The CKM analysis: inputs from theory

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This talk is dedicated to NICOLA CABIBBO

He was the "father of flavor physics“ [M .Wise]
Universality of weak interactions, the Cabibbo angle, the GIM mechanism, the CKM matrix, …

He has given fundamental contributions to Lattice QCD The Cabibbo-Marinari algorithm, Weak interactions on the lattice (Cabibbo-Martinelli-Petronzio), the APE project (Cabibbo, Parisi, …)

OUTLINE

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- **The 1st row unitarity test**

Processes: $K \to l\,\nu$, $K \to \pi\, l\, \nu$

Theory input: f_K/f_π, $f_+(0)$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- **The 2nd row unitarity test**

Processes: $D_{(s)} \to l\,\nu$, $D \to K/\pi\, l\, \nu$

Theory input: f_D, f_{Ds}, $f_+(0)$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- **The unitarity triangle analysis**

Processes: $B \to l\,\nu$, $B \to D/\pi\, l\, \nu$, $K - K$, $B_{(s)} - B_{(s)}$

Theory input: f_B, f_{Bs}, $f_+(0)$, B_K, $B_{B(s)}$

THE 1st ROW UNITARITY TEST

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

The most stringent unitarity test

$$\begin{pmatrix} \boxed{V_{ud} \quad V_{us} \quad V_{ub}} \\ V_{cd} \quad V_{cs} \quad V_{cb} \\ V_{td} \quad V_{ts} \quad V_{tb} \end{pmatrix}$$

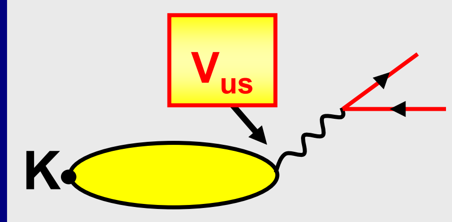
Processes: $K \to l\,\nu$, $K \to \pi\, l\, \nu$

Theory input: f_K/f_π, $f_+(0)$

Vus/Vud from Kμ2/πμ2 decays

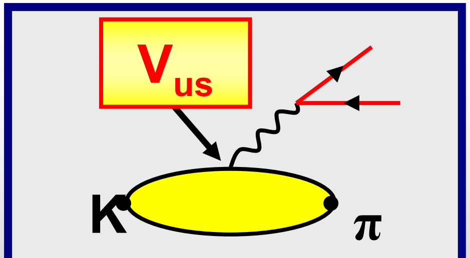
$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-\frac{m_\mu^2}{m_K^2})}{m_\pi(1-\frac{m_\mu^2}{m_\pi^2})} \times 0.9930(35)$$

[Marciano 04]



Vus from Kl3 decays

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{EW}[1+\Delta_{SU(2)}+2\Delta_{EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$



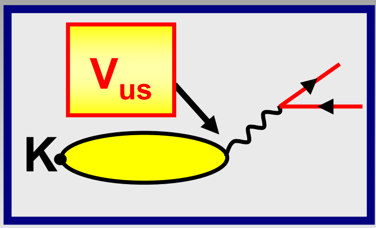
$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_k}{f_\pi} = 0.2758(5)$$

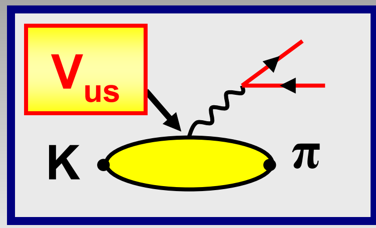
FlaviA net Kaon WG

$$|V_{us}|\, f_+(0) = 0.2163(5)$$

arXiv:1005.2323 [hep-ph]

LQCD independent estimates of f_K/f_π, $f_+(0)$





$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_k}{f_\pi} = 0.2758(5)$$ 1

$$|V_{us}|\, f_+(0) = 0.2163(5)$$ 2

FlaviA net Kaon WG

Assuming the Standard Model and combining with nuclear β decays

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$ 3

$$|V_{ud}| = 0.97425(22)$$ 4

From 20 superallowed transitions

[Hardy and Towner 08]

one obtains:

$$f_k/f_\pi = 1.192(6)$$

$$f_+(0) = 0.960(5)$$

and:

$$|V_{us}| = 0.2254(10)$$

The error is at the per mille level: a challenge for Lattice QCD

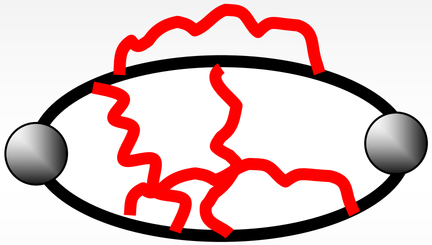
FLAG [Flavianet Lattice Averaging Group]

G.Colangelo, S.Dürr, A.Jüttner, L.Lellouch, H.Leutwyler, V.Lubicz, S.Necco, C.Sachrajda, S.Simula, T.Vladikas, U.Wenger, H.Wittig

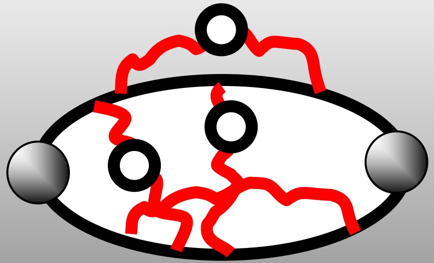
History of LQCD errors until 2006

For many years, uncertainties in lattice calculations have been dominated by the quenched approximation

	f_B [MeV]	$f_{Bs}\sqrt{B_s}$ [MeV]	ξ
J.Flynn **Latt'96**	175(25) 14%	----	----
C.Bernard **Latt'00**	200(30) 15%	267(46) 17%	1.16(5) 4%
L.Lellouch **Ichep'02**	193(27)(10) 15%	276(38) 14%	1.24(4)(6) 6%
Hashimoto **Ichep'04**	189(27) 14%	262(35) 13%	1.23(6) 5%
N.Tantalo **CKM'06**	223(15)(19) 11%	246(16)(20) 10%	1.21(2)(5) 4%



QUENCHED

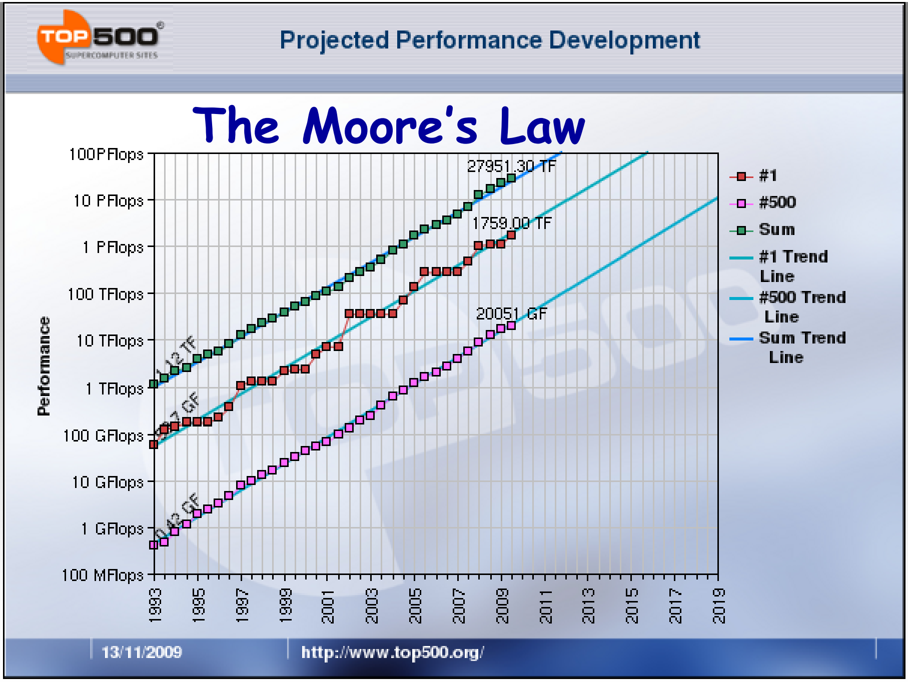


UNQUENCHED

THE "PRECISION ERA" OF LATTICE QCD

1) Increasing of computational power

TeraFlops machines are required to perform unquenched simulations. Available only since few years.



2) Algorithmic improvements

CPU cost of a simulation (for Nf=2 Wilson fermions):

- Ukawa 2001

$$\text{TFlops-years} \simeq 3.1 \left(\frac{N_{conf}}{100}\right) \left(\frac{L_s}{3\text{ fm}}\right)^5 \left(\frac{L_t}{2L_s}\right) \left(\frac{0.2}{\hat{m}/m_s}\right)^3 \left(\frac{0.1\text{ fm}}{a}\right)^7$$

- Del Debbio et al. 2006

$$\text{TFlops-years} \simeq 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{L_s}{3\text{ fm}}\right)^5 \left(\frac{L_t}{2L_s}\right) \left(\frac{0.2}{\hat{m}/m_s}\right) \left(\frac{0.1\text{ fm}}{a}\right)^6$$

$M_\pi \approx$ 200-300 MeV ⟶ Light quark masses in the ChPT regime

FLAVOUR PHYSICS ON THE LATTICE

Collaboration	Quark action	Nf	a [fm]	$(M_\pi)^{min}$ [MeV]	Observables
MILC + FNAL, HPQCD,…	Improved staggered	2+1	≥ 0.045	230	f_K, B_K, $f_{D(s)}$, $D\to\pi/K\,l\nu$, $f_{B(s)}$, $B_{B(s)}$, $B\to D/\pi\,l\nu$
PACS-CS	Clover (NP)	2+1	0.09	156	f_K
RBC/UKQCD	DWF	2+1	≥ 0.08	290	$f_+(0)$, f_K, B_K
BMW	Clover smeared	2+1	≥ 0.07	190	f_K
JLQCD	Overlap	2 2+1	0.12	290	B_K
ETMC	Twisted mass	2 2+1+1	≥ 0.07	260	$f_+(0)$, f_K, B_K, $f_{D(s)}$, $D\to\pi/K\,l\nu$, $f_{B(s)}$
QCDSF	Clover (NP)	2	≥ 0.06	300	$f_+(0)$, f_K

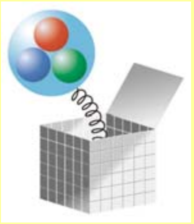
PRECISION FLAVOUR PHYSICS

Experiments 2010

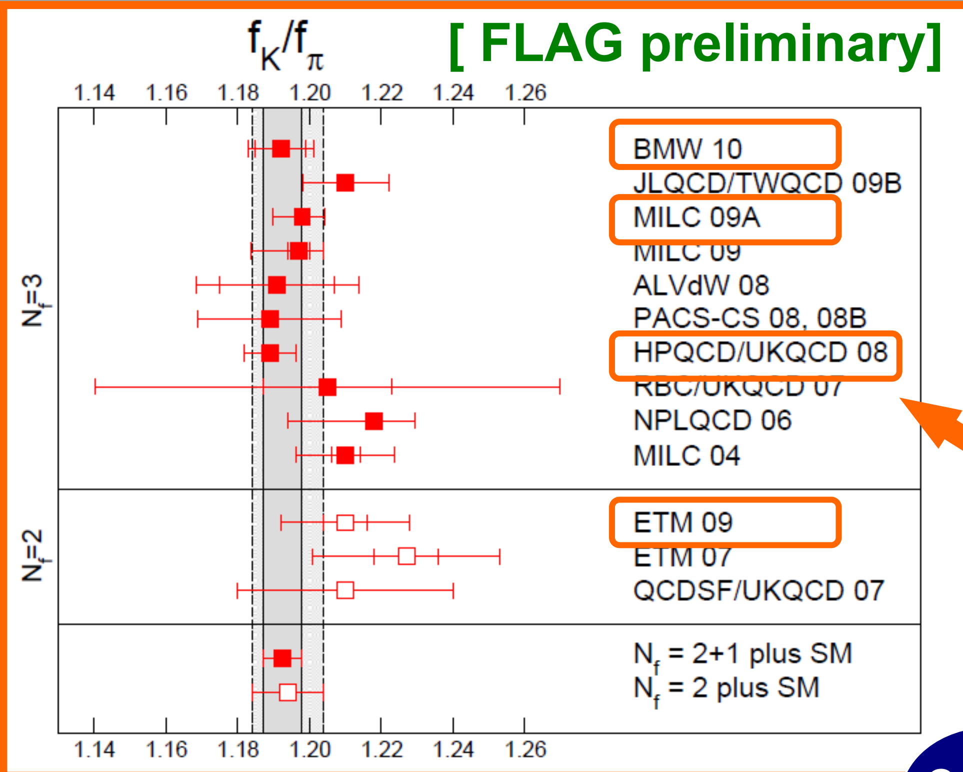
$\|V_{us}\| f_+(0)$	0.21661 ± 0.00047	0.2%
$\dfrac{\|V_{us}\|\ F_K}{\|V_{ud}\|\ F_\pi}$	0.27599 ± 0.00059	0.2%
ε_K	$(2.228 \pm 0.011) \times 10^{-3}$	0.5%
Δm_d	$(0.507 \pm 0.005)\ \mathrm{ps}^{-1}$	1%
Δm_s	$(17.77 \pm 0.12)\ \mathrm{ps}^{-1}$	0.7%
$\mathrm{Sin}2\beta$	0.655 ± 0.027	4%

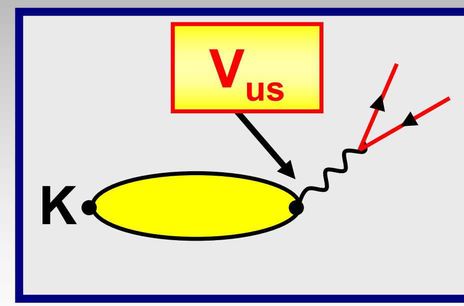
Lattice 2010 / 2006

	Lattice 2010	2006
$f_+(0)$	0.5%	0.9%
F_K / F_π	0.9%	1.1%
B_K	5%	11%
$f_B \sqrt{B_B}$	5%	13%
$f_{Bs} \sqrt{B_{Bs}}$	5%	13%
—	—	—



V_{us}/V_{ud} from leptonic K decays: f_K/f_π





XXXXXXX = included in the FLAG average

0.8%

$f_K/f_\pi = 1.196(10)$ [VL @ Lat2009]

$$f_K/f_\pi = \begin{cases} 1.193(6) & \text{[FLAG, Nf=2+1]} \\ 1.210(18) & \text{[FLAG, Nf=2]} \end{cases}$$

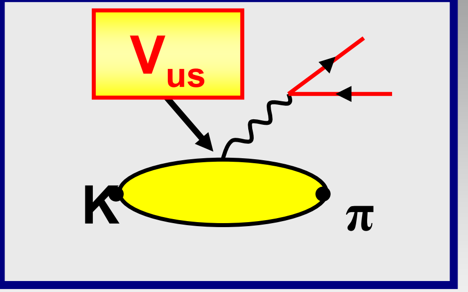
preliminary

Using $|V_{ud}|$ from nuclear β-decays:

- $|V_{us}|_{Kl2} = 0.2247(19)$
- $|V_{us}|_{Unitar.} = 0.2254(10)$

V_{us} from semileptonic K decays: $f_+^{K\pi}(0)$

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{EW}[1+\Delta_{SU(2)}+2\Delta_{EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$



Ademollo-Gatto: $f_+(0) = 1 - O(m_s - m_u)^2$ ← $O(1\%)$. But represents the largest theoret. uncertainty

SU(3)-ChPT

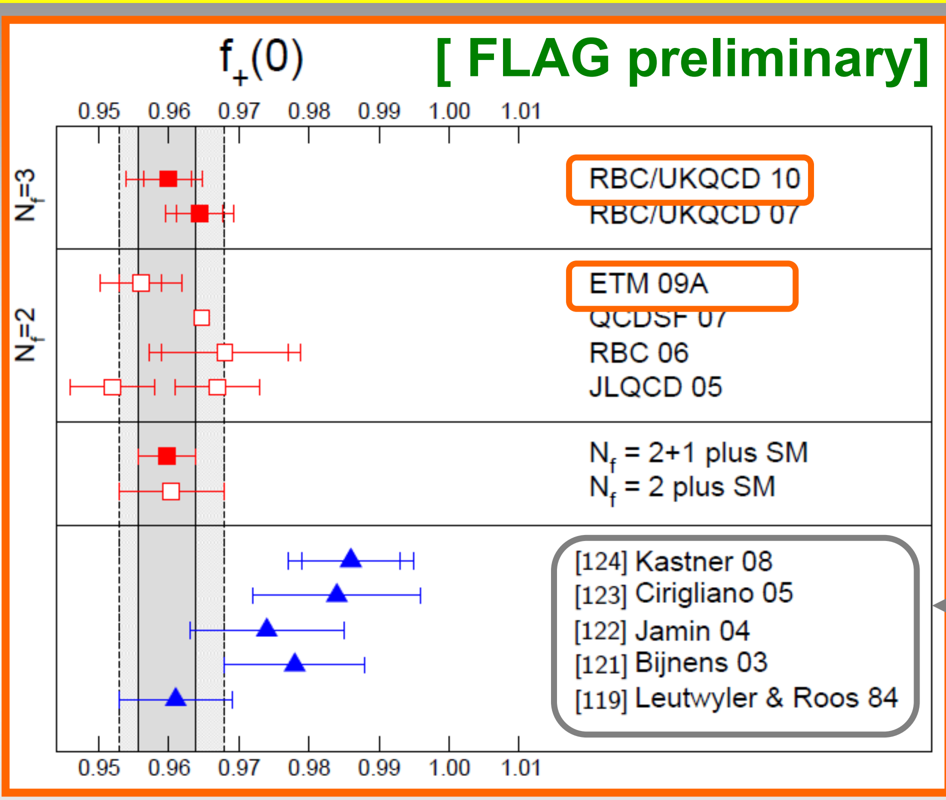
$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

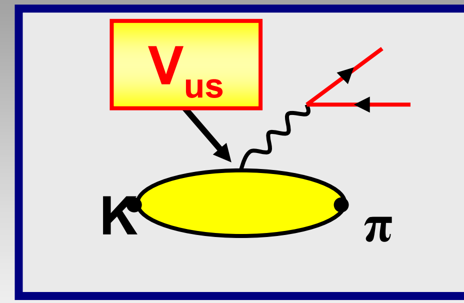
Vector Current Conservation	$f_2 = -0.023$ Independent of L_i (Ademollo-Gatto)	THE LARGEST UNCERTAINTY

Old standard estimate:

Leutwyler, Roos (1984)
(QUARK MODEL)
$f_4 = -0.016 \pm 0.008$

V_{us} from semileptonic K decays: $f_+^{K\pi}(0)$





Analytical model calculations tends to give larger predictions than lattice results

$f_+(0) = 0.960(9)$ Nf = 0 SPQcdR 04

The first lattice calculation

Nf is not the only parameter in a lattice simulation

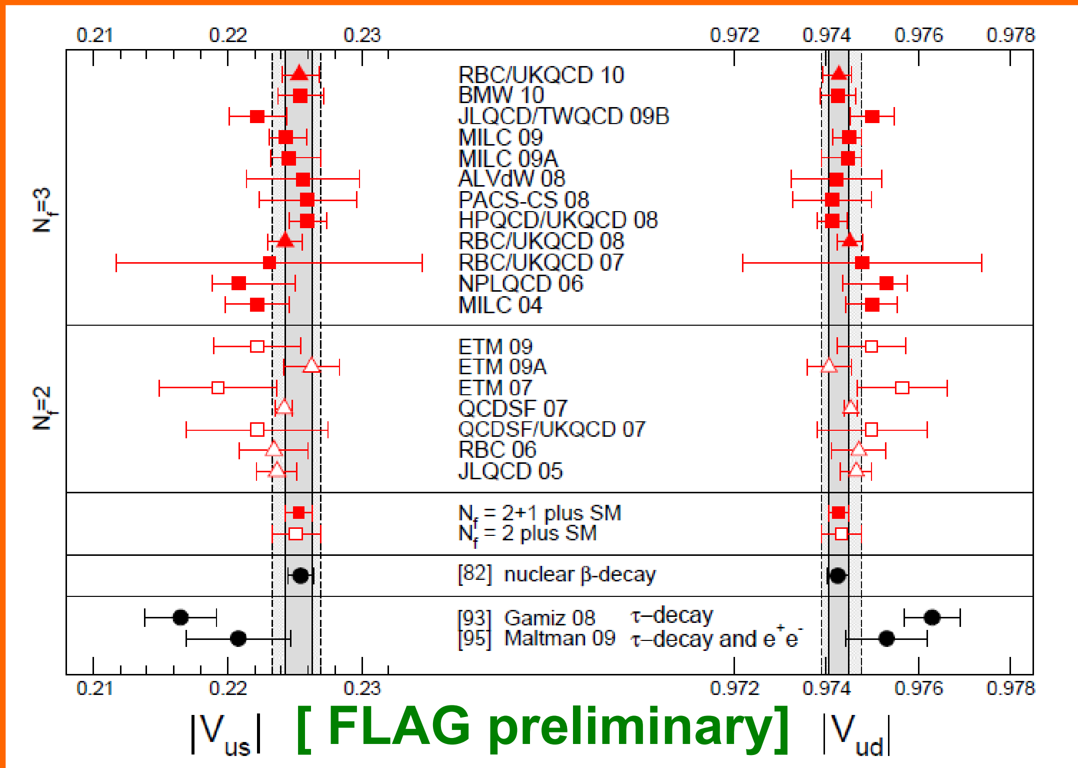
$f_+(0) = 0.956(8)$ Nf = 2 ETM 09

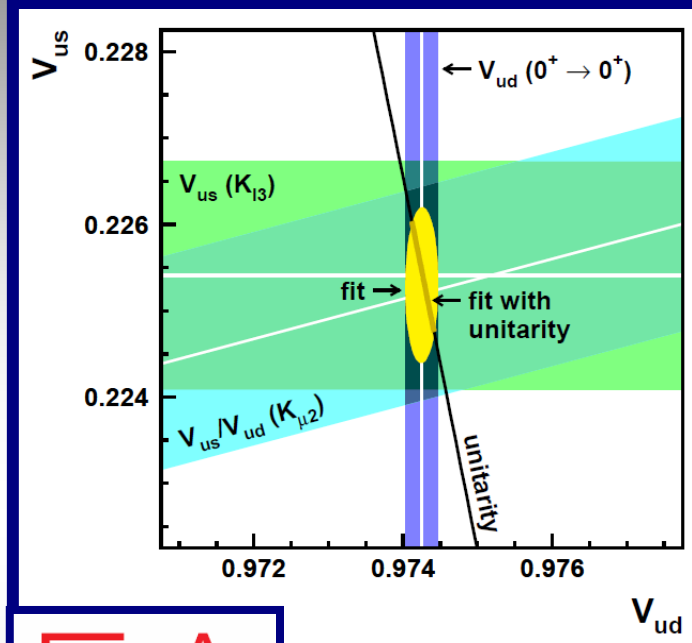
$f_+(0) = 0.959(5)$ Nf = 2+1 RBC/UKQCD 10

0.8%

$f_+(0) = 0.956\ (8)$ [FLAG prelim.]

The 1st row unitarity test





FlaviaNet Kaon WG

Combining with $|V_{ud}|$ from nuclear β decays

- $|V_{us}|_{Kl3} = 0.2263(20)$
- $|V_{us}|_{Kl2} = 0.2247(19)$
- $|V_{us}|_{Kl3+Kl2} = 0.2255(14)$
- $|V_{us}|_{Unitar.} = 0.2254(10)$

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = (0 \pm 8)\cdot 10^{-4}$$

THE 2nd ROW UNITARITY TEST

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$\Delta_{CKM} = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.14(13)$$ [PDG 08]

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \boxed{V_{cd} \quad V_{cs} \quad V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Processes: $D_{(s)} \to l\nu$, $D \to K/\pi\, l\nu$

Theory input: f_D, f_{Ds}, $f_+(0)$

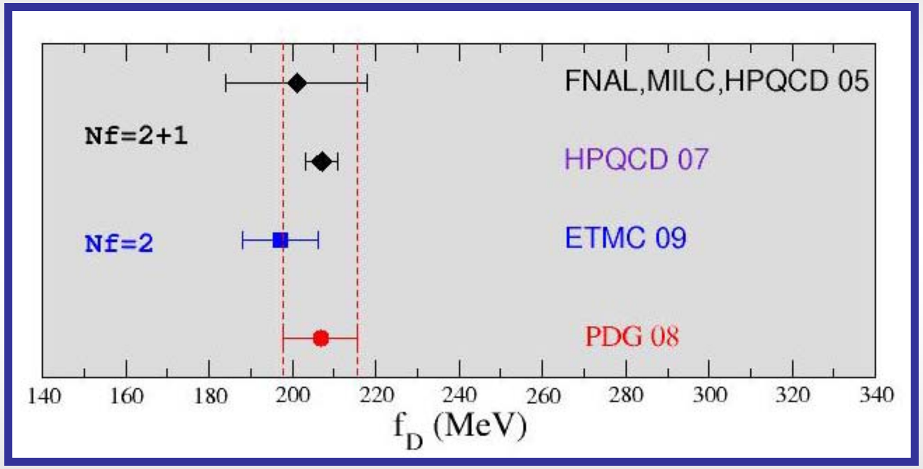
Leptonic $D_{(s)}$ mesons decays: f_D, f_{Ds}

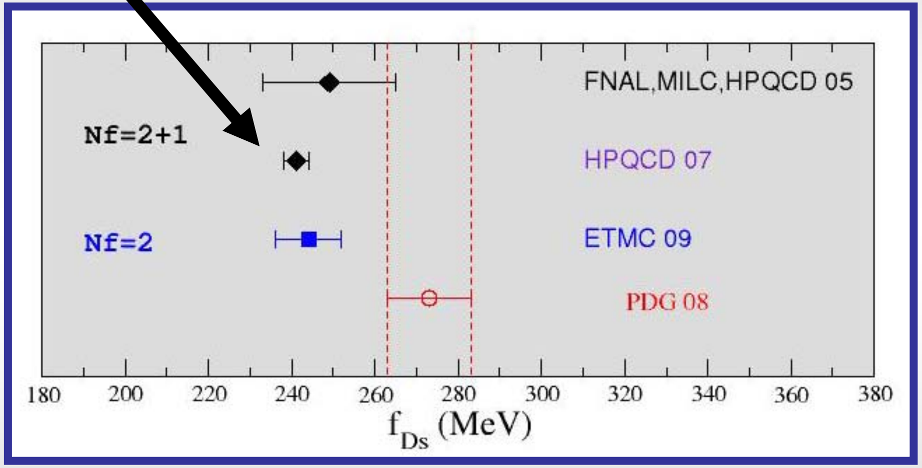
The 2008 summer puzzle

f_{Ds} = 241 ± 3 MeV HPQCD 07

f_{Ds} = 273 ± 10 MeV PDG 08

3 sigma deviation

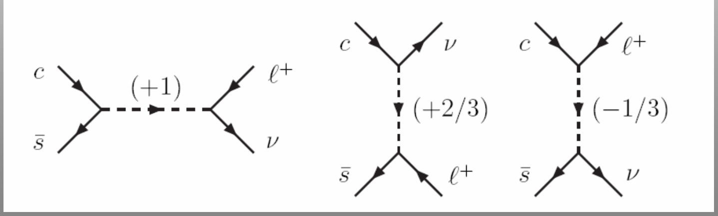




The discrepancy led to speculations about new physics in leptonic D-decays

B.Dobrescu, A.Kronfeld, 0803.0512

"Evidence for nonstandard leptonic decays of Ds mesons"



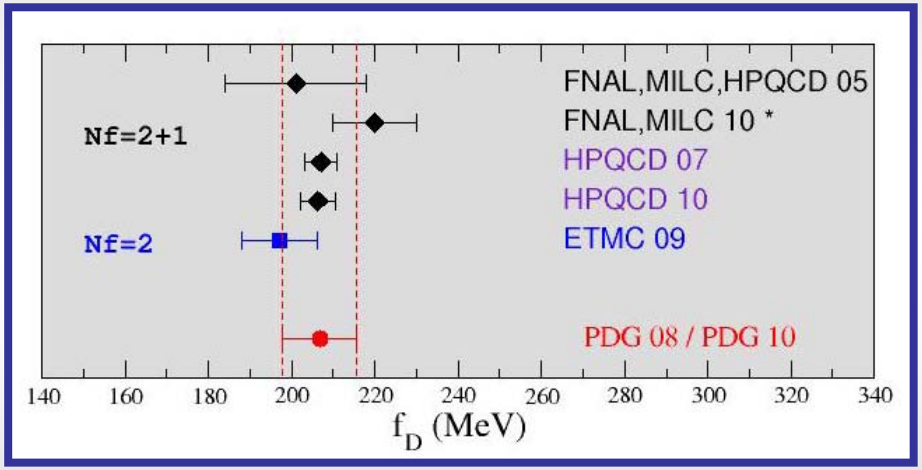
Leptonic $D_{(s)}$ mesons decays: f_D, f_{D_s}

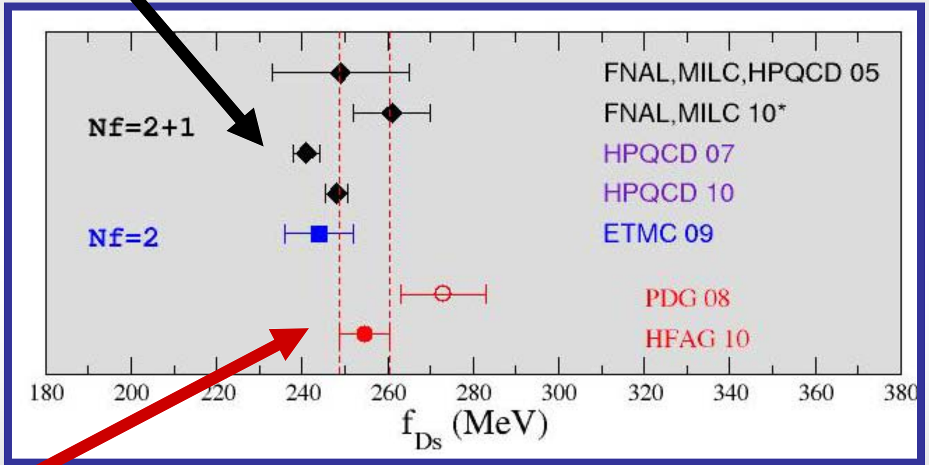
The 2008 summer puzzle

f_{D_s} = 241 ± 3 MeV HPQCD 07

f_{D_s} = 273 ± 10 MeV PDG 08

3 sigma deviation





2010: the puzzle solution

f_{D_s} = 248.0 ± 2.5 MeV HPQCD 10 ↑ + 3% (2.3 σ)

f_{D_s} = 254.6 ± 5.9 MeV HFAG 10 ↓ - 7% (1.8 σ)

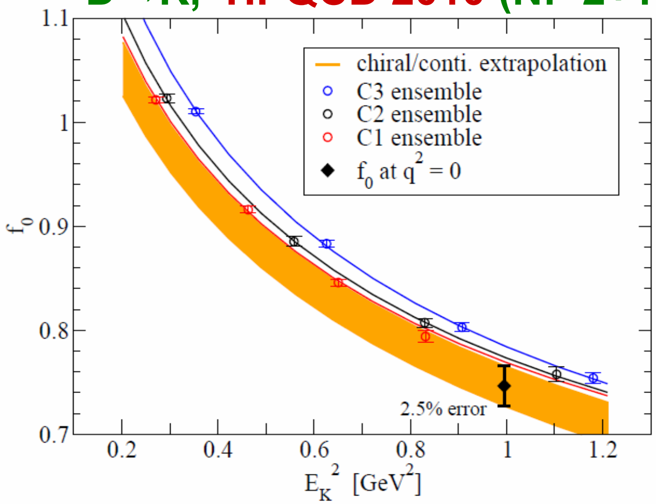
(CLEO-c, Belle, BaBar)

1 sigma agreement

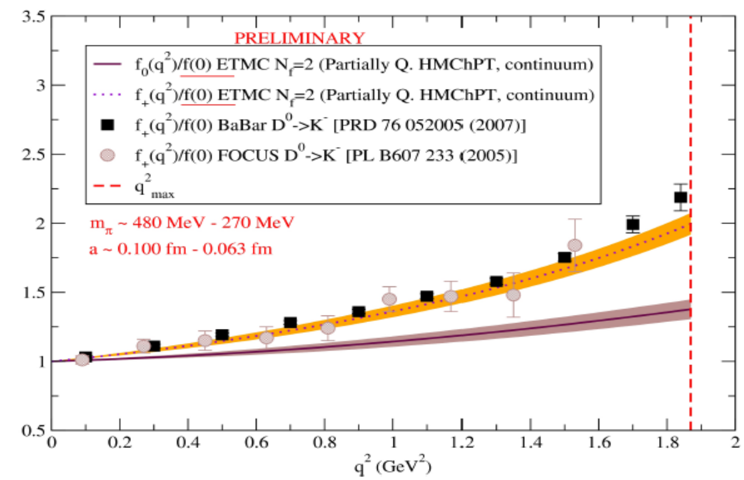
V_{cs} and V_{cd} from semileptonic D decays

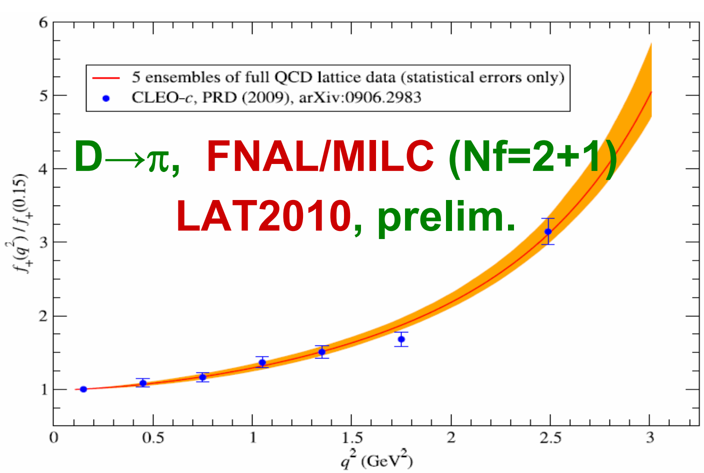
New lattice calculations: HPQCD, FNAL/MILC, ETMC

D→K, HPQCD 2010 (Nf=2+1)



D→K, ETMC (Nf=2) LAT2010, prelim.





	$f^{D\to\pi}(0)$	$f^{D\to K}(0)$
FNAL/MILC/ HPQCD 2004	0.64(7)	0.73(8)
ETMC 2010 (preliminary)	0.66(6)	0.76(4)
HPQCD 2010	-----	0.75(2)

The 2nd row and 2nd column unitarity test

Very precise experimental results from BaBar and CLEO-c

$$|V_{cs}|\, f_+^{D\to K}(0) = 0.718(8)$$

CLEO-c + BaBar average

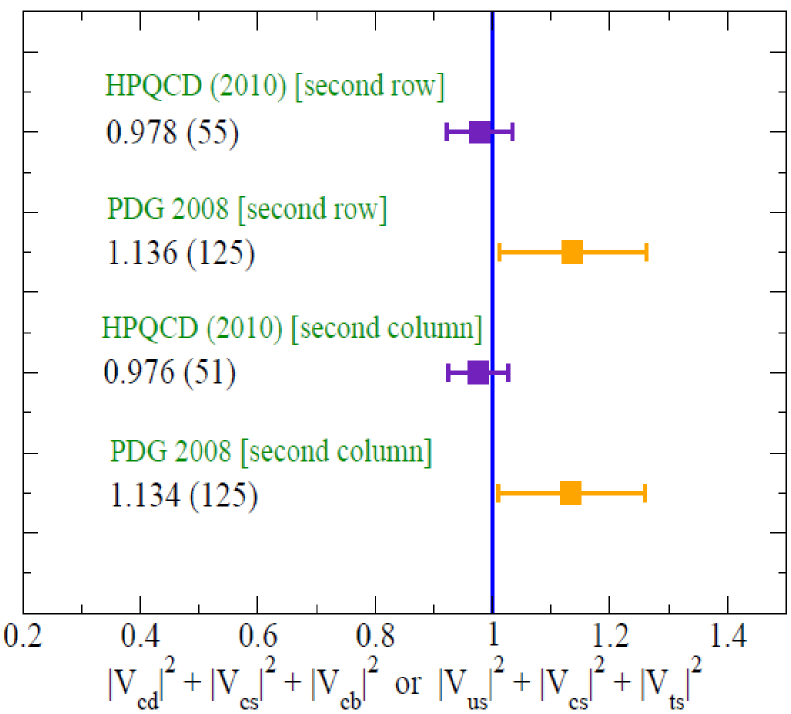
Using the precise lattice determination by **HPQCD 2010**

$$f_+^{D\to K}(0) = 0.747(19)$$

$$|V_{cs}| = 0.961 \pm 0.011_{\text{exp}} \pm 0.024_{\text{LQCD}}$$

It will be interesting to compare with the forthcoming FNAL/MILC and ETMC results

The unitarity tests



THE UNITARITY TRIANGLE ANALYSIS

$$V_{ud}\,V^*_{ub} + V_{cd}\,V^*_{cb} + V_{td}\,V^*_{tb} = 0$$

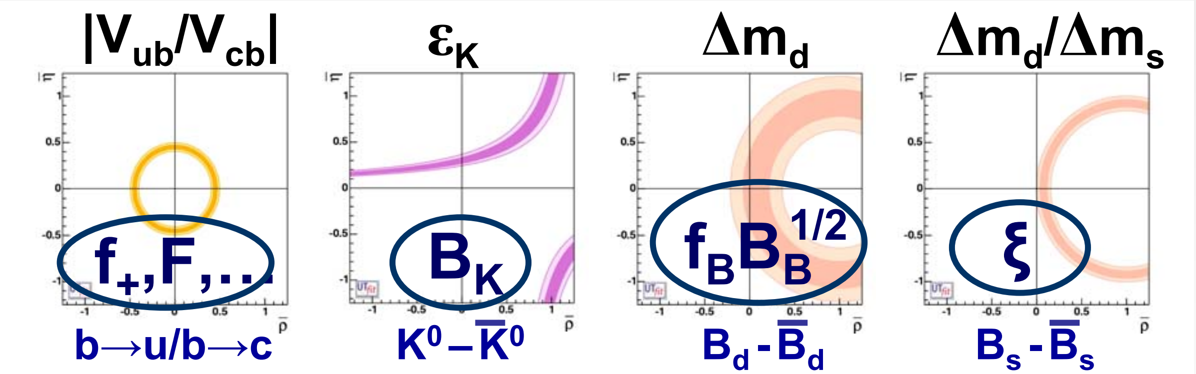
$$\begin{pmatrix} \boxed{V_{ud}} & V_{us} & \boxed{V_{ub}} \\ \boxed{V_{cd}} & V_{cs} & \boxed{V_{cb}} \\ \boxed{V_{td}} & V_{ts} & \boxed{V_{tb}} \end{pmatrix}$$

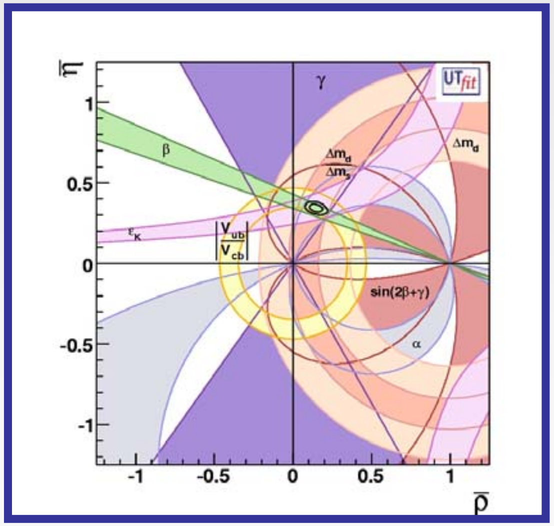
Processes: $B \rightarrow l\,\nu$, $B \rightarrow D/\pi\, l\,\nu$, $K - K$, $B_{(s)} - B_{(s)}$

Theory input: f_B, f_{Bs}, $f_+(0)$, B_K, B_B, ...

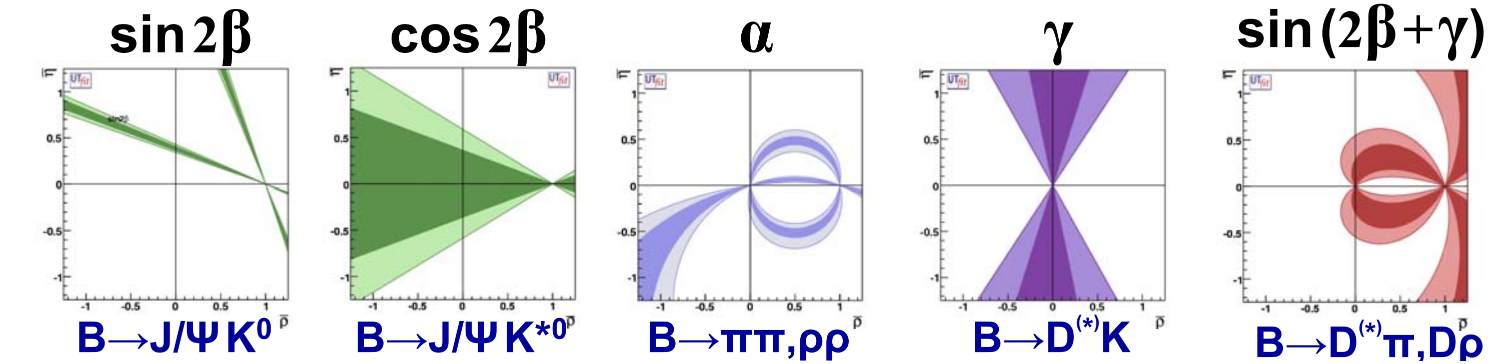
THE UTA CONSTRAINTS

UT-LATTICE

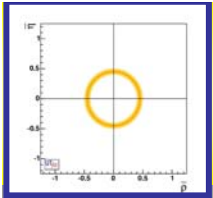




UT-ANGLES



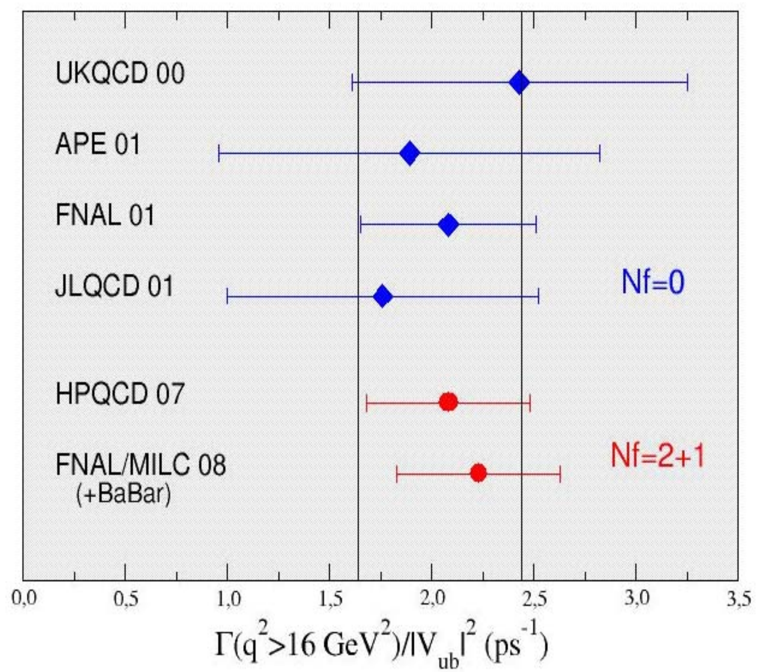
See also talks by C. Tarantino and L. Silvestrini

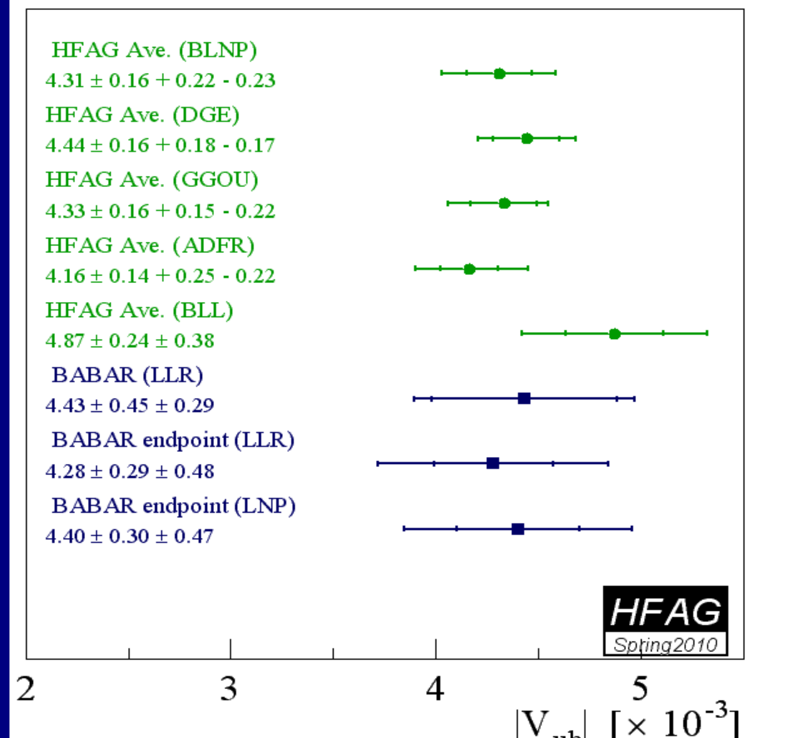


Exclusive vs Inclusive Vub

THEORETICALLY CLEAN BUT MORE LATTICE CALCULATIONS ARE WELCOME

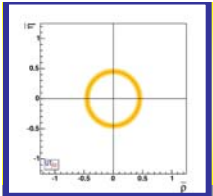
IMPORTANT LONG DISTANCE CONTRIBUTIONS. THE RESULTS ARE MODEL DEPENDENT





$|V_{ub}|_{excl.} = (35.0 \pm 4.0)\ 10^{-4}$

$|V_{ub}|_{incl.} = (42.0 \pm 1.5 \pm 5.0)\ 10^{-4}$



Exclusive vs Inclusive Vub

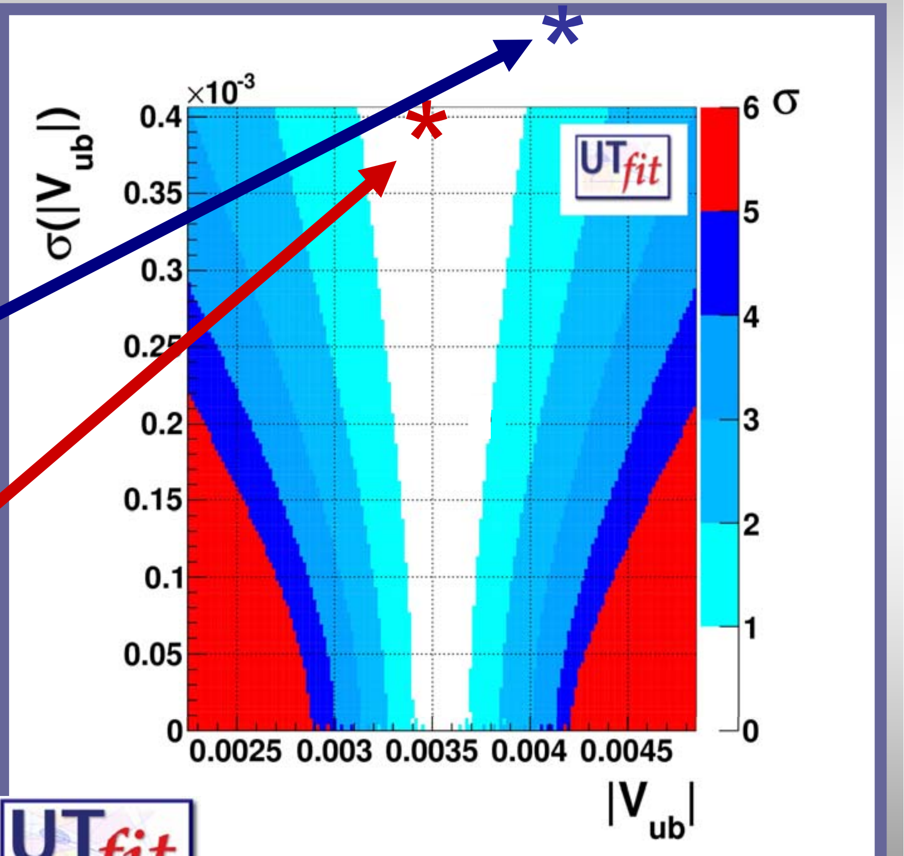
- The uncertainty of inclusive Vub estimated from the spread among different models. This is questionable

- The fit in the SM favors a low value of Vub, as indicated by exclusive decays

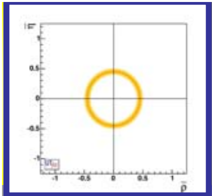
$|V_{ub}|_{incl.} = (42.0 \pm 1.5 \pm 5.0)\ 10^{-4}$

$|V_{ub}|_{excl.} = (35.0 \pm 4.0)\ 10^{-4}$

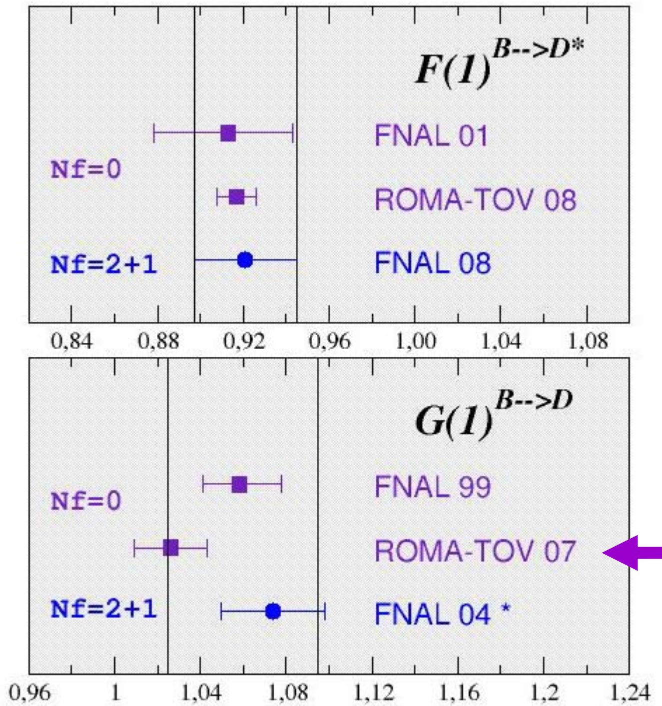
- Improve the accuracy of exclusive Vub in order to clarify the issue (see $D \to \pi/K\ l\nu$)



$|V_{ub}|_{SM\text{-}Fit} = (35.5 \pm 1.4)\ 10^{-4}$



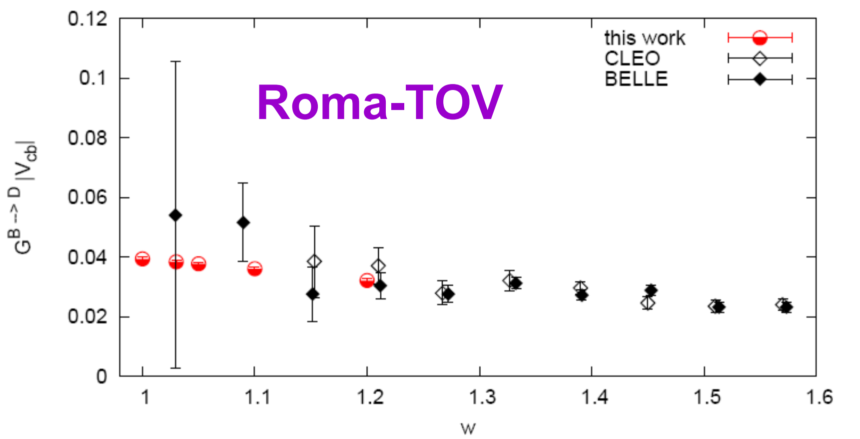
Exclusive Vcb



TWO DIFFERENT APPROACHES:

- "double ratios" (FNAL)
- "step scaling" (TOV)

Remarkable agreement

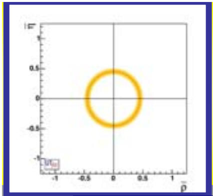


Averages from
VL, C.Tarantino 0807.4605

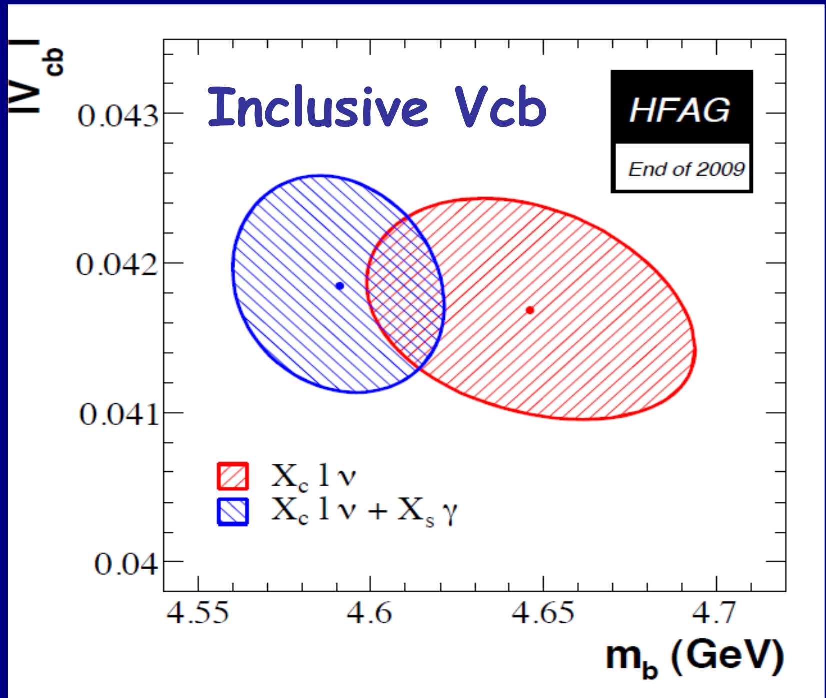
$F(1) = 0.924 \pm 0.022$ **2%**

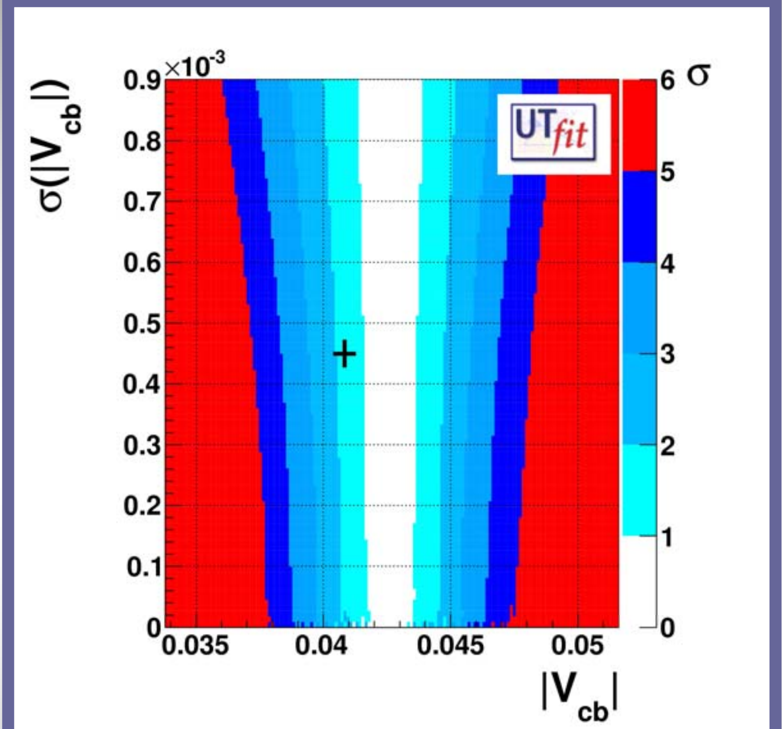
$G(1) = 1.060 \pm 0.035$ **3%**

$|V_{cb}|_{excl.} = (39.0 \pm 0.9)\ 10^{-3}$



Exclusive vs Inclusive Vcb





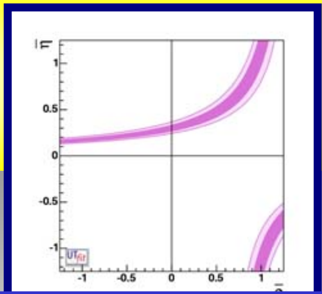
$|V_{cb}|_{incl.} = (41.7 \pm 0.7)\ 10^{-3}$

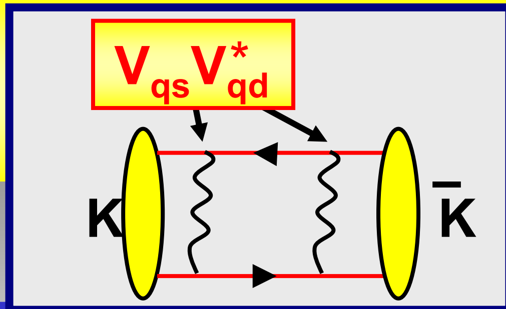
2.5 σ

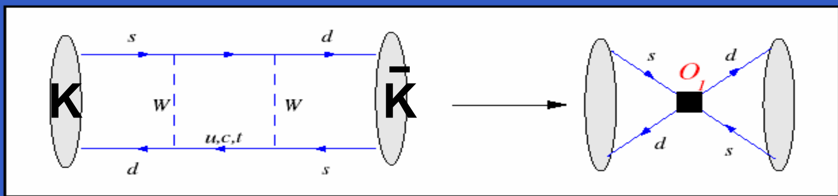
$|V_{cb}|_{excl.} = (39.0 \pm 0.9)\ 10^{-3}$

$|V_{cb}|_{SM\text{-}Fit} = (42.7 \pm 1.0)\ 10^{-3}$

K^0–$\bar{K}^0$ mixing: B_K







$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

Pre-history

QCD SR, Pich, De Rafael, **1985**:

$\hat{B}_K = 0.33 \pm 0.09$

1/Nc exp., Buras, Gerard, **1985**:

$\hat{B}_K = 0.75$

LQCD, Gavela et al., **1987**:

$\hat{B}_K = 0.90 \pm 0.20$

History

Quench. error

$\hat{B}_K = 0.90 \pm 0.03 \pm 0.15$

S.Sharpe@Latt'96 — 17%

$\hat{B}_K = 0.86 \pm 0.05 \pm 0.14$

L.Lellouch@Latt'00 — 17%

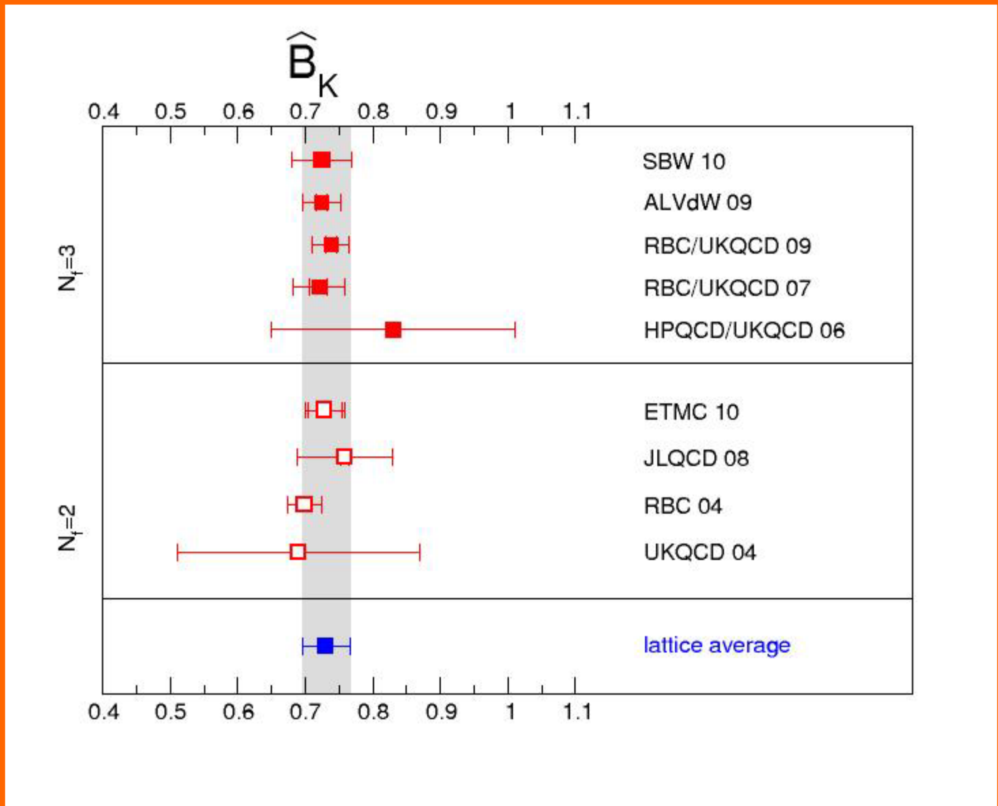
$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$

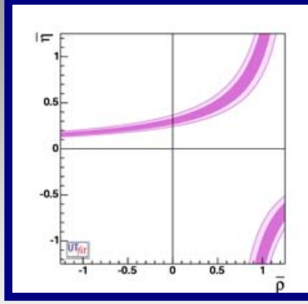
C.Dawson@Latt'05 — 11%

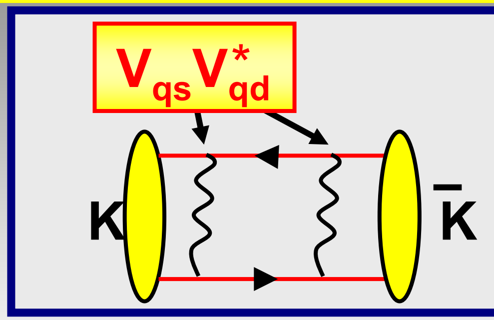
$\hat{B}_K = 0.731 \pm 0.036$

V.Lubicz@Latt'09 — 5%

K^0–$\bar{K}^0$ mixing: B_K







From the UT fit, assuming the Standard Model

$\hat{B}_K = 0.85\,(7)$

(with Kε = 0.94(2), A.Buras, D.Guadagnoli, G.Isidori)

$\hat{B}_K = 0.724\,(12)\,(43)$ [Nf=2+1, SBW 10]

$\hat{B}_K = 0.724\,(8)\,(28)$ [Nf=2+1, ALVdW 09]

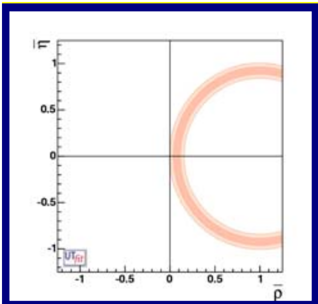
$\hat{B}_K = 0.738\,(8)\,(25)$ [Nf=2+1, RBC/UKQCD 09]

$\hat{B}_K = 0.729\,(25)\,(16)$ [Nf=2, ETMC 10]

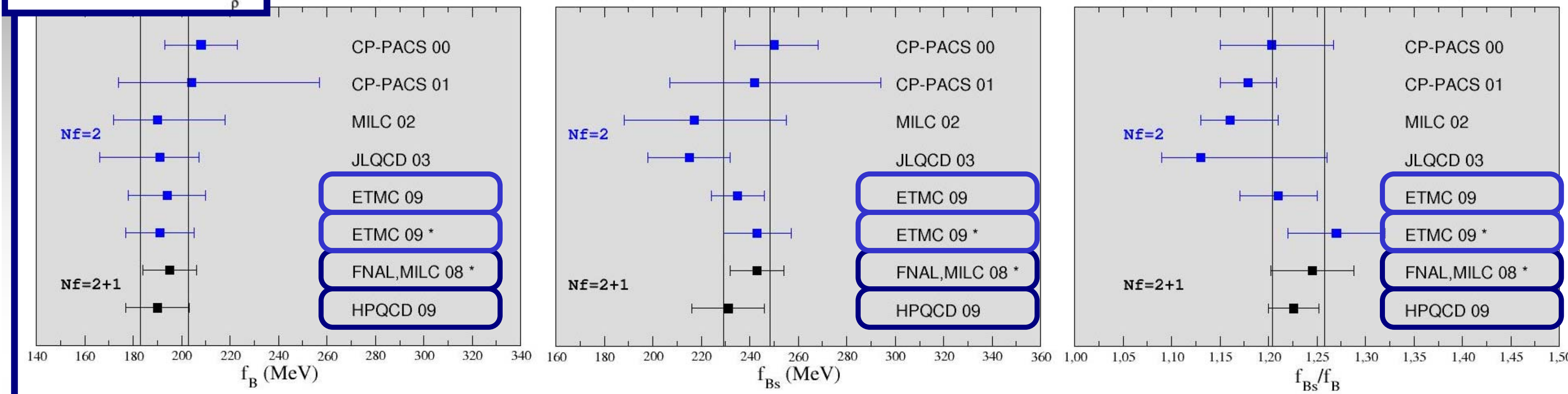
$\hat{B}_K = 0.731\,(7)\,(35)$

[VL @ Lat2009]

5%



B-mesons decay constants f_B, f_{Bs} and B-$\bar{B}$ mixing, $B_{Bd/s}$



$f_{Bs} = 238.8 \pm 9.5$ MeV

$f_B = 192.8 \pm 9.9$ MeV

4-5%

$f_{Bs}/f_B = 1.231 \pm 0.027$

2%

Combining with the only modern calculation HPQCD [0902.1815]: $\hat{B}_{Bd} = 1.26 \pm 0.11$, $\hat{B}_{Bs} = 1.33 \pm 0.06$

$f_{Bs}\sqrt{\hat{B}_{Bs}} = 275 \pm 13$ MeV

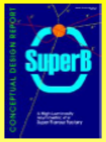
5%

$\xi = 1.243 \pm 0.028$

2%

Averages from J.Laiho, E.Lunghi, R.Van de Water, 0910.2928

A look at the future



V.Lubicz @
Villa Mondragone
Monte Porzio Catone - Italy
13 - 15 November 2006

Hadronic matrix element	Lattice error in 2006	Lattice error in 2009	6 TFlop Year [2009]	60 TFlop Year [2011 LHCb]	1-10 PFlop Year [2015 SuperB]
$f_+^{K\pi}(0)$	0.9%	**0.5%**	0.7%	0.4%	**< 0.1%**
$\hat{B}_K$	11%	**5%**	5%	3%	**1%**
f_B	14%	**5%**	3.5 - 4.5%	2.5 - 4.0%	**1 – 1.5%**
$f_{Bs}B_{Bs}^{1/2}$	13%	**5%**	4 - 5%	3 - 4%	**1 – 1.5%**
ξ	5%	**2%**	3%	1.5 - 2 %	**0.5 – 0.8 %**
$\mathcal{F}_{B\to D/D^*l\nu}$	4%	**2%**	2%	1.2%	**0.5%**
$f_+^{B\pi},\ldots$	11%	**11%**	5.5 - 6.5%	4 - 5%	**2 – 3%**
$T_1^{B\to K^*/\rho}$	13%	**13%**	----	----	**3 – 4%**

Cost of the "SuperB" lattice simulation

Simulation parameters

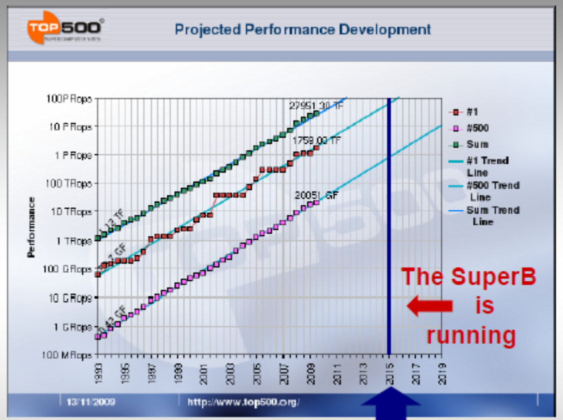
Nconf = 120

a = 0.033 fm
[$1/a$ = 6.0 GeV]

$\hat{m}/m_s$ = 1/12
[M_π = 200 MeV]

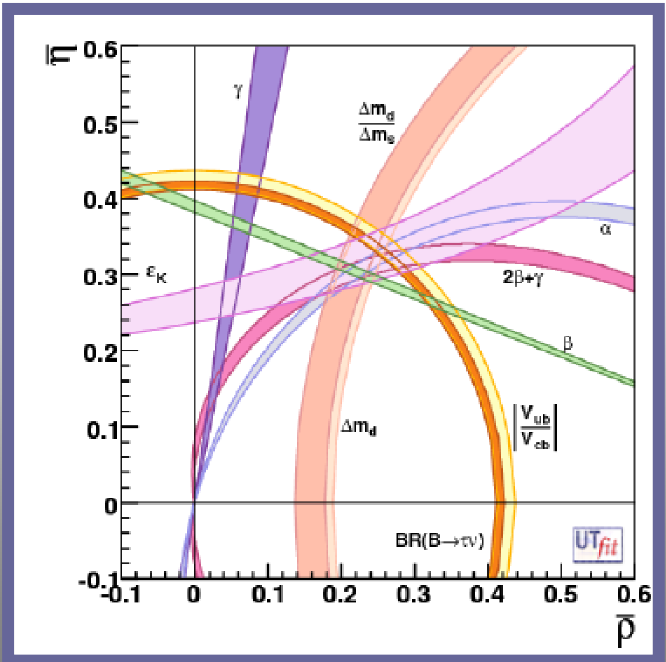
L_s = 4.5 fm
[V = $136^3 \times 270$]

~ 3 PFlop-years

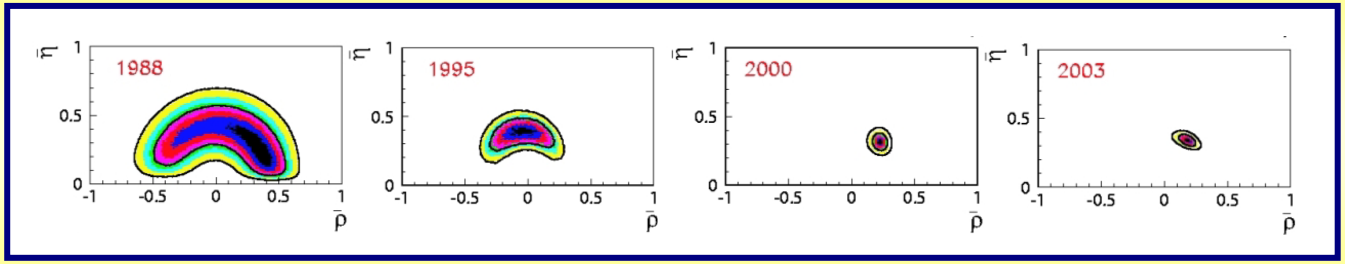


Affordable with 1-10 PFlops available for Lattice QCD in 2015 !

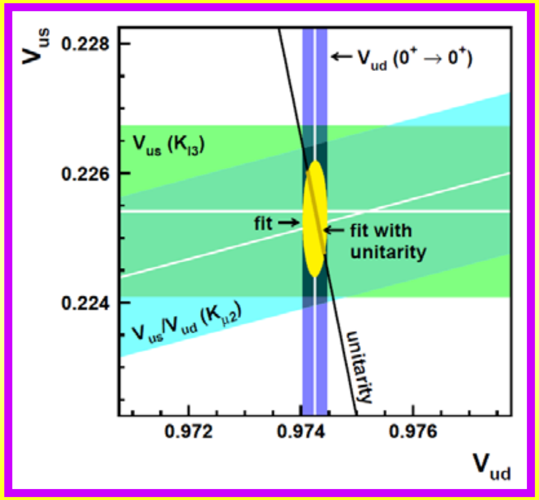
VL @

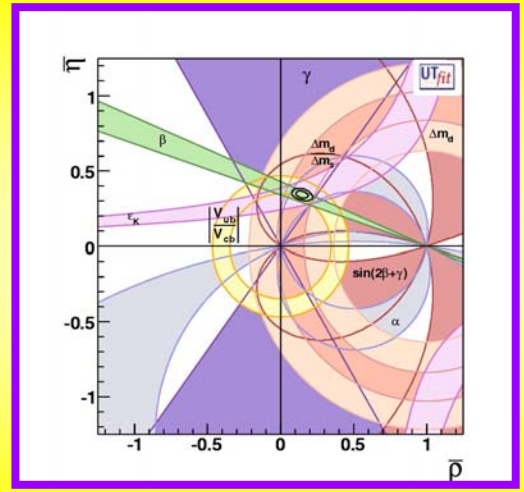


The past



the present





and the future