

efficiency : $\varepsilon_{i}=\frac{N_{i}}{N_{i}^{0}}=\frac{\text { total yield in } i^{\text {th }} q^{2} \text { bin after all cuts }}{\text { total yield in } i^{\text {th }} q^{2} \text { bin before any cut }}$
factor of 2 since signal yield \& eff. obtained for sum of e \& muons
$N_{B}=N_{B^{+}}+N_{B^{-}}=2 N_{B B} B\left(\Upsilon(4 S) \rightarrow B^{+} B^{-}\right) \quad$ [2: charge conjugate]
$N_{B}=N_{B^{0}}+N_{\bar{B}^{0}}=2 N_{B B} B\left(\Upsilon(4 S) \rightarrow B^{0} \bar{B}^{0}\right)$
$N_{B B}$ : total number of $B \bar{B}$ pairs observed
Need large amounts of MC events to obtain efficiencies, to optimize the cuts, to get shapes of signal and background distributions

## $\pi-\eta$ analysis : Loose Neutrino Reconstruction


>> Untagged analysis with loose neutrino cuts
>> event's missing momentum ~ signal $v$

$$
P_{v}=P_{\text {beam }}-\sum P_{i}=\left(\left|\vec{p}_{\text {miss }}\right|, \vec{p}_{\text {miss }}\right)
$$

>> High signal yield allows to fit in a large (12) number of $q^{2}$ bins
>> Large background yield allows to fit in several bins


We measure $q^{2}=\left(P_{B}-P_{\text {meson }}\right)^{2}$ in Y -average frame approximation, $P_{Y}=P_{\pi}+P_{\ell}$, as the mean of 4 different values of $\mathrm{q}^{2}$ corrresponding to 4 different directions of $p_{B}$ on the surface of a cone


Use of Y -average approximation improves $q^{2}$ resolution

The 2D $q^{2}$ distribution yields the detector response matrix used to unfold the measured $q^{2}$ distribution onto the true $q^{2}$ distribution


## $\pi-\rho$ analysis : $q^{2}$ resolution


similar to results in $\pi-\eta$ analysis

## $\pi-\eta$ analysis : Event selection

All cuts are optimized to maximize $S / \sqrt{S+B} \approx S / \sigma$




$\pi-\eta$ analysis : Backgrounds

## 3 main sources of background :

- semileptonic $b \rightarrow c \ell \nu$ decays, mostly $B \rightarrow D \ell v$
- semileptonic $b \rightarrow u \ell \nu$ decays $(B \rightarrow \rho \ell \nu, B \rightarrow \omega \ell \nu$, etc...)
- continuum events

$$
\begin{aligned}
& \Delta E=\left(P_{B} \cdot P_{\text {beans }}-s / 2\right) / \sqrt{s} \\
& m_{E S}=\sqrt{\left(s / 2+\vec{p}_{B} \cdot \vec{p}_{\text {beans }}\right)^{2} / E_{\text {beans }}^{2}-\vec{p}_{B}^{2}}
\end{aligned}
$$

$B^{0} \rightarrow \pi^{-} \ell^{+} \nu$

same $B: \ell$ and $\pi$ originate from the same $B$ meson. both $B: \ell$ and $\pi$ originate from different $B$ mesons.
$\pi-\eta$ analysis : Backgrounds categories \& fit parameters

| Categories | Decay mode |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi_{l v}$ | $\eta_{l}(\gamma \gamma \& 3 \pi)$ | $\eta_{\nu}(\gamma \gamma)$ | $\eta_{\nu}(3 \pi)$ | $\eta^{\prime} \ell v(\gamma \gamma)$ |
| Signal | 12 | 3 | 3 | 1 | 1 |
| $b \rightarrow u l \nu$ same $B$ | 2 | fixed | fixed | fixed | fixed |
| $b \rightarrow u l \nu$ both $B$ | 2 | - | - | - | - |
| other $B B$ same $B$ | 2 | 1 | 1 | 1 | 1 |
| other $B \bar{B}$ both $B$ | 2 | - | - | - | - |
| Continuum | 2 | 1 | 1 | fixed | fixed |

For each bin of $q^{2}$ :

$$
B^{0} \rightarrow \pi^{-} \ell^{+} \nu(34 \text { bins }) \quad B \rightarrow \eta^{()} \ell \nu \text { (19 bins) }
$$



## $\pi-\eta$ analysis : Fit strategy \& results

Use $\triangle E-m_{E s}$ histograms, from MC as 2D PDFs, in our fit to the data to extract yields of signal and backgrounds as function of $\mathrm{q}^{2}$

| mode | mev | nev | n'ev |
| :---: | :---: | :---: | :---: |
| efficiency \% | 8.3-14.6 | 1.4-2.6 | 0.6 |
| signal | 11778(435) | 888(98) | 141(46) |
| $b \rightarrow u \ell v$ | 27793(929) | 2201 (fixed) | 204(fixed) |
| other | 80185(963) | 17429(247) | 2660(82) |
| continuum | 27790(814) | 3435(195) | 517(fixed) |
| fit ( $\mathrm{X}^{2} / \mathrm{ndf}$ ) | 411/386 | 56/52 | 19/17 |

$\pi-\eta$ analysis : Data/MC ratios in 34 bins of $\Delta \mathrm{E}-\mathrm{m}_{\mathrm{ES}}$ Before the fit

$2<\mathrm{q}^{2}<4 \mathrm{GeV}^{2} / \kappa^{4}$

$8<q^{\prime}<10 \mathrm{GeV}$ f ${ }^{*}$

$16<q^{\prime}<18 \mathrm{GeV}^{2} \mathrm{t}^{*}$

$18<\mathrm{q}^{\prime}<20 \mathrm{GeV}^{1} /{ }^{\prime}$ *

$\pi-\eta$ analysis : Data/MC ratios in 34 bins of $\Delta E-m_{E S}$ Before the fit After the fit
$B^{0} \rightarrow \pi^{-} \ell^{+} V$


$2<q^{2}<4 \mathrm{GeV}^{2} \mathrm{c}^{4}$

$8<q^{\prime}<10 \mathrm{GeV} / \mathrm{c}$;

$10<q^{\prime}<12 \mathrm{GeV}^{1} / \mathrm{c}^{*}$

$8<q^{\prime}<10 \mathrm{GeV} k+$

$10<q^{\prime}<12 \mathrm{GeV}^{1} / \mathrm{c}^{*}$


## $\pi-\eta$ analysis : Fit projections

Signal enhanced region : $-0.16<\Delta \mathrm{E}<0.20 \mathrm{GeV} ; \mathrm{m}_{\mathrm{ES}}>5.268 \mathrm{GeV}$
$B^{0} \rightarrow \pi^{-} \ell^{+} v$

$B \rightarrow \eta \ell \nu$


$$
B \rightarrow \eta^{\prime} \ell \nu
$$



Good agreement between data and fitted MC distributions

## $\pi-\rho$ analysis : Fit projections

$$
B^{0} \rightarrow \pi^{-} \ell^{+} v
$$







12 October $20 \cdot 10$



$\pi-\eta$ analysis : Systematic uncertainties \& their correlations

- Systematic uncertainties in the partial BF values and their correlations among the $q^{\wedge} 2$ bins investigated
- For each parameter, generate one hundred MC event samples
- Each MC sample contains new $\Delta E-m_{E S}$ distributions generated by varying randomly only the parameter of interest over a full gaussian distribution whose $\sigma$ is given by the uncertainty on the parameter investigated
- Each MC sample is analyzed the same way as the real data to yield values of partial and total BF
- The contribution of the parameter to the systematic uncertainty is given by the RMS of the distribution of these values over the 100 samples


## $\pi-\eta$ analysis : Statistical \& systematic relative uncertainties

| Decay mode | $\pi^{-} \ell^{+} \nu$ |  |  |  |  | $\eta \ell^{+} \nu$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $q^{2}<12$ | $q^{2}<16$ | $q^{2}>16$ | full $q^{2}$ range | $q^{2}<16$ | $\eta^{\prime} \ell^{+} \nu$ |
| Yield | 6541.6 | 8422.1 | 3355.4 | 11777.6 | 887.9 | 141.0 |
| BF $\left(10^{-4}\right)$ | 0.83 | 1.09 | 0.33 | 1.42 | 0.36 | 0.24 |
| Fit error | 3.9 | 3.7 | 7.6 | 3.5 | 12.5 | 32.8 |
| Detector effects | 3.1 | 3.5 | 6.1 | 4.0 | 8.0 | 8.8 |
| Continuum bkg | 2.3 | 1.9 | 4.0 | 2.4 | 0.3 | 7.1 |
| $B \rightarrow X_{u} \ell \nu$ bkg | 2.0 | 1.7 | 4.2 | 2.0 | 7.6 | 6.7 |
| $B \rightarrow X_{c} \ell \nu$ bkg | 0.6 | 0.7 | 1.8 | 1.0 | 1.2 | 2.6 |
| Other effects | 2.3 | 2.2 | 3.2 | 2.3 | 3.4 | 4.6 |
| Total uncertainty | 6.3 | 6.2 | 12.0 | 6.7 | 17.0 | 35.8 |

- Complete tables and correlation matrices available in backup slides
$\pi-\eta$ analysis : Theoretical \& experimental $\Delta \mathrm{B}\left(\mathrm{q}^{2}\right)$ distributions

$$
B^{0} \rightarrow \pi^{-} \ell^{+} v
$$



| Function | Fit | $\pi^{-} \ell^{+} \nu$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Ref. | Parameter | value | $\chi^{2} /$ ndf | Prob. |
| BK | $\alpha_{B K}$ | $0.51 \pm 0.04$ | $10.5 / 10$ | $39.6 \%$ |
| BGL | $a_{1} / a_{0}$ | $-1.58 \pm 0.13$ | $19.3 / 10$ | $3.7 \%$ |
| BGL | $a_{1} / a_{0}$ | $-0.64 \pm 0.30$ | $3.8 / 9$ | $92.2 \%$ |
|  | $a_{2} / a_{0}$ | $-6.8 \pm 1.8$ |  |  |

QCD calculations agree with data in their ranges of validity
arXiv:1010.0987, submitted to Phys. Rev. D
$\pi-\rho$ analysis : Theoretical \& experimental $\Delta \mathrm{B}\left(\mathrm{q}^{2}\right)$ distributions
$B \rightarrow \pi \ell \nu$ (combine charged and neutral pions in fit)

Fit to expt data only

Fit to expt data \& theor values

| Parametrization | $\chi^{2} /$ ndf | $\operatorname{Prob}\left(\chi^{2} /\right.$ ndf $)$ | Fit parameters |
| :--- | :---: | :---: | :--- |
| BK | $6.8 / 4$ | 0.148 | $\alpha_{B K}=+0.310 \pm 0.085$ |
| BGL (2 par.) | $6.6 / 4$ | 0.156 | $a_{1} / a_{0}=-0.94 \pm 0.20$ |
| BGL (3 par.) | $6.3 / 3$ | 0.100 | $a_{1} / a_{0}=-0.82 \pm 0.29$ |
|  |  |  | $a_{2} / a_{0}=-1.14 \pm 1.81$ |

arXiv:1005.3288, accepted by Phys. Rev. D

## Comparison of the two $B \rightarrow \pi \ell v$ results



Results are consistent. Statistical correlations between the 2 data sets fairly low (estimated < 20 \%)

Form factor shapes for other decay channels
$\pi-\eta$ analysis



LCSR calculations compatible with data

## Main results

Branching fractions :
$\pi-\eta$ analysis
${ }\left(B^{0} \rightarrow \pi^{-} \ell^{+} \nu\right)=\left(1.42 \pm 0.05_{\text {stat }} \pm 0.08_{\text {syst }}\right) \times 10^{-4} } }$
$\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)=\left(3.61 \pm 0.45_{\text {stat }} \pm 0.44_{\text {syst }}\right) \times 10^{-5}$
$\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)=\left(2.43 \pm 0.80_{\text {stat }} \pm 0.34_{\text {syst }}\right) \times 10^{-5}$

$$
\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right) / \mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)=0.67 \pm 0.24_{\text {stat }} \pm 0.11_{\text {syst }}
$$

$\pi-\rho$ analysis
$\mathcal{B}\left(B^{0} \rightarrow \pi^{-} \ell^{+} \nu\right)=(1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$
$\mathcal{B}\left(B^{0} \rightarrow \rho^{-} \ell^{+} \nu\right)=(1.75 \pm 0.15 \pm 0.27) \times 10^{-4}$
|Vub|


## Summary

- New precise BF measurements for

$$
B \rightarrow \pi \ell \nu, B \rightarrow \eta^{(\cdot)} \ell \nu \text { and } B \rightarrow \rho \ell \nu
$$

- Two different BaBar analyses consistent with each other
- All three QCD calculations of the form factor for $B \rightarrow \pi \ell \nu$ decays are consistent with data and yield values of |Vub| consistent with the value of |Vub| measured in inclusive semileptonic decays

|  | $\pi-\eta$ analysis | $\pi-\rho$ analysis |
| :---: | :---: | :---: |
| $\left\|V_{u b}\right\|\left(10^{-3}\right)$ |  |  |
| HPQCD | $3.24 \pm 0.13 \pm 0.16$ | $3.21 \pm 0.17_{-0.36}^{+0.55}$ |
| FNAL | $3.14 \pm 0.12 \pm 0.16$ | (2.95 $\pm 0.31)$ |
| LCSR | $3.70 \pm 0.07 \pm 0.09$ | $3.78 \pm 0.13_{-0.40}^{+0.55}$ |

Inclusive value of $\left|\mathrm{V}_{\mathrm{ub}}\right|$

$$
\left|\mathrm{V}_{\mathrm{ub}}\right|=(4.27 \pm 0.38) \times 10^{-3}
$$

## The end

## Backup slides

## Backup Slides for $B \rightarrow D_{S} K \ell v$ analysis

Puzzle in exclusive $B \rightarrow D^{(*, *)} \ell v$ branching fractions $B F\left(B \rightarrow X_{c} \ell v\right)>B F(B \rightarrow D \ell v)+B F\left(B \rightarrow D^{*} \ell v\right)+B F\left(B \rightarrow D^{* *} \ell v\right)$


$$
M_{m}^{2}=\left(E_{\text {beam }}-E_{Y}\right)^{2}-\left|\vec{p}_{Y}\right|^{2}=m_{v}^{2}
$$

Measure $B F\left(B \rightarrow D_{s} K \ell v\right)$


$$
Y=D_{S} K \ell \text { candidate }
$$

$B F\left(B^{-} \rightarrow D_{S}^{+} K^{-} \ell^{-} \bar{v}_{\ell}\right)=\left(6.13_{-1.03 s \operatorname{sit}}^{+1.04} \pm 0.43_{\text {syst }} \pm 0.51\left(B F\left(D_{S}\right)\right) \times 10^{-4}\right.$
BF too small to solve the BF puzzle
(presented at ICHEP2010)

## $B\left(B^{-} \rightarrow D_{s}^{(*)+} K^{-} \ell^{-} \bar{\nu}_{\ell}\right)$

Puzzle in exclusive $B \rightarrow D^{(*, *)} \mid v$ Branching Fractions:

$$
\mathcal{B}\left(B \rightarrow X_{c} \ell \nu\right)>\mathcal{B}(B \rightarrow D \ell \nu)+\mathcal{B}\left(B \rightarrow D^{*} \ell \nu\right)+\mathcal{B}\left(B \rightarrow D^{* *} \ell \nu\right)
$$



- experimentally: similar rate for broad and narrow (QCD sum rule: narrow >> broad)
- small statistics doesn' $\dagger$ allow to separate broad from non resonant
- $B \rightarrow D_{s}^{(*)}$ Klv similar to $B \rightarrow D^{(*)} \pi l v$. Study hadronic mass spectrum above $2.46 \mathrm{GeV} / \mathrm{c}^{2}$
- $\mathcal{B}\left(B \rightarrow D_{s}^{\left({ }^{*}\right.} \mathrm{Klv}\right)$ expected $\sim 10^{-3}$
A. Petrella ICHEP 2010


12 October 2010
P.Taras/Heavy Quark \& Leptons

- Exclusive reconstruction

$$
\left.\begin{array}{l}
D_{s} \rightarrow \phi\left(K^{+} K^{-}\right) \pi \\
D_{s} \rightarrow \bar{K}^{* 0}\left(K^{ \pm} \pi^{\mp}\right) K
\end{array}\right\} \longleftarrow \begin{aligned}
& \text { Feed Forward NN to } \\
& \text { suppress combinatorial } \\
& \text { bkg. }
\end{aligned}
$$

- lepton ( $\mathrm{p}_{\text {lep }}>0.8 \mathrm{GeV} / \mathrm{c}$ ) and Kaon added to $\mathrm{D}_{s}$ candidate
- Bkg from $B \rightarrow D D_{s}$ reduced using angular correlation between $D_{s}$ and $D$ (signal events no correlation)
~30\% of
$D_{s}$ bkg
rejected




## $B\left(B^{-} \rightarrow D_{s}^{(*)+} K^{-} \ell^{-} \bar{\nu}_{\ell}\right)$

- Signal yields extracted via unbinned extended maximum likelihood fit to Missing mass

$$
M_{m}^{2}=\left(E_{\text {beam }}-E_{Y}\right)^{2}-\left|\vec{p}_{Y}\right|^{2}=m_{\nu}^{2} \quad Y=D_{s} K l \text { candidate }
$$

electron channel



- leading systematic uncertainty: signal MC modelling ( $\sim 3 \%-8 \%$ depending on channel) Signal MC statistics (~2\%)

$$
\mathcal{B}\left(B \rightarrow D_{s}^{+} K^{-} \ell^{-} \bar{\nu}_{\ell}\right)=\left(6.13_{-1.03}^{+1.04} \text { stat. } \pm 0.43_{\text {syst. }} \pm 0.51\left(\mathcal{B}\left(D_{s}\right)\right)\right) \times 10^{-4}
$$

- Result in agreement with ARGUS measurement: $\mathcal{B}\left(B \rightarrow D_{s}^{+} K^{-} \ell^{-} \bar{\nu}_{\ell}\right)<5 \times 10^{-3}$
- $B R$ too small to solve the $B R$ puzzle


## Backup Slides for $\pi-\eta$ analysis

## Statistical and Systematic relative uncertainties



## Statistical and Systematic relative uncertainties

| Decay mode | $\pi / \ell^{+} \nu$ | $7 \ell^{+} \nu(3 \pi)$ | ${ }^{7} \ell^{+} \nu(\gamma \gamma)$ |  |  |  | ${ }_{7} \ell^{+} \nu(3 \pi$ and $\gamma \gamma$ combined) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q^{2}$ bins ( $\mathrm{GeV}^{2}$ ) | Total | Total | 0-4 | 4.8 | 8.16 | Total | 0.4 | 4.8 | 8-16 | Total |
| Fitted yield | 141.0 | 244.8 | 279.9 | 216.8 | 146.7 | 643.4 | 303.9 | 331.5 | 252.5 | 887.9 |
| Systematic error | 14.6 | 14.3 | 5.1 | 5.8 | 34.9 | 9.6 | 5.8 | 6.6 | 28.1 | 10.3 |
| Fit error | 32.8 | 25.6 | 13.9 | 17.2 | 33.9 | 12.0 | 14.1 | 14.2 | 26.6 | 11.0 |
| Efficiency | 0.61 | 0.59 | 2.01 | 2.55 | 1.42 | - | 2.53 | 3.41 | 1.94 | - |
| Unfolded yield | 141.0 | 244.8 | 299.1 | 210.9 | 133.3 | 643.4 | 319.3 | 334.8 | 233.9 | 887.9 |
| $\Delta \mathcal{B}$ | 242.5 | 431.5 | 155.3 | 86.3 | 97.7 | 339.3 | 131.8 | 102.6 | 126.2 | 360.6 |
| Tracking efficiency | 5.2 | 4.1 | 3.2 | 2.4 | 14.6 | 2.6 | 2.1 | 2.0 | 11.1 | 2.8 |
| Photon efficiency | 5.6 | 3.1 | 10.1 | 4.3 | 27.4 | 7.0 | 8.0 | 3.8 | 9.0 | 5.7 |
| $K_{L}^{0}$ efficiency | 2.5 | 0.7 | 8.6 | 2.9 | 27.2 | 3.2 | 1.0 | 0.5 | 2.2 | 0.6 |
| $K_{L}^{0}$ production spectrum | 2.7 | 1.4 | 4.7 | 1.5 | 16.2 | 2.5 | 0.8 | 0.5 | 2.3 | 1.0 |
| $K_{L}^{0}$ energy | 1.1 | 1.4 | 0.6 | 0.5 | 2.5 | 0.9 | 0.6 | 0.4 | 2.3 | 1.0 |
| $\ell$ identification | 2.0 | 1.8 | 0.1 | 2.7 | 3.9 | 1.8 | 0.2 | 1.9 | 3.4 | 1.8 |
| $\pi$ identification | 0.6 | 0.5 | - | - | - | - | 0.1 | 0.2 | 0.5 | 0.3 |
| Bremustrahlung | 0.5 | 0.2 | 1.6 | 2.7 | 22.2 | 8.0 | 0.3 | 0.7 | 12.3 | 4.2 |
| Continuum yield | 4.9 | 1.1 | - | - | - | - | - | - | - | - |
| $q^{2}$ continuum shape | 5.2 | 2.6 | 2.6 | 1.5 | 4.5 | 0.5 | 2.4 | 0.7 | 2.8 | 0.3 |
| $\mathcal{B}\left(B^{0} \rightarrow \pi^{-} \ell^{+} \nu\right)$ | 0.0 | 0.1 | 0.0 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 |
| $\mathcal{B}\left(B^{+} \rightarrow \pi^{0} \ell^{+} \nu\right)$ | 0.2 | 0.0 | 0.4 | 0.9 | 5.2 | 1.9 | 0.3 | 0.6 | 2.9 | 1.3 |
| $\mathcal{B}\left(B^{+} \rightarrow \eta^{(\prime)} \ell^{+} \nu\right)$ | 0.4 | 0.4 | 0.0 | 0.1 | 0.8 | 0.2 | 0.1 | 0.1 | 1.0 | 0.4 |
| $\mathcal{B}\left(B^{\circ} \rightarrow \rho^{-\ell^{+} \nu}\right)$ | 0.3 | 0.5 | 0.1 | 1.1 | 6.9 | 2.3 | 0.1 | 0.6 | 4.2 | 1.7 |
| $\mathcal{B}\left(B^{+} \rightarrow \rho^{0} \ell^{+} \nu\right)$ | 0.0 | 0.3 | 0.1 | 0.1 | 0.5 | 0.1 | 0.0 | 0.1 | 0.8 | 0.2 |
| $\mathcal{B}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ | 0.8 | 1.1 | 0.1 | 0.2 | 2.6 | 0.8 | 0.1 | 0.1 | 2.6 | 0.9 |
| Non resonamt $b \rightarrow u \ell \nu \mathrm{BF}$ | 2.3 | 3.5 | 0.4 | 0.9 | 9.5 | 3.1 | 0.5 | 0.6 | 8.6 | 3.4 |
| 7 BF | 3.1 | 1.2 | 0.5 | 0.7 | 0.7 | 0.6 | 0.5 | 0.6 | 0.7 | 0.5 |
| SF parameters | 4.3 | 6.3 | 1.4 | 2.7 | 16.8 | 6.1 | 1.5 | 2.5 | 14.3 | 6.2 |
| $B \rightarrow \rho l \nu \mathrm{FF}$ | 0.1 | 0.7 | 0.1 | 2.3 | 1.7 | 0.9 | 0.1 | 1.5 | 0.9 | 0.5 |
| $\mathrm{B}^{+} \rightarrow \eta^{(\prime)} \ell^{+}{ }_{\nu} \mathrm{FF}$ | 1.1 | 1.0 | 0.1 | 0.1 | 1.4 | 0.4 | 0.1 | 0.1 | 1.5 | 0.6 |
| Other scalar FF | 2.9 | 4.2 | 7.7 | 1.4 | 0.1 | 3.2 | 0.7 | 0.1 | 0.0 | 0.2 |
| $B \rightarrow \omega \ell \nu \mathrm{FF}$ | 1.2 | 2.1 | 0.1 | 0.5 | 2.8 | 0.7 | 0.1 | 0.4 | 3.9 | 1.3 |
| $\mathcal{B}(B \rightarrow D \ell \nu)$ | 1.6 | 0.7 | 0.3 | 0.7 | 0.6 | 0.3 | 0.3 | 0.7 | 0.7 | 0.4 |
| $\mathcal{B}\left(B \rightarrow D^{*} \ell \nu\right)$ | 0.3 | 0.4 | 0.1 | 0.8 | 1.2 | 0.4 | 0.1 | 0.7 | 1.0 | 0.4 |
| $\mathcal{B}\left(B \rightarrow D^{* *} \psi\right)$ | 2.0 | 1.2 | 0.6 | 0.9 | 2.5 | 0.7 | 0.6 | 0.7 | 2.6 | 0.9 |
| Non resonant $b \rightarrow c / \nu \mathrm{BF}$ | 0.1 | 0.1 | 0.2 | 0.1 | 0.8 | 0.2 | 0.3 | 0.1 | 0.4 | 0.2 |
| $B \rightarrow D<\nu \mathrm{FF}$ | 0.1 | 0.3 | 0.1 | 0.1 | 0.5 | 0.2 | 0.1 | 0.1 | 0.7 | 0.3 |
| $B \rightarrow D^{*} \& \nu \mathrm{FF}$ | 0.6 | 0.9 | 0.5 | 0.9 | 1.3 | 0.4 | 0.5 | 1.2 | 1.2 | 0.4 |
| $\mathcal{B}\left(T(4 S) \rightarrow B^{0} B^{0}\right)$ | 1.1 | 1.2 | 1.4 | 1.1 | 0.9 | 1.2 | 1.4 | 1.2 | 1.0 | 1.2 |
| Secondary lepton | 4.2 | 5.0 | 1.3 | 0.7 | 9.1 | 2.1 | 1.2 | 1.6 | 9.3 | 3.0 |
| B counting | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| Signal MC stat error | 1.2 | 1.1 | 1.4 | 1.6 | 1.2 | 0.7 | 1.3 | 1.3 | 1.0 | 0.5 |
| Total systernatic error | 14.3 | 12.4 | 17.0 | 8.7 | 55.4 | 14.1 | 9.3 | 6.6 | 28.7 | 11.6 |
| Fit error | E2. | Elfels) | 174.6 | [felil | -30\% | 190) |  | 16.6 | 30.3 | 12.5 |
| Total error | 35.8 | 28.4 | 22.4 | 22.7 | 67.9 | 19.6 | 17.8 | 17.8 | 41.8 | 17.0 |

## Correlation Matrices

## $B^{0} \rightarrow \pi^{-} \ell^{+} v$ Statistical

| $q^{2}$ bins $\left(\mathrm{GeV}^{2}\right)$ | $0-2$ | $2-4$ | $4-6$ | $6-8$ | $8-10$ | $10-12$ | $12-14$ | $14-16$ | $16-18$ | $18-20$ | $20-22$ | $22-26.4$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-2$ | 1.00 | -0.16 | 0.17 | 0.02 | -0.02 | 0.03 | 0.01 | 0.04 | 0.05 | 0.02 | 0.04 | -0.00 |
| $2-4$ | -0.16 | 1.00 | -0.32 | 0.11 | 0.00 | -0.00 | -0.01 | 0.01 | 0.01 | -0.00 | 0.00 | -0.00 |
| $4-6$ | 0.17 | -0.32 | 1.00 | -0.30 | 0.15 | 0.02 | 0.06 | 0.06 | 0.07 | 0.00 | 0.01 | 0.01 |
| $6-8$ | 0.02 | 0.11 | -0.30 | 1.00 | -0.22 | 0.13 | 0.07 | 0.06 | 0.07 | 0.00 | 0.00 | 0.02 |
| $8-10$ | -0.02 | 0.00 | 0.15 | -0.22 | 1.00 | -0.22 | 0.16 | 0.05 | 0.08 | 0.01 | -0.00 | 0.02 |
| $10-12$ | 0.03 | -0.00 | 0.02 | 0.13 | -0.22 | 1.00 | -0.15 | 0.10 | 0.07 | -0.01 | 0.02 | 0.00 |
| $12-14$ | 0.01 | -0.01 | 0.06 | 0.07 | 0.16 | -0.15 | 1.00 | -0.16 | 0.13 | -0.01 | 0.05 | -0.00 |
| $14-16$ | 0.04 | 0.01 | 0.06 | 0.06 | 0.05 | 0.10 | -0.16 | 1.00 | -0.01 | 0.01 | -0.02 | -0.02 |
| $16-18$ | 0.05 | 0.01 | 0.07 | 0.07 | 0.08 | 0.07 | 0.13 | -0.01 | 1.00 | -0.17 | 0.09 | -0.08 |
| $18-20$ | 0.02 | -0.00 | 0.00 | 0.00 | 0.01 | -0.01 | -0.01 | 0.01 | -0.17 | 1.00 | 0.05 | -0.05 |
| $20-22$ | 0.04 | 0.00 | 0.01 | 0.00 | -0.00 | 0.02 | 0.05 | -0.02 | 0.09 | 0.05 | 1.00 | -0.35 |
| $22-26.4$ | -0.00 | -0.00 | 0.01 | 0.02 | 0.02 | 0.00 | -0.00 | -0.02 | -0.08 | -0.05 | -0.35 | 1.00 |

## $B^{0} \rightarrow \pi^{-} \ell^{+} v$ Systematic

|  | statistical |  |  |  | systematic |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q^{2}$ bins $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-16$ | $0-4$ | $4-8$ | $8-16$ |  |  |
| $0-4$ | 1.00 | -0.08 | 0.00 | 1.00 | 0.36 | 0.05 |  |  |
| $4-8$ | -0.08 | 1.00 | -0.06 | 0.36 | 1.00 | 0.29 |  |  |
| $8-16$ | 0.00 | -0.06 | 1.00 | 0.05 | 0.29 | 1.00 |  |  |


| $q^{2}$ bins $\left(\mathrm{GeV}^{2}\right)$ | 0－2 | 2－4 | 4－6 | 6－8 | 8－10 | 10－12 | 12－14 | 14－16 | 16－18 | 18－20 | 20－22 | 22－26．4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0－2 | 1.00 | －0．19 | 0.41 | 0.33 | 0.49 | 0.42 | 0.49 | 0.35 | 0.39 | 0.14 | 0.47 | 0.56 |
| 2－4 | －0．19 | 1.00 | －0．17 | 0.08 | －0．20 | －0．07 | －0．14 | －0．16 | －0．31 | 0.41 | －0．24 | 0.05 |
| 4－6 | 0.41 | －0．17 | 1.00 | 0.78 | 0.82 | 0.76 | 0.67 | 0.68 | 0.53 | 0.33 | 0.72 | 0.48 |
| 6－8 | 0.33 | 0.08 | 0.78 | 1.00 | 0.71 | 0.74 | 0.65 | 0.63 | 0.47 | 0.49 | 0.55 | 0.38 |
| 8－10 | 0.49 | －0．20 | 0.82 | 0.71 | 1.00 | 0.74 | 0.70 | 0.70 | 0.49 | 0.36 | 0.66 | 0.39 |
| 10－12 | 0.42 | －0．07 | 0.76 | 0.74 | 0.74 | 1.00 | 0.74 | 0.80 | 0.61 | 0.36 | 0.61 | 0.40 |
| 12－14 | 0.49 | －0．14 | 0.67 | 0.65 | 0.70 | 0.74 | 1.00 | 0.69 | 0.73 | 0.29 | 0.55 | 0.36 |
| 14－16 | 0.35 | －0．16 | 0.68 | 0.63 | 0.70 | 0.80 | 0.69 | 1.00 | 0.71 | 0.35 | 0.64 | 0.37 |
| 16－18 | 0.39 | －0．31 | 0.53 | 0.47 | 0.49 | 0.61 | 0.73 | 0.71 | 1.00 | －0．01 | 0.62 | 0.29 |
| 18－20 | 0.14 | 0.41 | 0.33 | 0.49 | 0.36 | 0.36 | 0.29 | 0.35 | －0．01 | 1.00 | 0.04 | 0.20 |
| 20－22 | 0.47 | －0．24 | 0.72 | 0.55 | 0.66 | 0.61 | 0.55 | 0.64 | 0.62 | 0.04 | 1.00 | 0.52 |
|  |  |  | 0.48 | 0.38 | 0.39 | 0.40 | $\mathrm{C}_{3} \mathrm{~S}_{2}$ | ${ }_{0}^{0} 37$ | 星的呺 | $2 . \hat{2}$ | Off | 1．80 |

3939
$B^{0} \rightarrow \pi^{-} \ell^{+} \nu \quad$ Projections on $\Delta E$ axis


$14<\mathrm{q}^{2}<16 \mathrm{GeV}^{2} / \mathrm{c}^{4}$



## Branching fraction results

Total BF for $B^{+} \rightarrow \eta^{\prime} \ell^{+} v:\left(2.43 \pm 0.80_{\text {stat }} \pm 0.34_{\text {syst }}\right) \times 10^{-5}$, significance of $2.67 \sigma$ an order of magnitude smaller than most recent result of CLEO

$$
\left(2.66 \pm 0.80_{\text {stat }} \pm 0.56_{\text {syst }}\right) \times 10^{-4}
$$

Total BF for $B^{+} \rightarrow \eta \ell^{+} v:\left(3.61 \pm 0.45_{\text {stat }} \pm 0.44_{\text {syst }}\right) \times 10^{-5}$, most precise, compatible with previous BaBar result:

$$
\left(3.7 \pm 0.6_{\text {stat }} \pm 0.7_{\text {syst }}\right) \times 10^{-5}
$$

$B F\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} v\right) / B F\left(B^{+} \rightarrow \eta \ell^{+} v\right)=0.67 \pm 0.27$ allows an important gluonic singlet contribution to the $\eta^{\prime}$ form factor

## Backup Slides for $\pi-\rho$ analysis

## Statistical and Systematic relative uncertainties

| $B \rightarrow \pi \ell \nu$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q^{2}$ range ( $\mathrm{GeV}^{2}$ ) | 0-4 | 4-8 | 8-12 | 12-16 | 16-20 | >20 | 0-26.4 |
| Track efficiency | 3.4 | 1.5 | 2.3 | 0.1 | 1.5 | 2.8 | 1.9 |
| Photon efficiency | 0.1 | 1.4 | 1.0 | 4.6 | 2.8 | 0.3 | 1.8 |
| Lepton identification | 3.8 | 1.6 | 1.9 | 1.8 | 1.9 | 3.0 | 1.8 |
| $K_{L}$ efficiency | 1.0 | 0.1 | 0.5 | 4.5 | 0.4 | 2.0 | 1.4 |
| $K_{L}$ shower energy | 0.1 |  | 0.1 | 0.8 | 0.9 | 3.8 | 0.7 |
| $K_{L}$ spectrum | 1.6 | 1.9 | 2.2 | 3.1 | 4.4 | 2.3 | 2.5 |
| $B \rightarrow \pi \ell \nu F F f_{+}$ | 0.5 | 0.5 | 0.5 | 0.6 | 1.0 | 1.0 | 0.6 |
| $B \rightarrow \rho \ell \nu F F A_{1}$ | 1.7 |  | 3.4 | 2.0 | 0.1 | 1.6 | 1.7 |
| $B \rightarrow \rho \ell \nu F F A_{2}$ | 1.3 |  | 2.6 | 1.0 | 0.1 | 0.4 | 1.1 |
| $B \rightarrow \rho \ell \nu F F V$ | 0.2 | 0.3 | 0.9 | 0.7 | 0.1 | 0.5 | 0.5 |
| $\overline{\mathcal{B}}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ | 0.1 |  | 0.1 | 0.2 | 0.3 | 1.5 | 0.2 |
| $\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)$ | 0.1 |  | 0.2 | 0.2 | 0.2 | 0.5 | 0.2 |
| $\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.1 |
| $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$ | 0.2 |  | 0.1 | 0.1 | 1.1 | 1.6 | 0.4 |
| $B \rightarrow X_{u} \ell \nu$ SF param. | 0.4 | 0.1 | 0.2 | 0.2 | 0.5 | 4.2 | 0.7 |
| $B \rightarrow D \ell \nu$ FF $\rho_{D}^{2}$ | 0.2 | 0.1 | 0.5 | 0.3 | 0.2 | 0.7 | 0.3 |
| $B \rightarrow D^{*} \ell \nu$ FF $R_{1}$ | 0.1 | 0.4 | 0.8 | 0.6 | 0.3 | 0.6 | 0.5 |
| $B \rightarrow D^{*} \ell \nu$ FF $R_{2}$ | 0.5 | 0.2 | 0.1 | 0.2 | 0.1 | 0.4 | 0.2 |
| $B \rightarrow D^{*} \ell \nu$ FF $\rho_{D}^{2}$. | 0.7 |  | 0.6 | 0.8 | 0.4 | 1.1 | 0.6 |
| $\mathcal{B}(B \rightarrow$ 生 $)$ | 0.2 |  | 0.3 | 0.4 | 0.5 | 0.5 | 0.3 |
| $\mathcal{B}\left(B \rightarrow D^{*} \ell \nu\right)$ | 0.4 | 0.1 | 0.3 | 0.3 | 0.3 | 0.7 | 0.3 |
| $\mathcal{B}\left(B \rightarrow D^{* *} \ell \nu\right)_{\text {narrow }}$ | 0.4 | 0.1 | 0.1 | 0.3 | 0.1 | 0.5 | 0.2 |
| $\mathcal{B}\left(B \rightarrow D^{* *} \ell \nu\right)_{\text {broad }}$ | 0.1 | 0.1 | 0.1 | 0.5 | 0.1 | 0.2 | 0.2 |
| Secondary leptons | 0.5 | 0.2 | 0.3 | 0.2 | 0.2 | 0.7 | 0.3 |
| Continuum | 5.3 | 1.0 | 2.6 | 1.8 | 3.1 | 6.1 | 2.0 |
| Bremsstrahlung | 0.3 | 0.1 | 0.1 | 0.1 | 0.1 | 0.4 | 0.2 |
| Radiative corrections | 0.5 | 0.1 | 0.1 | 0.2 | 0.2 | 0.6 | 0.3 |
| $N_{B \bar{B}}$ | 1.2 | 1.0 | 1.2 | 1.2 | 1.1 | 1.6 | 1.2 |
| $B$ lifetimes | 0.3 |  | 0.3 | 0.3 | 0.3 | 0.7 | 0.3 |
|  |  |  |  | $0.8$ |  |  | $0.8$ |


| $B \rightarrow \rho \ell \nu$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $q^{2}$ range ( $\mathrm{GeV}^{2}$ ) | 0-8 | 8-16 | >16 | 0-20.3 |
| Track efficiency | 3.2 | 2.9 | 0.3 | 2.5 |
| Photon efficiency | 2.6 | 2.0 | 2.6 | 2.4 |
| Lepton Identification | 5.7 | 3.0 | 4.0 | 3.4 |
| $K_{L}$ efficiency | 10.3 | 1.2 | 4.9 | 4.8 |
| $K_{L}$ shower energy | 1.6 | 0.8 | 1.0 | 1.1 |
| $K_{L}$ spectrum | 4.2 | 6.1 | 7.0 | 5.7 |
| $B \rightarrow \pi \ell \nu$ FF $f_{+}$ | 0.1 | 0.1 | 0.7 | 0.2 |
| $B \rightarrow \rho \ell \nu$ FF $A_{1}$ | 10.7 | 6.6 | 4.5 | 7.5 |
| $B \rightarrow \rho \ell \nu$ FF $A_{2}$ | 8.5 | 3.8 | 0.8 | 4.7 |
| $B \rightarrow \rho \ell \nu$ FF $V$ | 3.4 | 3.0 | 3.6 | 3.2 |
| $\overline{\mathcal{B}}\left(B^{+} \rightarrow \omega \ell^{+} \nu\right)$ | 0.7 | 0.7 | 3.4 | 1.2 |
| $\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right)$ | 0.8 | 0.1 | 0.6 | 0.4 |
| $\mathcal{B}\left(B^{+} \rightarrow \eta^{\prime} \ell^{+} \nu\right)$ | 0.8 | 0.5 | 1.2 | 0.7 |
| $\mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$ | 7.4 | 7.3 | 10.6 | 8.0 |
| $B \rightarrow X_{u} \ell \nu$ SF param. | 11.9 | 7.6 | 12.8 | 10.0 |
| $B \rightarrow D \ell \nu$ FF $\rho_{D}^{2}$ | 0.9 | 0.2 | 0.1 | 0.4 |
| $B \rightarrow D^{*} \ell \nu$ FF $R_{1}$ | 0.7 | 0.1 | 0.3 | 0.3 |
| $B \rightarrow D^{*} \ell \nu$ FF $R_{2}$ | 1.7 | 0.1 | 0.2 | 0.6 |
| $B \rightarrow D^{*} \ell \nu \mathrm{FF} \rho_{D^{*}}^{2}$ | 2.0 | 0.2 | 0.1 | 0.7 |
| $\mathcal{B}(B \rightarrow$ 生 $)$ | 1.6 | 0.3 | 0.1 | 0.7 |
| $\mathcal{B}\left(B \rightarrow D^{*} \ell \nu\right)$ | 0.5 | 0.1 | 0.3 | 0.3 |
| $\mathcal{B}\left(B \rightarrow D^{* *} \ell \nu\right)_{\text {narrow }}$ | 1.3 | 0.1 | 0.1 | 0.5 |
| $\mathcal{B}\left(B \rightarrow D^{* *} \ell \nu\right)_{\text {broad }}$ | 0.7 | 0.1 | 0.1 | 0.3 |
| Secondary leptons | 1.5 | 0.1 | 0.1 | 0.5 |
| Continuum | 8.9 | 3.8 | 5.0 | 4.0 |
| Bremsstrahlung | 0.9 | 0.1 | 0.2 | 0.4 |
| Radiative corrections | 1.3 | 0.1 | 0.7 | 0.6 |
| $\overline{N_{B \bar{B}}}$ | 2.7 | 2.0 | 2.5 | 2.3 |
| $B$ lifetimes | 1.5 | 0.4 | 0.4 | 0.7 |
| $f_{ \pm} / f_{00}$ | 1.2 | 0.1 | 0.1 | 0.4 |
| Hegutbyst. error | 26.1 | 16.1 | 21.3 | 15.7 |

## Correlation Matrices

$$
B \rightarrow \pi \ell \nu \text { Statistical }
$$

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-12$ | $12-16$ | $16-20$ | $>20$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $0-4$ | 1.000 | 0.191 | 0.050 | -0.005 | 0.068 | 0.057 |
| $4-8$ |  | 1.000 | 0.089 | 0.058 | 0.085 | 0.011 |
| $8-12$ |  |  | 1.000 | 0.197 | 0.127 | 0.005 |
| $12-16$ |  |  |  | 1.000 | 0.135 | -0.008 |
| $16-20$ |  |  |  |  |  | 1.000 |
| $>20$ |  |  |  |  |  | 0.032 |

$B \rightarrow \pi \ell \nu$ Systematic

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-4$ | $4-8$ | $8-12$ | $12-16$ | $16-20$ | $>20$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $0-4$ | 1.000 | 0.521 | 0.705 | 0.394 | -0.052 | 0.075 |
| $4-8$ |  | 1.000 | 0.853 | 0.687 | 0.605 | 0.478 |
| $8-12$ |  |  | 1.000 | 0.652 | 0.366 | 0.439 |
| $12-16$ |  |  |  | 1.000 | 0.637 | 0.367 |
| $16-20$ |  |  |  |  | 1.000 | 0.509 |
| $>20$ |  |  |  |  |  |  |

$$
\begin{array}{cc}
B \rightarrow \rho \ell v & B \rightarrow \rho \ell v \\
\text { Statistical } & \text { Systematic }
\end{array}
$$

| $q^{2}$ range $\left(\mathrm{GeV}^{2}\right)$ | $0-8$ | $8-16$ | $>16$ | $0-8$ | $8-16$ | $>16$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $0-8$ | 1.000 | 0.264 | 0.137 | 1.000 | 0.339 | 0.692 |
| $8-16$ |  | 1.000 | 0.189 |  | 1.000 | 0.296 |
| $>16$ |  | 1.000 |  | 1.000 |  |  |

