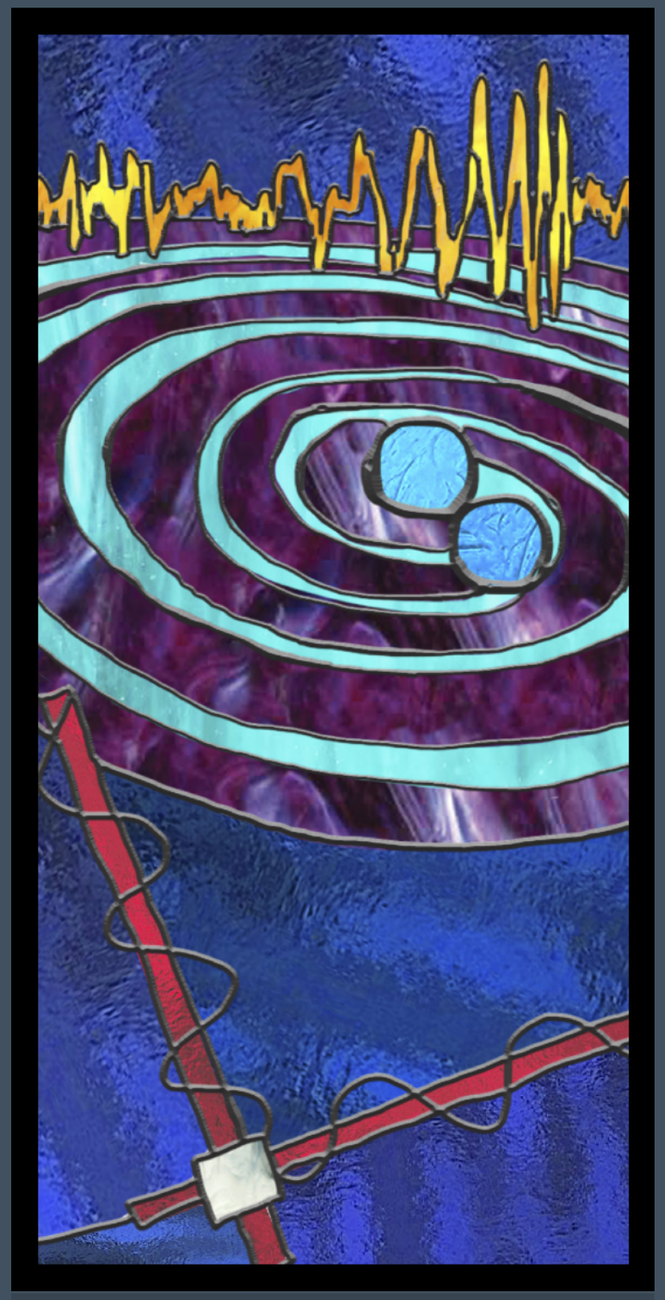
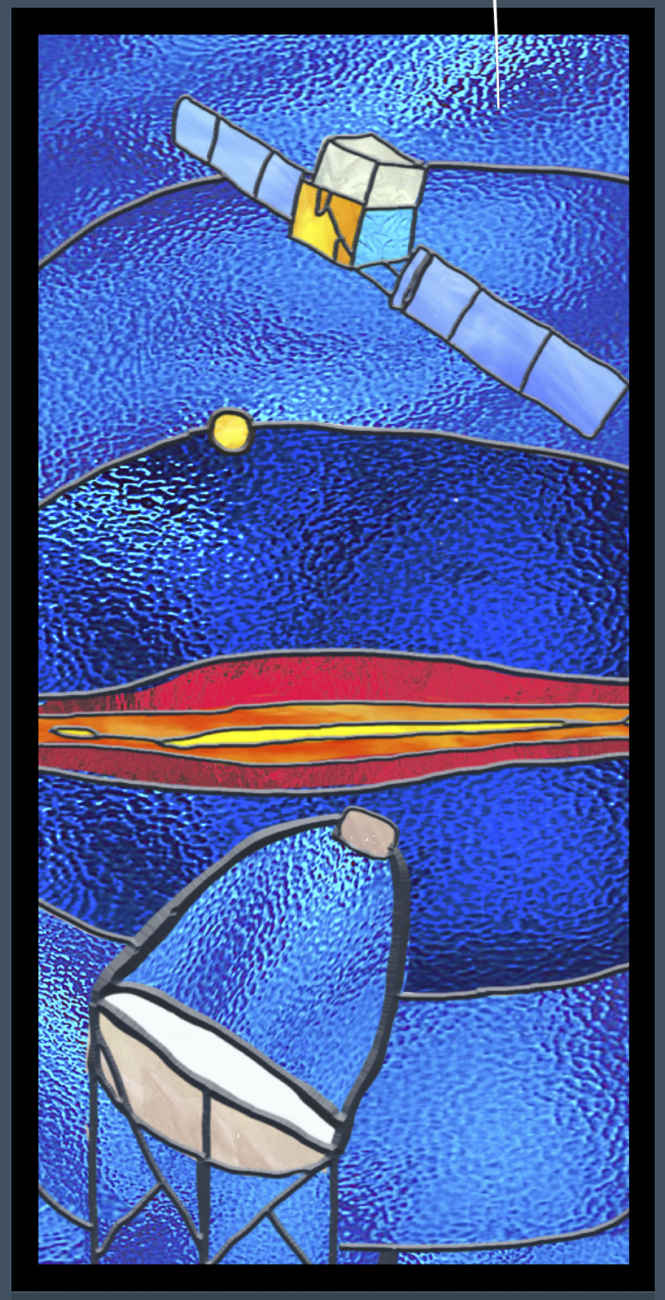




CR Sources & Radiative Processes

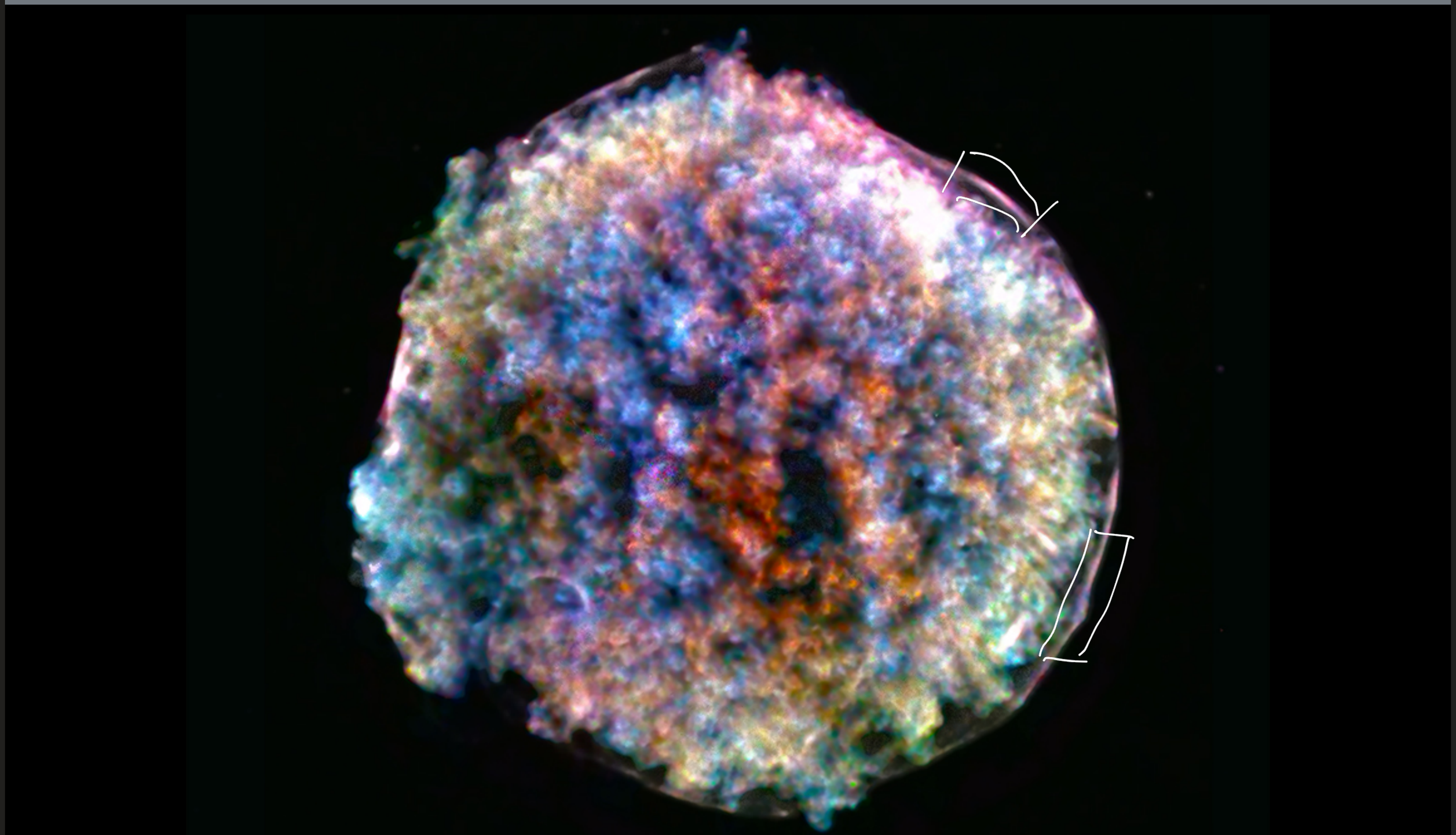
Markus Ahlers

March 17 & 18, 2021

VILLUM FONDEN

KØBENHAVNS UNIVERSITET

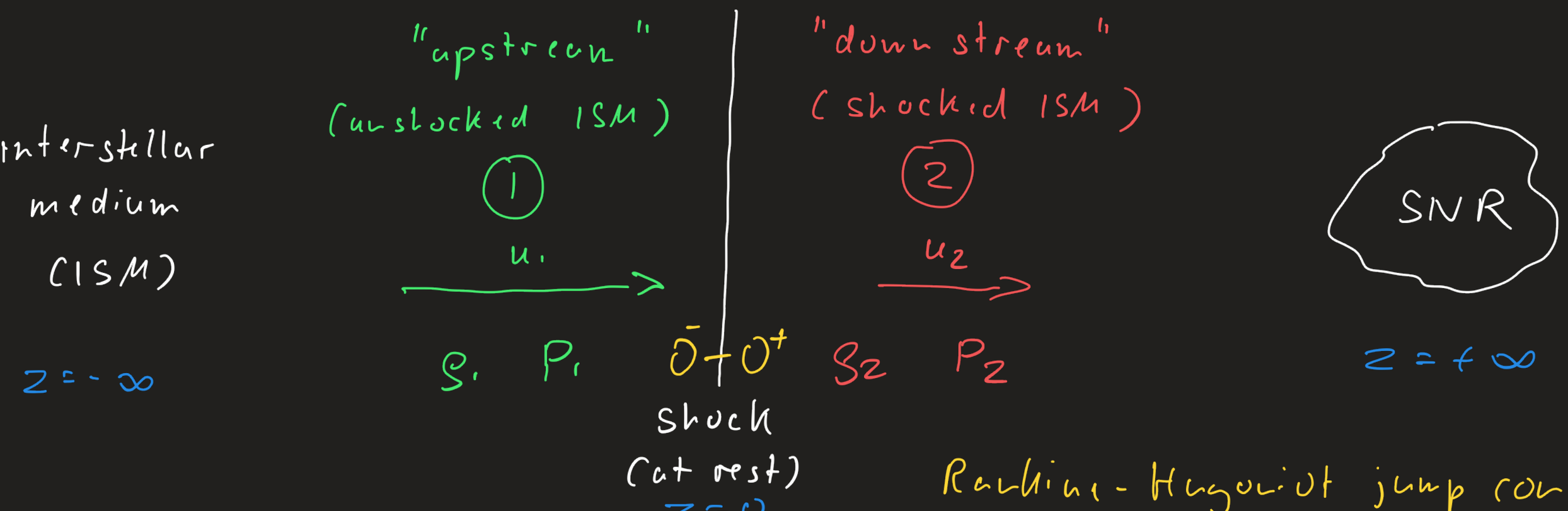
Supernova Remnants



Tycho's Supernova Remnant (SN 1572) in X-rays (Chandra)

Diffusive Shock Acceleration

The SNR blast wave forms eventually a collisionless shock:



Velocity u, density ρ and pressure P on both sides of the shock are determined by continuity of mass, momentum & energy:

Rankine-Hugoniot jump cond.

mass: $\frac{\partial}{\partial z}(\rho \cdot u) = 0$

momentum: $\frac{\partial}{\partial z}(\rho u^2 + P) = 0$

energy: $\frac{\partial}{\partial z}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma-1} u P\right) = 0$

adiabatic index

$P = (\gamma - 1) \cdot E$

monatomic: $\gamma = \frac{5}{3}$

diatomic: $\gamma = \frac{7}{5}$

solving R.H. jump condition:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)\cdot M^2}{(\gamma-1)M^2+2}$$

Mach number:

$$M \equiv \frac{u_1}{c_s}$$

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{\gamma P}{\rho} \Rightarrow c_s = \left(\frac{\gamma P}{\rho}\right)^{\frac{1}{2}}$$

$M \gg 1$ $\quad \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1} = \frac{\frac{5}{3}+1}{\frac{5}{3}-1} = \frac{8}{2} = 4$

stationary condition $\langle PSD \rangle_\Omega = \phi$

$$u\frac{\partial \phi}{\partial z} = \frac{\partial}{\partial z}\left[\kappa \frac{\partial \phi}{\partial z}\right] + \frac{1}{3}\frac{du}{dz} p \frac{\partial \phi}{\partial p} + q(z,p) \qquad (*)$$

$$\frac{du}{dz} = (u_2 - u_1)\,\delta(z)$$

assume: $q(z,p) = q_0(p)\,\delta(z)$

(1) shock region: integrate between 0^- ($-\varepsilon$) and 0^+ ($+\varepsilon$)

$$\left[\kappa \frac{\partial \psi}{\partial z}\right]_+ - \left[\kappa \frac{\partial \psi}{\partial z}\right]_- + \frac{1}{3}(u_2 - u_1)\, p \frac{\partial \phi}{\partial p} + q_0(p) = 0$$

(2) "downstream" $\frac{\partial \phi}{\partial z} = 0$ ✓ { repeated scattering

(3) "upstream" $\frac{\partial}{\partial z}\left[u\phi - \kappa \frac{\partial \phi}{\partial z}\right] = 0$

integrate between $-\infty$ and 0^-

$$\left[u_1 \phi - \kappa \frac{\partial \psi}{\partial z}\right]_{0^-} - \left[u_1 \phi - \kappa \frac{\partial \psi}{\partial z}\right]_{-\infty} = 0$$

~ 0

$$\Rightarrow \left[\kappa \frac{\partial \psi}{\partial z}\right]_{0^-} = u_1 \phi_0$$

$\leftarrow$ CR distribution in slock

$$\Rightarrow \quad u_1 \phi_0 = \frac{1}{3}(u_2 - u_1)\, p \frac{\partial \phi_0}{\partial p} + q_0(p) \qquad (*)$$

assume: $q_0(p) = \dfrac{\eta \cdot n_1 \cdot u_1}{4\pi p_{inj}^2}\, \delta(p - p_{inj})$

CR acceleration efficiency

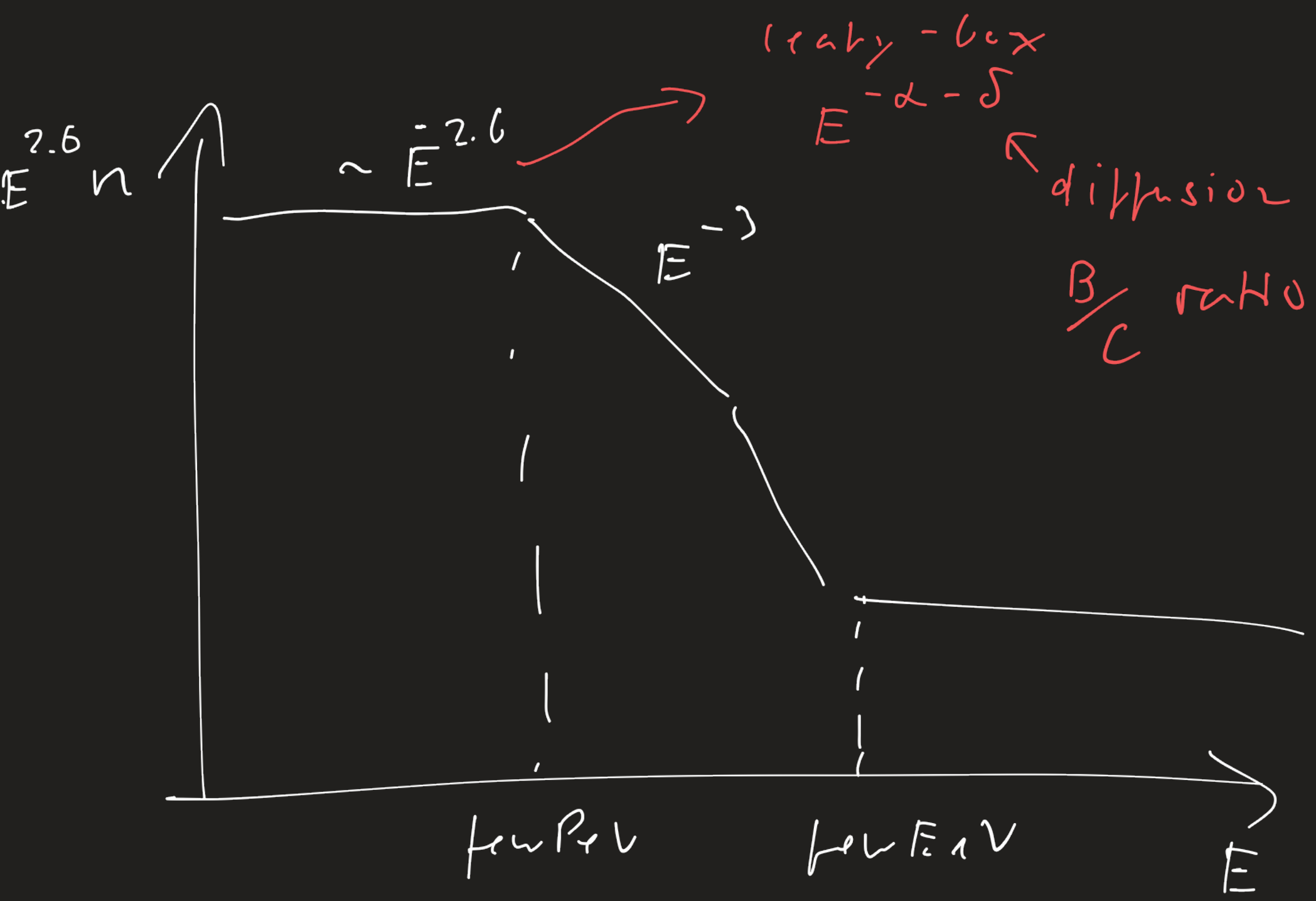
$$\int dp\, q_0(p) = \eta \cdot u_1 \cdot u_1$$

(*) result: $\phi_0 = \dfrac{3r}{r-1} \dfrac{\eta \cdot n_u}{4\pi p_{inj}^2} \left(\dfrac{p}{p_{inj}}\right)^{-\frac{3r}{r-1}} \longrightarrow \dfrac{3 \cdot 4}{4-1} = 4$

$r \equiv \dfrac{u_1}{u_2} \to 4$

$\Rightarrow \phi_0 \sim p^{-4}$

$\to n = \dfrac{4\pi}{1} p^2 \phi_0(p) \sim (p_0)^{-2}$

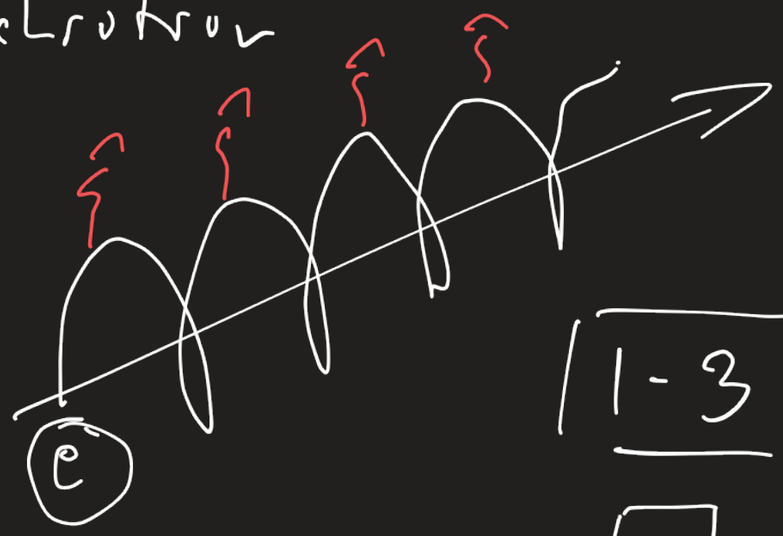


$$2.6 - \frac{1}{3} \sim \underbrace{2.2 - 2.3}_{>2}$$

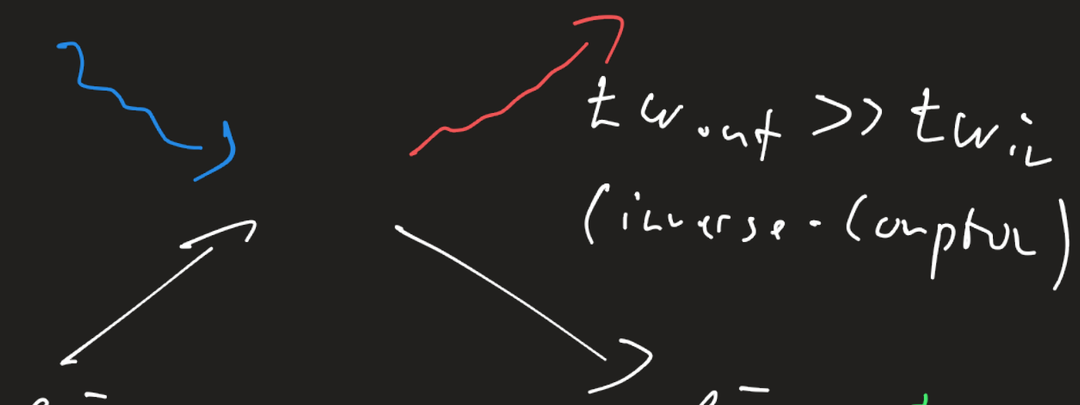
Radiative Processes

Energetic CRs (electrons, protons, heavy nuclei) interact magnetic fields, background photon fields (internal & external) and matter. Important for H.E. γ-ray emission:

(1) synchrotron



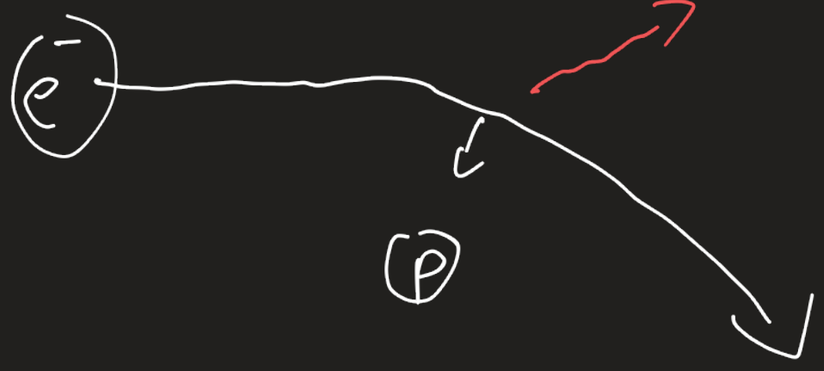
(2) Compton scattering



1-3 : hadronic

4 : hadronic

(3) Bremsstahlung



π^0 production

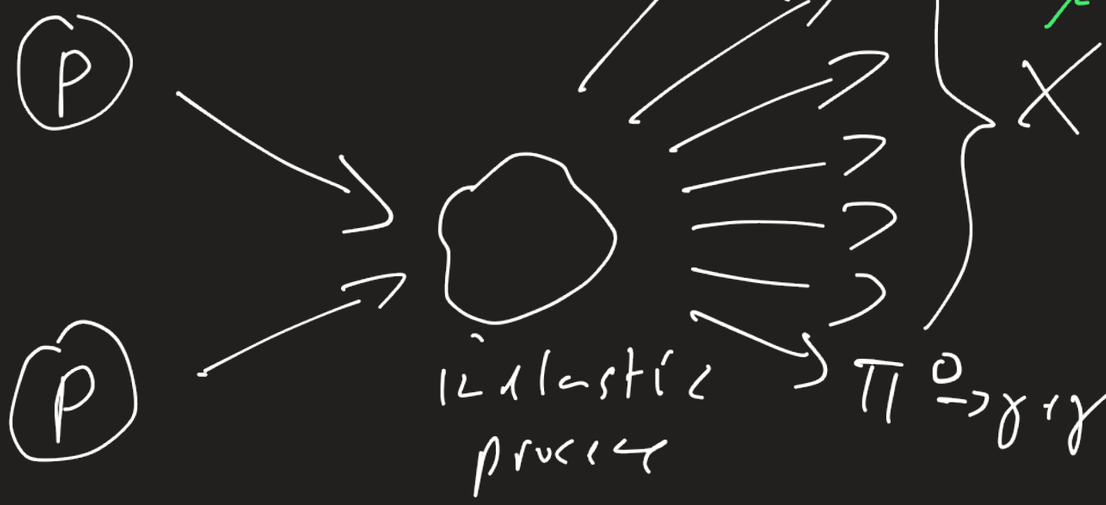
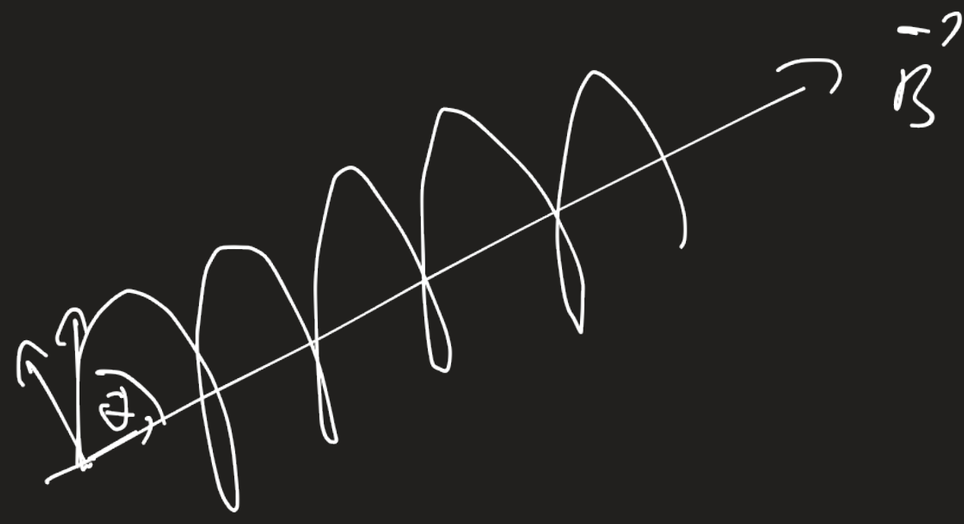


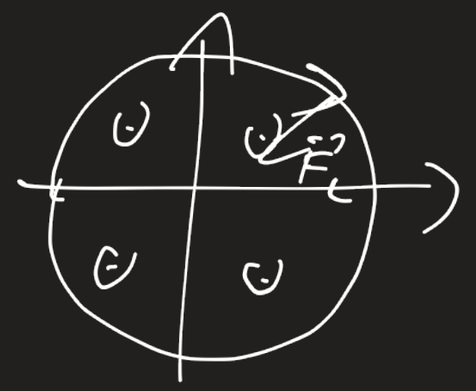
Photo processes:

- $\gamma + \gamma_{bgr} \rightarrow e^+ + e^-$
- $p + \gamma_{bgr} \rightarrow p + e^+ + e^-$ (Bethe-Heitler pair production)
- $p + \gamma_{bgr} \rightarrow \Delta^+ \rightarrow n + \pi^+$
  - $\hookrightarrow p + \pi^0 \rightarrow$ hadronic emission
- $A + \gamma_{bgr} \rightarrow (A-1)^* + N$
  - $\hookrightarrow (A-2) + N + N$

synchrotron radiation:



$$\cos\theta = \hat{B}\cdot\hat{p}$$



Larmor formula: (non-relativistic)

$$P_L = \frac{2}{3}\frac{1}{4\pi} q^2 (\dot{\beta})^2 = \frac{2}{3}\alpha(\dot{\beta})^2$$

relativistic generalization: $P_L = \frac{2}{3}\alpha\gamma^4\left[\gamma^2(\dot{\beta}_{\parallel})^2 + (\dot{\beta}_{\perp})^2\right]$

$$\dot{\vec{p}} = \vec{p}\times\vec{\Omega} \qquad |\vec{\Omega}| = \frac{eB}{\gamma m}$$

$$= \left(\frac{eB}{\gamma m}\right)^2 \cdot (\sin\theta)^2$$

$$P_L = \frac{2}{3}\alpha\gamma^4\left(\frac{eB}{\gamma m_e}\right)^2 = \frac{2}{3}\frac{\alpha^2}{m_e^2}4\pi\gamma^2 B^2\cdot\sin^2\theta$$

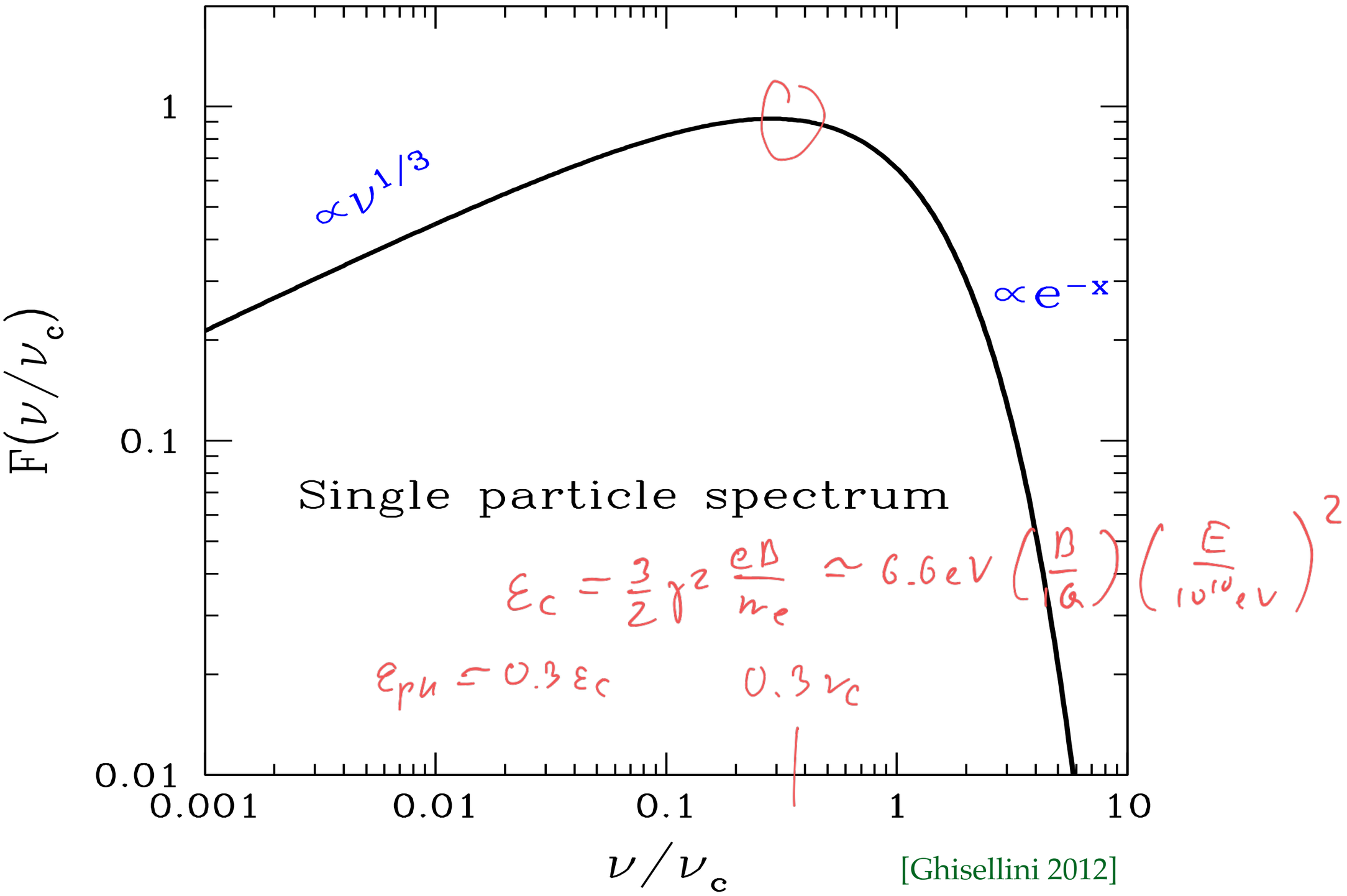
$\times\sin^2\theta$

$$\frac{\alpha}{m_e} = r_e = 2.8\ \text{fm} \qquad \sigma_T = \frac{8\pi}{3} r_e^2 \simeq 0.67\ \text{barn} = 0.67\times 10^{-28}\ m^2$$

$$P_L = \frac{4}{3}\sigma_T \underbrace{\frac{B^2}{2}}_{U_B} \gamma^2\beta^2 = \frac{4}{3}\sigma_T U_B \gamma^2\beta^2 = \frac{dE_e}{dt}$$

$$\tau_{sync} = E_e \Big/ \frac{dE_e}{dt} = 4\times10^4 s \left(\frac{B}{1G}\right)^{-2} \left(\frac{E_e}{10^{10} eV}\right)^{-1}$$

Synchrotron Emission



[Ghisellini 2012]

low : $n_e(E) = n_e(E_0)\left(\frac{E}{E_0}\right)^{-p}$ D.S.A. : $p \simeq 2$

approximate : $\hat{n}(\varepsilon, E) = K \cdot \delta(\varepsilon - \varepsilon_{pk})$

$$\varepsilon_{pk} = A \cdot E_{pk}^2$$

$$n(\varepsilon) \sim K \int dE\, n_e(E)\, \delta(\varepsilon - \varepsilon_{pk})$$

$$= K \int dE\, n_e(E)\, \frac{\delta(E - E_{pk})}{2 \cdot A \cdot E_{pk}}$$

$$\left|\frac{d\varepsilon_{pk}}{dE}\right|_{E_{pk}} = 2 \cdot A \cdot E_{pk}$$

$$E_{pk} \sim \sqrt{\varepsilon_{pk}}$$

E_{pk}^{-p-1}

$$n(\varepsilon) \sim \varepsilon^{-\frac{(p+1)}{2}}$$

$p = 2 \Rightarrow \frac{p+1}{2} = \frac{3}{2}$

Conton scaHering

$$P_L = \frac{2}{3}\alpha(\dot{\beta})^2$$

Thomson scaHering ($\varepsilon_\gamma \ll m_e$)

$$\langle \dot{p} \rangle_T = \frac{1}{T}\int_0^T dt\ \sin^2(\omega t)\left(\frac{e\cdot E_0}{m_e}\right)^2 = \frac{e^2}{2m_e^2}E_0^2$$

$$P_L = \underbrace{\frac{2}{3}\alpha^2 4\pi \frac{1}{m_e^2}}_{\sigma_T}\ \underbrace{\frac{E_0^2}{2}}_{U_\gamma} = \sigma_T \cdot U_\gamma$$

relativistic case: $\dfrac{\varepsilon'}{m_e} = \dfrac{\varepsilon/m_e}{1+\varepsilon/m_e(1-\cos\theta)}$

Compton scattering

relativistic case: $P_L = \frac{dE_\gamma}{dt} = \sigma_T \cdot U_\gamma \cdot \gamma^2 \left(1 + \frac{\beta^2}{3}\right)$

energy loss of electron:

$$\gamma^2 = \frac{1}{1-\beta^2}$$

$$\frac{dE_e}{dt} = \frac{dE_\gamma}{dt} - \sigma_T \cdot U_\gamma = \sigma_T U_\gamma \underbrace{\left[\frac{\gamma^2\beta^2}{3} + \gamma^2 - 1 \right]}_{\frac{4}{3}\beta^2\gamma^2}$$

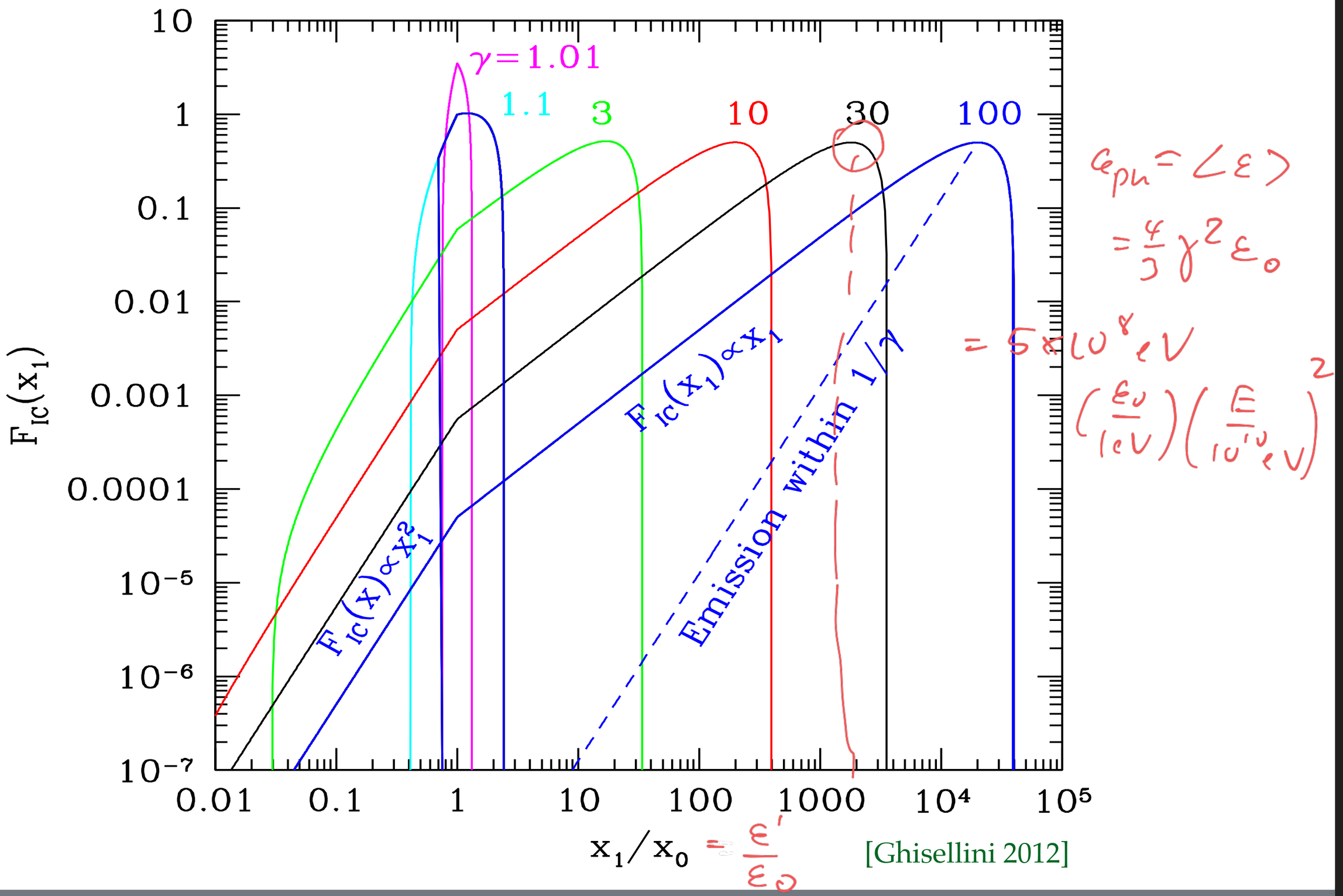
$$\left(\frac{dE_e}{dt}\right)_{IC} = \frac{4}{3}\beta^2\gamma^2\sigma_T \cdot U_\gamma$$

$$\left(\frac{dE_e}{dt}\right)_{sync} = \frac{4}{3}\beta^2\gamma^2\sigma_T U_B$$

$$\frac{U_\gamma}{U_B} = \frac{P_{IC}}{P_{sync}}$$

$$\tau_{IC} = \frac{E_e}{\left(\frac{dE_e}{dt}\right)_{IC}} = 10^{15}\,s \left(\frac{U_\gamma}{1\,eV/cm^3}\right)^{-1} \left(\frac{E_e}{10^{10}\,eV}\right)^{-1}$$

Inverse-Compton Emission

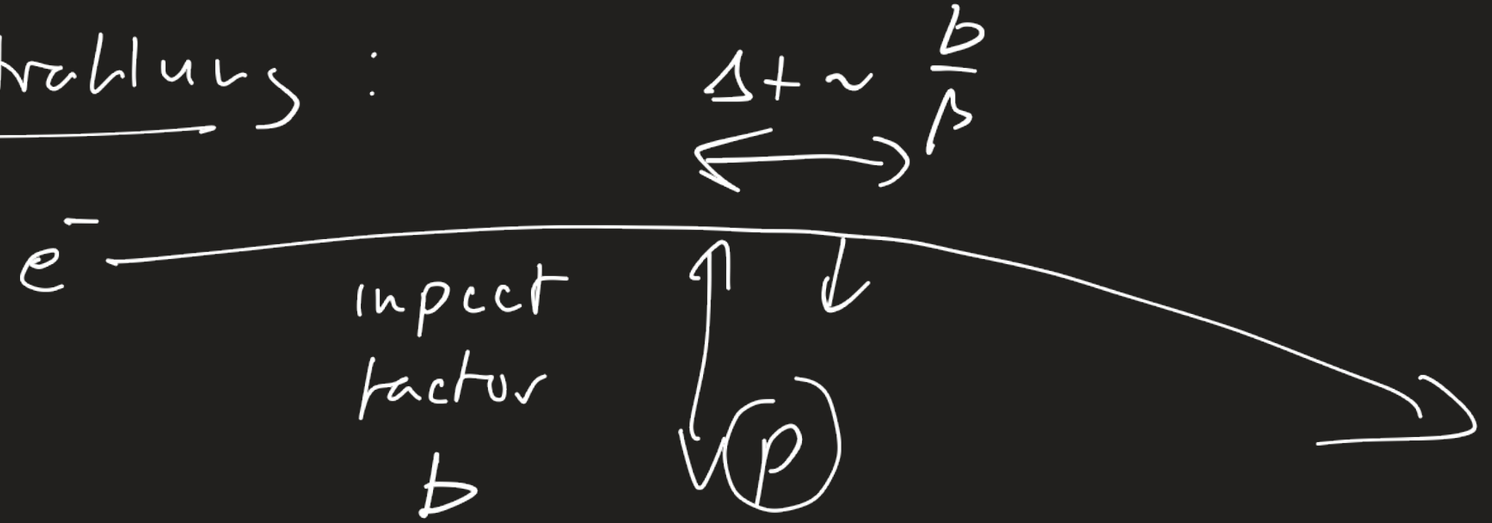


[Ghisellini 2012]

aycin : $n_e(E) \sim E^{-p}$

$$\Rightarrow n_{IC}(\varepsilon) \sim \varepsilon^{-\frac{(p+1)}{2}}$$

Bremsstrahlung :



acceleration : $\frac{F}{m_e} = \frac{\alpha}{m_e} \frac{1}{b^2} = \dot{\beta}$

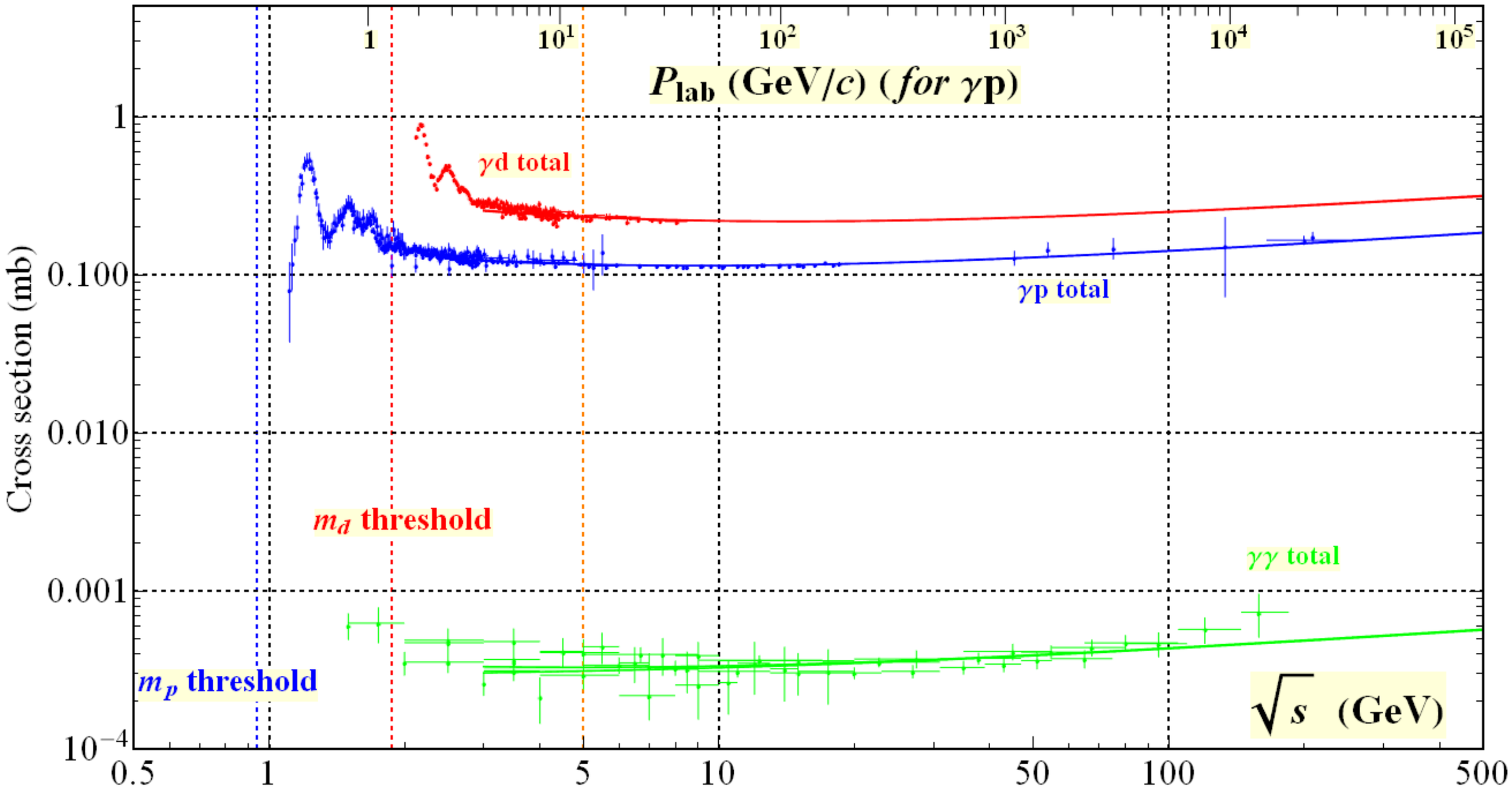
Larmor formula: $P_L = \frac{2}{3} \alpha (\dot{\beta})^2 = \frac{2}{3} \alpha \left(\frac{\alpha}{m_e b^2} \right)^2$

$= \frac{2}{3} \alpha^3 \frac{1}{m_e^2} \frac{1}{b^4}$

typically : $b = \frac{1}{(np)^{\frac{1}{3}}}$ $\Rightarrow \frac{P_L}{\varepsilon} = \frac{2}{3} \alpha^3 \frac{1}{m_e^2} \frac{n_p}{\beta}$

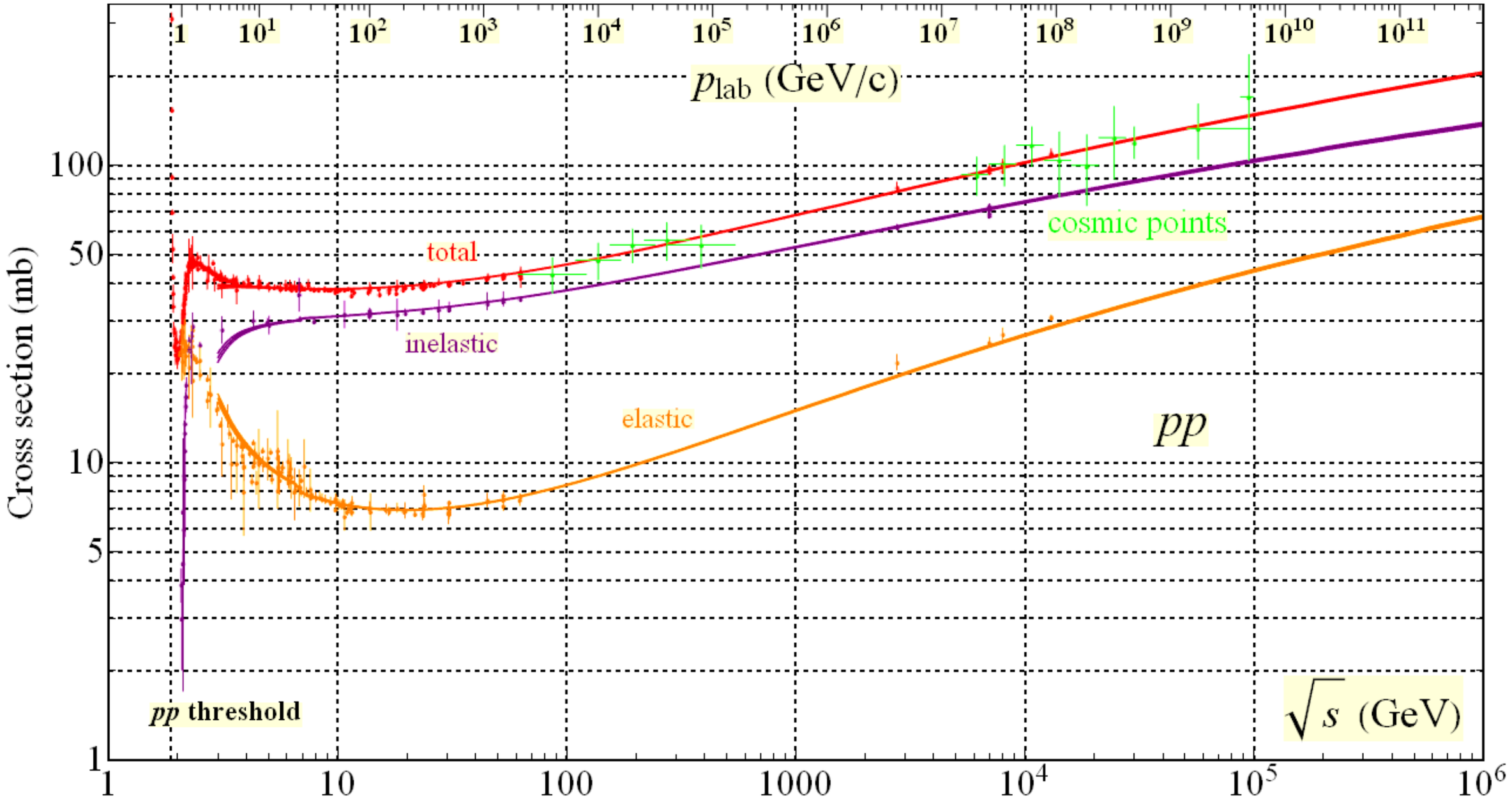
characteristic frequency : $\varepsilon = \frac{\beta}{b}$

Hadronic Interactions



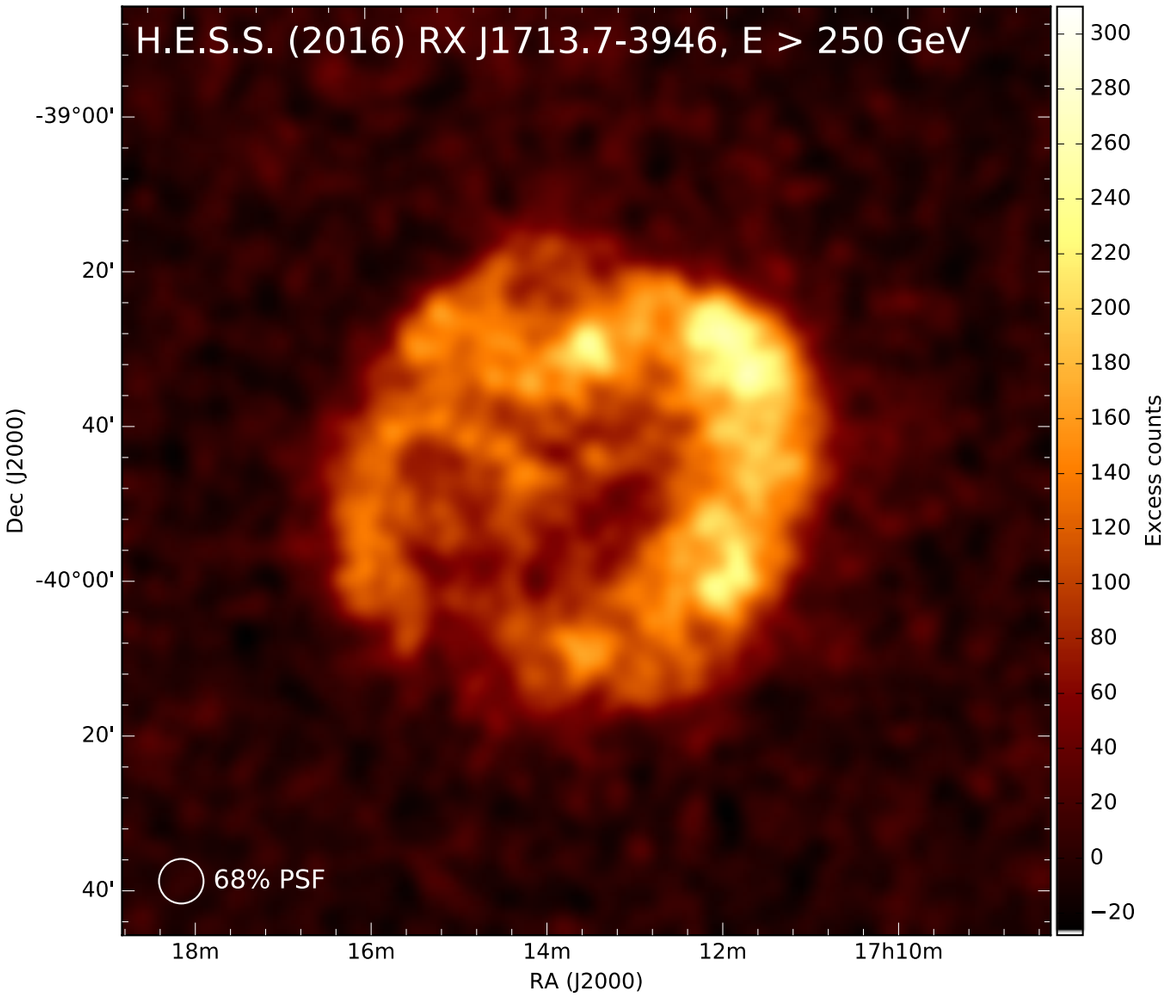
[Particle Data Group; pdg.lbl.gov]

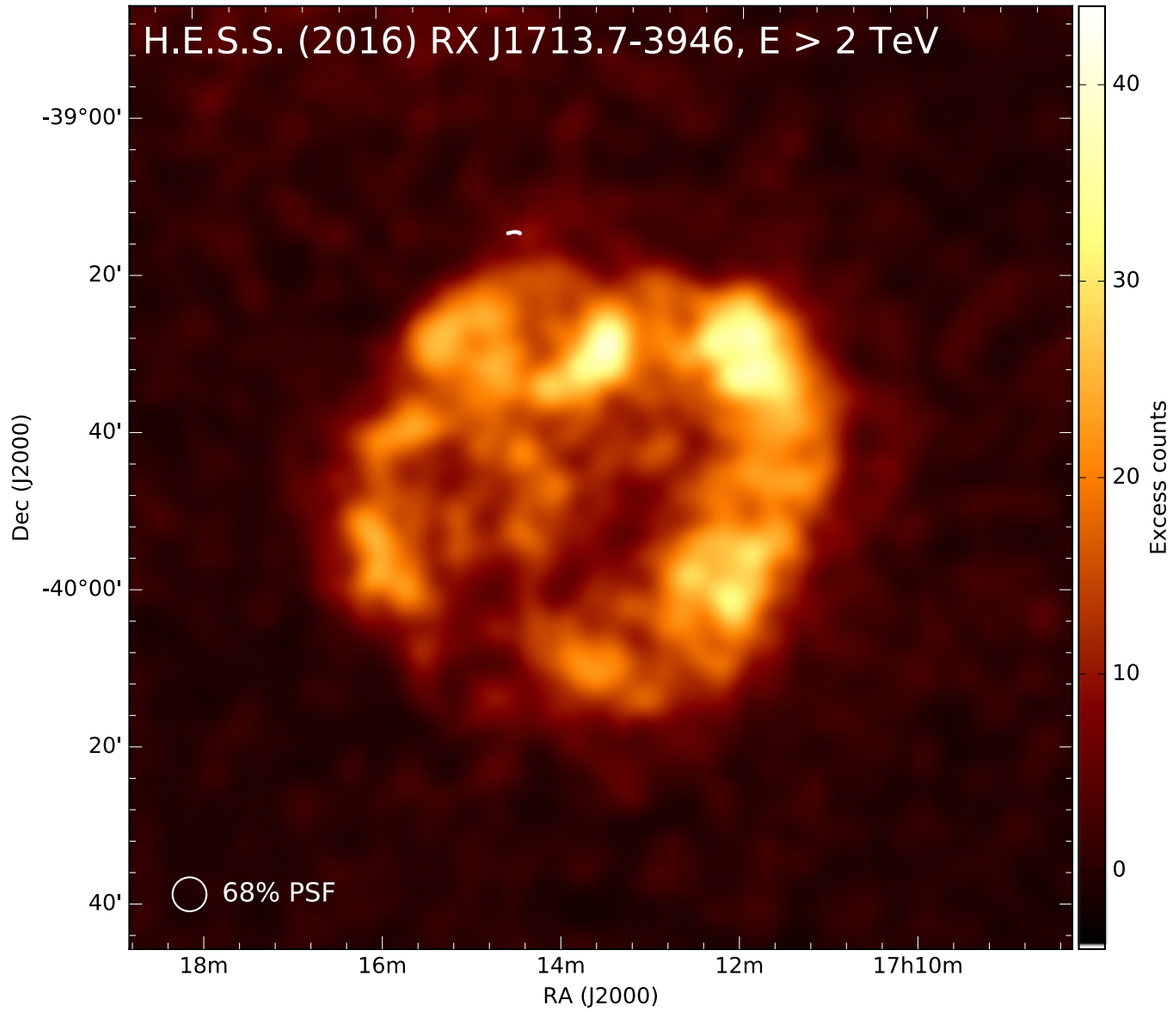
Hadronic Interactions



[Particle Data Group; pdg.lbl.gov]

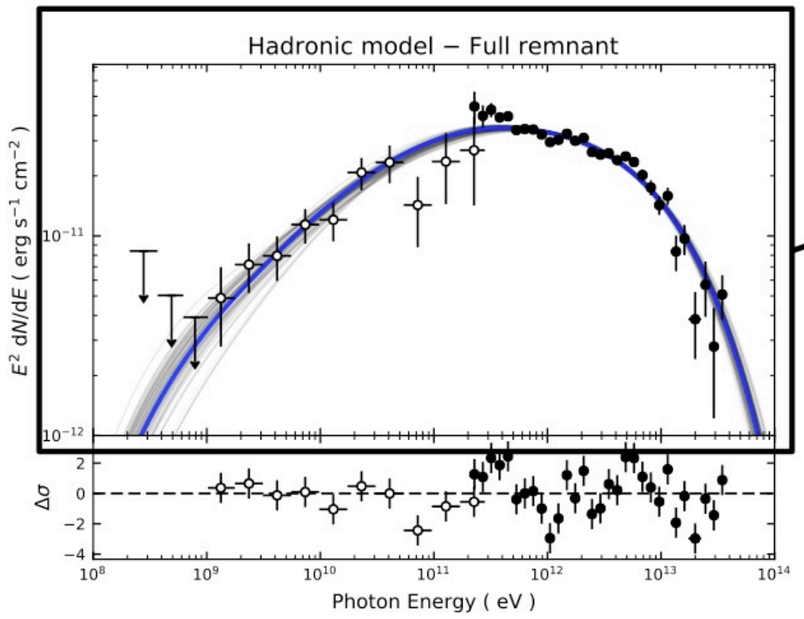
RX J1713.7-3946 SNR

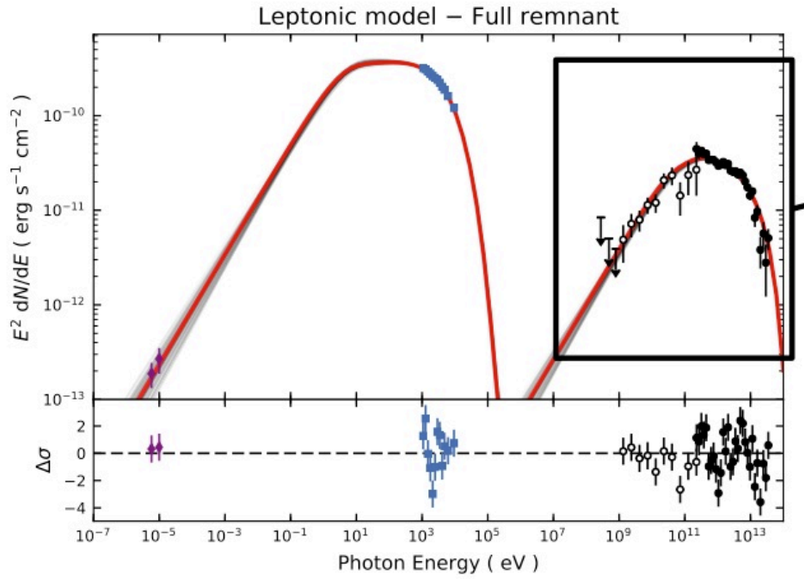


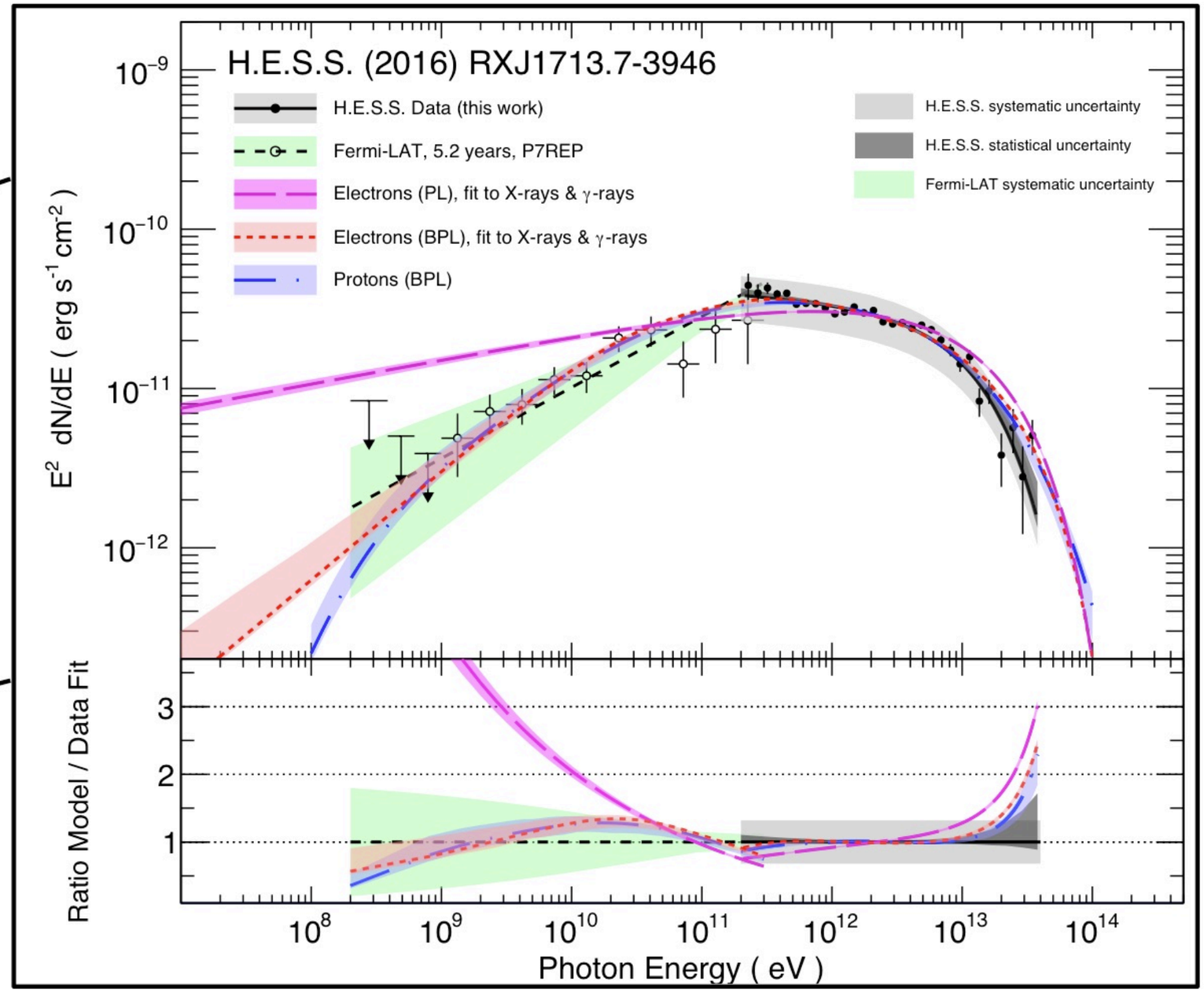


H.E.S.S. observation in γ-rays of a young ($\mathcal{O}(1000)$ yrs) SNR.

RX J1713.7-3946 SNR







[H.E.S.S. 2018]