

**European Research Council**

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**MARIA UBIALI**  
UNIVERSITY OF CAMBRIDGE

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# LHC RUN III AND THE PRECISION FRONTIER

# NEW PHENO CHALLENGES AT THE LHC PRECISION FRONTIER

▶ **Lecture I:**

"LHC Run III and the precision frontier"

▶ **Lecture II:**

"New frontiers in the determination of the proton structure"

▶ **Lecture III:**

"Hints for new physics from precision physics"

**AIM: GIVE A PERSONAL PERSPECTIVE ABOUT WHAT I IDENTIFY AS MOST EXCITING CHALLENGES THAT MODERN COLLIDER PHENOMENOLOGY FACES.**

**DISCLAIMER: IMPOSSIBLE TO GO INTO ALL DETAILS THAT TOPICS DESERVE NOR TO COVER ALL RELEVANT TOPICS.**

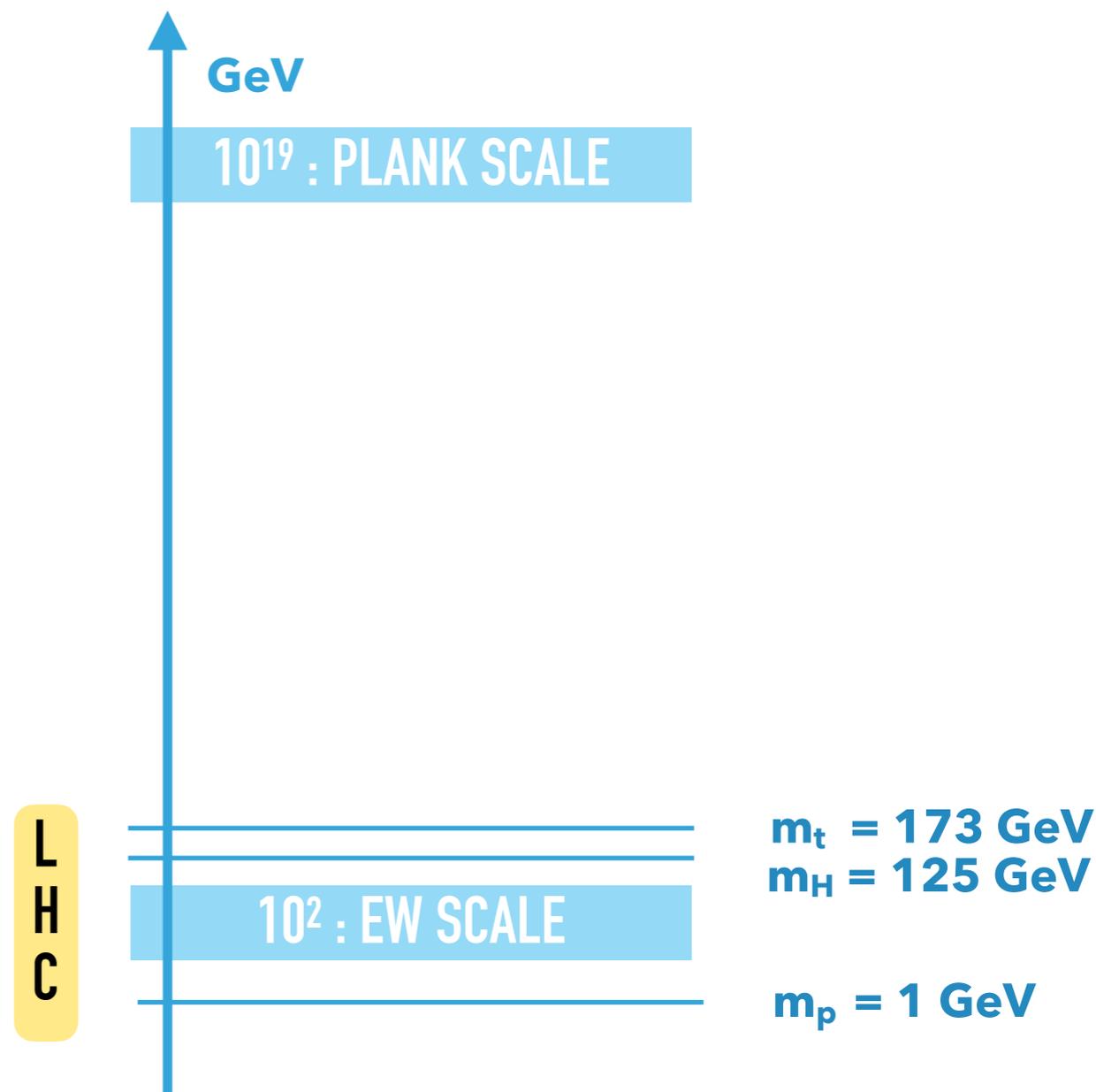
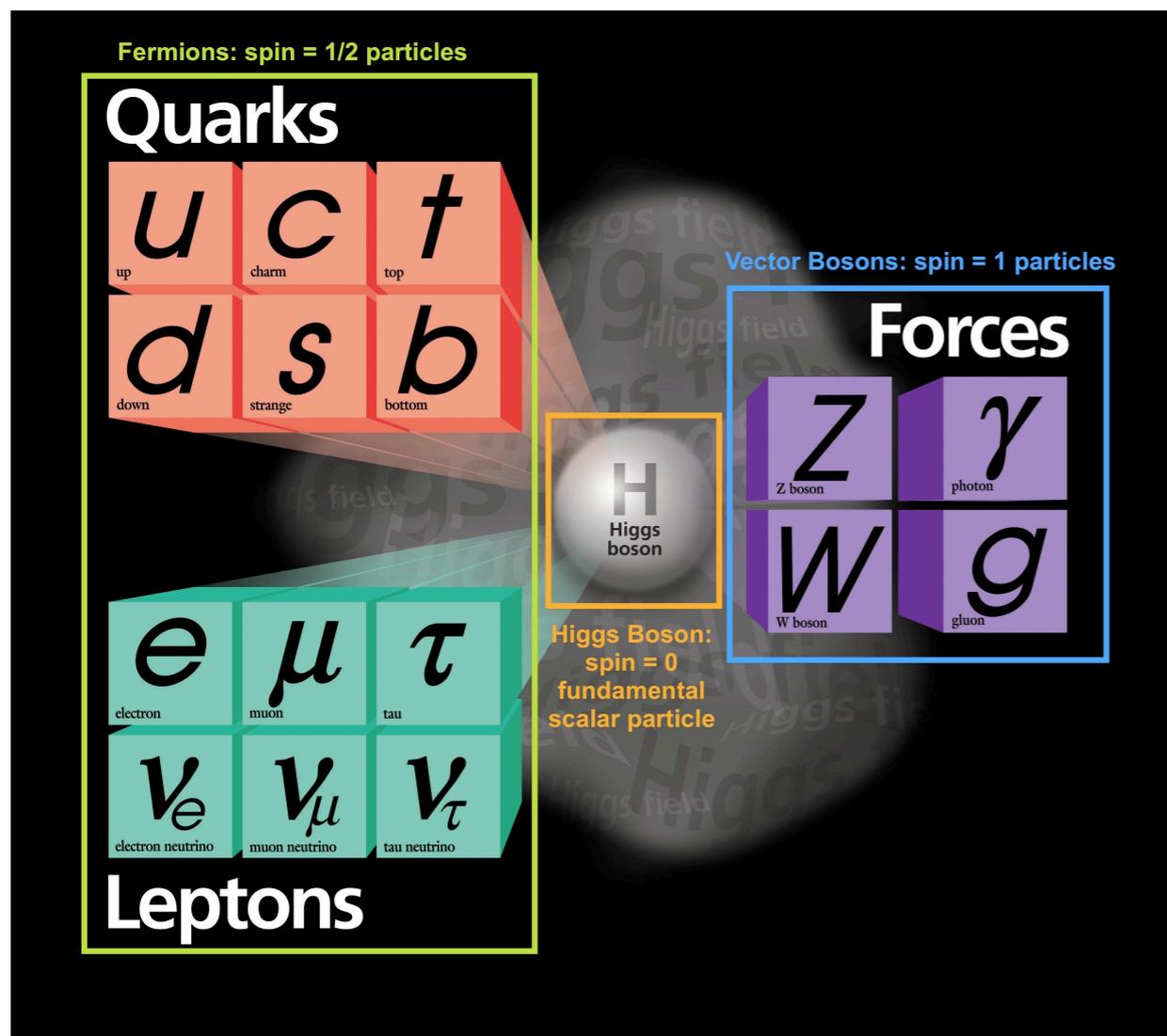
# LHC RUN III AND THE PRECISION FRONTIER

- I. Introduction
  - I. Basics of collider phenomenology
  - II. Basics of QCD
- II. The precision ingredients at the LHC
  - I. The strong coupling constant
  - II. QCD fixed order computations
  - III. Beyond fixed order QCD



# INTRODUCTION

# EXPLORING THE HIGH ENERGY FRONTIER AT THE LHC...



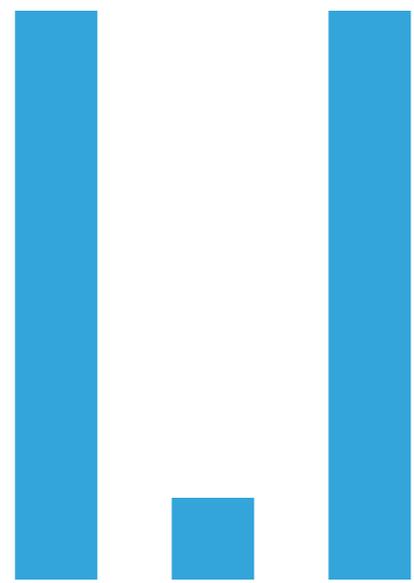
- ▶ Huge gap between EW scale and Planck scale and many outstanding unanswered questions (neutrino masses, dark matter...)
- ▶ The discovery of the Higgs at the LHC opens a new era of exploration of the high energy frontier at particle colliders

## ...VIA PRECISION PHYSICS

*"Precision* is the keystone to *consolidate our description of nature*, increase the sensitivity to SM deviations, give credibility to discovery claims[...].

The LHC has also proven to be a ***discovery machine***, and in a context broader than just Higgs and BSM phenomena. Altogether, it delivered results that could not have been obtained otherwise, immensely enriching our understanding of nature. "

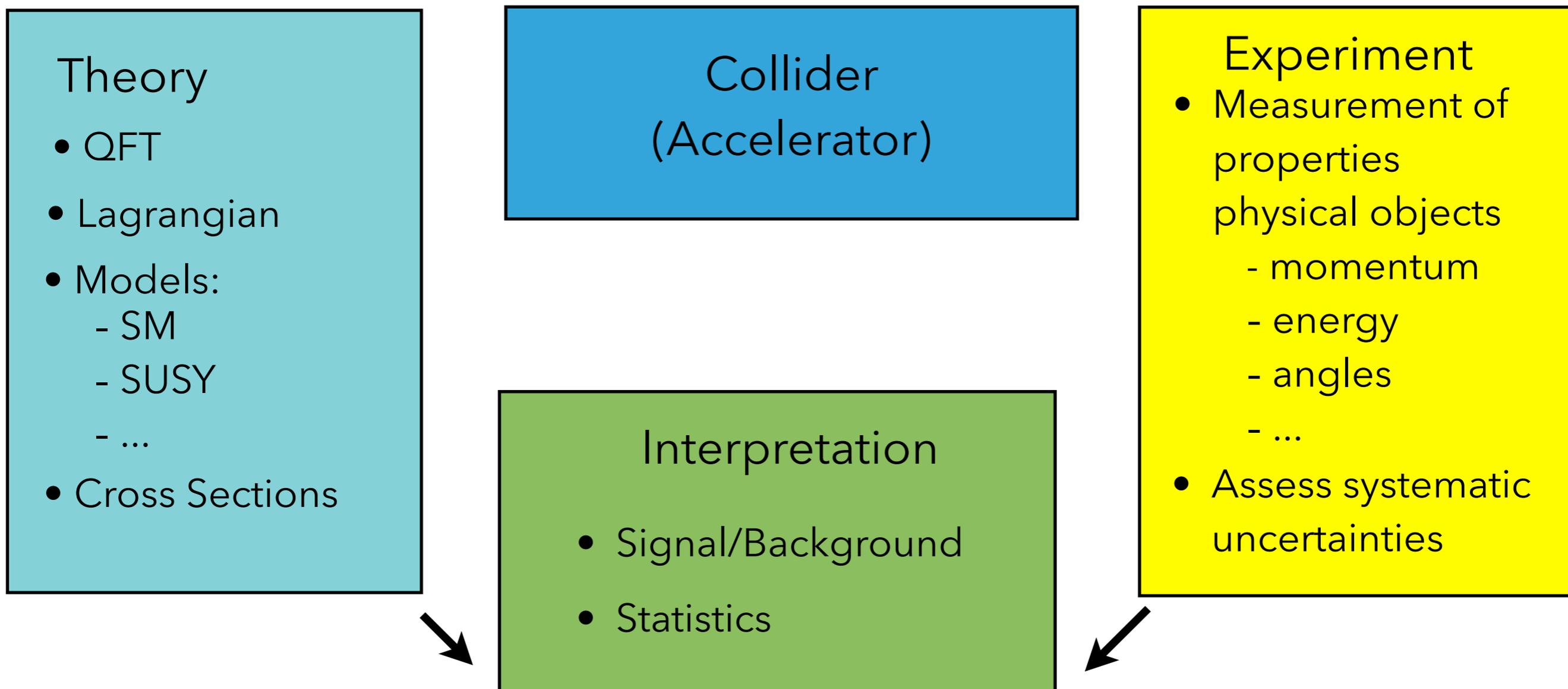
M. Mangano, "The LHC at 10: the physics legacy", CERN courier



# BASICS OF COLLIDER PHENOMENOLOGY

# COLLIDER PHYSICS

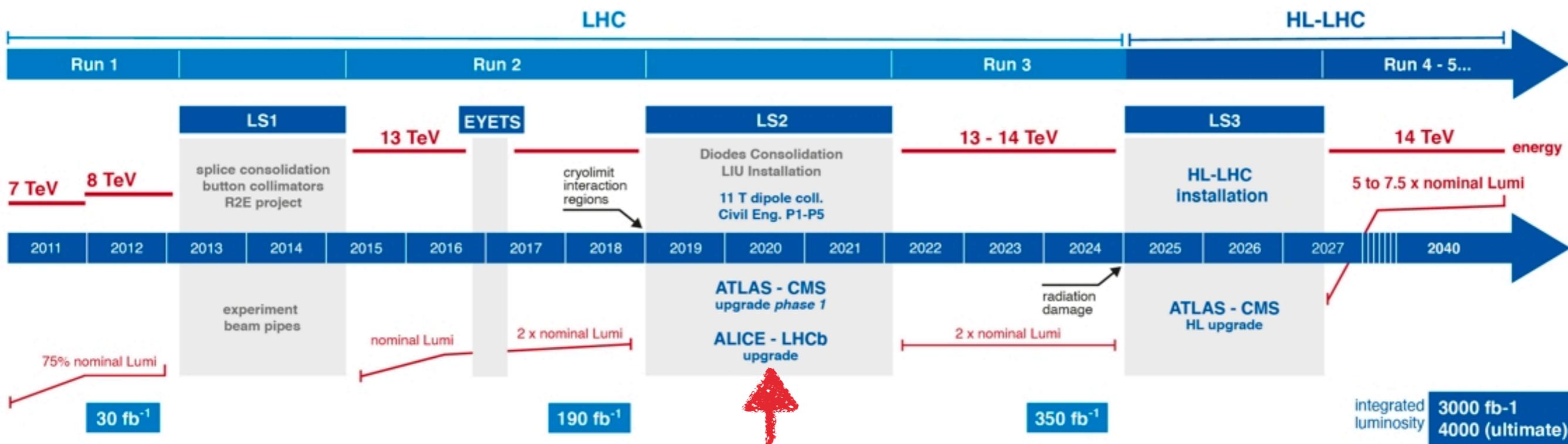
The purpose of collider phenomenology is to test theoretical predictions experimentally in a controllable environment



# THE LHC SCHEDULE



## LHC / HL-LHC Plan



### HL-LHC TECHNICAL EQUIPMENT:



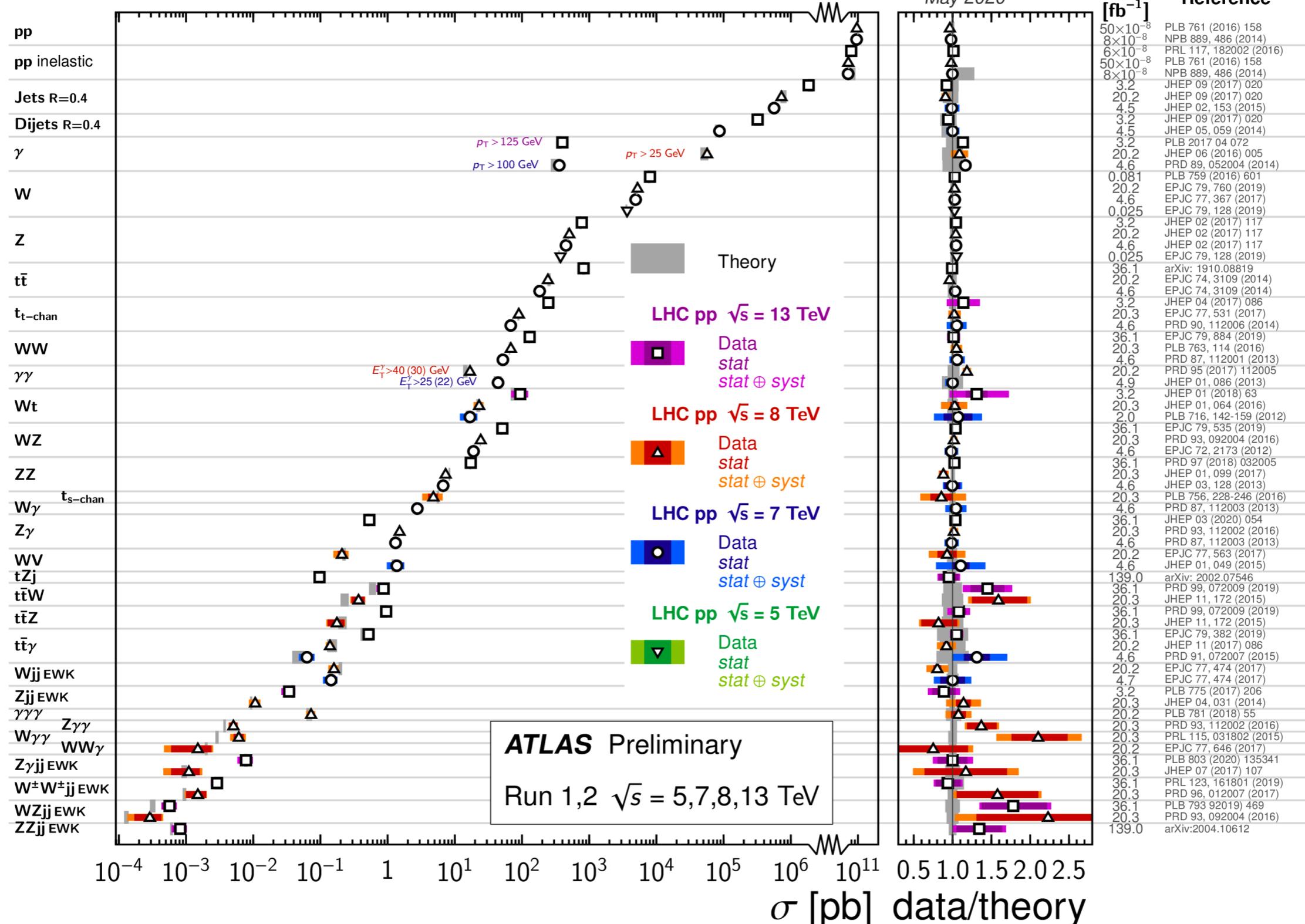
### HL-LHC CIVIL ENGINEERING:



**20+ YEARS OF CUTTING-EDGE PHENOMENOLOGY AHEAD**

# THE PRECISION FRONTIER

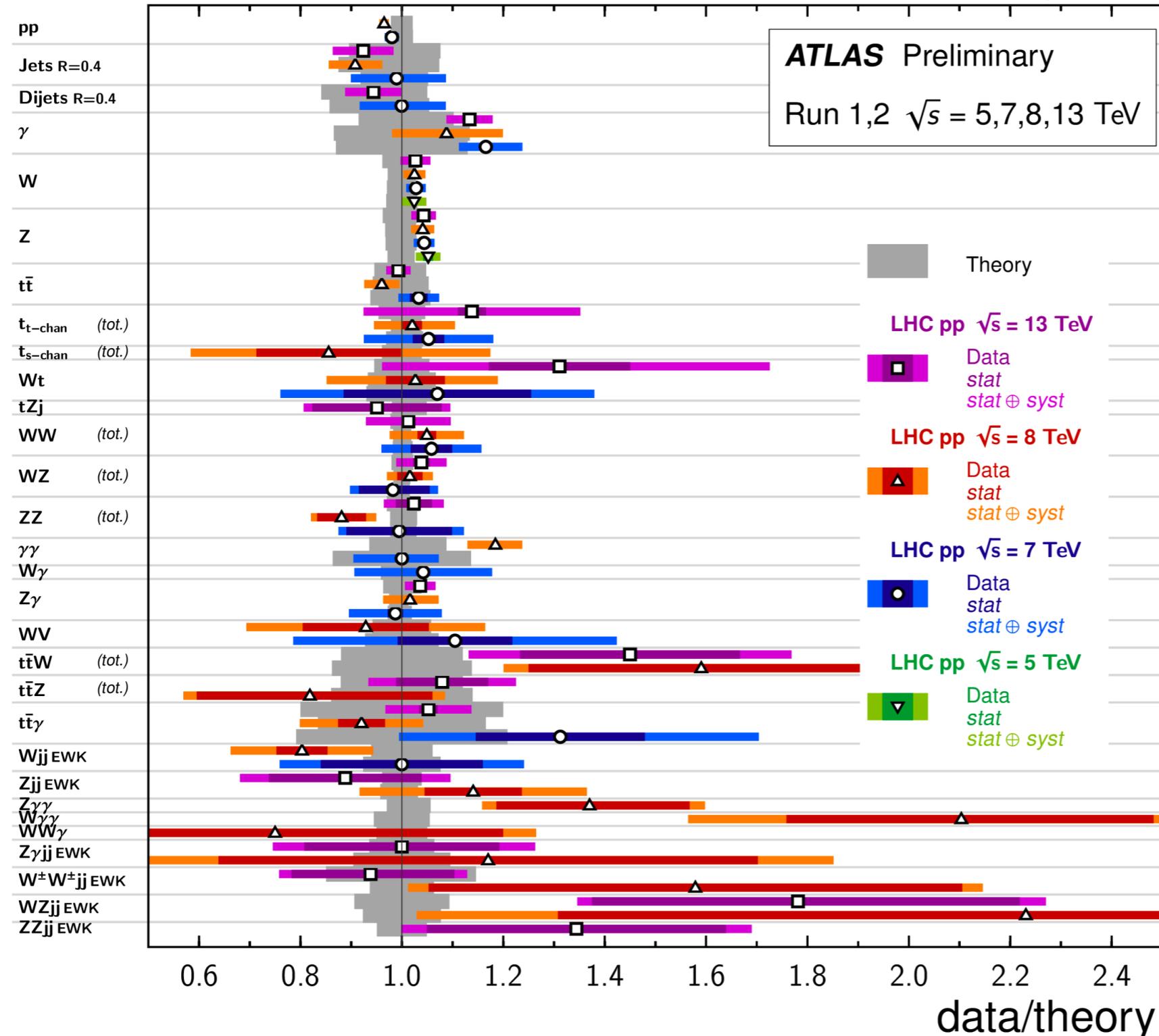
## Standard Model Production Cross Section Measurements



# THE PRECISION FRONTIER

LHC = QCD +  $\epsilon$

**Standard Model Production Cross Section Measurements** *Status: May 2020*



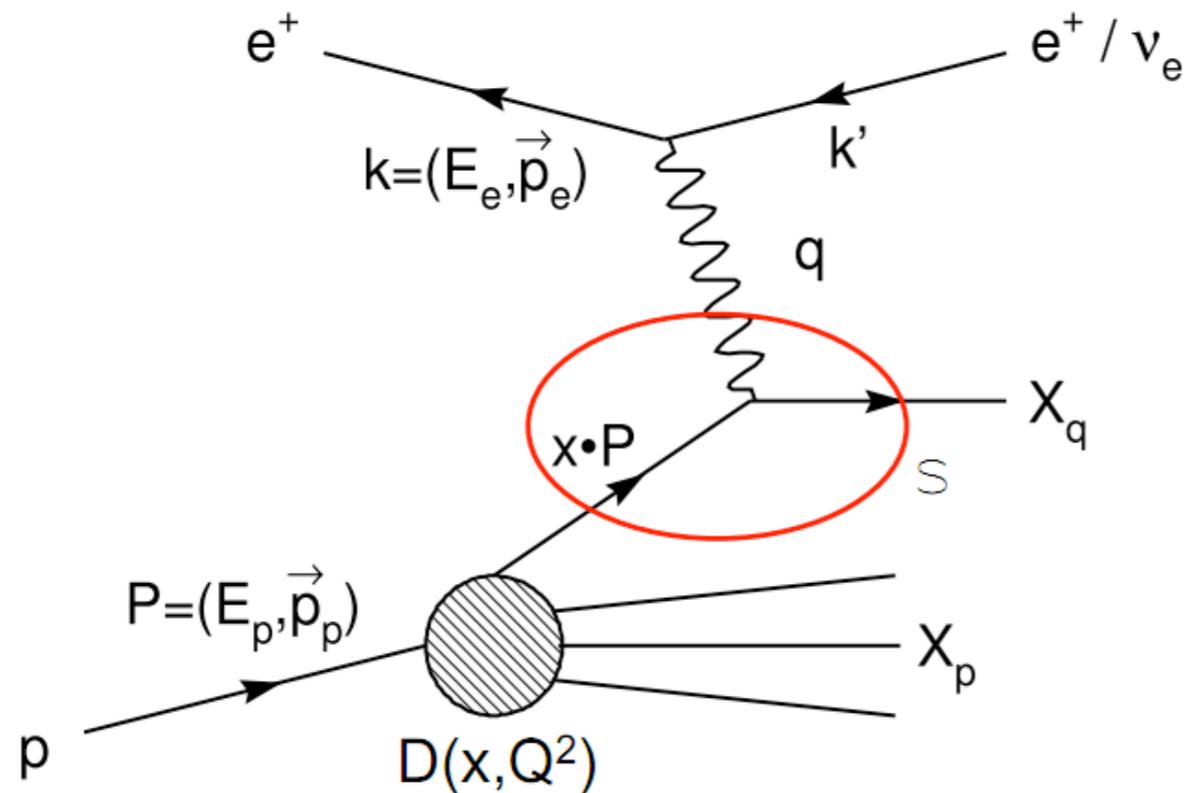
$\int \mathcal{L} dt$ [fb <sup>-1</sup> ]	Reference
$50 \times 10^{-8}$	PLB 761 (2016) 158
$8 \times 10^{-8}$	NPB 889, 486 (2014)
3.2	JHEP 09 (2017) 020
20.2	JHEP 09 (2017) 020
4.5	JHEP 02, 153 (2015)
3.2	JHEP 09 (2017) 020
4.5	JHEP 05, 059 (2014)
3.2	PLB 2017 04 072
20.2	JHEP 06 (2016) 005
4.6	PRD 89, 052004 (2014)
0.081	PLB 759 (2016) 601
20.2	EPJC 79, 760 (2019)
4.6	EPJC 77, 367 (2017)
0.025	EPJC 79, 128 (2019)
3.2	JHEP 02 (2017) 117
20.2	JHEP 02 (2017) 117
4.6	JHEP 02 (2017) 117
0.025	EPJC 79, 128 (2019)
36.1	arXiv: 1910.08819
20.2	EPJC 74, 3109 (2014)
4.6	EPJC 74, 3109 (2014)
3.2	JHEP 04 (2017) 086
20.3	EPJC 77, 531 (2017)
4.6	PRD 90, 112006 (2014)
20.3	PLB 756, 228-246 (2016)
3.2	JHEP 01 (2018) 63
20.3	JHEP 01, 064 (2016)
2.0	PLB 716, 142-159 (2012)
139.0	arXiv: 2002.07546
36.1	EPJC 79, 884 (2019)
20.3	PLB 763, 114 (2016)
4.6	PRD 87, 112001 (2013)
36.1	EPJC 79, 535 (2019)
20.3	PRD 93, 092004 (2016)
4.6	EPJC 72, 2173 (2012)
36.1	PRD 97 (2018) 032005
20.3	JHEP 01, 099 (2017)
4.6	JHEP 03, 128 (2013)
20.2	PRD 95 (2017) 112005
4.9	JHEP 01, 086 (2013)
4.6	PRD 87, 112003 (2013)
36.1	JHEP 03 (2020) 054
20.3	PRD 93, 112002 (2016)
4.6	PRD 87, 112003 (2013)
20.2	EPJC 77, 563 (2017)
4.6	JHEP 01, 049 (2015)
36.1	PRD 99, 072009 (2019)
20.3	JHEP 11, 172 (2015)
36.1	PRD 99, 072009 (2019)
20.3	JHEP 11, 172 (2015)
36.1	EPJC 79, 382 (2019)
20.2	JHEP 11 (2017) 086
4.6	PRD 91, 072007 (2015)
20.2	EPJC 77, 474 (2017)
4.7	EPJC 77, 474 (2017)
3.2	PLB 775 (2017) 206
20.3	JHEP 04, 031 (2014)
20.3	PRD 93, 112002 (2016)
20.3	PRL 115, 031802 (2015)
20.2	EPJC 77, 646 (2017)
36.1	PLB 803 (2020) 135341
20.3	JHEP 07 (2017) 107
36.1	PRL 123, 161801 (2019)
20.3	PRD 96, 012007 (2017)
36.1	PLB 793 92019) 469
20.3	PRD 93, 092004 (2016)
139.0	arXiv:2004.10612

- Theory
- LHC pp  $\sqrt{s} = 13$  TeV
- Data
- stat
- stat  $\oplus$  syst
- LHC pp  $\sqrt{s} = 8$  TeV
- Data
- stat
- stat  $\oplus$  syst
- LHC pp  $\sqrt{s} = 7$  TeV
- Data
- stat
- stat  $\oplus$  syst
- LHC pp  $\sqrt{s} = 5$  TeV
- Data
- stat
- stat  $\oplus$  syst



# BASICS OF QCD

# DEEP INELASTIC SCATTERING



$$s = (P + k)^2 \quad \text{c.o.m. energy}^2$$

$$Q^2 = -(k - k')^2 \quad \text{momentum transfer}^2$$

$$x = Q^2 / 2(P \cdot q) \quad \text{scaling variable}$$

$$\frac{d\sigma_{\text{elastic}}}{dq^2} = \left( \frac{d\sigma}{dq^2} \right)_{\text{point}} \cdot F_{\text{elastic}}^2(q^2) \delta(1 - x) dx$$

$$\frac{d\sigma_{\text{inelastic}}}{dq^2} = \left( \frac{d\sigma}{dq^2} \right)_{\text{point}} \cdot F_{\text{inelastic}}^2(q^2, x) dx$$

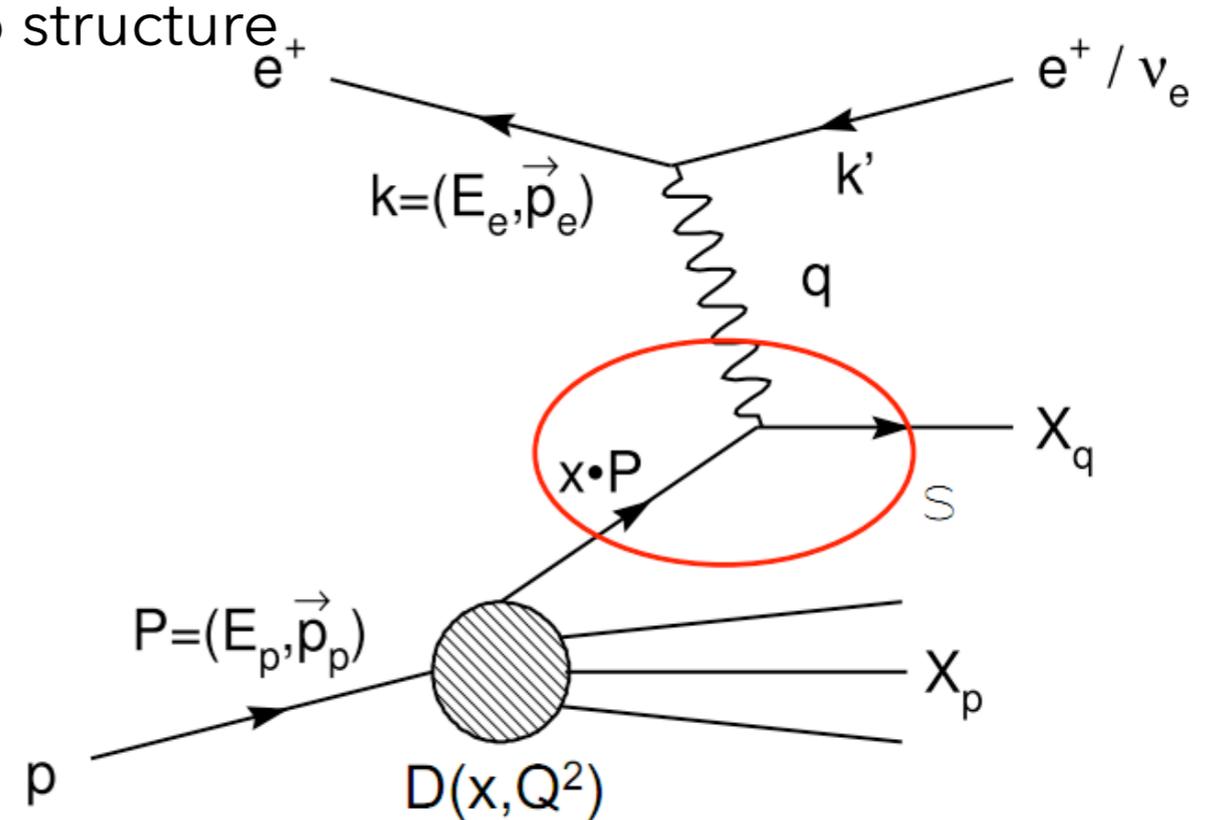
What should one expect for  $F_{\text{inelastic}}(q^2, x)$ ?

# THE PARTON MODEL

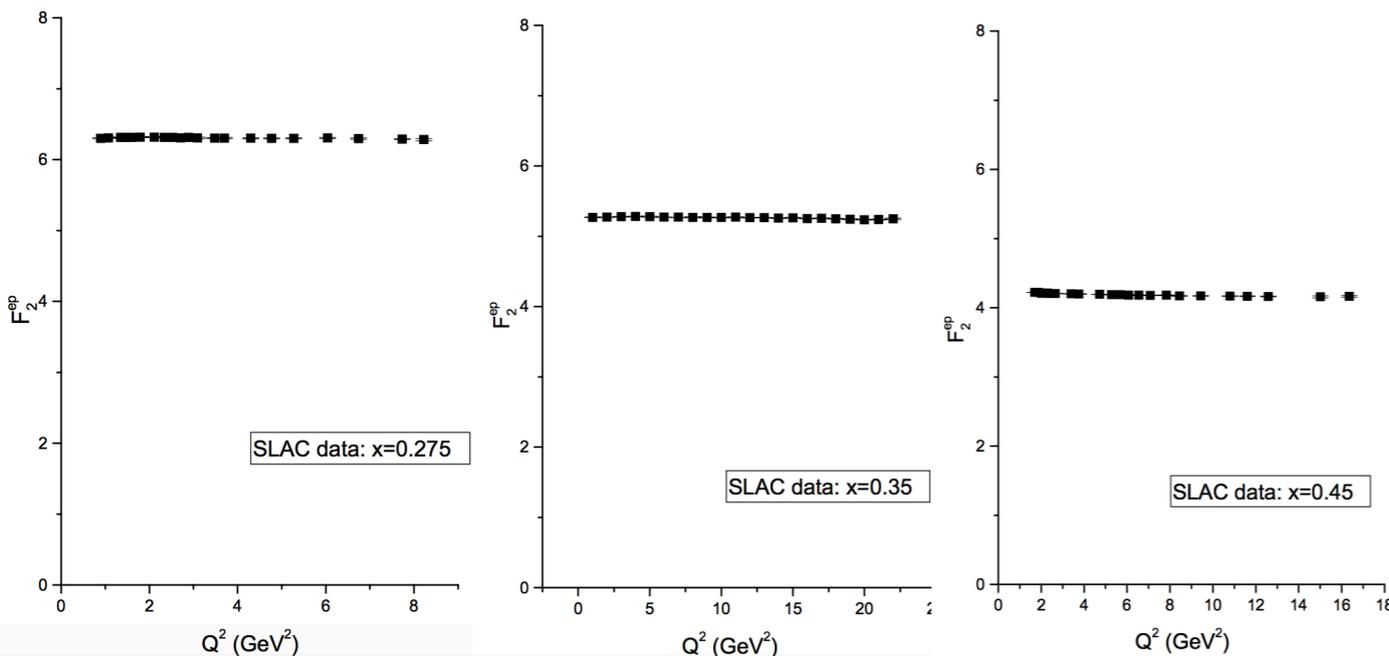
Feynman, 1969

- Virtual photon scatters incoherently off massless, free, point like, spin 1/2 partons
- The functions  $q(x)$  are the Parton Distribution Functions encode probability that a parson carries a fraction  $x$  of parent proton's
- They encode information about proton deep structure

$$F_2^{\gamma,Z}(x) = x \sum_{i=1}^{n_f} c_i [q_i(x) + \bar{q}_i(x)]$$



**THIS MOTIVATED THE SEARCH FOR A WEAKLY-COUPLED THEORY AT HIGH ENERGY!**



## QCD CRUCIAL FEATURES

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\Psi}_i^{(f)} \left( i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \Psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$D_{ij}^\mu = \partial_\mu \delta_{ij} + ig_s t_{ij}^a A_a^\mu \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c$$

Asymptotic freedom  
(UV property)

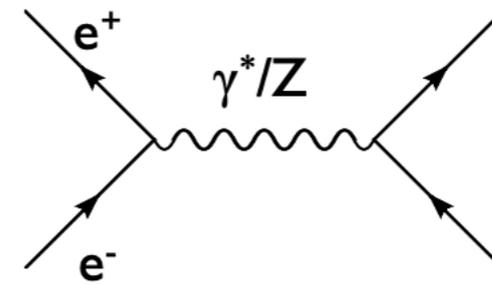
Universality  
(IR property)

- ▶ Perturbative calculation
- ▶ Parton model
- ▶ ...
- ▶ Collinear Factorisation (PDFs & FFs)
- ▶ Parton Showers
- ▶ Resummation
- ▶ ...

# QCD CRUCIAL FEATURE #1: ASYMPTOTIC FREEDOM

- ▶ One of the first tests of QCD was the measurement of the R-ratio, defined as

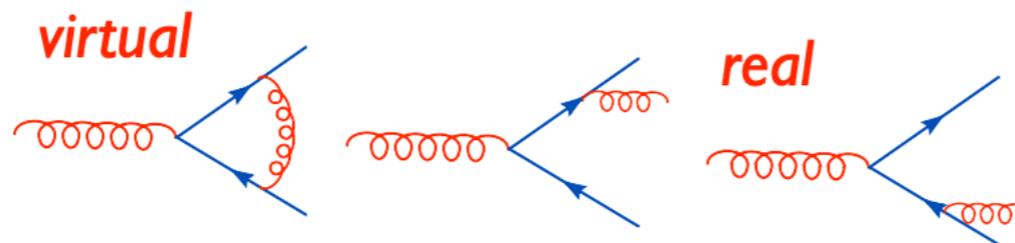
$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



- ▶ At leading order in QCD

$$R^{(0)} = N_c \sum_i e_i^2$$

- ▶ First order QCD corrections (Next-to-Leading Order)

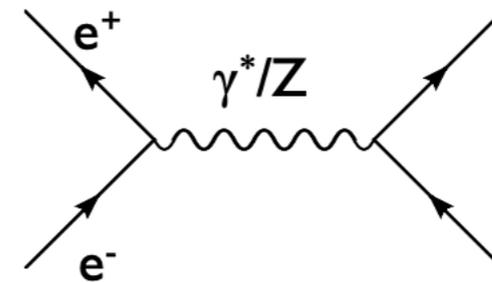


$$R^{(1)} = R^{(0)} \left( 1 + \frac{\alpha_S}{\pi} \right) \quad \alpha_S \equiv \frac{g_S^2}{4\pi}$$

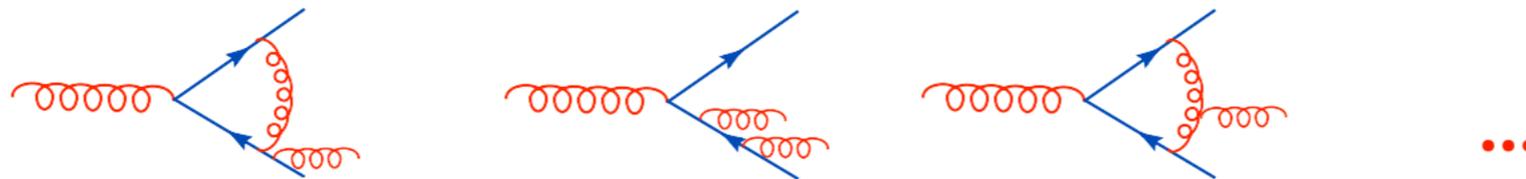
# QCD CRUCIAL FEATURE #1: ASYMPTOTIC FREEDOM

- ▶ One of the first tests of QCD was the measurement of the R-ratio, defined as

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



- ▶ Second order QCD correction (NNLO = next-to-next-to-leading order)



$$R^{(2)} = R^{(0)} \left( 1 + \frac{\alpha_S}{\pi} + \left( \frac{\alpha_S}{\pi} \right)^2 \left( c + \pi b_0 \log \left( \frac{M_{UV}^2}{Q^2} \right) \right) \right) \quad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

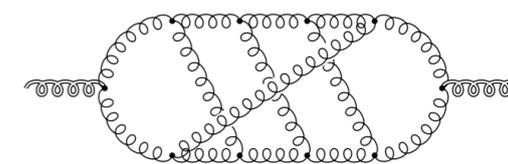
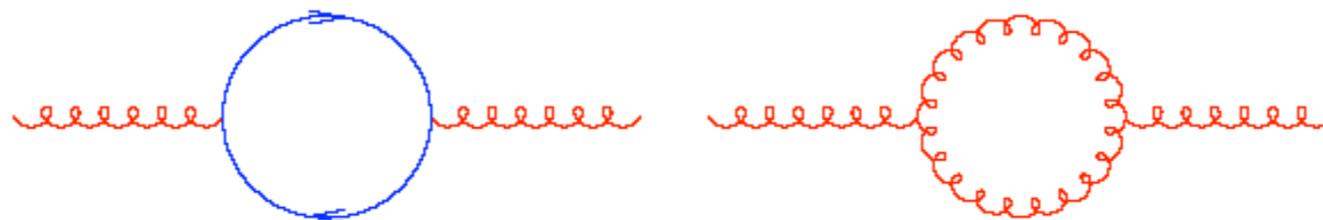
- ▶ UV divergences do not cancel => Renormalisation procedure: the UV divergence is dealt with renormalisation of bare coupling

$$\alpha_S(\mu) = \alpha_S^{\text{bare}} + b_0 \log \left( \frac{M_{UV}^2}{\mu^2} \right) (\alpha_S^{\text{bare}})^2 \quad \longrightarrow \quad \mu^2 \frac{d\alpha_S(\mu)}{d\mu^2} = -b_0 \alpha_S^2(\mu) + \dots$$

# QCD CRUCIAL FEATURE #1: ASYMPTOTIC FREEDOM

$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -(b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \dots)$$

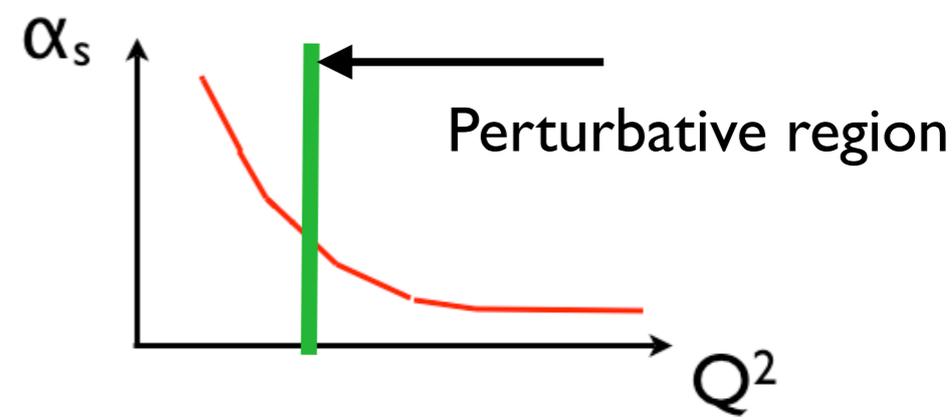
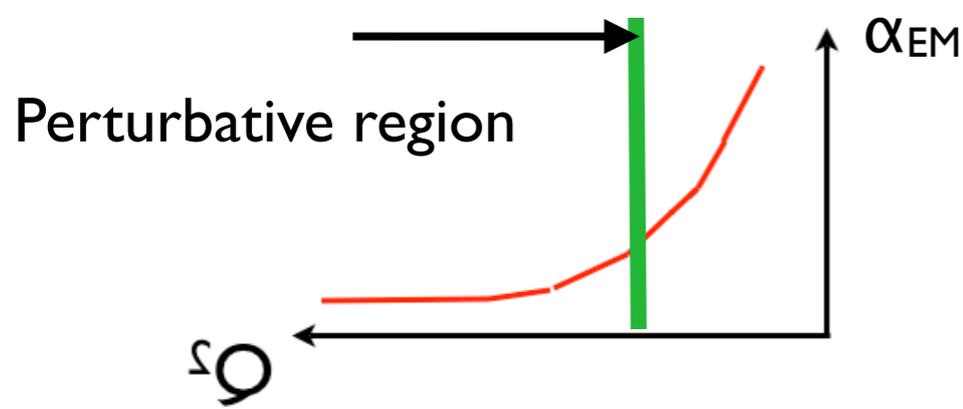
Known up to  $b_4$  (5 loops):  
[arXiv: 1606.08659, ...](https://arxiv.org/abs/1606.08659),  
[arXiv: 1709.08541](https://arxiv.org/abs/1709.08541)

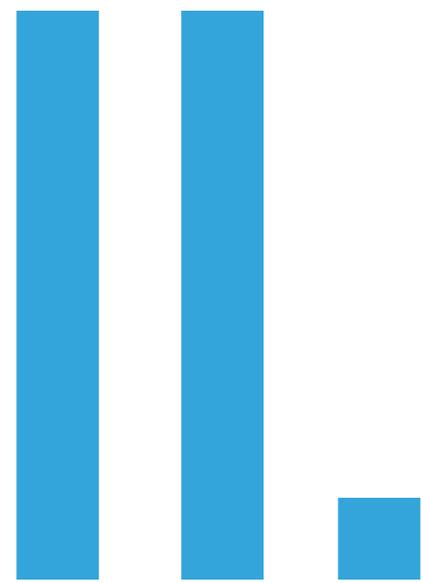


Roughly speaking, quark loop diagrams contribute with  $N_f$  negative terms in  $b_0$ , while the gluon loop, diagram gives a positive contribution proportional to  $N_c$ , which is dominant and make the overall beta function negative.

$$b_0 = \frac{11N_c - 2n_f}{12\pi} > 0 \quad \Rightarrow \quad \beta(\alpha_s) < 0 \quad \text{in QCD}$$

$$b_0 = -\frac{n_f}{3\pi} > 0 \quad \Rightarrow \quad \beta(\alpha_{EM}) > 0 \quad \text{in QED}$$





# THE PRECISION INGREDIENTS AT THE LHC

# EVENTS AT HADRON COLLIDERS

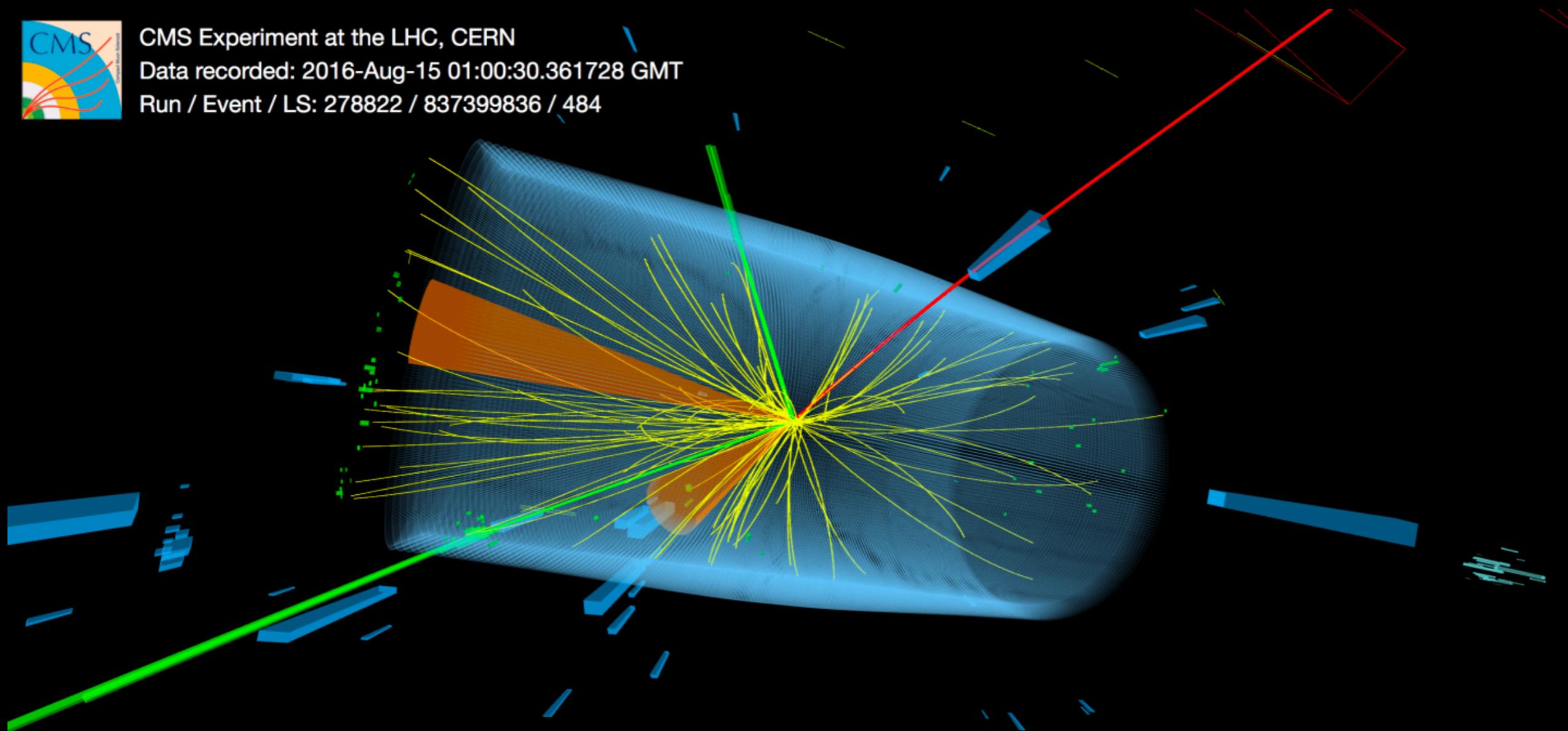
$pp \rightarrow t ( b W (\mu\nu) ) Z(ee) q$



CMS Experiment at the LHC, CERN

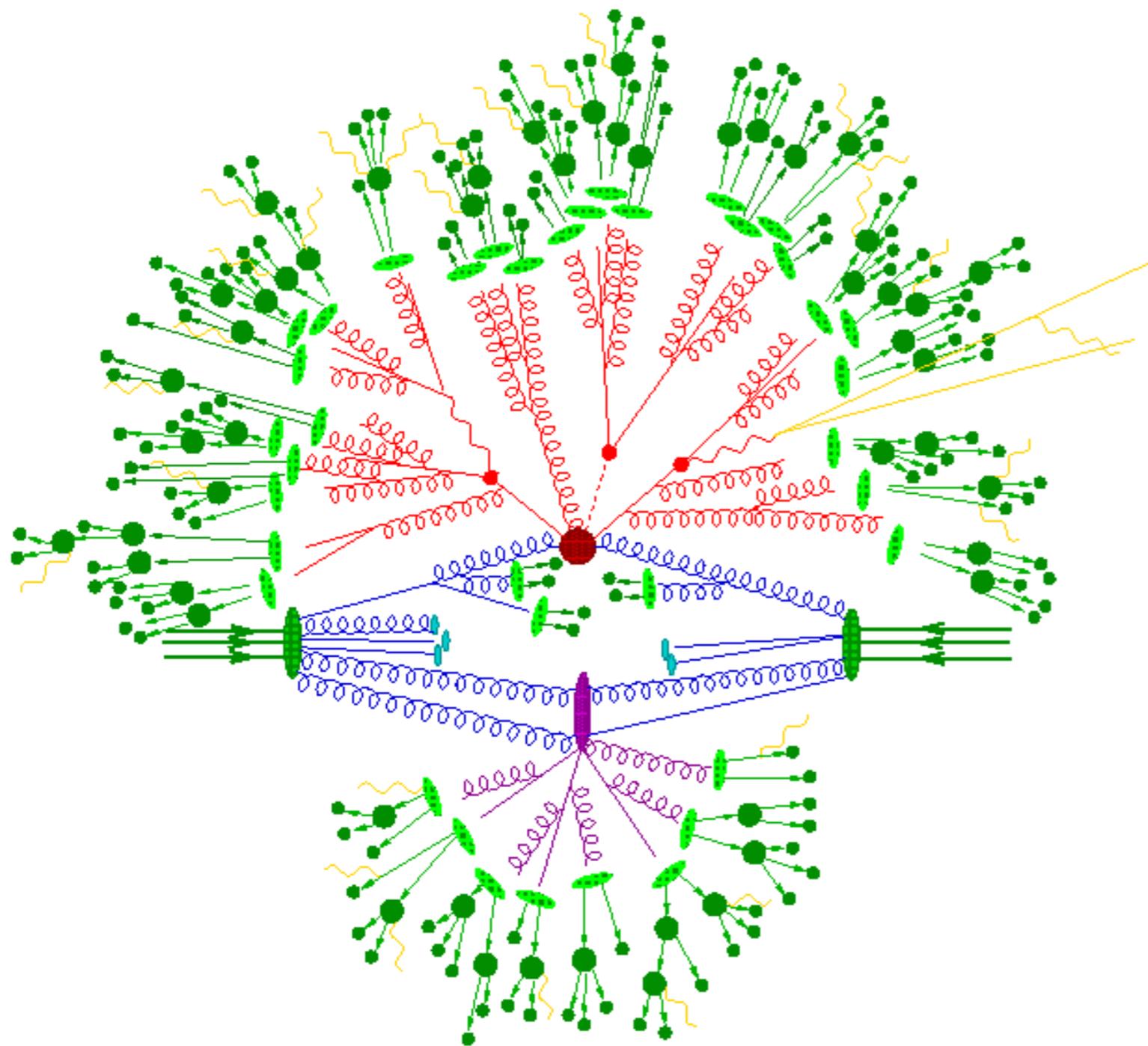
Data recorded: 2016-Aug-15 01:00:30.361728 GMT

Run / Event / LS: 278822 / 837399836 / 484

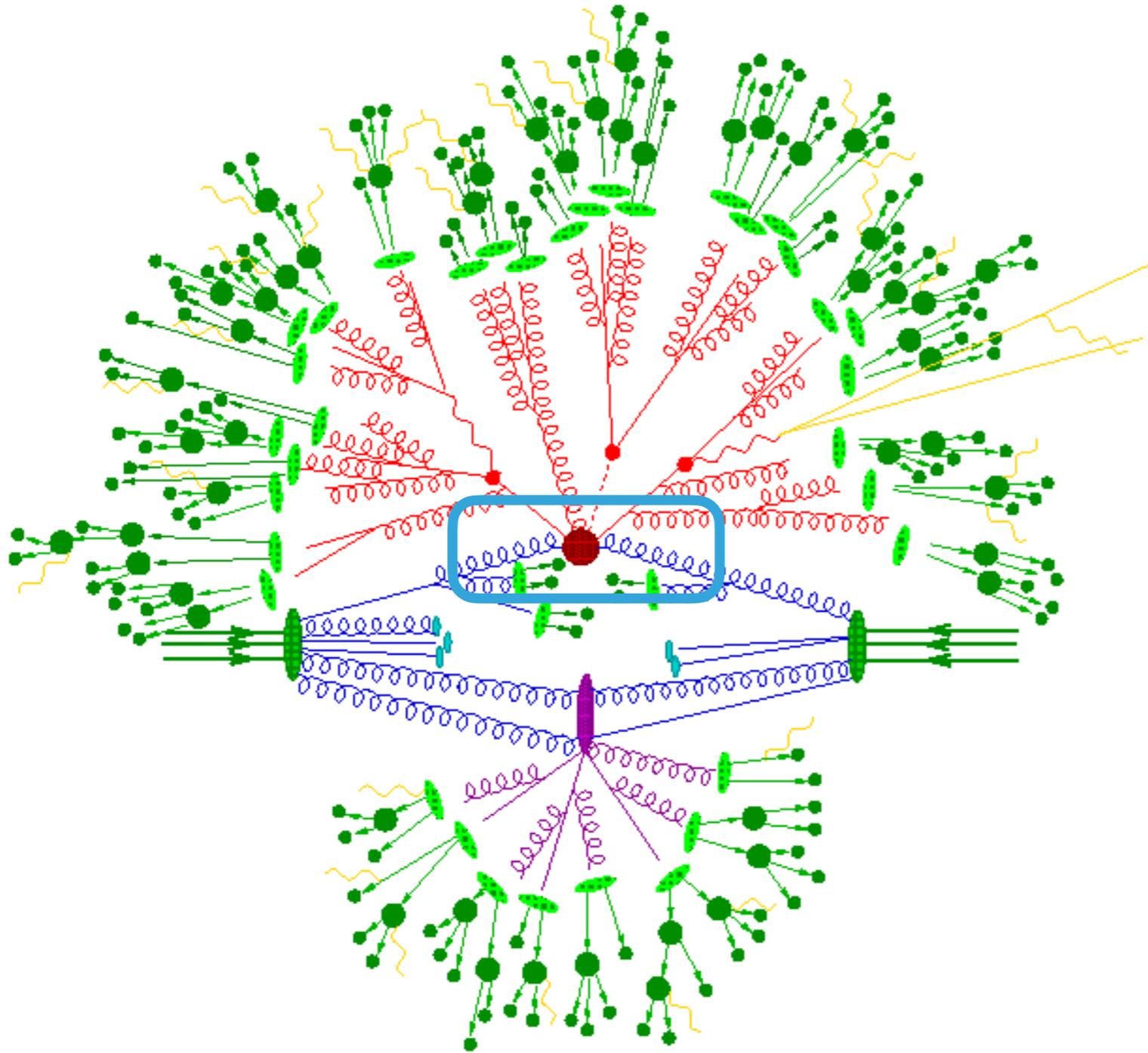


A top + Z candidate collision recorded by CMS. The  $tZq$  state is characterised by three leptons (in this case two electrons and one muon), a jet produced from decay of a bottom quark, and a forward jet that is close to the LHC beam direction (Image: CMS/CERN)

# EVENTS AT HADRON COLLIDERS

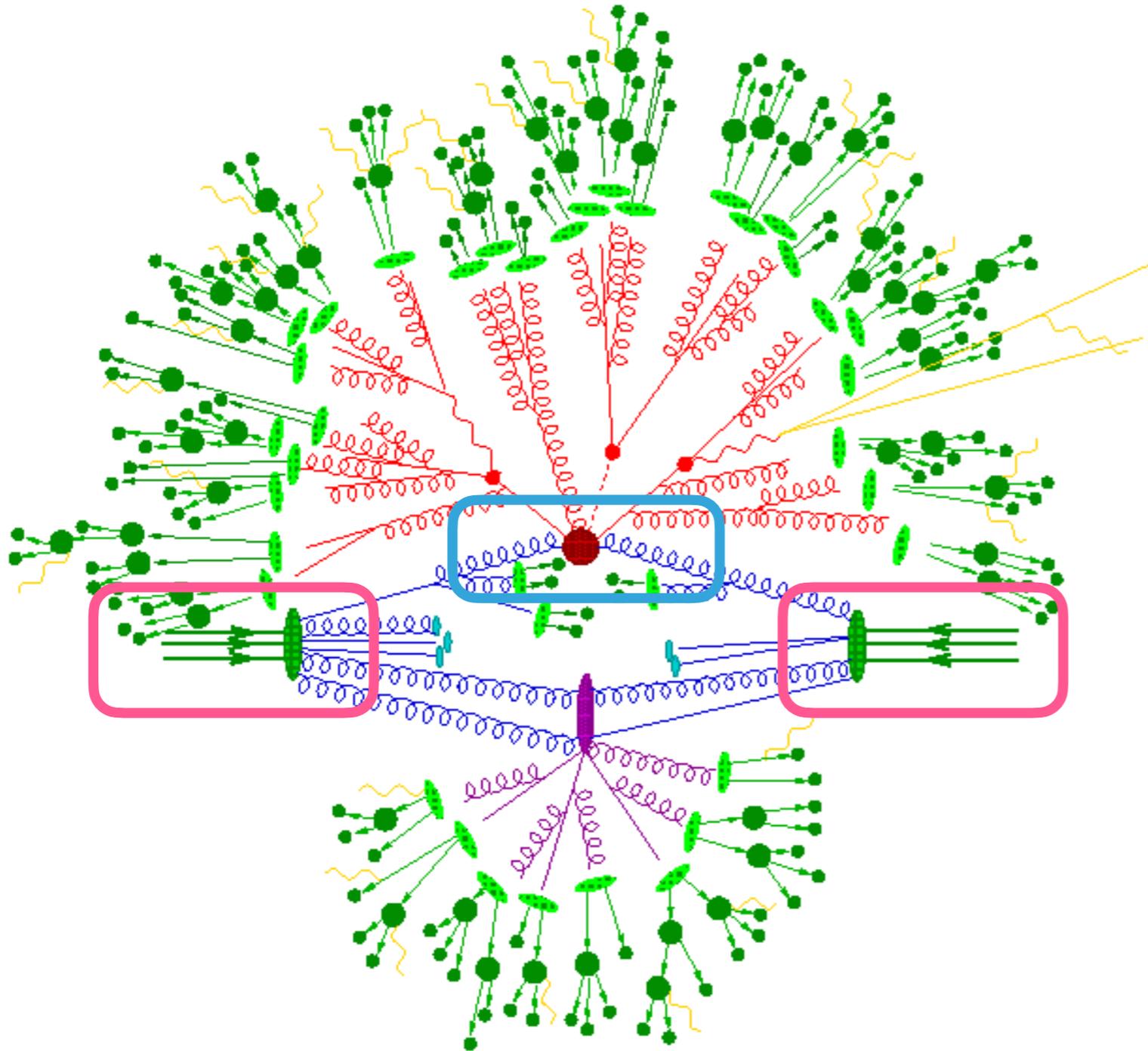


# EVENTS AT HADRON COLLIDERS



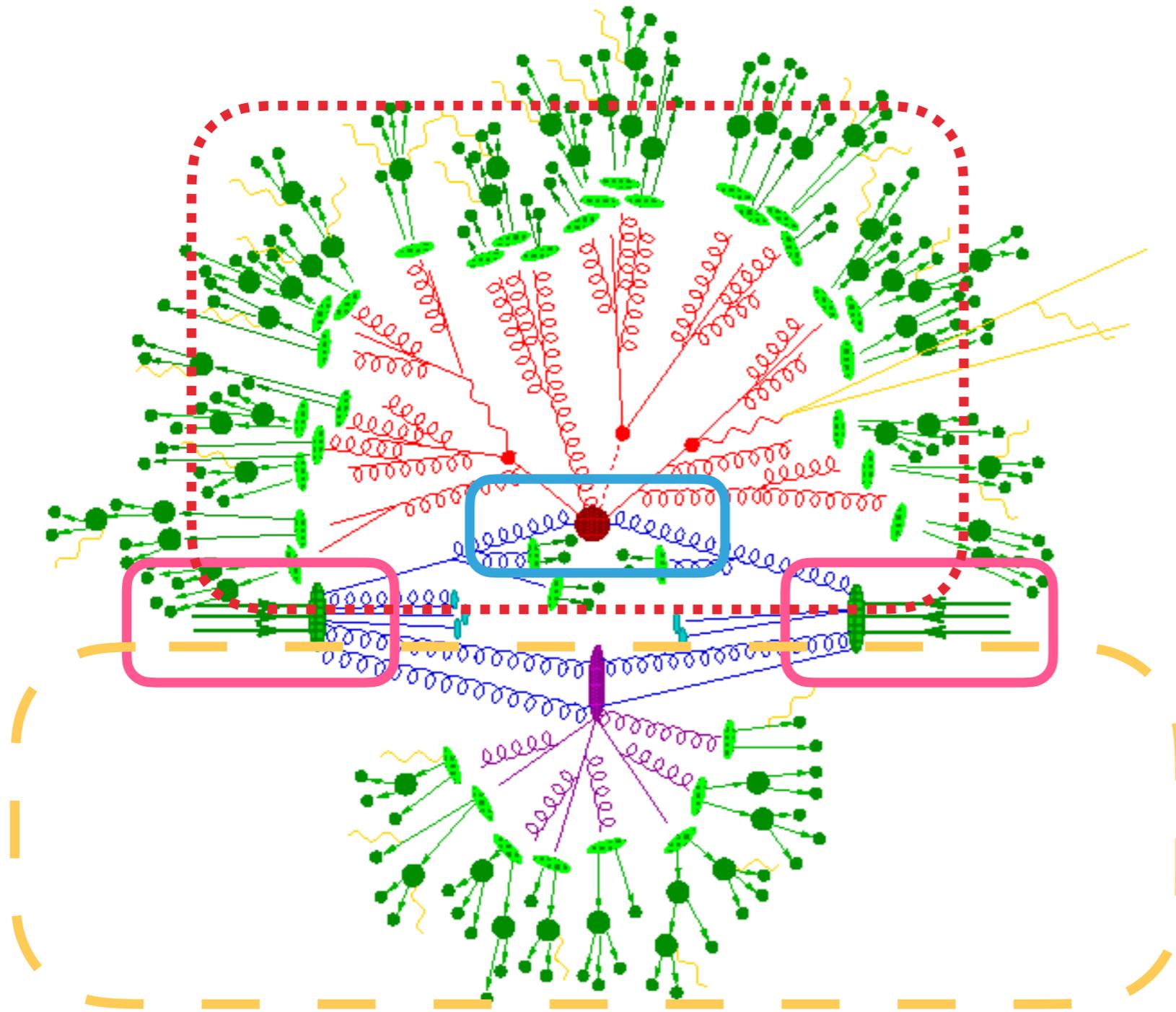
- Hard scattering of partons (Perturbative QCD+EW)

# EVENTS AT HADRON COLLIDERS



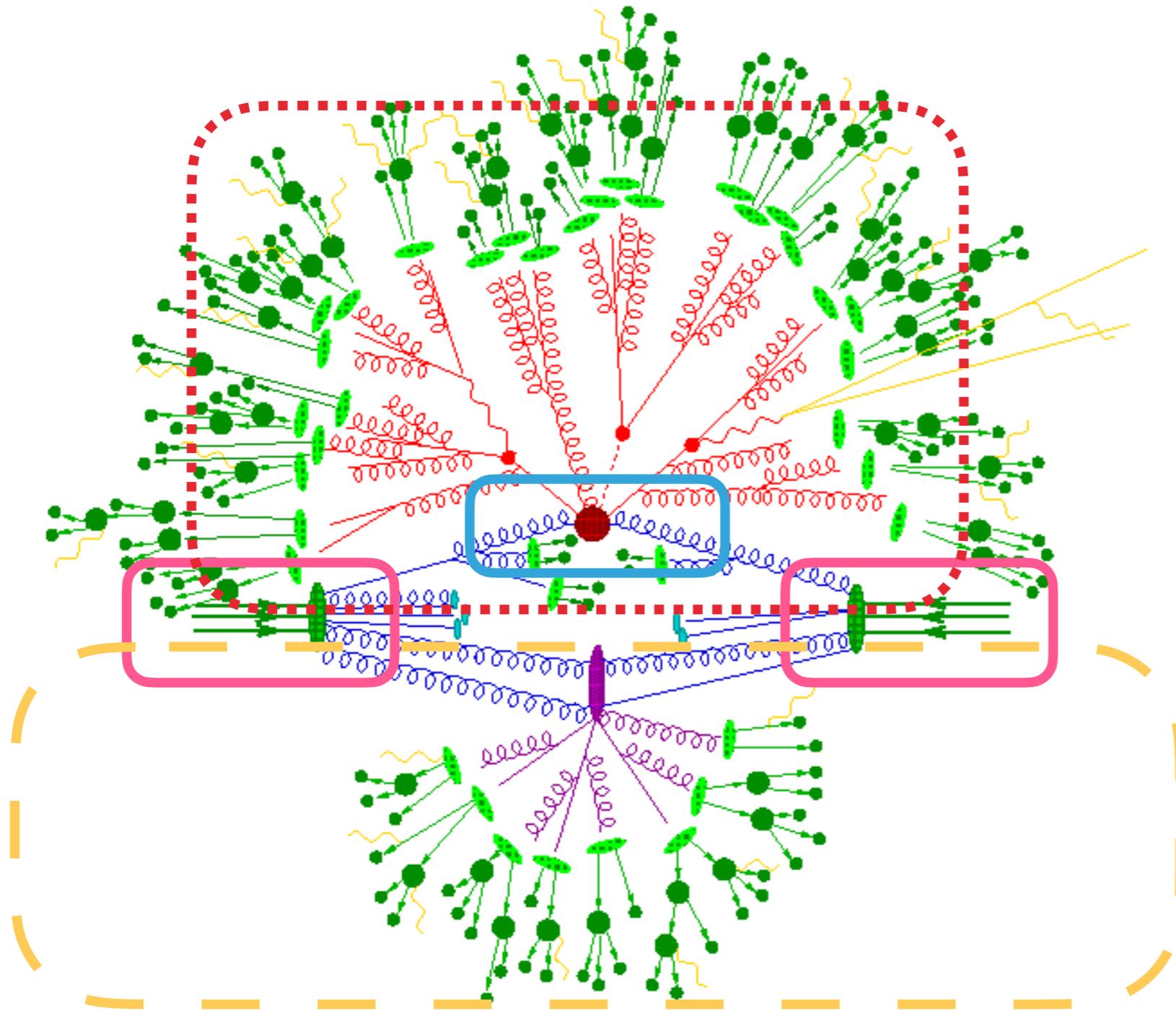
- Hard scattering of partons (Perturbative QCD+EW)
- The structure of the proton: parton distribution functions

# EVENTS AT HADRON COLLIDERS



- Hard scattering of partons (Perturbative QCD+EW)
- The structure of the proton: parton distribution functions
- Parton Showering and Hadronization
- Multiple Parton Interaction, Underlying Events

# EVENTS AT HADRON COLLIDERS



- Hard scattering of partons (Perturbative QCD+EW)
- The structure of the proton: parton distribution functions
- Parton Showering and Hadronization
- Multiple Parton Interaction, Underlying Events

**DIVIDE ET IMPERA!**

# FACTORIZATION: THE LHC MASTER FORMULA

$$\sigma^{pp \rightarrow ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1 dz_2 f_i(z_1, \mu_F) f_j(z_2, \mu_F) \hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

The diagram illustrates the LHC Master Formula, which is a factorization theorem. The formula is presented as a sum of three terms, each enclosed in a colored box and connected to a descriptive text block by a colored arrow:

- PDFs:** A pink box highlights the parton distribution functions  $f_i(z_1, \mu_F) f_j(z_2, \mu_F)$ . A pink arrow points to the text: "PDFs: universal functions that parametrise the proton structure".
- Partonic cross section:** A green box highlights the partonic cross section  $\hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F)$ . A green arrow points to the text: "Partonic cross section computed in perturbative QCD + EW corrections".
- Soft stuff:** A yellow box highlights the higher-order terms  $\mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$ . A yellow arrow points to the text: "Soft stuff: higher twists, multiple parton interaction, underlying event".

Additionally, a red arrow points from the green box to the text: "Parton showers, hadronisation", indicating that the partonic cross section is followed by these non-perturbative processes.



# THE STRONG COUPLING CONSTANT

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$$\sigma^{pp \rightarrow ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1 dz_2 f_i(z_1, \mu_F) f_j(z_2, \mu_F) \hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$\alpha_s(\mu_F)$        $\alpha_s(\mu_F)$        $\alpha_s(\mu_R)$

- ▶ In pQCD all theoretical predictions are expressed in terms of the renormalised coupling  $\alpha_s(\mu^2)$ , a function of unphysical renormalization and factorization scales  $\mu_R$  and  $\mu_F$
- ▶ When one takes  $\mu_R$  close to the scale of the momentum transfer  $Q$  in a given process, then  $\alpha_s(Q^2)$  is indicative of the effective strength of the strong interaction in that process
- ▶ Beside the quark masses, the only free parameter in the QCD Lagrangian is  $\alpha_s$

# THE STRONG COUPLING CONSTANT

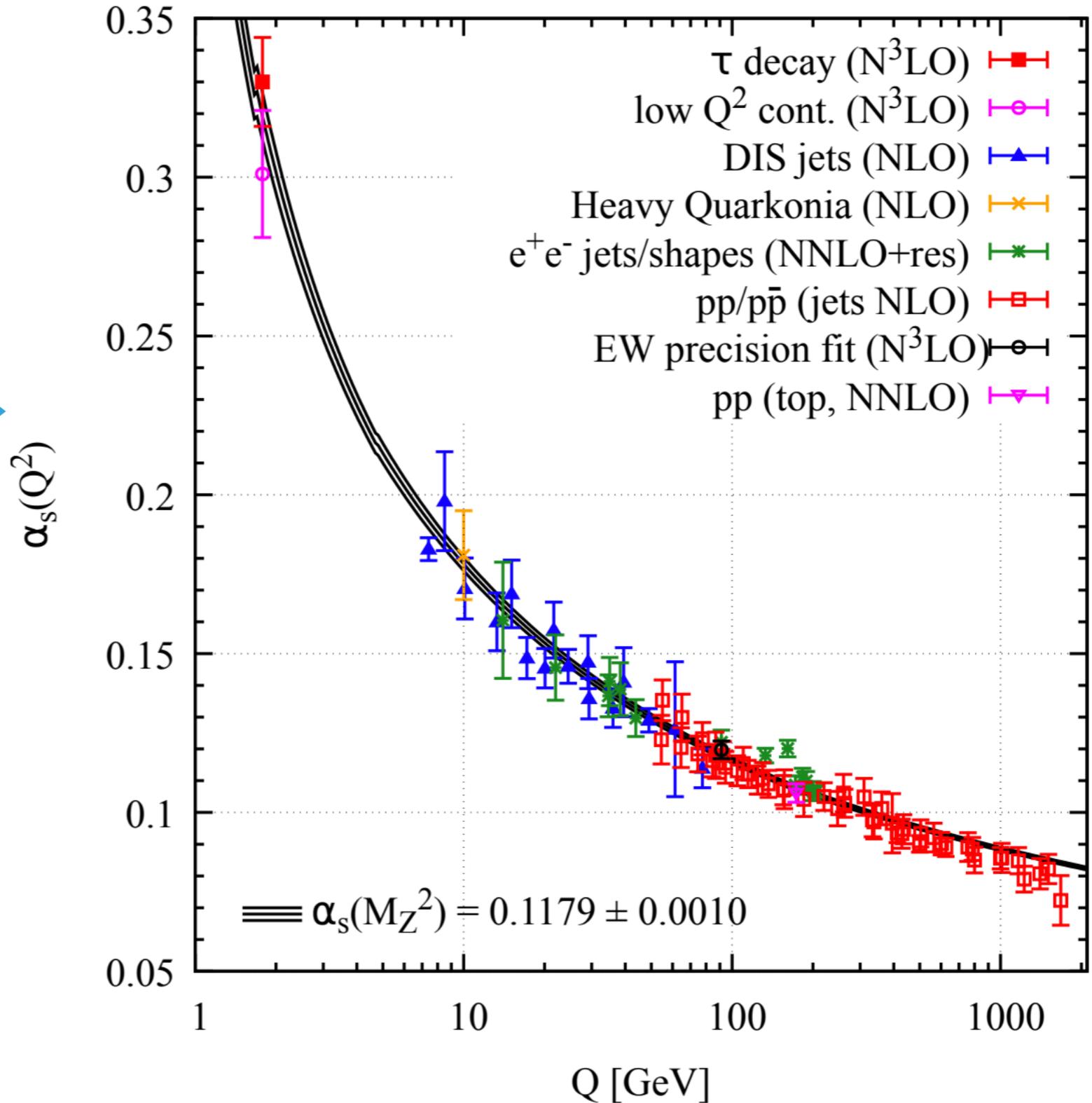
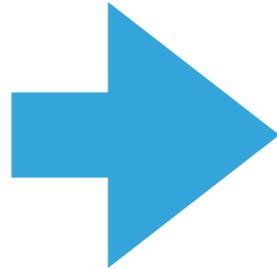
$$\sigma^{pp \rightarrow ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1 dz_2 f_i(z_1, \mu_F) f_j(z_2, \mu_F) \hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

$\alpha_s(\mu_F)$        $\alpha_s(\mu_F)$        $\alpha_s(\mu_R)$

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- ▶ Beside the quark masses, the only free parameter in the QCD Lagrangian is  $\alpha_s$
- ▶ The coupling constant not a physical observable, rather quantity defined in the context of perturbation theory, which enters predictions for measurable observables.

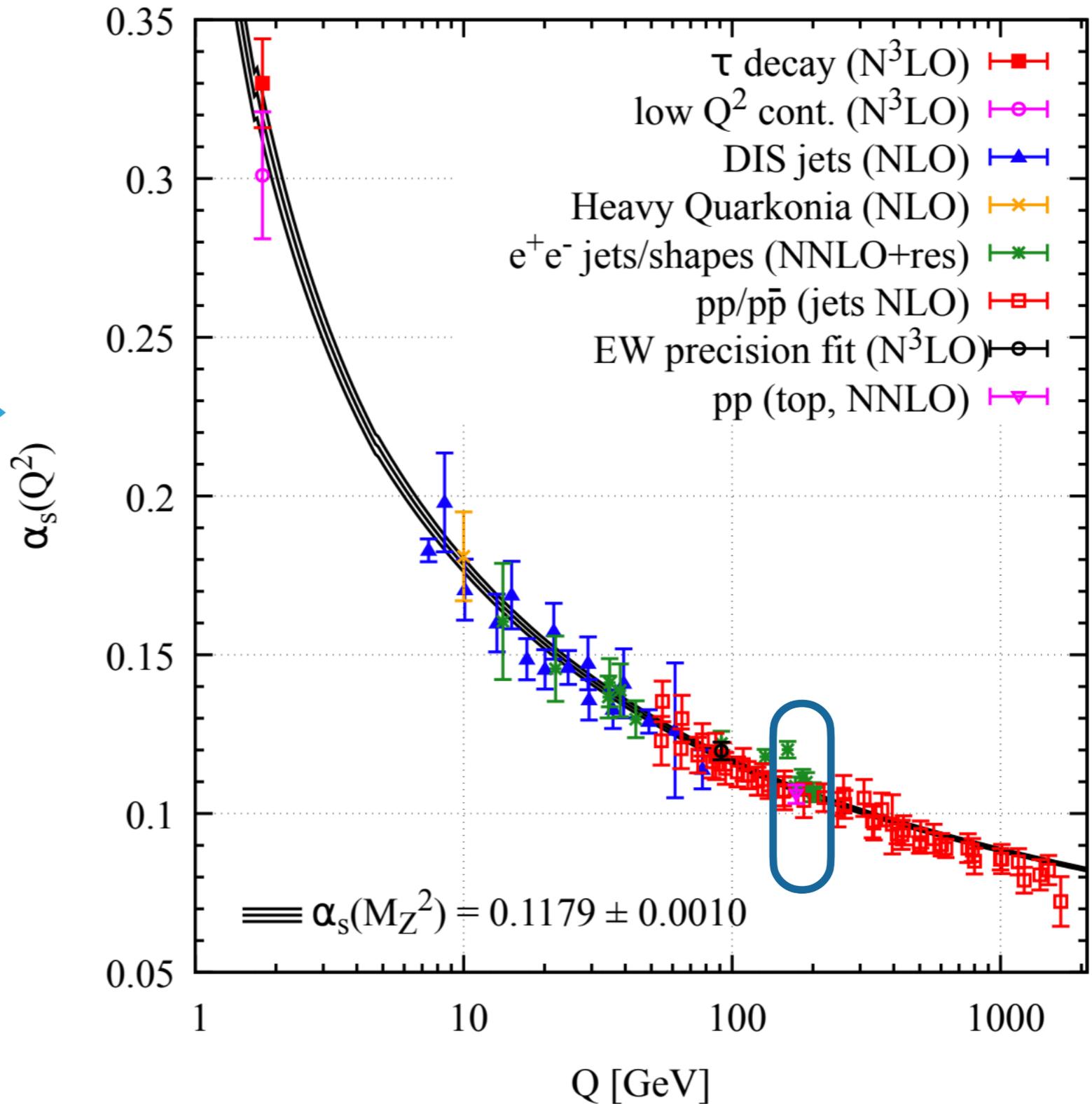
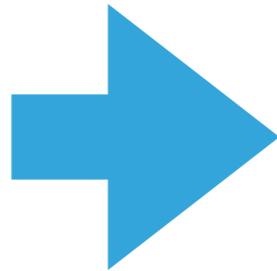
# THE STRONG COUPLING CONSTANT

PDG 2020 average  
from fit of specific  
experimental  
observables



# THE STRONG COUPLING CONSTANT

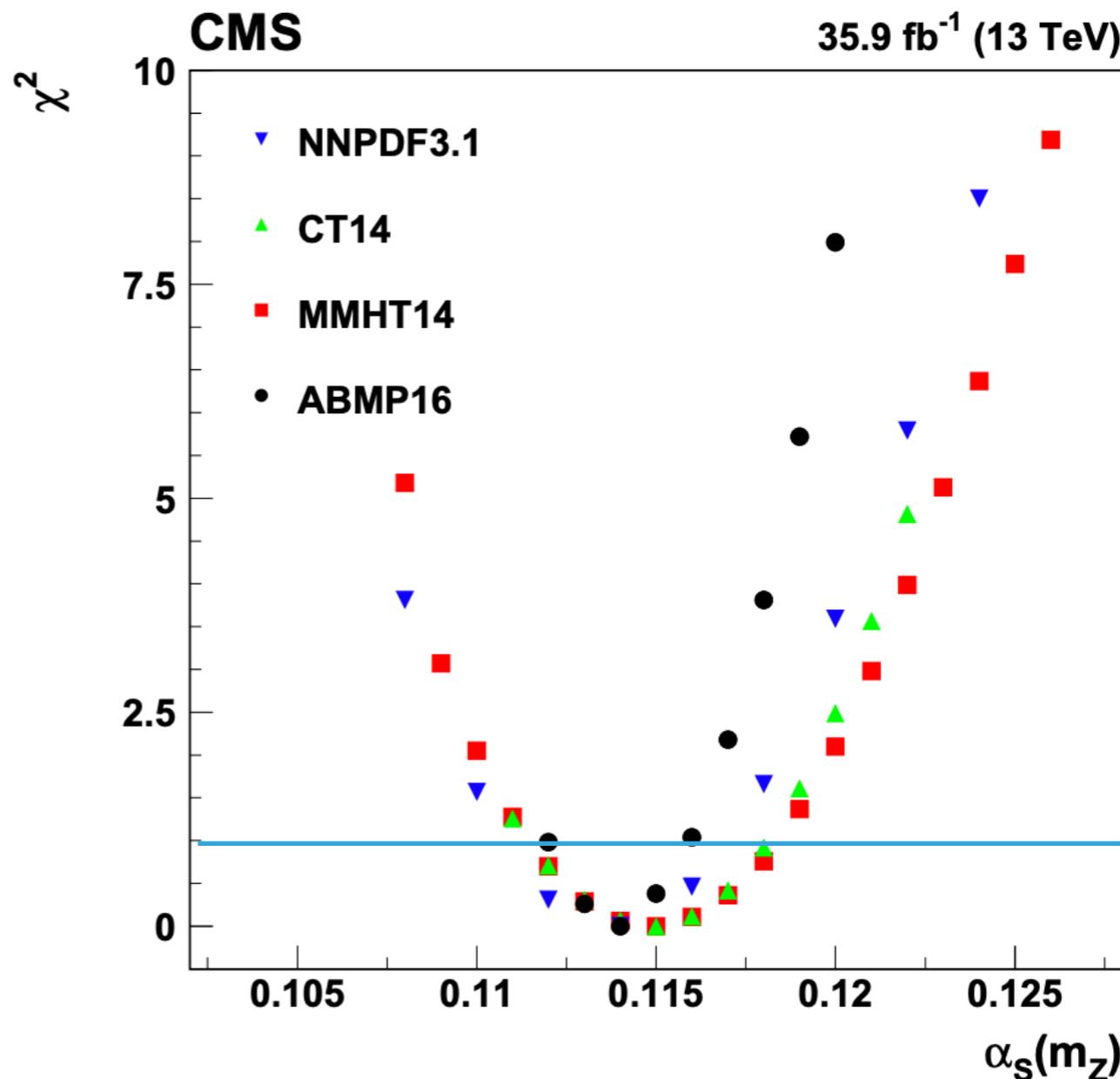
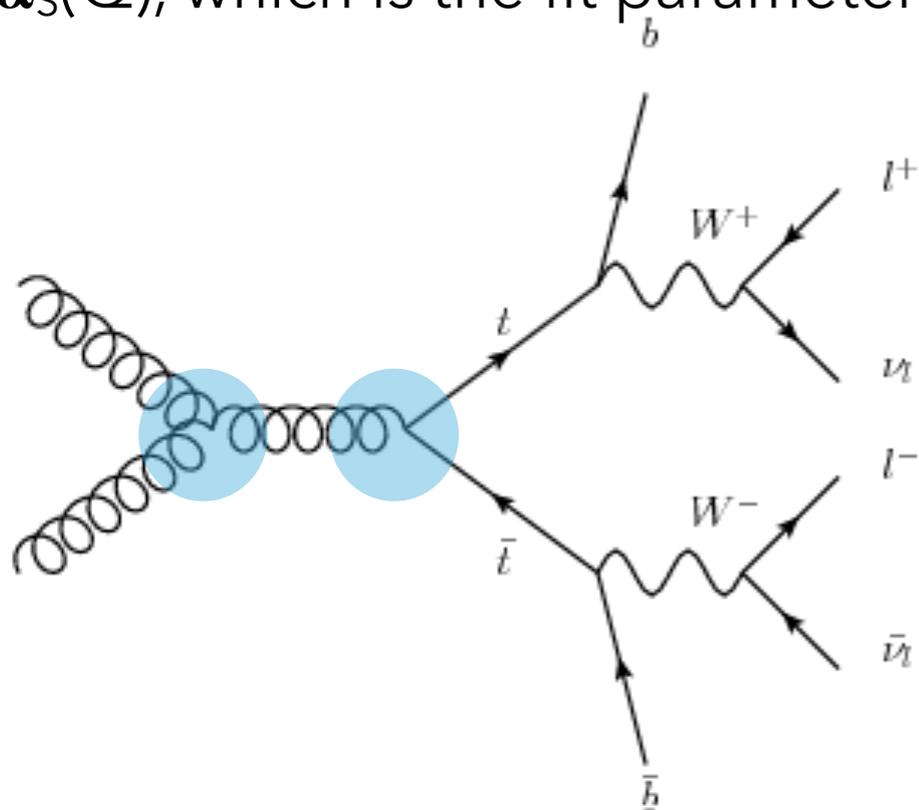
PDG 2020 average  
from fit of specific  
experimental  
observables



# THE STRONG COUPLING CONSTANT: AN EXAMPLE

CMS collaboration  
arXiv:1812.10505

- ▶ Example: CMS determination of  $m_t$  and  $\alpha_s(M_Z)$  from  $t\bar{t}$  production cross section in leptonic channel
- ▶ Results depend on experimental systematics (in  $D_i$  and  $\text{cov}_{ij}$ ) and theoretical prediction  $T_i$ , which, for a given perturbative order depends on  $\alpha_s(Q)$ , which is the fit parameter



$$\chi^2 = \sum_{i,j=1}^{N_{\text{dat}}} (D_i - T_i) (\text{cov})_{ij}^{-1} (D_j - T_j)$$



# FIXED-ORDER QCD COMPUTATIONS

$$\sigma^{pp \rightarrow ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1 dz_2 f_i(z_1, \mu_F) f_j(z_2, \mu_F) \hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

# PARTONIC CROSS SECTIONS

The partonic cross section is calculated by

$$\hat{\sigma}^{ij \rightarrow ab} = \frac{1}{2s} \int \left[ \prod_{k=1}^n \frac{d^3 \vec{q}_k}{(2\pi)^3 2E_k} \right] \left[ (2\pi)^2 \delta^{(4)} \left( \sum_k q_k^\mu - (p_1 + p_2)^\mu \right) \right] |\mathcal{M}_{ij \rightarrow ab}(\mu_F, \mu_R)|^2$$

↑
↑
↑
  
 [flux factor]      ×      [phase space]      ×      [squared matrix element]

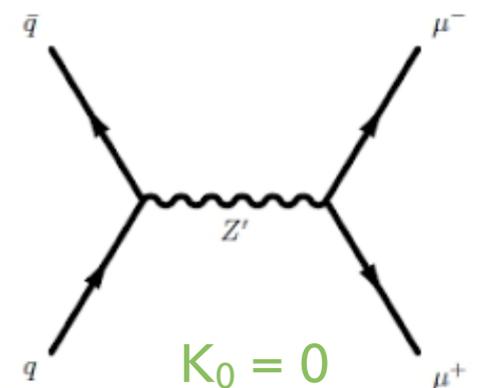
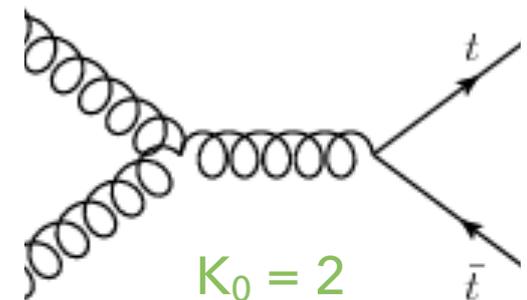
Short distance coefficients as an expansion in  $\alpha_s$  (from theory)

$$\hat{\sigma}^{ij \rightarrow ab} = (\alpha_s)^{k_0} \sigma_0 + (\alpha_s)^{k_0+1} \sigma_1 + (\alpha_s)^{k_0+2} \sigma_2 + \dots$$

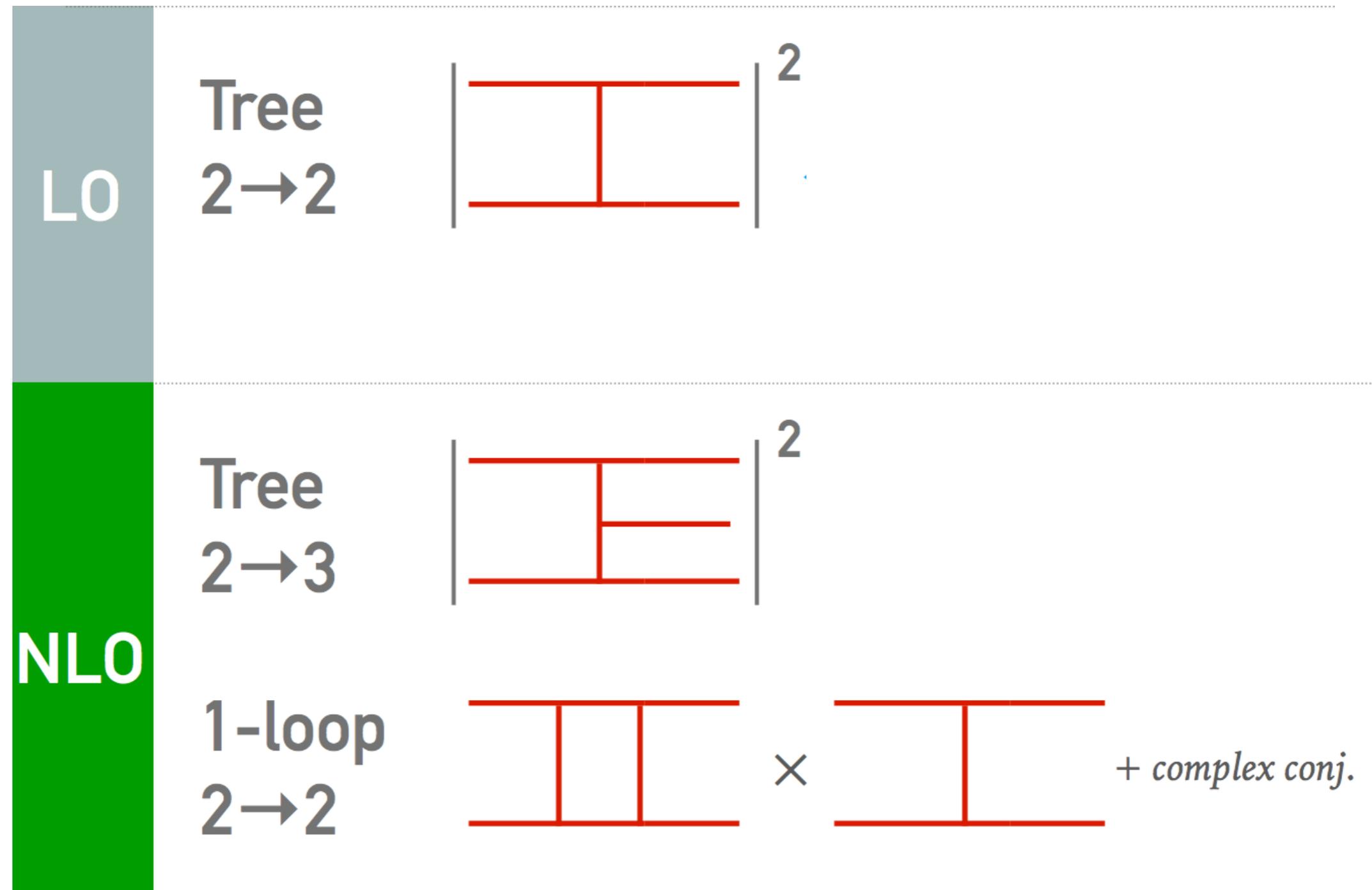
Leading Order

Next-to-Leading Order

Next-to-Next-to-Leading Order

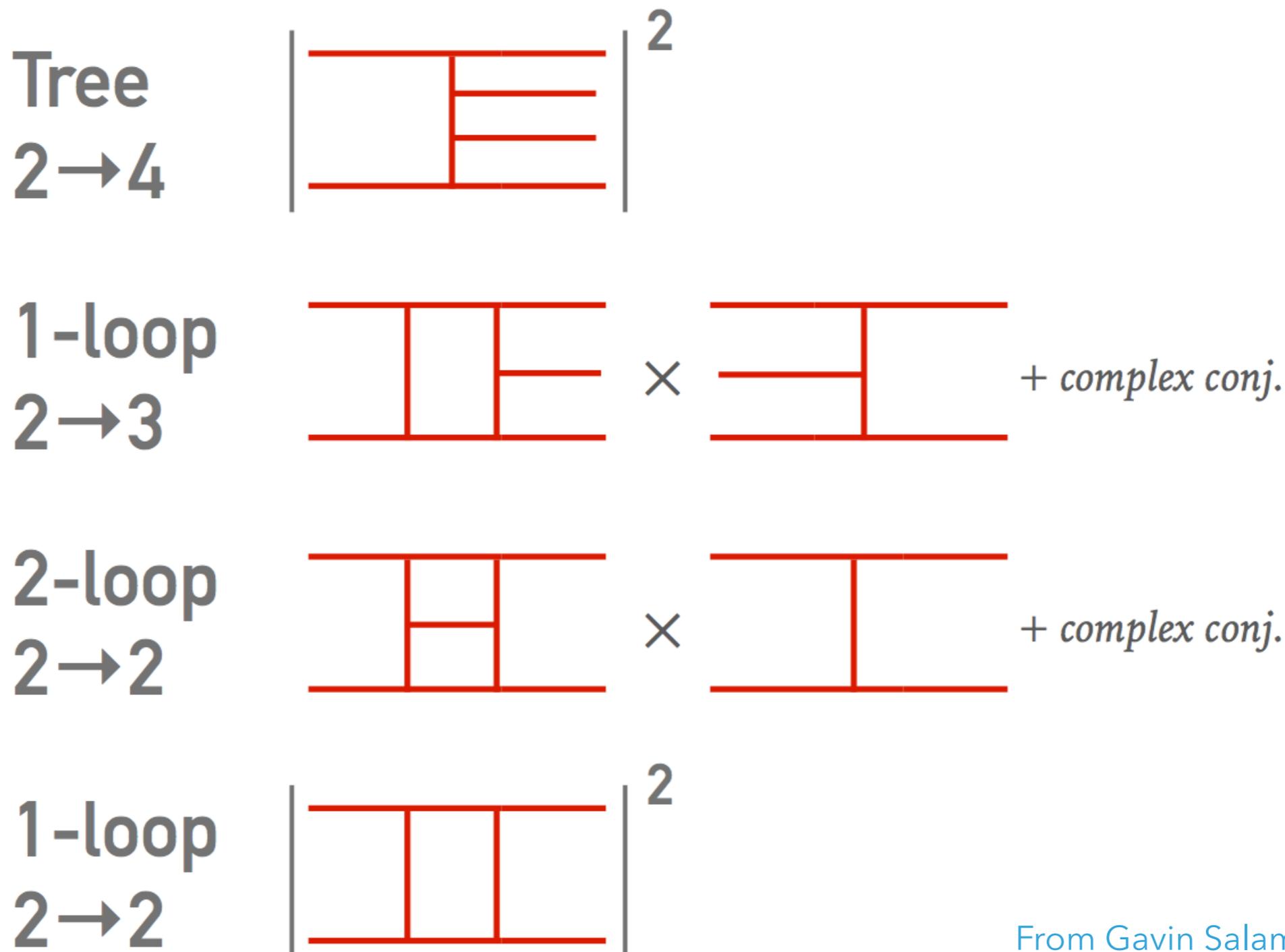


# THE PARTONIC CROSS SECTION



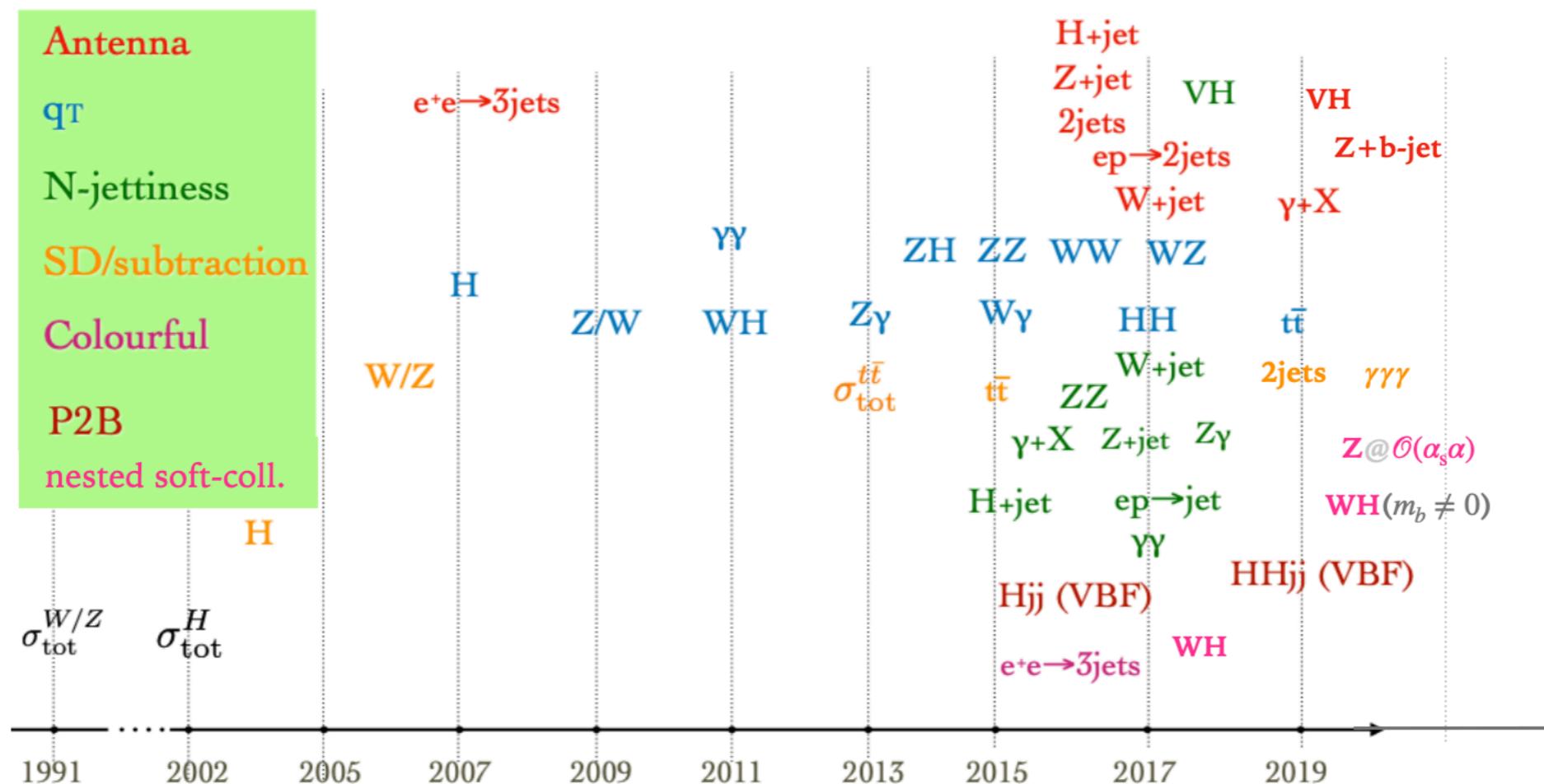
# THE PARTONIC CROSS SECTION

NNLO



# PARTONIC CROSS SECTION COMPUTATION: STATE OF THE ART

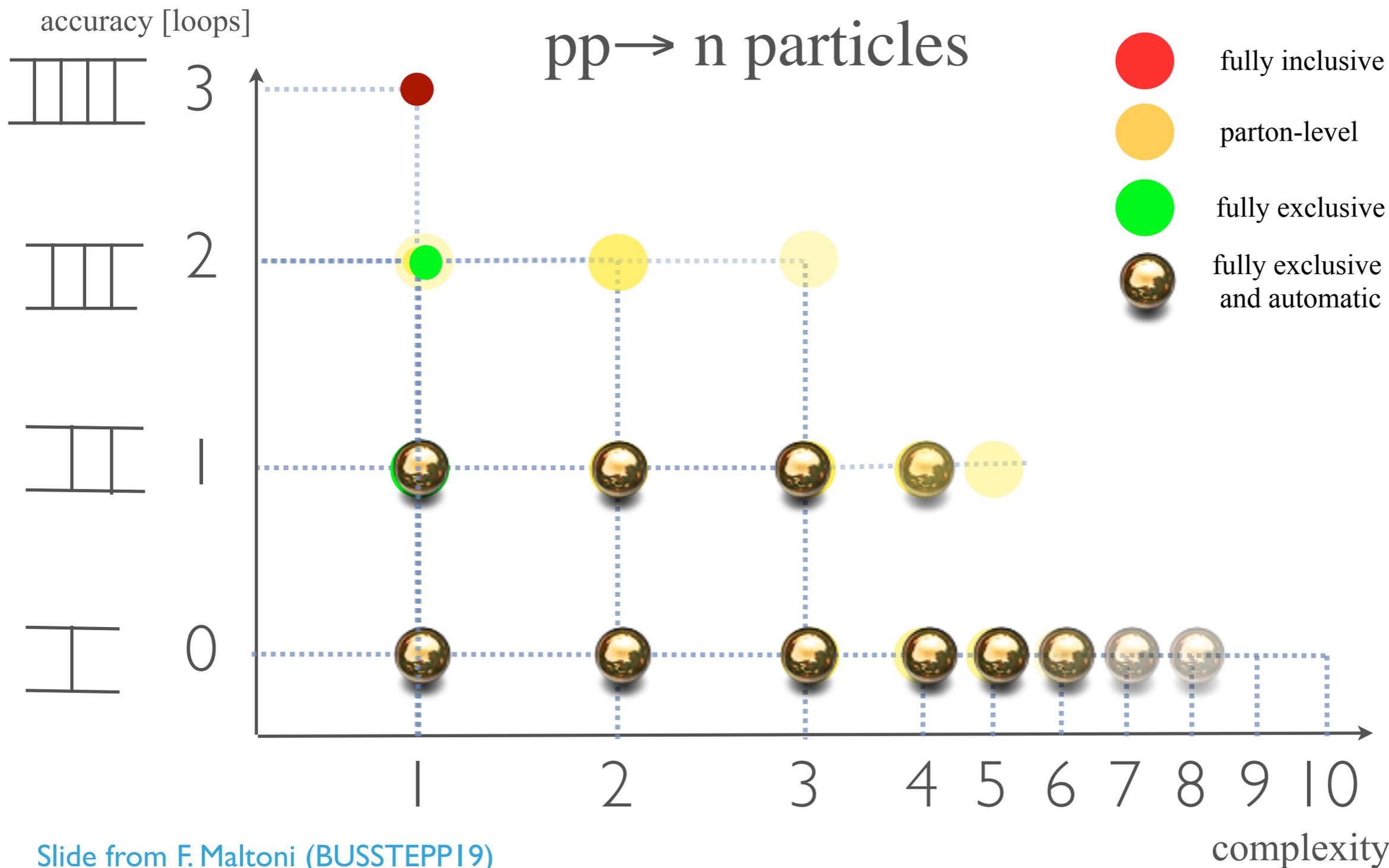
## TIMELINE FOR NNLO



- LO: almost all processes
- NLO: most processes (automated calculations)
- NNLO: all  $2 \rightarrow 1$  and  $2 \rightarrow 2$  (explosion of calculations in the past few years)
- N3LO: five processes so far: strong indication about perturbative series convergence!

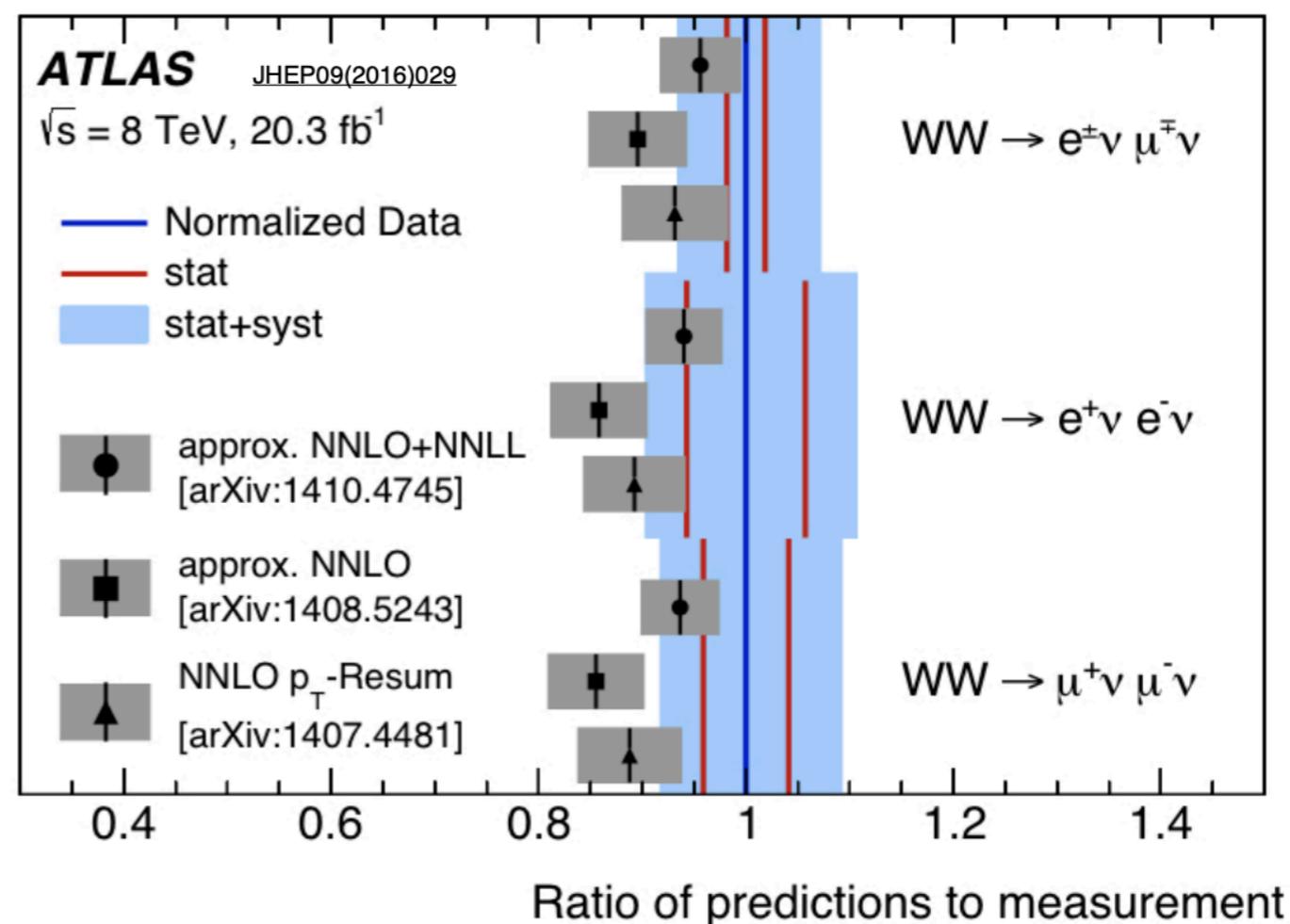
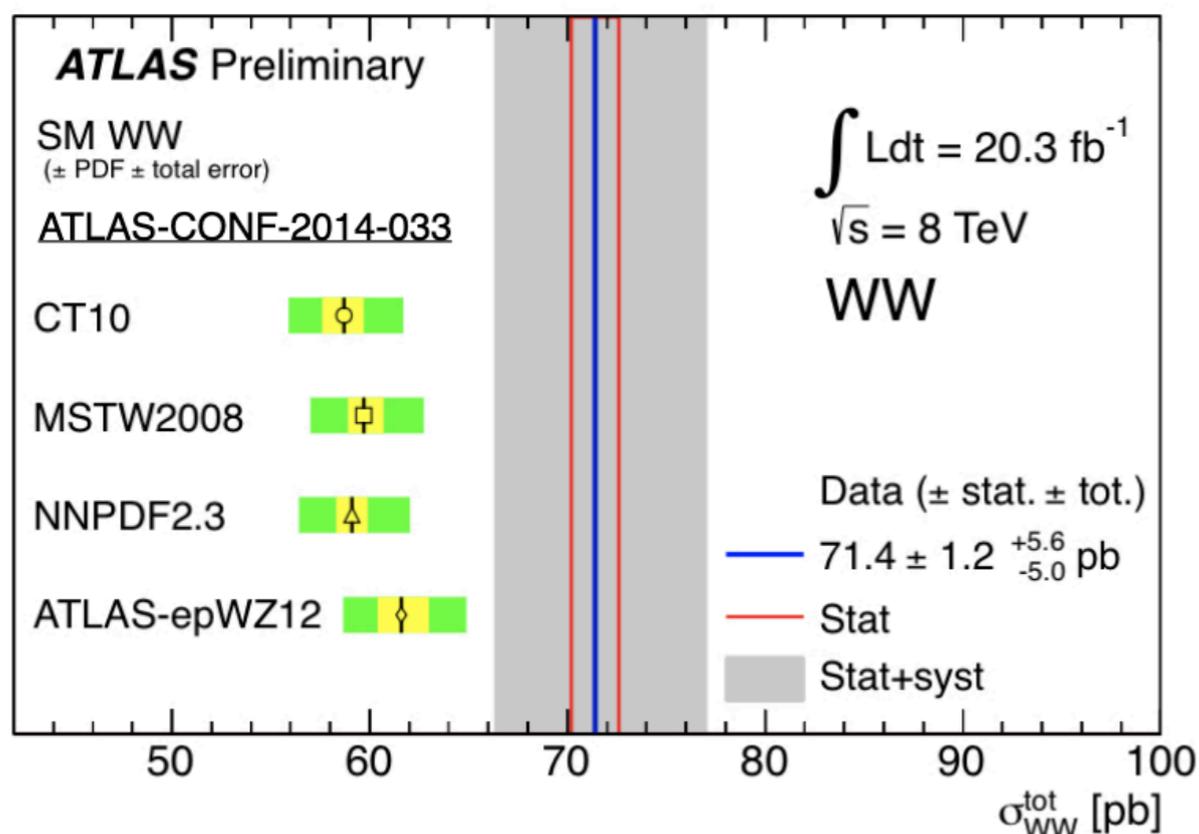
A. Huss, QCD@LHC-X 2020

# PARTONIC CROSS SECTION COMPUTATION: STATE OF THE ART



## EXAMPLE OF IMPACT OF NNLO CORRECTIONS

- Early ATLAS and CMS measurement (2013) of WW production off (NLO) theoretical predictions by about  $3\sigma$   $\rightarrow$  SUSY chargino production?
- Once NNLO (Gerhmann et al 2014) and resummation effects on jet-veto efficiency (Maede et al, Monni et al 2014) discrepancy is strongly reduced

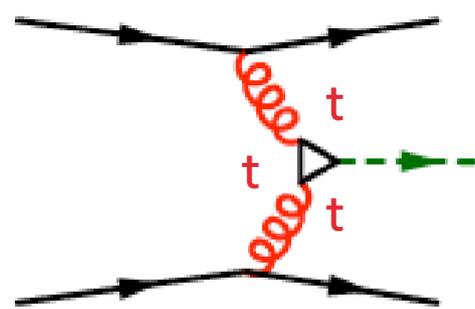


## N3LO: GLUON FUSION INTO HIGGS

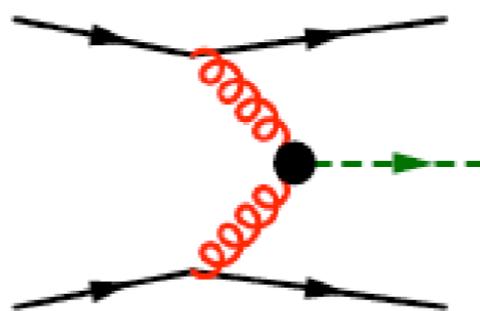
$pp \rightarrow H$  (via gluon fusion) is the main Higgs production channel at the LHC and is one of only four hadron collider processes known at N3LO (Anastasiou et al, [arXiv:1602.00695](https://arxiv.org/abs/1602.00695)) along with

- ★ Higgs production via vector boson fusion [Dreyer et al, arXiv:1811.07906](https://arxiv.org/abs/1811.07906)
- ★ Bottom pair fusion into Higgs [Duhr et al, arXiv:1904.09990](https://arxiv.org/abs/1904.09990)
- ★ Photon-mediated NC + CC Drell-Yan [Duhr et al, arXiv:2001.07717](https://arxiv.org/abs/2001.07717), [2007.13313](https://arxiv.org/abs/2007.13313)

$$\sigma(pp \rightarrow H) = (961 \text{ pb}) \times (\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \dots)$$



$m_t \rightarrow \infty$



Effective  
vertex

$$\alpha_s \equiv \alpha_s(M_H/2)$$

$$\sqrt{s_{pp}} = 13 \text{ TeV}$$

Here the perturbative series does not converge very well!

## N3LO: GLUON FUSION INTO HIGGS

- The perturbative series is written in terms of powers of  $\alpha_S(\mu_R)$ , with renormalisation scale  $\mu_R = \mu_0 = M_H/2$  and for a factorization scale  $\mu_F = \mu_0 = M_H/2$
- But  $\mu_R$  and  $\mu_F$  are arbitrarily chosen  $\sim M_H$  at any given perturbative order. Their variation introduces terms of higher perturbative order.

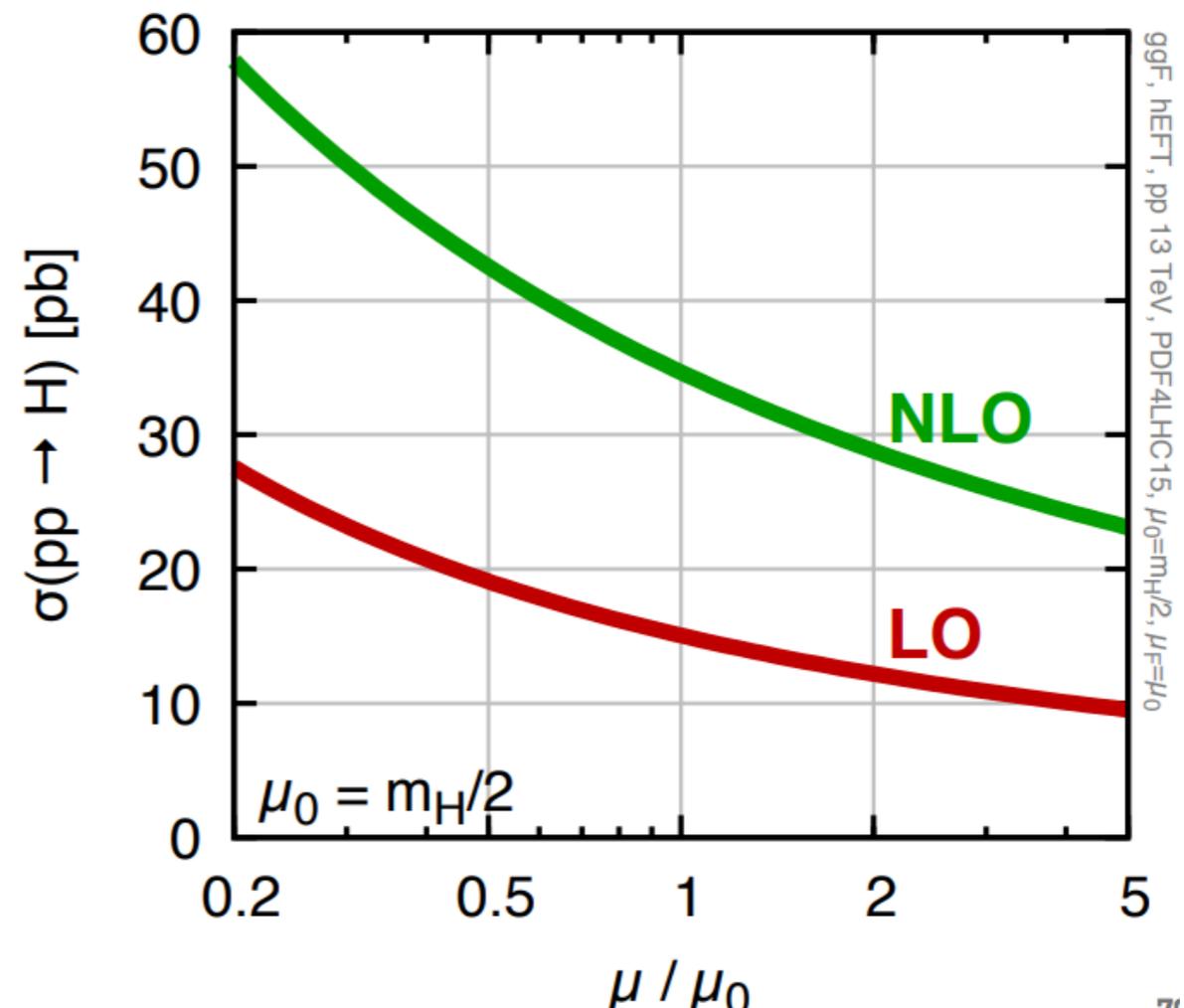
$$\alpha_S(\tilde{\mu}^2) = \alpha_S(\mu^2) - b_0 \log\left(\frac{\tilde{\mu}^2}{\mu^2}\right) \alpha_S^2(\mu^2)$$

## N3LO: GLUON FUSION INTO HIGGS

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- But  $\mu_R$  and  $\mu_F$  are arbitrarily chosen  $\sim M_H$  at any given perturbative order. Their variation introduces terms of higher perturbative order.
- Free to rewrite cross section in terms of  $\alpha_s$  for any choice of  $\mu_R$  (here keep  $\mu_F$  fixed for simplicity)

**NLO**

$$\sigma(pp \rightarrow H) = \sigma_0 \times \left( \alpha_s^2(\mu) + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2}) \alpha_s^3(\mu) \right)$$

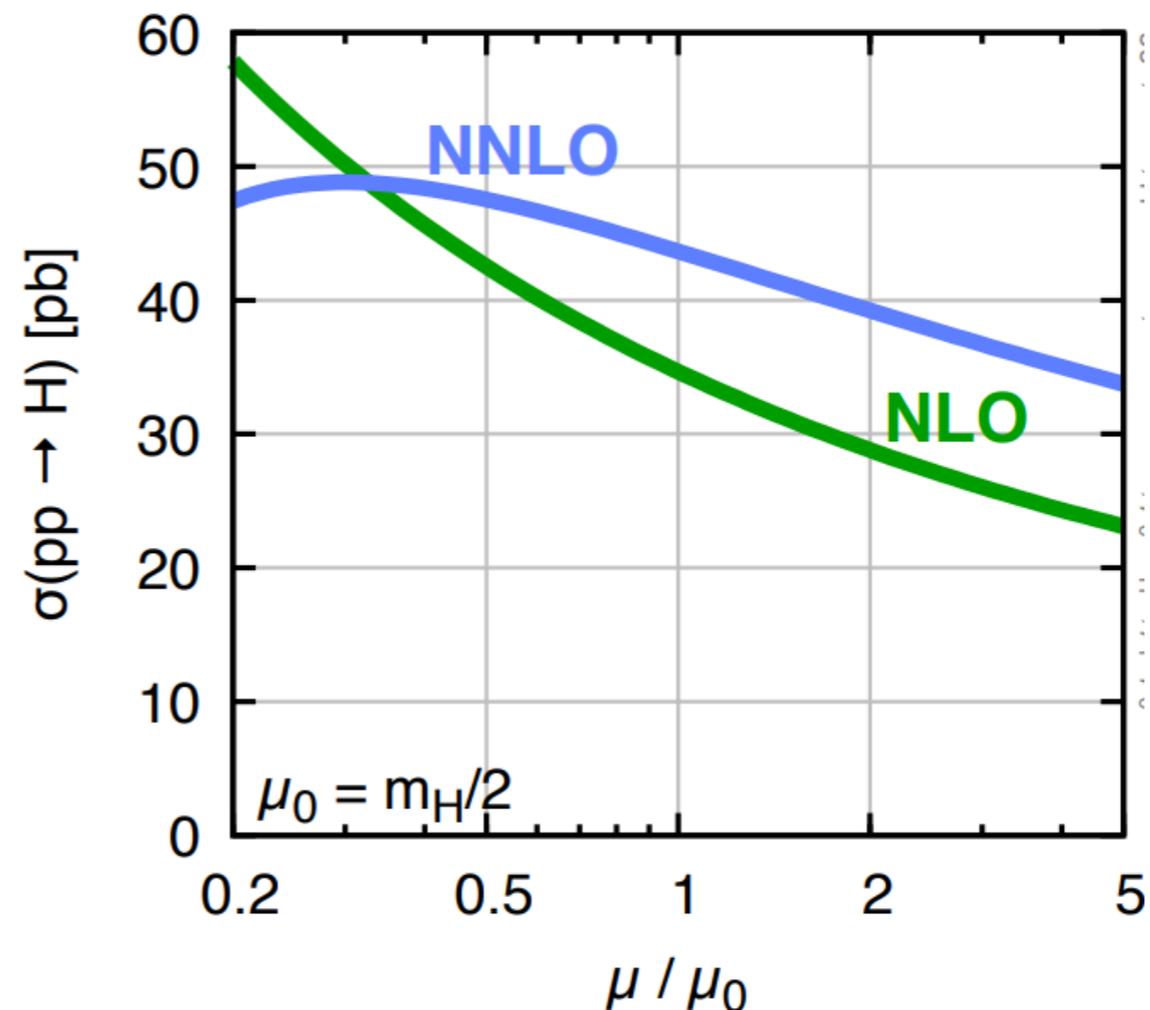


## N3LO: GLUON FUSION INTO HIGGS

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- Free to rewrite cross section in terms of  $\alpha_s$  for any choice of  $\mu_R$  (here keep  $\mu_F$  fixed for simplicity)

**NNLO**

$$\begin{aligned} \sigma(pp \rightarrow H) = & \sigma_0 \times \left( \alpha_s^2(\mu) \right. \\ & \left. + (10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2}) \alpha_s^3(\mu) \right. \\ & \left. + c_4(\mu) \alpha_s^4(\mu) \right) \end{aligned}$$

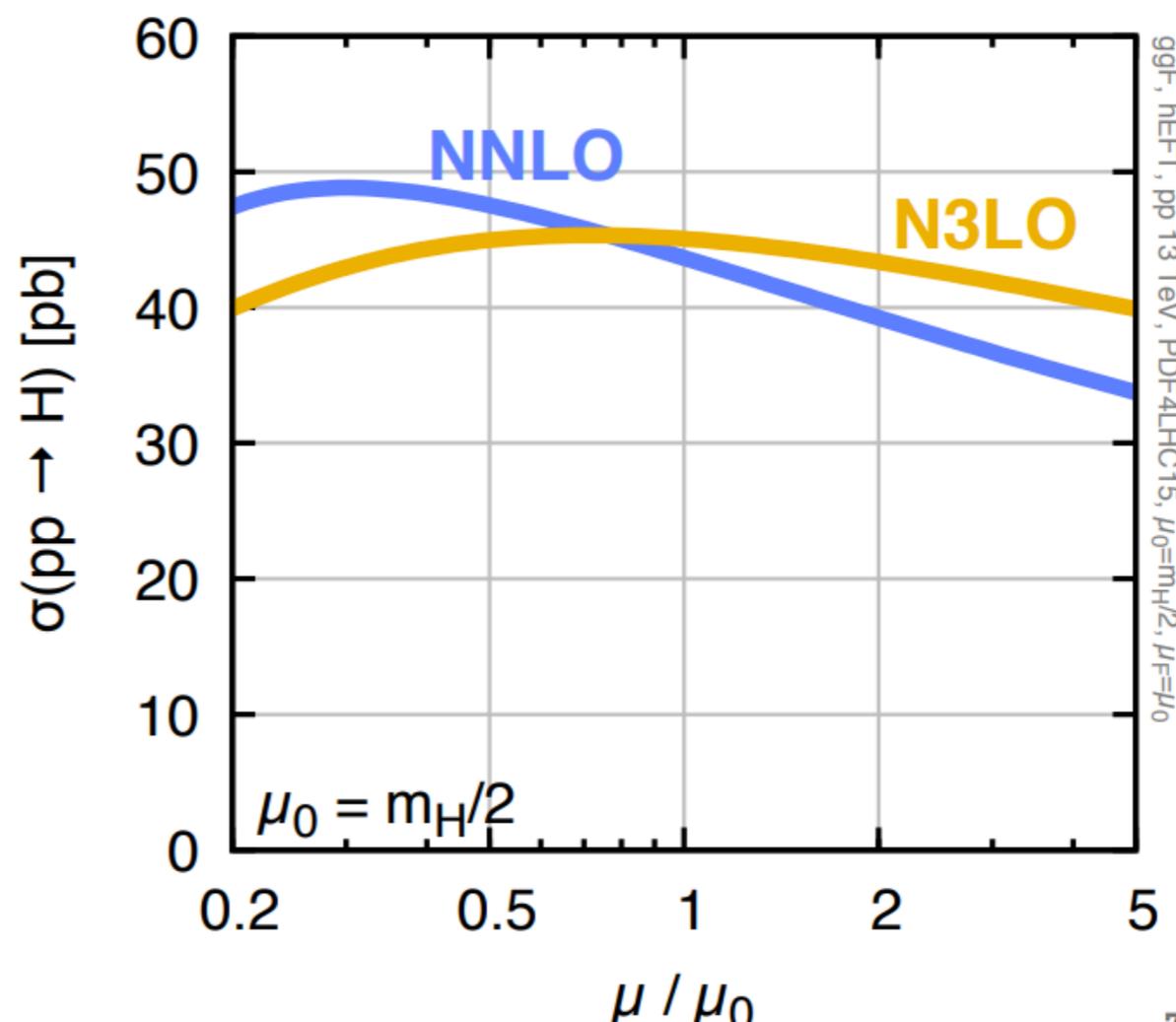


## N3LO: GLUON FUSION INTO HIGGS

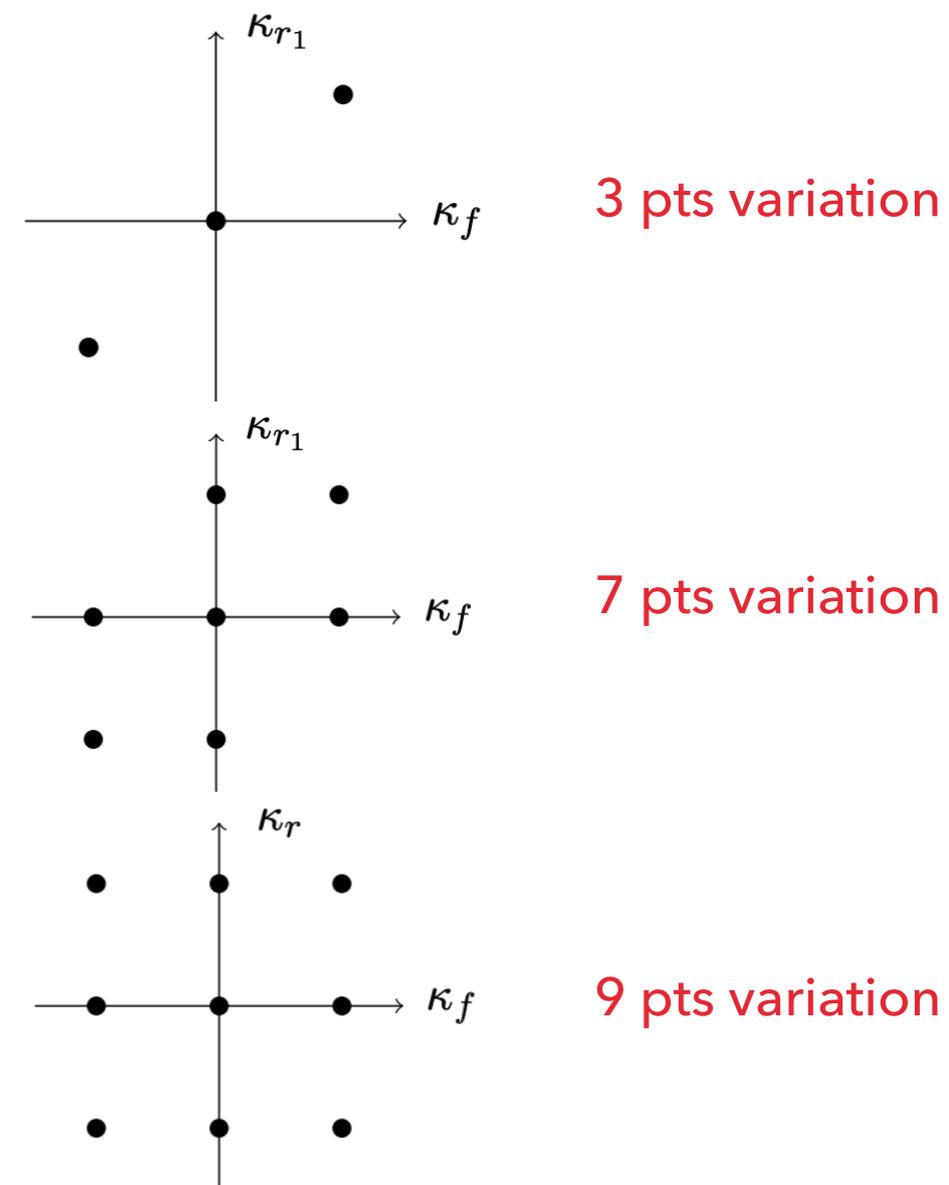
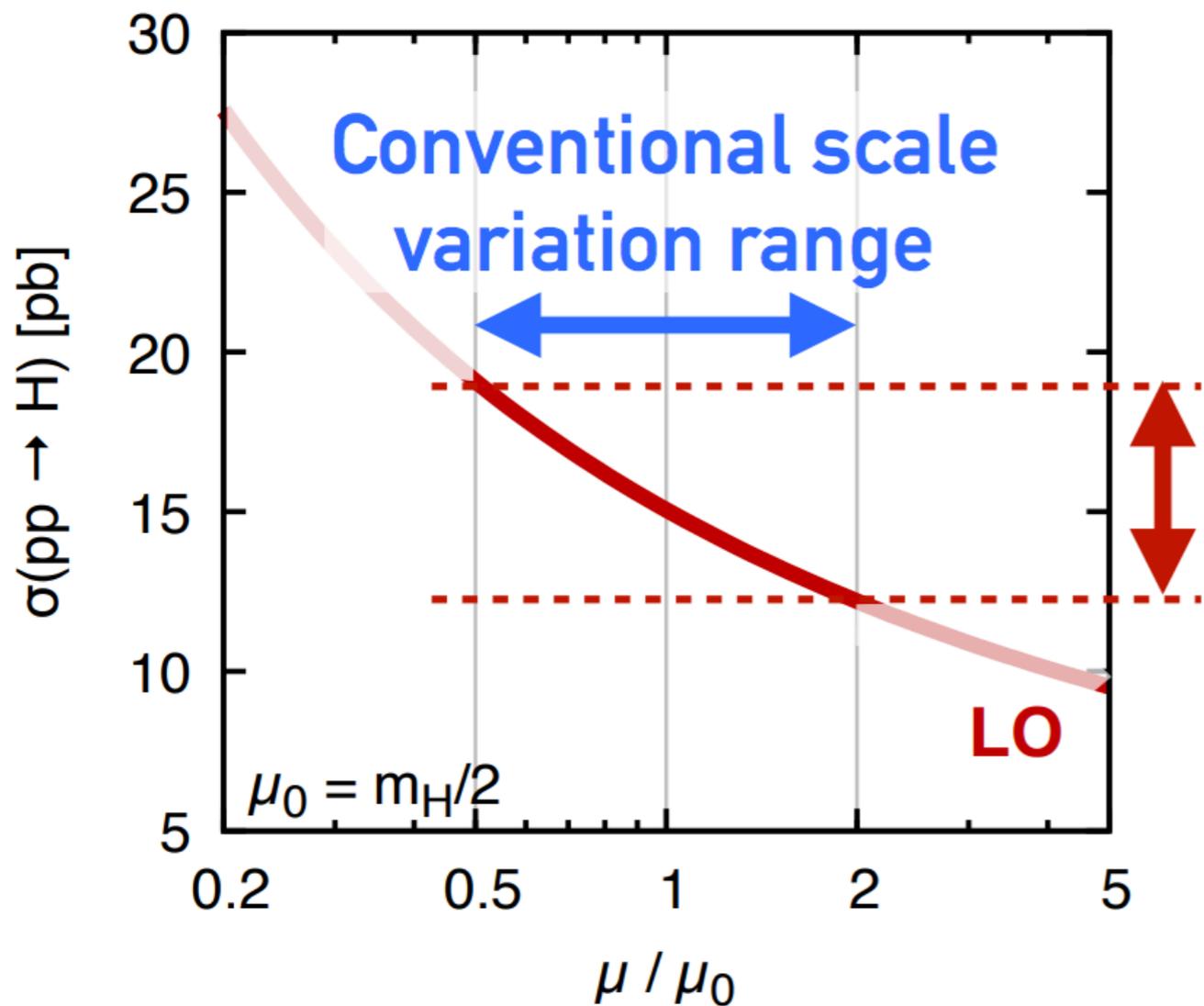
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### N3LO

$$\begin{aligned} \sigma(pp \rightarrow H) = & \sigma_0 \times \left( \alpha_s^2(\mu) \right. \\ & + \left( 10.4 + 2b_0 \ln \frac{\mu^2}{\mu_0^2} \right) \alpha_s^3(\mu) \\ & \left. + c_4(\mu) \alpha_s^4(\mu) + c_5(\mu) \alpha_s^5(\mu) \right) \end{aligned}$$

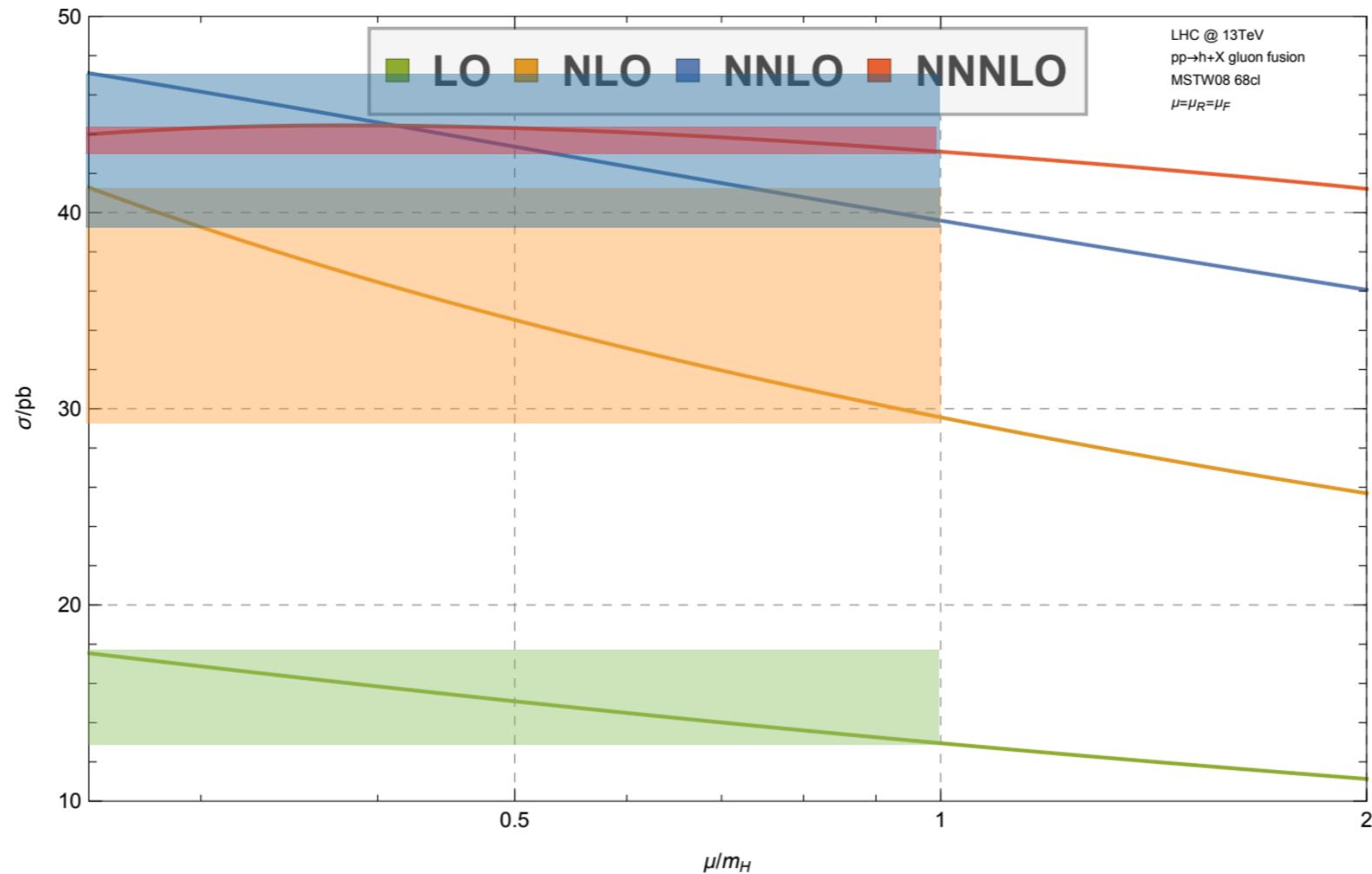


# N3LO: GLUON FUSION INTO HIGGS



Convention: missing higher order uncertainties (or scale uncertainty or theory uncertainty) is estimated by change of cross section when varying  $\mu_R$  and  $\mu_F$  scales up and down by a factor of 2.

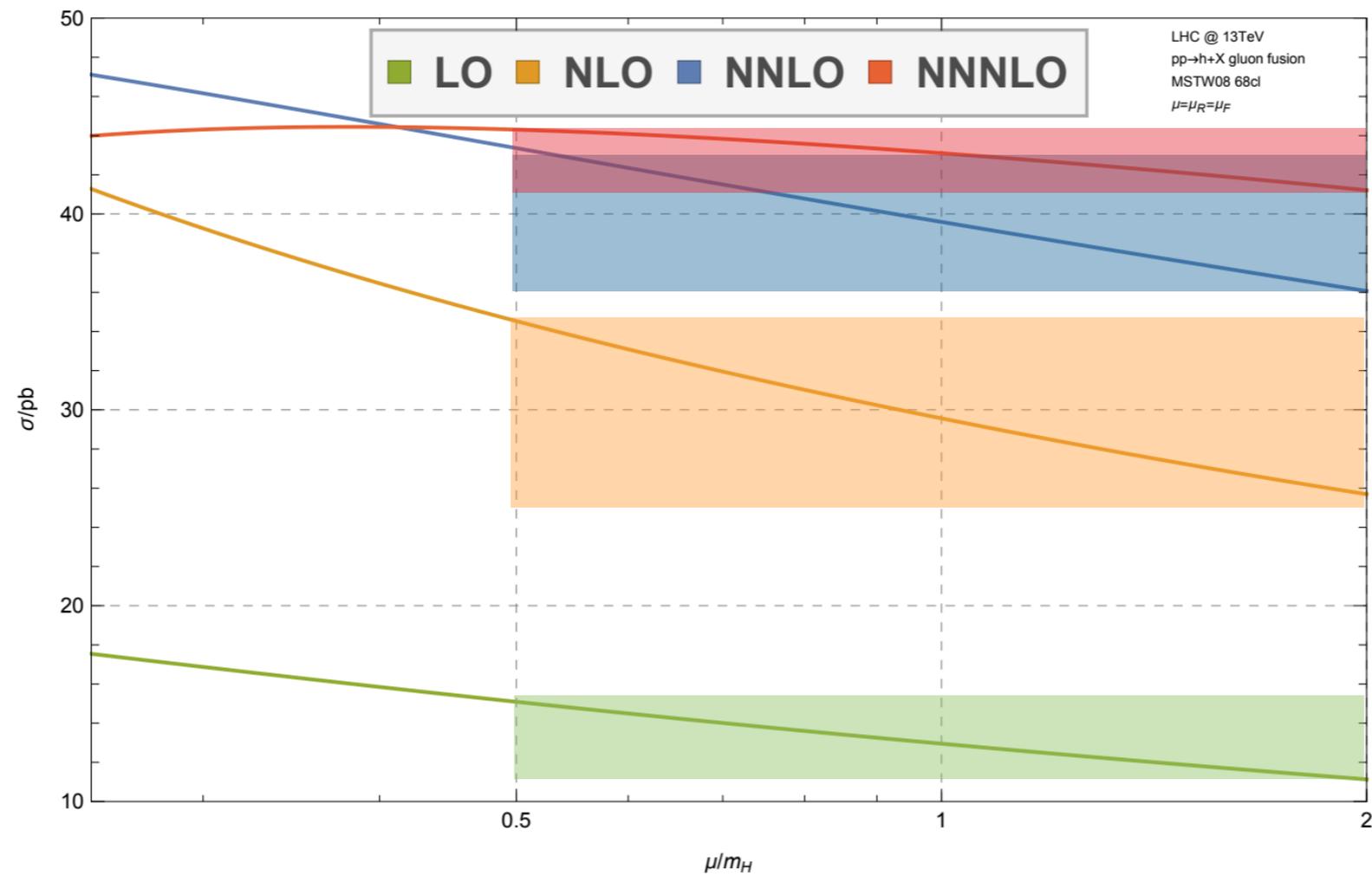
# N3LO: GLUON FUSION INTO HIGGS



$$\mu_0 = \frac{M_H}{2}$$

The perturbative series displays convergence only at NNLO for  $\mu_0 = M_H/2$   
 Good news is that, at N3LO, the choice of  $\mu_R$  and  $\mu_F$  does not matter much

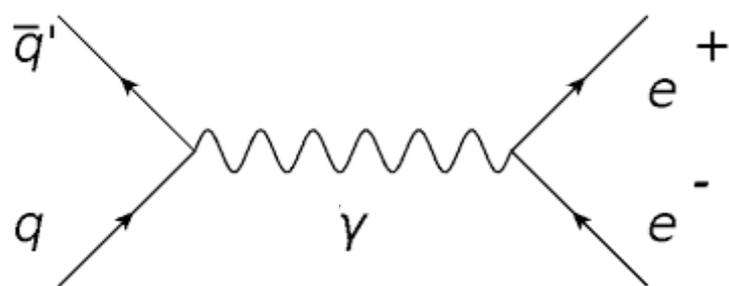
# N3LO: GLUON FUSION INTO HIGGS



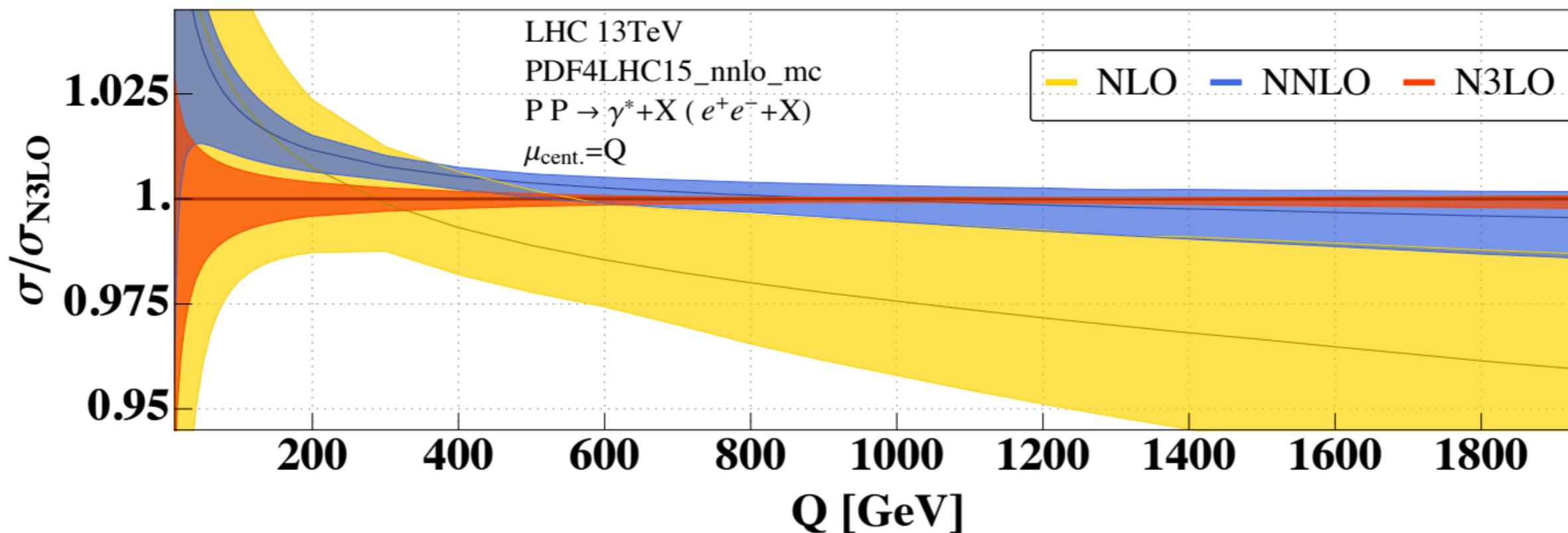
$$\mu_0 = M_H$$

# N3LO: DRELL-YAN PRODUCTION

Durh et al 2001.07717

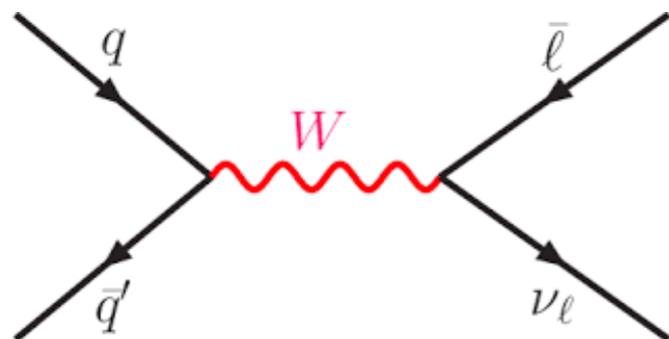


- ▶ Results at NLO and NNLO indicated a good convergence of the perturbative series. Size of the MHOU estimated at % level at NNLO.
- ▶ In contrast to the corrections to Higgs production, the shift of the predicted value of the DY cross section due to the inclusion of N3LO corrections is not contained in the naive scale variation bands of NNLO predictions for  $Q \lesssim 400$  GeV.

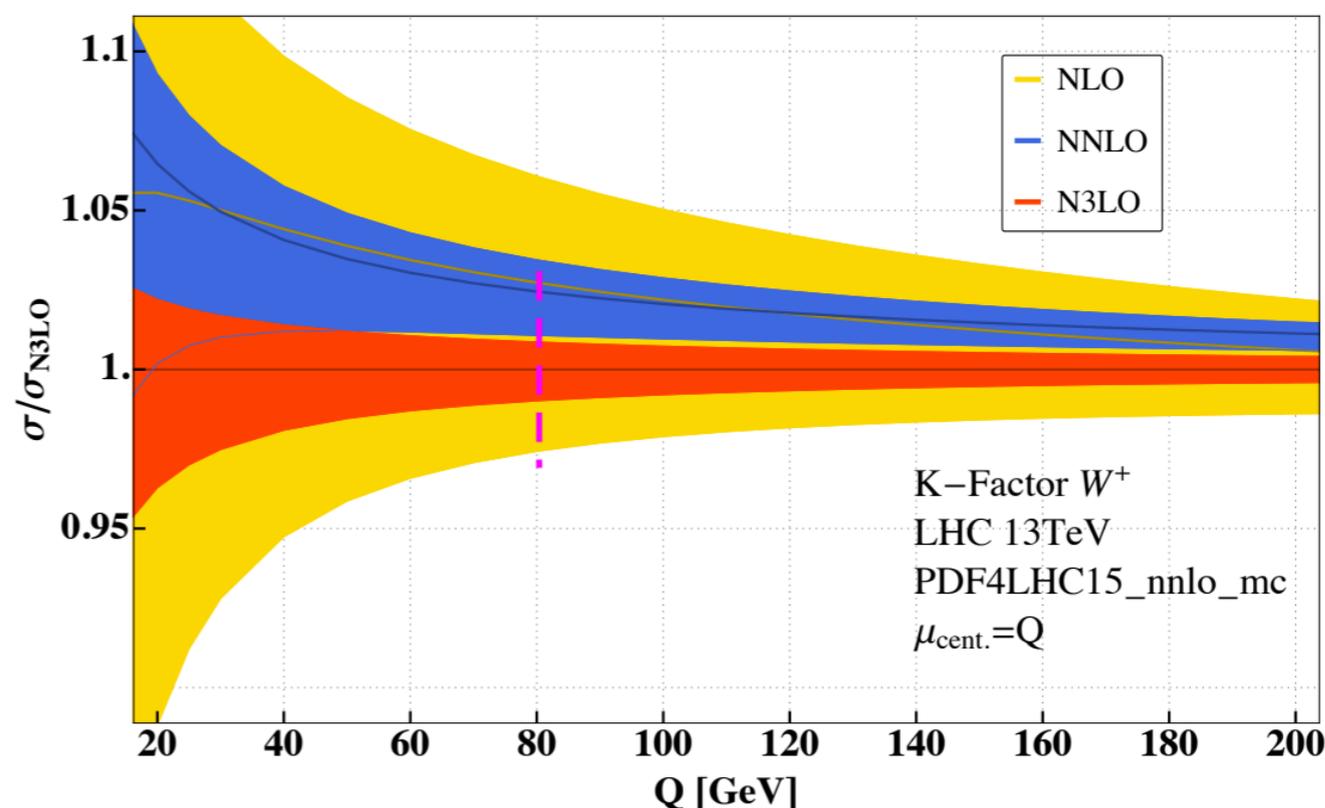
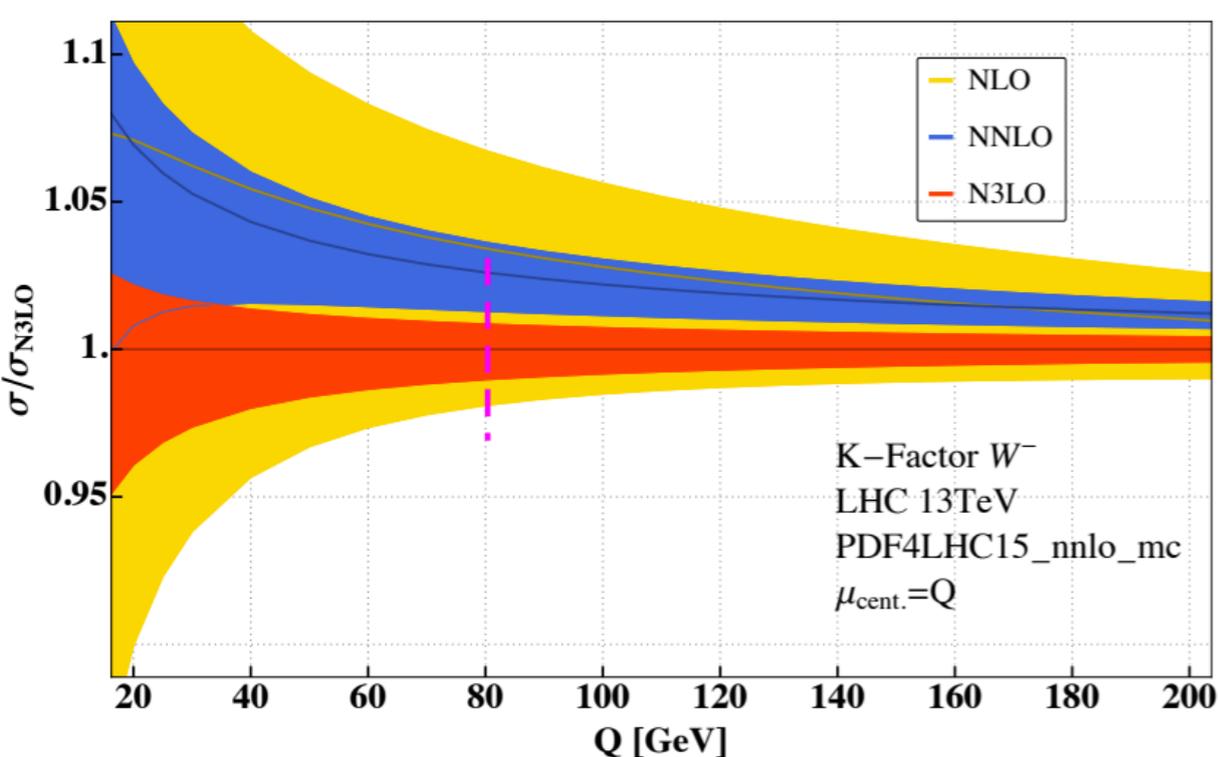


# N3LO: DRELL-YAN PRODUCTION

Durh et al, 2007.13313



- ▶ Similar situation for  $W^+$  and  $W^-$  production
- ▶ Rather than failure of perturbative series, results probably indicate underestimate of MHOUs. Improved method to estimate it? See for example [Bonvini 2006.16293](#)
- ▶ Also mismatch between perturbative order in partonic cross section and PDFs (see tomorrow's lecture)



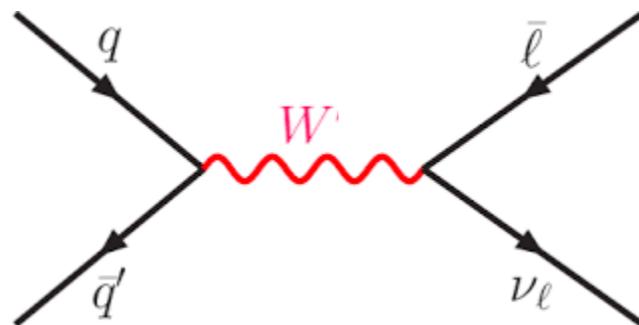


# BEYOND FIXED-ORDER QCD

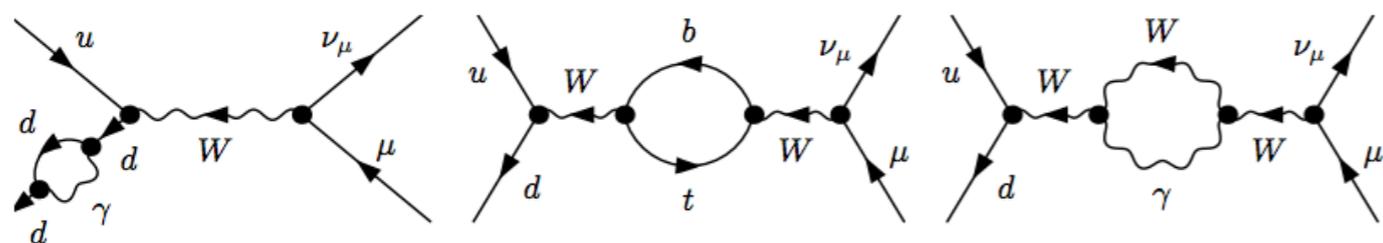
$$\sigma^{pp \rightarrow ab} = \sum_{i,j=-n_f}^{n_f} \int dz_1 dz_2 f_i(z_1, \mu_F) f_j(z_2, \mu_F) \hat{\sigma}^{ij \rightarrow ab}(z_1 z_2 S, \alpha_s(\mu_R), \mu_F) + \mathcal{O}\left(\frac{\Lambda^n}{S^n}\right)$$

# #1: EW CORRECTIONS

- Because  $\alpha_{EM}(Mz) \sim \alpha_S(Mz)/10 \implies$  NLO EW corrections  $\sim$  NNLO QCD corrections



$$\mathcal{O}(\alpha_{EM}^2) \times [1]$$



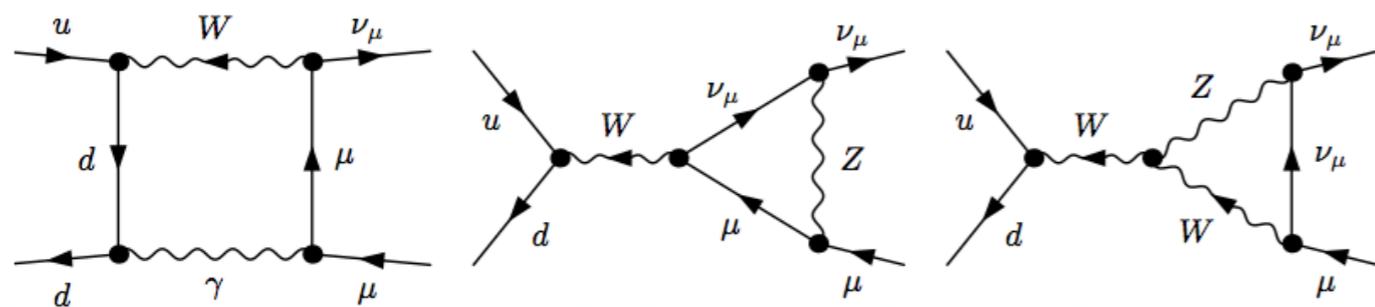
(a)

(b)

(c)

Virtual corrections

$$\mathcal{O}(\alpha_{EM}^2) \times [1 + \alpha_{EM} C_1]$$

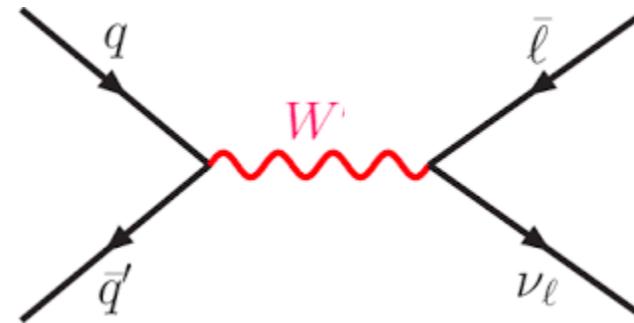


$$+ \frac{3\alpha_{EM}}{\pi M_W^2} \log\left(\frac{s}{M_W^2}\right)$$

At large  $s$  these logs can become large

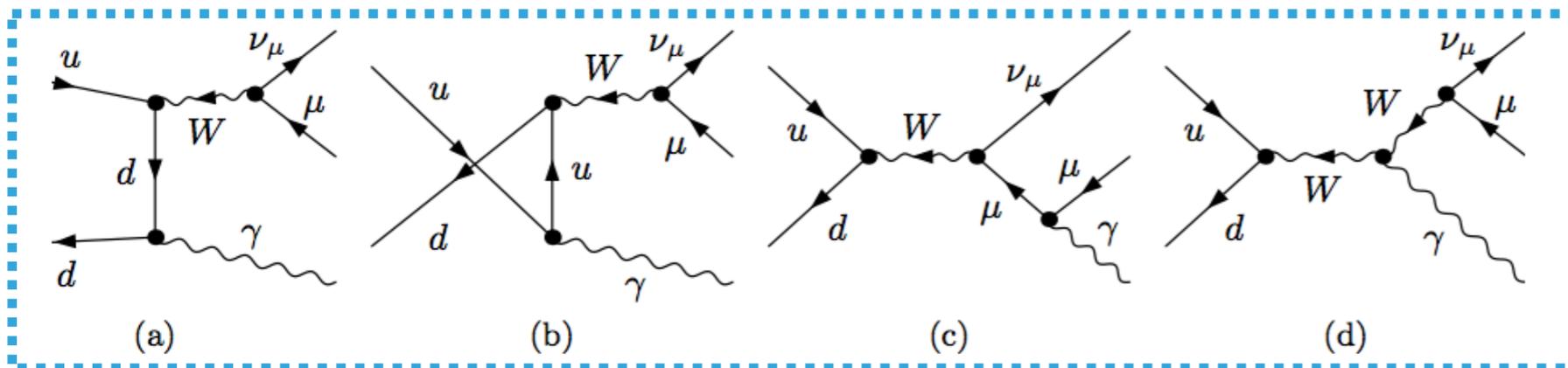
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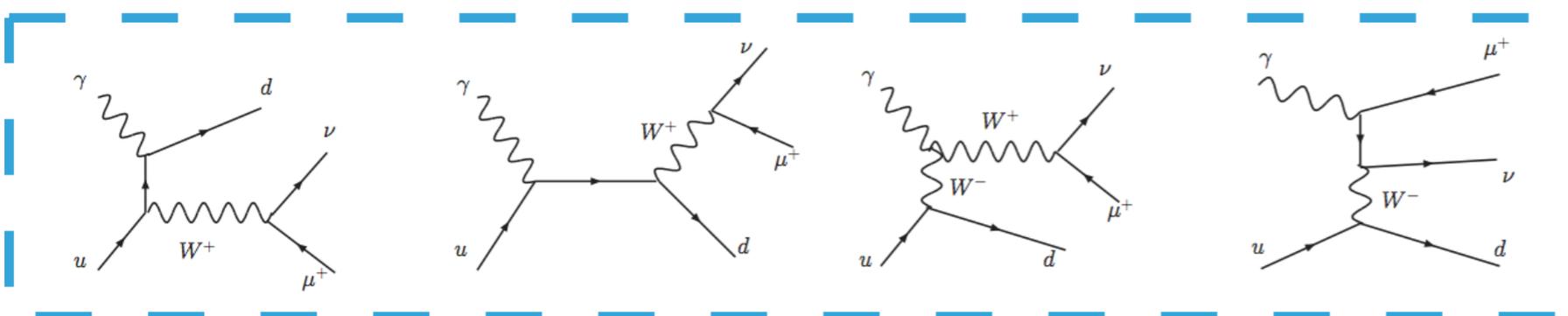


$$\mathcal{O}(\alpha_{EM}^2) \times [1]$$

Real corrections - quark initiated

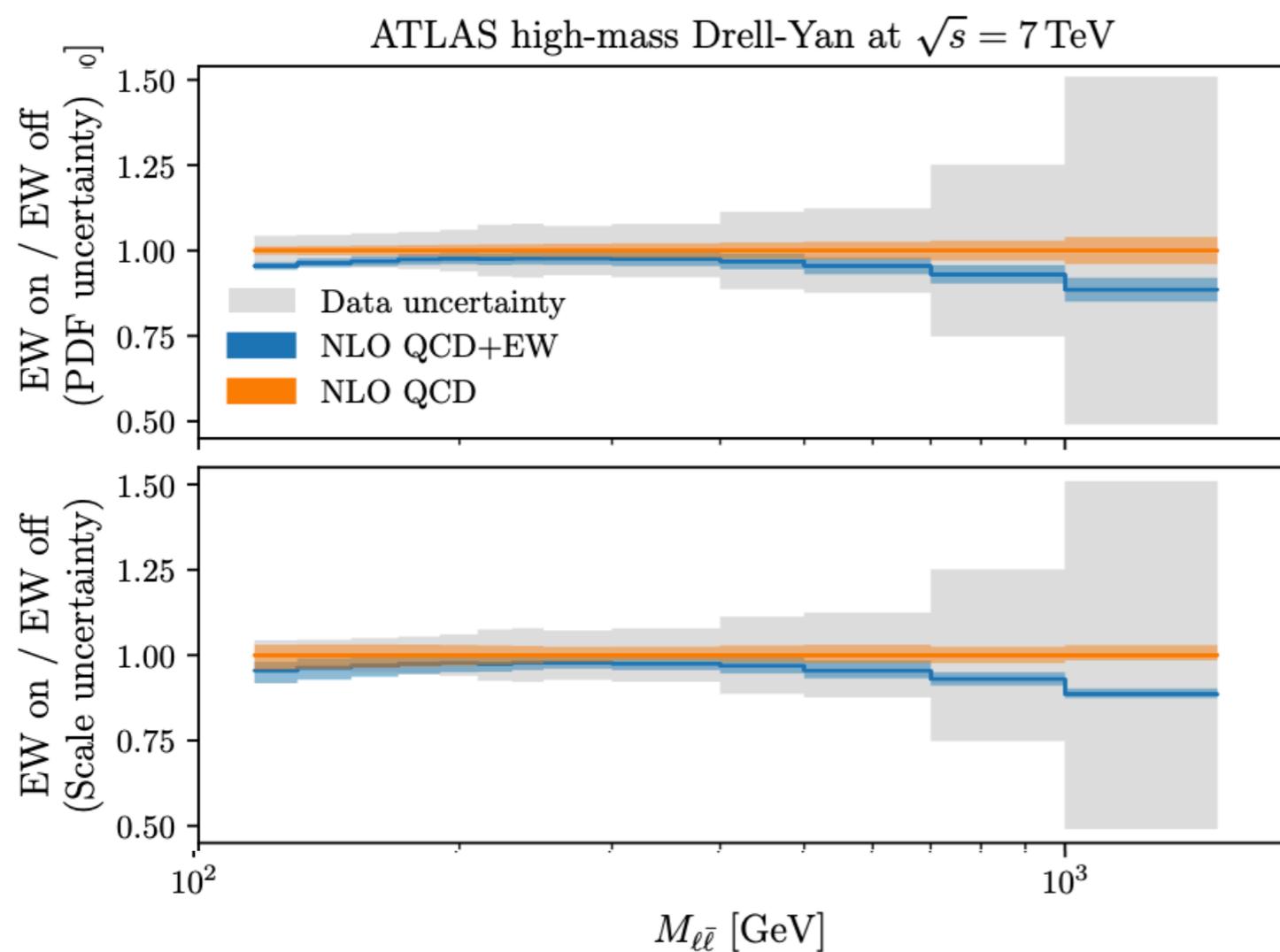


Real corrections - photon initiated

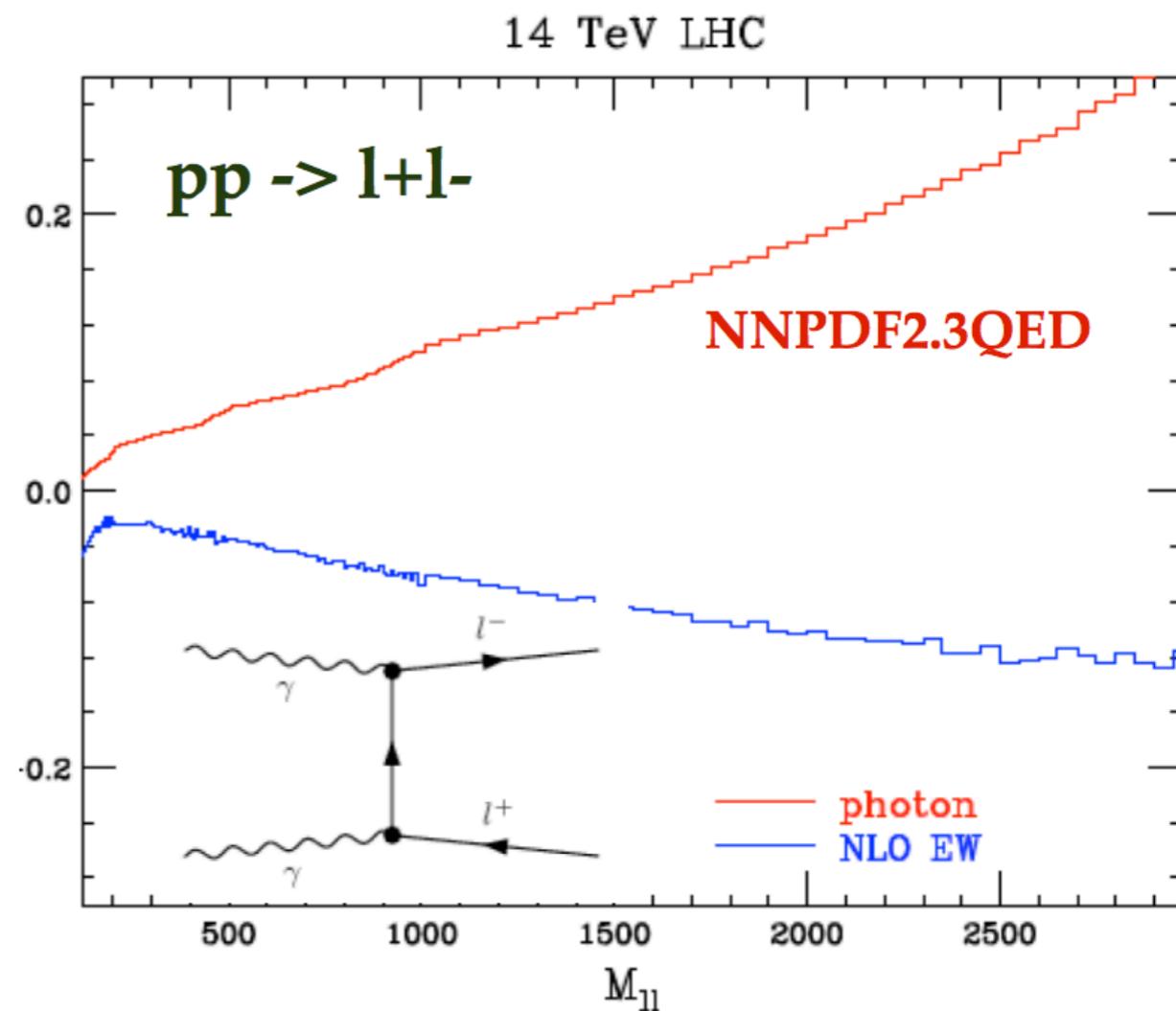


# #1: EW CORRECTIONS

- NLO EW corrections become large in the large M and pT region of lepton but partially compensated by photon-initiated real corrections

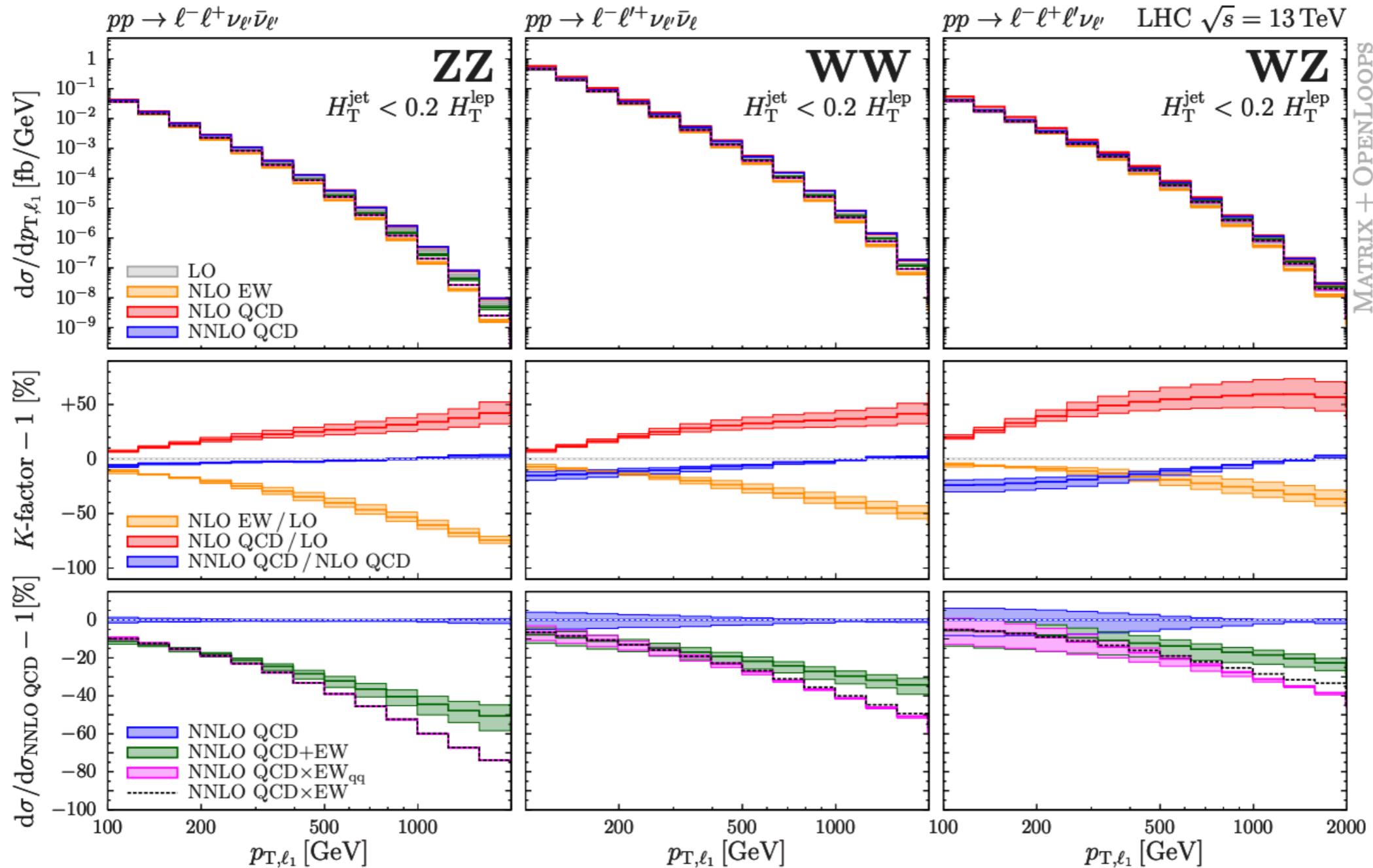


Carrazza et al, arXiv:2008.12789



Boughezal et al Phys.Rev. D89 (2014)3, 034030

# #1: FRONTIER: NNLO QCD + NLO EW

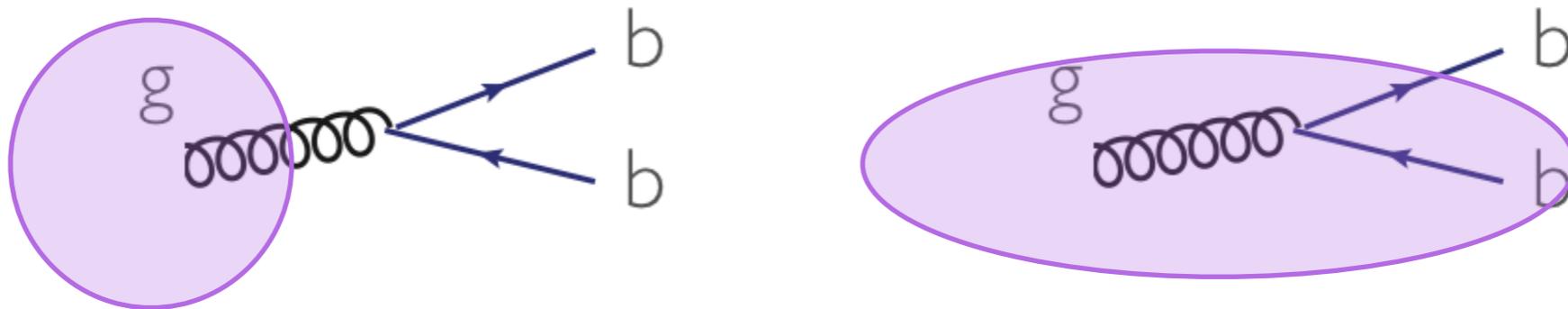


MATRIX + OPENLOOPS

- Lots of activity in merging NNLO QCD and NLO EW calculations  
[Grazzini et al 1912.0068](#), [Bonciani et al 1911.06200](#), [Bonciani et al 2007.06518](#),...

## #2: 4FS VERSUS 5FS SCHEMES

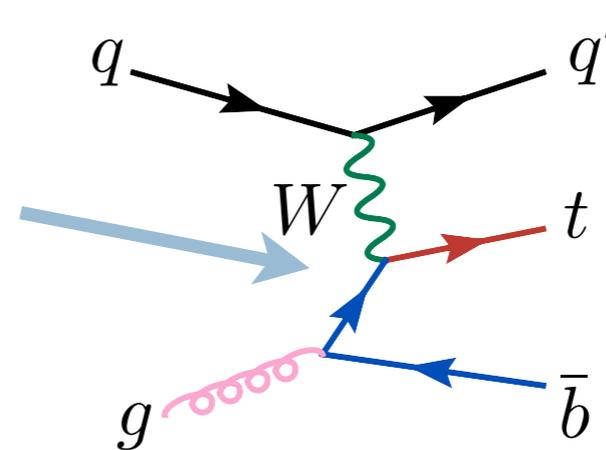
- For all processes that feature bottom quarks at the hard-process level there are two ways of performing computations: **4F (four-flavour)** and **5F(five-flavour)** schemes



- Take a 4F scheme, in which bottom quark does not belong to the proton wavefunction

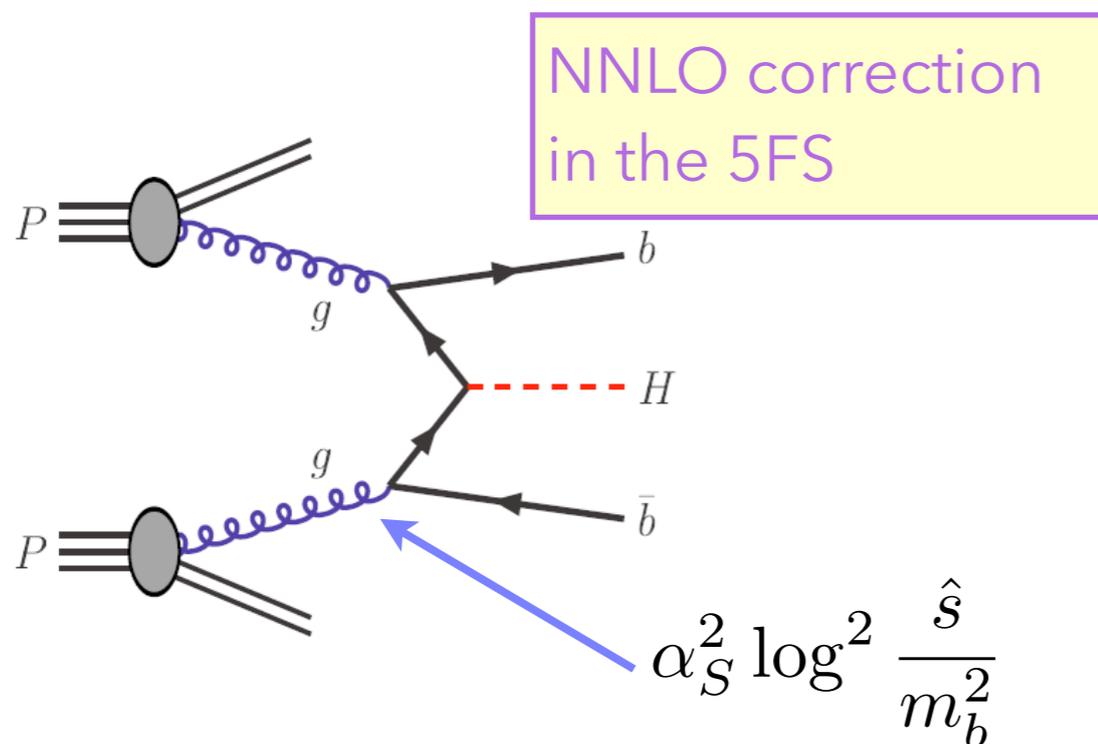
$$\frac{1}{t - m_b^2} \sim \frac{1}{p_T^2 + m_b^2}$$

$$t = (p_{\bar{b}} - p_g)^2, p_T^2 = p_{T,\bar{b}}^2$$



$$\int_0^{p_{T,max}^2} \frac{dp_T^2}{p_T^2 + m_b^2} = \log \left( \frac{p_{T,max}^2}{m_b^2} \right)$$

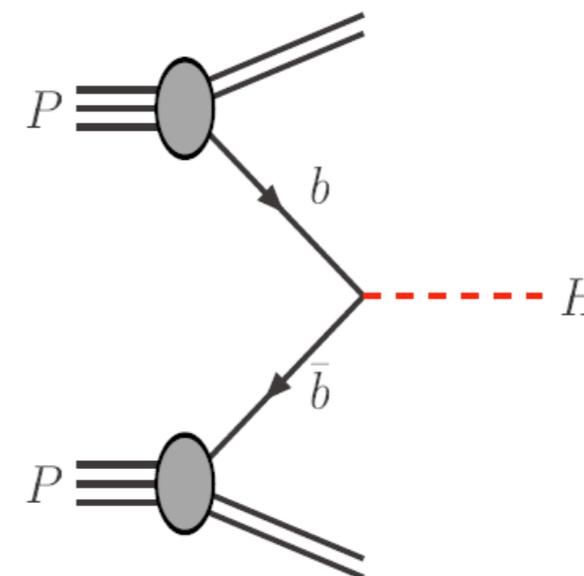
## #2: 4FS VERSUS 5FS SCHEMES



4F scheme

- ✗ It does not resum possibly large logs, yet it has them explicitly
- ✗ Computing higher orders is more difficult
- ✓ Mass effects are there at any order
- ✓ Straightforward implementation in MC event generators at LO and NLO

Decoupling or massive scheme

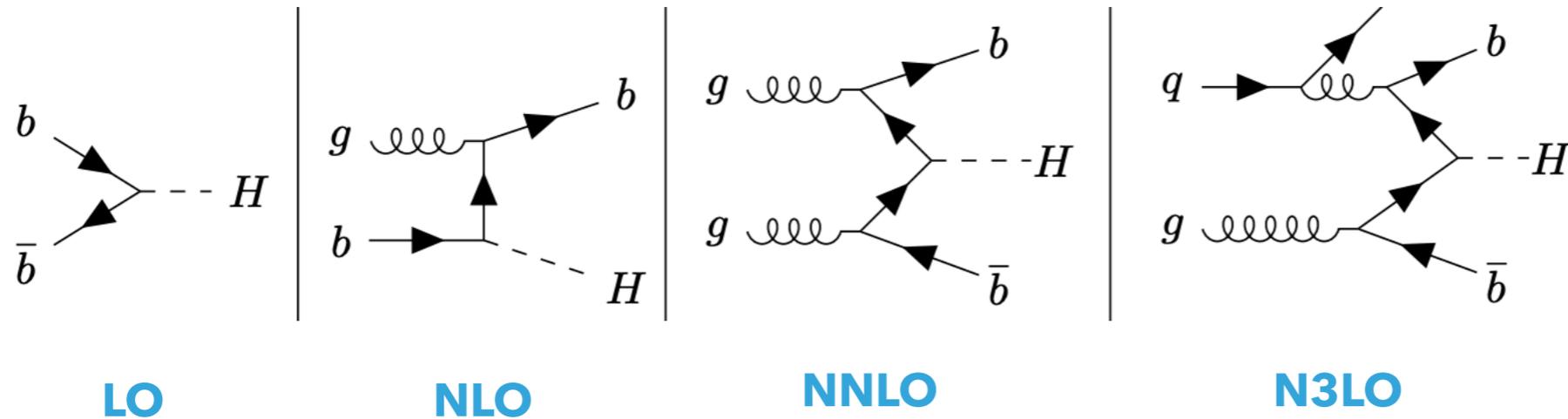


5F scheme

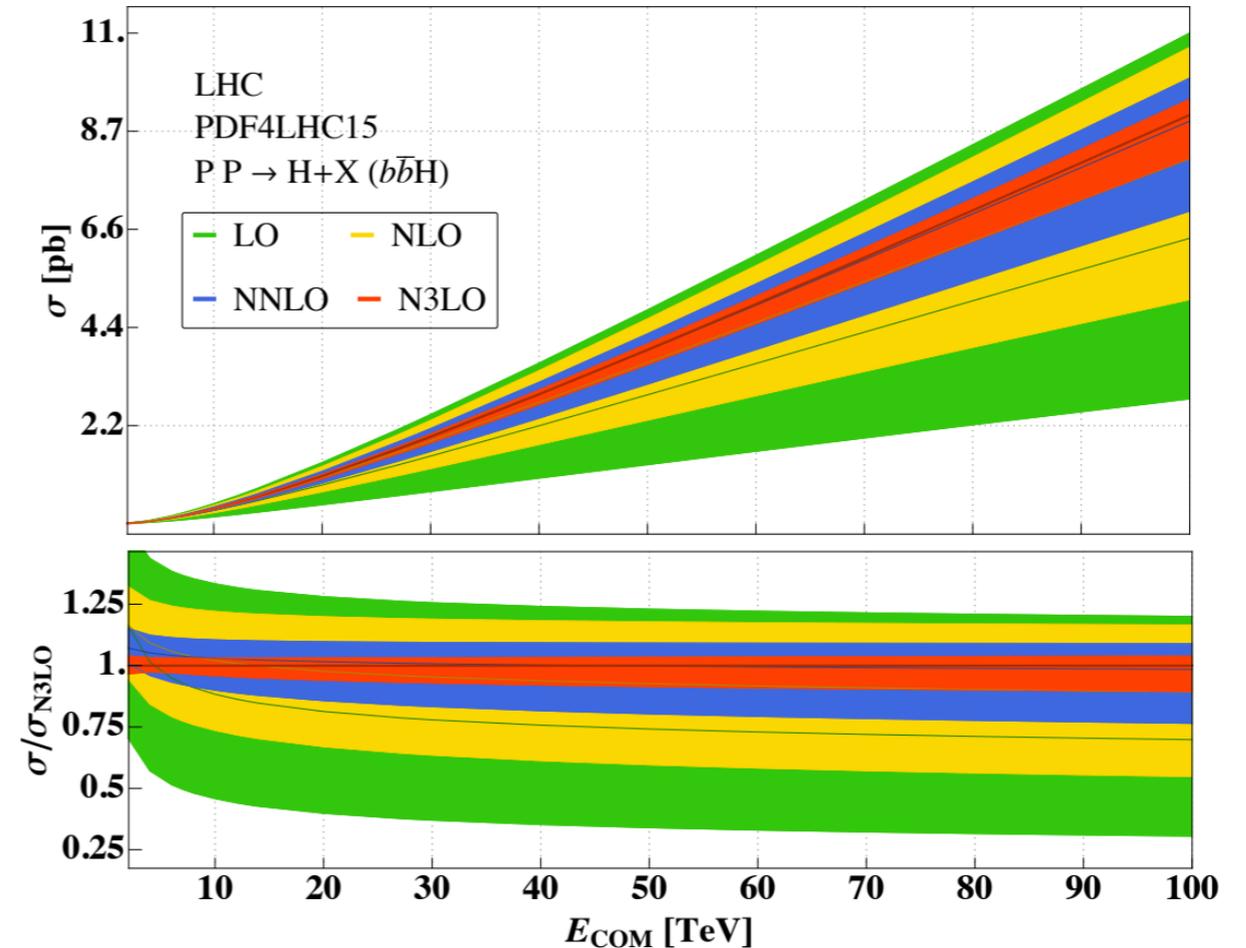
- ✓ It resums initial state large logs into b-PDFs leading to more stable predictions
- ✓ Computing higher orders is easier
- ✗  $p_T$  of bottom enters at higher orders
- ✗ Implementation in MC depends on the gluon splitting model in the PS

Massless scheme

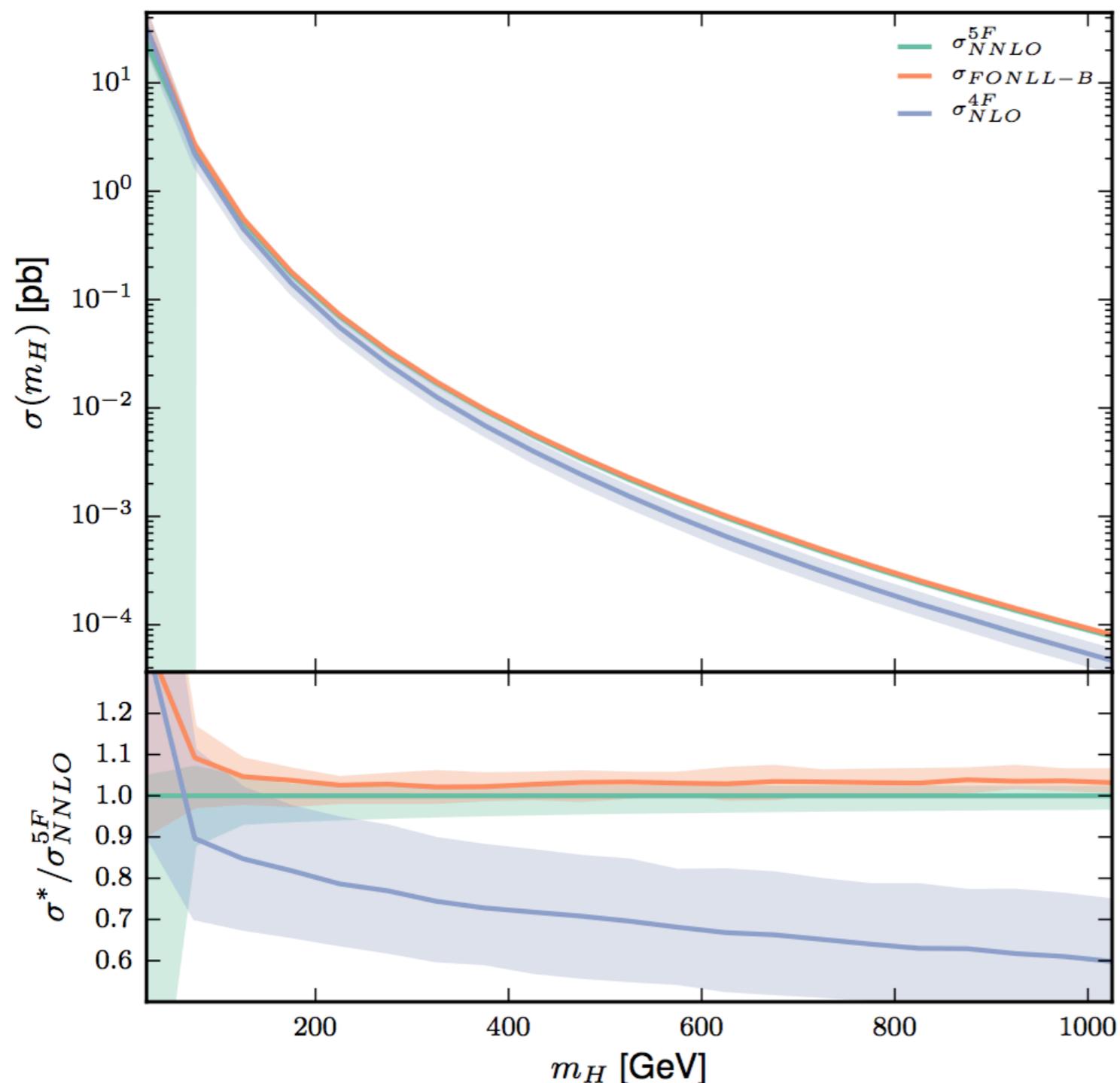
## #2: 4FS VERSUS 5FS - BOTTOM FUSION INTO HIGGS



- Bottom quark fusion into Higgs is suppressed in the SM, but enhanced in new physics models in which there are two Higgs doublets (2HDM) or a triplet
- Here perturbative series converges well in the 5-Flavour-Scheme in which bottom quark ( $m_b \sim 4.5$  GeV) is neglected and  $b$  treated as massless parton



## #2: FRONTIER: 4FS VERSUS 5FS MATCHING



- Predictions in the 4F and 5F schemes consistently combined in consistent matching of bottom quark mass effects with resummation of collinear logarithms
- Lots of activity in matching processing in massive versus massless schemes in heavy quarks initiate processes, both at inclusive and differential level

Forte et al, arXiv:1508.01529

Forte et al, arXiv:1607.00389

Duhr, et al, arXiv:2004.04752

## #3: RESUMMATIONS

- ▶ Many observables studied at the LHC depend on more than one scale; single or double logs of the ratio of those scales at all orders in perturbation theory.
- ▶ If the logarithms are large the convergence of the series is spoiled

Fixed order

$$\frac{\sigma}{\sigma_0} = 1 \quad \text{LO}$$

$$+ c_1 \alpha \quad \text{NLO}$$

$$+ c_2 \alpha^2 \quad \text{NNLO}$$

$$+ \dots$$

All orders (L=some large log)

$$\ln \frac{\sigma}{\sigma_0} = \alpha^n L^{n+1} \quad \text{LL}$$

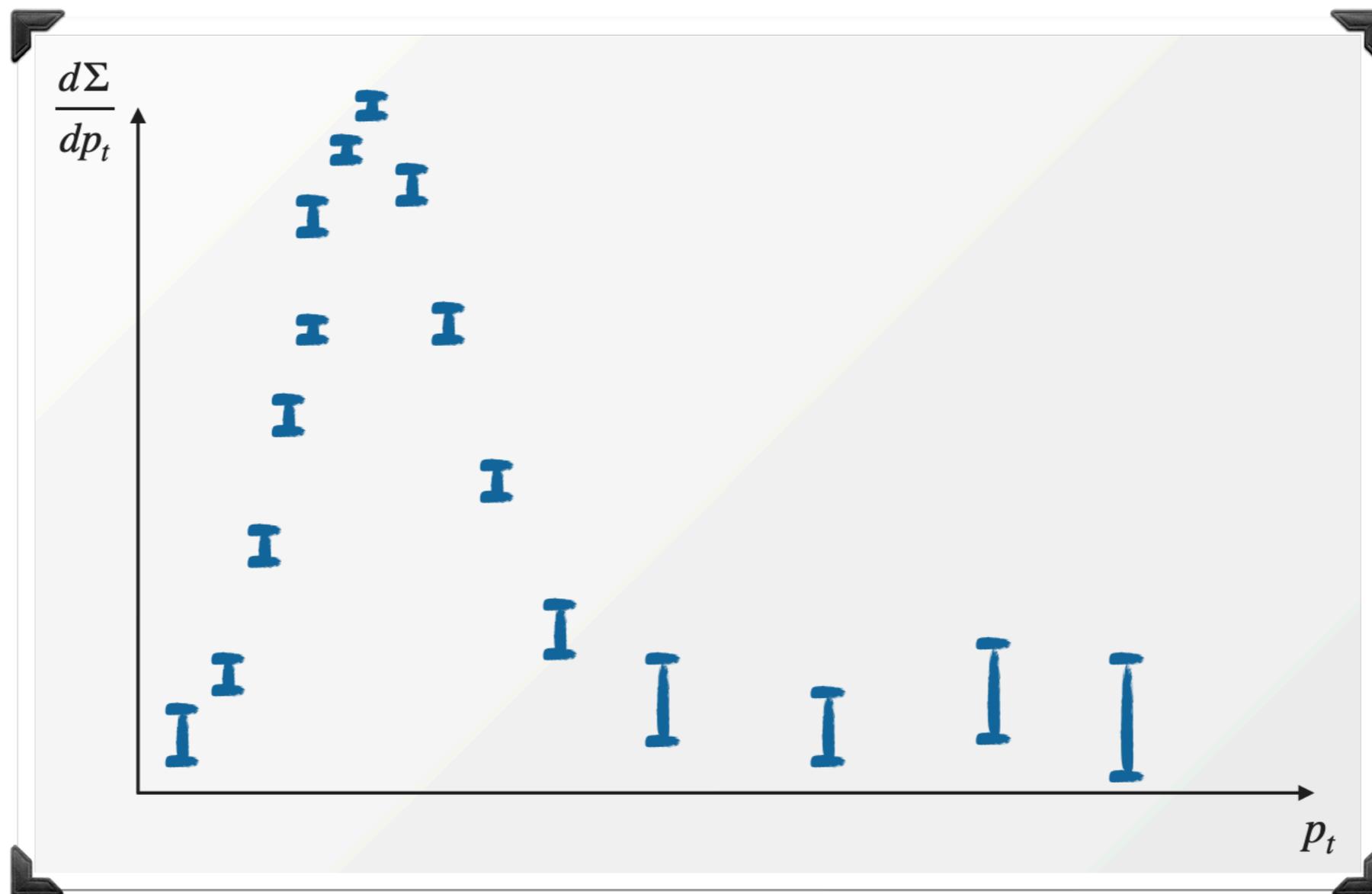
$$+ \alpha^n L^n \quad \text{NLL}$$

$$+ \alpha^n L^{n-1} \quad \text{NNLL}$$

$$+ \dots$$

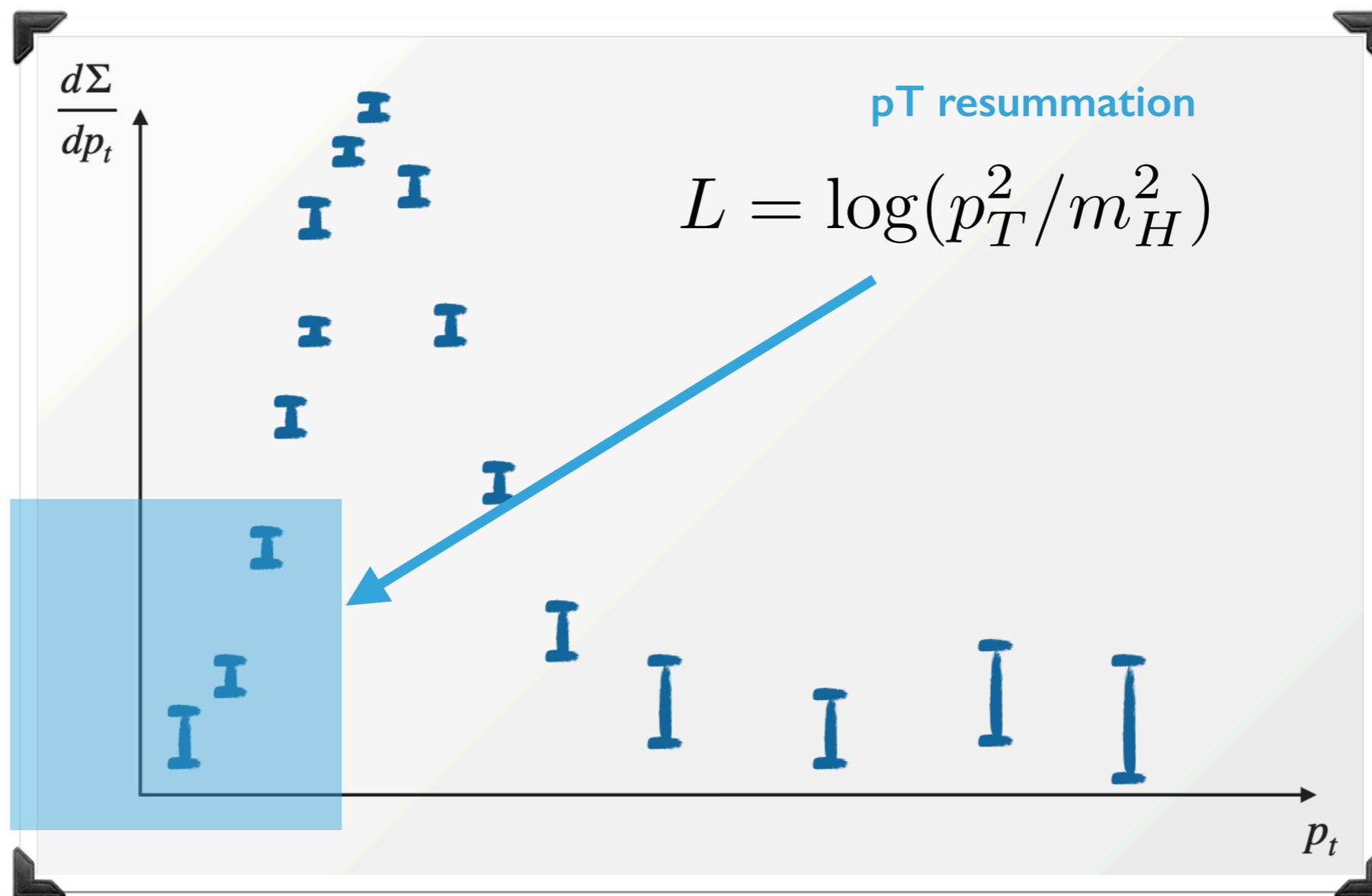
## #3: RESUMMATIONS

Take the Higgs transverse momentum spectrum



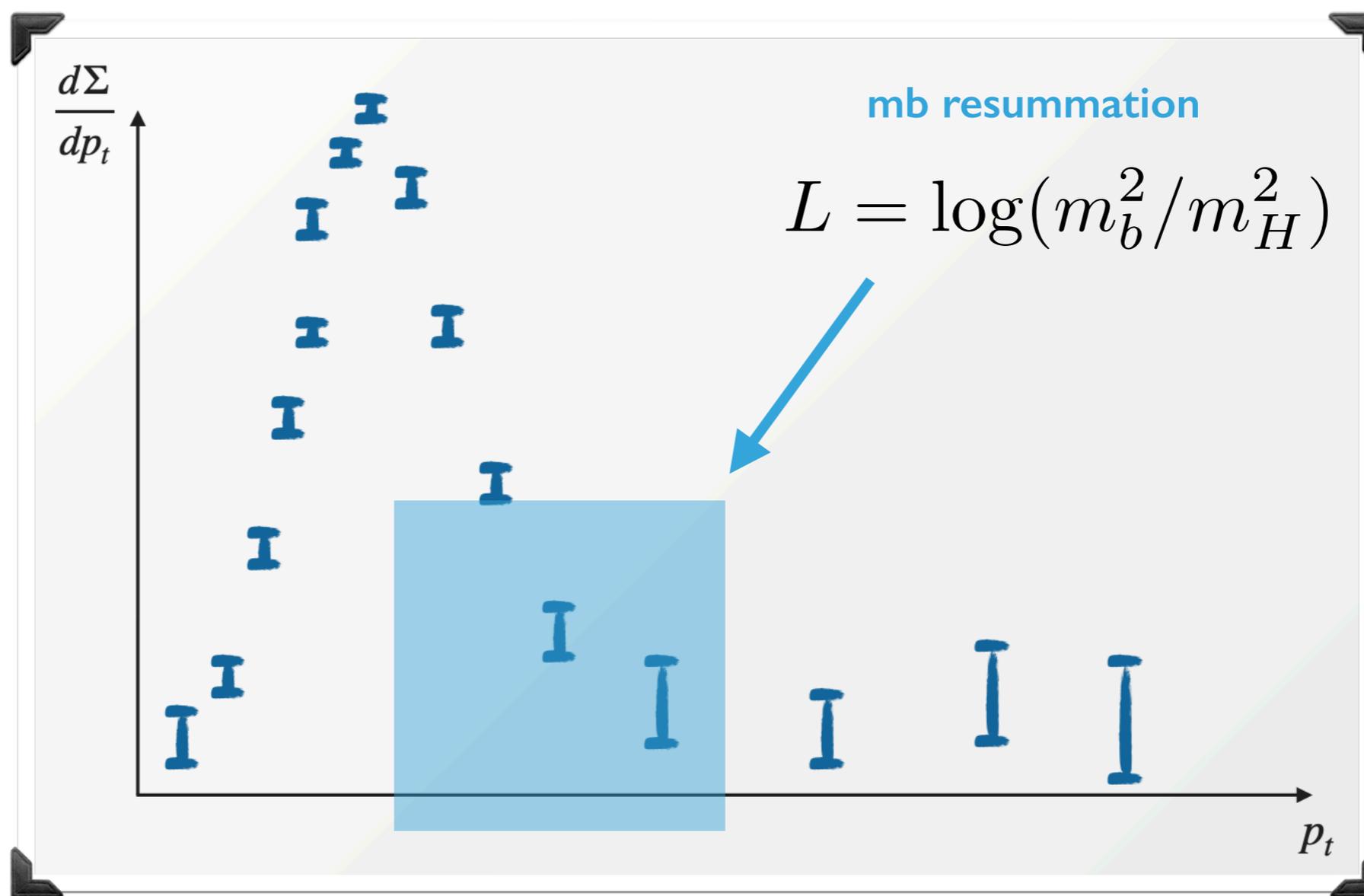
## #3: RESUMMATIONS

Take the Higgs transverse momentum spectrum



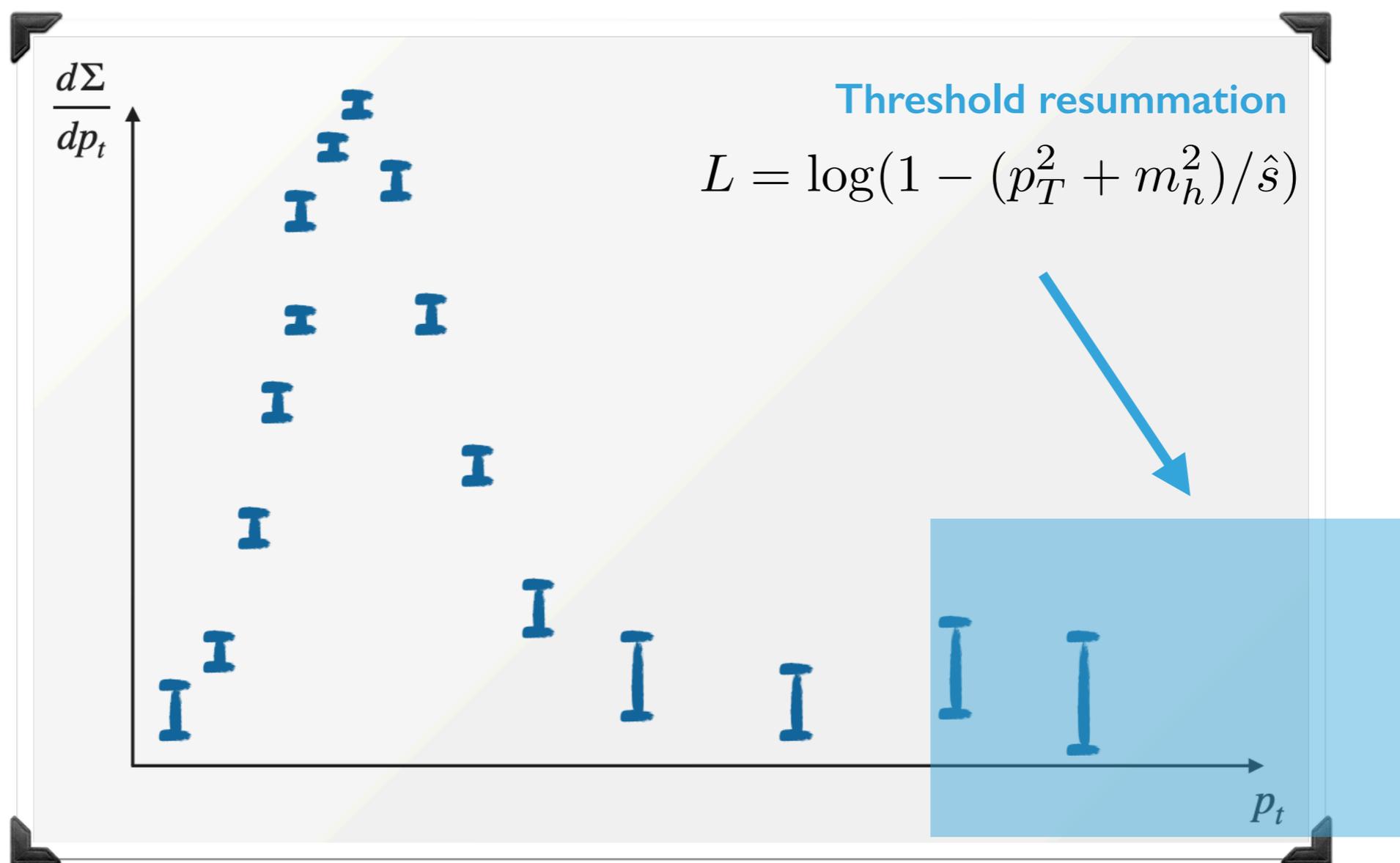
## #3: RESUMMATIONS

Take the Higgs transverse momentum spectrum



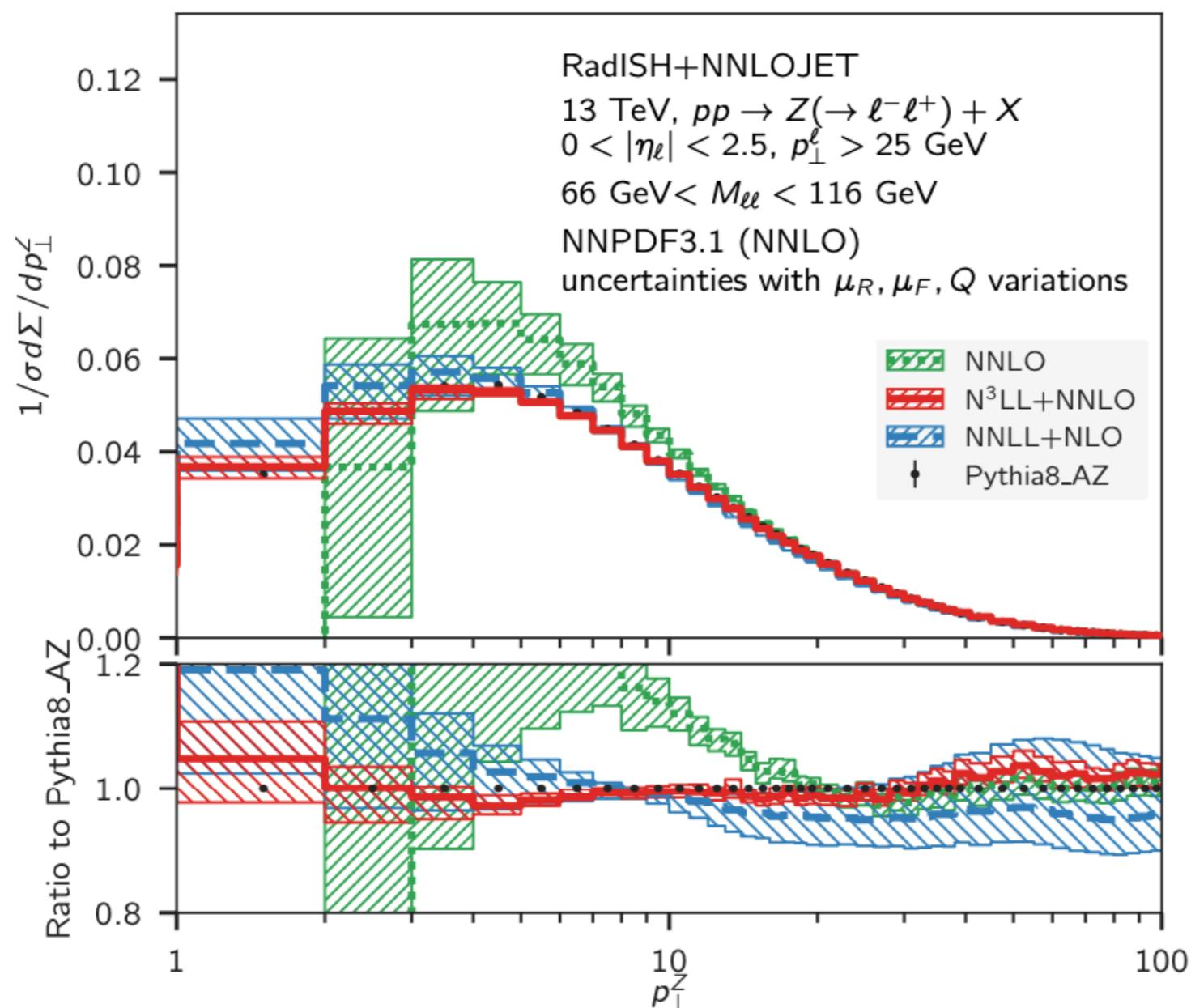
## #3: RESUMMATIONS

Take the Higgs transverse momentum spectrum



## #3: FRONTIER: COMBINED RESUMMATIONS

Bizon et al, 1905.05171



- Huge recent progress in precision thanks to resummation, for example:
- Transverse Z pT resummation at NNNLL matched with NNLO [Bizon et al, 1905.05171]
- Combined QED and QCD pT resummation [Cieri et al, 1805.11948]
- Higgs pT at NNNLL+NNLO [Bizon et al, 1705.09127]

- Huge progress also in combined resummation: pT and small-x [Forte et al 1511.05561], pT and large-x [Marzani 1511.06039, Muselli et al 1701.01464], small-x and large-x [Bonvini et al 1802.07758], pT and jet radius [Banfi et al 1511.02886] ...

## REALISTIC EVENT SIMULATION

- Only time to talk about fixed order perturbative QCD+EW calculations (some matched with large log resummation), where the final state looks clean and well defined
- But real events look much more messy, and we need to connect fixed-order (resummed) QCD+EW calculations to realistic simulation of the hadronic final state.
- This is achieved by parton showers Monte Carlo event generators that bridge the gap from the hard interaction scale  $Q$  down to the hadronisation scale  $O(1 \text{ GeV})$
- Need to be able to describe an arbitrarily number of parton branchings, i.e. need to 'dress' partons (and leptons) with radiation and turn partons into hadrons (hadronization).

# HUGE PROGRESS IN MONTE CARLO TOOLS

- NLO+ (leading-log) PS well-established , automated and used in all advanced LHC analyses [MG5\_aMCatNLO, POWHEG, SHERPA...]
- Frontiers: NNLO + PS and NLL parton showers

Lagrangian  
Matrix elements  
(QCD and EW)  
Resummation  
Parton showers  
Hadronisation  
Jet algorithms  
MPI ...

Theory



Detector simulation  
Pions, Kaons, ...  
Reconstruction  
B-tagging efficiency  
Boosted decision  
tree  
Neural network  
...

Experiment

## (PARTIAL) CONCLUSIONS

- Precision physics opens up new fascinating challenges
- I hope I conveyed the feeling that, as the community has move forward at an incredible rate, still a lot of interesting and necessary developments to work on
- Only time to mention few of them: NNLO for higher multiplicities, N3LO, theory uncertainties, EW corrections and matching with QCD corrections, heavy flavour schemes, large log resummations and combined resummations, NNLO + PS, NLO parton showers.
- Did not have time to mention huge progress in jets physics and Monte Carlo simulations
- Tomorrow: PDFs
- On Wednesday: precision as a key to new physics

**THANK YOU FOR YOUR ATTENTION!**

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**EXTRA MATERIAL**

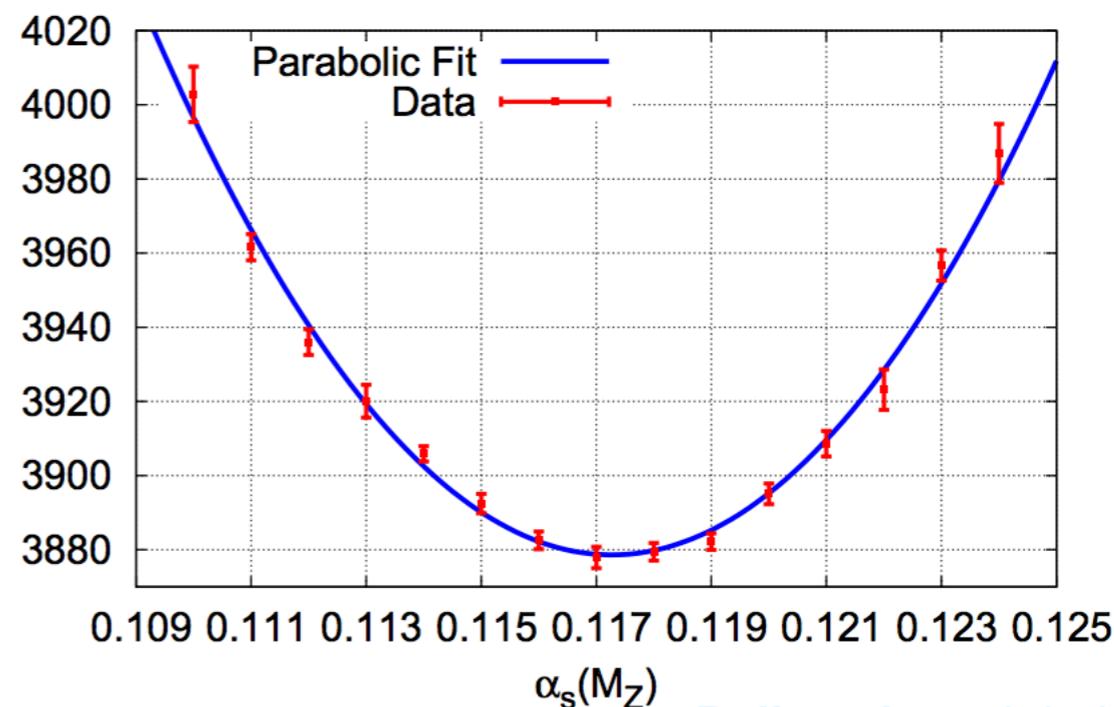
Collider	Site	Initial State	Energy	Discovery / Target
SPEAR (1972)	SLAC	$e^+e^-$	4 GeV	charm quark, tau lepton
PETRA (1978)	DESY	$e^+e^-$	38 GeV	gluon
SppS (1981)	CERN	$p\bar{p}$	600 GeV	W, Z bosons
LEP (1989)	CERN	$e^+e^-$	210 GeV	SM: elw and QCD
SLC (1989)	SLAC	$e^+e^-$	90 GeV	elw SM
HERA (1992)	DESY	$ep$	320 GeV	quark/gluon structure of proton
Tevatron (1987)	FNAL	$p\bar{p}$	2 TeV	top quark
BaBar / Belle (1999)	SLAC / KEK	$e^+e^-$	10 GeV	quark mix / CP violation
LHC (2010)	CERN	$pp$	7/8/14 TeV	Higgs boson, elw. sb, New Physics
RHIC	Brookhaven	$pp, pA$	200-510 GeV	Heavy ions, polarised beams
EIC	Fermilab	$ep, eA$	80 GeV	Nuclear, spin structure
ILC/CLIC			> 200 GeV / 3-5 TeV	hi. res of elw sb / Higgs couplings
FCC			100 TeV	disc. multi-TeV physics

# THE STRONG COUPLING CONSTANT: A WARNING

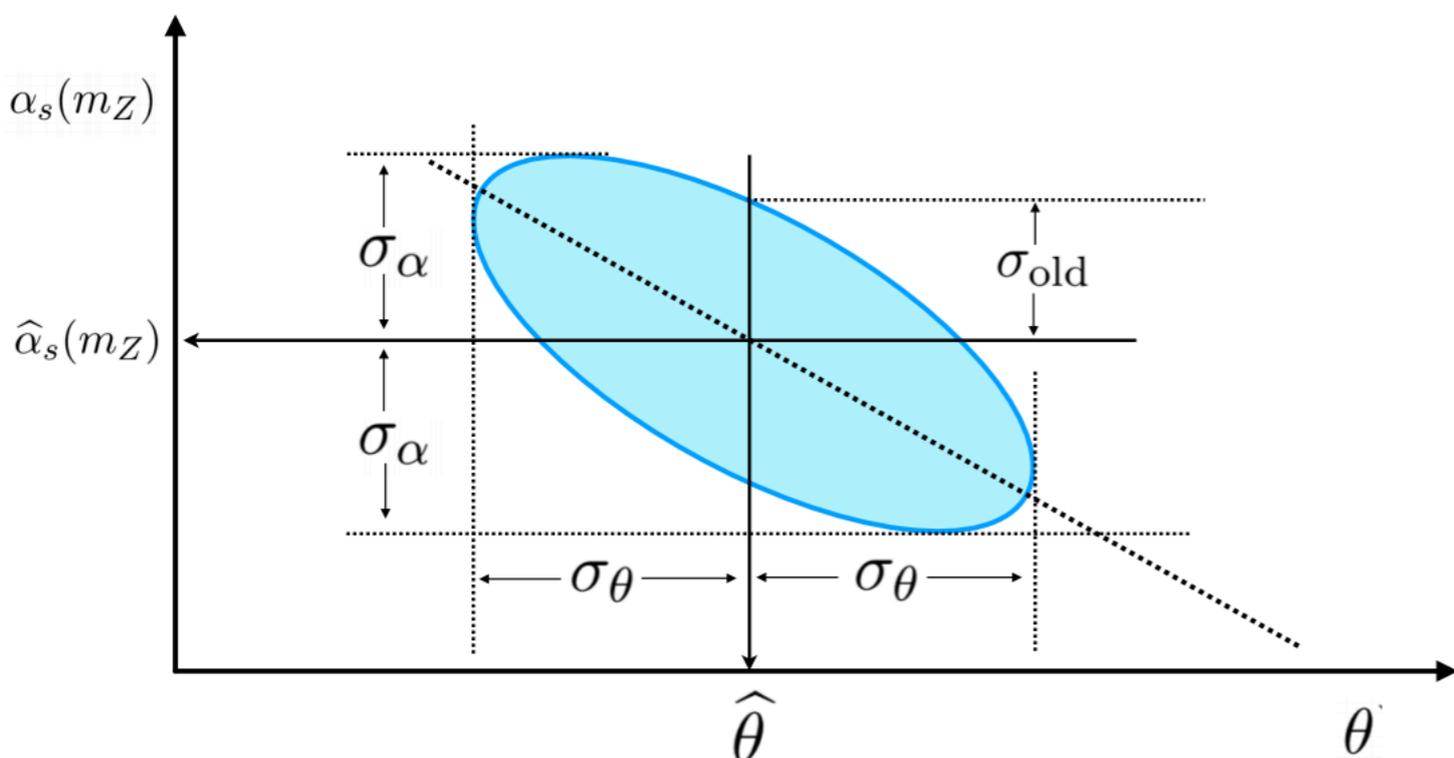
- PDFs and  $\alpha_s$  strongly correlated (PDF evolution with the scale and hard cross sections)
- Cleanest determinations of  $\alpha_s$  from processes that do not require knowledge of the PDFs
- A determination of  $\alpha_s$  jointly with the PDFs has advantage that it is driven by the combination of many experimental measurements from several different processes.

$\chi^2$

NNPDF2.1 NNLO Global



Ball et al, 1110.2483



- Early determinations involve a scan over  $\alpha_s$  and ignored PDF and  $\alpha_s$  correlation in the fit
- Recent simultaneous determination of PDF and  $\alpha_s$  using correlated replica method
- Many determination of  $\alpha_s$  from analyses of specific LHC processes have been published recently ( from  $t\bar{t}$ , Z and W production, jets)
- How reliable are such partial determination of  $\alpha_s$ ?