Sterile Neutrinos in Physics, Astrophysics, Cosmology

Part III: Theory of Dirac and Majorana Neutrino Masses and Mixing

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Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation

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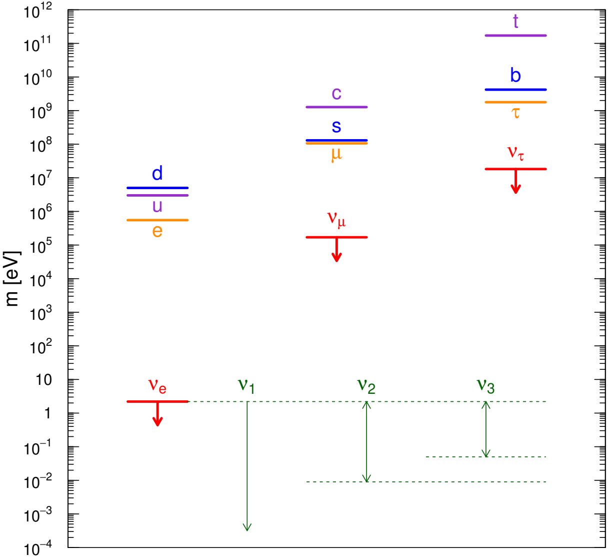
Fermion Mass Spectrum



Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0$ with $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$

$$x^\mu = (x^0, x^1, x^2, x^3) = (x^0, \vec{x}) = (t, \vec{x}), \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$x_\mu = g_{\mu\nu}\, x^\nu, \qquad g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

- 4×4 Dirac matrices defined by

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\, g^{\mu\nu} \quad \text{and} \quad \gamma^0\,\gamma^{\mu\,\dagger}\,\gamma^0 = \gamma^\mu$$

Useful properties ($\mu = 0, 1, 2, 3$ and $k = 1, 2, 3$):

$$(\gamma^0)^\dagger = \gamma^0, \quad (\gamma^k)^\dagger = -\gamma^k, \quad (\gamma^0)^2 = \mathbb{1}, \quad (\gamma^k)^2 = -\mathbb{1}$$

- Dirac Lagrangian: $\mathscr{L}_{\mathrm{D}}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$ with $\overline{\nu} \equiv \nu^\dagger \gamma^0$

- γ_5 matrix: $\gamma_5 \equiv \gamma^5 \equiv i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$

Useful properties: $\{\gamma^5, \gamma^\mu\} = 0, \quad (\gamma_5)^2 = \mathbb{1}, \quad \gamma_5^\dagger = \gamma_5$

- Chiral Left-handed and Right-handed Projectors:

$$P_L \equiv \frac{1-\gamma_5}{2}, \qquad P_R \equiv \frac{1+\gamma_5}{2}$$

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = P_R P_L = 0$$

- Chiral decomposition: $\nu = \nu_L + \nu_R$

with $\nu_L \equiv P_L \nu$ and $\nu_R \equiv P_R \nu$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right)$$

- In SM only ν_L by assumption $\Longrightarrow$ no neutrino mass
  Note that all the other elementary fermion fields (charged leptons and quarks) have both left and right-handed components

- Oscillation experiments have shown that neutrinos are massive

- Simplest and natural extension of the SM: consider also ν_R as for all the other elementary fermion fields

- ν_R is a sterile neutrino field! No SM weak interactions!

[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]

Higgs Mechanism in the Standard Model

- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix} \quad \Longrightarrow \quad |\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$

- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$

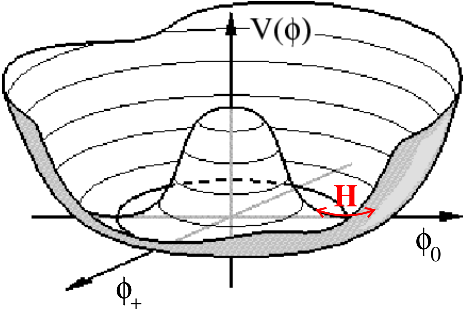
- Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$

- $\mu^2 < 0$ and $\lambda > 0 \quad \Longrightarrow \quad V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$

$$v \equiv \sqrt{-\frac{\mu^2}{\lambda}} = \left(\sqrt{2} G_{\text{F}} \right)^{-1/2} \simeq 246\,\text{GeV}$$

- Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \Longrightarrow \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

- Spontaneous Symmetry Breaking: $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_Q$



- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \Longrightarrow |\Phi|^2 = \frac{v^2}{2} + vH + \frac{1}{2} H^2$

- $V = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \simeq 126\,\text{GeV}$$

$$-\mu^2 \simeq (89\,\text{GeV})^2 \qquad \lambda = -\frac{\mu^2}{v^2} \simeq 0.13$$

SM Extension: Dirac Neutrino Masses

		I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
SM left-handed lepton doublet	$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
SM right-handed charged lepton singlet	ℓ_R	0	0	-2	-1
BSM right-handed neutrino singlet	ν_R	0	0	0	0
SM Higgs doublet	$\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

Third component of weak isospin: $I_3 = \frac{\sigma_3}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$I_3 L_L = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \begin{pmatrix} (1/2)\,\nu_L \\ (-1/2)\,\ell_L \end{pmatrix}$$

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
SM left-handed lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
SM right-handed charged lepton singlet ℓ_R	0	0	-2	-1
BSM right-handed neutrino singlet ν_R	0	0	0	0
SM Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

Lepton-Higgs Yukawa Lagrangian:

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\,\overline{L_L}\,\Phi\,\ell_R - y^{\nu}\,\overline{L_L}\,\widetilde{\Phi}\,\nu_R + \mathrm{H.c.}$$

Y: $+1 +1 -2$ $+1 -1\ 0$

with

$$\widetilde{\Phi} = i\sigma_2\Phi^* = \begin{pmatrix} \phi_0^*(x) \\ -\phi_+^*(x) \end{pmatrix} \quad \leftarrow \quad Y = -1$$

Invariance under $\mathrm{SU}(2)_L$

- $\mathrm{SU}(2)_L$ transformation of doublets: $L_L \to U L_L$ and $\Phi \to U\Phi$ with

$$U = \exp\left(\frac{i}{2}\sum_{k=1}^{3}\theta^k \sigma_k\right)$$

- Pauli matrices: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$(\sigma_k)^2 = 1 \qquad (\sigma_k)^\dagger = \sigma_k \qquad (\sigma_k)^* = -\sigma_2 \sigma_k \sigma_2$$

Therefore: $U^* = \sigma_2 U \sigma_2$

- $\widetilde{\Phi} = i\sigma_2 \Phi^* \to i\sigma_2 U^* \Phi^* = i\sigma_2 \sigma_2 U \sigma_2 \Phi^* = U i\sigma_2 \Phi^* = U\widetilde{\Phi}$

- Lepton-Higgs Yukawa terms:

$$\overline{L_L}\,\Phi\,\ell_R \to \overline{L_L}\,U^\dagger\,U\,\Phi\,\ell_R = \overline{L_L}\,\Phi\,\ell_R$$
$$\overline{L_L}\,\widetilde{\Phi}\,\nu_R = \overline{L_L}\,U^\dagger\,U\,\widetilde{\Phi}\,\nu_R = \overline{L_L}\,\widetilde{\Phi}\,\nu_R$$

Dirac Mass Generation

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^\ell \, \overline{L_L} \, \Phi \, \ell_R - y^\nu \, \overline{L_L} \, \widetilde{\Phi} \, \nu_R + \mathrm{H.c.}$$

Spontaneous Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & - \frac{y^\ell}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & - \frac{y^\nu}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \mathrm{H.c.} \end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\,\frac{v}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R - y^{\nu}\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R$$

$$-\frac{y^{\ell}}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R\,H - \frac{y^{\nu}}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R\,H + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell}\,\frac{v}{\sqrt{2}} \qquad\qquad m_{\nu}^{\mathrm{D}} = y^{\nu}\,\frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_{\ell}}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_{\nu}^{\mathrm{D}}}{v}$$

$$v = \left(\sqrt{2}\,G_{\mathrm{F}}\right)^{-1/2} = 246\,\mathrm{GeV}$$

PROBLEM

Extremely small Yukawa couplings are needed to get $m_{\nu}^{\mathrm{D}} \lesssim 1\,\mathrm{eV}$:

$$y^{\nu} \lesssim 10^{-11} \ll y^{e} \sim 10^{-6}$$

It is considered unnatural, unless there is a protecting BSM symmetry.

Majorana Mass

- Neutrinos can have a Dirac mass if the singlet ν_R is added to the SM ν_L.

- Can ν_L alone describe a massive neutrino?

Yes! (E. Majorana, 1937)

- Trick: ν_R and ν_L are not independent: $\boxed{\nu_R = \nu_L^c = \mathcal{C}\,\overline{\nu_L}^T}$

charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$

useful properties: $\begin{cases} \mathcal{C}^\dagger = \mathcal{C}^{-1} \\ \mathcal{C}^T = -\mathcal{C} \\ \mathcal{C}\,\gamma_5^T\,\mathcal{C}^{-1} = \gamma_5 \end{cases}$

- The relation between ν_R and ν_L must satisfy two requirements:
  - It must have the correct chirality.

    This is satisfied, because ν_L^c is right-handed: $P_R \nu_L^c = \nu_L^c \quad P_L \nu_L^c = 0$

    To check, let us expand: $\nu_L^c = \mathcal{C}\,\overline{\nu_L}^T = \mathcal{C}\,(\nu_L^\dagger \gamma_0)^T = \mathcal{C}\,\gamma_0^T\,\nu_L^*$

$$P_R \nu_L^c = \frac{1+\gamma_5}{2}\,\mathcal{C}\,\gamma_0^T\,\nu_L^* = \mathcal{C}\,\gamma_0^T\,\frac{1-\gamma_5}{2}\,\nu_L^* = \mathcal{C}\,\gamma_0^T\,\nu_L^* = \nu_L^c$$
$$P_L \nu_L^c = \frac{1-\gamma_5}{2}\,\mathcal{C}\,\gamma_0^T\,\nu_L^* = \mathcal{C}\,\gamma_0^T\,\frac{1+\gamma_5}{2}\,\nu_L^* = 0$$

  - It must be compatible with the chiral Dirac equations

$$i\gamma^\mu \partial_\mu \nu_L = m\,\nu_R$$
$$i\gamma^\mu \partial_\mu \nu_R = m\,\nu_L$$

    Check:

$$\begin{aligned} i\gamma^\mu \partial_\mu \nu_R &= i\gamma^\mu \partial_\mu \mathcal{C}\overline{\nu_L}^T = i\mathcal{C}\mathcal{C}^{-1}\gamma^\mu \mathcal{C}\partial_\mu \overline{\nu_L}^T = -i\mathcal{C}(\gamma^\mu)^T \partial_\mu \overline{\nu_L}^T \\ &= -i\mathcal{C}(\partial_\mu \overline{\nu_L}\gamma^\mu)^T = m\,\mathcal{C}\overline{\nu_R}^T = m\,\nu_L \qquad \text{OK} \end{aligned}$$

- It can be shown that $\nu_R = \nu_L^c$ is the only relation that satisfies these two requirements.

Majorana Lagrangian

Dirac Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{D}} &= \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu \\
&= \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R)
\end{aligned}
$$

$$\nu_R \quad \to \quad \nu_L^c = \mathcal{C}\,\overline{\nu_L}^{\,T}$$

$$\frac{1}{2}\,\mathscr{L}^{\mathrm{D}} \quad \to \quad \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^{\,T}\right)$$

Majorana Lagrangian

$$
\begin{aligned}
\mathscr{L}^{\mathrm{M}} &= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^{\,T}\right) \\
&= \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(\overline{\nu_L^c}\,\nu_L + \overline{\nu_L}\,\nu_L^c\right)
\end{aligned}
$$

- Two-component chiral Majorana field: ν_L or ν_R
- Four-component Majorana field:

$$\nu = \nu_L + \nu_L^c \quad \text{or} \quad \nu = \nu_R^c + \nu_R$$

- A four-component Majorana field is characterized by the Majorana condition

$$\boxed{\nu = \nu^c}$$

- $\nu = \nu^c$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\nu}\gamma^\mu\nu = \overline{\nu^c}\gamma^\mu\nu^c = -\nu^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\nu}^T = \overline{\nu}\mathcal{C}\gamma^{\mu\,T}\mathcal{C}^\dagger\nu = -\overline{\nu}\gamma^\mu\nu = 0$$

- Majorana Lagrangian: $\mathscr{L}^{\text{M}} = \frac{1}{2}\,\overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu|_{\nu=\nu^c}$

Total Lepton Number

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu = \nu^c} \quad \longrightarrow \quad \cancel{L = -1}$$

$$\nu_L \quad \Longrightarrow \quad L = +1 \qquad\qquad \nu_L^c \quad \Longrightarrow \quad L = -1$$

$$\mathscr{L}^{\mathrm{M}} = \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2} \left(\overline{\nu_L^c}\, \nu_L + \overline{\nu_L}\, \nu_L^c \right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta^-_{0\nu})$$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta^+_{0\nu})$$

Majorana Antineutrino Terminology

- A Majorana neutrino is the same as a Majorana antineutrino
- Neutrino interactions are described by the CC and NC Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_L} \, \gamma^{\mu} \, \ell_L \, W_{\mu} + \overline{\ell_L} \, \gamma^{\mu} \, \nu_L \, W_{\mu}^{\dagger} \right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2 \cos \vartheta_{\mathrm{W}}} \, \overline{\nu_L} \, \gamma^{\mu} \, \nu_L \, Z_{\mu}$$

- Dirac: $\nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{cases}$
- Majorana: $\nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{cases}$
- Common implicit definitions:

left-handed Majorana neutrino $\equiv$ neutrino
right-handed Majorana neutrino $\equiv$ antineutrino

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T\,\mathcal{C}^\dagger\,\nu_L - \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T\,\mathcal{C}^\dagger\,\nu_L$ has $I_3 = 1$ and $Y = -2$ $\Longrightarrow$ needed $Y = 2$ Higgs triplet ($I = 1$, $I_3 = -1$)
- Compare with Dirac Mass Term $\propto \overline{\nu_R}\nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \to v$ with $I_3 = -1/2$ and $Y = +1$

Beyond the Standard Model

- Since a ν_L Majorana neutrino mass violates the SM symmetries, in order to get a neutrino mass term we have only the Dirac option, with the introduction of ν_R.
- The introduction of ν_R leads us anyway Beyond the Standard Model because they can have the Majorana mass term

$$\mathcal{L}_{\text{M}} \sim m_{\text{M}} \overline{\nu_R} \nu_R^c \qquad \text{singlet under SM symmetries!}$$

- This is beyond the Standard Model because m_{M} is not generated by the Higgs mechanism of the Standard Model $\Rightarrow$ new physics is required.
- The Majorana mass term can be avoided by imposing lepton number conservation which should anyway be explained by some physics beyond the Standard Model.

One-Generation Dirac-Majorana Mass Term

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{L}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$\mathscr{L}^{\text{D}}_{\text{mass}} = -m_{\text{D}}\,\overline{\nu_R}\,\nu_L + \text{H.c.}$$

Standard Dirac Mass Term

$$\mathscr{L}^{L}_{\text{mass}} = -\frac{1}{2}\,m_L\,\overline{\nu_L^c}\,\nu_L + \text{H.c.}$$

ν_L Majorana Mass Term
Forbidden in the SM,
can be generated BSM

$$\mathscr{L}^{R}_{\text{mass}} = -\frac{1}{2}\,m_R\,\overline{\nu_R}\,\nu_R^c + \text{H.c.}$$

ν_R Majorana Mass Term
Allowed in the SM

- Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2}\, \overline{N_L^c}\, M\, N_L + \text{H.c.} \quad \text{with} \quad M = \begin{pmatrix} m_L & m_\text{D} \\ m_\text{D} & m_R \end{pmatrix}$$

- The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- Diagonalization: $N_L = U\, n_L$ with $n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2}\, \overline{n_L^c}\, U^T\, M\, U\, n_L + \text{H.c.}$$

$$U^T\, M\, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{with real} \quad m_k \geq 0$$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_{kL}^c}\, \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_k}\, \nu_k$$

$$\nu_k = \nu_{kL} + \nu_{kL}^c \quad \Longrightarrow \quad \boxed{\nu_k = \nu_k^c}$$

Massive neutrinos are Majorana!

Diagonalization of the Dirac-Majorana mass matrix

- Let us consider for simplicity $m_L = 0 \quad \Longrightarrow \quad$ real m_{D} and m_R

$$M = \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \quad \Longrightarrow \quad U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{with} \quad m_k \geq 0$$

- $U = \mathcal{O}\,\rho \quad$ with $\quad \mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \quad$ and $\quad \rho = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$

$$\tan 2\vartheta = 2\,\frac{m_{\mathrm{D}}}{m_R} \quad \Longrightarrow \quad \mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}$$

$$m'_{2,1} = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 + 4\,m_{\mathrm{D}}^2} \right]$$

- ρ is needed because m'_1 is negative:

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} -m'_1 & 0 \\ 0 & m'_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

One-Generation Active-Sterile Neutrino Mixing

- The left-handed active ν_L and left-handed sterile ν_R^c are superpositions of the left-handed massive neutrino fields ν_{1L} and ν_{2L}:

$$N_L = U\, n_L \quad \Longrightarrow \quad \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$$

- We obtain explicit one-generation active-sterile neutrino mixing for ν_e, or ν_μ, or ν_τ with the simple change of notation $(\alpha = e, \mu, \tau)$

$$\nu_{\alpha L} \equiv \nu_L \quad \text{and} \quad \nu_{sL} \equiv \nu_R^c \quad \Longrightarrow \quad \begin{pmatrix} \nu_{\alpha L} \\ \nu_{sL} \end{pmatrix} = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$$

- In other words, a charge-conjugated right-handed neutrino field is ν_R^c is equivalent to a left-handed sterile neutrino field ν_{sL} that can mix with any of the active neutrinos ν_e, ν_μ, ν_τ.

Summary of 1G Dirac-Majorana Mixing

$$N_L = U\, n_L \quad \Longrightarrow \quad \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$$

$$U^T M U = \mathrm{diag}(m_1, m_2)$$

$$\Downarrow$$

$$\begin{pmatrix} i\cos\vartheta & -i\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$\boxed{\tan 2\vartheta = 2\,\frac{m_{\mathrm{D}}}{m_R}}$$

$$\boxed{\begin{aligned} m_1 &= \frac{1}{2}\left[\sqrt{m_R^2 + 4\,m_{\mathrm{D}}^2} - m_R\right] \\ m_2 &= \frac{1}{2}\left[\sqrt{m_R^2 + 4\,m_{\mathrm{D}}^2} + m_R\right] \end{aligned}}$$

Dirac Limit

$$\boxed{m_R = 0}$$

- $m'_{2,1} = \pm m_{\text{D}} \implies m_1 = m_2 = m_{\text{D}}$
- $\tan 2\vartheta \to \infty \implies \vartheta = \pi/4$ maximal mixing

$$\nu_L = \frac{i\nu_{1L} + \nu_{2L}}{\sqrt{2}} \quad \text{and} \quad \nu_R^c = \frac{-i\nu_{1L} + \nu_{2L}}{\sqrt{2}} \implies \nu_R = \frac{i\nu_{1L}^c + \nu_{2L}^c}{\sqrt{2}}$$

- The two Majorana fields $\nu_1 = \nu_{1L} + \nu_{1L}^c$ and $\nu_2 = \nu_{2L} + \nu_{2L}^c$ can be combined to give one Dirac field:

$$\nu = \nu_L + \nu_R = \frac{i(\nu_{1L} + \nu_{1L}^c) + (\nu_{2L} + \nu_{2L}^c)}{\sqrt{2}} = \frac{i\nu_1 + \nu_2}{\sqrt{2}}$$

- A Dirac field ν can always be split in two Majorana fields ν_1 and ν_2:

$$\begin{aligned}\nu &= \frac{1}{2}\left[(\nu - \nu^c) + (\nu + \nu^c)\right] \\ &= \frac{i}{\sqrt{2}}\left(-i\,\frac{\nu - \nu^c}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\nu + \nu^c}{\sqrt{2}}\right) = \frac{i\nu_1 + \nu_2}{\sqrt{2}}\end{aligned}$$

- A Dirac field is equivalent to two Majorana fields with the same mass.
- In this limit, obviously, there is no sterile neutrino.

Pseudo-Dirac (or Quasi-Dirac) Neutrinos

$$\boxed{m_R \ll m_\mathrm{D}}$$

- $m'_{2,1} \simeq \dfrac{m_R}{2} \pm m_\mathrm{D} \quad \Longrightarrow \quad m_{2,1} \simeq m_\mathrm{D} \pm \dfrac{m_R}{2}$
- The two massive Majorana neutrinos are almost degenerate in mass
- The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations $\nu_L \longleftrightarrow \nu_R^c \equiv \nu_{sL}$ due to the small squared-mass difference

$$\Delta m^2 = m_2^2 - m_1^2 \simeq \left(m_\mathrm{D} + \frac{m_R}{2} \right)^2 - \left(m_\mathrm{D} - \frac{m_R}{2} \right)^2 = 2 m_\mathrm{D} m_R$$

- The oscillations occur with practically maximal mixing:

$$\tan 2\vartheta = 2 \frac{m_\mathrm{D}}{m_R} \gg 1 \quad \Longrightarrow \quad \vartheta \simeq \frac{\pi}{4}$$

Type-I Seesaw Mechanism

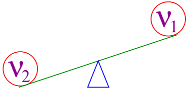
[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$\boxed{m_R \gg m_D}$$

- $m_D \lesssim v \sim 100\,\text{GeV}$ is generated by SM Higgs Mechanism and is protected by the SM symmetries

- m_R is not protected by the SM symmetries $\Longrightarrow$ $m_R \gg v$

- $$\left.\begin{array}{l} m_1' \simeq -\dfrac{m_D^2}{m_R} \\ m_2' \simeq m_R \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} m_1 \simeq \dfrac{m_D^2}{m_R} \\ m_2 \simeq m_R \end{array}\right.$$



- Natural explanation of smallness of neutrino masses

- The mixing angle is very small:

$$\tan 2\vartheta = 2\,\frac{m_D}{m_R} \ll 1 \quad \Longrightarrow \quad \vartheta \simeq \frac{m_D}{m_R} \ll 1$$

- ν_1 is composed mainly of active ν_L: $\nu_{1L} \simeq -i\,\nu_L$ observable

- ν_2 is composed mainly of sterile ν_R: $\nu_{2L} \simeq \nu_R^c$ decoupled

General 3+N_s Active-Sterile Neutrino Mixing

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \mathscr{L}_{\text{mass}}^{\text{D}} + \mathscr{L}_{\text{mass}}^{L} + \mathscr{L}_{\text{mass}}^{R}$$

$$\mathscr{L}_{\text{mass}}^{\text{D}} = -\sum_{i=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{s_{iR}}\, M_{i\alpha}^{\text{D}}\, \nu_{\alpha L} + \text{H.c.} \quad \rightarrow \quad M^{\text{D}} \text{ complex } N_S \times 3$$

$$\mathscr{L}_{\text{mass}}^{L} = -\frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \overline{\nu_{\alpha L}^{c}}\, M_{\alpha\beta}^{L}\, \nu_{\beta L} + \text{H.c.} \quad \rightarrow \quad M^{L} \text{ complex symmetric } 3 \times 3$$

$$\mathscr{L}_{\text{mass}}^{R} = -\frac{1}{2} \sum_{i,j=1}^{N_S} \overline{s_{iR}}\, M_{ij}^{R}\, s_{jR}^{c} + \text{H.c.} \quad \rightarrow \quad M^{R} \text{ complex symmetric } N_S \times N_S$$

$$\boldsymbol{N}_L \equiv \begin{pmatrix} \boldsymbol{\nu}_L \\ \boldsymbol{s}_R^C \end{pmatrix} \quad \text{with} \quad \boldsymbol{\nu}_L \equiv \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad \text{and} \quad \boldsymbol{s}_R^C \equiv \begin{pmatrix} s_{1R}^C \\ \vdots \\ s_{N_S R}^C \end{pmatrix} \equiv \begin{pmatrix} \nu_{s_1 L} \\ \vdots \\ \nu_{s_{N_S} L} \end{pmatrix}$$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \overline{\boldsymbol{N}_L^c}\, M^{\text{D+M}}\, \boldsymbol{N}_L + \text{H.c.} \quad \text{with} \quad M^{\text{D+M}} = \begin{pmatrix} M^L & {M^{\text{D}}}^T \\ M^{\text{D}} & M^R \end{pmatrix}$$

- $M^{\text{D+M}} = \begin{pmatrix} M^L & M^{\text{D}\,T} \\ M^{\text{D}} & M^R \end{pmatrix}$ complex symmetric $\mathcal{N} \times \mathcal{N}$ with $\mathcal{N} = 3 + N_S$

- Diagonalization: $\boldsymbol{N}_L = \mathscr{U}\, \boldsymbol{n}_L$ with $\boldsymbol{n}_L = \begin{pmatrix} \nu_{1L} \\ \vdots \\ \nu_{\mathcal{N}L} \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2}\, \overline{\boldsymbol{n}_L^c}\, \mathscr{U}^T\, M^{\text{D+M}}\, \mathscr{U}\, \boldsymbol{n}_L + \text{H.c.}$$

- $\mathscr{U}^T\, M^{\text{D+M}}\, \mathscr{U} = M$ where

$$M = \text{diag}(m_1, \ldots, m_{\mathcal{N}}) \quad \text{with real} \quad m_k \geq 0$$

- $\mathcal{N} \times \mathcal{N}$ mixing matrix:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{s_1 L} \\ \vdots \\ \nu_{s_{N_s} L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & \cdots & U_{s_1\mathcal{N}} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{s_{N_s} 1} & U_{s_{N_s} 2} & U_{s_{N_s} 3} & U_{s_{N_s} 4} & \cdots & U_{s_{N_s}\mathcal{N}} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \\ \nu_{4L} \\ \vdots \\ \nu_{\mathcal{N}L} \end{pmatrix}$$

- The $\mathcal{N} \times \mathcal{N} = (3 + N_s) \times (3 + N_s)$ mixing matrix can be written as

$$\mathscr{U} = \begin{pmatrix} V & W \\ Y & Z \end{pmatrix}$$

- The 3×3 square submatrix V describes the mixing of the three active flavor neutrinos ν_e, ν_μ, ν_τ with the three "standard" light massive neutrinos ν_1, ν_2, ν_3:

$$V = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

  In the effective three-neutrino mixing approximation $V \to U$ is assumed to be unitary.

- The $3 \times N_s$ rectangular submatrix W describes the mixing of ν_e, ν_μ, ν_τ with the "non-standard" massive neutrinos ν_4, $\ldots$, $\nu_{\mathcal{N}}$:

$$W = \begin{pmatrix} U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix}$$

$$M^{\mathrm{D+M}} = \begin{pmatrix} M^L & {M^{\mathrm{D}}}^T \\ M^{\mathrm{D}} & M^R \end{pmatrix} \qquad\qquad \mathscr{U} = \begin{pmatrix} V & W \\ Y & Z \end{pmatrix}$$

- In general $M^L \ll M^{\mathrm{D}}$: suppressed by SM symmetries.
- If $M^R \gg M^{\mathrm{D}} \Longrightarrow$ seesaw mechanism with $|W|, |Y| \ll |V|, |Z|$ and $m_{k>3} \gg m_{k\leq 3} \Longrightarrow$ phenomenological decoupling of the sterile neutrinos that are practically equivalent to the heavy $\nu_{k>3} \Longrightarrow$ small non-unitarity of the effective 3×3 low-energy mixing matrix V

Non-Unitary Lepton Mixing

Standard Light Massive Neutrinos

ν_1, ν_3, ν_3

Heavy Neutral Leptons ($m_k \gtrsim 100\,\text{GeV}$)

$\nu_4, \ldots, \nu_N$

$N_s = N - 3$ "Heavy" Sterile Neutrinos

$\nu_{s_1}, \ldots, \nu_{N_s}$

$$U^{N \times N} = \begin{array}{c} N \\ \left(\begin{array}{ccccc} \boxed{\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{matrix}} & & & \cdots & \begin{matrix} U_{eN} \\ U_{\mu N} \\ U_{\tau N} \end{matrix} \\ \vdots \quad \vdots \quad \vdots & & & \ddots & \vdots \\ U_{s_{N_s}1} \; U_{s_{N_s}2} \; U_{s_{N_s}3} & & & \cdots & U_{s_{N_s}N} \end{array} \right) \end{array}$$

Effective Low-Energy Mixing of Active Neutrinos ($\alpha = e, \mu, \tau$)

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U^{N\times N}_{\alpha k} |\nu_k\rangle = \sum_{k=1}^{3} N_{\alpha k} |\nu_k\rangle$$

Non-Unitary Effective 3×3 Mixing Matrix N

$$P_{\nu_\alpha \to \nu_\beta} = |(NN^\dagger)_{\alpha\beta}|^2 - 4 \sum_{k>j} \mathrm{Re}\left[N_{\alpha k}^* N_{\beta k} N_{\alpha j} N_{\beta j}^*\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$
$$+ 2 \sum_{k>j} \mathrm{Im}\left[N_{\alpha k}^* N_{\beta k} N_{\alpha j} N_{\beta j}^*\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta}(L=0) = |(NN^\dagger)_{\alpha\beta}|^2 \neq \delta_{\alpha\beta}}$$

Small zero-distance flavor conversion!

It is due to the non-orthogonality of the active flavors:

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} N_{\alpha k} |\nu_k\rangle \quad \Longrightarrow \quad \begin{cases} \langle \nu_\beta | \nu_\alpha \rangle = \displaystyle\sum_{j,k=1}^{3} \langle \nu_j | N_{\beta j}^* N_{\alpha k} | \nu_k \rangle \\ \qquad\qquad = \displaystyle\sum_{k=1}^{3} N_{\beta k}^* N_{\alpha k} \neq \delta_{\alpha\beta} \end{cases}$$

Weak Interactions

- Neutrino interactions are described by the charged-current (CC) and neutral-current (NC) weak interaction Lagrangians

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}}\left(\overline{\boldsymbol{\ell}_L}\,\gamma^\mu\,\boldsymbol{\nu}_L\,W_\mu^\dagger + \overline{\boldsymbol{\nu}_L}\,\gamma^\mu\,\boldsymbol{\ell}_L\,W_\mu\right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\,\overline{\boldsymbol{\nu}_L}\,\gamma^\mu\,\boldsymbol{\nu}_L\,Z_\mu$$

- Mixing: $\boldsymbol{N}_L = \mathscr{U}\,\boldsymbol{n}_L \quad \Longrightarrow \quad \begin{pmatrix} \boldsymbol{\nu}_L \\ \boldsymbol{s}_R^C \end{pmatrix} = \begin{pmatrix} V & W \\ Y & Z \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \vdots \\ \nu_{\mathcal{N}L} \end{pmatrix}$

- Only the upper rectangular $3 \times \mathcal{N}$ submatrix $U = \begin{pmatrix} V & W \end{pmatrix}$ enters in CC and NC weak interactions: $\boldsymbol{\nu}_L = U\,\boldsymbol{n}_L$

$$\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}}\left(\overline{\boldsymbol{\ell}_L}\,\gamma^\mu\,U\,\boldsymbol{n}_L\,W_\mu^\dagger + \overline{\boldsymbol{n}_L}\,U^\dagger\,\gamma^\mu\,\boldsymbol{\ell}_L\,W_\mu\right)$$

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\,\overline{\boldsymbol{n}_L}\,U^\dagger\,\gamma^\mu\,U\,\boldsymbol{n}_L\,Z_\mu$$

- Since sterile neutrinos do not interact, the lower rectangular $N_s \times \mathcal{N}$ submatrix $\begin{pmatrix} Y & Z \end{pmatrix}$ is not measurable and hence phenomenologically irrelevant.

- Measurable rectangular $3 \times \mathcal{N}$ mixing matrix:

$$U = \begin{pmatrix} V & W \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix}$$

- Note that U is not unitary: $UU^\dagger = \mathbb{1}$, but $U^\dagger U \neq \mathbb{1}$!

$$UU^\dagger = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix} \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \\ U_{e4}^* & U_{\mu 4}^* & U_{\tau 4}^* \\ \vdots & \vdots & \vdots \\ U_{e\mathcal{N}}^* & U_{\mu\mathcal{N}}^* & U_{\tau\mathcal{N}}^* \end{pmatrix} = \mathbb{1}_{3\times 3}$$

- Measurable rectangular $3 \times \mathcal{N}$ mixing matrix:

$$U = \begin{pmatrix} V & W \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix}$$

- Note that U is not unitary: $UU^\dagger = \mathbb{1}$, but $U^\dagger U \neq \mathbb{1}$!

$$U^\dagger U = \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \\ U_{e4}^* & U_{\mu 4}^* & U_{\tau 4}^* \\ \vdots & \vdots & \vdots \\ U_{e\mathcal{N}}^* & U_{\mu\mathcal{N}}^* & U_{\tau\mathcal{N}}^* \end{pmatrix} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix} \neq \mathbb{1}_{\mathcal{N}\times\mathcal{N}}$$

- The GIM mechanism for neutral-current weak interactions does not work in active-sterile neutrino mixing:

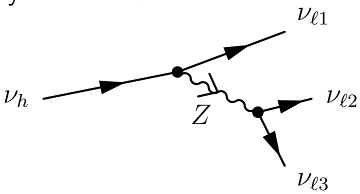
$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\,\overline{\boldsymbol{n}_L}\,U^{\dagger}\,U\,\gamma^{\mu}\,\boldsymbol{n}_L\,Z_{\mu}$$

- There can be neutral-current transitions among different massive neutrinos.
- This effect is sometimes called flavor-changing neutral current (FCNC) in analogy with that of quarks, that however are different, because all quarks are defined as mass eigenstates.
- There is no lepton FCNC, because $\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}}$ is diagonal in the flavor basis:

$$\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\mu}\,\boldsymbol{\nu}_L\,Z_{\mu} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}}\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\mu}\,\nu_{\alpha L}\,Z_{\mu}$$

- There can be Z-mediated decay of heavy neutrinos:

$$\nu_h \to \nu_{\ell 1} + \nu_{\ell 2} + \nu_{\ell 3}$$

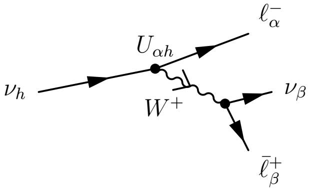


[Schechter, Valle, PRD 22 (1980) 2227, PRD 25 (1982) 774]

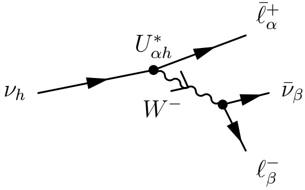
- The neutral-current decay $\nu_h \to \nu_{\ell 1} + \nu_{\ell 2} + \nu_{\ell 3}$ is interesting, but practically invisible.

- Heavy neutrinos (also called heavy neutral leptons) can have detectable charged-current decays: [See: Levy, arXiv:1805.06419]

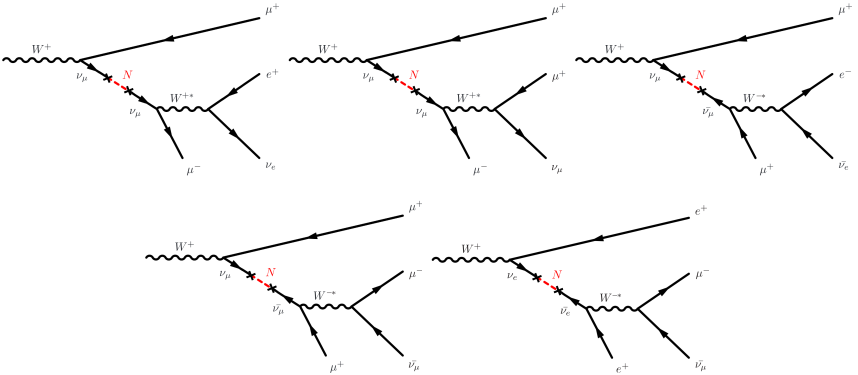
$$\nu_h \to \ell_\alpha^- + \bar{\ell}_\beta^+ + \nu_\beta$$



$$\nu_h \to \bar{\ell}_\alpha^+ + \ell_\beta^- + \bar{\nu}_\beta$$



Example: Search for heavy neutral leptons in decays of W bosons produced in 13 TeV pp collisions using prompt and displaced signatures with the ATLAS detector [arXiv:1905.09787]



Parameterization of the $3 + N_s$ Mixing Matrix

- Effective rectangular $3 \times \mathcal{N}$ mixing matrix, with $\mathcal{N} = 3 + N_s$:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots & U_{e\mathcal{N}} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \cdots & U_{\mu\mathcal{N}} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \cdots & U_{\tau\mathcal{N}} \end{pmatrix}$$

- The number of physical mixing parameters is smaller than the number necessary to parameterize the $\mathcal{N} \times \mathcal{N}$ unitary matrix $\mathscr{U}$.

- This is due to the arbitrariness of the mixing in the sterile sector, which does not affect weak interactions. Any linear combination of the sterile neutrinos is equivalent.

- The effective rectangular $3 \times \mathcal{N}$ mixing matrix is not unitary:

$$UU^\dagger = \mathbb{1}_{3\times3}, \qquad \text{but} \qquad U^\dagger U \neq \mathbb{1}_{\mathcal{N}\times\mathcal{N}}$$

- How many mixing parameters?

- A rectangular $3 \times \mathcal{N}$ matrix depends on $6\mathcal{N}$ real parameters, but

$$UU^\dagger = \mathbb{1}_{3\times 3} \quad \Longrightarrow \quad 9 \quad \text{constraints}$$

$$N_{\text{real parameters}} = 6\mathcal{N} - 9 = 6\,(3 + N_s) - 9 = 9 + 6N_s$$

- But how many mixing angles and physical CP-violating phases?

- For example, we know that for $N_s = 0$ three phases can be eliminated by rephasing the charged lepton fields and we have

  3 mixing angles

  3 physical CP-violating phases (one Dirac and 2 Majorana)

- Standard parameterization of the mixing matrix in three-neutrino mixing:

$$U^{(3\nu)} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- The unitary $\mathcal{N} \times \mathcal{N}$ matrix $\mathscr{U}$ can be written as

$$\mathscr{U} = \mathrm{diag}\left(e^{i\omega_e}, e^{i\omega_\mu}, e^{i\omega_\tau}, e^{i\omega_{s_1}}, \ldots, e^{i\omega_{s_{N_s}}}\right) \left[\prod_{a=1}^{\mathcal{N}} \prod_{b=a+1}^{\mathcal{N}} W^{ab}(\vartheta_{ab}, \delta_{ab})\right]$$

- Complex rotation in the $a - b$ plane:

$$\begin{aligned}\left[W^{ab}(\vartheta_{ab}, \delta_{ab})\right]_{rs} = \delta_{rs} &+ (c_{ab} - 1)\left(\delta_{ra}\delta_{sa} + \delta_{rb}\delta_{sb}\right) \\ &+ s_{ab}\left(e^{-i\delta_{ab}}\delta_{ra}\delta_{sb} - e^{i\delta_{ab}}\delta_{rb}\delta_{sa}\right)\end{aligned}$$

- Example:

$$W^{12}(\vartheta_{12}, \delta_{12}) = \begin{pmatrix} \cos\vartheta_{12} & \sin\vartheta_{12} e^{-i\delta_{12}} & 0 & 0 & \cdots & 0 \\ -\sin\vartheta_{12} e^{i\delta_{12}} & \cos\vartheta_{12} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

- The effective $3 \times \mathcal{N}$ mixing matrix U is made of the first 3 rows of $\mathscr{U}$:
  Truncation of the phases $e^{i\omega_{s_1}}, \ldots, e^{i\omega_{s_{N_s}}}$
  Truncation of the complex rotations $W^{ab}(\vartheta_{ab}, \delta_{ab})$ with $b > a > 3$

- Effective rectangular $3 \times \mathcal{N}$ mixing matrix:

$$U = \operatorname{diag}\left(e^{i\omega_e}, e^{i\omega_\mu}, e^{i\omega_\tau}\right) \left[\prod_{a=1}^{3} \prod_{b=a+1}^{\mathcal{N}} W^{ab}(\vartheta_{ab}, \delta_{ab})\right]_{3\times\mathcal{N}}$$

- The three phases ω_1, ω_2, ω_3 can be eliminated by rephasing the charged lepton fields.

$$\mathscr{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{\mathcal{N}} \overline{\ell_{\alpha L}} \gamma^\rho U_{\alpha k} \nu_{kL} W_\rho^\dagger + \text{H.c.}$$

$$\ell_{\alpha L} \to e^{i\omega_\alpha} \ell_{\alpha L}$$

$$\mathscr{L}_{\text{CC}} \to -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{\mathcal{N}} \overline{\ell_{\alpha L}} \gamma^\rho e^{-i\omega_\alpha} U_{\alpha k} \nu_{kL} W_\rho^\dagger + \text{H.c.}$$

- Physical effective rectangular $3 \times \mathcal{N}$ mixing matrix:

$$U = \left[\prod_{a=1}^{3} \prod_{b=a+1}^{\mathcal{N}} W^{ab}(\vartheta_{ab}, \delta_{ab}) \right]_{3\times\mathcal{N}}$$

- How many complex rotations?

- For each value of $a = 1, 2, 3$ there are $\mathcal{N} - a$ values of b:

$$\begin{aligned} N_{\text{complex rotations}} &= (\mathcal{N} - 1) + (\mathcal{N} - 2) + (\mathcal{N} - 3) \\ &= 3\mathcal{N} - 6 = 3\,(3 + N_s) = 3 + 3N_s \end{aligned}$$

$3 + 3N_s$ mixing angles

$3 + 3N_s$ physical CP-violating phases

$\mathcal{N} - 1 = 2 + N_s$ phases are Majorana

$1 + 2N_s$ phases are Dirac

- Note that in the case under consideration none of the phases of the complex rotations can be eliminated, because the Majorana mass Lagrangian

$$\mathscr{L}_{\text{mass}} = \frac{1}{2} \sum_{k=1}^{\mathcal{N}} m_k \left(\nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} - \overline{\nu_{kL}} \mathcal{C} \overline{\nu_{kL}}^T \right)$$

is not invariant under rephasing of the neutrino fields

$$\nu_{kL} \to e^{i\varphi_k} \nu_{kL}$$

- We distinguish the Majorana phases as those that could be eliminated by rephasing the neutrino fields when the Majorana neutrino masses can be neglected.

- Therefore the physical effects of the Majorana phases appear only in $|\Delta L| = 2$ processes that are induced by the Majorana mass Lagrangian.

- Why there are only $\mathcal{N} - 1$ Majorana phases when there are $\mathcal{N}$ massive neutrino fields?

- In general only $3+\mathcal{N}-1$ of the $3+\mathcal{N}$ phases of the 3 charged lepton fields and $\mathcal{N}$ massive neutrino fields can be used to eliminate phases in the neutrino mixing matrix.

- Weak Charged Current: $j^{\rho\dagger}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{\mathcal{N}} \overline{\ell_{\alpha L}} \, \gamma^{\rho} \, U_{\alpha k} \, \nu_{kL}$

$$\ell_{\alpha} \to e^{i\varphi_{\alpha}} \, \ell_{\alpha} \quad (\alpha = e, \mu, \tau) \qquad\qquad \nu_{k} \to e^{i\varphi_{k}} \, \nu_{k} \quad (k = 1, 2, 3)$$

$$j^{\rho\dagger}_{W,\mathrm{L}} \to 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{\mathcal{N}} \overline{\ell_{\alpha L}} \, e^{-i\varphi_{\alpha}} \, \gamma^{\rho} \, U_{\alpha k} \, e^{i\varphi_{k}} \, \nu_{kL}$$

$$j^{\rho\dagger}_{W,\mathrm{L}} \to 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{\mathcal{N}} \overline{\ell_{\alpha L}} \, \underbrace{e^{-i(\varphi_{\alpha}-\varphi_{1})}}_{3} \, \gamma^{\rho} \, U_{\alpha k} \, \underbrace{e^{i(\varphi_{k}-\varphi_{1})}}_{\mathcal{N}-1} \, \nu_{kL}$$

- A common rephasing of the massive neutrino fields is equivalent to a common rephasing of the charged lepton fields, which can only eliminate an overall phase in $\mathrm{diag}\big(e^{i\omega_e}, e^{i\omega_\mu}, e^{i\omega_\tau}\big)$, which has already been eliminated.

- Convenient parameterization scheme:

$$U = \left[\left(\prod_{a=1}^{3} \prod_{b=4}^{\mathcal{N}} W^{ab} \right) R^{23} W^{13} R^{12} \right]_{3\times\mathcal{N}} \mathrm{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{\mathcal{N}1}}\right)$$

- Real rotation in the $a-b$ plane: $R^{ab} = W^{ab}(\theta_{ab}, 0)$.

- In the product of $W^{ab}(\vartheta_{ab}, \delta_{ab})$ matrices one can eliminate an unphysical phase δ_{ab} for each value of the index $b = 4, \ldots, \mathcal{N}$.

- For $N_s = 0$ we recover the standard parameterization in three-neutrino mixing:

$$\begin{aligned} U^{(3\nu)} &= \left[R^{23} W^{13} R^{12}\right]_{3\times3} \mathrm{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}\right) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix} \end{aligned}$$

- It is convenient to choose the order of the real or complex rotations for each index $b \geq 4$ such that the rotations in the $3-b$, $2-b$ and $1-b$ planes are ordered from left to right.
- In this way, the first two lines, which are relevant for the study of the oscillations of the experimentally more accessible flavor neutrinos ν_e and ν_μ, are independent of the mixing angles and Dirac phases corresponding to the rotations in all the $3-b$ planes for $b \geq 4$.
- Moreover, the first line, which is relevant for the study of ν_e disappearance, is independent also of the mixing angles and Dirac phases corresponding to the rotations in the $2-b$ planes for $b \geq 3$.
- Example:

$$U = \left[W^{3\mathcal{N}} R^{2\mathcal{N}} W^{1\mathcal{N}} \cdots W^{34} R^{24} W^{14} R^{23} W^{13} R^{12}\right]_{3\times\mathcal{N}} \times \operatorname{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{\mathcal{N}1}}\right)$$

- Another example:

$$U = \left[W^{3\mathcal{N}} \cdots W^{34} W^{2\mathcal{N}} \cdots W^{24} R^{1\mathcal{N}} \cdots R^{14} R^{23} W^{13} R^{12}\right]_{3\times\mathcal{N}} \times \operatorname{diag}\left(1, e^{i\lambda_{21}}, \ldots, e^{i\lambda_{\mathcal{N}1}}\right)$$

- $3+1$ mixing:

$$U = \left[W^{34}R^{24}W^{14}R^{23}W^{13}R^{12}\right]_{3\times 4}\operatorname{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

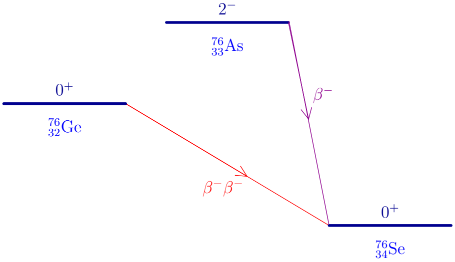
$$= \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \cdots & \cdots & \begin{array}{c} c_{13}c_{24}s_{23} \\ -s_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{13})} \end{array} & c_{14}s_{24} \\ \cdots & \cdots & \cdots & c_{14}c_{24}s_{34}e^{-i\delta_{34}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

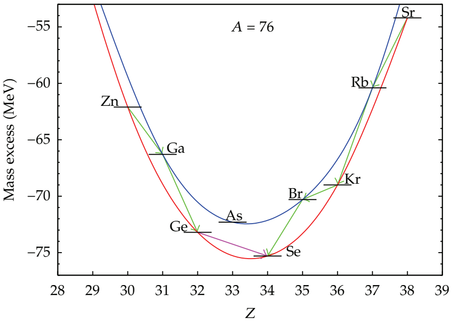
- $3+2$ mixing:

$$U = \left[W^{35}R^{25}W^{15}W^{34}R^{24}W^{14}R^{23}W^{13}R^{12}\right]_{3\times 5}\cdots$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14}c_{15} & s_{12}c_{13}c_{14}c_{15} & c_{14}c_{15}s_{13}e^{-i\delta_{13}} & c_{15}s_{14}e^{-i\delta_{14}} & s_{15}e^{-i\delta_{15}} \\ \cdots & \cdots & \cdots & \begin{array}{c} c_{14}c_{25}s_{24} \\ -s_{14}s_{15}s_{25}e^{i(\delta_{15}-\delta_{14})} \end{array} & c_{15}s_{25} \\ \cdots & \cdots & \cdots & \cdots & c_{15}c_{25}s_{35}e^{-i\delta_{35}} \end{pmatrix}\cdots$$

Neutrinoless Double-Beta Decay

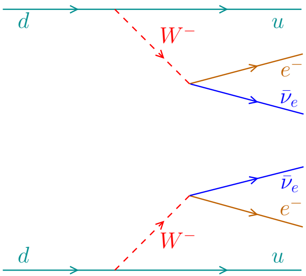




Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

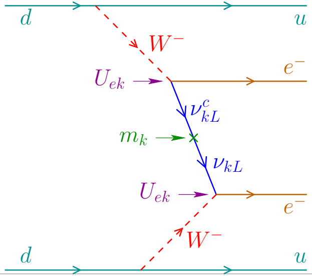


$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

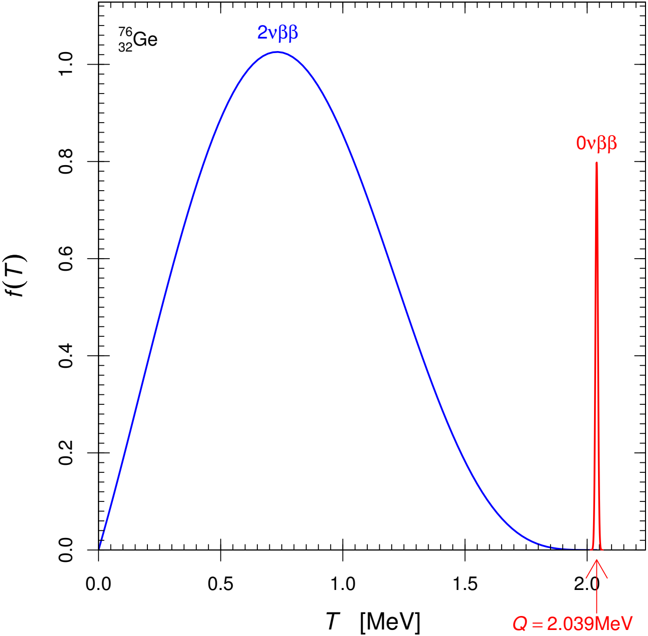
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$



$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$



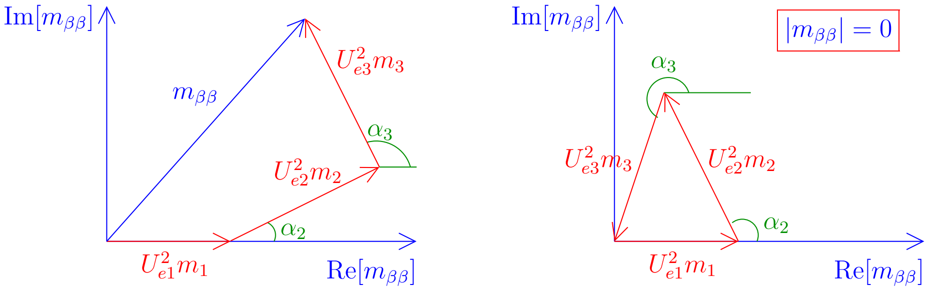
$^{76}_{32}$Ge
2νββ
0νββ
f(T)
T [MeV]
Q = 2.039MeV
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0

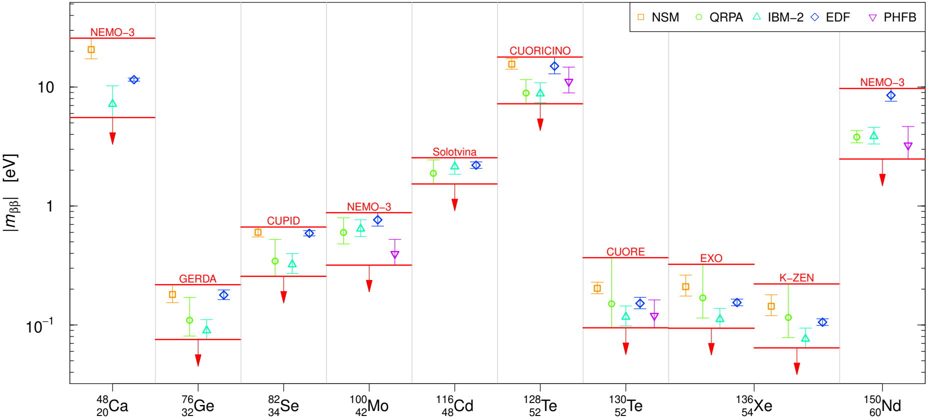
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

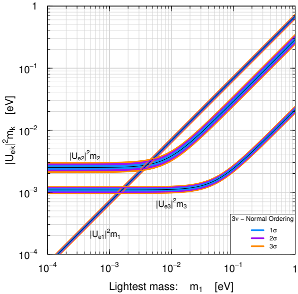
$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$

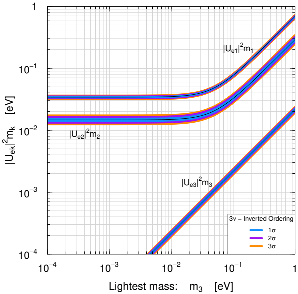


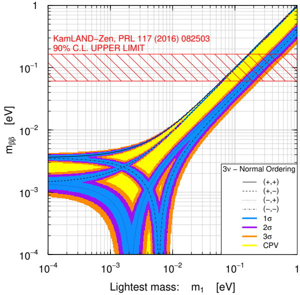

NSM
QRPA
IBM−2
EDF
PHFB
NEMO−3
GERDA
CUPID
NEMO−3
Solotvina
CUORICINO
CUORE
EXO
K−ZEN
NEMO−3
$|m_{\beta\beta}|$ [eV]
10
1
10^{-1}
$^{48}_{20}$Ca
$^{76}_{32}$Ge
$^{82}_{34}$Se
$^{100}_{42}$Mo
$^{116}_{48}$Cd
$^{128}_{52}$Te
$^{130}_{52}$Te
$^{136}_{54}$Xe
$^{150}_{60}$Nd

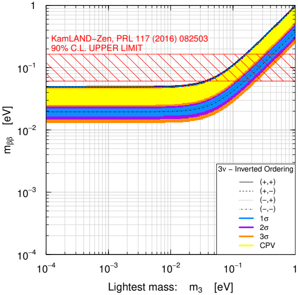
Predictions of 3ν-Mixing Paradigm

$$m_{\beta\beta} = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3 \right|$$



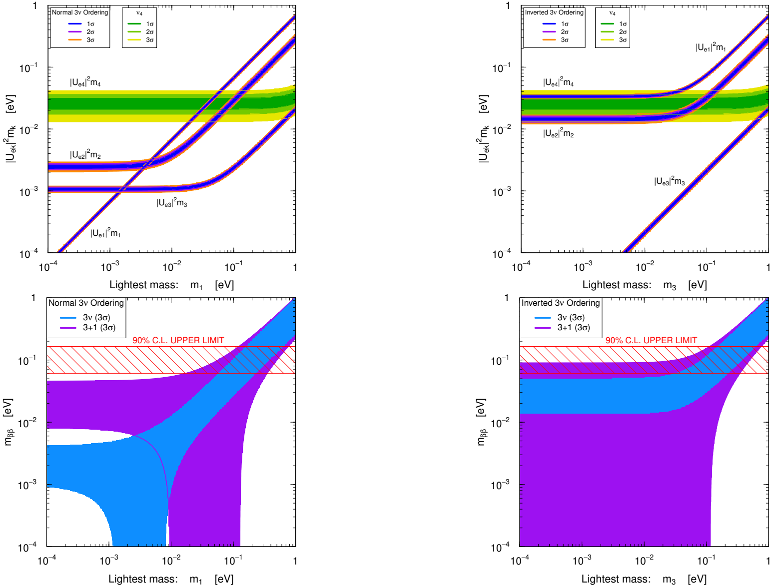






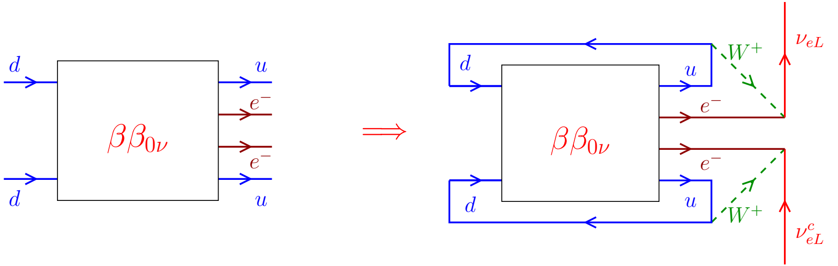
Predictions of 3+1 Active-Sterile Mixing

$$m_{\beta\beta} = \left| |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3 + |U_{e4}|^2 e^{i\alpha_{41}} m_4 \right|$$



$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

- $|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among the ν_1, ν_2, ν_3 contributions or because neutrinos are Dirac particles.
- However, $\beta\beta_{0\nu}$ decay can be generated by another mechanism beyond the Standard Model.
- In this case, a Majorana mass for ν_e is generated by radiative corrections:



[Schechter, Valle, PRD 25 (1982) 2951; Takasugi, PLB 149 (1984) 372]

- Majorana Mass Term: $\boxed{\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu^c_{eL}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu^c_{eL} \right)}$
- Very small four-loop diagram contribution: $m_{ee} \sim 10^{-24}\,\mathrm{eV}$

[Duerr, Lindner, Merle, JHEP 06 (2011) 091 (arXiv:1105.0901)]

- In any case finding $\beta\beta_{0\nu}$ decay is important for
  - Finding total Lepton number violation $(\Delta L = \pm 2)$.
  - Establishing the Majorana (or pseudo-Dirac) nature of neutrinos.
- On the other hand, even if $\beta\beta_{0\nu}$ decay is not found, it is not possible to prove experimentally that neutrinos are Dirac particles, because
  - A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  - It is impossible to prove experimentally that the mass splitting is exactly zero.

Conclusions of Part III

- The most general BSM scenario with massive neutrinos is $3 + N_s$ active-sterile neutrino mixing.
- Diagonalization of the $3 + N_s$ Dirac-Majorana Mass Term $\Longrightarrow$ $3 + N_s$ massive Majorana neutrinos.
- If the mass matrix of the right-handed neutrino fields is generated by very high-energy BSM physics, we have the seesaw mechanism that is a very attractive and compelling way to generate small neutrino masses.
- In this case the sterile neutrinos are decoupled from observable low-energy physics (standard effective three-neutrino mixing), with the only possible observable effect of very small non-unitarity of the mixing matrix.
- However, in general there is no constraint on the number and mass scale of the sterile neutrinos.
- It is possible and interesting that there is low-energy new physics (maybe connected with dark matter).
- Light neutral BSM fermions can mix with neutrinos: they are the sterile neutrinos.

Possible scenarios (in principle not incompatible):

- Very light sterile neutrinos with mass scale $\ll 1\,\text{eV}$: may have effects in solar neutrino phenomenology [de Holanda, Smirnov, PRD 69 (2004) 113002; PRD 83 (2011) 113011; Das, Pulido, Picariello, PRD 79 (2009) 073010]
  Experimental Daya Bay [arXiv:2002.00301] and RENO [arXiv:2006.07782] constraints for $10^{-3} \lesssim \Delta m^2 \lesssim 10^{-1}\,\text{eV}^2$

- Light sterile neutrinos with mass scale $\sim 1\,\text{eV} \Longrightarrow$ short-baseline neutrino oscillation anomalies (main topic of these lectures).

- Heavy sterile neutrinos with mass scale $\gg 1\,\text{eV}$: could be Warm Dark Matter at the keV scale [Asaka, Blanchet, Shaposhnikov, PLB 631 (2005) 151; Asaka, Shaposhnikov, PLB 620 (2005) 17; Asaka, Shaposhnikov, Kusenko, PLB 638 (2006) 401; Asaka, Laine, Shaposhnikov, JHEP 0606 (2006) 053, JHEP 0701 (2007) 091] [Reviews: Kusenko, Phys. Rept. 481 (2009) 1; Boyarsky, Ruchayskiy, Shaposhnikov, Ann. Rev. Nucl. Part. Sci. 59 (2009) 191; Boyarsky, Iakubovskyi, Ruchayskiy, Phys. Dark Univ. 1 (2012) 136; Drewes, IJMPE, 22 (2013) 1330019; White Paper, arXiv:1602.04816; Boyarsky, Drewes, Lasserre, Mertens, Ruchayskiy, arXiv:1807.07938] or be detectable at LHC at the TeV scale (heavy neutral leptons) [Reviews: Abada, Teixeira, arXiv:1812.08062; Senjanovic, arXiv:2011.01264]