Highlights in Kaon Decays

A Theorists Perspective

Heavy Quarks & Leptons
INFN, Laboratori Nazionali di Frascati
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What can we learn from Kaons?

➡ Determination of fundamental parameters

- CKM unitarity
- Lepton universality
- Mass determination

➡ Test suppression of top-dominated FCNCs

- Rare decays
- CP violation

Many more interesting things – only 30 minutes

Leptonic and Semileptonic

Observables: $K(\pi) \to l\bar{\nu}_l$ & $K \to \pi\, l\bar{\nu}_l$

$$\frac{\Gamma(K^{\pm}_{\ell 2(\gamma)})}{\Gamma(\pi^{\pm}_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right)^2 \times (1 + \delta_{\rm em})$$

[Marciano `04]

Lattice

[Cirigliano, Giannotti, Neufeld `08]

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{\rm ew} |V_{us}|^2 f_+(0)^2\, I_K^\ell(\lambda_{+,0}) \left(1 + \delta^K_{SU(2)} + \delta^{K\ell}_{\rm em}\right)^2$$

Isospin breaking effects: Flavianet `10

Lattice input: Talk by Vittorio Lubicz

Talk on measurement of semileptonic form factors: Hita Hochgesand

CKM Unitarity

$\Gamma(K_{l3})$ $\quad |V_{us}|f_+(0) = 0.2163(5)$

$\frac{\Gamma(K_{l2})}{\Gamma(\pi_{l2})}$ $\quad \frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2758(5)$

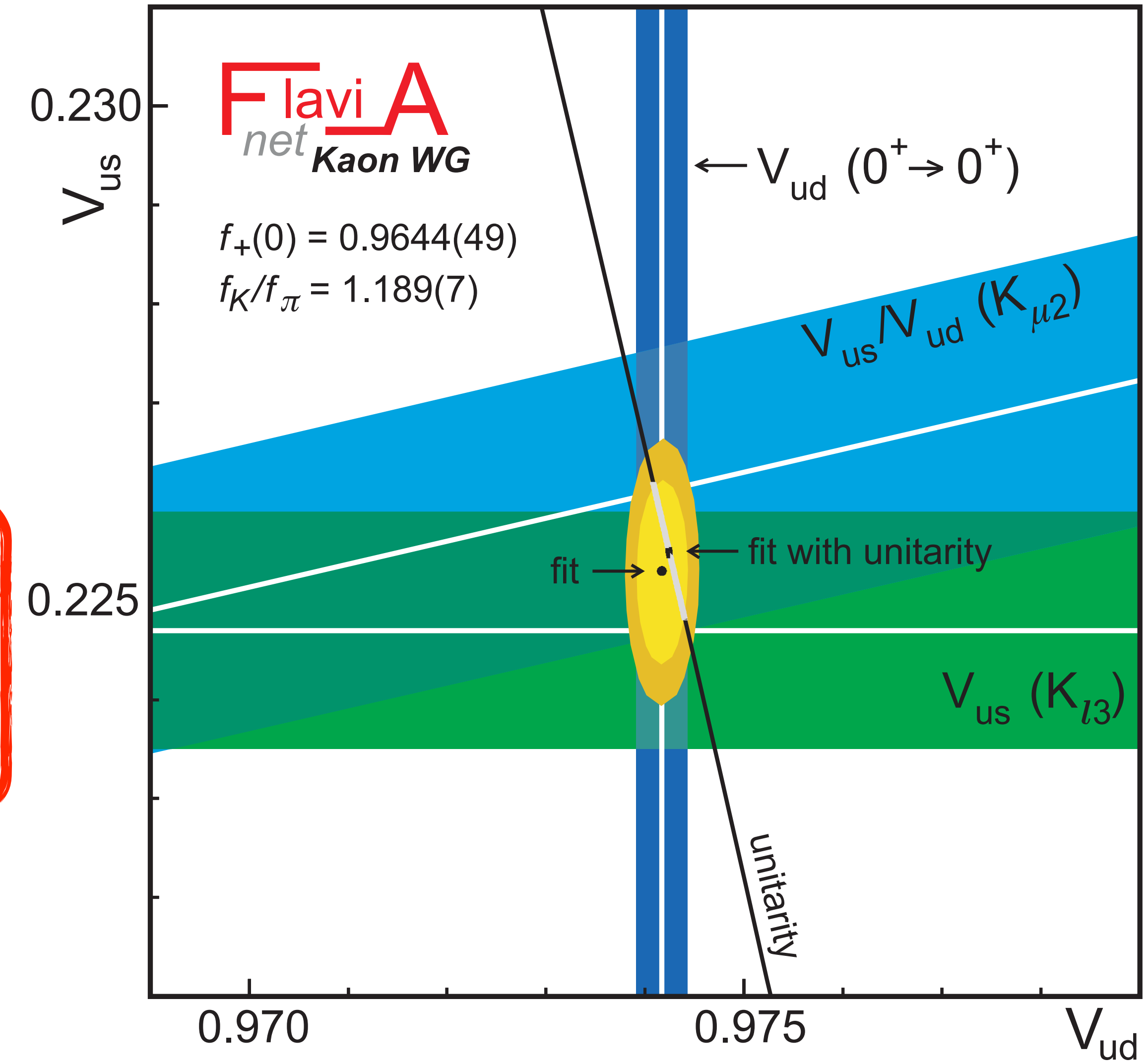
[Flavianet `10]

and nuclear β decay

$$V_{ud} = 0.97425(22)$$

[Hardy, Towner `08]

$$\Delta_{CKM} = |V_{ud}^2| + |V_{us}^2| + |V_{ub}^2| - 1 = (0.1 \pm 0.6) \times 10^{-3}$$



4 relations 4 parameters

CKM Unitarity (Model Independent)

[Cirigliano et. al. `09]

$\Lambda_{NP} \gg M_W$ Neglect $\mathcal{O}\left(\frac{M_W}{\Lambda_{NP}}\right)$ corrections

Use SU(2)⊗U(1) invariant operators [Buchmüller-Wyler `06]

(plus $U(3)^5$ flavour symmetry)

$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q) \qquad O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l)$$

Constrained from EW precision data [Han, Skiba `05]

Redefine

$$G_F(\mu \to e \nu \bar{\nu}) \to G_F(1 - 2\bar{\alpha}_{ll}^{(3)}) \longrightarrow G_F^\mu$$

$$G_F(d \to u e \bar{\nu}) \to G_F(1 - 2\bar{\alpha}_{lq}^{(3)}) \longrightarrow G_F^{SL}$$

CKM Unitarity (Model Independent)

$$V^{PDG}_{ud_i} = \frac{G_F^{SL}}{G_F^{\mu}} V_{ud_i} \longrightarrow \Delta_{CKM} = 4\left(\bar{\alpha}^{(3)}_{ll} - \bar{\alpha}^{(3)}_{lq} + \dots\right)$$

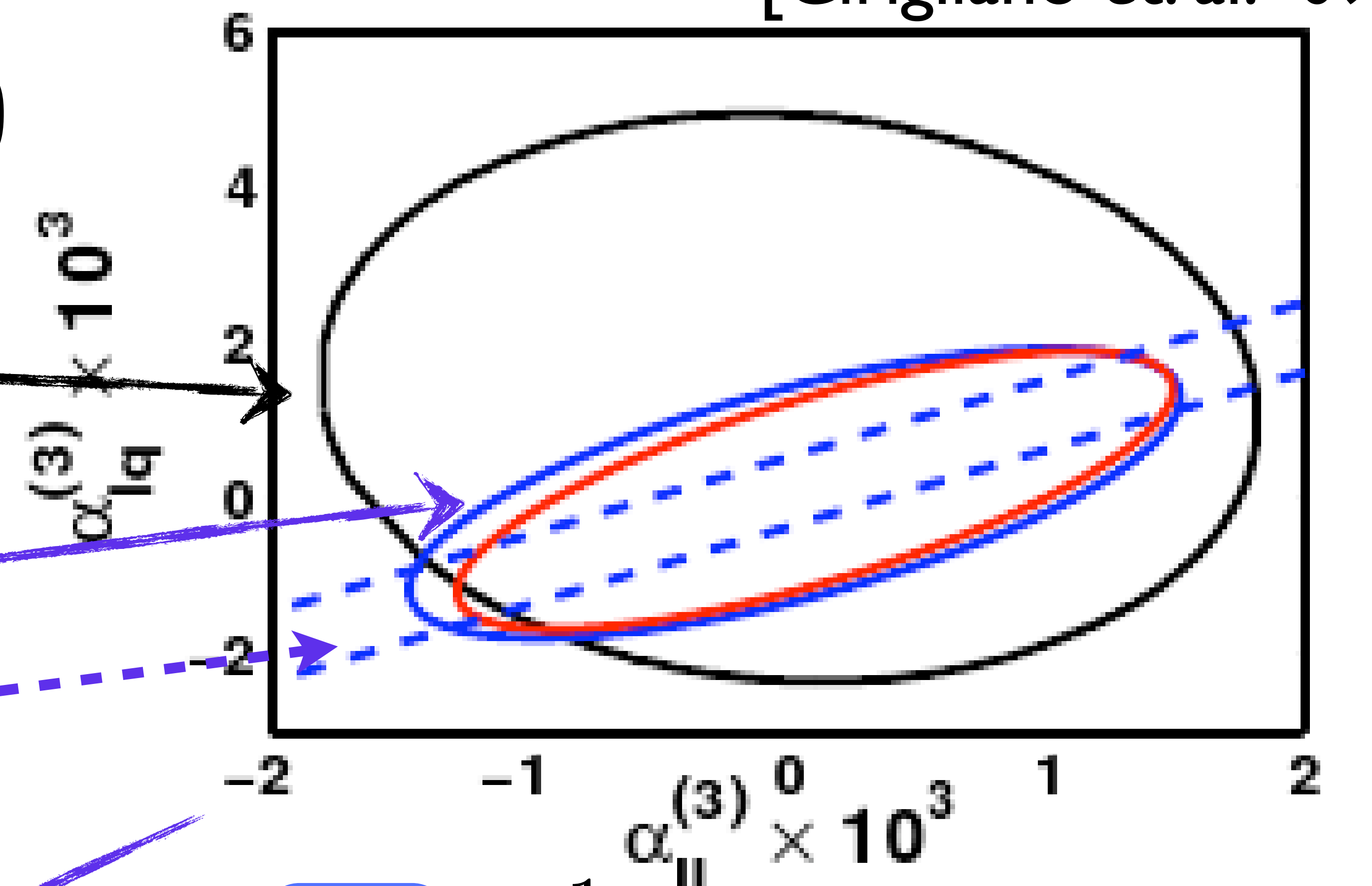
[Cirigliano et. al. `09]

$$O^{(3)}_{lq} = (\bar{l}\gamma^{\mu}\sigma^{a}l)(\bar{q}\gamma_{\mu}\sigma^{a}q)$$

from HEP

HEP + CKM

CKM



$$\Lambda_{NP} > 10\,\mathrm{TeV}$$

$$O^{(3)}_{ll} = \frac{1}{2}(\bar{l}\gamma^{\mu}\sigma^{a}l)(\bar{l}\gamma_{\mu}\sigma^{a}l)$$

Leptonic and Semileptonic

$$K(\pi) \to l \bar{\nu}_l \quad \& \quad K \to \pi\, l \bar{\nu}_l$$

Observables

$$R_K = \frac{\Gamma(K \to e \bar{\nu})}{\Gamma(K \to \mu \bar{\nu})} \qquad R_K^{SM} = 2.477(1) \times 10^{-5}$$

[Cirigliano, Rosell `07]

See also[Marciano, Sirlin `93]

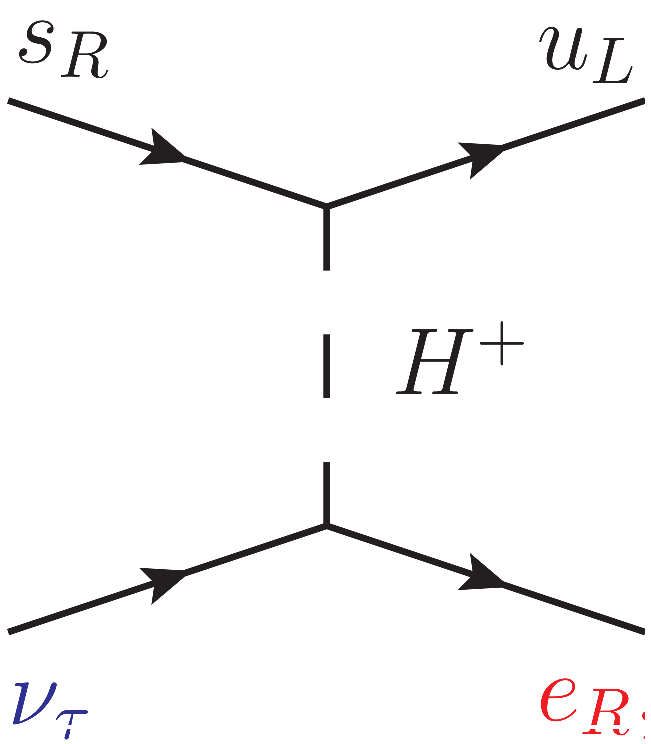
$$R_K^{NA62} = 2.486(11)(7) \times 10^{-5}$$ [NA62 June `10]

$$R_K^{KLOE} = 2.493(25)(19) \times 10^{-5}$$ [EPJ C64 (2009) 627]

Test of lepton universality violation driven by experimental precision

Experimental talks by Antoni Sergi and Barbara Sciascia

Lepton Universality in the MSSM



LF Conserving: ~ lepton mass

Lepton Flavour Violation: $\Delta_R^{31} \sim \frac{g_2^2}{16\pi^2} \delta_{RR}^{31}$

[Masiero, Paradisi, Petronzio `08]

$$R_K^{LFV} = \frac{\Gamma_{SM}(K \to e\nu_e) + \Gamma_{SM}(K \to e\nu_\tau)}{\Gamma_{SM}(K \to \mu\nu_\mu)}$$

$$\Delta r_K \sim \frac{m_K^4}{m_{H^+}^4} \frac{m_\tau}{m_e} |\Delta_R^{31}|^2 \tan_\beta^6 \longrightarrow \text{can reach } 10^{-2}$$

But: finetuning of m_e necessary [Girrbach et. al. `09]

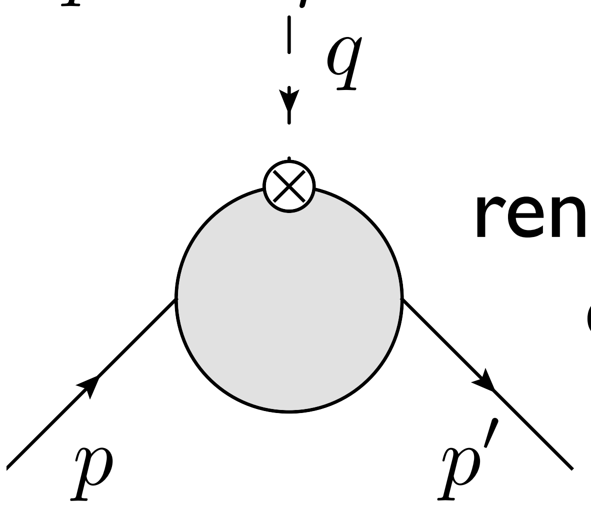
Model independent MLFV and GUT analysis [Isidori et. al. `09]

Light-Quark Masses from Lattice QCD

Extract $m_{u,d}$ & m_s from $\frac{M_\pi}{f_\pi}$ & $\frac{M_K}{M_\pi}$ using Lattice QCD

→Connect lattice and $\overline{\text{MS}}$ renormalization scheme:
Find a scheme good for lattice & loops: RI/SMOM [Sturm et. al. `09]

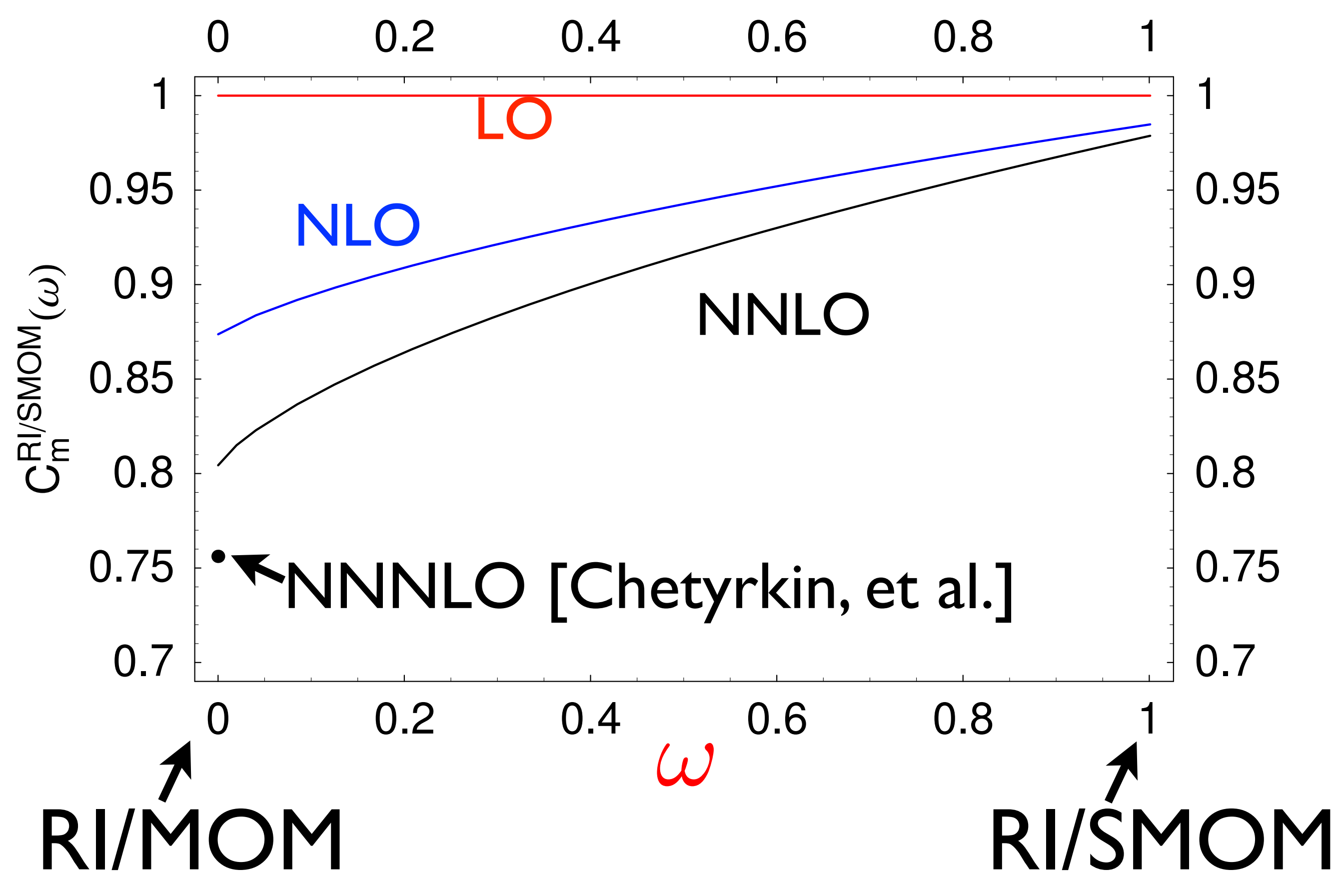
$q^2 = -\mu^2 \times \omega$



$p^2 = p'^2 = -\mu^2$

off-shell
3-point
renormalization
condition

RI/SMOM: ω=1



Prove that good convergence at NLO is no accident
→NNLO: error on mass 2% [Gorbahn, Jäger `10]

Rare Kaon Decays

FCNCs which are dominated by top-quark loops:

$b \to s:$	$b \to d:$	$s \to d:$
$\lvert V_{tb}^* V_{ts}\rvert \propto \lambda^2$	$\lvert V_{tb}^* V_{td}\rvert \propto \lambda^3$	$\lvert V_{ts}^* V_{td}\rvert \propto \lambda^5$

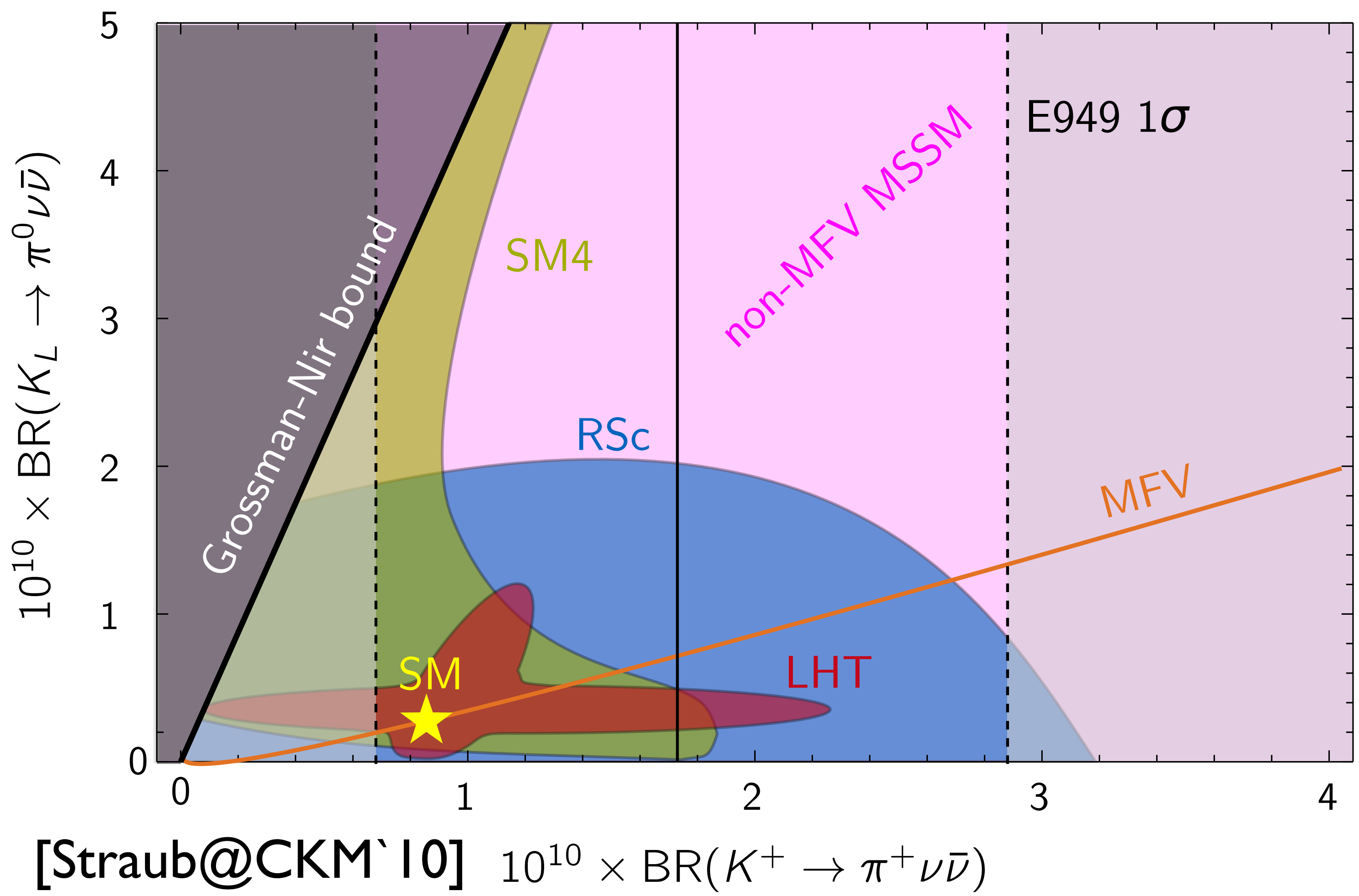
CKM suppression: enhanced sensitivity to NP

$$\underbrace{V_{ts}^* V_{td}}_{\lambda^5} + \underbrace{V_{cs}^* V_{cd}}_{\lambda} = -\underbrace{V_{us}^* V_{ud}}_{\lambda}$$

how can we suppress the light quark contribution?

Quadratic GIM: $\lambda \frac{m_c^2}{M_W^2}$

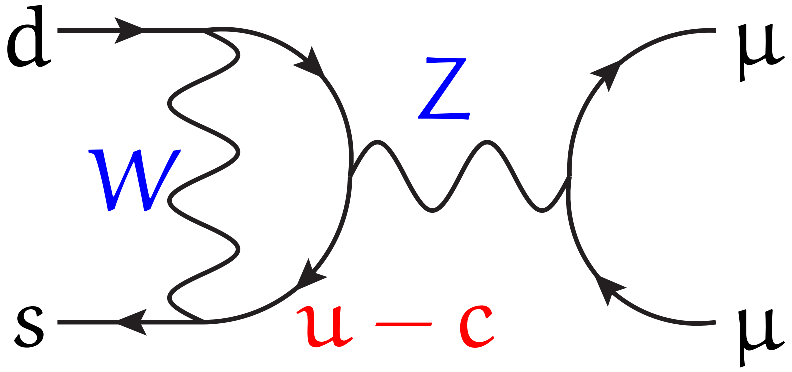
CP violation: $\mathrm{Im}(V_{cs}^* V_{cd})$

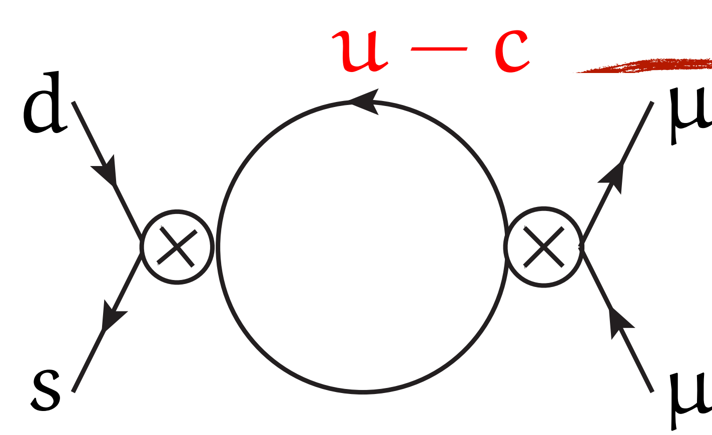


[Straub@CKM`10]

GIMnastics

Quadratic GIM suppresses light quark contribution





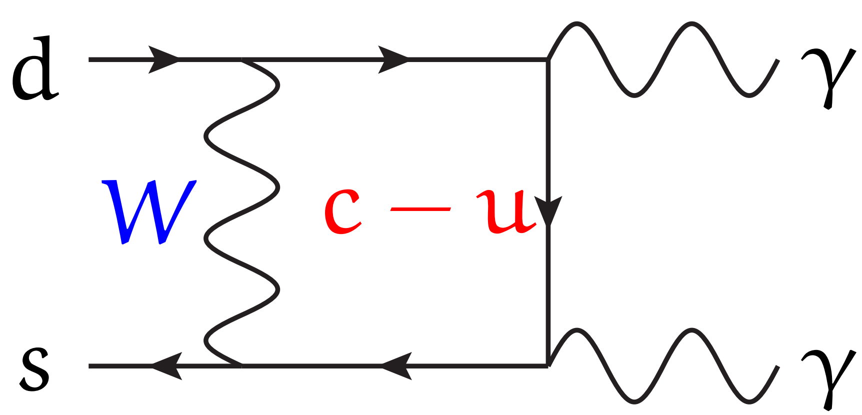
$$m_c^2 - m_u^2$$

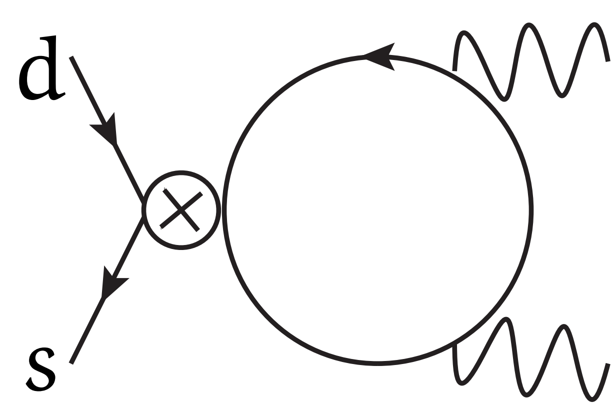
$$m_c^2 G_F^2 \log \frac{m_c}{M_w}$$

NNLO

[MG, Haisch `07]

No quadratic suppression for $K_L \to \gamma\gamma$

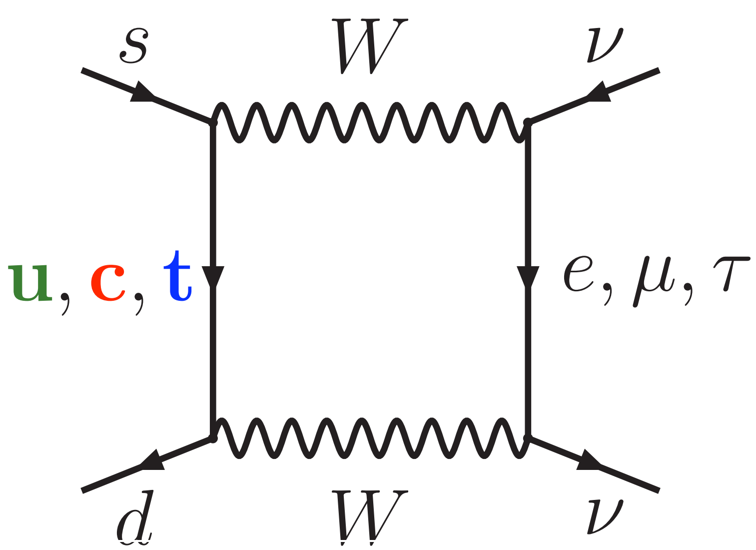




$$G_F \log \frac{\Lambda_{QCD}}{m_c}$$

$\frac{\alpha}{4\pi} \times K_L \to \gamma\gamma$ **also contributes to:** $K_L \to \mu^+\mu^-$

No couplings to γ s: $K \to \pi \nu \bar{\nu}$

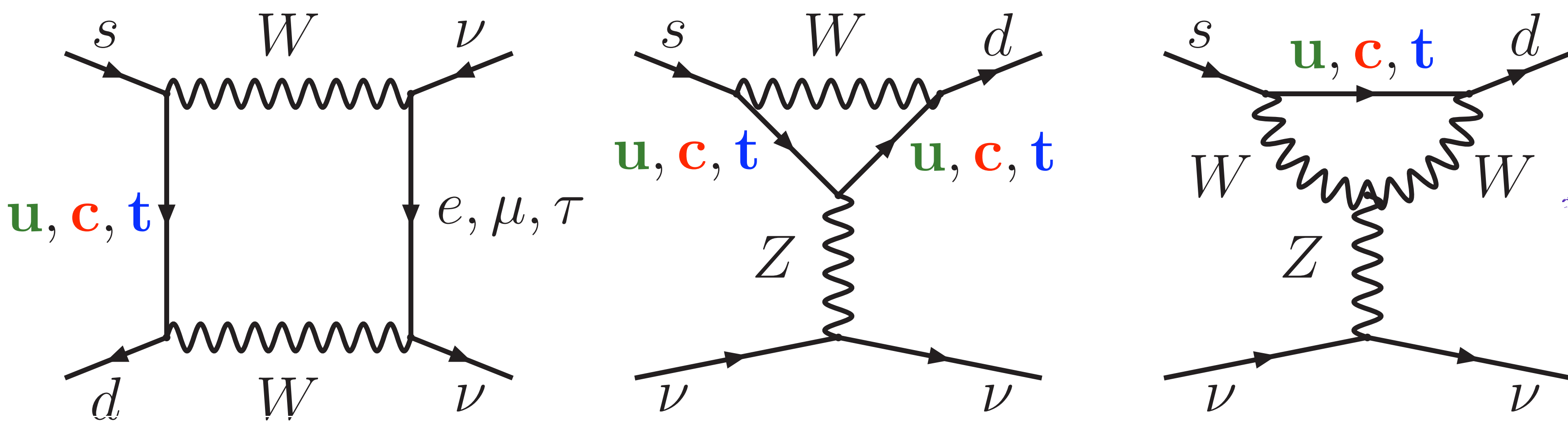


- Dominant Operator: $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda^2}{M_W^2}$

$K_L \to \pi^0 \bar{\nu}\nu$: Effective Hamiltonian



including
NLO EW
[Bord, Gorbahn, Stamou `10]

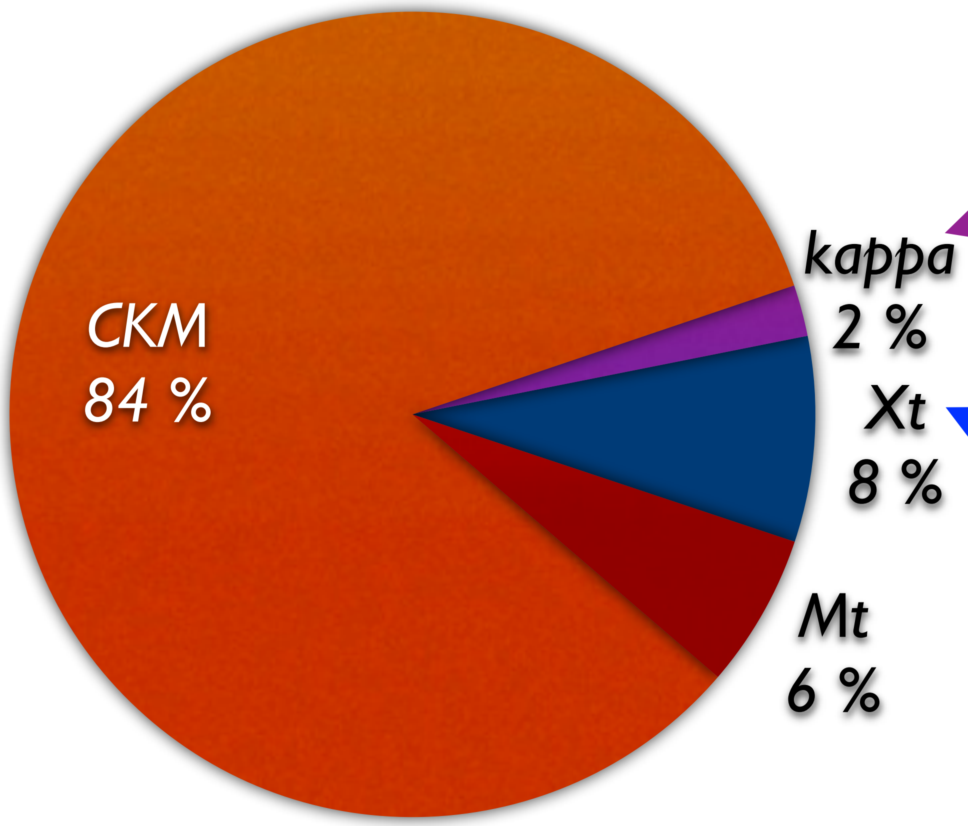
CP violating

Only top quark contributes: $H_{eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2\Theta_W} X(x_t) Q_\nu$

Use isospin symmetry and normalise to: $K^+ \to \pi^0 e^+ \nu$

$$\mathcal{B}r(K_L \to \pi^0 \bar{\nu}\nu) = \kappa_L \left(\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} X(x_t) \right)^2$$

$K_L \to \pi^0 \nu \bar{\nu}$: Theoretical Status



Matrix element extracted from K_{l3} decays. $N^{\frac{3}{2}}LO\ \chi PT$
[Mescia, Smith ´07; Bijnens, Ghorbani ´07]

$X(x_t)$: Full NLO electroweak corrections
[Brod, Gorbahn, Stamou ´10]
Reduce theory uncertainty by factor of 2

$$\mathcal{B}r_{K_L} = 2.57(37)(4) \times 10^{-11}$$

Experiment: $< 6.7 \times 10^{-8}$ [E391a ´08]

=> K0T0

$K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \pi^0 \bar{\nu}\nu$

Different from $K_L \to \pi^0 \bar{\nu}\nu$

- CP conserving: Top & charm contribute

$$\mathcal{B}r\left(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)\right) = \kappa_+ (1 + \Delta_{EM})$$

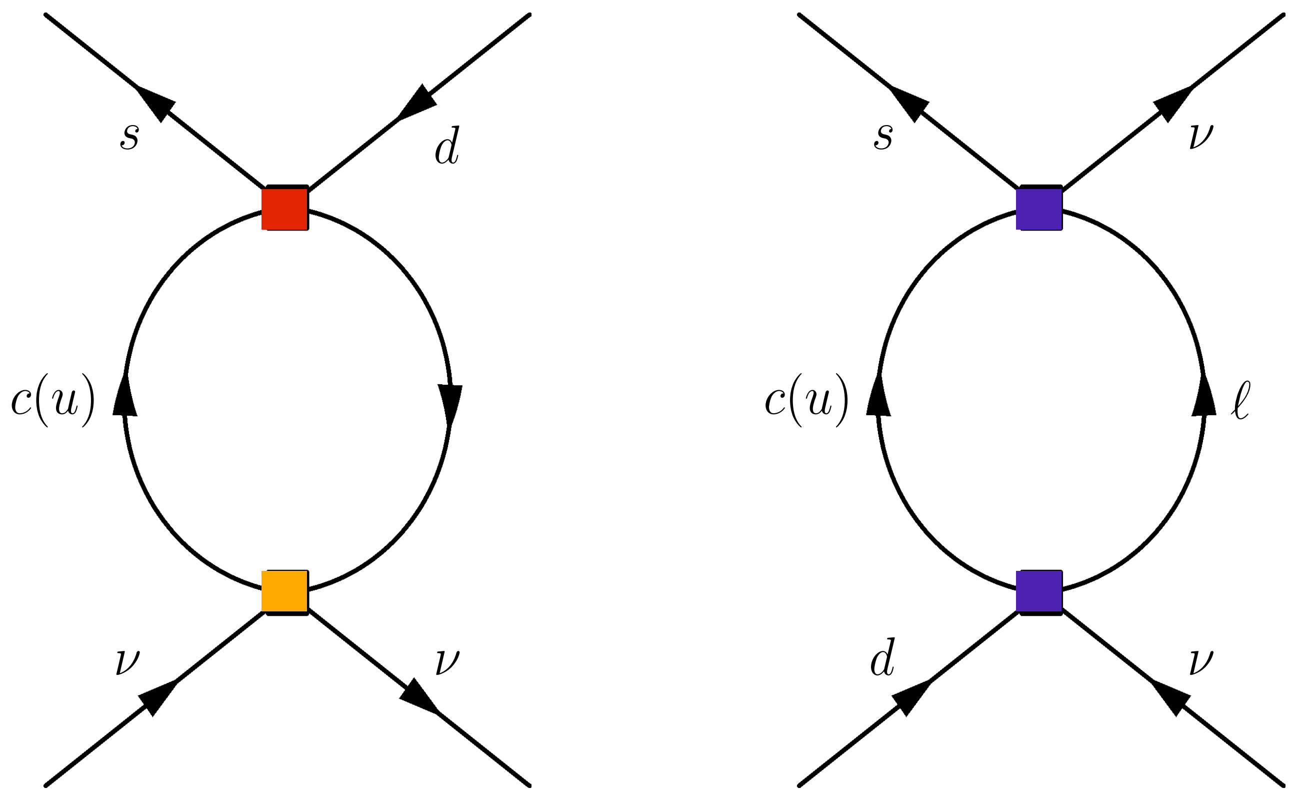
$$\times \left| \frac{V_{ts}^* V_{td} X_t(m_t^2) + \lambda^4 \mathrm{Re} V_{cs}^* V_{cd} \left(P_c(m_c^2) + \delta P_{c,u}\right)}{\lambda^5} \right|^2 .$$

$\frac{m_c^2}{M_W^2}$ suppression lifted by $\log(\frac{m_c}{M_W})\frac{1}{\lambda^4}$

Like in $K_L \to \pi^0 \bar{\nu}\nu$

- Only Q_ν: Quadratic GIM & Isospin symmetry
- Top quark contribution like in $K_L \to \pi^0 \bar{\nu}\nu$

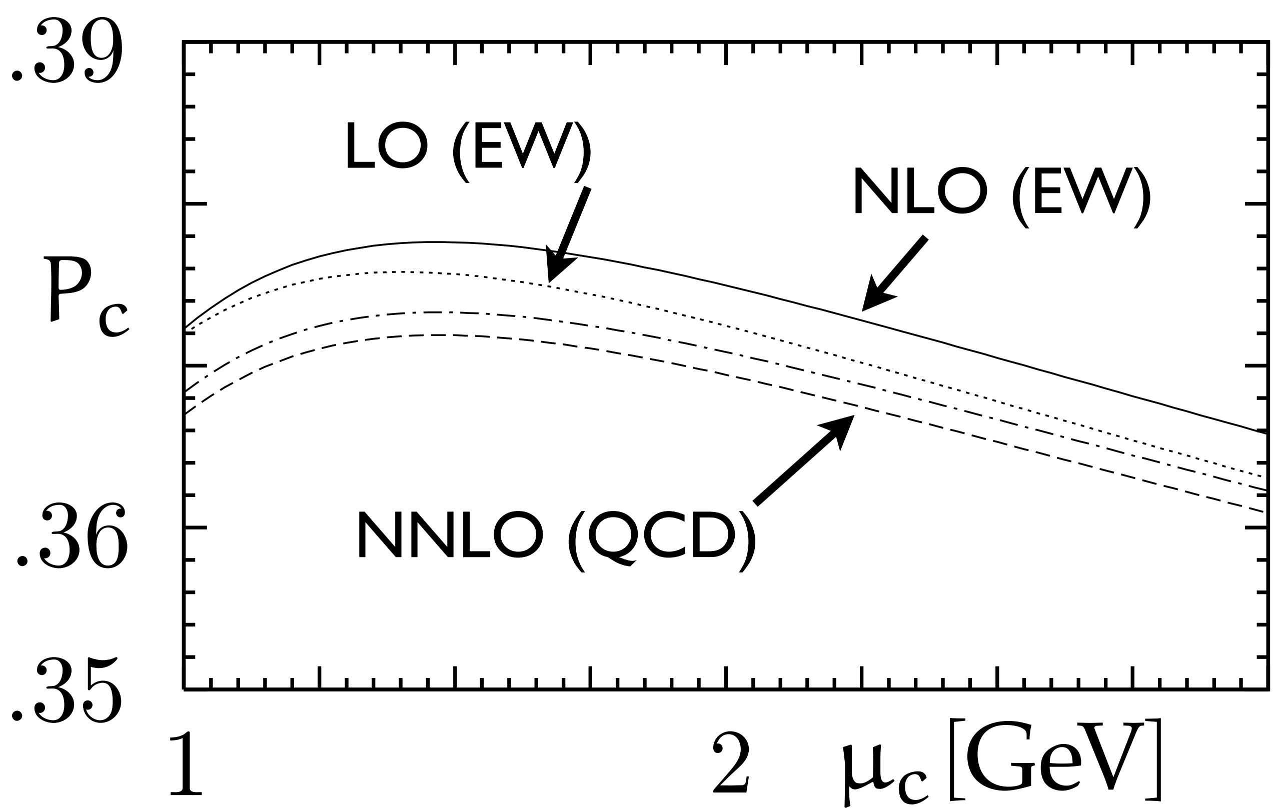
$K^+ \to \pi^+ \bar{\nu}\nu$ charm contribution



$$\bar{s}_i\gamma^\mu(1-\gamma_5)q_j \ \bar{q}_j\gamma_\mu(1-\gamma_5)d_i$$
$$\bar{s}_i\gamma^\mu(1-\gamma_5)q_i \ \bar{q}_j\gamma_\mu(1-\gamma_5)d_j$$

$$\bar{c}\gamma^\mu\gamma_5 c \ \ \bar{\nu}\gamma^\mu(1-\gamma_5)\nu$$

$$\bar{s}\gamma^\mu(1-\gamma_5)q \ \bar{\nu}_l\gamma_\mu(1-\gamma_5)l$$
$$\bar{l}\gamma^\mu(1-\gamma_5)\nu_l \ \bar{q}\gamma_\mu(1-\gamma_5)d$$

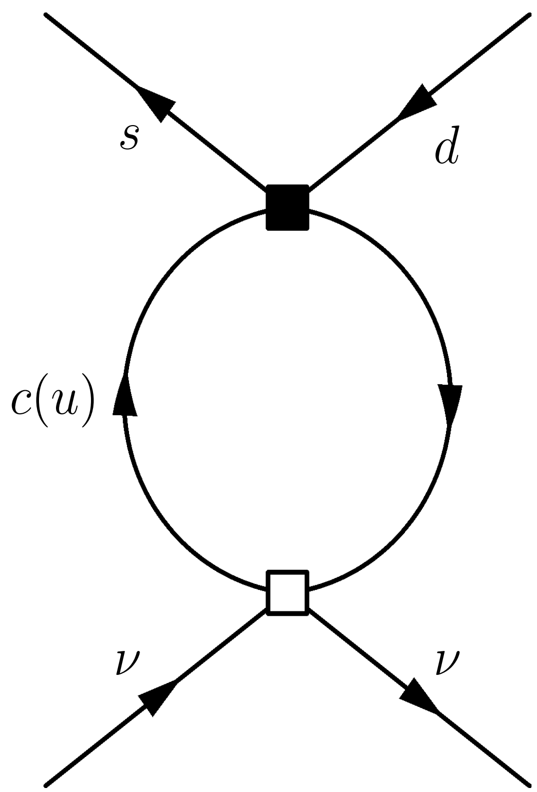


- Resum $\log\frac{m_c}{M_W}$ in P_c

P_c at NNLO: ±2.5% (theory)
[Buras, Gorbahn, Haisch, Nierste ´06]

NLO EW [Brod, Gorbahn`08]

Long Distance Contribution



No GIM below the charm quark mass scale

q^2/m_c^2 higher dimensional operators

UV scale dependent

One loop CHPT calculation approximately cancels this scale dependence [Isidori, Mescia, Smith `05]

Also: box-type diagrams considered (from two semileptonic operator insertions) cancelation is more complicated

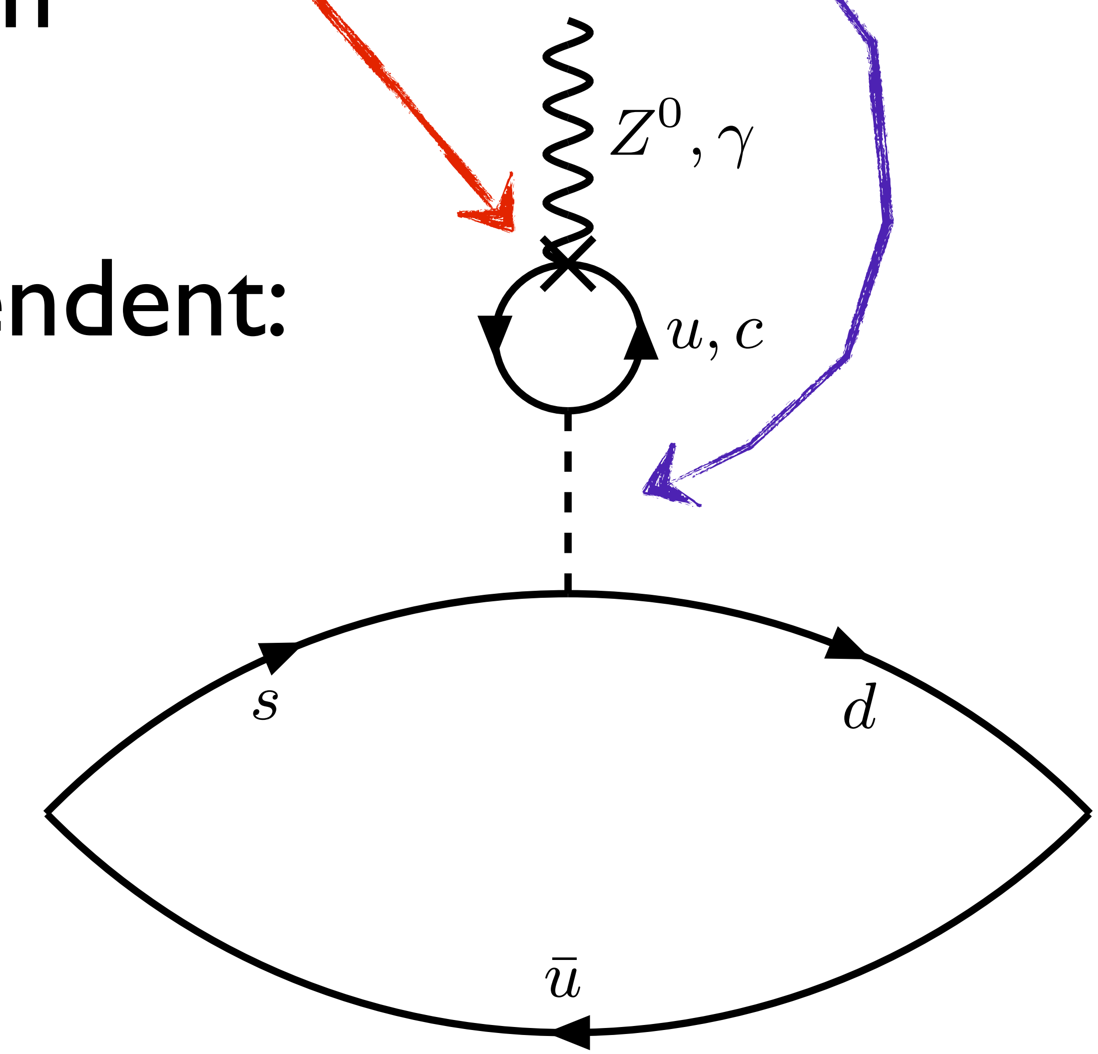
$$\delta P_{c,u} = 0.04 \pm 0.02$$ [Isidori, Mescia, Smith `05]

One Current & One Operator

$$\left(\mathcal{T}_i^j\right)^\mu_{\rm em}(q^2) \;=\; -i\int d^4x\, e^{-i\,q\cdot x}\,\langle\pi^j(p)|T\left\{J^\mu_{\rm em}(x)\left[Q_i^u(0)-Q_i^c(0)\right]\right\}|K^j(k)\rangle$$

Current and operator insertion

[Isidori, Martinelli, Turchetti `06]



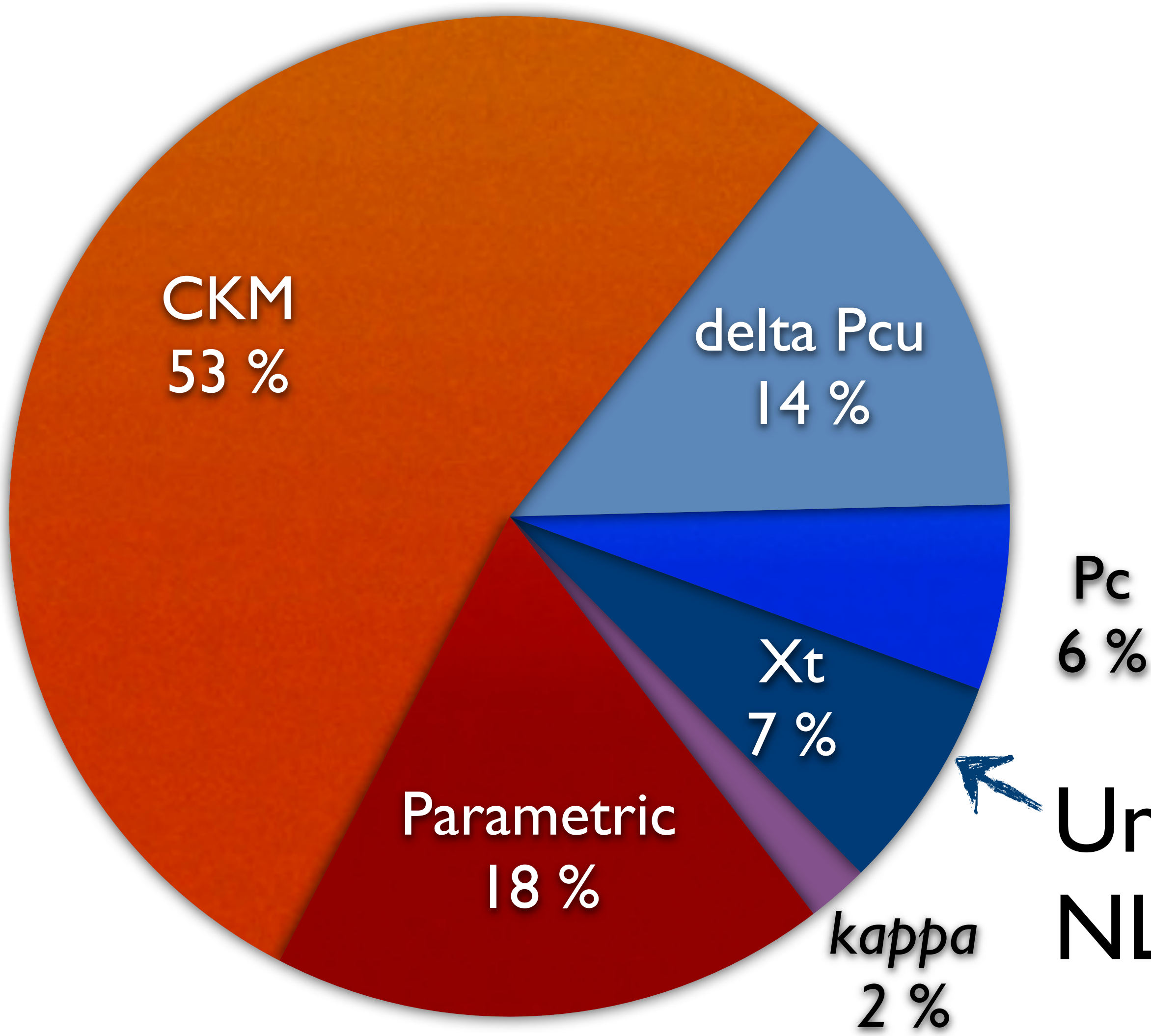
$\mathcal{O}\left(\frac{1}{a^2}\right)$ divergence mass independent: cancelled by GIM

$\mathcal{O}\left(\frac{1}{a}\right)$ appear → maximally twisted fermions

also: no semileptonic operators discussed

$K^+ \to \pi^+ \bar{\nu}\nu$ Error Budget

Theory error budget $\mathcal{B}_{K^+} = 0.822(69)(29) \times 10^{-10}$



Experiment [E787, E949 ´08]

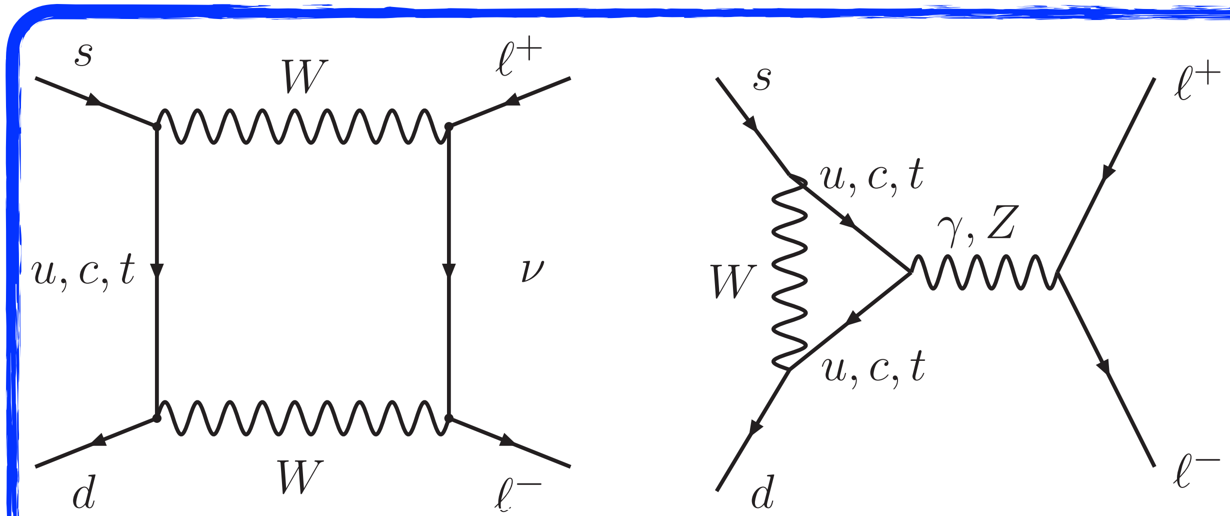
$\mathcal{B}r_{K^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

Talk by Giuseppe Ruggiero on NA62 => 10%

Uncertainty reduced by a factor 2 by NLO e.w. calculation[Brod, Gorbahn, Stamou ´10]

Uncertainty reduced by a factor 7 by (N)NLO χPT calculation[Mescia, Smith ´07]

$K_L \to \pi^0 l^+ l^-$: Three Contributions



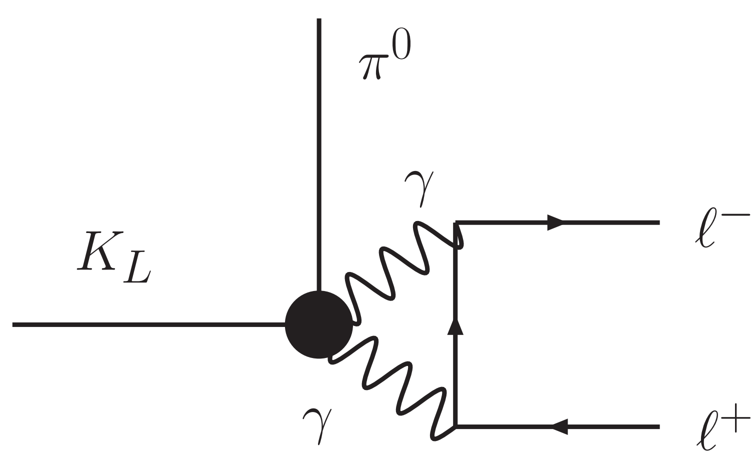
$$Q_{7V} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu l) \to 1^{--}$$

$$Q_{7A} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu \gamma_5 l)$$

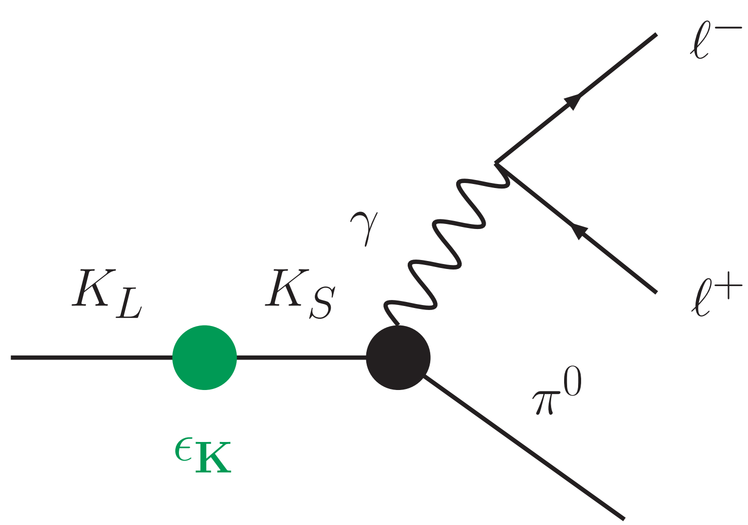
$$\to 1^{++},\ 0^{-+}$$

Direct CP Violating

Wilson Coefficients: y_{7V}, y_{7A} at NLO [Buchalla et al. ´96]

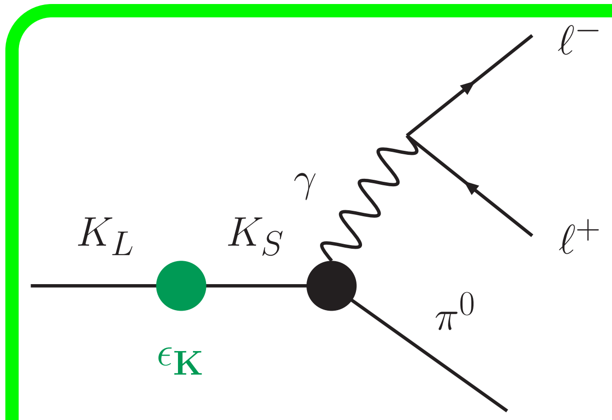


CP Conserving



Indirect CP Violating

$K_L \to \pi^0 l^+ l^-$: Three Contributions

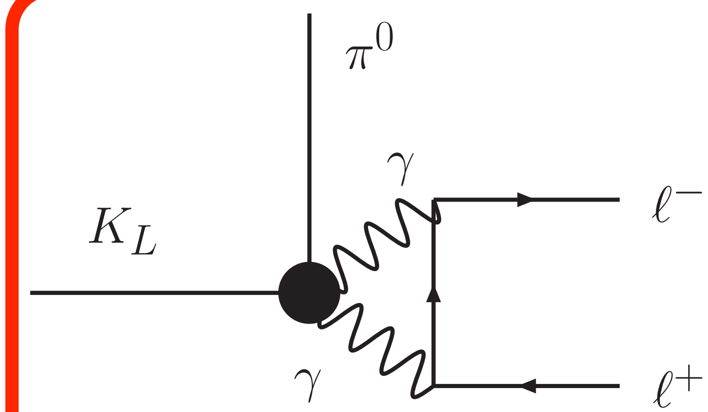


Counterterm $|a_S| = 1.2 \pm 0.2$ from $K_S \to \pi^0 l^+ l^-$

[D´Ambrosio et. al. ´98, Mescia et. al. ´06]

For 1^{--} interference with Q_{7V}

[Buchalla et. al. ´03, Friot et al. ´04]



Estimate from $K_L \to \pi^0 \gamma\gamma$

[Isidori et. al. ´04]

$$\mathcal{B}r(K_L \to \pi^0 \ell^+ \ell^-) = \left(C^\ell_{dir} \pm C^\ell_{int}|a_S| + C^\ell_{mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \times 10^{-12}$$

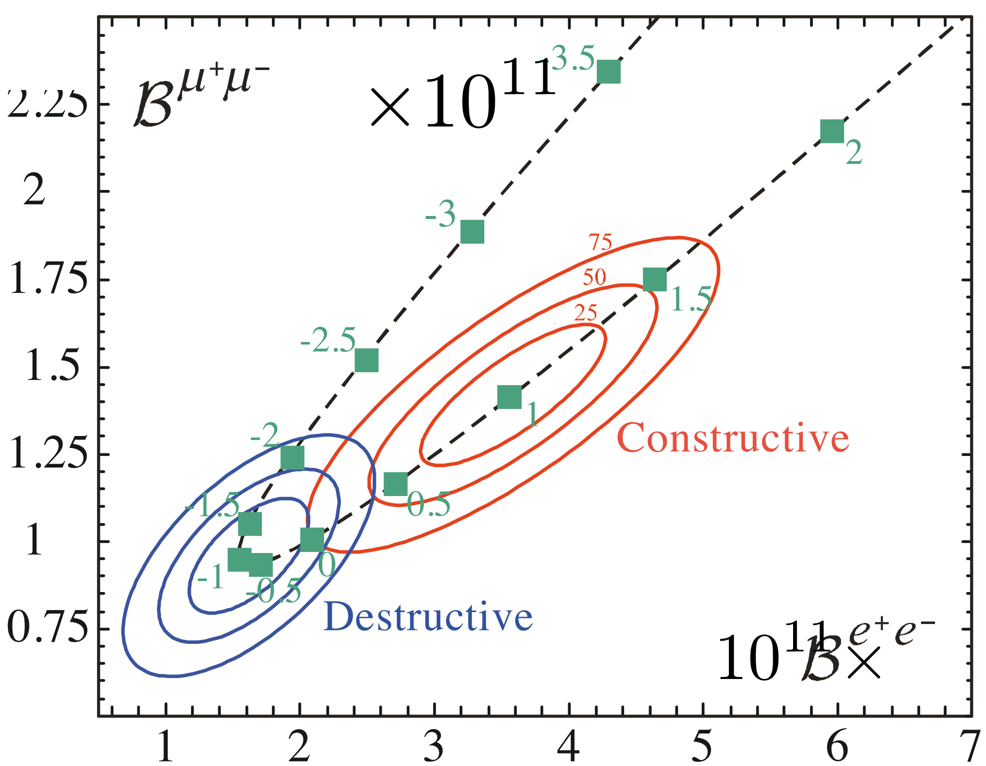
ℓ	C^ℓ_{dir}	C^ℓ_{int}	C^ℓ_{mix}	$C^\ell_{\gamma\gamma}$
e	$(4.62 \pm 0.24)(y_V^2 + y_A^2)$	$(11.3 \pm 0.3)y_V$	14.5 ± 0.5	≈ 0
μ	$(1.09 \pm 0.05)(y_V^2 + 2.32 y_A^2)$	$(2.63 \pm 0.06)y_V$	3.36 ± 0.20	5.2 ± 1.6

$K_L \to \pi^0 l^+ l^-$: Improvements

- Measure both $\mathcal{B}r_{e^+e^-}$ and $\mathcal{B}r_{\mu^+\mu^-}$: [Mescia et. al. ´06]
  Disentangle short distance contribution $(y_{7V},\ y_{7A})$

- Dominant theory error in a$_s$:
  Forward backward asymmetry. [Mescia, Smith, Trine ´06]
  Better measurement of $K_S \to \pi^0 l^+ l^-$

Talk on radiative decays by Monica Pepe

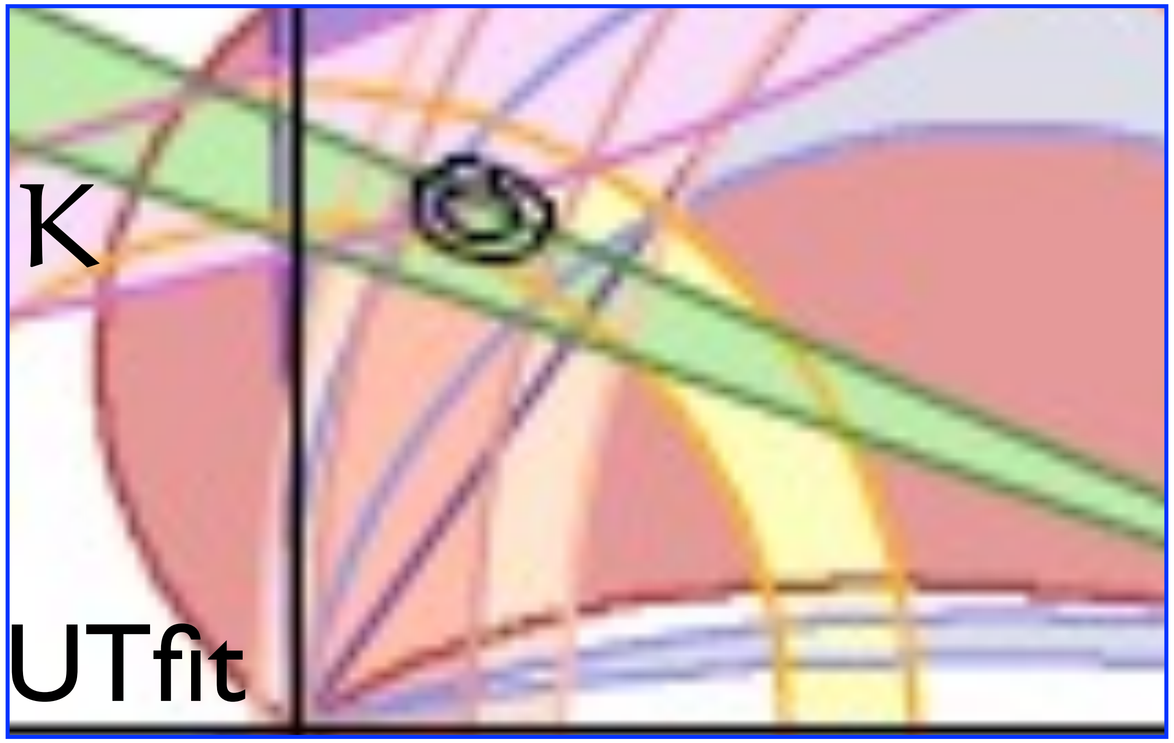


[Mescia et. al. ´06]

Lattice: $K \to \pi(\gamma/Z)$ contribution similar to $K \to \pi\nu\bar{\nu}$ calculation

[KTEV ´04]	[KTEV ´00]
$Br_{e^+e^-}$	$Br_{\mu^+\mu^-}$
$< 28 \times 10^{-11}$	$< 38 \times 10^{-11}$

ϵ_K:Indirect CP Violation



Talk by Cecilia Tarantino: -1.7 σ Pull

$$\epsilon_K \simeq \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle}$$

$$\frac{\Im A_0}{\Re A_0}$$

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

- In almost all old analysis: $\phi_\epsilon = 45^\circ$ **and** $\xi = 0$
- In reality: $\xi \neq 0$ $\phi_\epsilon \neq 45^\circ$ [Nierste; Andriyash; Buras, Guadagnoli]

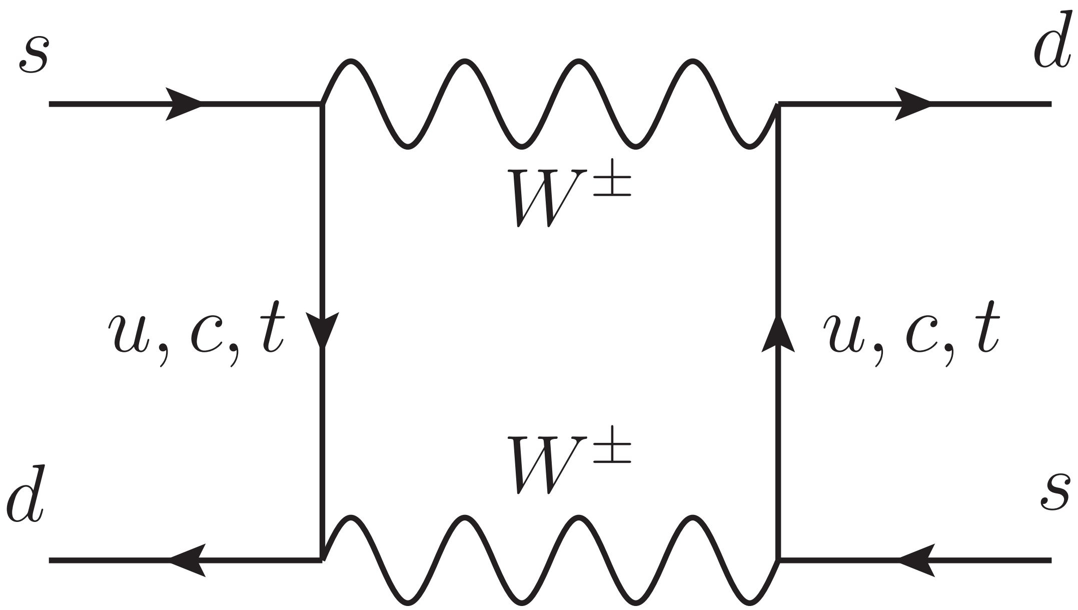
$$|\epsilon_K^{SM}| = \kappa_\epsilon |\epsilon_K|(\phi_\epsilon = 45^\circ, \xi = 0)$$

+ similar contribution as $\delta P_{c,u}$ in ϵ_K

$$\kappa_\epsilon = 0.94 \pm 0.02$$ [Buras, Guadagnoli, Isidori `10]

Calculation of $M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$

Box diagram
with internal u,c,t



$$\lambda_i \lambda_j A(x_i, x_j)$$

$$\lambda_i = V_{is}^* V_{id}$$

plus GIM:

$$\lambda_c + \lambda_t = -\lambda_u$$

Gives three different contributions for

$$M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$$

Caveat: first only SD

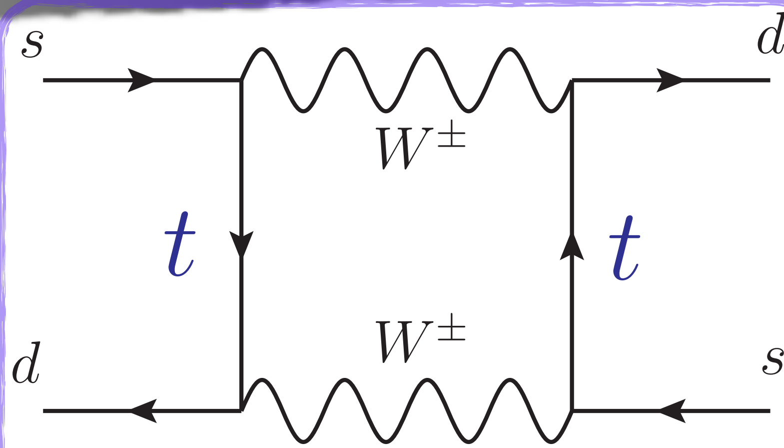
$$\mathcal{H} \propto \Big[\lambda_t^2 \eta_t S(x_t)$$ top

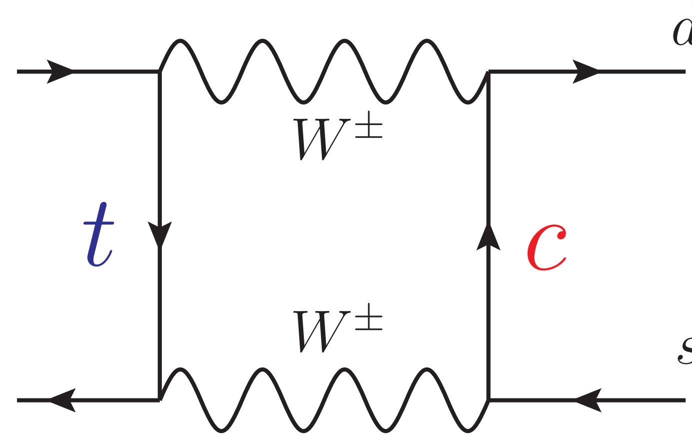
$$+ 2\lambda_c \lambda_t \eta_{ct} S(x_c, x_t)$$ charm top

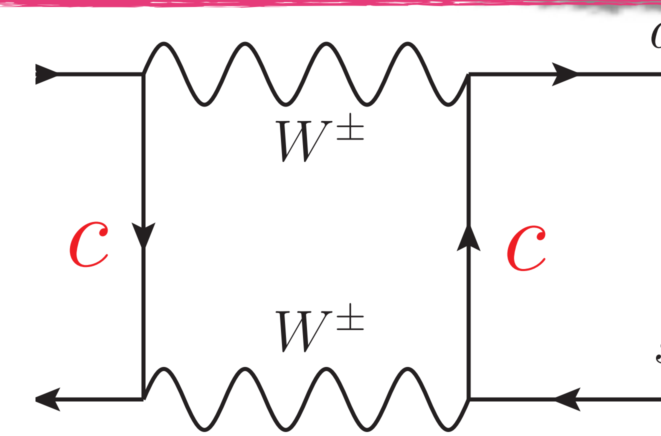
$$+ \lambda_c^2 \eta_c S(x_c) \Big] b(\mu) \tilde{Q}$$ charm

$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$$

Calculation of $M_{12}^{K} = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$

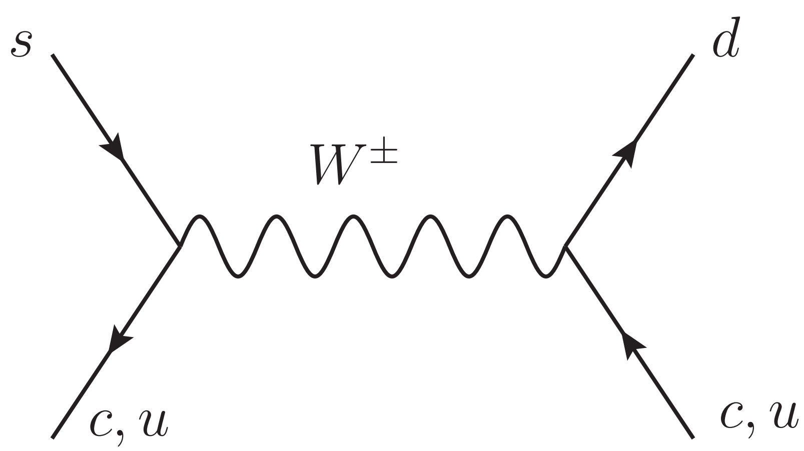


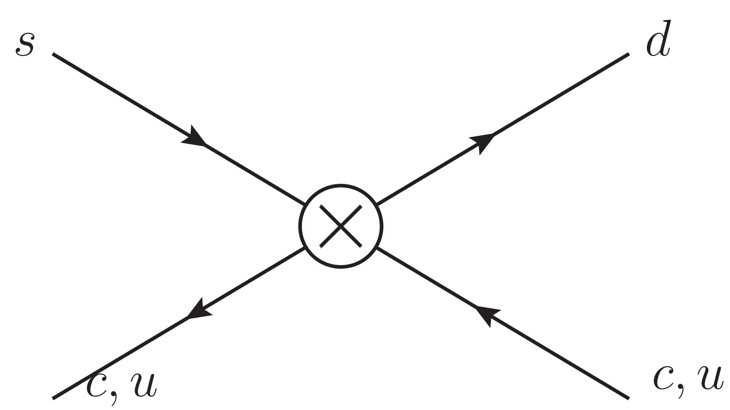


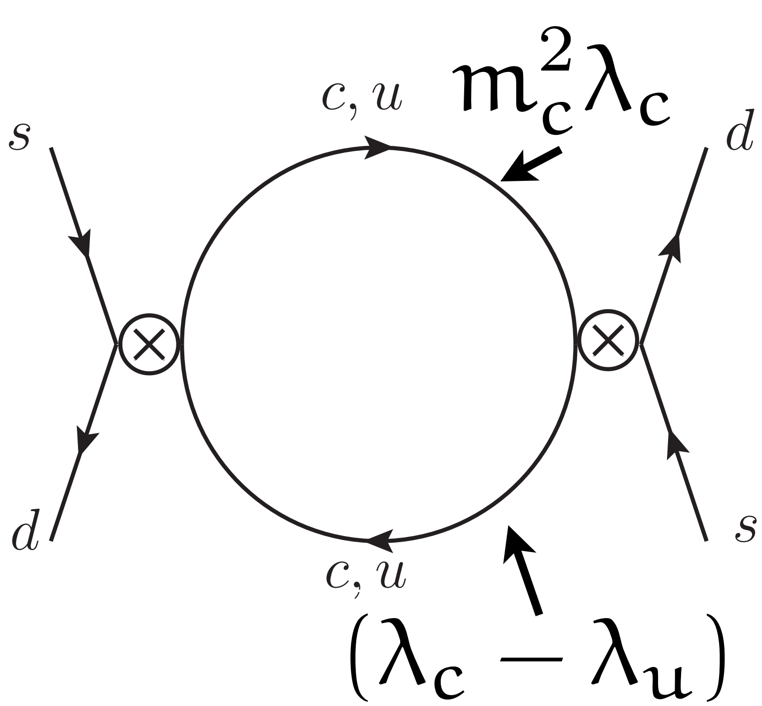


	top	charm top	charm
	$\log x_t$	$\log x_c$	$(\log x_c)^0$ ← hard GIM
LO	$(\alpha_s \log x_c)^n$	$(\alpha_s \log x_c)^n \log x_c$	$(\alpha_s \log x_c)^n$
NLO	$\alpha_s(\alpha_s \log x_c)^n$	$(\alpha_s \log x_c)^n$	$\alpha_s(\alpha_s \log x_c)^n$
ϵ_K	75%	37%	-12%
scale	1.8%	16%	17.7%

η_{ct} :Charm Top at LO







- The Leading Order result

  $$(\alpha_s \log x_c)^n \log x_c$$

  starts with a $\log x_c$

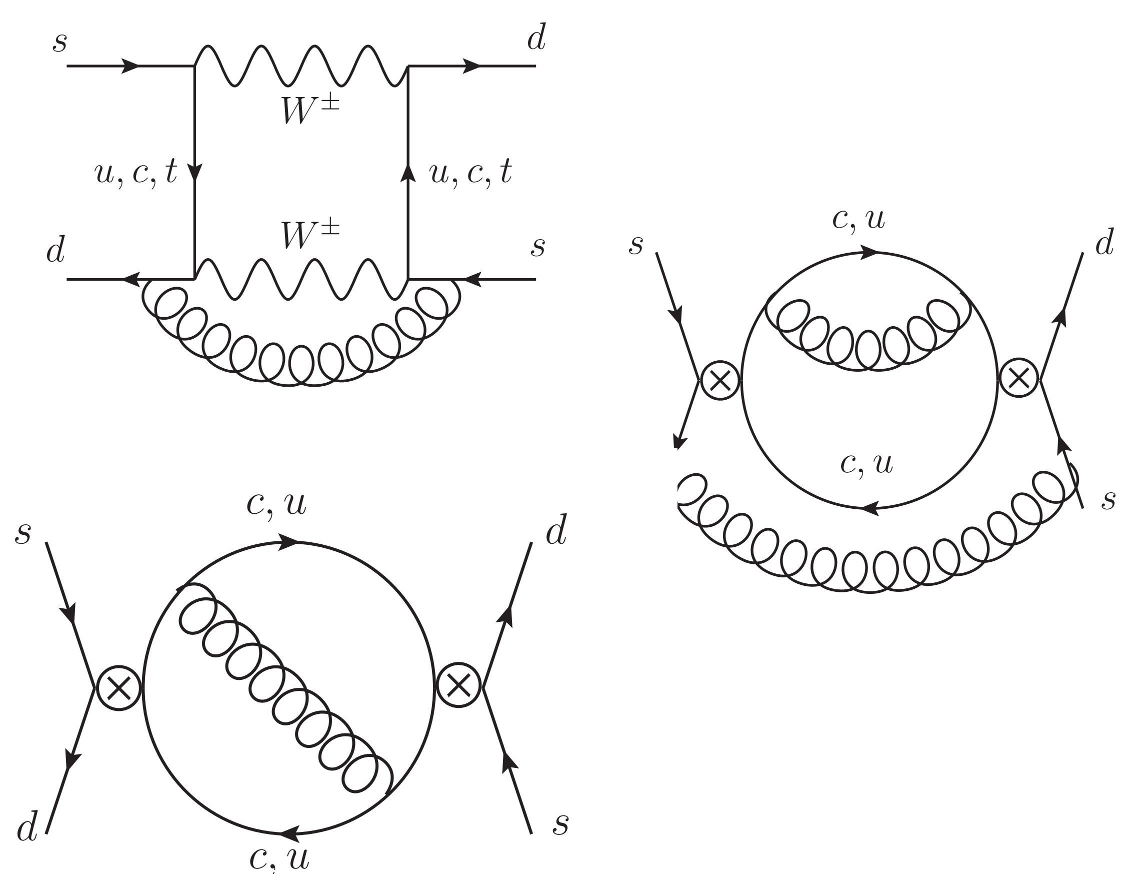
- Tree level matching

- One-loop Renormalistion Group Equation

  $$m_c^2\lambda_c(\lambda_c - \lambda_u) \log \frac{m_c}{M_W}$$

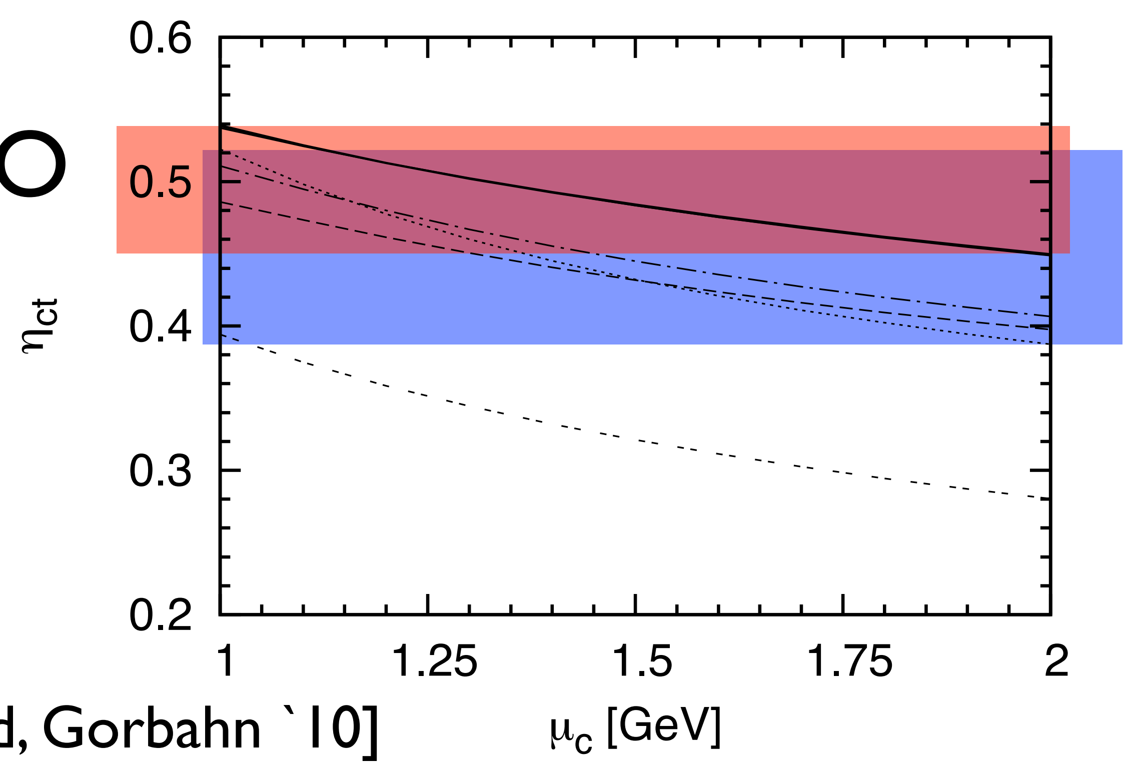
  $$\to m_c^2\lambda_c\lambda_t\tilde{Q} \log x_c$$

η_{ct} :Charm Top beyond LO



- NLO [Herrlich, Nierste]
- NNLO: RGE and matching for d=6 operators RGE: [MG, Haisch `04], Matching: [Bobeth, et. al. `00]
- O(10000) diagrams were calculated [Brod, Gorbahn `10]

NNLO



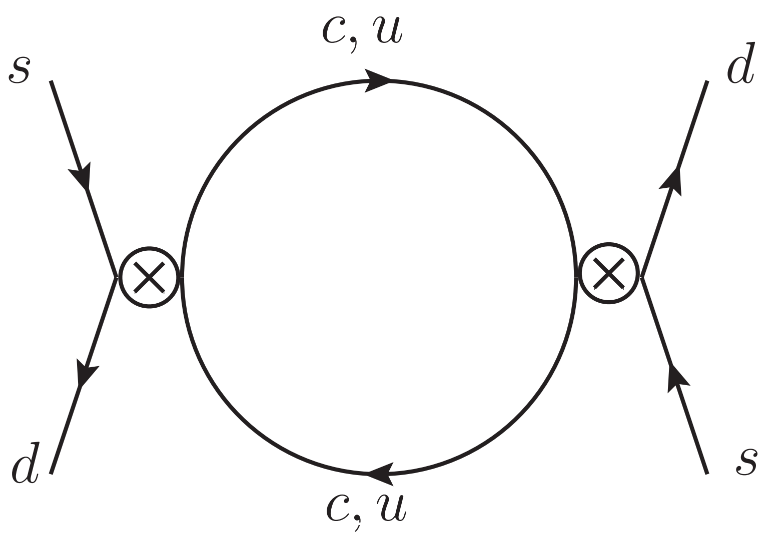
NLO

[Brod, Gorbahn `10]

Long Distance Contribution

ε_K the matrix element B_K is known precisely

[D.J.Antonio et al `07; Aubin, Laiho, de Water `09]



$$\int d^4x \, \langle K^0 | \, H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) \, |\bar{K}^0\rangle$$

dispersive part

absorptive part

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

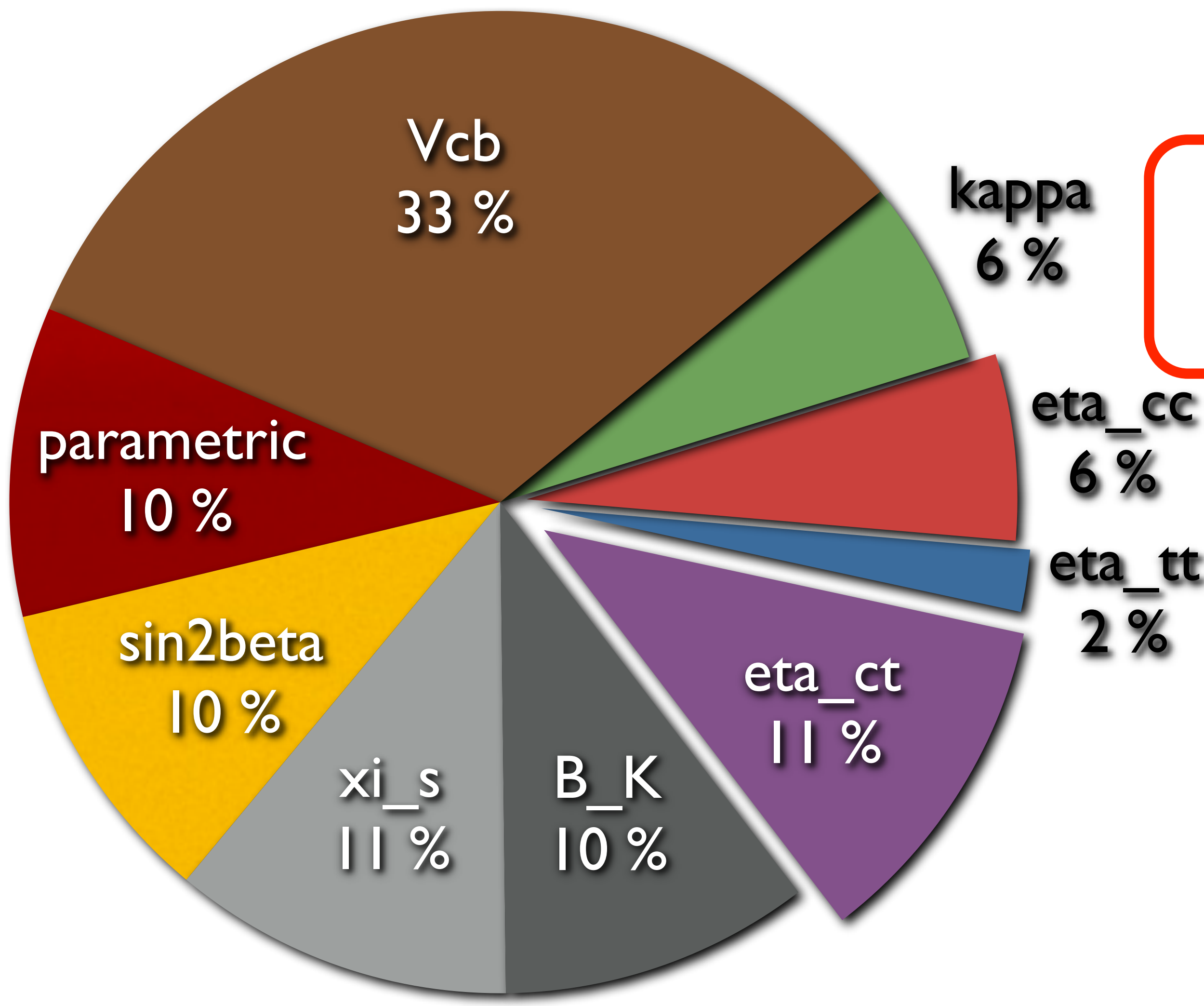
estimated form ϵ'

dispersive part estimated in CHPT

no higher dimensional operators and scale cancellation

put everything in: $\kappa_\epsilon = 0.94 \pm 0.02$ [Buras, Isidori, Guadgnoli `10]

|εK| and Error Budget



New input [PDG `10]

$$|\epsilon_K| = 1.90(26) \times 10^{-3}$$

using

$$\eta_{ct} = 0.496 \pm 0.047$$

$$|V_{cb}| = 406(13) \times 10^{-4}$$

Experiment [PDG `10]:

$$|\epsilon_K| \overset{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

Conclusions

High precision in experiment and theory:
extraction of fundamental parameters =>
CKM unitarity, lepton universality & quark masses

Rare kaon decays:
$K \to \pi \nu \bar{\nu}$: very clean and sensitive to short distances

ε_K: CP-violation in kaon mixing

Improvement from lattice => discrepancy with SM

slightly lifted by new long distance & NNLO contribution