## **Recent Results from BESIII**

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Recent Results from BESIII

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### Outline

- BESIII data
- Light Hadron Spectroscopy:
  - → Measurement of the matrix element for the decay  $\eta'(958) \rightarrow \eta \pi^+ \pi^-$
  - → Study of  $a_0(980) f_0(980)$  mixing
- ✤ Confirmation of X(1835) and observation of two new structures Charmonium decay:
  - → Evidence for  $\psi(2S)$  decays into  $\gamma\pi^0$  and  $\gamma\eta$
  - → Two-photon transition from  $\psi(2S)$  to  $J/\psi$
  - ✤ Study of  $\chi_{cJ}$  radiative decays into a vector meson
  - → Obervation of  $\chi_{cJ} \rightarrow VV (V = \omega, \phi)$

All results are preliminary!

- **→ → →** 

## World $J/\psi$ and $\psi(2S)$ Samples (×10<sup>6</sup>)



BESIII:  $J/\psi$  2009: ~226M, and  $\psi$ (2S) 2009: ~106M.

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Recent Results from BESIII

Measurement of the Matrix Element for the Decay  $\eta'(958) \rightarrow \eta \pi^+ \pi^-$ 

Motivation:

- Important for deeper insight into the dynamics of the process and the structure of the particles.
- Important for studies devoted to chiral theory, the effect of the gluon component, and the possible nonet of light scalars.
- → Important for the determination of a possible contribution from  $f_0(600)$  (or  $\sigma$ ) resonance (even though the  $a_0(980)$  is also present).
- → Precision measurements on  $\eta$  and  $\eta'(958)$  provide useful information in understanding low energy QCD.



 $\mathcal{B}(J/\psi 
ightarrow \gamma \eta') = (4.84 \pm 0.03(stat) \pm 0.25(sys)) imes 10^{-3}$ 

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#### Measurement of the Matrix Element:

- $X = \frac{\sqrt{3}}{Q}(T_{\pi^+} T_{\pi^-}),$   $Y = \frac{m_\eta + 2m_\pi}{m_\pi} \frac{T_\eta}{Q} 1,$  $T_{\pi,\eta}$  denote the kinetic energies of mesons in the  $\eta'(958)$  rest frame and  $Q = T_\eta + T_{\pi^+} + T_{\pi^-} = m_{\eta'(958)} - m_\eta - 2m_\pi.$
- general parametrization:  $M^2 = A(1 + aY + bY^2 + cX + dX^2)$
- linear parametrization: M<sup>2</sup> = A(|1 + αY|<sup>2</sup> + cX + dX<sup>2</sup>)
   α is a complex parameter. A non-zero value of α may represent the contribution of a gluon component in the wave function of the η'(958) in the decay.



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•  $\chi^2(N, a, b, c, d) = \sum_{i}^{n_{bin}} \frac{(D_i - NM_i)^2}{\sigma_i^2}$ 

 $M_i$  and  $D_i$  are the numbers of (weighted) entries in the *i*-th bin of the 2-d Dalitz plot for MC and data, respectively.

• general parametrization:  $M_i = \sum_{j=1}^{N_{ev}} (1 + aY_j + bY_j^2 + cX_j + dX_j^2)$ *j* is an index over the MC events, and  $X_j$  and  $Y_j$  are the true generated values of Dalitz variables. Similarly for the linear parametrization



Experimental distributions of the variable Y in various intervals of X with fitting function (histogram) for the general decomposition parametrization.

Table: The left four columns are for a,b,c and d. The right for  $Re(\alpha)$ ,  $Im(\alpha)$ , c and d.

VES <sup>1</sup>	Theory	This work	CLEO	VES <sup>2</sup>	This work
$-0.127 \pm 0.018$	$-0.116 \pm 0.011$	$-0.047 \pm 0.012$	$-0.021 \pm 0.025$	$-0.072 \pm 0.014$	$-0.033 \pm 0.006$
$-0.106 \pm 0.032$	$-0.042 \pm 0.034$	$-0.068 \pm 0.021$	0.000 (fixed)	$0.000 \pm 0.100$	$0.000 \pm 0.050$
$+0.015 \pm 0.018$	-	$+0.020 \pm 0.012$	0.000 (fixed)	$+0.020 \pm 0.019$	$+0.018 \pm 0.010$
$-0.082 \pm 0.019$	$+0.010 \pm 0.019$	$-0.073 \pm 0.013$	0.000 (fixed)	$-0.066 \pm 0.034$	$-0.058 \pm 0.013$

VES<sup>1</sup>: Phys. Lett. B 651, 22 (2007) Theory: Eur. Phys. J A 26, 383 (2005) CLEO: Phys. Rev. Lett. 84, 26 (2000) VES<sup>2</sup>: Phys. Atom. Nucl. 68, 372 (2005).

Some comments:

- The errors of our fitted parameter values are smaller than previous published results.
- In the general parametrization, the values of *a* and *b* are consistent with the results from GAMS-4 $\pi$  (PLB177,115), however the values of *c* and *d* are consistent with the results from VES<sup>1</sup>.
- A negative value of the coefficient *b* indicates that two kinds of parametrization are not equivalent. This conclusion is consistent with that from GAMS- $4\pi$ . VES<sup>1</sup> found the fit with linear parametrization yields unsatisfactory  $\chi^2/NDF = 170.5/114$  ratio.
- The quadratic term in X is unambiguously different from zero. Similarly for the quadratic term in Y. The dynamical nature of this term needs clarification.
- The value of the parameter *c* testing C parity violation in strong interaction is consistent with zero within  $2\sigma$  in both parametrizations.

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Recent Results from BESIII

## Study of $a_0(980) - f_0(980)$ mixing

Motivation:

- → There has been much argument whether the  $a_0^0(980)$  and the  $f_0(980)$  are part of the ground-state quark-antiquark family or whether they are 4-quark states, hybrids or  $K\overline{K}$  molecules.
- → The mixing between  $a_0^0(980)$  and  $f_0(980)$  is expected to shed light on the nature of these two resonances.
- → Two kinds of mixing intensities  $\xi_{af}$  and  $\xi_{fa}$  for the  $a_0^0(980) \rightarrow f_0(980)$  and  $f_0(980) \rightarrow a_0^0(980)$  transitions are expressed as:

$$\xi_{fa} = \frac{Br(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \eta \pi^0)}{Br(J/\psi \to \phi f_0 \to \phi \pi \pi)},$$

$$\xi_{at} = \frac{Br(\psi' \to \gamma \chi_{c1} \to \gamma \pi^0 a_0^0(980) \to \gamma \pi^0 f_0(980) \to \gamma \pi^0 \pi^+ \pi^-)}{Br(\psi' \to \gamma \chi_{c1} \to \gamma \pi^0 a_0^0 \to \gamma \pi^0 \pi^0 \eta)}$$

Measurement of  $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \eta \pi^0$  and  $\psi' \rightarrow \gamma \chi_{c1} \rightarrow \gamma \pi^0 a_0^0(980) \rightarrow \gamma \pi^0 f_0(980) \rightarrow \gamma \pi^0 \pi^+ \pi^-$ 



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#### Discussion:

The mixing intensity  $\xi_{fa}$  for the  $f_0(980) \rightarrow a_0^0(980)$  transition is calculated to be:

$$\xi_{ta} = \frac{Br(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \eta \pi^0)}{Br(J/\psi \to \phi f_0 \to \phi \pi \pi)}$$
$$= 0.6 \pm 0.2(stat.) \pm 0.2(sys.)\%$$

The mixing intensity  $\xi_{af}$  for the  $a_0^0(980) \rightarrow f_0(980)$  transition is calculated to be:

$$\xi_{at} = \frac{Br(\psi' \to \gamma \chi_{c1} \to \gamma \pi^0 a_0^0(980) \to \gamma \pi^0 f_0(980) \to \gamma \pi^0 \pi^+ \pi^-)}{Br(\psi' \to \gamma \chi_{c1} \to \gamma \pi^0 a_0^0 \to \gamma \pi^0 \pi^0 \eta)} = 0.3 \pm 0.2(stat.) \pm 0.1(sys.)\%$$



The mixing intensities and predictions with various theoretical predictions. The shaded region is our measurement with error bars and the red lines are our limits.

# Confirmation of X(1835) and observation of two new structures in $J/\psi \rightarrow \gamma \eta'(958)\pi^+\pi^-$

Motivation:

- ✤ Confirmation of X(1835) is necessary with high statistic data sample.
- $\rightarrow$  LQCD predicts the 0<sup>-+</sup> glueball mass is 2.3 GeV/ $c^2$ .
- → A 0<sup>-+</sup> glueball may have similar property as  $\eta_c$  (the main  $\eta_c$  decay mode is  $\eta'(958)\pi^+\pi^-$ ).



BESII results: signal significance is 7.7  $\sigma$   $M = 1833.7 \pm 6.1(stat) \pm 2.7(sys)$ MeV/ $c^2$   $\Gamma = 67.7 \pm 20.3(stat) \pm 7.7(sys)$ MeV/ $c^2$ Phys. Rev. Lett. 95, 262001 (2005)

## Mass spectrum of $\pi^+\pi^-\eta'$



> X(1835) and  $\eta_c$  is observed.

- Two additional structures at M~2.1GeV and 2.3GeV
- > There maybe some  $f_1(1510)$ .



#### Fitting the mass spectrum:

- Three background components:
  - (1) Contribution from non- $\eta'$  events estimated by  $\eta'$  mass sideband
  - 2 Contribution from  $J/\psi \to \pi^0 \pi^+ \pi^- \eta'(\eta' \to \gamma \rho)$  with re-weighting method
  - ③ Contribution from "PS background"  $f_{bkg}(x) = (x - m_0)^{1/2} + a_0 (x - m_0)^{5/2} + a_1 (x - m_0)^{5/2}, m_0 = 2m_{\pi} + m_{\pi'}$



, Red line: estimated contribution of (1+2)

Black line: total background

resonance	$M(\text{ MeV}/c^2)$	$\Gamma(\text{ MeV}/c^2)$	Stat. sig.
X(1835)	$1838.1\pm2.8$	$179.5 \pm 9.1$	$> 25\sigma$
X(2120)	$2124.8\pm5.6$	$101 \pm 14$	$> 7.2\sigma$
X(2370)	$2371.0\pm6.4$	$108 \pm 15$	$> 6.7\sigma$

Stat. sig. is conservatively estimated: fit range, background shape, contribution of extra resonances

- X(1835) resonance is confirmed at BESIII, but the width is significantly larger than that measured at BESII with one resonance in the fit.
- Two new resonances, X(2120) and X(2370), are observed.
- PWA is needed

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## Evidence for $\psi(2S)$ decays into $\gamma\pi^0$ and $\gamma\eta$

#### Motivation:

- → Important tests for various phenomenological mechanisms, such as vector meson dominance model, two-gluon couplings to  $q\bar{q}$  states, mixing of  $\eta_c \eta^{(\prime)}$ , and final-state radiation by light quarks.
- → The ratio of  $R_{J/\psi} = \mathcal{B}(J/\psi \rightarrow \gamma \eta)/\mathcal{B}(J/\psi \rightarrow \gamma \eta')$  can be predicted by the first order of perturbation theory, and  $R_{J/\psi} = R_{\psi(2S)}$  is expected (CLEO:  $R_{\psi(2S)} < 1.8\%$  at 90% C.L. and  $R_{J/\psi} = (21.1 \pm 09)\%$ . PRD79,111101 (2009))
- → The decay  $\psi(2S) \rightarrow \gamma \pi^0$  is suppressed because the photon can only be from final state radiation off one of the quarks.
- →  $\mathcal{B}(\psi(2S) \to \gamma \pi^0) = 2.19 \times 10^{-7}$ : calculated in PRD79,097301. CLEO: < 5.0 × 10<sup>-6</sup> at 90% C.L. (PRD79,111101).
- →  $e^+e^- \rightarrow \psi(2S)/\gamma^* \rightarrow \gamma \pi^0$  will be very useful in testing the form factor for timelike photons  $Q^2 = -q^2 < 0$  (PRD79,097301).

#### **Results:**



(a) 
$$\gamma \pi^{0}$$
; (b)  $\gamma \eta (\pi^{+}\pi^{-}\pi^{0})$ ; (c)  $\gamma \eta (3\pi^{0})$ ;  
(d)  $\gamma \eta' (958) [\pi^{0} \pi^{0} \eta (\gamma \gamma)]$ ; and (e)  
 $\gamma \eta' (958) (\gamma \pi^{+} \pi^{-})$ .

Signal significances are 4.1  $\sigma$  for  $\psi(2S) \rightarrow \gamma \pi^0$ , 3.2  $\sigma$  for  $\psi(2S) \rightarrow \gamma \eta$ 

 $R_{\psi(2S)} = (1.10 \pm 0.38 \pm 0.07)\%$ : it is the first measurement and it is much smaller than  $R_{J/\psi} = (21.1 \pm 0.9)\%$ .

laple: Branching fractions (10 <sup>-0</sup> ).				
Mode	BESIII	Combined BESIII	PDG	
$\psi(2S) \rightarrow \gamma \pi^0$	$1.58 \pm 0.40 \pm 0.13$	$1.58 \pm 0.40 \pm 0.13$	$\leq 5$	
$\psi(2S) \rightarrow \gamma \eta (\pi^+ \pi^- \pi^0)$	$1.78 \pm 0.72 \pm 0.17$			
$\rightarrow \gamma \eta (\pi^0 \pi^0 \pi^0)$	$1.07 \pm 0.65 \pm 0.08$	$1.38 \pm 0.48 \pm 0.09$	$\leq 2$	
$\psi(2S) \rightarrow \gamma \eta'(958)(\pi^+\pi^-\eta)$	120 $\pm$ 5 $\pm$ 8			
$\rightarrow \gamma \eta'$ (958)( $\pi^+\pi^-\gamma$ )	$129\pm3\pm8$	$126\pm3\pm8$	121 ± 8	

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## Two-photon transition from $\psi(2S)$ to $J/\psi$

Motivation:

On experimental side:

- not seen previously in  $\psi(2S)$  decays
- analogous process to positronium and hydrogen two-photon transition
- CLEO reported  $\Upsilon(3S) \rightarrow \gamma \gamma \Upsilon(2S)$  (Phys. Rev. D 49, 40 (1994))

On theoretical side:

- order  $\alpha^2$  QED transition between two hadrons
- Similar process has been studied in heavy-light quark system
- improve understanding of heavy quarkonium such as spectrum, decay et al, and the strong interaction
- possibility of testing the hadron-loop effect

## **Signal Estimation**

unbinned maximum likelihood fit with composition of three PDFs:

- signal (red): shape from phase-space-like MC simulation
- ψ(2S) bkg.(blue): shape and magnitude from exclusive MC simulation

• other bkg.(green): 1st-order polynominal



Combined with  $e^+e^-$  and  $\mu^+\mu^-$  modes, the branching fraction is measured to be  $\mathcal{B}(\psi(2S) \rightarrow \gamma\gamma J/\psi) = (1.02 \pm 0.05(\text{stat})^{+0.18}_{-0.20}(\text{sys})) \times 10^{-3}$ .

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## Study of $\chi_{cJ}$ radiative decays into a vector meson

#### Motivation:

- →  $\psi \rightarrow \gamma X \rightarrow \gamma \gamma V$  ( $\rho^0$ ,  $\omega$ ,  $\phi$ ) provide a favorable place to extract information on the flavor content of the *C*-even resonance *X* to study gluon hadronization dynamics.
- → By including hadronic loop contributions, a recent pQCD calculation (arXiv:1005.0066) obtains results in agreement with the experimental measurements of  $\mathcal{B}(\chi_{c1} \rightarrow \gamma V)$ .

Table: Theoretical predictions(in units of  $10^{-6}$ ) and results from the CLEO.

Mode	CLEO <sup>1</sup>	pQCD <sup>2</sup>	QCD <sup>3</sup>	QCD+QED <sup>3</sup>
$\chi_{c0} \rightarrow \gamma \rho^0$	< 9.6	1.2	3.2	2.0
$\chi_{c1} \rightarrow \gamma \rho^0$	$243\pm19\pm22$	14	41	42
$\chi_{c2} \rightarrow \gamma \rho^0$	< 50	4.4	13	38
$\chi_{c0} \rightarrow \gamma \omega$	< 8.8	0.13	0.35	0.22
$\chi_{c1} \rightarrow \gamma \omega$	$83\pm15\pm12$	1.6	4.6	4.7
$\chi_{c2} \rightarrow \gamma \omega$	< 7.0	0.5	1.5	4.2
$\chi_{c0} \rightarrow \gamma \phi$	< 6.4	0.46	1.3	0.03
$\chi_{c1} \rightarrow \gamma \phi$	< 26	3.6	11	11
$\chi_{c2} \rightarrow \gamma \phi$	< 13	1.1	3.3	6.5

1. PRL 101,151801 (2008). 2. Chin. Phys. Lett. 23, 2376 (2006). 3. hep-ph/0701009



Invariant mass distributions of (a)  $\gamma\phi$ , (b)  $\gamma\rho^0$ , and (c)  $\gamma\omega$ . Dots with error bars are data; histograms are the best fit; dashed lines are signal shapes; and the shaded histograms are vector meson sideband background plus a 2nd order polynomial background.

Decay	Number of	Efficiency	Systematic	Branching	Statistical
mode	events	(%)	error (%)	fraction ( $\times 10^{-6}$ )	significance
$\chi_{c0} \rightarrow \gamma \phi$	$15.0 \pm 6.6$	32.4	8.1	$9.5\pm4.2\pm0.8$	$2.9\sigma$
$\chi_{c1} \rightarrow \gamma \phi$	$42.6\pm8.6$	34.6	7.8	$25.8 \pm 5.2 \pm 2.0$	$6.4\sigma$
$\chi_{c2} \rightarrow \gamma \phi$	$4.6 \pm 4.9$	32.6	8.8	<8.0	
$\chi_{c0} \rightarrow \gamma \rho^0$	$6\pm12$	22.6	7.4	<10.2	
$\chi_{c1} \rightarrow \gamma \rho^0$	$432\pm25$	19.4	7.2	$228\pm13\pm16$	$\gg$ 10 $\sigma$
$\chi_{c2} \rightarrow \gamma \rho^0$	$13 \pm 11$	15.7	7.9	<20.4	
$\chi_{c0} \rightarrow \gamma \omega$	$5\pm11$	18.6	8.3	<12.7	
$\chi_{c1} \rightarrow \gamma \omega$	136 $\pm$ 14	22.7	8.0	$69.7 \pm 7.2 \pm 5.6$	$\gg 10\sigma$
$\chi_{\rm C2} \to \gamma \omega$	$1\pm 6$	19.2	8.9	<6.0	

able: Results of $\chi_{cJ} \rightarrow \gamma V$ .	The upper limits are	at the 90% C.L.
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The longitudinal (transverse) polarization exhibits a  $\cos^2 \Theta$  ( $\sin^2 \Theta$ ) dependence, and the angular distribution is expressed as:

$$rac{dN}{d\cos heta} \propto \left|A_L
ight|^2 \cos^2 \Theta + rac{1}{2} \left|A_T
ight|^2 \sin^2 \Theta \; ,$$

where  $A_L$  and  $A_T$  are the longitudinal and transverse polarization amplitudes, and  $\Theta$  is defined as the angle between the vector meson flight direction in the  $\chi_{cJ}$  rest frame and either the  $\pi^+/K^+$  direction in the  $\rho^0/\phi$  rest frame or the normal to the  $\omega$  decay plane in the  $\omega$  rest frame.



**Results:** The transverse component fraction:  $f_T = |A_T|^2/(|A_T|^2 + |A_L|^2) = N_T/(N_T + R * N_L)$ , where  $R = \varepsilon_T/\varepsilon_L$  $f_T$  are  $0.29^{+0.13+0.10}_{-0.12-0.09}$  for  $\chi_{c1} \rightarrow \gamma \phi$ ,  $0.158 \pm 0.034^{+0.015}_{-0.014}$  for  $\chi_{c1} \rightarrow \gamma \rho^0$ , and  $0.247^{+0.090+0.044}_{-0.087-0.026}$  for  $\chi_{c1} \rightarrow \gamma \omega$ .

### Observation of $\chi_{cJ} \rightarrow \omega \omega, \phi \phi$ and $\omega \phi$

#### Motivation:

#### Important laboratory to test QCD:

- **Previous measurements from BESII.**
- $\hfill\square$  Only  $\chi_{c0}$  and  $\chi_{c2}$  decays into  $\phi\phi$  and  $\omega\omega$  are observed.

BR(10 <sup>-3</sup> )	Xco	Xc2	
$\rightarrow \phi \phi$	$0.94{\pm}0.21{\pm}0.13$	$1.70 \pm 0.30 \pm 0.25$	BESII, PLB 642, 197 (2006)
$\rightarrow \omega \omega$	$2.29 \pm 0.58 \pm 0.41$	1.77±0.47±0.36	BESII, PLB 630, 7 (2005)

 $\Box \chi_{c1} \rightarrow V V$  is suppressed due to helicity selection rule in pQCD

$$\operatorname{Br}[\chi_{c1} \to V(\lambda_1)V(\lambda_2)] \sim \left(\frac{\Lambda_{QCD}^2}{m_c^2}\right)^{|\lambda_1+\lambda_2|+2} \qquad \operatorname{Nucl. Phys. B201,492}$$

P-parity conservation requires the two vectors

$$\begin{array}{c} T \downarrow & \longleftarrow L \\ \downarrow & \downarrow \\ V(\lambda_1) & \chi_{c1} & V(\lambda_2) \end{array}$$

having different polarization, so it is suppressed.

 $\Box \chi_{cl} \rightarrow \omega \phi$  is doubly OZI suppressed.

So  $\lambda_1 + \lambda_2 \neq 0$ 

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First observation of  $\omega \phi$  which is a doubly OZI suppressed decay, long distance contribution may be important in charmonium decays.

# Other recent results from BESIII that have not been included here.

Solution Analysis of  $J/\psi \rightarrow \omega \eta \pi^+ \pi^-$ : A structure denoted as X(1870) is seen in  $\eta \pi^+ \pi^-$  mass spectrum. For details, see Yanping HUANG's report at ICHEP10:

http://indico.cern.ch/contributionDisplay.py?contribId=1210&sessionId=46&confId=73513

Solution Analysis of  $\chi_{cJ}$  → 4π<sup>0</sup>: it is the first measurement. For details, see Ronggang PING's report at ICHEP10.

http://indico.cern.ch/contributionDisplay.py?contribId=1233&sessionId=50&confId=73513

## Summary

Some preliminary results from BESIII have been shown here.

- Interpretent plot of  $\eta'(958) \rightarrow \eta \pi^+ \pi^-$  decay is studied in a generalized and a linear representation.
- $\checkmark$  We perform direct measurements of  $a_0^0(980) f_0(980)$  mixing.
- ∠  $X(1835) \rightarrow \eta'(958)\pi^+\pi^-$  is confirmed and two new resonances, X(2120) and X(2370), are observed.
- ∠  $\psi(2S) \rightarrow \gamma \pi^0$  and  $\psi(2S) \rightarrow \gamma \eta$  are observed for the first time with signal significance of 4.1*σ* and 3.2*σ*, respectively.
- A significant enhancement of two-photon transition of ψ (2S) to J/ ψ was observed for the first time
- In the decays  $\chi_{cJ} \rightarrow \gamma V$  ( $V = \phi, \rho^0, \omega$ ) are studied. The fractions of the longitudinal polarization component of V in  $\chi_{c1} \rightarrow \gamma V$  are measured.
- ∠  $\chi_{cJ}$  signals are observed in the decays  $\chi_{cJ} → ωω, φφ$  and ωφ.

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The corresponding projections on variables *X* and *Y* in (b) and (c), respectively, where the dashed histograms are from MC signal sample with  $\eta'(958) \rightarrow \eta \pi^+ \pi^-$  events produced with phase space and the blank histograms are the fitted results described in the text.