$K^{\pm}_{\mu 3}$ Form Factor Measurement at NA48/2

Manuel Hita-Hochgesand

Johannes Gutenberg Universität Mainz

*on behalf of the **NA48/2** Collaboration:*

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Heavy Quarks & Leptons 2010

Outline

- Physics Motivation
- NA48/2-Experiment
- $K^{\pm}_{\mu3}$ Form Factor Analysis
  - Event selection
  - Background sources
  - Radiative corrections
  - DATA-MC comparison
  - Preliminary result
  - Systematic checks
- $K^{\pm}_{\mu3}$ Form Factors at NA62
- Summary

Physical Motivation

K_{l3} decays provide the **most accurate** and **theoretical cleanest** way to access $|V_{us}|$.
The master formula for K_{l3} decay rates:

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |V_{us}| |f_+(0)|^2 I_K^l(\lambda_{+0}) (1 + \delta_{SU(2)}^l + \delta_{EM}^l)^2$$

Experimental Inputs:

- $\Gamma(K_{l3(\gamma)})$ Branching ratios and Kaon lifetimes.
- $I_K^l(\lambda_{+0})$ Phase space integral depends on the form factors.

Theory Inputs:

- S_{EW} Universal short distance EW corrections (1.0232 ± 0.0003).
- $f_+(0)$ Form factor at zero momentum transfer.
- $\delta_{SU(2)}^l$ Form factor correction for isospin breaking (charged mode only).
- δ_{EM}^l Long distance EM effects.

Physical Motivation

K_{l3} decays are described by **two form factors** $f_{\pm}(t)$ and the **matrix element** can be written as:

$$M = \frac{G_F}{2} V_{us}(f_+(t)(P_K + P_\pi)^\mu \bar{u}_l \gamma_\mu (1+\gamma_5) u_\nu + f_-(t) m_l \bar{u}_l (1+\gamma_5) u_\nu)$$

$t = q^2$ is the square of the four-momentum transfer to the lepton neutrino system.
$f_-(t)$ can only be measured in $K_{\mu 3}$ decays because of $m_e << m_K$.

$f_+(t)$ is the **vector form factor** and $f_0(t)$ the **scalar form factor** which is a linear combination of:

$$f_0(t) = f_+(t) + \frac{t}{(m_K^2 - m_\pi^2)} f_-(t)$$

By construction $f_+(0) = f_0(0)$. $f_+(0)$ cannot be measured directly, therefore the form factors are normalised to $f_+(0)$:

$$\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)} \qquad \bar{f}_0(t) = \frac{f_0(t)}{f_+(0)} \qquad \bar{f}_+(t) = \bar{f}_0(t)$$

Physical Motivation

Form Factor Parametrizations

Parametrizations who make use of **physical quantities** are called **class 1** parametrizations, which depend on free parameters which have a physical meaning.

Pole Parametrization:

Describes the exchange of K^* resonances with spin-parity $1^-/0^+$ and mass m_V/m_S.
$f_+(t)$ can be described by $K^*(892)$, for $f_0(t)$ no obvious dominance is seen.

$$\bar{f}_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t}$$

Dispersive Parametrization:

This parametrization is based on a dispersive approach with the free parameters Λ_+ and $\ln C$.

Accurate polynomial approximations for the dispersive integrals $G(t)$ and $H(t)$ are available.

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ + H(t))\right] \quad \bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - G(t))\right]$$

(PLB 638(2006) 480, PRD 80(2009) 034034)

Physical Motivation

Form Factor Parametrizations

Parametrizations without a **physical meaning** are called **class 2** parametrizations. They require more free parameters and are mathematical expansions in the momentum transfer.

Linear and quadratic parametrization:

The expansion in the momentum transfer t is widely used:

$$\bar{f}_{+,0}(t) = \left[1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right] \quad \textbf{Linear}$$

$$\bar{f}_{+,0}(t) = \left[1 + \lambda'_{+,0}\frac{t}{m_\pi^2} + \frac{1}{2}\lambda''_{+,0}\left(\frac{t}{m_\pi^2}\right)^2\right] \quad \textbf{Quadratic}$$

- More free parameters to be determined → **Correlations**!
- No sensitivity to determine λ''_0 with current experiment → $\bar{f}_+$ quadratic / $\bar{f}_0$ linear.

Z-fit parametrization:

The parametrization function depending on t and $t_+ = (m_K + m_\pi)^2$ sums an infinite number of terms, transforming the original series, naively an expansion involving $t/t_+ \lesssim 0.3$, into a series with much smaller expansion parameters *(PRD74(2006) 096006)*.

Physical Motivation

SM Test using form factors

The dispersive parametrization provides a link from the experimental accessible t region to the Callan-Treiman point $\Delta_{K\pi} = (m_K^2 - m_\pi^2)$. The value for $\bar{f}_0(\Delta_{K\pi})$ is given by the CT-theorem:

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{f_{K^+}}{f_{\pi^+} f_+(0)} + \Delta_{CT}$$

Δ_{CT} is evaluated in NLO in ChPT *(Gasser and Leutwyler (85))* $\Delta_{CT}^{\mathrm{NLO}} = (-3.5 \pm 8.0) \times 10^{-3}$.

It is possible to calculate C by measuring $\mathrm{Br}(\mathrm{K}_{l2}/\pi_{l2})$ and $\Gamma(K_{e3})$:

$$C = \bar{f}_0(\Delta_{K\pi}) = B_{\exp} r + \Delta_{CT} = \frac{f_K |V_{us}|}{f_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}| r + \Delta_{CT}$$

Compare the value of C from this calculation to those obtained by the $K_{\mu 3}^{\pm}$ analysis.

In the Standard model $r = 1$ is expected ➔ $B_{\exp} = 1.2446 \pm 0.0041$

Physics beyond the Standard model can lead to small modifications in the measured value of $B_{\exp}$.

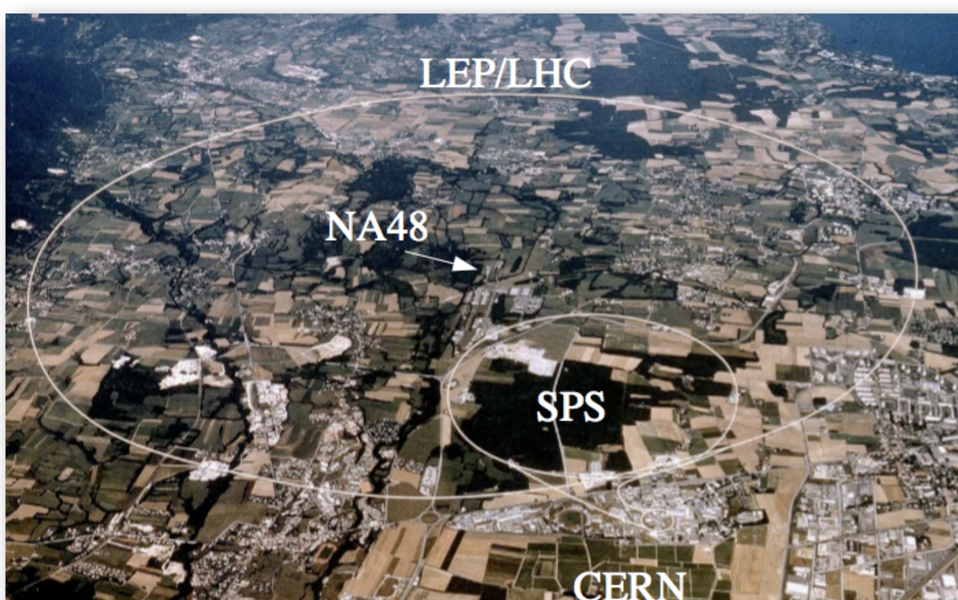
NA48/2-Experiment

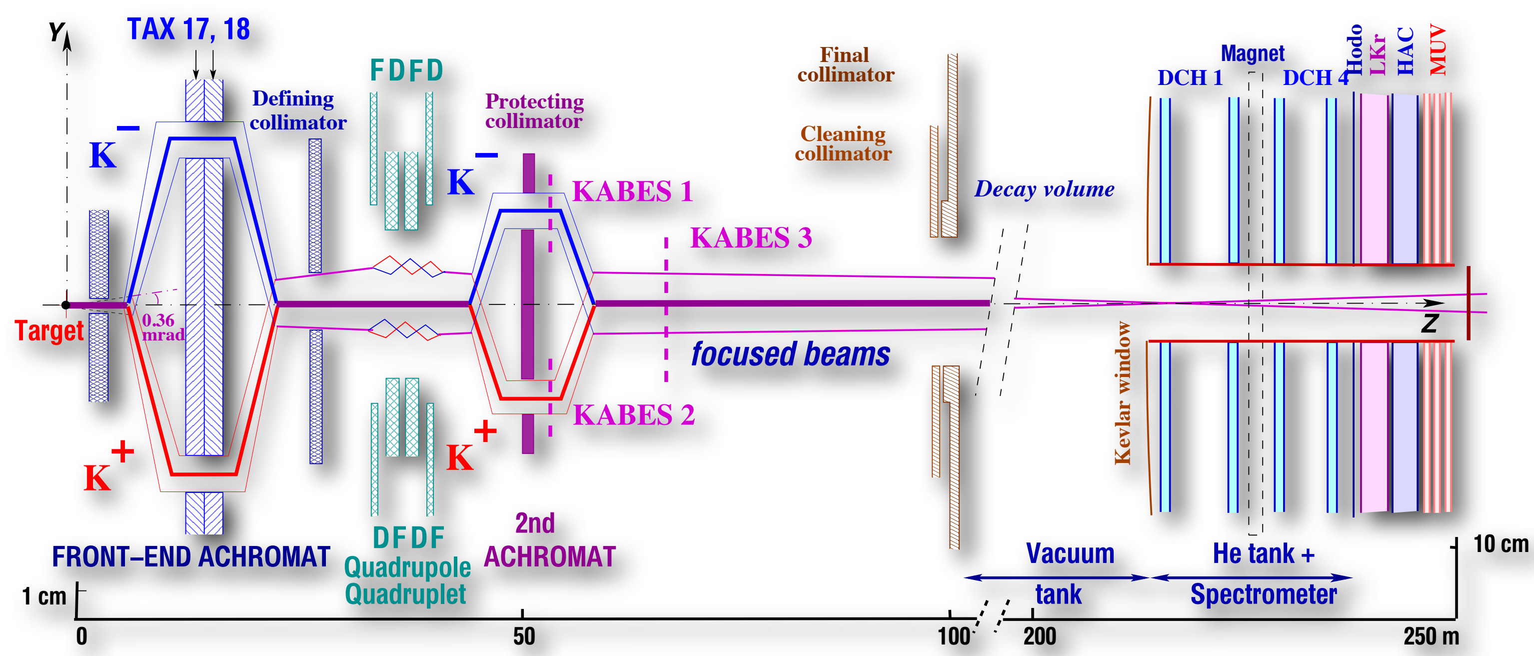
- **NA48/2:** is a **fixed target** experiment in the **N**orth **A**rea of CERN the **SPS.** In 2004 the main purpose was the search of direct CP violation in $K^{\pm} \to 3\pi$ decays.

- The beamline offered simultaneous K^{+} and K^{-} beams. The beams were coinciding within 1 mm along the 114 meter long decay volume.

- For the form factor measurement a dedicated three-day run with minimum bias trigger and low intensity was used.

- The beam momentum was $(60 \pm 1.8)\ \mathrm{GeV/c}$ in this special run.





NA48/2-Experiment

Main detector components:

- **Magnetic Spectrometer**

$$\frac{\sigma_p}{p} = 1.02\% \oplus 0.044\% \frac{p}{\mathrm{GeV}/c}$$

~ 1% resolution for charged particles with p=20 GeV/c

- **Hodoscope**

two planes of scintillator for fast triggering.
$\sigma_t \sim 150$ ps

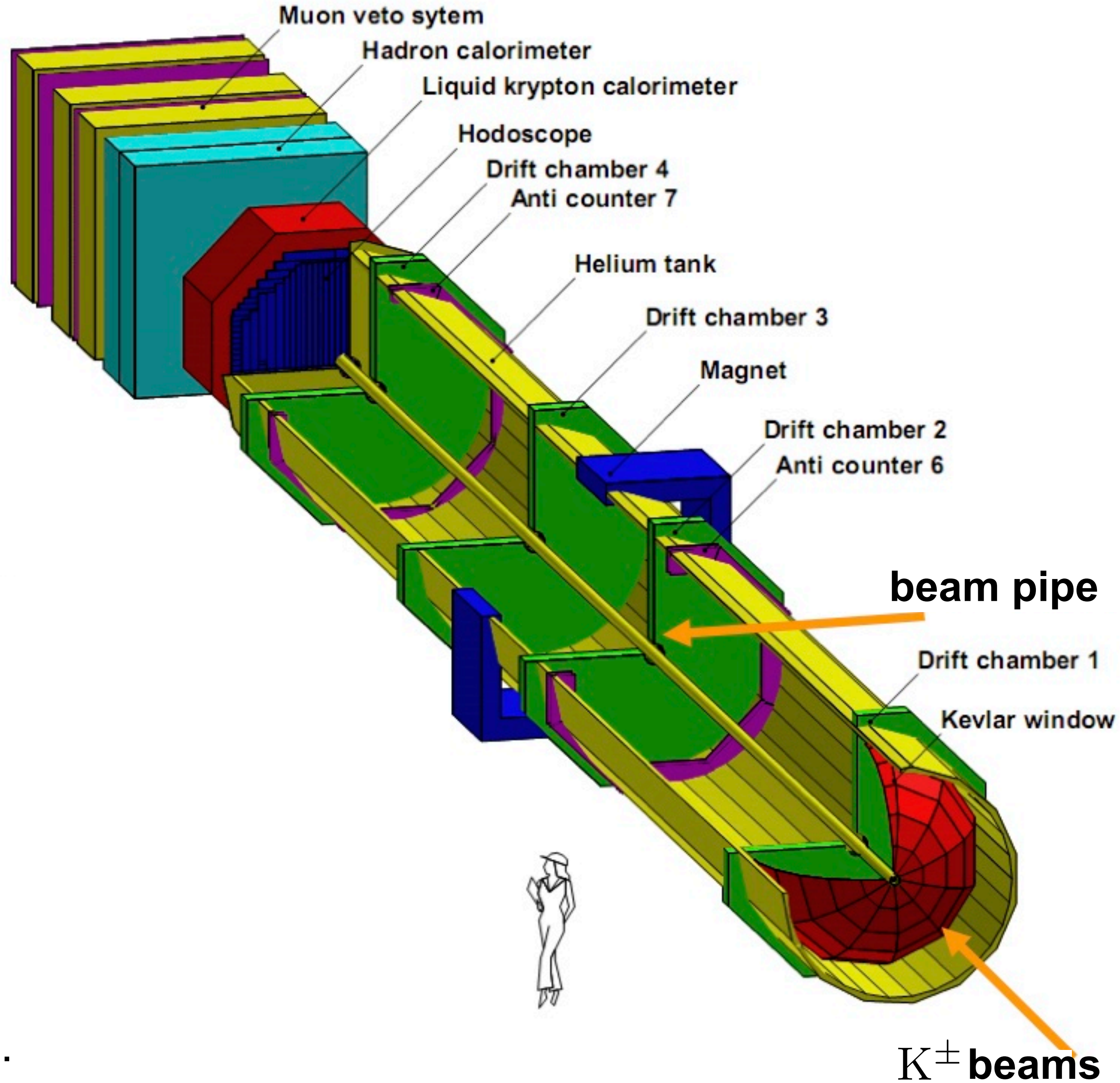
- **Liquid Krypton EM Calorimeter**

$$\frac{\sigma_E}{E} = \frac{3.2\%}{\sqrt{E/\mathrm{GeV}}} \oplus \frac{9.0\%}{E/\mathrm{GeV}} \oplus 0.42\% \sim 1\%$$

1.4% resolution for particles with E=20 GeV

- **Muon veto system**

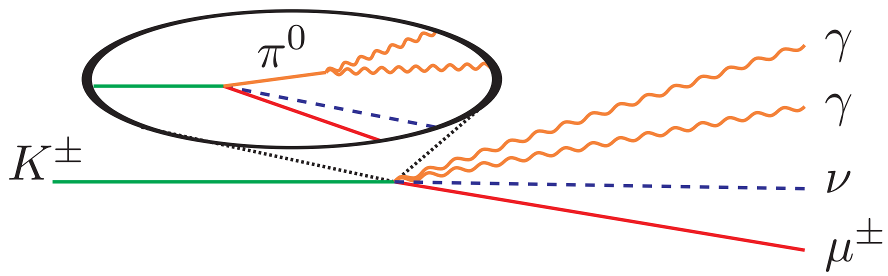
three planes of scintillators, each shielded by 80 cm iron.
99.9% efficient.
$\sigma_t \sim 350$ ps



Min Bias Trigger: Coincidence of two Hodoscope hits $\times E_{\mathrm{LKr}} > 10$ GeV

$K^{\pm}_{\mu3}$ Form Factor Analysis

Event selection:

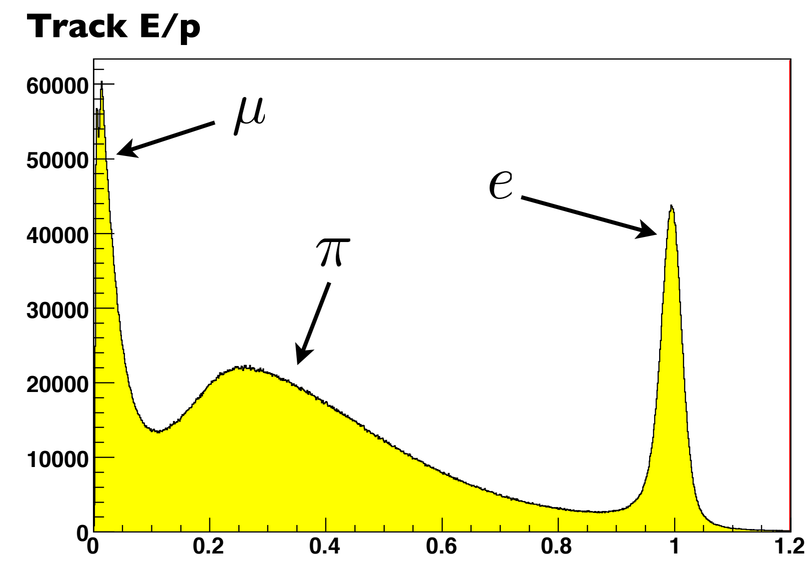


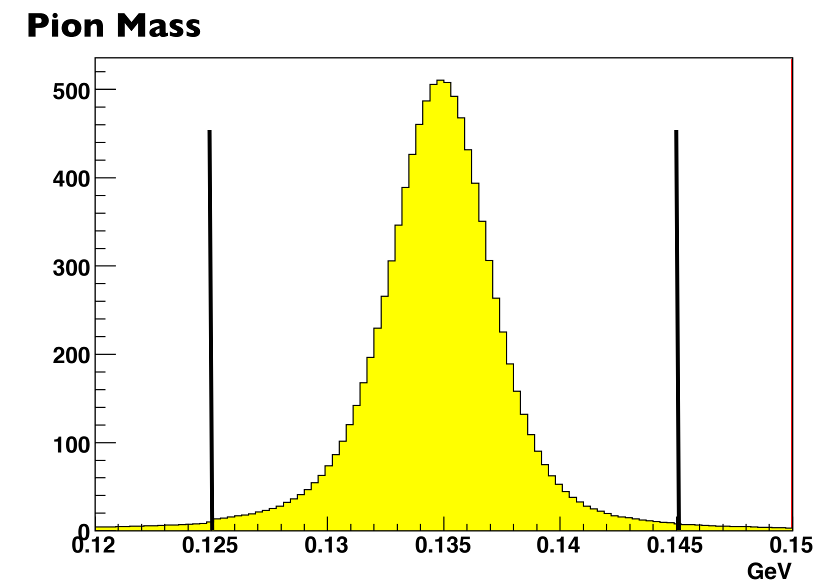
- **1 good track.**
  - Muon identification (muon veto + E/P)
  - $P_\mu > 10\ \text{GeV}$
- **1 good $\pi^0 \to \gamma\gamma$.**
  - Pion mass cut: $|m_{\gamma\gamma} - m_{\pi^0}^{PDG}| < 10\ \text{MeV}$
- **Event reconstruction**
  - Event timing between clusters and muon track
  - Missing mass calculation with $K^{\pm}_{\mu3}$ hypothesis

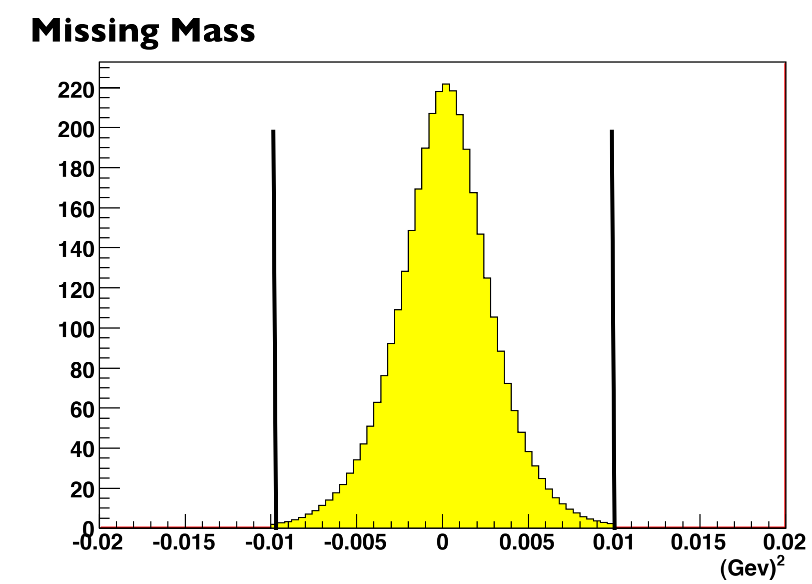
$$\mathrm{MM}^2_{\mathrm{K}_{\mu3}} = (\mathrm{P_K} - \mathrm{P}_\mu - \mathrm{P}_{\pi^0})^2$$

  - Missing mass cut: $|\mathrm{MM}_{\mathrm{K}_{\mu3}}|^2 < 10\ \mathrm{MeV}^2$

$3.4 \times 10^6 K^{\pm}_{\mu3}$ events selected



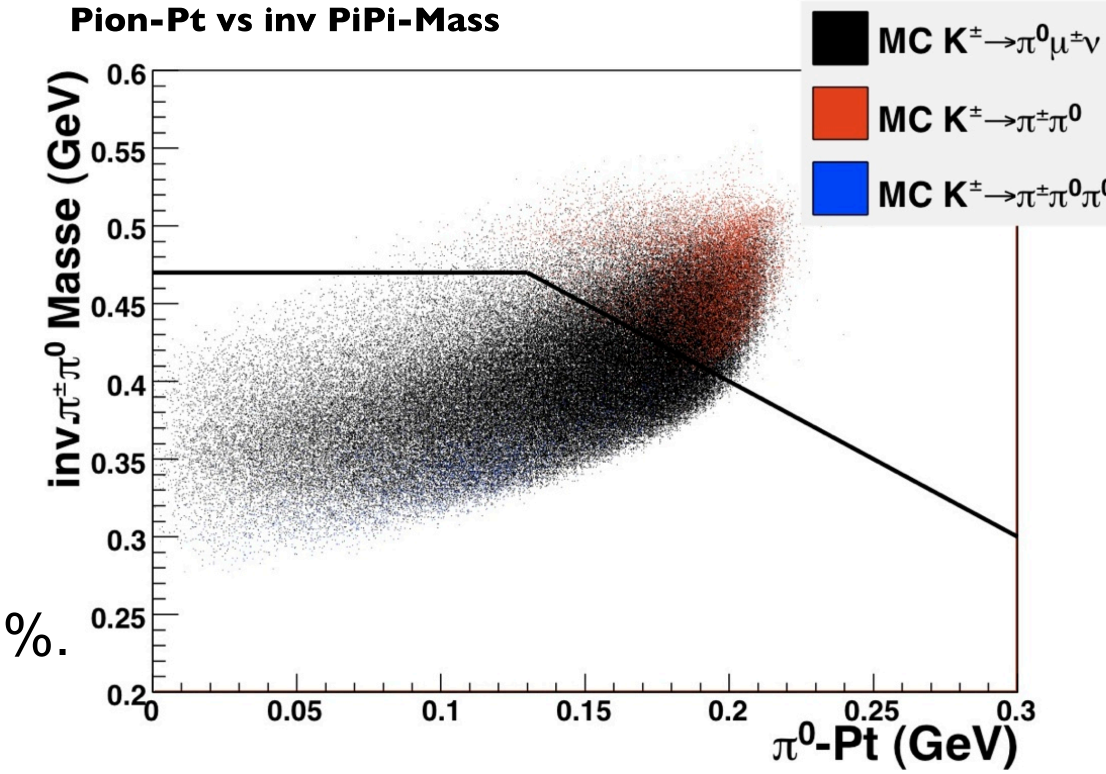




$K_{\mu3}^{\pm}$ Form Factor Analysis

$\pi^{\pm}\pi^{0}$ Background:

Decay	BR(%)	P $(\pi^{\pm}\pi^{0} \rightarrow K_{\mu3})$(%)
$K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$	20.66 ±0.08	19.8

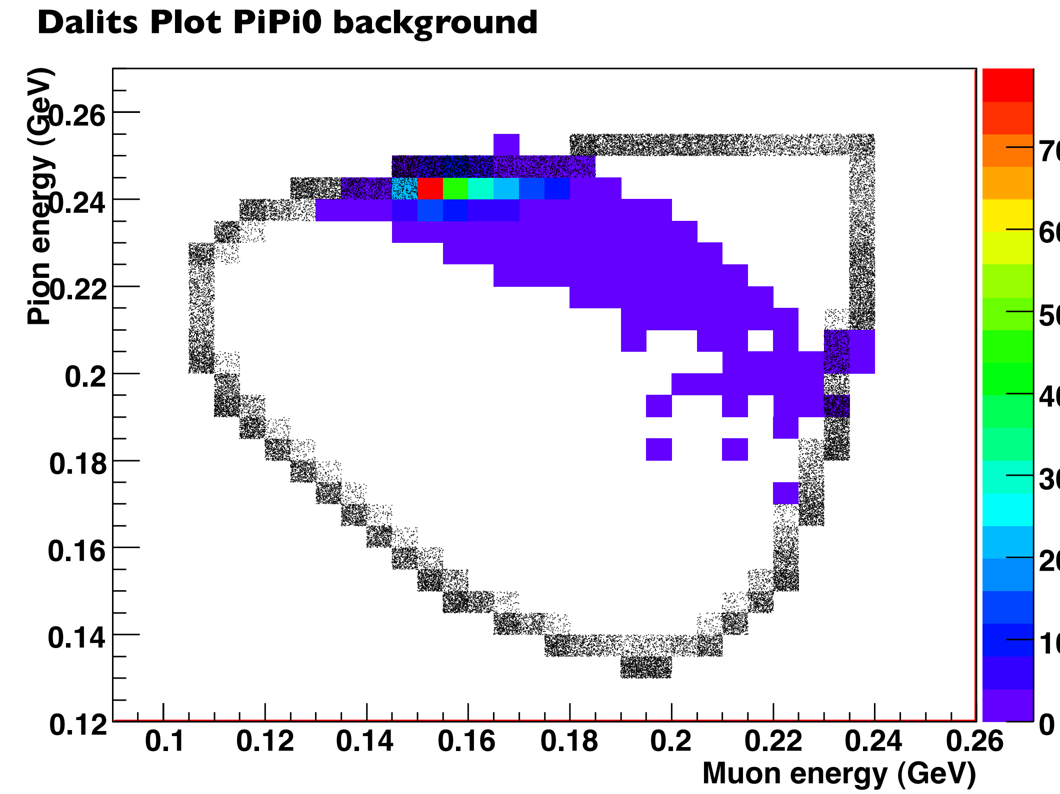


- $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ with $\pi \rightarrow \mu$ can fake $K_{\mu3}^{\pm}$.
- Without suppression, $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ bkg at the level of 20%.

- Cut in the **invariant** $\pi^{\pm}\pi^{0}$**- mass** and the **transverse momentum of the pion**:

  ➞ about 24% lost of $K_{\mu3}^{\pm}$ events.

  ➞ **Background contamination reduced to 0.6%.**

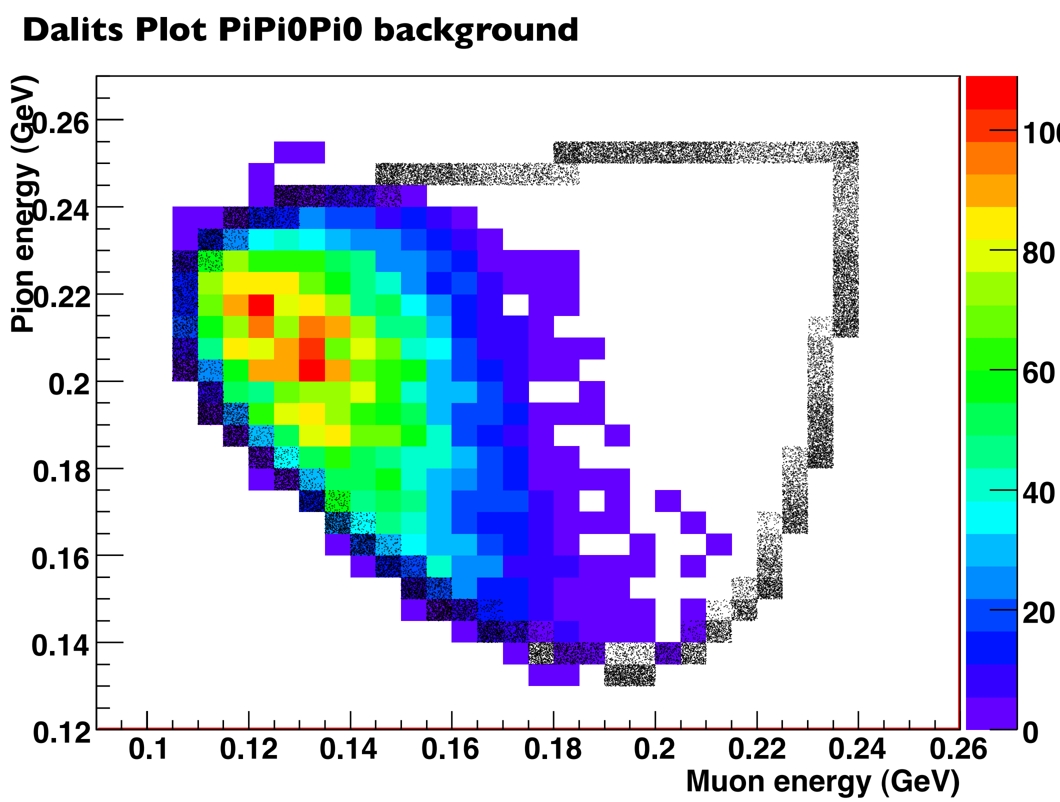


- **Background** is well **localised** in the Dalitz plot.

$K^{\pm}_{\mu 3}$ Form Factor Analysis

$\pi^{\pm}\pi^0\pi^0$ Background:

Decay	BR(%)	P $(\pi^{\pm}\pi^0\pi^0 \rightarrow K_{\mu 3})$(%)
$K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$	1.761 ± 0.022	0.14



- $\pi \longrightarrow \mu$ **decay** with lost **photons** from π^0-decays.
- **Small contamination** but introduce slope in the Dalitz plot.
- **No dedicated cut** to reduce the background is applied.
- In the fit a correction is applied to take the background into account.
- Without the correction the result shifts about $\simeq 0.5\ \sigma_{\text{stat}}$.

$K^{\pm}_{\mu3}$ Form Factor Analysis

Radiative effects:

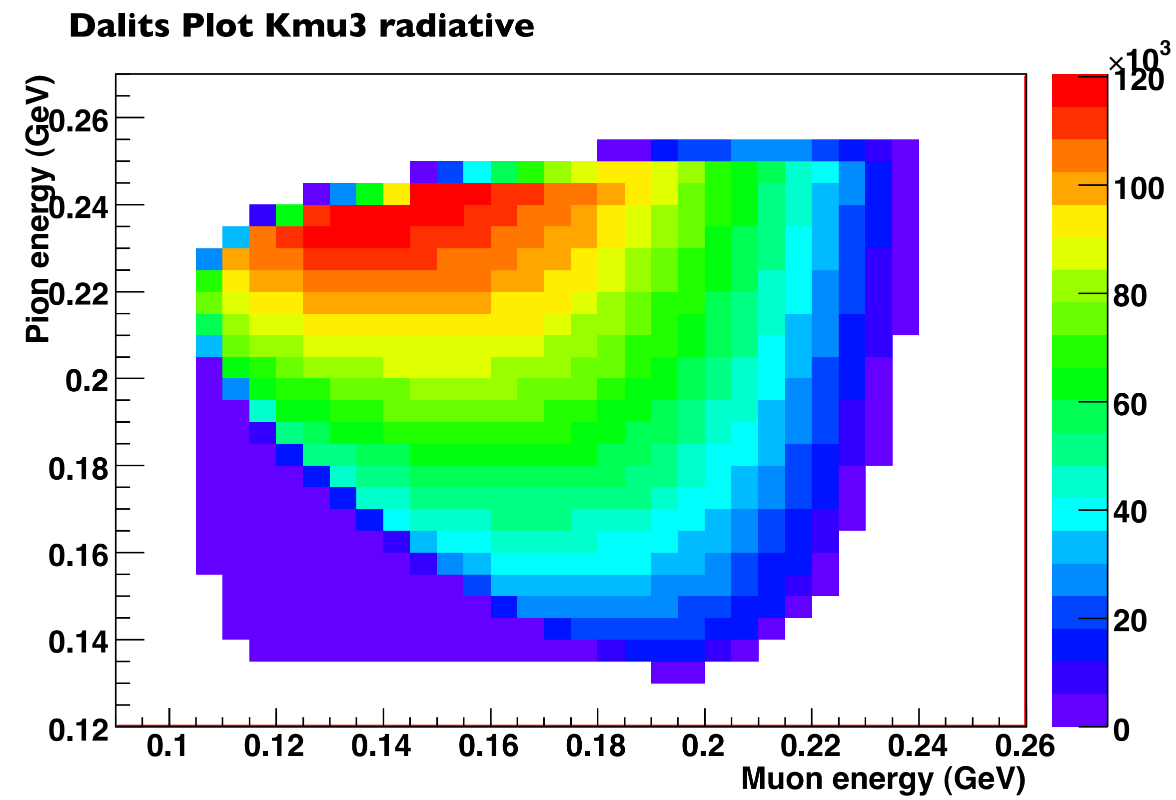
The K_{l3} decay rate including first order radiative corrections can be written as:

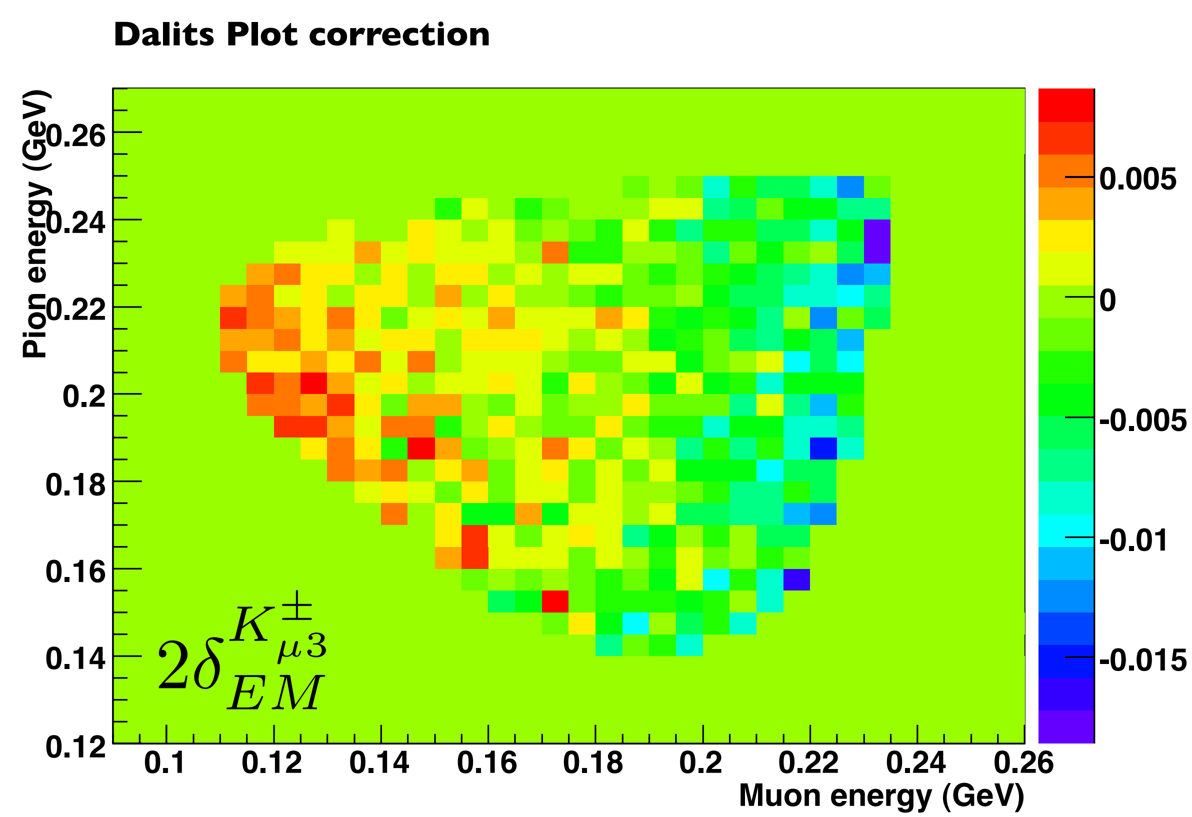
$$\Gamma_{K_{l3}} = \Gamma^0_{K_{l3}} + \Gamma^1_{K_{l3}} = \Gamma^0_{K_{l3}}(1 + 2\delta^{Kl}_{EM})$$

- Simulation with C. Gatti code provided by KLOE *(EPJ C45 (2006) 417)*
- For the normalisation on different decay modes used parameters are: *(JHEP 11 (2008) 006)*

Mode	$\delta^{Kl}_{EM}(\%)$
K^0_{e3}	0.495 ± 0.110
$K^{\pm}_{e3}$	0.050 ± 0.125
$K^0_{\mu3}$	0.700 ± 0.110
$K^{\pm}_{\mu3}$	0.008 ± 0.125

- For $K^{\pm}_{\mu3}$ small effects on the acceptance.
- percent effect on the Dalitz plot slope.





$K^{\pm}_{\mu3}$ Form Factor Analysis

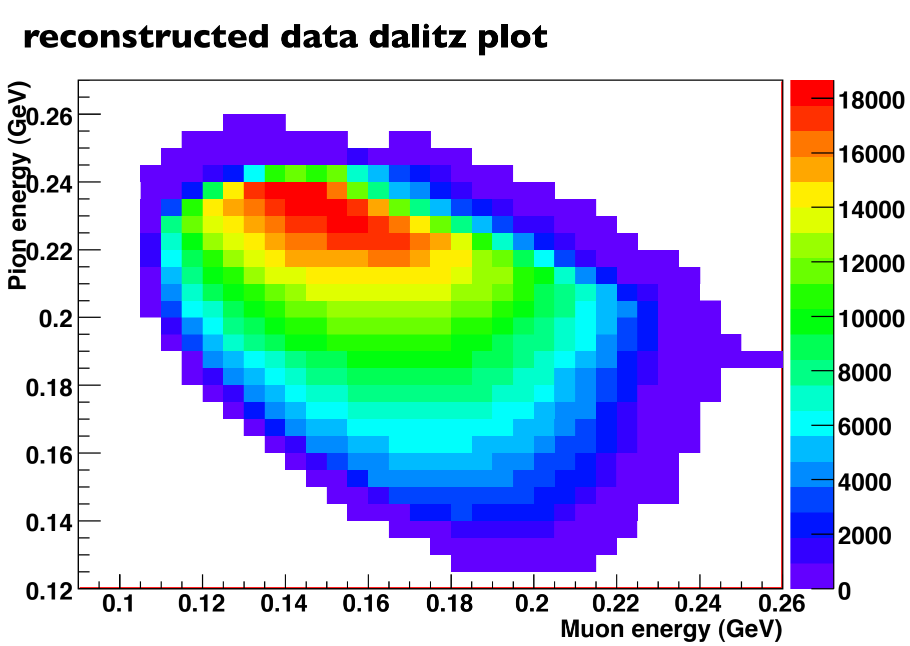
Fitting Procedure:

To extract the form factors a fit to the Dalitz Plot density is performed.

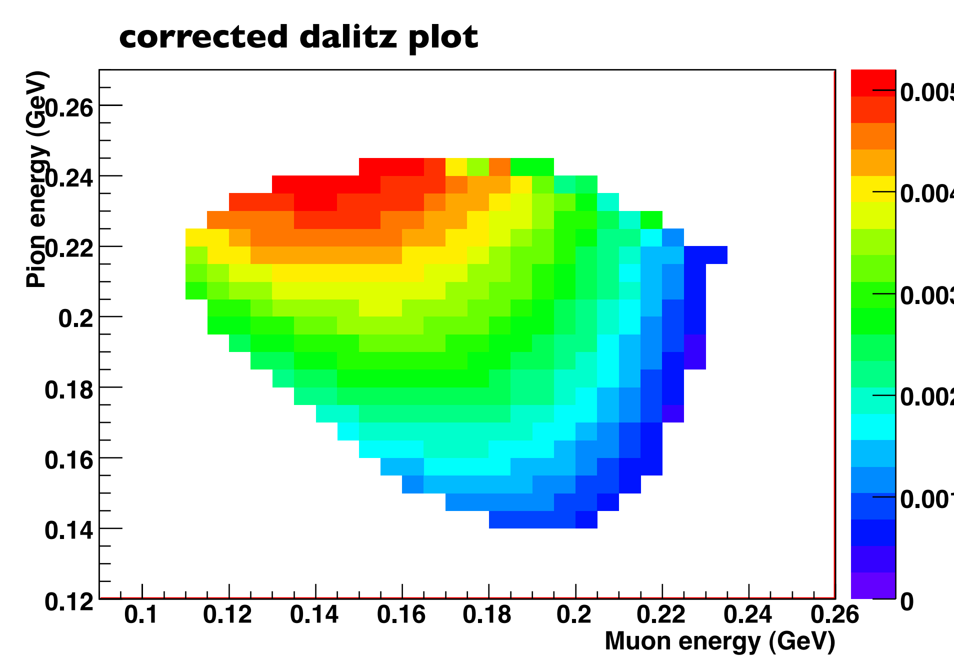
$$\rho(E^*_\mu, E^*_\pi) = \frac{d^2N(E^*_\mu, E^*_\pi)}{dE^*_\mu dE^*_\pi} \propto Af^2_+(t) + Bf_+(t)(f_0 - f_+)\frac{m^2_K - m^2_\pi}{t} + C\left[(f_0 - f_+)\frac{m^2_K - m^2_\pi}{t})\right]^2$$

- E^*_μ and E^*_π are the energy of the muon and the pion in the CMS of the kaon.
- *A*, *B* and *C* are kinematical terms.

- The fit is performed in cells of $5 \times 5\ \mathrm{MeV}^2$
- Cells who are outside or crossing the border of the physical region of the Dalitz Plot are not used in the fit.



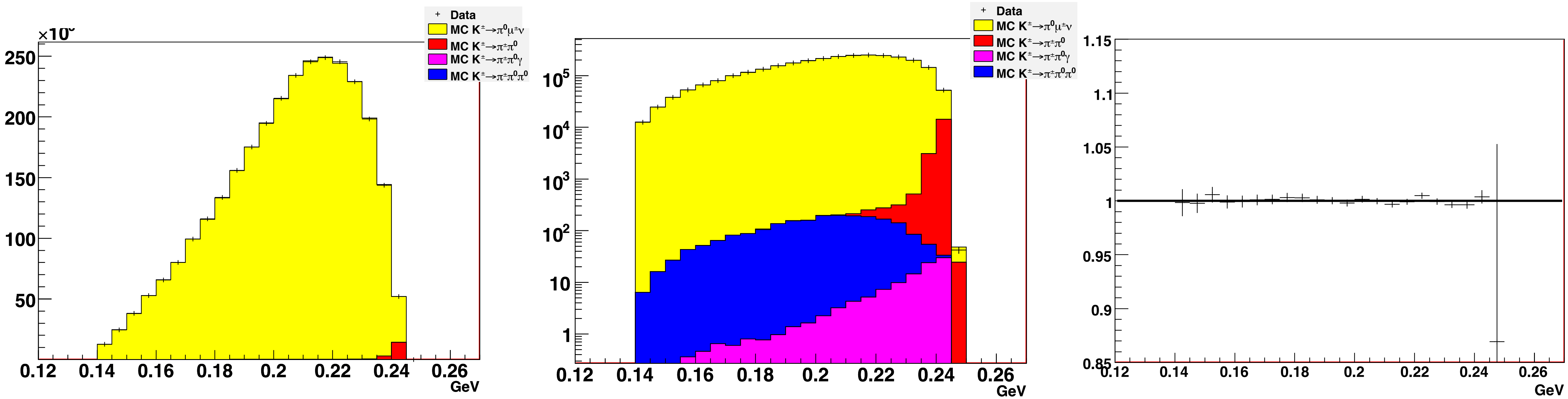
- **Applied corrections:**
  - Background subtraction.
  - Acceptance.
  - Radiative corrections.



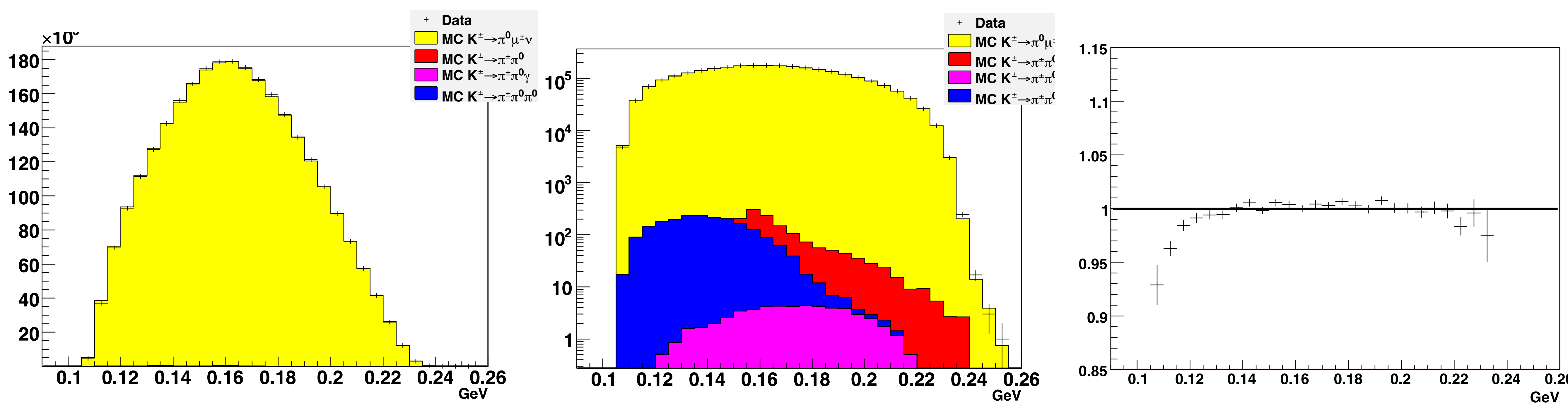
$K^{\pm}_{\mu 3}$ Form Factor Analysis

DATA-MC Comparison

- **Pion energy in the Kaon CMS:**



- **Muon energy in the Kaon CMS:**



$K^{\pm}_{\mu 3}$ Form Factor Analysis

Preliminary NA48/2 Result:

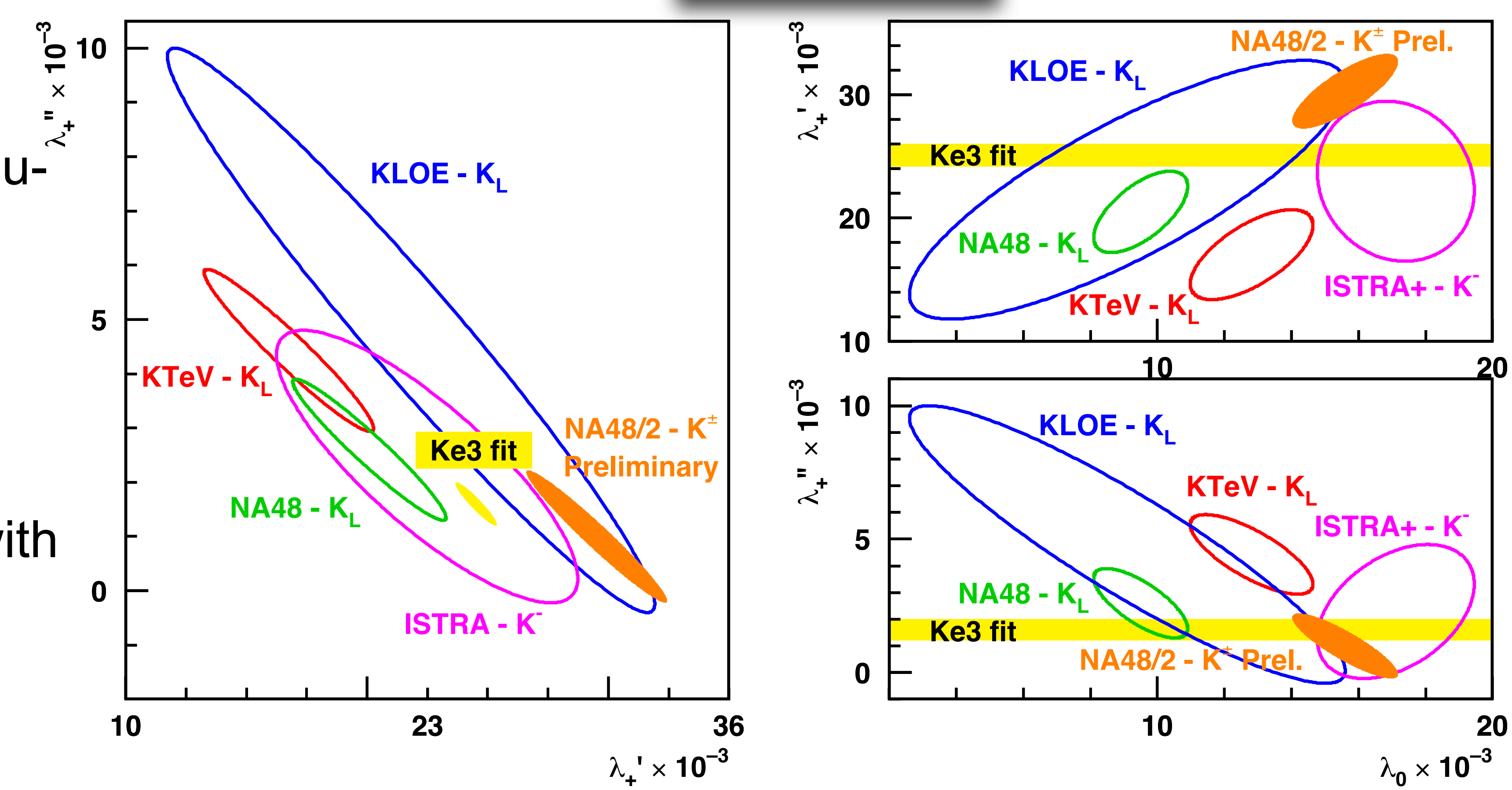
Quadratic ($\times 10^{-3}$)	λ'_{+}	λ''_{+}	λ_0
	$30.3 \pm 2.7 \pm 1.4$	$1.0 \pm 1.0 \pm 0.7$	$15.6 \pm 1.2 \pm 0.9$
Pole (MeV/c^2)	m_V		m_S
	$836 \pm 7 \pm 9$		$1210 \pm 25 \pm 10$
Dispersive ($\times 10^{-3}$)	Λ_{+}		lnC
	$28.5 \pm 0.6 \pm 0.7 \pm 0.5$		$188.8 \pm 7.1 \pm 3.7 \pm 5.0$

- First uncertainty is statistical, second is systematical.
- For the dispersive result the theoretical error is added *(v. Bernard et al. PRD80 (2009) 034034).*
- z-fit in progress....

1 σ contours

Experimental situation:

- $K^0_{\mu 3}$ results from KLOE, KTeV and NA48, $K^-_{\mu 3}$ from ISTRA+. First measurement which uses $K^+_{\mu 3}$.
- NA48/2 preliminary result with high precision - very competitive with the other results.
- Small quadratic term - Larger λ_0 with respect to NA48 $K^0_{\mu 3}$ result.



$K^{\pm}_{\mu3}$ Form Factor Analysis

Preliminary Systematics:

	$\Delta\lambda'_{+}$	$\Delta\lambda''_{+}$ ($\times10^{-3}$)	$\Delta\lambda_0$	Δm_V	Δm_S (MeV/c^2)	Λ_+	LnC ($\times10^{-3}$)
$K^{\pm}$ Energy	±0.7	±0.5	±0.6	±7	±2	±0.5	±2.6
Vertex	±1.0	±0.4	±0.6	±2	±4	±0.1	±1.1
Acceptance	±0.3	±0.1	±0.2	±2	±7	±0.1	±1.8
$\pi \to \mu$ scale	±0.4	±0.2	±0.2	±1	±1	±0.0	±0.0
2^{nd} analysis	±0.4	±0.1	±0.2	±6	±6	±0.5	±1.5
Total Systematic	±1.4	±0.7	±0.9	±10	±10	±0.7	±3.7
Statistical	±2.7	±1.0	±1.3	±7	±26	±0.6	±7.1
Theory						±0.5	±5.0
Total Uncertainties	±3.0	±1.2	±1.6	±12	±28	±1.0	±10.1

Total uncertainties mostly dominated by statistics.

Outlook: $K^{\pm}_{\mu 3}$ Form Factors at NA62

In the year 2007 NA62 collected data for a dedicated measurement of $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ and test of the future $K^+ \to \pi^+ \nu\bar{\nu}$ experiment.

- 4 month of data taking with a minimum bias trigger $Q1 \times E_{\mathrm{Lkr}} > 10\ \mathrm{GeV}$.
- Simultaneous K^+ and K^- beams with a beam momentum of $P_K = (74 \pm 1.6)\mathrm{GeV/c}$.
- Transverse momentum kick of the magnetic spectrometer was doubled
  ➔ Improvement in the track momentum resolution.
- Collected about 150000 K_{e2} events.
- First results on 40% of the present statistics presented at BEACH2010 and ICHEP2010.
- The expected precession for the full data sample is $\sigma(R_K)/R_K \simeq \pm 0.4\%$.

Form Factors from NA62 2007 data

- Huge statistics in $K^{\pm}_{\mu 3}$ and $K^{\pm}_{e3}$ of $\mathcal{O}(10^7)$ events.
- Special K_L run (15 h) to measure electron ID efficiency.
  ➔ $K^0_{\mu 3}$ and K^0_{e3} statistics of $\mathcal{O}(10^6)$ events.

NA48 analyses of K^0_{l3} and $K^{\pm}_{l3}$ can be repeated with different/larger data sets.

Summary and outlook

- **NA48/2** provides new results on the $K^{\pm}_{\mu 3}$ form factors in the quadratic, dispersive and Pole parametrization.
- For the **first time** a result is presented which studied K^{+} **and** K^{-} decays.
- High precision measurement which is very competitive with other results.
- A new result on $K^{\pm}_{e3}$ is in progress and will appear soon.
- **NA62 is ready to give its contribution with high statistics in K^{0}_{l3} and $K^{\pm}_{l3}$.**