Sterile Neutrinos in Physics, Astrophysics, Cosmology

Part I: Active-Sterile Neutrino Oscillations

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Theoretical Aspects of Astroparticle Physics, Cosmology and Gravitation

Galileo Galilei Institute for Theoretical Physics
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Plan of the Lectures

Part I: Active-Sterile Neutrino Oscillations

Part II: Light Active and Sterile Neutrinos in Cosmology

Part III: Theory of Dirac and Majorana Neutrino Masses and Mixing

Plan of the Tutorials

Stefano Gariazzo

Tutorial 1: Short-Baseline Reactor Neutrino Oscillations

Tutorial 2: Sterile Neutrinos in Cosmology

Why Neutrino Physics

- Neutrinos are special among the known particles (fermions):
  - Neutrinos are the only neutral fermions.
  - Neutrinos interact very weakly, only with left-handed weak interactions (and of course gravitational interactions).
  - Their mass is extremely small.
- In the Standard Model neutrinos are special: they are assumed to be left-handed and massless.
- The observation of neutrino oscillations implies that neutrinos are massive.
- Neutrino masses are so far the only certain phenomenon Beyond the Standard Model (BSM).
- Neutrino masses, possible non-standard sterile neutrino states, non-standard neutrino properties, and non-standard neutrino interactions are windows on the physics Beyond the Standard Model.
- Neutrinos are powerful astrophysical messengers (from Sun, supernovae, AGNs, ...) thanks to their extremely weak interactions and neutrality.

Why Sterile Neutrinos

- Sterile neutrinos are neutral BSM fermions that do not have weak interactions. [Pontecorvo, Sov. Phys. JETP 26 (1968) 984]
- A very plausible way to generate neutrino masses requires the introduction of right-handed neutrinos that are sterile. (Part III)
- Very heavy right-handed sterile neutrinos can explain the smallness of the neutrino masses, but are decoupled from low-energy phenomenology (small non-unitarity of the effective mixing matrix). (Part III)
- New BSM physics can include sterile neutrinos at all mass scales.

- Heavy Sterile neutrinos at the TeV mass scale may be connected with low-energy seesaw and may be detected at LHC or future high-energy colliders. (Not treated in these lectures)
- Medium-heavy Sterile neutrinos at the keV mass scale may be dark matter. (Not treated in these lectures)
- Light Sterile neutrinos at the eV mass scale can generate short-baseline neutrino oscillations and explain some experimental anomalies. (Part I)

  They can also have cosmological effects (Part II)
- Very light Sterile neutrinos below the eV mass scale may generate small deviations from the standard three-neutrino oscillation framework of solar, atmospheric and long-baseline neutrino experiments. (Not treated in these lectures)

Standard Three-Neutrino Mixing

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{\text{CC}} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

Fields $\quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U^*_{\alpha k} |\nu_k, -\rangle \quad$ States

$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3×3 Unitary Mixing Matrix: $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1\,\text{MeV}$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered} \nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2} \end{gathered}}$$

$$\begin{array}{ll}
\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\
\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\
\bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\
\nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\
\nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV}
\end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

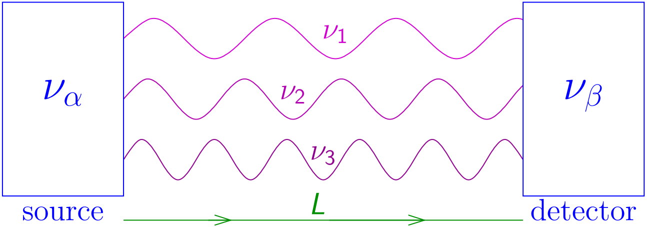
Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO) , $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_1, m_2, m_3 \lesssim 1\,\text{eV}}$

For oscillations we consider light sterile neutrinos $\Longrightarrow$ $m_{k>3} \lesssim 10\,\text{eV}$

Neutrino Oscillations in Vacuum

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^{*}\,|\nu_1\rangle + U_{\alpha 2}^{*}\,|\nu_2\rangle + U_{\alpha 3}^{*}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{\alpha 1}^{*}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{\alpha 2}^{*}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{\alpha 3}^{*}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$|\nu(L)\rangle = e^{-ipL}\sum_{k=1}^{3} U_{\alpha k}^{*}\,e^{-i(E_k - p)L}\,|\nu_k\rangle$$

$$E_k \simeq p + \frac{m_k^2}{2p} \qquad \Longrightarrow \qquad E_k - p \simeq \frac{m_k^2}{2p} \equiv \frac{m_k^2}{2E}$$

$$|\nu(L)\rangle = e^{-iEL} \sum_k U_{\alpha k}^* \exp\left(-i \frac{m_k^2 L}{2E}\right) |\nu_k\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L) \rangle|^2$$

$$= \left| \cancel{e^{-iEL}} \langle \nu_\beta | \sum_k U_{\alpha k}^* \exp\left(-i \frac{m_k^2 L}{2E}\right) |\nu_k\rangle \right|^2$$

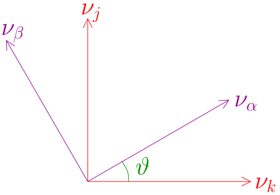
$$|\nu_\beta\rangle = \sum_k U_{\beta k}^* |\nu_k\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j} U_{\beta k} U_{\alpha k}^* U_{\beta j}^* U_{\alpha j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

The oscillation probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

Effective Two-Neutrino Mixing Approximation

$$\boxed{\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle \end{aligned}}$$



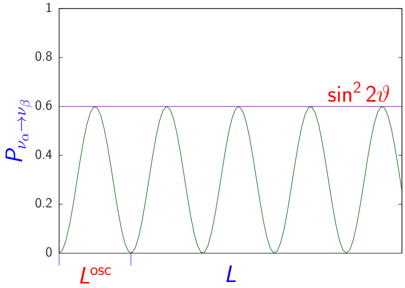
$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m_k^2 - m_j^2$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

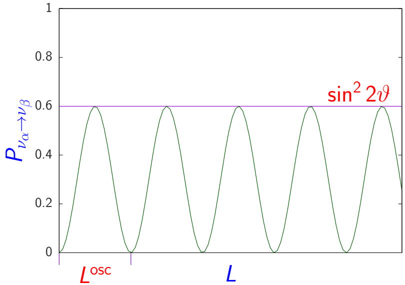
Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

2ν-mixing: $P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \implies L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$



- The effect of a tiny Δm^2 can be amplified by a large distance L.
- A tiny Δm^2 generates oscillations observable at macroscopic distances!
- Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

2ν-mixing: $\quad P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}\right)$



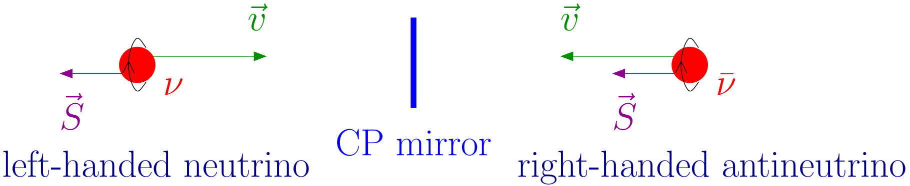
$$\frac{L}{E} \gtrsim \begin{cases} 10 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1} \, \text{eV}^2 \\ 10^3 \, \frac{\text{m}}{\text{MeV}} \, \left(\frac{\text{km}}{\text{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3} \, \text{eV}^2 \\ 10^4 \, \frac{\text{km}}{\text{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4} \, \text{eV}^2 \\ 10^{11} \, \frac{\text{m}}{\text{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11} \, \text{eV}^2 \end{cases}$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu_{\alpha L}}^T$$

$$\begin{array}{lcl} \mathsf{C} & \Longrightarrow & \text{Particle} \leftrightarrows \text{Antiparticle} \\ \mathsf{P} & \Longrightarrow & \text{Left-Handed} \leftrightarrows \text{Right-Handed} \end{array}$$



$$\text{Fields: } \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$$

$$\text{States: } |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

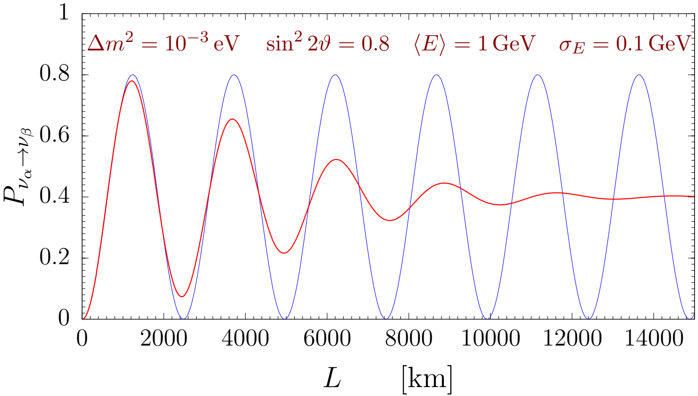
$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

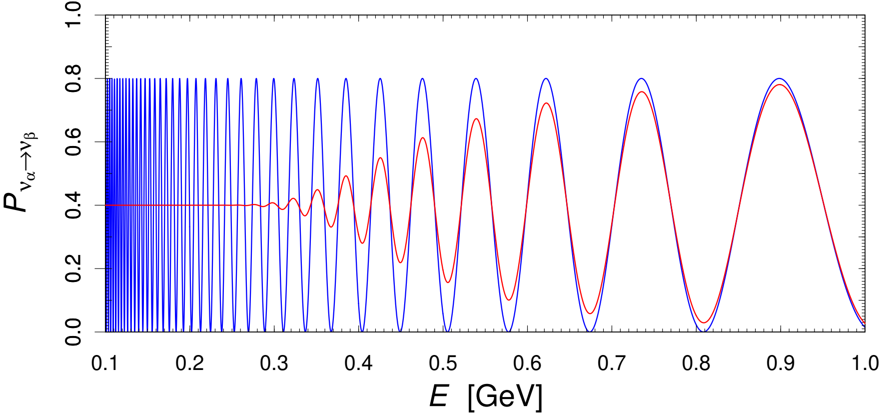
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

$$\Downarrow$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E)\rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\, dE\right] \qquad (\alpha \neq \beta)$$





$\Delta m^2 = 10^{-3}\,\text{eV} \qquad \sin^2 2\vartheta = 0.8 \qquad L = 10^4\,\text{km} \qquad \sigma_E = 0.01\,\text{GeV}$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E \right] \qquad (\alpha \neq \beta)$$

Three-Neutrino Mixing Matrix

Standard Parameterization of Mixing Matrix (as CKM)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad s_{ab} \equiv \sin\vartheta_{ab}$ $\quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\quad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS $\left\{ \begin{array}{l} \text{3 Mixing Angles: } \vartheta_{12},\ \vartheta_{23},\ \vartheta_{13} \\ \text{1 CPV Dirac Phase: } \delta_{13} \\ \text{2 independent } \Delta m^2_{kj} \equiv m^2_k - m^2_j\text{: } \Delta m^2_{21},\ \Delta m^2_{31} \end{array} \right.$

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes

Three-Neutrino Mixing Parameters

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

(SNO, Borexino / Super-Kamiokande / GALLEX/GNO, SAGE / Homestake, Kamiokande)

VLBL Reactor
$\bar{\nu}_e$ disappearance

(KamLAND)

$\to$

$$\Delta m^2_{\rm S} = \Delta m^2_{21} = (7.36 \pm 0.155) \times 10^{-5}\,\mathrm{eV}^2$$
($\sim$ 2.3% accuracy)

$$\sin^2\vartheta_{\rm S} = \sin^2\vartheta_{12} = 0.303 \pm 0.013$$
($\sim$ 4.5% accuracy)

[A. Marrone, talk at NeuTel 2021]
[Capozzi, Di Valentino, Lisi, Marrone, Melchiorri, Palazzo, arXiv:2003.08511]
[de Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortola, Valle, arXiv:2006.11237]
[Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, arXiv:2007.14792]

Three-Neutrino Mixing Parameters

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

Atmospheric
$\nu_\mu \to \nu_\tau$
(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)

LBL Accelerator
ν_μ disappearance
(K2K, MINOS, T2K, NOνA)

LBL Accelerator
$\nu_\mu \to \nu_\tau$
(OPERA)

$\to$

$$\Delta m^2_{\text{A}} = |\Delta m^2_{31} + \Delta m^2_{32}|/2$$
$$= (2.475 \pm 0.028) \times 10^{-3}\,\text{eV}^2$$
$(\sim 1.1\%$ accuracy$)$ (NO)
$$= (2.455 \pm 0.028) \times 10^{-3}\,\text{eV}^2$$
$(\sim 1.2\%$ accuracy$)$ (IO)

$$\sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{23}$$
$$= 0.569 \pm 0.017$$
$(\sim 5.4\%$ accuracy$)$

[A. Marrone, talk at NeuTel 2021]
[Capozzi, Di Valentino, Lisi, Marrone, Melchiorri, Palazzo, arXiv:2003.08511]
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LBL Accelerator $\nu_\mu \to \nu_e$ (T2K, MINOS, NOνA)

LBL Reactor $\bar{\nu}_e$ disappearance (Daya Bay, RENO, Double Chooz)

$$\rightarrow \begin{cases} \Delta m^2_{\text{A}} = |\Delta m^2_{31} + \Delta m^2_{32}|/2 \\ \sin^2\vartheta_{\text{A}} = \sin^2\vartheta_{13} = 0.0223 \pm 0.0006 \quad (\sim 2.9\% \text{ accuracy}) \end{cases}$$

[A. Marrone, talk at NeuTel 2021]
[Capozzi, Di Valentino, Lisi, Marrone, Melchiorri, Palazzo, arXiv:2003.08511]
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CP Violation

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} \underbrace{- 4 \sum_{k>j} \mathrm{Re}\left[U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}\right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)}_{\text{CP conserving}}$$

$$\underbrace{+ 2 \sum_{k>j} \mathrm{Im}\left[U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}_{\text{CP violating}}$$

- The oscillation probabilities depend on the quartic rephasing invariants

$$U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}$$

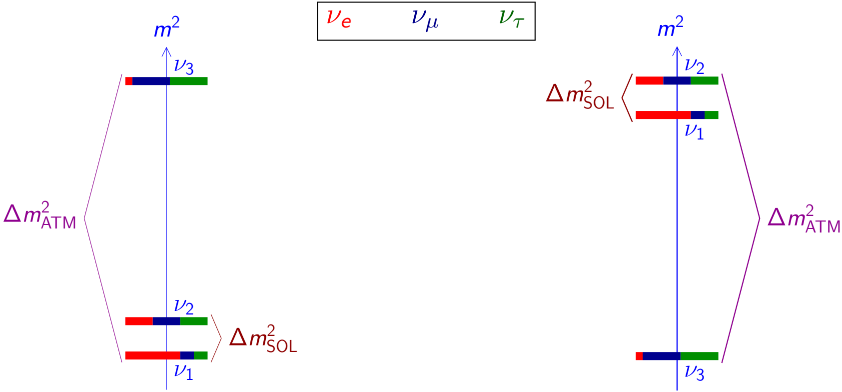
- CP violation depends on the Jarlskog invariants

$$\mathrm{Im}\left[U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}\right]$$

- In three-neutrino mixing there is only one Jarlskog invariant, corresponding to the Dirac CP-violating phase:

$$J_{\mathrm{CP}} = \pm\, \mathrm{Im}\left[U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}\right] = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$$

Neutrino Mass Ordering



Normal Ordering

$$\Delta m^2_{31} > \Delta m^2_{32} > 0$$

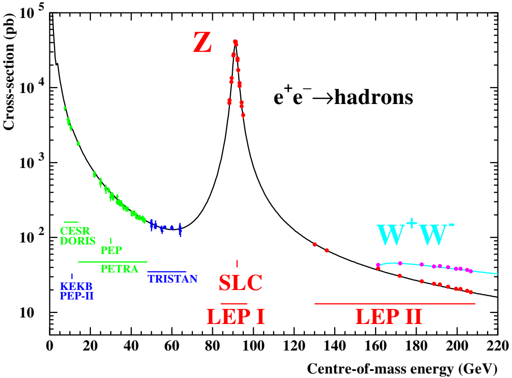
Inverted Ordering

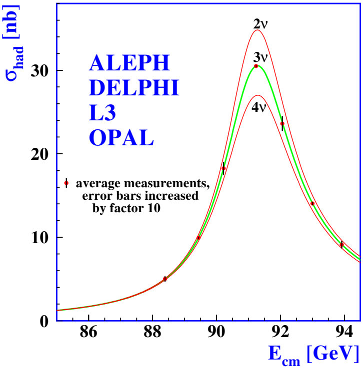
$$\Delta m^2_{32} < \Delta m^2_{31} < 0$$

absolute scale is not determined by neutrino oscillation data

- In the standard framework of three-neutrino mixing there are two independent Δm^2's:
  - $\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\text{eV}^2$
  - $\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$
- Atmospheric and solar neutrino oscillations are detectable at the distances
  - $L^{\text{osc}}_{\text{ATM}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{ATM}}} \approx 1\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
  - $L^{\text{osc}}_{\text{SOL}} \gtrsim \dfrac{E_\nu}{\Delta m^2_{\text{SOL}}} \approx 50\,\text{km}\,\dfrac{E_\nu}{\text{MeV}}$
- The atmospheric and solar neutrino oscillations cannot explain flavor neutrino transitions at shorter distances.

Number of Flavor and Massive Neutrinos?





[LEP, Phys. Rept. 427 (2006) 257, arXiv:hep-ex/0509008]

$$\Gamma_Z = \sum_{\ell=e,\mu,\tau} \Gamma_{Z\to\ell\bar{\ell}} + \sum_{q\neq t} \Gamma_{Z\to q\bar{q}} + \Gamma_{\text{inv}} \qquad \Gamma_{\text{inv}} = N_\nu \, \Gamma_{Z\to\nu\bar{\nu}}$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

Improved cross section: $N_\nu = 2.9975 \pm 0.0074$ [Janot, Jadach, arXiv:1912.02067]

$$e^+e^- \to Z \xrightarrow{\text{invisible}} \sum_{a=\text{active}} \nu_a \bar{\nu}_a \quad \Longrightarrow \quad \nu_e \; \nu_\mu \; \nu_\tau$$

3 light active flavor neutrinos

mixing $\Rightarrow$ $\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$

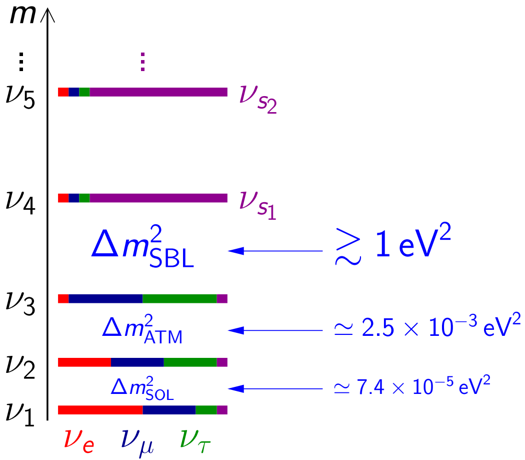
$N \geq 3$

no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau, s_1, s_2, \ldots}$$

Beyond Three-Neutrino Mixing: Sterile Neutrinos



$$\boxed{L^{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}}$$

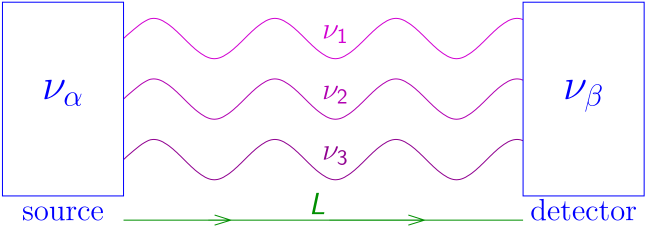
Terminology: a eV-scale sterile neutrino

means: a eV-scale massive neutrino which is mainly sterile

Short-Baseline Neutrino Oscillations?

Three-Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle$$



$$\begin{aligned} |\nu_{\text{detector}}\rangle &\simeq U_{\alpha 1}^* e^{-iEL} |\nu_1\rangle + U_{\alpha 2}^* e^{-iEL} |\nu_2\rangle + U_{\alpha 3}^* e^{-iEL} |\nu_3\rangle \\ &= e^{-iEL} \left(U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle \right) = e^{-iEL} |\nu_\alpha\rangle \end{aligned}$$

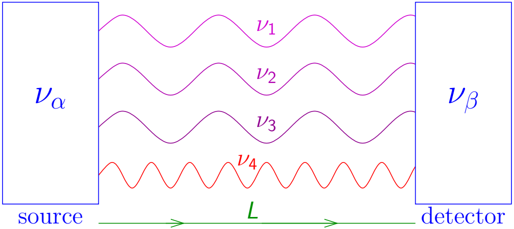
$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \simeq |e^{-iEL} \langle \nu_\beta | \nu_\alpha \rangle|^2 = \delta_{\alpha\beta}$$

No Short-Baseline Neutrino Oscillations!

Short-Baseline Neutrino Oscillations?

3+1 Neutrino Mixing

$$|\nu_{\text{source}}\rangle = |\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle + U_{\alpha 4}^* |\nu_4\rangle$$



$$|\nu_{\text{detector}}\rangle \simeq e^{-iEL} \left(U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle \right) + U_{\alpha 4}^* e^{-iE_4 L} |\nu_3\rangle \not\propto |\nu_\alpha\rangle$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu_{\text{detector}} \rangle|^2 \neq \delta_{\alpha\beta}$$

Short-Baseline Neutrino Oscillations!

The oscillation probabilities depend on U and

$$\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$$

- Some authors that probably did not think about the quantum mechanics of neutrino oscillations present $\nu_\mu \to \nu_e$ short-baseline transitions due to sterile neutrinos as

$$\nu_\mu \to \nu_s \to \nu_e$$

- This is wrong!

THERE IS NO INTERMEDIATE ν_s !

Two possible interpretations of $\nu_\mu \to \nu_s \to \nu_e$:

1. There is a transition from ν_μ to ν_s, and then to ν_e: wrong!
   Because the intermediate determination of the neutrino flavor interrupts the quantum evolution.
   Moreover, ν_s is not detectable!

2. There is an intermediate linear combination of massive neutrinos that corresponds to $|\nu_s\rangle$: wrong!
   This is possible only with the mixing $(|a|^2 + |b|^2 + |c|^2 = 1)$

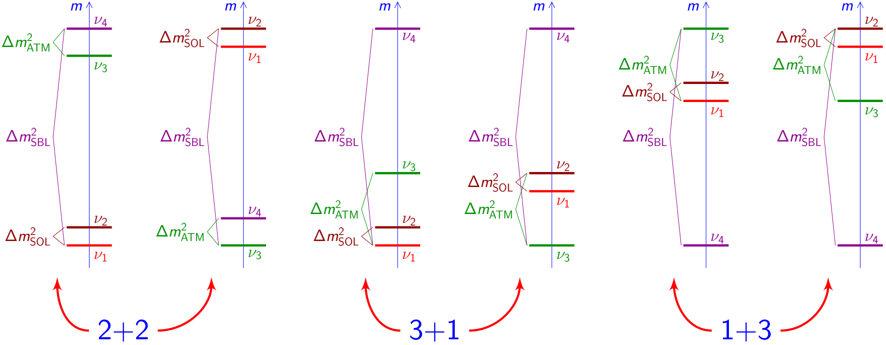
$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_s\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cdots & \cdots & \cdots & 0 \\ a & b & c & 1 \\ \cdots & \cdots & \cdots & 0 \\ -a & -b & -c & 1 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \end{pmatrix}$$

$$|\nu(L)\rangle = \frac{e^{-iEL}}{\sqrt{2}} \left[a\,|\nu_1\rangle + b\,|\nu_2\rangle + c\,|\nu_3\rangle + e^{-i(E_4-E)L}\,|\nu_4\rangle \right]$$

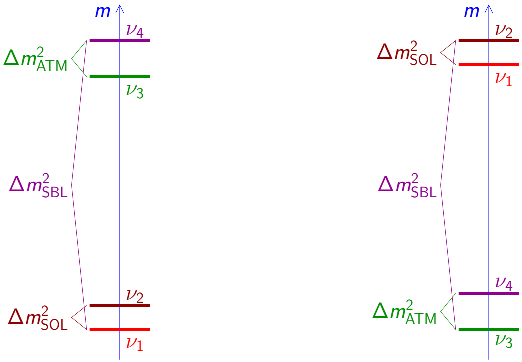
$|\nu(L)\rangle = |\nu_\mu\rangle$ for $L = 0$ and $|\nu(L)\rangle \propto |\nu_s\rangle$ for $e^{-i(E_4-E)L} = -1$

but in this case there are no SBL $\nu_\mu \to \nu_e$ transitions!

Four-Neutrino Schemes: 2+2, 3+1 and 1+3



2+2 Four-Neutrino Schemes

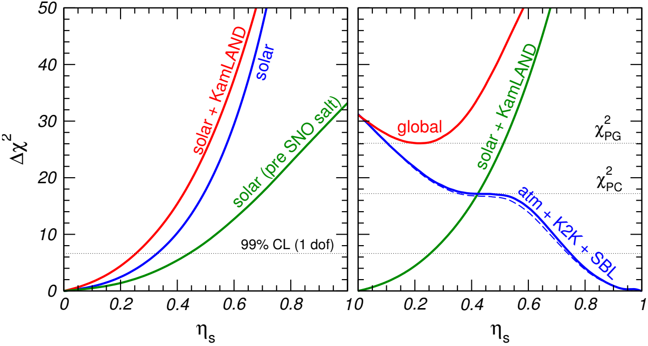


- After LSND (1995) 2+2 was preferred to 3+1, because of the 3+1 appearance-disappearance tension

  [Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

- This is not a perturbation of 3-ν Mixing $\Longrightarrow$ Large active–sterile oscillations for solar or atmospheric neutrinos!

2+2 Schemes are Strongly Disfavored



Solar: Matter Effects + SNO NC

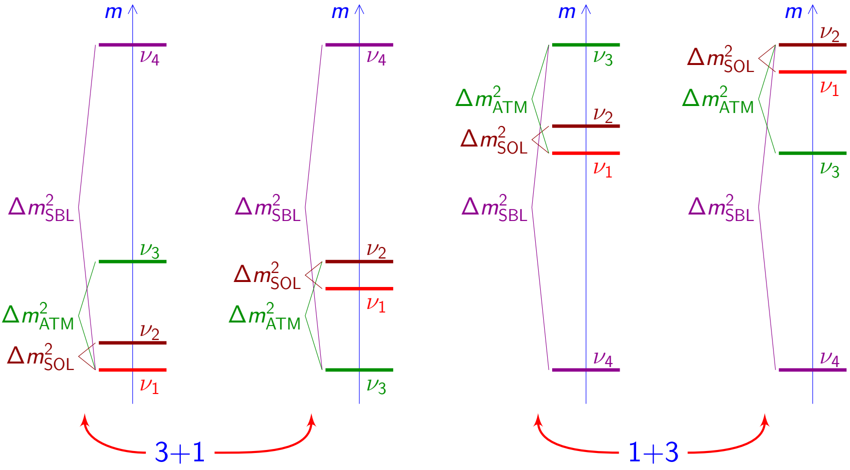
Atmospheric: Matter Effects

$$\eta_s = |U_{s1}|^2 + |U_{s2}|^2 = 1 - |U_{s3}|^2 + |U_{s4}|^2$$

$$99\%\ \text{CL:} \quad \begin{cases} \eta_s < 0.25 & \text{(Solar + KamLAND)} \\ \eta_s > 0.75 & \text{(Atmospheric + K2K)} \end{cases}$$

[Maltoni, Schwetz, Tortola, Valle, New J. Phys. 6 (2004) 122]

3+1 and 1+3 Four-Neutrino Schemes



- Perturbation of 3-ν Mixing: $|U_{e4}|^2, |U_{\mu 4}|^2, |U_{\tau 4}|^2 \ll 1 \quad |U_{s4}|^2 \simeq 1$
- 1+3 schemes are disfavored by cosmology (ΛCDM):

$$\sum_{k=1}^{3} m_k \lesssim 0.12\,\text{eV} \qquad (95\%\ \text{CL})$$ [Planck, arXiv:1807.06209]

Effective 3+1 SBL Oscillation Probabilities

$$|\nu_\alpha\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} |\nu_k\rangle \quad \xrightarrow{t} \quad |\nu_\alpha(t)\rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} e^{-iE_k t} |\nu_k\rangle$$

$$A_{\nu_\alpha \to \nu_\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \qquad (\langle \nu_\beta | \nu_k \rangle = U_{\beta k})$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 = \left| e^{iE_1 t} \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$= \underbrace{\left| e^{iE_1 t} \right|^2}_{1} \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$= \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1) t} \right|^2$$

$$P_{\nu_\alpha\to\nu_\beta} = \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2$$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} \quad \Longrightarrow \quad E_k - E_1 \simeq \frac{\Delta m_{k1}^2}{2p}$$

$$E = p \qquad\qquad t \simeq L$$

$$P_{\nu_\alpha\to\nu_\beta} \simeq \left| \sum_{k=1}^{4} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$P_{\nu_\alpha\to\nu_\beta} = \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} \exp\left(-i \frac{\Delta m^2_{21} L}{2E} \right) \right.$$
$$\left. + U_{\alpha 3}^* U_{\beta 3} \exp\left(-i \frac{\Delta m^2_{31} L}{2E} \right) + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m^2_{41} L}{2E} \right) \right|^2$$

SBL $\Longrightarrow$ $\frac{\Delta m^2_{21} L}{2E} \ll 1 \qquad \frac{\Delta m^2_{31} L}{2E} \ll 1$

$$P^{\text{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq \left| U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4} \exp\left(-i \frac{\Delta m^2_{41} L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3} = \delta_{\alpha\beta} - U_{\alpha 4}^* U_{\beta 4}$$

$$
\begin{aligned}
P^{\mathrm{SBL}}_{\nu_\alpha\to\nu_\beta} &\simeq \left|\delta_{\alpha\beta} - U^*_{\alpha 4} U_{\beta 4}\left[1-\exp\left(-i\frac{\Delta m^2_{41}L}{2E}\right)\right]\right|^2 \\
&= \delta_{\alpha\beta} + |U_{\alpha 4}|^2 |U_{\beta 4}|^2 \left(2 - 2\cos\frac{\Delta m^2_{41}L}{2E}\right) \\
&\qquad - 2\delta_{\alpha\beta}|U_{\alpha 4}|^2\left(1-\cos\frac{\Delta m^2_{41}L}{2E}\right) \\
&= \delta_{\alpha\beta} - 2|U_{\alpha 4}|^2\left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\left(1-\cos\frac{\Delta m^2_{41}L}{2E}\right) \\
&= \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2\left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right)\sin^2\frac{\Delta m^2_{41}L}{4E}
\end{aligned}
$$

$$
\alpha \neq \beta \quad \Longrightarrow \quad P^{\mathrm{SBL}}_{\nu_\alpha\to\nu_\beta} \simeq 4|U_{\alpha 4}|^2|U_{\beta 4}|^2 \sin^2\left(\frac{\Delta m^2_{41}L}{4E}\right)
$$

$$
\alpha = \beta \quad \Longrightarrow \quad P^{\mathrm{SBL}}_{\nu_\alpha\to\nu_\alpha} \simeq 1 - 4|U_{\alpha 4}|^2\left(1-|U_{\alpha 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41}L}{4E}\right)
$$

Appearance ($\alpha \neq \beta$) Disappearance

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

SBL

- 6 mixing angles
- 3 Dirac CP phases
- 3 Majorana CP phases

- $\Delta m^2_{\text{SBL}} = \Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$
- CP violation is not observable in SBL experiments!
- Observable in LBL accelerator exp. sensitive to Δm^2_{ATM} [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122] and solar exp. sensitive to Δm^2_{SOL} [Long, Li, CG, PRD 87, 113004 (2013) 113004]

Common Parameterization of 4ν Mixing

$$U = \left[W^{34} R^{24} W^{14} R^{23} W^{13} R^{12} \right] \mathrm{diag}\left(1, e^{i\lambda_{21}}, e^{i\lambda_{31}}, e^{i\lambda_{41}}\right)$$

$$= \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ \cdots & \cdots & \cdots & c_{14}s_{24} \\ \cdots & \cdots & \cdots & c_{14}c_{24}s_{34}e^{-i\delta_{34}} \\ \cdots & \cdots & \cdots & c_{14}c_{24}c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 & 0 \\ 0 & 0 & e^{i\lambda_{31}} & 0 \\ 0 & 0 & 0 & e^{i\lambda_{41}} \end{pmatrix}$$

$$|U_{e4}|^2 = \sin^2\vartheta_{14} \quad \Rightarrow \quad \sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) = \sin^2 2\vartheta_{14}$$

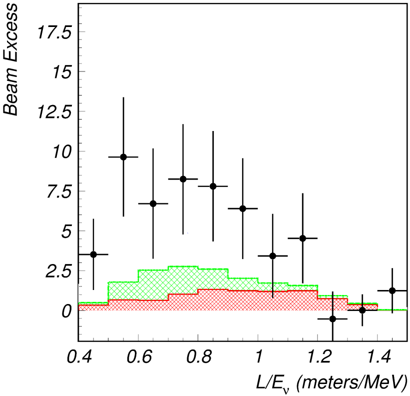
$$|U_{\mu4}|^2 = \cos^2\vartheta_{14}\sin^2\vartheta_{24} \simeq \sin^2\vartheta_{24} \Rightarrow \sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq \sin^2 2\vartheta_{24}$$

Short-Baseline Neutrino Oscillation Anomalies

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e \qquad 20\,\text{MeV} \le E \le 52.8\,\text{MeV}$



$$\Delta m^2_{\text{SBL}} \gtrsim 0.1\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$$

- Well-known and pure source of $\bar{\nu}_\mu$

$$\underset{800\,\text{MeV}}{p} + \text{target} \to \pi^+ \xrightarrow{\text{at rest}} \mu^+ + \nu_\mu$$

$$\mu^+ \xrightarrow[\text{at rest}]{} e^+ + \nu_e + \bar{\nu}_\mu$$

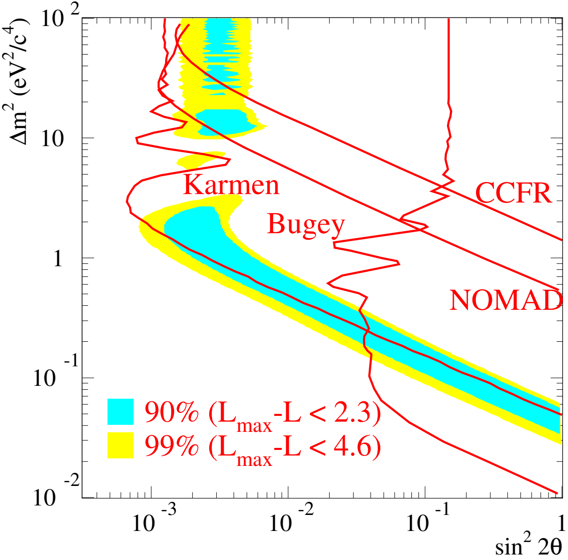
$L \simeq 30\,\text{m}$

$$\bar{\nu}_e + p \to n + e^+$$

Well-known detection process of $\bar{\nu}_e$

- $\approx 3.8\sigma$ excess
- But signal not seen by KARMEN at $L \simeq 18\,\text{m}$ with the same method

[PRD 65 (2002) 112001]



$$\Delta m^2_{\rm SBL} \gtrsim 3 \times 10^{-2}\,{\rm eV}^2 \gg \Delta m^2_{\rm ATM} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2 \gg \Delta m^2_{\rm SOL}$$

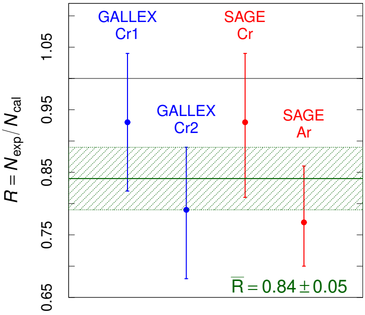
Gallium Anomaly

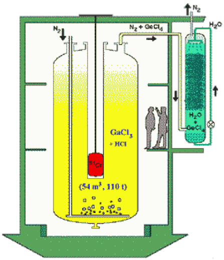
Gallium Radioactive Source Experiments: GALLEX and SAGE

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $\quad e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

$E \simeq 0.75\,\mathrm{MeV}$ $\quad E \simeq 0.81\,\mathrm{MeV}$

Test of Solar ν_e Detection: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$





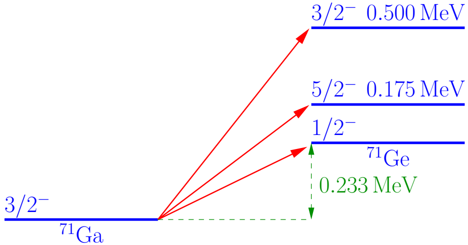
$\approx 2.9\sigma$ deficit

$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$ $\quad \langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$

$\Delta m^2_{\mathrm{SBL}} \gtrsim 1\,\mathrm{eV}^2 \gg \Delta m^2_{\mathrm{ATM}}$

[SAGE, PRC 73 (2006) 045805; PRC 80 (2009) 015807; Laveder et al, Nucl.Phys.Proc.Suppl. 168 (2007) 344, MPLA 22 (2007) 2499, PRD 78 (2008) 073009, PRC 83 (2011) 065504]

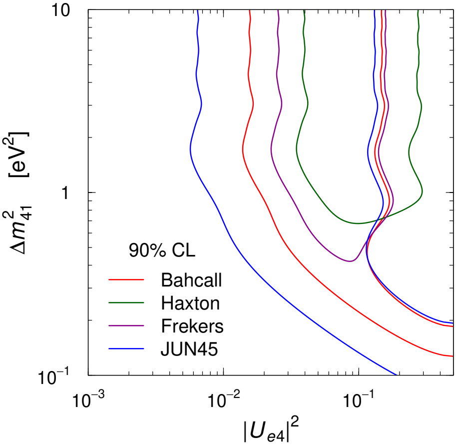
- Deficit could be due to an overestimate of
$$\sigma(\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-)$$
- First calculation: Bahcall, PRC 56 (1997) 3391



- $\sigma_{\mathrm{G.S.}}$ from $T_{1/2}({}^{71}\mathrm{Ge}) = 11.43 \pm 0.03\,\mathrm{days}$ [Hampel, Remsberg, PRC 31 (1985) 666]

$$\sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) = 55.3 \times 10^{-46}\,\mathrm{cm}^2\,(1 \pm 0.004)_{3\sigma}$$

- $\sigma({}^{51}\mathrm{Cr}) = \sigma_{\mathrm{G.S.}}({}^{51}\mathrm{Cr}) \left(1 + 0.669\,\frac{\mathrm{BGT}_{175}}{\mathrm{BGT}_{\mathrm{G.S.}}} + 0.220\,\frac{\mathrm{BGT}_{500}}{\mathrm{BGT}_{\mathrm{G.S.}}}\right)$
- The contribution of excited states is only $\sim 5\%$, but it is crucial for the size of the Gallium anomaly!



Cross sections in units of $10^{-45}\,\mathrm{cm}^2$:

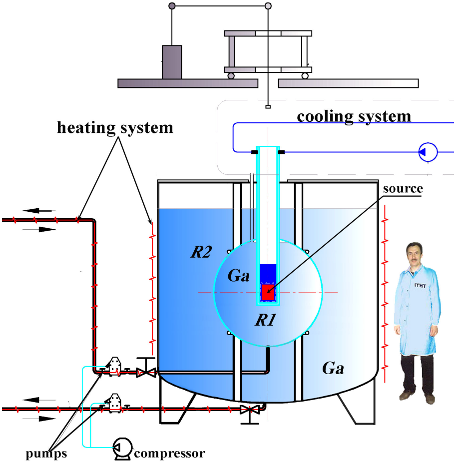
	$\sigma(^{51}\mathrm{Cr})$	$\sigma(^{37}\mathrm{Ar})$
Bahcall	5.81 ± 0.16	7.00 ± 0.21
Haxton	6.39 ± 0.65	7.72 ± 0.81
Frekers	5.92 ± 0.11	7.15 ± 0.14
JUN45	5.67 ± 0.06	6.80 ± 0.08

[Kostensalo, Suhonen, Giunti, Srivastava, arXiv:1906.10980]

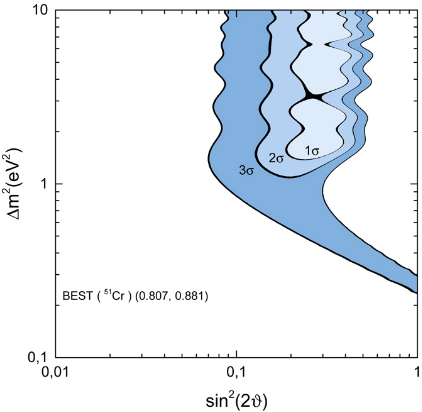
BEST

[arXiv:1006.2103, arXiv:1602.03826, arXiv:1710.06326, arXiv:1807.02977, arXiv:1905.07437]

Direct test of the Gallium anomaly with ^{51}Cr source.



$$R_1 = 0.66\,\text{m}, \qquad R_2 = 1.096\,\text{m}$$



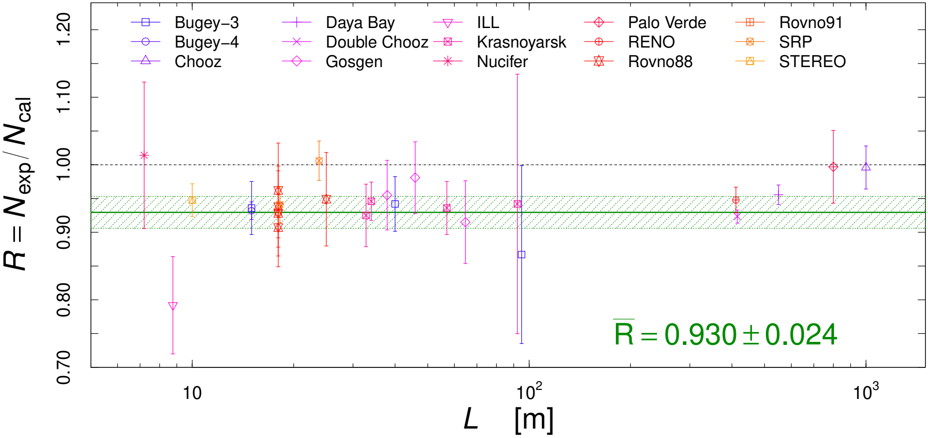
Allowed regions of oscillations parameters if the result of the BEST experiment corresponds to the best fit point for combining the SAGE + GALLEX. The numbers in parentheses indicate the most probable ratios R of observed-to-expected without sterile neutrinos germanium atoms in the two vessels.

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

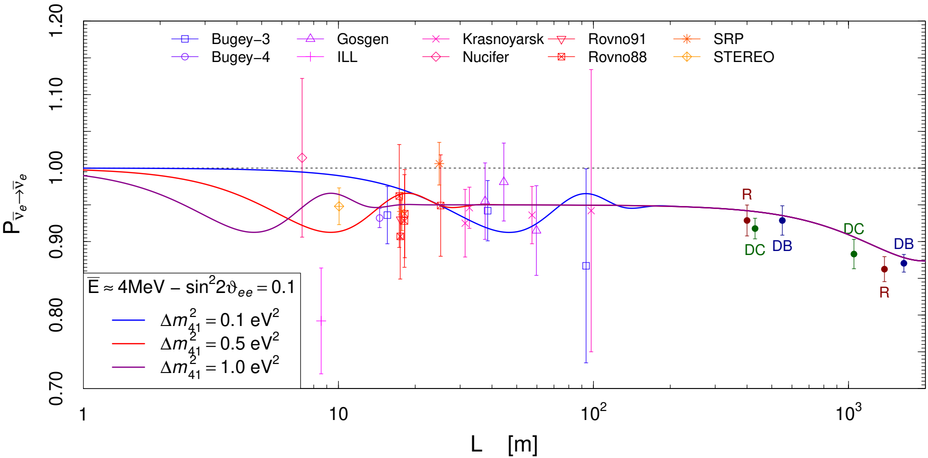
2011: new reactor $\bar{\nu}_e$ fluxes: Huber-Mueller (HM)

[Mueller et al, PRC 83 (2011) 054615; Huber, PRC 84 (2011) 024617]



$\approx 3.0\,\sigma$ deficit $\Longrightarrow$ Anomaly!

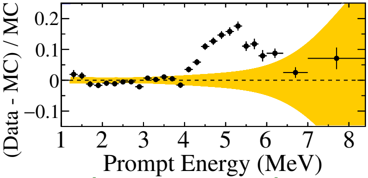
Short-Baseline Reactor Neutrino Oscillations



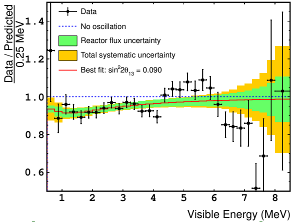
$$\Delta m^2_{\text{SBL}} \gtrsim 0.5\,\text{eV}^2 \gg \Delta m^2_{\text{ATM}}$$

- SBL oscillations are averaged at the Daya Bay, RENO, and Double Chooz near detectors $\Longrightarrow$ no spectral distortion

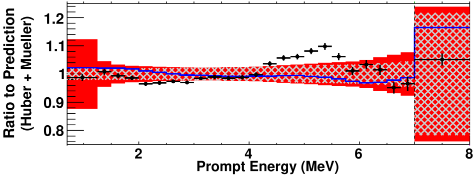
Reactor Antineutrino 5 MeV Bump



[RENO, arXiv:1511.05849]



[Double Chooz, arXiv:1406.7763]



[Daya Bay, arXiv:1508.04233]

- Cannot be explained by neutrino oscillations (SBL oscillations are averaged in RENO, DC, DB).

- If it is due to a theoretical miscalculation of the spectrum, it can have opposite effects on the anomaly:

  [see: Berryman, Huber, arXiv:1909.09267]

  - If it is a 4-6 MeV excess it increases the anomaly: new HKSS flux calculation

    [Hayen, Kostensalo, Severijns, Suhonen, arXiv:1908.08302]

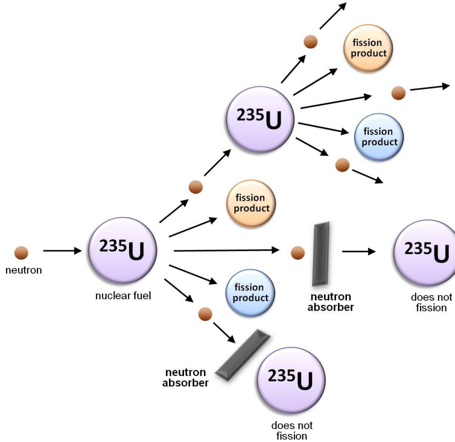
  - If it is a 1-4 MeV suppression it decreases the anomaly: new EF flux calculation

    [Estienne, Fallot, et al, arXiv:1904.09358]

Reactor Neutrinos

- Nuclear reactors are the most intense terrestrial sources of

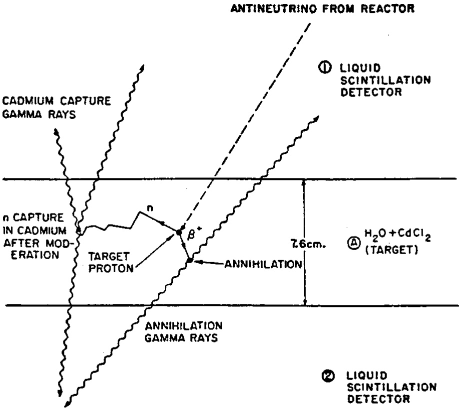
  electron antineutrinos $\bar{\nu}_e$



- $n + {}^{235}\text{U} \to A + B + 2.5n + 6e^- + 6\bar{\nu}_e + 200\,\text{MeV}$

- $N_{\bar{\nu}_e} \simeq 2 \times 10^{20}\,\mathrm{s}^{-1}\,\mathrm{GW}_{\mathrm{th}}^{-1}$
- $\Phi_{\bar{\nu}_e} \simeq 1.6 \times 10^{13}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{GW}_{\mathrm{th}}^{-1}$ at 10 m
- Comparison: $\Phi_{\nu_e}^{\mathrm{Sun}} \simeq 6.4 \times 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ on Earth.
- Reactor Neutrinos are a great opportunity for Neutrino Physics!
- Indeed neutrinos were detected for the first time by Cowan and Reines in 1956 at the Savannah River nuclear reactor.
- Further advantages:
  - The $\bar{\nu}_e$ flux is under control: background measurement when reactor is off.
  - The $\bar{\nu}_e$ detection cross section is well-known.

Detection: Inverse beta Decay



$$\bar{\nu}_e + p \to n + e^+$$

Cowan and Reines 1956

- The delayed ($\lesssim 200\,\mu$s) neutron capture signal is crucial for the background suppression.
- Well-known cross section obtained by crossing from the neutron lifetime.
- Neutrino energy measurement: $E_{\bar{\nu}_e} \simeq T_e + 1.8\,\text{MeV}$

$$T_e = E_{\text{prompt}} - 2m_e$$

E_{prompt} is total visible prompt energy from positron annihilation

Nuclear Fuel

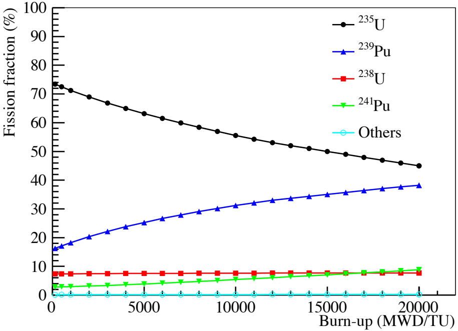
- Nuclear reactor energy is produced by the fissions of

$$^{235}\mathrm{U} \qquad ^{238}\mathrm{U} \qquad ^{239}\mathrm{Pu} \qquad ^{241}\mathrm{Pu}$$

- ^{235}U, ^{239}Pu, and ^{241}Pu are fissile nuclides, i.e capable of sustaining a nuclear fission chain reaction.
- They have large fission cross section and small neutron capture cross section for slow "thermal" neutrons ($E_n \approx 0.025\,\mathrm{eV}$).
- ^{238}U can be fissioned by the fast neutrons ($E_n \approx 2\,\mathrm{MeV}$) emitted in fissions but it has a small fission cross section and a large neutron capture cross section.
- ^{235}U is the only natural fissile nuclide. Natural Uranium: 0.72% of ^{235}U.
- Neutrons are slowed down by the moderator (H_2O, D_2O, C).
- In typical light water reactors (LWR) the moderator is H_2O that has a significant neutron capture cross section.
- LWR use Low Enriched Uranium (LEU) with 3-5% of ^{235}U to sustain the nuclear chain reactions.

Commercial Light Water Reactors

- In a commercial LWR nuclear power plant as Daya Bay a reactor burning cycle (18 months) starts with the replacement of 1/3 of the fuel elements with fresh LEU.



[Daya Bay, Chin. Phys. C 41 (2017) 013002]

- $^{239}\mathrm{Pu}$ is generated from $^{238}\mathrm{U}$:

$$
\begin{array}{l}
n + {}^{238}\mathrm{U} \to {}^{239}\mathrm{U} + \gamma \\
\qquad\qquad\qquad \downarrow \\
\qquad\qquad\qquad {}^{239}\mathrm{Np} + e^- + \bar{\nu}_e \\
\qquad\qquad\qquad \downarrow \\
\qquad\qquad\qquad {}^{239}\mathrm{Pu} + e^- + \bar{\nu}_e
\end{array}
$$

- $^{241}\mathrm{Pu}$ is generated from $^{239}\mathrm{Pu}$:

$$n + {}^{239}\mathrm{Pu} \to {}^{240}\mathrm{Pu} + \gamma$$
$$n + {}^{240}\mathrm{Pu} \to {}^{241}\mathrm{Pu} + \gamma$$

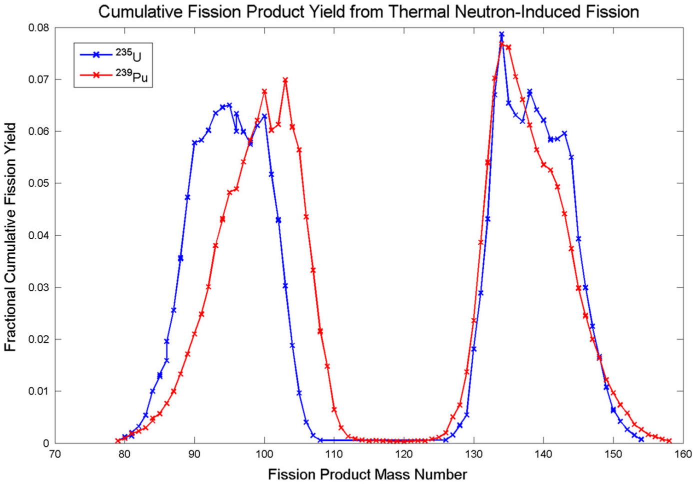
Research Reactors

- Optimized as neutron sources for testing of materials and production of radioisotopes.
- Use Highly Enriched Uranium (HEU): about 93% of ^{235}U (weapons grade).
- The burning cycle is short (about 1 month), minimizing the production of ^{239}Pu and ^{241}Pu.
- The ^{235}U fission fraction is larger than 99%.
- Small core sizes (good for neutrino oscillation measurements).
- The frequent reactor-off periods during refueling allow a precise background determination.

Reactor $\bar{\nu}_e$ Flux Calculation

Reactor $\bar{\nu}_e$ flux produced by the β decays of the fission products of

^{235}U ^{238}U ^{239}Pu ^{241}Pu



[Dayman, Biegalski, Haas, Rad. Nucl. Chem. 305 (2015) 213]

- For each allowed β decay the electron spectrum is

$$S_\beta(E_e) = K p_e E_e \left(E_e - E_0\right)^2 F(Z, E_e) \qquad (E_\nu = E_0 - E_e)$$

$$S_\nu(E_\nu) = K \sqrt{(E_0 - E_e)^2 - m_e^2}\,(E_0 - E_e)\, E_\nu^2 F(Z, E_e)$$

- Aggregate reactor spectrum (electron or neutrino):

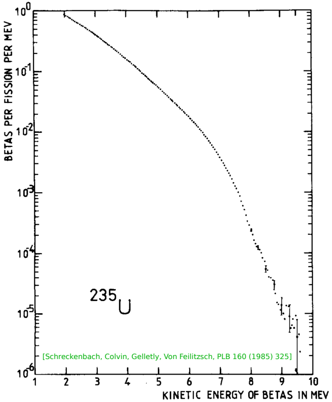
$$S_{\text{tot}}(E, t) = \sum_k \underset{\substack{\uparrow \\ \text{fission fractions}}}{F_k(t)}\, S_k(E) \qquad (k = 235, 238, 239, 241)$$

$$S_k(E) = \sum_n \underset{\substack{\uparrow \\ \text{cumulative} \\ \text{fission yield}}}{Y_n^k} \sum_b \mathrm{BR}_n^b\, S_n^b(E)$$

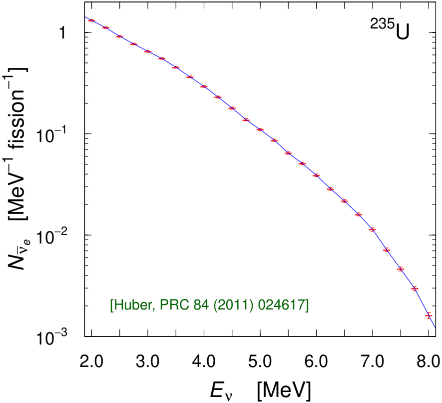
- The *ab initio* calculation of each $S_k^\nu(E_\nu)$ requires knowledge of about 1000 spectra and branching ratios $(k = 235, 238, 239, 241)$.

- Nuclear data tables are incomplete and sometimes inexact.

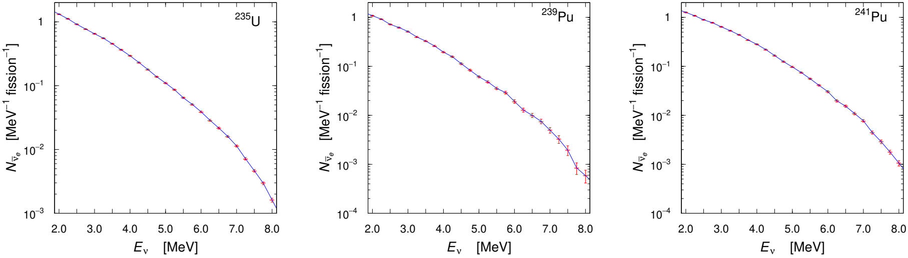
- Semi-empirical method: conversion of the aggregate β spectra $S_k^\beta(E_e)$ measured at ILL in the 80's with ~ 30 virtual β branches.



$\Longrightarrow$



- In the 80's Schreckenbach et al. measured the aggregate β spectra of ^{235}U, ^{239}Pu, and ^{241}Pu exposing thin foils to the thermal neutron flux of the ILL reactor in Grenoble.
- The standard reactor $\bar{\nu}_e$ fluxes and spectra from ^{235}U, ^{239}Pu, and ^{241}Pu were obtained with the virtual-branches conversion method:



[Huber, PRC 84 (2011) 024617]

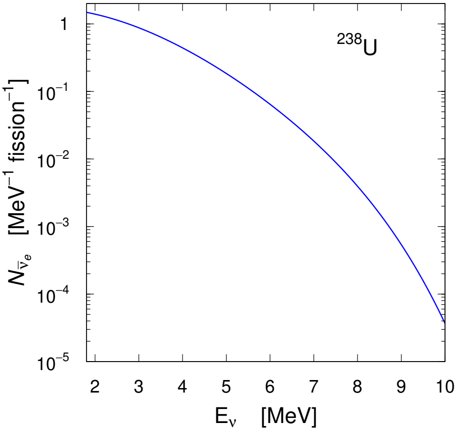
- The conversion method was estimated to have about 1% uncertainty.

[Vogel, PRC 76 (2007) 025504]

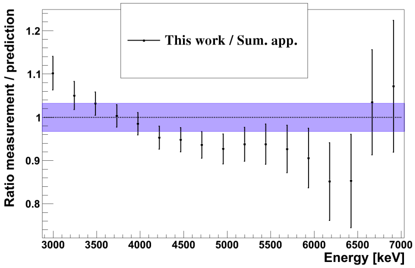
- Estimated total uncertainties on the neutrino detection rates:

$$2.4\%(^{235}\text{U}) \qquad 2.9\%(^{239}\text{Pu}) \qquad 2.6\%(^{241}\text{Pu})$$

- The ^{238}U $\bar{\nu}_e$ flux was calculated ab initio with estimated 8% uncertainty. [Mueller et al, PRC 83 (2011) 054615]
- Approximate agreement with the 2014 β spectrum measurement at FRM II in Garching using a fast neutron beam. [Haag et al, PRL 112 (2014) 122501]



[Mueller et al, PRC 83 (2011) 054615]



[Haag et al, PRL 112 (2014) 122501]

Updated Summation Model: An Improved Agreement with the Daya Bay Antineutrino Fluxes

M. Estienne,[1,*] M. Fallot,[1] A. Algora,[2,3] J. Briz-Monago,[1] V. M. Bui,[1] S. Cormon,[1] W. Gelletly,[4] L. Giot,[1] V. Guadilla,[1] D. Jordan,[2] L. Le Meur,[1] A. Porta,[1] S. Rice,[4] B. Rubio,[2] J. L. Taín,[2] E. Valencia,[2] and A.-A. Zakari-Issoufou[1]

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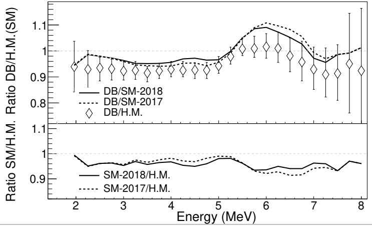
[2]*Instituto de Física Corpuscular, CSIC-Universitat de València, E-46071 València, Spain*

[3]*Institute of Nuclear Research of the Hungarian Academy of Sciences, H-4026 Debrecen, Hungary*

[4]*Department of Physics, University of Surrey, GU2 7XH Guildford, United Kingdom*

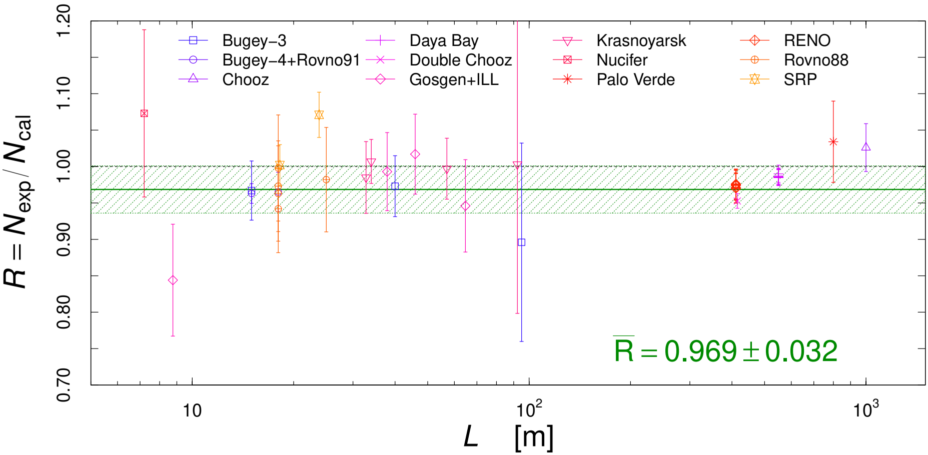
A new summation method model of the reactor antineutrino energy spectrum is presented. It is updated with the most recent evaluated decay databases and with our total absorption gamma-ray spectroscopy measurements performed during the last decade. For the first time, the spectral measurements from the Daya Bay experiment are compared with the antineutrino energy spectrum computed with the updated summation method without any renormalization. The results exhibit a better agreement than is obtained with the Huber-Mueller model in the 2–5 MeV range, the region that dominates the detected flux. A systematic trend is found in which the antineutrino flux computed with the summation model decreases with the inclusion of more pandemonium-free data. The calculated flux obtained now lies only 1.9% above that detected in the Daya Bay experiment, a value that may be reduced with forthcoming new pandemonium-free data, leaving less room for a reactor anomaly. Eventually, the new predictions of individual antineutrino spectra for the ^{235}U, ^{239}Pu, ^{241}Pu, and ^{238}U are used to compute the dependence of the reactor antineutrino spectral shape on the fission fractions.



[arXiv:1904.09358]

2019: new ab initio reactor $\bar{\nu}_e$ fluxes: Estienne, Fallot, et al (EF)

[Estienne, Fallot, et al, arXiv:1904.09358]

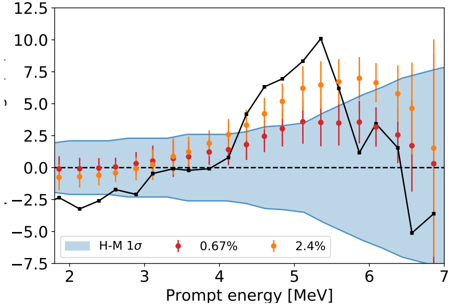


$\approx 1.0\,\sigma$ deficit $\Longrightarrow$ No Anomaly!

First-forbidden transitions in the reactor anomaly

L. Hayen,[1,*] J. Kostensalo,[2] N. Severijns,[1] and J. Suhonen[2]
[1] Instituut voor Kern- en Stralingsfysica, KU Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium
[2] Department of Physics, University of Jyväskylä, P.O. Box 35, FI-40014 University of Jyväskylä, Finland

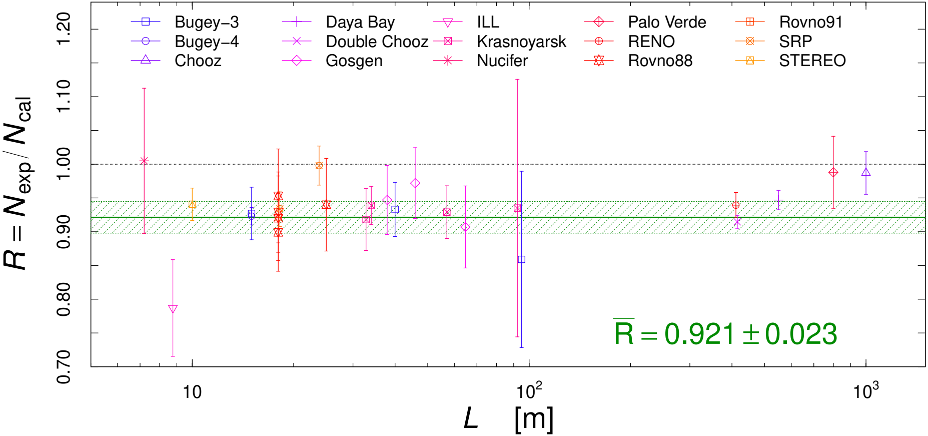
We describe here microscopic calculations performed on the dominant forbidden transitions in reactor antineutrino spectra above 4 MeV using the nuclear shell model. By taking into account Coulomb corrections in the most complete way, we calculate the shape factor with the highest fidelity and show strong deviations from allowed approximations and previously published results. Despite small differences in the *ab initio* electron cumulative spectra, large differences on the order of several percent are found in the antineutrino spectra. Based on the behavior of the numerically calculated shape factors we propose a parametrization of forbidden spectra. Using Monte Carlo techniques we derive an estimated spectral correction and uncertainty due to forbidden transitions. We establish the dominance and importance of forbidden transitions in both the reactor anomaly and spectral shoulder analysis with their respective uncertainties. Based on these results, we conclude that a correct treatment of forbidden transitions is indispensable in both the normalization anomaly and spectral shoulder.



[arXiv:1908.08302]

2019: new conversion reactor $\bar{\nu}_e$ fluxes: Hayen, Kostensalo, Severijns, Suhonen (HKSS)

[Hayen, Kostensalo, Severijns, Suhonen, arXiv:1908.08302]



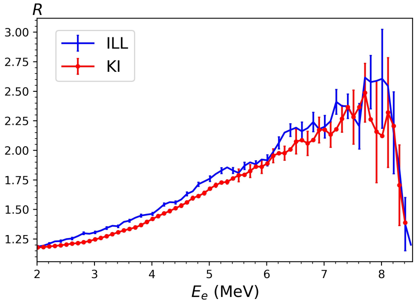
$\approx 3.4\,\sigma$ deficit $\Longrightarrow$ Anomaly larger than the $\approx 3.0\,\sigma$ HM anomaly!

Reevaluating reactor antineutrino spectra with new measurements of the ratio between ^{235}U and ^{239}Pu β spectra

V. Kopeikin,[1] M. Skorokhvatov,[1,2] and O. Titov[1,*]

[1] *National Research Centre Kurchatov Institute, 123182, Moscow, Russia*
[2] *National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 115409, Moscow, Russia*
(Dated: March 3, 2021)

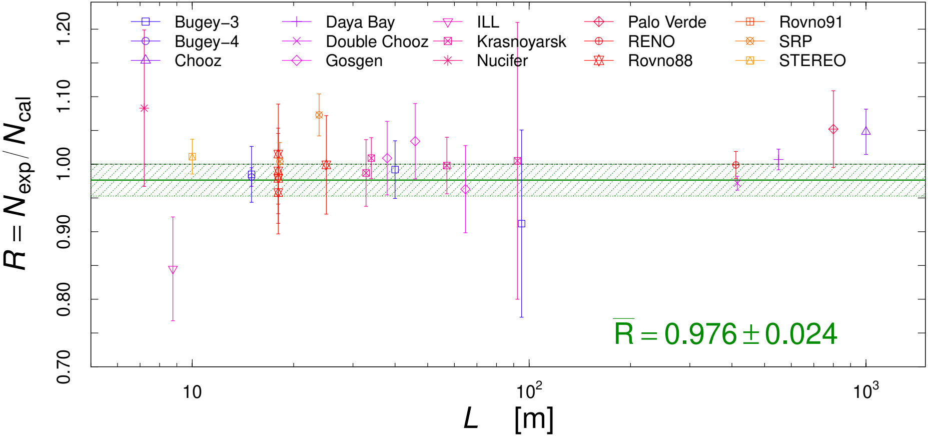
We report a reanalysis of the reactor antineutrino energy spectra based on the new relative measurements of the ratio $R = ^eS_5/^eS_9$ between cumulative β spectra from ^{235}U and ^{239}Pu, performed at a research reactor in National Research Centre Kurchatov Institute (KI). A discrepancy with the β spectra measured at Institut Laue-Langevin (ILL) was observed, indicating a steady excess of the ILL ratio by the factor of 1.054 ± 0.002. We find a value of the ratio between inverse beta decay cross section per fission for ^{235}U and ^{239}Pu: $(^5\sigma_f/^9\sigma_f)_{KI} = 1.45 \pm 0.03$, and then we reevaluate the converted antineutrino spectra for ^{235}U and ^{238}U. We conclude that the new predictions are consistent with the results of Daya Bay and STEREO experiments.



[arXiv:2103.01684]

2021: new converted reactor $\bar{\nu}_e$ fluxes: Kurchatov Institute (KI)

[Kopeikin, Skorokhvatov, Titov, arXiv:2103.01684]



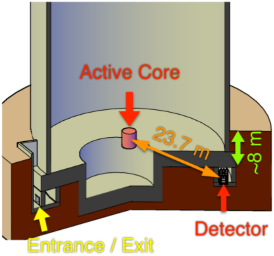
$\approx 1.0\,\sigma$ deficit $\Longrightarrow$ No Anomaly!

Approximate agreement with ab initio EF fluxes!

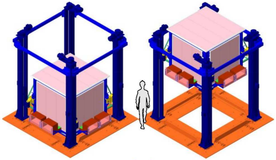
Model Indep. Measurements of Reactor ν Osc.

Ratios of spectra at different distances

NEOS

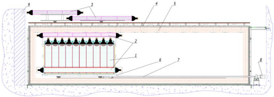


DANSS

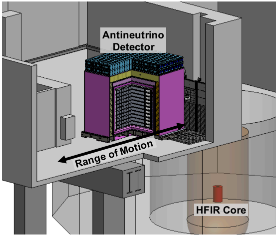


DANSS on a lifting platform

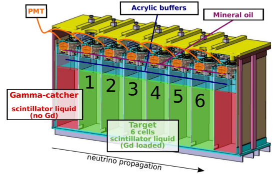
Neutrino-4



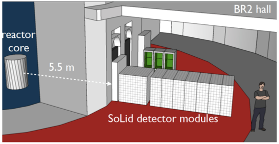
PROSPECT

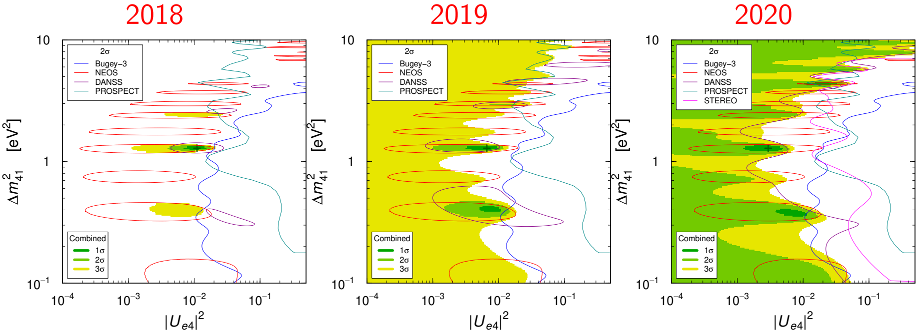


STEREO



SoLid





- 2018: remarkable agreement of the DANSS and NEOS best-fit regions at $\Delta m^2_{41} \approx 1.3\,\mathrm{eV}^2 \Longrightarrow$ model independent indication in favor of SBL oscillations. [Gariazzo, Giunti, Laveder, Li, arXiv:1801.06467]

  [Dentler, Hernandez-Cabezudo, Kopp, Machado, Maltoni, Martinez-Soler, Schwetz, arXiv:1803.10661]

- 2019: decreased agreement between NEOS and DANSS allowed regions.

  [Giunti, Y.F. Li, Y.Y. Zhang, arXiv:1912.12956]

- 2020: No 2σ DANSS allowed regions (exclusion curve).
  No compelling indication of oscillations.
  In practice these reactor experiments exclude large values of $|U_{e4}|^2$ for

$$0.1 \lesssim \Delta m^2_{41} \lesssim 10\,\mathrm{eV}^2$$

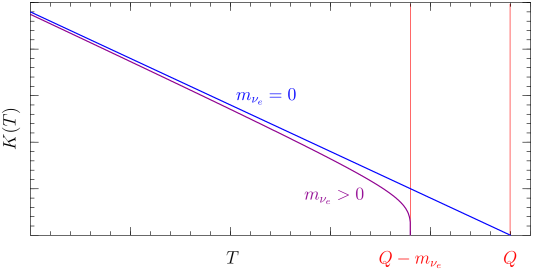
Tritium Beta-Decay

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_\mathrm{F})^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, p\, E\, K^2(T)$$

Kurie function: $$K(T) = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$



$$\boxed{m_{\nu_e} < 1.1\,\mathrm{eV} \quad (90\%\ \mathrm{C.L.})}$$

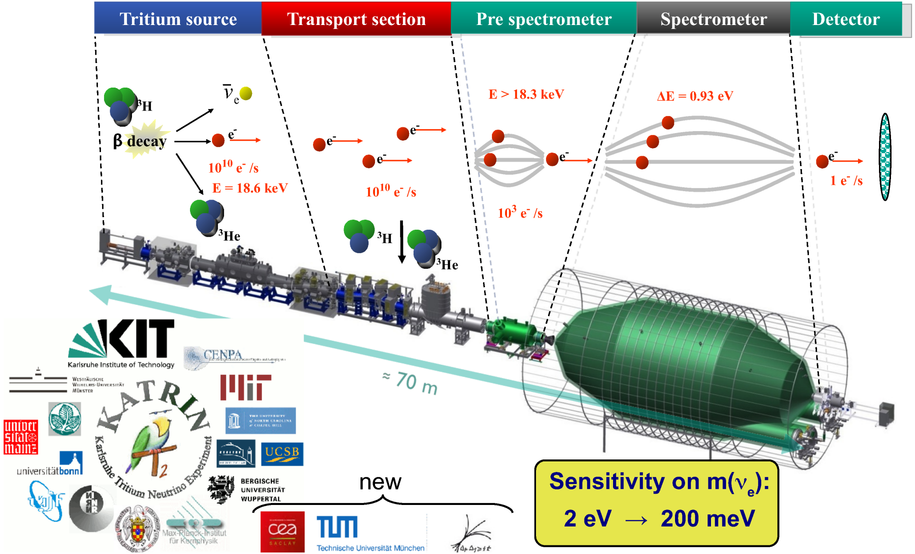
KATRIN

[PRL 123 (2019) 221802, arXiv:1909.06048]

Expected final sensitivity:

$$m_{\nu_e} \approx 0.2\,\mathrm{eV}$$

The Karlsruhe Tritium Neutrino Experiment KATRIN - overview

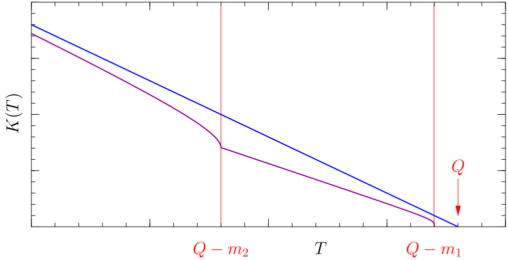


Sensitivity on $m(\nu_e)$:

2 eV → 200 meV

Transport of the KATRIN spectrometer from the Rhine river to the Karlsruhe Institute of Technology (November 2006).

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N - 1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

if experiment is not sensitive to masses $(m_1, m_2, m_3 \ll Q - T)$

effective mass: $\boxed{m_\beta^2 = \sum_{k=1}^{3} |U_{ek}|^2 m_k^2}$

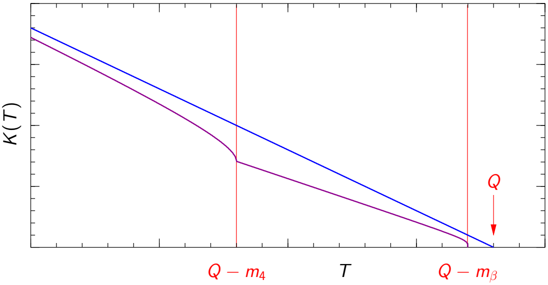
$$K^2 = (Q-T)^2 \sum_{k=1}^{3} |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_{k=1}^{3} |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

Robust kinematical probe of $\nu_e - \nu_s$ mixing

$$\frac{K^2(T)}{Q-T} = \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2}\,\theta(Q-T-m_k)$$

$$m_4 \gg m_{1,2,3} \Rightarrow \simeq \left(1-|U_{e4}|^2\right)\sqrt{(Q-T)^2 - m_\beta^2}\,\theta(Q-T-m_\beta)$$
$$+|U_{e4}|^2 \sqrt{(Q-T)^2 - m_4^2}\,\theta(Q-T-m_4)$$



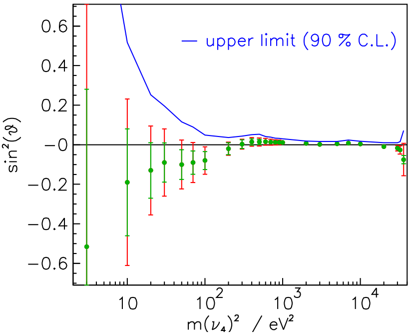
$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

$$m_\beta^2 = \sum_{k=1}^{3} |U_{ek}|^2 m_k^2$$

Mainz and Troitsk Limit on $\Delta m^2_{41} \simeq m^2_4$

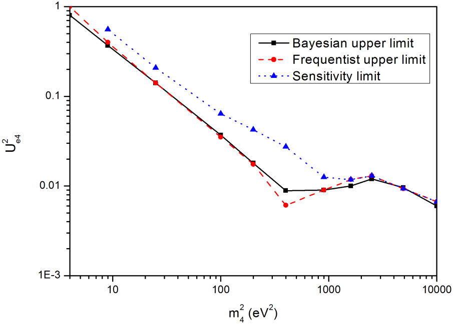
$$m_4 \gg m_{1,2,3} \quad \Longrightarrow \quad \Delta m^2_{41} \equiv m^2_4 - m^2_1 \simeq m^2_4$$

Mainz



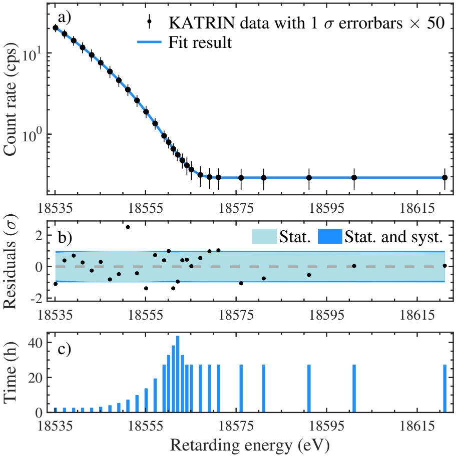
[Kraus, Singer, Valerius, Weinheimer, arXiv:1210.4194]

Troitsk



[Belesev et al, arXiv:1307.5687]

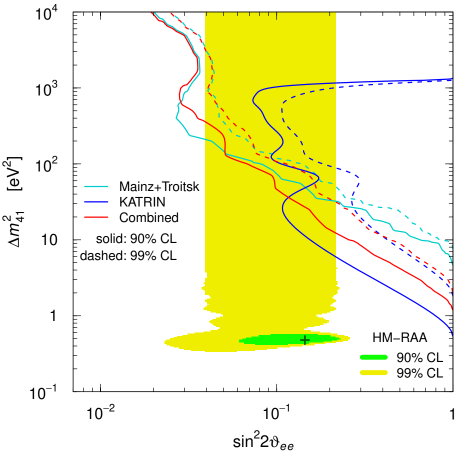
Bound from first KATRIN data



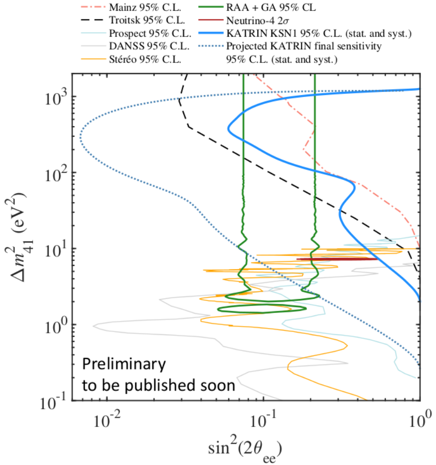
[KATRIN, arXiv:1909.06048]

$$m_\beta^2 = \sum_{k=1}^{3} |U_{ek}|^2 m_k^2$$

- $T_2 \to {}^3\mathrm{HeT}^+ + e^- + \bar{\nu}_e$
- Electron spectrum measurement until $\approx Q - 40\,\mathrm{eV}$
- We can probe the mixing of ν_4 with $m_4 \lesssim 40\,\mathrm{eV}$
- $R_\beta(E) = (1 - |U_{e4}|^2)\, R_\beta(E, m_\beta) + |U_{e4}|^2\, R_\beta(E, m_4)$
- $R_\beta(E, m_\nu) \propto \sum_{i,j} |U_{ei}|^2 \zeta_j \varepsilon_j \times \sqrt{\varepsilon_j^2 - m_\nu^2}\, \Theta(\varepsilon_j - m_\nu)$
- $\varepsilon_j = E_0 - E - V_j$



[Giunti, Y.F. Li, Y.Y. Zhang, arXiv:1912.12956]



[KATRIN @ Neutrino 2020]

[arXiv:2011.05087]

$$\Delta m^2_{41} \simeq m^2_4$$

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2 \quad \text{for} \quad |U_{e4}|^2 \ll 1$$

3+1: Appearance vs Disappearance

- SBL Oscillation parameters: Δm^2_{41} $|U_{e4}|^2$ $|U_{\mu 4}|^2$ $(|U_{\tau 4}|^2)$
- Amplitude of ν_e disappearance:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- Amplitude of ν_μ disappearance:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2 \left(1 - |U_{\mu 4}|^2\right) \simeq 4|U_{\mu 4}|^2$$

- Amplitude of $\nu_\mu \to \nu_e$ transitions:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

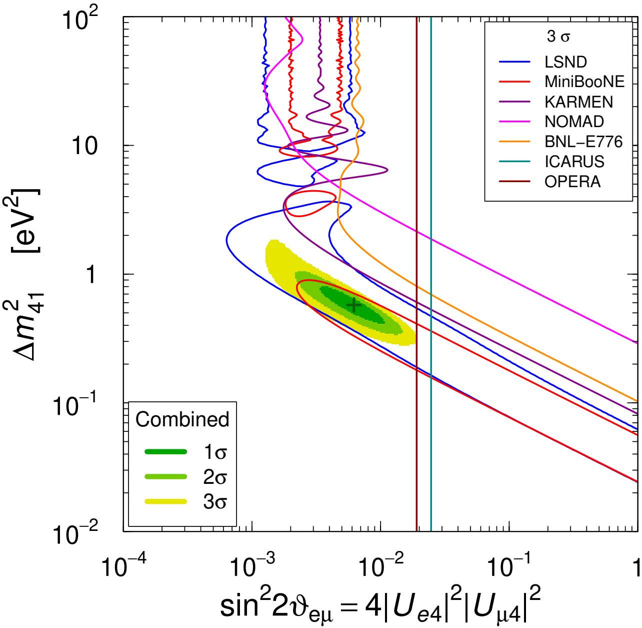
quadratically suppressed for small $|U_{e4}|^2$ and $|U_{\mu 4}|^2$

$\Downarrow$

Appearance-Disappearance Tension

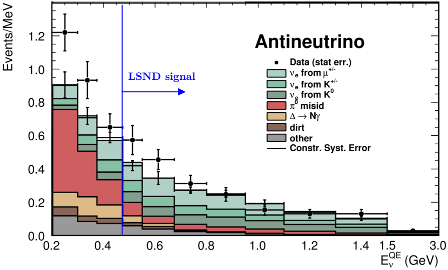
[Okada, Yasuda, IJMPA 12 (1997) 3669; Bilenky, CG, Grimus, EPJC 1 (1998) 247]

$\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ Appearance

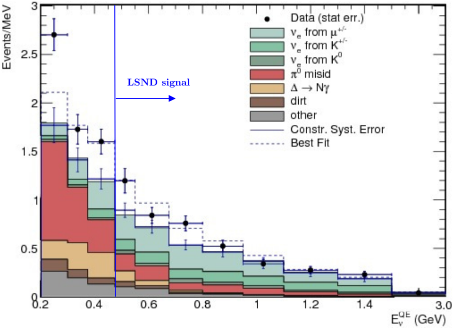


MiniBooNE

$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



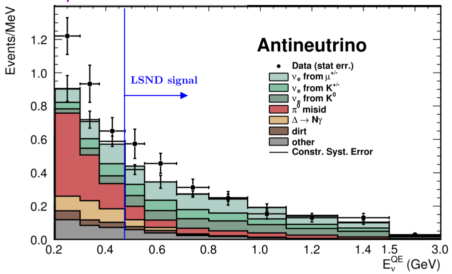
$\nu_\mu \to \nu_e$ [PRL 121 (2018) 221801]



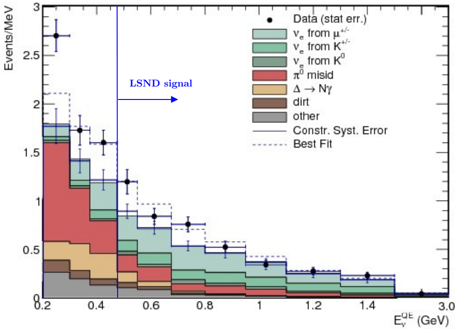
- Purpose: check the LSND signal
- Different $L \simeq 540\,\text{m}$
- Different $200\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$
- Similar $L/E \Longrightarrow$ Oscillations Smoking Gun?
- No money, no Near Detector
- Large beam-related background
- Large flux and cross section uncertainties

MiniBooNE

$\bar{\nu}_\mu \to \bar{\nu}_e$ [PRL 110 (2013) 161801]



$\nu_\mu \to \nu_e$ [PRL 121 (2018) 221801]



- LSND signal?
- LSND: excess only for

$$\frac{L}{E} \lesssim 1.2 \, \frac{m}{\text{MeV}}$$

- MiniBooNE: the LSND excess should be at

$$E \gtrsim \frac{540\,\text{m}}{1.2\,\text{m}} \, \text{MeV} \simeq 450\,\text{MeV}$$

- New large excess for

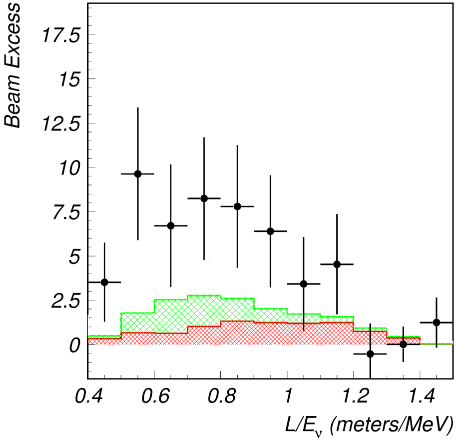
$$E \lesssim 450\,\text{MeV}$$

MiniBooNE low-energy anomaly

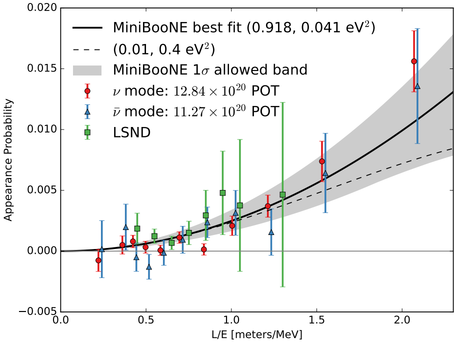
Maybe due to additional
$\Delta \to N + \gamma$ background

[Ioannisian et al, arXiv:1909.08571, arXiv:1912.01524]

▶ The MiniBooNE low-energy excess is at larger L/E than LSND.

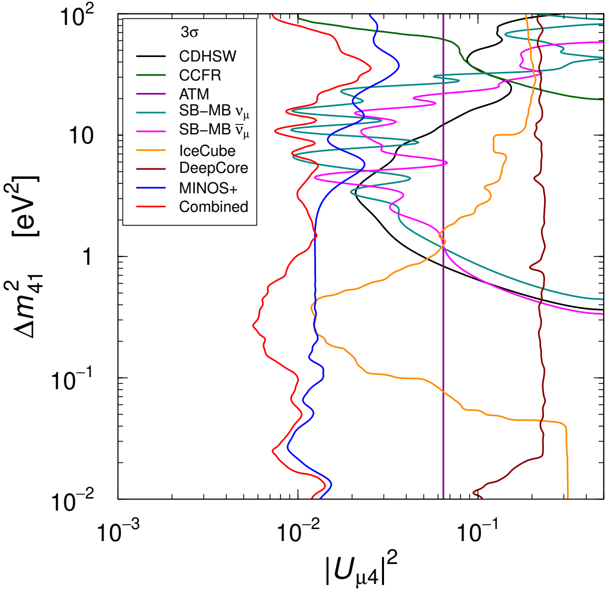


[LSND, PRD 64 (2001) 112007]

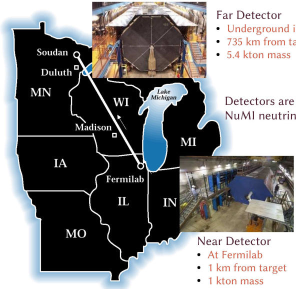


[MiniBooNE, PRL 121 (2018) 221801]

ν_μ and $\bar{\nu}_\mu$ Disappearance



MINOS+

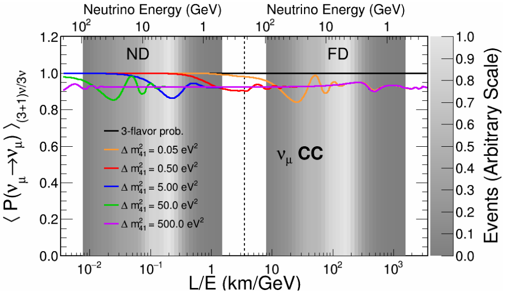


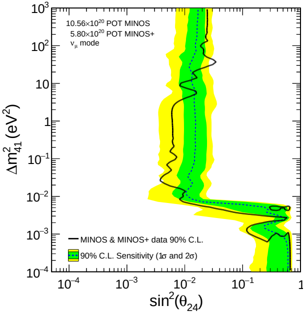
Far Detector
- Underground in Soudan mine
- 735 km from target
- 5.4 kton mass

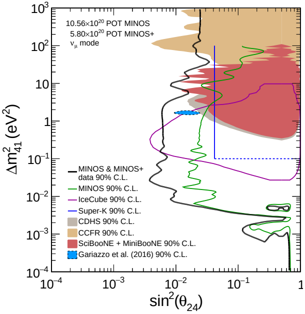
Detectors are on-axis for NuMI neutrino beam

Near Detector
- At Fermilab
- 1 km from target
- 1 kton mass

[PRL 122 (2019) 091803, arXiv:1710.06488]





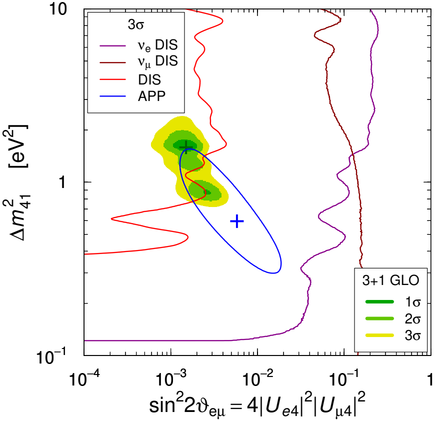


Global Appearance-Disappearance Tension

ν_e DIS
$\sin^2 2\vartheta_{ee} \simeq 4|U_{e4}|^2$

ν_μ DIS
$\sin^2 2\vartheta_{\mu\mu} \simeq 4|U_{\mu4}|^2$

$\nu_\mu \to \nu_e$ APP
$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4}\sin^2 2\vartheta_{ee}\sin^2 2\vartheta_{\mu\mu}$$



- $\nu_\mu \to \nu_e$ is quadratically suppressed!
- 2016 Global Fit:

  $\chi^2/\mathrm{NDF} = 304.0/275$

  $\mathrm{GoF} = 11\%$

  $\chi^2_{\mathrm{PG}}/\mathrm{NDF}_{\mathrm{PG}} = 15.0/2$

  $\mathrm{GoF}_{\mathrm{PG}} = 6 \times 10^{-4}$ $\leftarrow$ ☹
- Similar tension in

  $3+2, \quad 3+3, \quad \ldots, \quad 3+N_s$

[Giunti, Zavanin, arXiv:1508.03172]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$$N_{\text{D}} = \text{Number of Data} \qquad N_{\text{P}} = \text{Number of Fitted Parameters}$$

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF} \qquad \text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, dz \qquad p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020 (arXiv:hep-ph/0304176)

- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, dz$

Appearance vs Disappearance in $N = 3 + N_s$ Mixing

[Giunti, Zavanin, arXiv:1508.03172]

$$\frac{\Delta m^2_{21} L}{4E} \ll \frac{\Delta m^2_{31} L}{4E} \ll 1$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \delta_{\alpha\beta} - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(\delta_{\alpha\beta} - |U_{\beta k}|^2 \right) \sin^2 \Delta_{k1}$$
$$+8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \left| U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k} \right| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\Delta_{jk} = \frac{\Delta m^2_{jk} L}{4E} \qquad \eta_{\alpha\beta jk} = \arg\left[U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \right]$$

Survival Probabilities

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \sin^2 \Delta_{k1}$$
$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin \Delta_{j1} \sin \Delta_{k1} \cos \Delta_{jk}$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha$ disappearance due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$|U_{\alpha k}|^2 \ll 1 \qquad (\alpha = e, \mu, \tau; \quad k = 4, \ldots, N)$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \simeq 1 - \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 \Delta_{k1}$$

Appearance Probabilities $(\alpha \neq \beta)$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq 4 \sum_{k=4}^{N} |U_{\alpha k}|^2 |U_{\beta k}|^2 \sin^2 \Delta_{k1}$$

$$+ 8 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \left| U_{\alpha j} U_{\beta j} U_{\alpha k} U_{\beta k} \right| \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

Effective amplitude of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transitions due to $\nu_\alpha - \nu_k$ mixing:

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4 |U_{\alpha k}|^2 |U_{\beta k}|^2$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \simeq \sum_{k=4}^{N} \sin^2 2\vartheta^{(k)}_{\alpha\beta} \sin^2 \Delta_{k1}$$

$$+ 2 \sum_{k=4}^{N} \sum_{j=k+1}^{N} \sin 2\vartheta^{(k)}_{\alpha\beta} \sin 2\vartheta^{(j)}_{\alpha\beta} \sin \Delta_{k1} \sin \Delta_{j1} \cos(\Delta_{jk} \overset{(+)}{-} \eta_{\alpha\beta jk})$$

$$\sin^2 2\vartheta^{(k)}_{\alpha\alpha} = 4|U_{\alpha k}|^2 \left(1 - |U_{\alpha k}|^2\right) \simeq 4|U_{\alpha k}|^2$$

$$\sin^2 2\vartheta^{(k)}_{\alpha\beta} = 4|U_{\alpha k}|^2|U_{\beta k}|^2$$

$$\boxed{\sin^2 2\vartheta^{(k)}_{\alpha\beta} \simeq \frac{1}{4} \sin^2 2\vartheta^{(k)}_{\alpha\alpha} \sin^2 2\vartheta^{(k)}_{\beta\beta}}$$

$$\left.\begin{array}{l} \sin^2 2\vartheta^{(k)}_{ee} \ll 1 \\ \sin^2 2\vartheta^{(k)}_{\mu\mu} \ll 1 \end{array}\right\} \quad \Rightarrow \quad \sin^2 2\vartheta^{(k)}_{e\mu} \quad \text{is quadratically suppressed}$$

on the other hand, observation of $\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}$ transitions due to Δm^2_{k1} imply that the corresponding $\overset{(-)}{\nu}_{\alpha}$ and $\overset{(-)}{\nu}_{\beta}$ disappearances must be observed

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\Delta_{kj} = \Delta m^2_{kj} L / 4E$$

$$\eta = \arg[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}]$$

$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 \Delta_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 \Delta_{51}$$
$$+ 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin \Delta_{41} \sin \Delta_{51} \cos(\Delta_{54} \overset{(+)}{-} \eta)$$

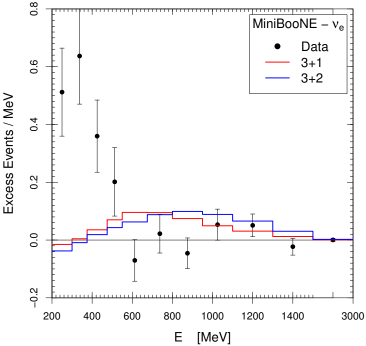
$$P^{\text{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \Delta_{41} + |U_{\alpha 5}|^2 \sin^2 \Delta_{51})$$
$$- 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \sin^2 \Delta_{54}$$

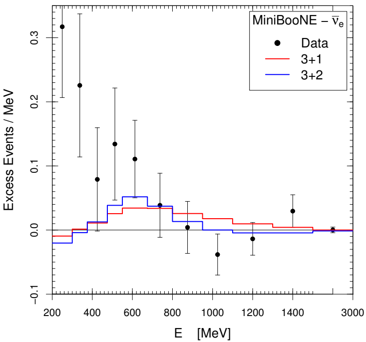
[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Archidiacono et al, PRD 86 (2012) 065028; Jacques, Krauss, Lunardini, PRD 87 (2013) 083515; Conrad et al, AHEP 2013 (2013) 163897; Archidiacono et al, PRD 87 (2013) 125034; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050; Giunti, Laveder, Y.F. Li, H.W. Long, PRD 88 (2013) 073008; Girardi, Meroni, Petcov, JHEP 1311 (2013) 146]

- Good: CP violation
- Bad: Two massive sterile neutrinos at the eV scale!

  4 more parameters: $\underbrace{\Delta m^2_{41}, |U_{e4}|^2, |U_{\mu 4}|^2,}_{3+1} \Delta m^2_{51}, |U_{e5}|^2, |U_{\mu 5}|^2, \eta$

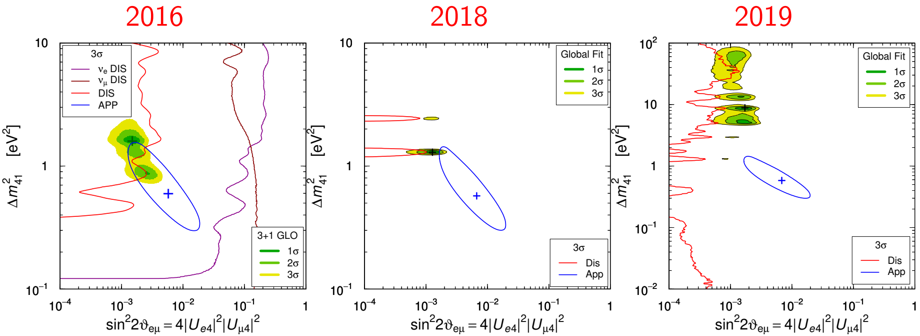
3+2 cannot fit MiniBooNE Low-Energy Excess





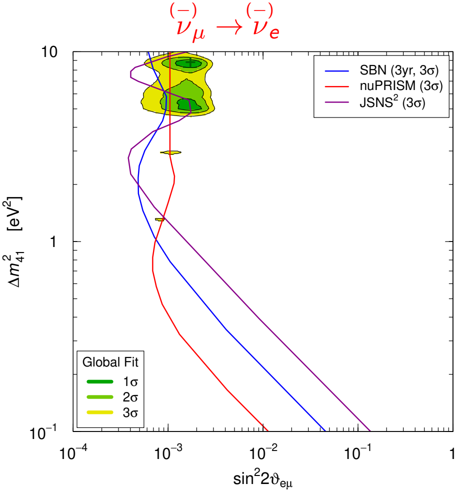
- Note difference between 3+2 ν_e and $\bar{\nu}_e$ histograms due to CP violation
- 3+2 can fit slightly better the small $\bar{\nu}_e$ excess at about 600 MeV
- 3+2 fit of low-energy excess as bad as 3+1
- Claims that 3+2 can fit low-energy excess do not take into account constraints from other data
- Conclusion: 3+2 is not needed

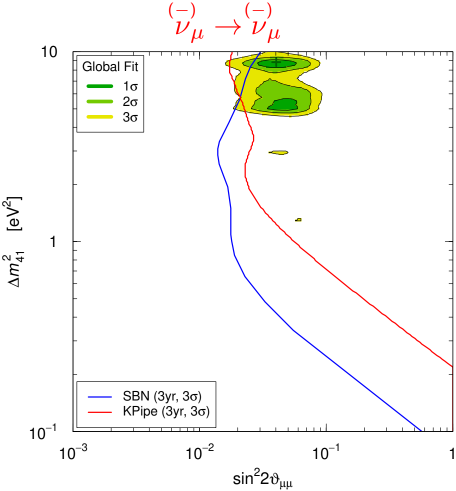
Global Appearance-Disappearance Tension



- 2016: Global Fit: $\text{GoF}_{\text{PG}} \approx 6 \times 10^{-4}$ [arXiv:1602.01390, arXiv:1606.07673]
- 2018: Global Fit: $\text{GoF}_{\text{PG}} \approx 2 \times 10^{-7}$ [arXiv:1801.06467, arXiv:1803.10661, arXiv:1901.08330]
- 2019: Global Fit: $\text{GoF}_{\text{PG}} \approx 7 \times 10^{-11}$

New Dedicated Experiments





Effective 3+1 LBL Oscillation Probabilities

[de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, JHEP 1602 (2016) 111, JHEP 1609 (2016) 016, PRL 118 (2017) 031804; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122; Capozzi et al, PRD 95 (2017) 033006]

$$|U_{e3}| \simeq \sin\vartheta_{13} \simeq 0.15 \sim \varepsilon \quad \Longrightarrow \quad \varepsilon^2 \sim 0.03$$

$$|U_{e4}| \simeq \sin\vartheta_{14} \simeq 0.17 \sim \varepsilon$$

$$|U_{\mu4}| \simeq \sin\vartheta_{24} \simeq 0.11 \sim \varepsilon$$

$$\alpha \equiv \frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \simeq \frac{7\times10^{-5}}{2.4\times10^{-3}} \simeq 0.031 \sim \varepsilon^2$$

At order ε^3: [Klop, Palazzo, PRD 91 (2015) 073017] $\Delta_{kj} \equiv \Delta m^2_{kj} L/4E$

$$\begin{aligned}
P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & 4\sin^2\vartheta_{13}\sin^2\vartheta_{23}\sin^2\Delta_{31} && \sim \varepsilon^2 \\
& +2\sin\vartheta_{13}\sin2\vartheta_{12}\sin2\vartheta_{23}(\alpha\Delta_{31})\sin\Delta_{31}\cos(\Delta_{32}+\delta_{13}) && \sim \varepsilon^3 \\
& +4\sin\vartheta_{13}\sin\vartheta_{14}\sin\vartheta_{24}\sin\vartheta_{23}\sin\Delta_{31}\sin(\Delta_{31}+\delta_{13}-\delta_{14}) && \sim \varepsilon^3
\end{aligned}$$

Alternative Explanations of MiniBooNE

- Generation by a particle X produced in the MiniBooNE target is excluded by the angular distribution of the ν_e-like events, that is not strongly forward peaked.

[Jordan, Kahn, Krnjaic, Moschella, Spitz, PRL 122 (2019) 081801]

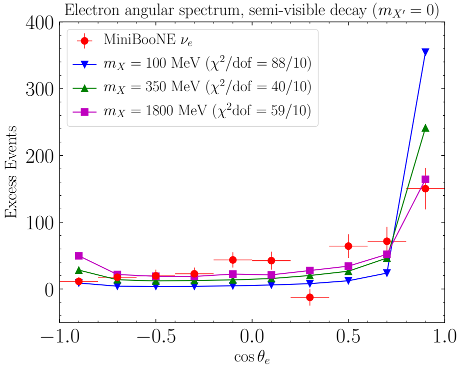
- Visible decays:

$$X \to e^+e^- \text{ or } X \to \gamma\gamma$$

$$\cos\theta_e > 0.9999$$

- Semi-visible decay:

$$X \to X' + p_{\text{EM}} \quad \longrightarrow$$



Heavy Neutrino Generation in the Detector

- Neutrino Neutral-Current Weak Interaction Lagrangian:

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{NC})} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} Z_\rho \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L}$$

- Sterile neutrinos: $\nu_{\alpha L} = \sum_{k=1}^{3+N_s} U_{\alpha k} \nu_{kL} \qquad (\alpha = e, \mu, \tau, s_1, \ldots, s_{N_s})$

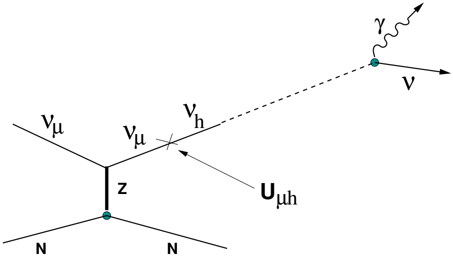
- No GIM: $\mathscr{L}_{\mathrm{I}}^{(\mathrm{NC})} = -\frac{g}{2\cos\vartheta_{\mathrm{W}}} Z_\rho \sum_{j=1}^{3+N_s} \sum_{k=1}^{3+N_s} \overline{\nu_{jL}} \gamma^\rho \nu_{kL} \sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k}$

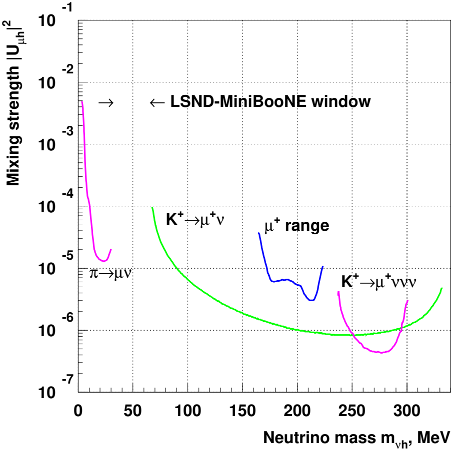
- $\sum_{\alpha=e,\mu,\tau,s_1,\ldots} U_{\alpha j}^* U_{\alpha k} = \delta_{jk}$ but $\sum_{\alpha=e,\mu,\tau} U_{\alpha j}^* U_{\alpha k} \neq \delta_{jk}$

- A heavy neutrino ν_h with $h \geq 4$ can be generated in the detector by neutral-current ν_μ scattering.

Heavy Sterile Neutrino Radiative Decay

[Gninenko, PRL 103 (2009) 241802, PRD 83 (2011) 015015, PRD 83 (2011) 093010, PLB 710 (2012) 86]





It may explain also LSND with

$$m_{\nu_h} \approx 40 - 80\,\text{MeV}$$

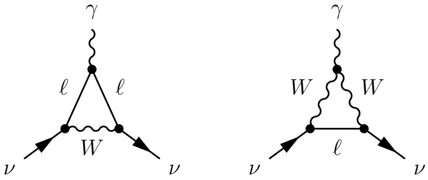
and

$$|U_{\mu h}|^2 \approx 10^{-3} - 10^{-2}$$

- It needs a fast radiative decay $\tau_{\nu_h} \lesssim 10^{-9}\,\mathrm{s}$ that can be generated by a transition magnetic moment $|\mu_{hi}| \gtrsim 10^{-8}\,\mu_{\mathrm{B}}$:

$$\Gamma_{\nu_h \to \nu_i + \gamma} = \frac{|\mu_{hi}|^2}{8\pi} m_{\nu_h}^3 \left(1 - \frac{m_{\nu_i}^2}{m_{\nu_h}^2}\right)^3$$

- Simplest extensions of the Standard Model:



$$|\mu_{hi}| \sim 10^{-11}\,\mu_{\mathrm{B}} \frac{m_{\nu_h}}{100\,\mathrm{MeV}} |U_{\ell h}| \sim 10^{-12}\,\mu_{\mathrm{B}} \qquad \text{not enough}$$

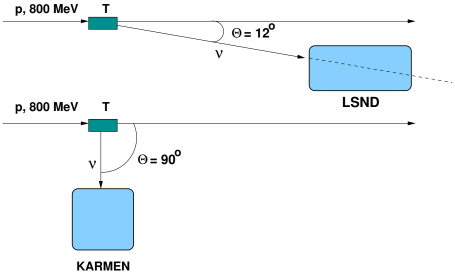
- More exotic extensions of the Standard Model may give the needed

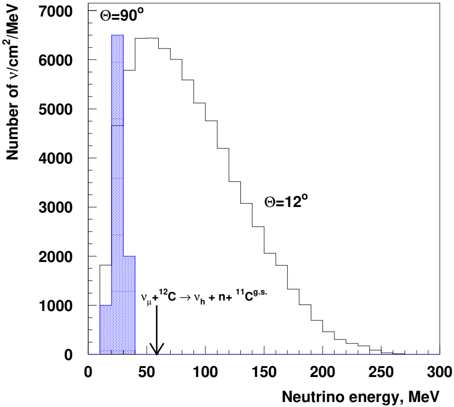
$$|\mu_{hi}| \gtrsim 10^{-8}\,\mu_{\mathrm{B}}$$

- It is interesting that this mechanism can explain why the LSND signal was not observed in KARMEN:

ν_μ from π^+ decay in flight

$$\nu_\mu + {}^{12}\mathrm{C} \to \nu_h + {}^{11}\mathrm{C} + n$$



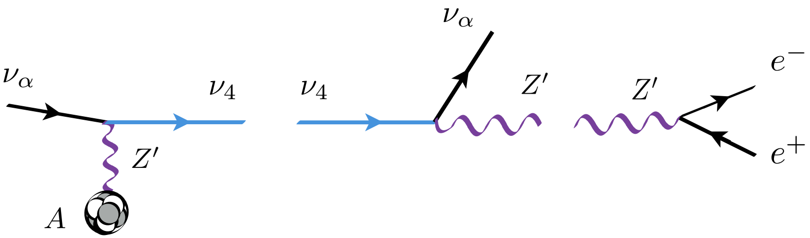


[Gninenko, PRD 83 (2011) 015015]

- This mechanism can be ruled out by Liquid Argon Time Projection Chamber (LArTPC) detectors that distinguish between electrons and photons: MicroBooNE, ICARUS, SBND (Fermilab Short-Baseline Neutrino Oscillation Program).

Interacting Heavy Sterile Neutrino

[Bertuzzo, Jana, Machado, Zukanovich Funchal, PRL 121 (2018) 241801]

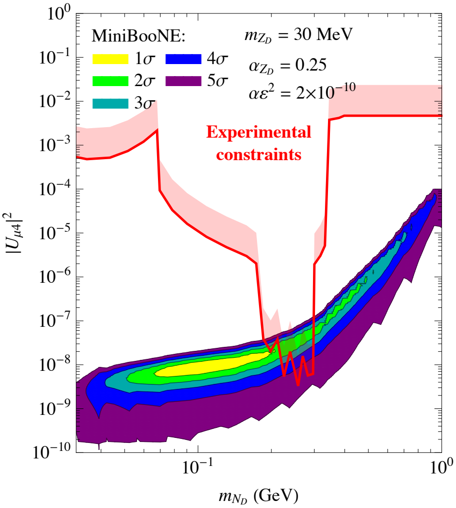


[Arguelles, Hostert, Tsai, arXiv:1812.08768]

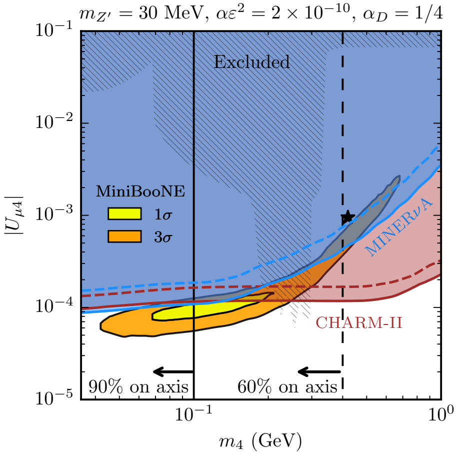
$$\mathcal{L} \supset \frac{m_{Z'}^2}{2} Z'_\mu Z'^\mu + g_{\mathcal{D}} Z'_\mu \overline{\nu}_s \gamma^\mu \nu_s + e\epsilon\, Z'^\mu\, J_\mu^{\mathrm{em}} + \frac{g}{c_W} \epsilon'\, Z'^\mu\, J_\mu^{\mathrm{Z}}$$

$$\Gamma_{\nu_4 \to Z' + \nu_\mu} = \frac{\alpha_{\mathcal{D}}}{2} |U_{\mu 4}|^2 \frac{m_{\nu_4}^3}{m_{Z'}^2} \left(1 - \frac{m_{Z'}^2}{m_{\nu_4}^2}\right) \left(1 + \frac{m_{Z'}^2}{m_{\nu_4}^2} - 2\frac{m_{Z'}^4}{m_{\nu_4}^4}\right)$$

$$\Gamma_{Z' \to e^+ e^-} \approx \frac{\alpha\, \epsilon^2}{3}\, m_{Z'}$$



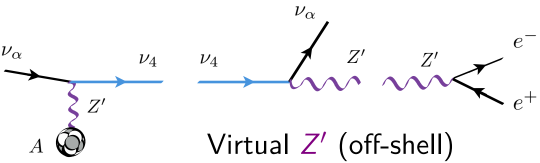
[Bertuzzo et al, PRL 121 (2018) 241801]



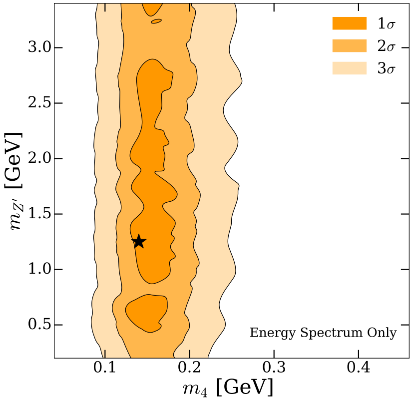
[Arguelles, Hostert, Tsai, arXiv:1812.08768]

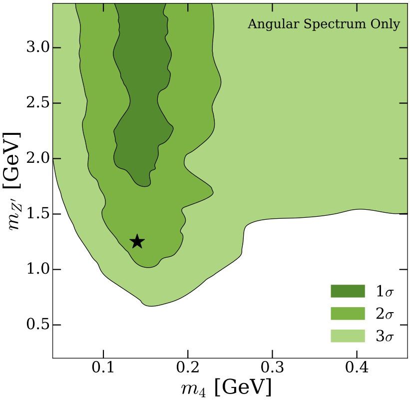
Heavy New Gauge Boson

[Ballett, Pascoli, Ross-Lonergan, PRD 99 (2019) 071701]



Virtual Z' (off-shell)





2016 (incomplete) list of papers on non-SBL effects of light sterile neutrinos:

- β Decay Experiments
  [Hannestad et al, JCAP 1102 (2011) 011, PRC 84 (2011) 045503; Formaggio, Barrett, PLB 706 (2011) 68; Esmaili, Peres, PRD 85 (2012) 117301; Gastaldo et al, JHEP 1606 (2016) 061]
- Neutrinoless Double-β Decay Experiments
  [Rodejohann et al, JHEP 1107 (2011) 091; Li, Liu, PLB 706 (2012) 406; Meroni et al, JHEP 1311 (2013) 146, PRD 90 (2014) 053002; Pascoli et al, PRD 90 (2014) 093005; CG, Zavanin, JHEP 1507 (2015) 171; Guzowski et al, PRD 92 (2015) 012002]
- Long-baseline Neutrino Oscillation Experiments
  [de Gouvea et al, PRD 91 (2015) 053005, PRD 92 (2015) 073012, arXiv:1605.09376; Palazzo et al, PRD 91 (2015) 073017, PLB 757 (2016) 142, JHEP 1602 (2016) 111, JHEP 1609 (2016) 016, PRL 118 (2017) 031804; Kayser et al, JHEP 1511 (2015) 039, JHEP 1611 (2016) 122; Pant et al, NPB 909 (2016) 1079, Choubey, Pramanik, PLB 764 (2017) 135]
- Solar neutrinos
  [Dooling et al, PRD 61 (2000) 073011, Gonzalez-Garcia et al, PRD 62 (2000) 013005; Palazzo, PRD 83 (2011) 113013, PRD 85 (2012) 077301; Li et al, PRD 80 (2009) 113007, PRD 87, 113004 (2013), JHEP 1308 (2013) 056; Kopp et al, JHEP 1305 (2013) 050]
- Atmospheric neutrinos
  [Goswami, PRD 55 (1997) 2931; Bilenky et al, PRD 60 (1999) 073007; Maltoni et al, NPB 643 (2002) 321, PRD 67 (2003) 013011; Choubey, JHEP 0712 (2007) 014; Razzaque, Smirnov, JHEP 1107 (2011) 084, PRD 85 (2012) 093010; Gandhi, Ghoshal, PRD 86 (2012) 037301; Barger et al, PRD 85 (2012) 011302; Esmaili et al, JCAP 1211 (2012) 041, JCAP 1307 (2013) 048, JHEP 1312 (2013) 014; Rajpoot et al, EPJC 74 (2014) 2936; Lindner et al, JHEP 1601 (2016) 124; Behera et al, arXiv:1605.08607]
- Supernova neutrinos
  [Caldwell, Fuller, Qian, PRD 61 (2000) 123005; Peres, Smirnov, NPB 599 (2001); Sorel, Conrad, PRD 66 (2002) 033009; Tamborra et al, JCAP 1201 (2012) 013; Wu et al, PRD 89 (2014) 061303; Esmaili et al, PRD 90 (2014) 033013]
- Cosmic neutrinos
  [Cirelli et al, NPB 708 (2005) 215; Donini, Yasuda, arXiv:0806.3029; Barry et al, PRD 83 (2011) 113012]
- Indirect dark matter detection [Esmaili, Peres, JCAP 1205 (2012) 002]
- Cosmology [see: Wong, ARNPS 61 (2011) 69; Archidiacono et al, AHEP 2013 (2013) 191047]

Conclusions of Part I

- Light sterile neutrinos can be powerful messengers of new physics beyond the SM.
- Historically, their existence is motivated by the reactor, Gallium and LSND short-baseline anomalies.
- The reactor antineutrino anomaly, discovered in 2011, is disappearing, because of new neutrino flux calculations and the absence of a clear model-independent signal in the new experiments (DANSS, PROSPECT, STEREO).
- The Gallium neutrino anomaly, discovered in 2007, is uncertain and needs a direct model-independent check.
- Important model-independent tests of the effect of m_4 in β-decay (KATRIN), electron-capture (ECHo, HOLMES) and $\beta\beta_{0\nu}$-decay experiments.

- In principle, the simplest explanation of the LSND and MiniBooNE ν_e-like excesses is neutrino oscillations, that requires a new Δm^2_{SBL} associated with a sterile neutrino.

- Unfortunately, the LSND and MiniBooNE ν_e-like excesses are too large to be compatible with the existing bounds on ν_e and ν_μ disappearance in the framework of $3 + N_s$ active-sterile neutrino mixing:

  APPEARANCE-DISAPPEARANCE TENSION

- Alternative explanations exist with a heavy sterile neutrino produced and decayed in the detector.

- Promising Fermilab SBN program aimed at a conclusive solution of the mystery with three Liquid Argon Time Projection Chamber (LArTPC): a near detector (LAr1-ND), an intermediate detector (MicroBooNE) and a far detector (ICARUS-T600).

- It is important that LArTPC detectors can distinguish a single ν_e-induced electron from a γ or a collimated e^+e^- pair.