

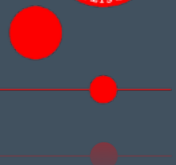
CR Sources & Radiative Processes

VILLUM FONDEN



Markus Ahlers
March 17 & 18, 2021

KØBENHAVNS
UNIVERSITET



Transport equations:

$$\phi \equiv \frac{1}{4\pi} \int d\hat{p} f(t, \vec{r}, p \cdot \hat{p})$$

$$\partial_t \phi + u_a \frac{\partial}{\partial r_a} \phi - \frac{1}{3} (\vec{v} \cdot \vec{u}) \rho \frac{\partial}{\partial \rho} \phi - \frac{\partial}{\partial r_a} \left(\kappa_{ab} \frac{\partial}{\partial r_b} \phi \right) - \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \tilde{\kappa} \frac{\partial}{\partial \rho} \phi \right) + (\text{collision terms}) = \eta(t, \vec{r}, \rho)$$

convection
momentum convection (adiabatic loss)
spatial diffusion
momentum diffusion
sources

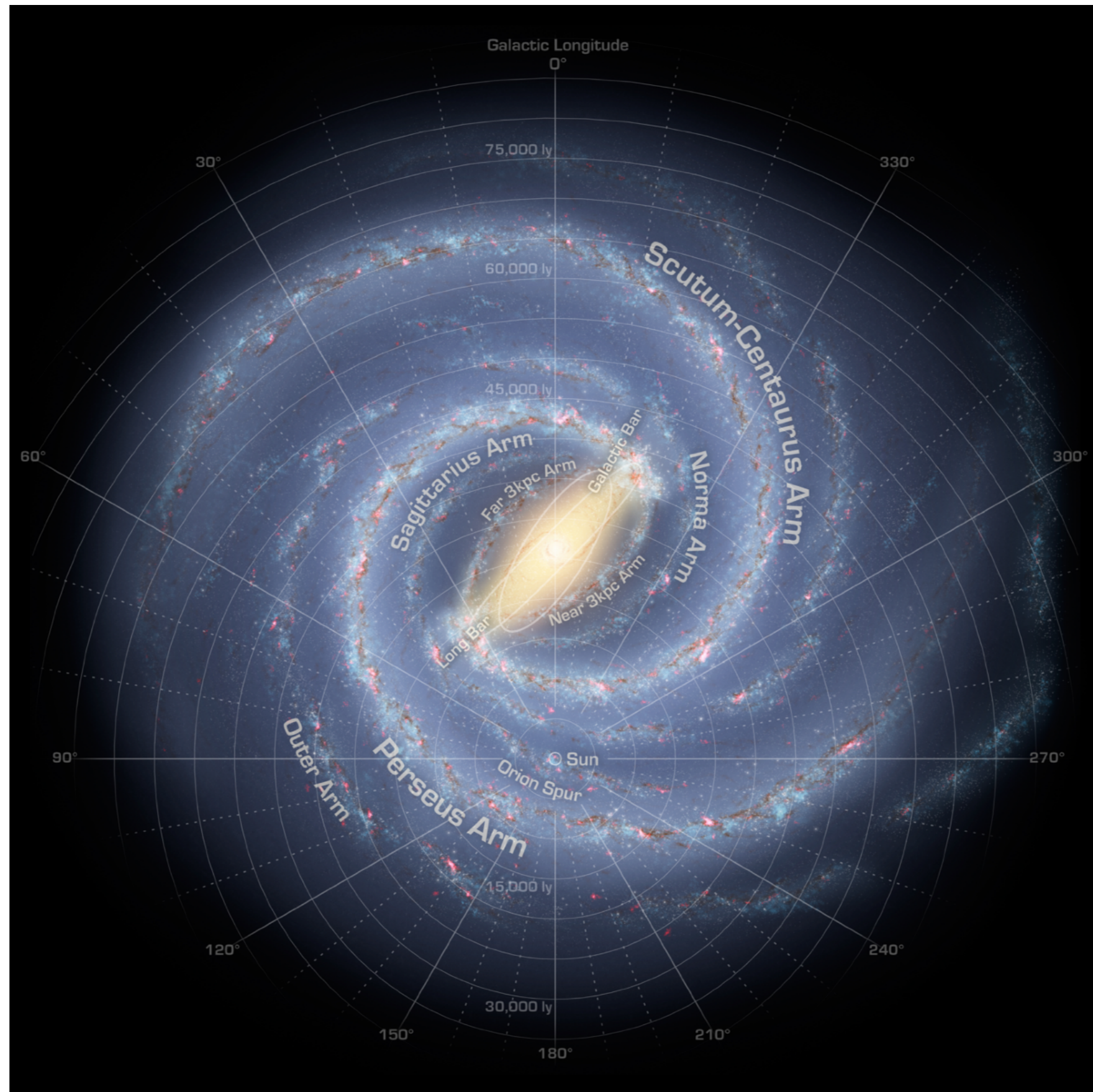
Or: $n(t, \vec{r}, p_0) \equiv \frac{4\pi}{\beta} p^2 \phi$ (spectral density)

$$\partial_t n + \vec{\nabla} \cdot (\vec{u} n) - \frac{1}{3} (\vec{v} \cdot \vec{u}) \frac{\partial}{\partial p_0} (\rho_0 n) - \frac{\partial}{\partial r_a} \left(\kappa_{ab} \frac{\partial}{\partial r_b} n \right) - \frac{\partial}{\partial p_0} \left(\rho_0^2 \tilde{\kappa} \frac{\partial}{\partial p_0} \left(\frac{n}{\rho_0^2} \right) \right) + (\text{collision terms}) = \underline{Q(t, \vec{r}, p_0)}$$

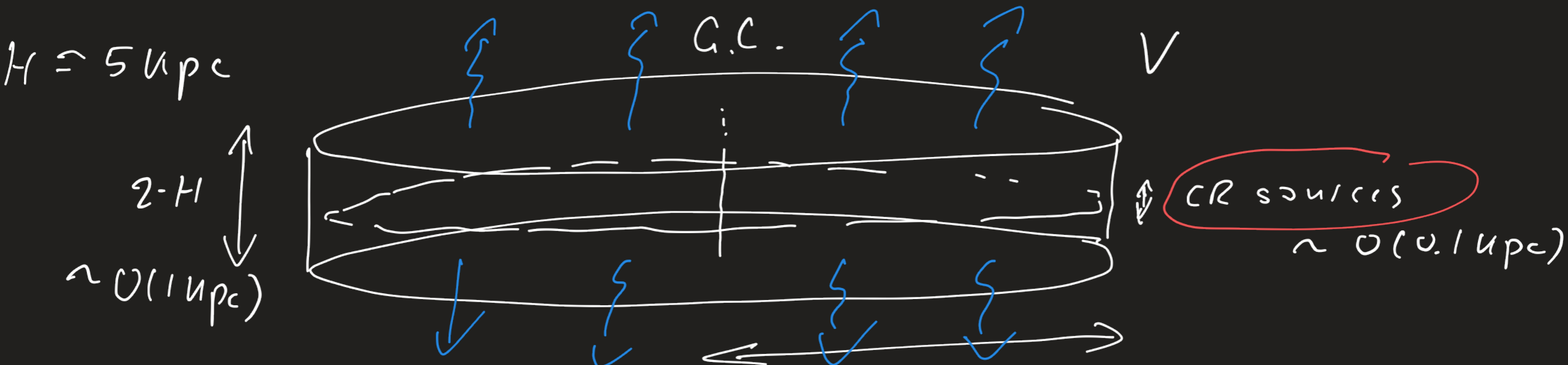
convection
mom. convection
spatial diff.
mom. diffusion
sources

Example: "Solar potential" (exercise 3 on Tuesday)

Galactic Cosmic Rays



Leaky-Box Model



volume: $V = 2 \cdot H \pi R^2$
 $\approx 7000 (\text{kpc})^3$

$$N_{CR}(t, p_0) = \int_V d^3r n(t, \vec{r}, p_0)$$

$$\dot{N}_{CR} = \underbrace{\int d\vec{r} \vec{v} (\kappa \cdot \vec{\nabla}_n)}_{\int_{\partial V} d\vec{s} \kappa \vec{\nabla}_n} + \underbrace{Q_{tot}}_{Q_{tot}} = - \frac{N_{CR}(t)}{\tau_{loss}(p_0)} + Q_{tot}$$

$$\int_{\partial V} d\vec{s} \kappa \vec{\nabla}_n + Q_{tot}$$

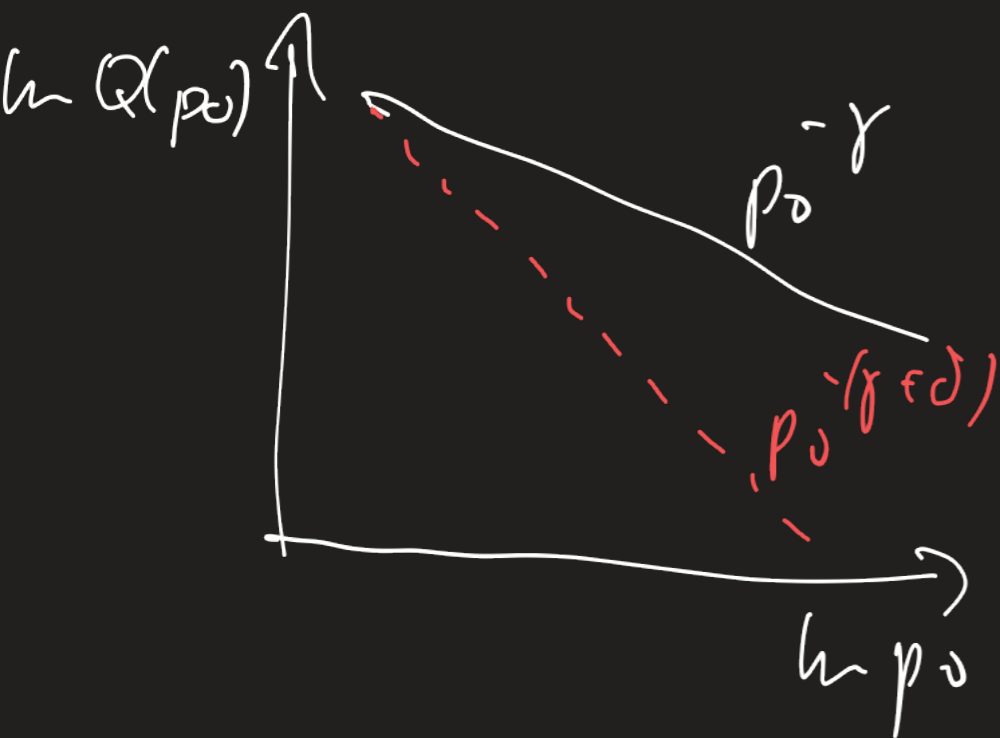
steady-state solution: $\dot{N}_{CR} = 0$ $\nearrow p_0^{-\gamma}$

$$\Rightarrow N_{CR}(t, p_0) = Q_{tot}(p_0) \cdot \tau_{loss}(p_0) \sim p_0^{-(\gamma+\delta)}$$

$$\tau_{loss}(p_0) \sim p_0^{-\delta}$$

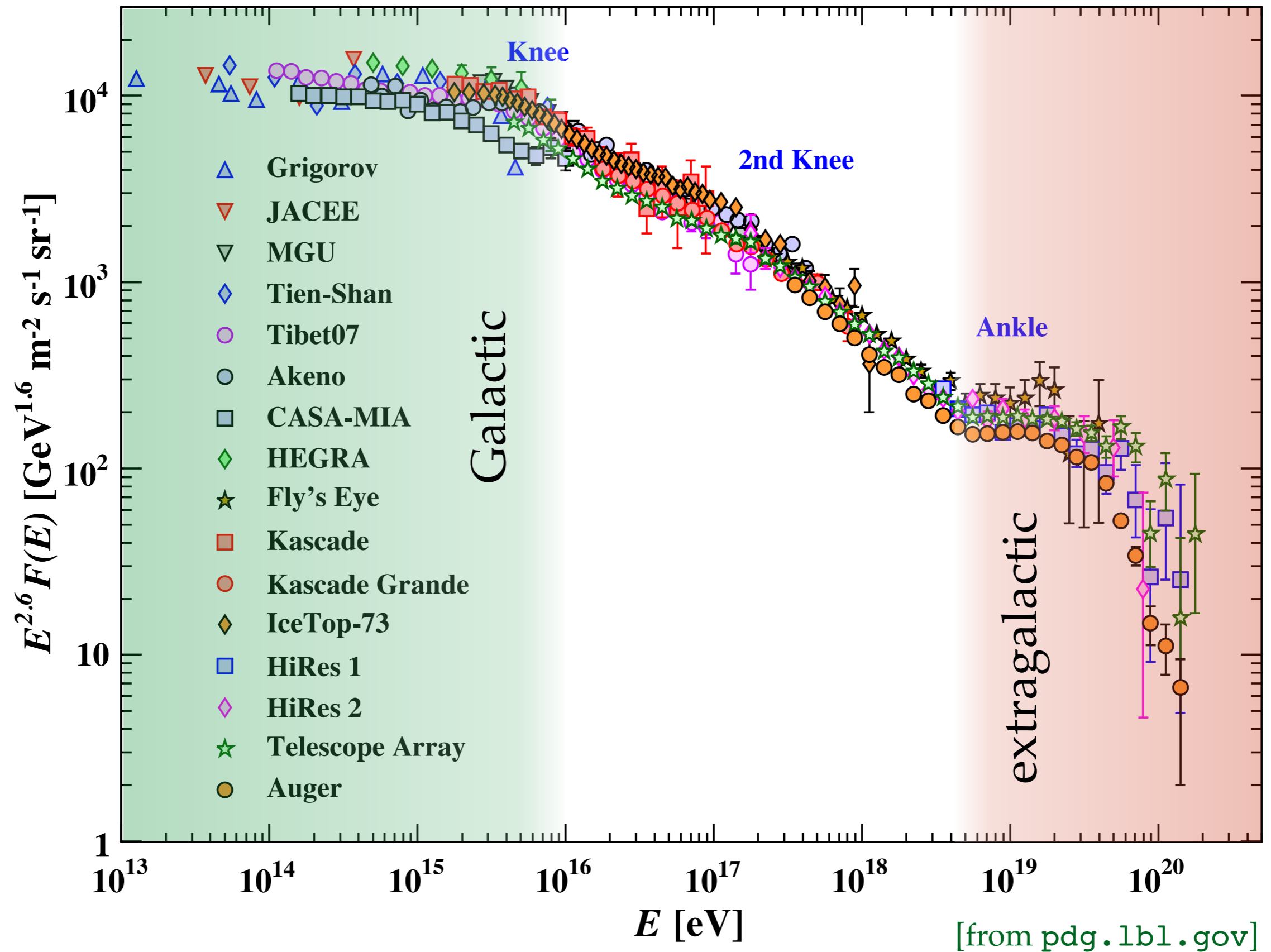
$$\delta = \frac{1}{3} \text{ (Kolmogorov)}$$

$$\delta = \frac{1}{2} \text{ (Kraichnan)}$$



observed	$N_{CR} \sim p_0^{-2.7}$
	$Q_{tot} \sim p_0^{-2.2}$

Very-High Energy Cosmic Rays



$$\partial_t N_s(p_0) = - \frac{N_s(p_0)}{\tau_{\text{loss}}(p_0)} + S_{\text{gas}} \cdot \underbrace{\sigma_{p \rightarrow s}}_{\text{primary}} N_p(p_0)$$

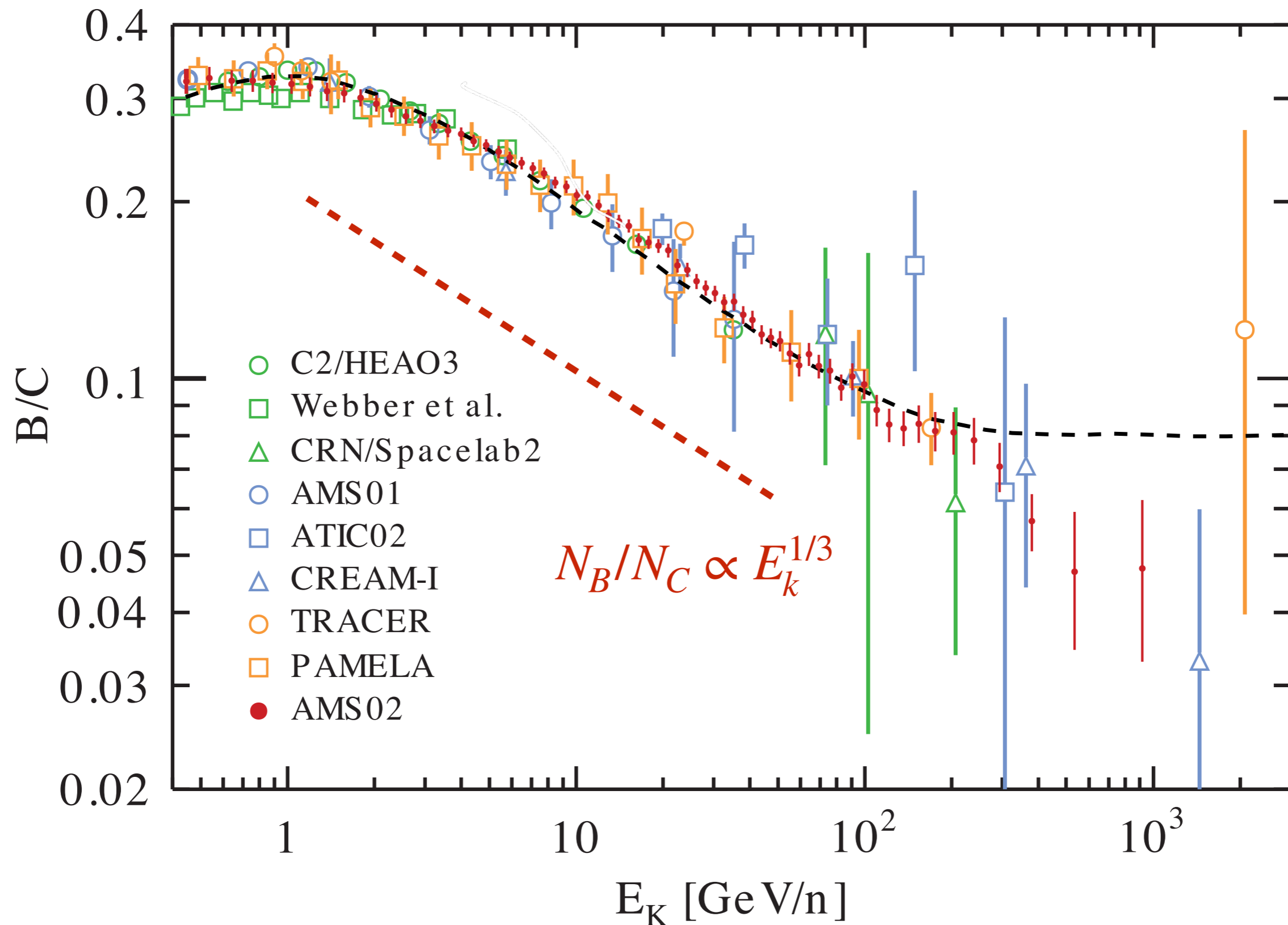
↑ secondary

$$\partial_t N_p(p_0) = - \frac{N_p(p_0)}{\tau_{\text{loss}}(p_0)} + Q_{\text{tot}}(p_0)$$

steady-state solution:

$$\frac{N_s(p_0)}{N_p(p_0)} = \tau_{\text{loss}}(p_0) \cdot \underbrace{S_{\text{gas}} \cdot \sigma_{p \rightarrow s}}_{\sim \text{const}} \sim p_0^{-5}$$

Boron-to-Carbon Ratio



relative abundance of radioactive isotopes



lifetime of $\tau = 1.5 \times 10^6 \text{ yr}$

$$\partial_t N_{10}(p_0) = - \frac{N_{10}(p_0)}{\tau_{\text{loss}}(p_0)} - \frac{N_{10}(p_0)}{\tau_{\text{decay}}(p_0)} + S_{\text{gas}} \sigma_{p \rightarrow s} N_p(p_0)$$

steady state:

$$N_{10}(p_0) = \frac{S_{\text{gas}} \sigma_{p \rightarrow s}}{\tau_{\text{loss}}^{-1} + \tau_{\text{decay}}^{-1}} N_p(p_0)$$

stable isotopes: ${}^7\text{Be}$ & ${}^9\text{Be}$

$$\boxed{\frac{N_{10}}{N_7 + N_9}}$$

$$= \frac{\tau_{\text{loss}}^{-1}(p_0)}{\tau_{\text{loss}}^{-1}(p_0) + \tau_{\text{decay}}^{-1}(p_0)}$$

$$\left\{ \frac{\sigma_{p \rightarrow 10}}{\sigma_{p \rightarrow 7} + \sigma_{p \rightarrow 9}} \right\}$$

CR measurement

laboratory measurement

observed loss time $p_0 = 100 \text{ MeV}$ of $t_{\text{loss}} = 15 \text{ Myr}$

$$w_{\text{CR}} = \int dp_0 p_0 n_{\text{CR}}(p_0) \approx 1 \frac{\text{eV}}{\text{cm}^3}$$

$$L_{\text{CR}} = \frac{V \times w_{\text{CR}}}{\tau_{\text{loss}}} \approx 5 \times 10^{53} \frac{\text{eV}}{\text{s}} \approx 7 \times 10^{41} \frac{\text{erg}}{\text{s}}$$

Becker & Zuckerman 30s

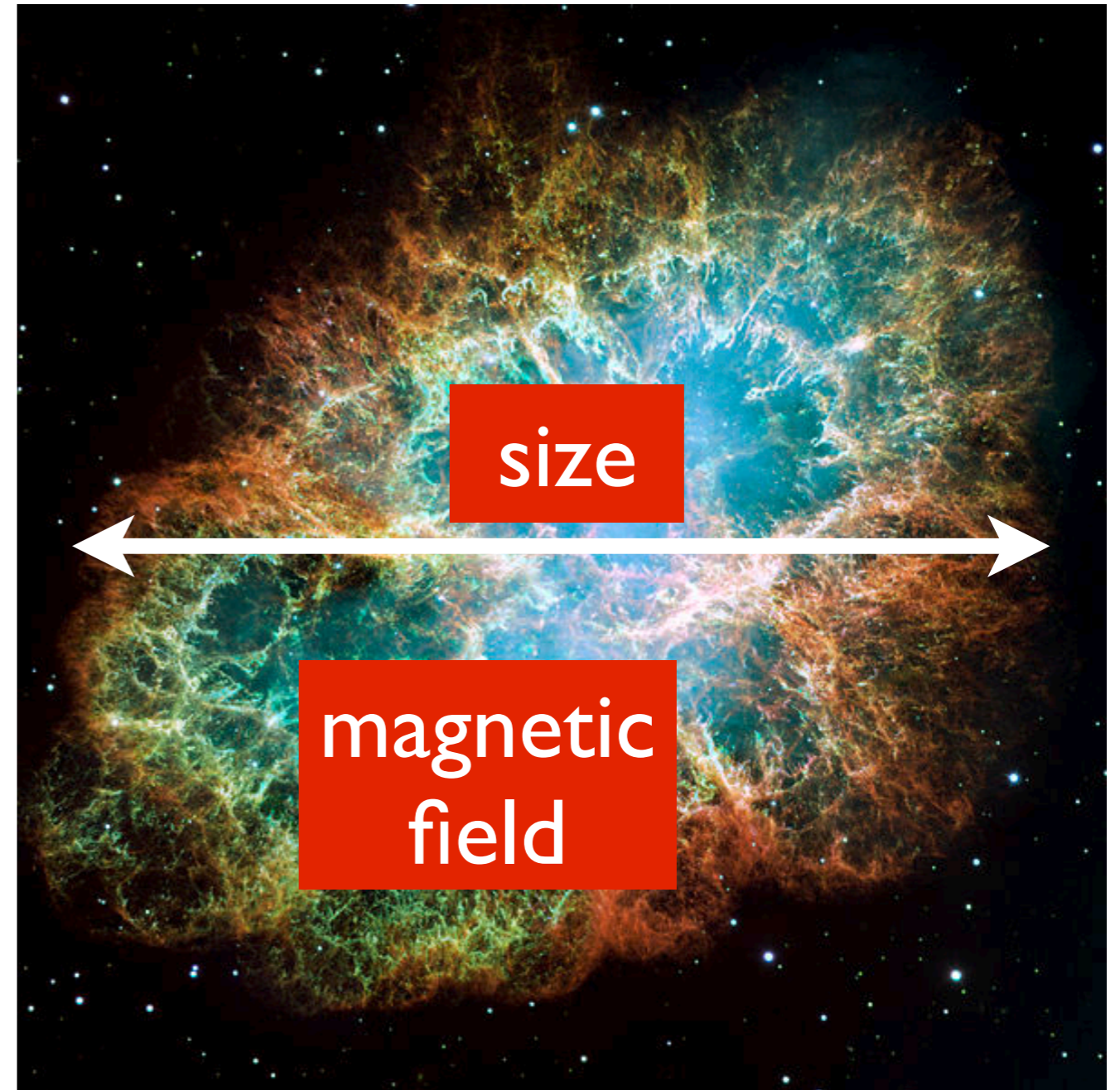
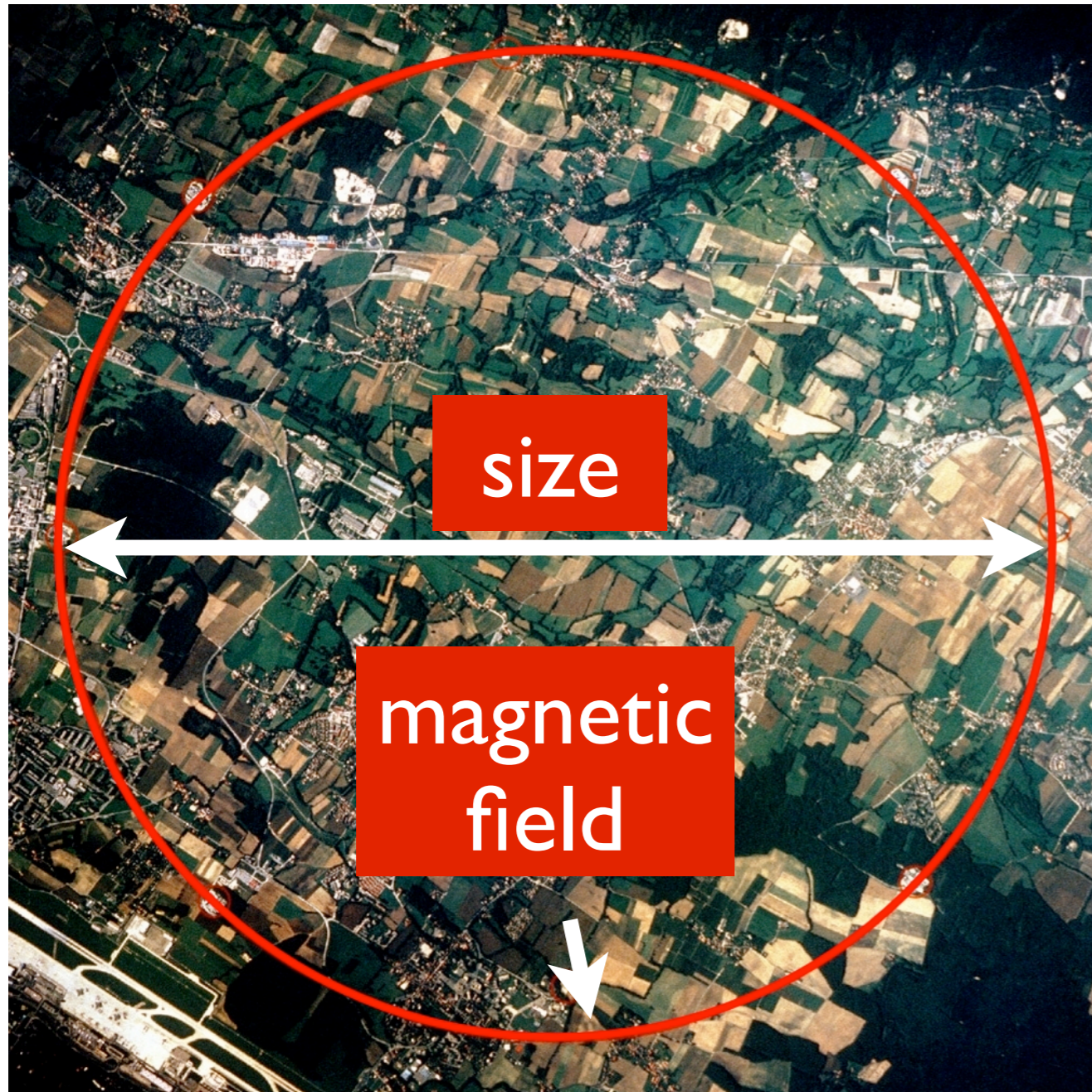
Supernova remnants $R = \frac{1 \text{ SN}}{30 \text{ yrs}}$

kinetic energy of ejecta $E_{\text{kin}} = 3 \times 10^{51} \text{ erg}$

$$L_{\text{SN}} = \left(\frac{3}{0.2} \right) 7 \times 10^{41} \frac{\text{erg}}{\text{s}}$$

Hillas Criterion

Hillas Criterion

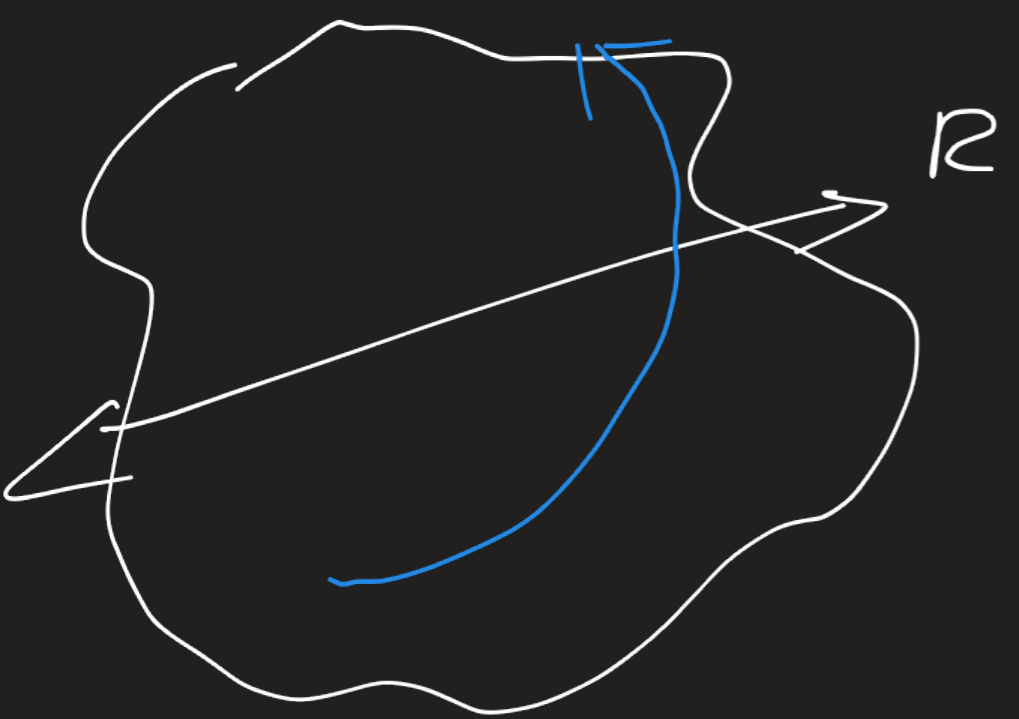


Larmor radius: $r_L = \frac{1.1 \text{ pc}}{Z} \left(\frac{p_0}{\text{PeV}} \right) \left(\frac{B}{\mu\text{G}} \right)^{-1}$

$\Rightarrow E_{\text{max}} = 0.4 \cdot Z \left(\frac{R}{\text{pc}} \right) \left(\frac{B}{\mu\text{G}} \right) \text{PeV}$

geometric condition

Hillas criterion



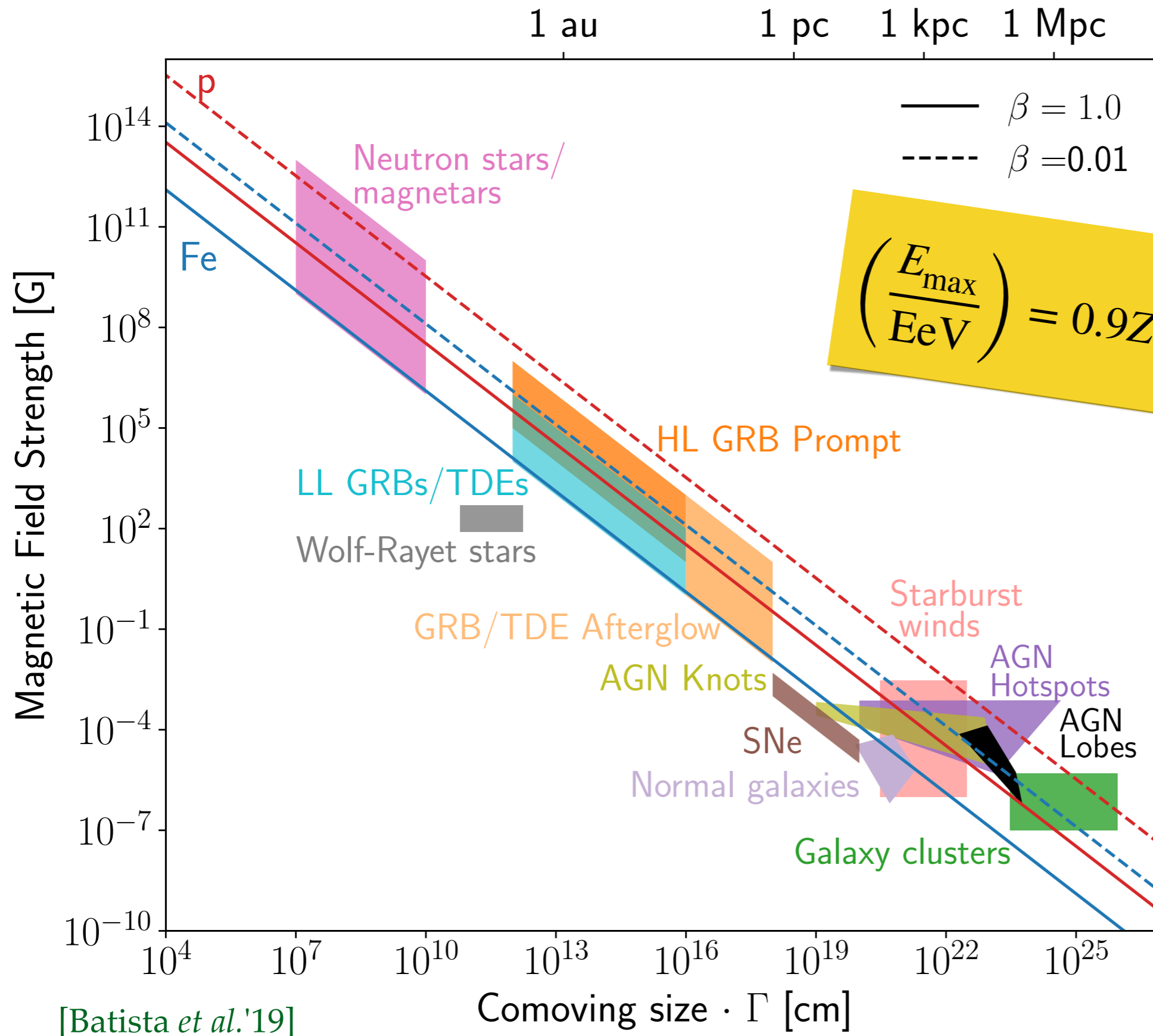
SNR:

$E_{\text{max}} \sim 1 \text{ PeV}$

LHC: $R = 4 \text{ km}$ $B = 8 \text{ T}$

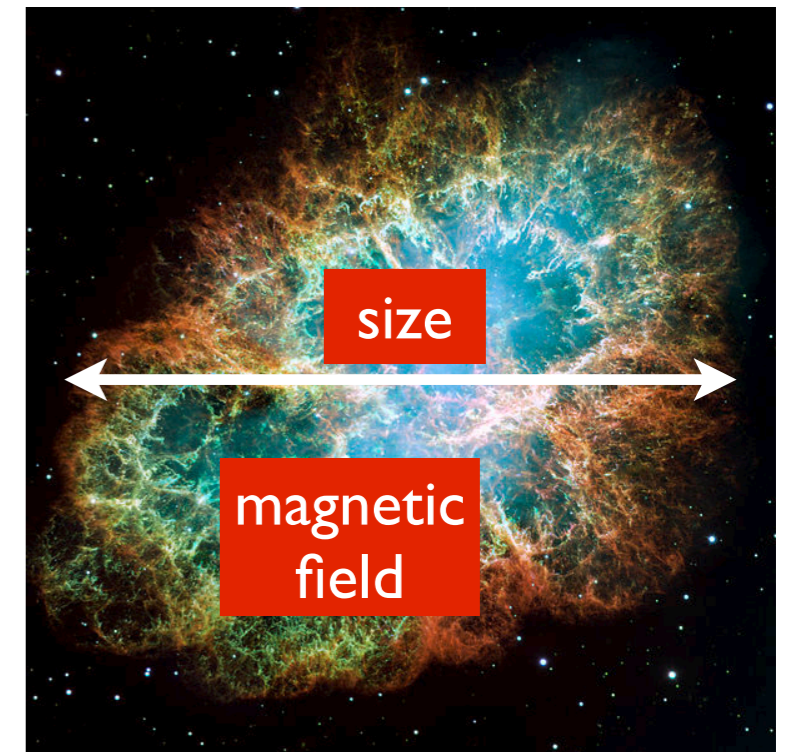
$E_{\text{max}} = 9 \text{ TeV} \left(\frac{R}{4 \text{ km}} \right) \left(\frac{B}{8 \text{ T}} \right)$

Hillas Criterion



"Hillas plot"

$$\left(\frac{E_{\max}}{\text{EeV}}\right) = 0.9Z(\beta_{\text{sh}}) \left(\frac{\Gamma R_{\text{acc}}}{\text{kpc}}\right) \left(\frac{B_{\text{acc}}}{\mu\text{G}}\right)$$

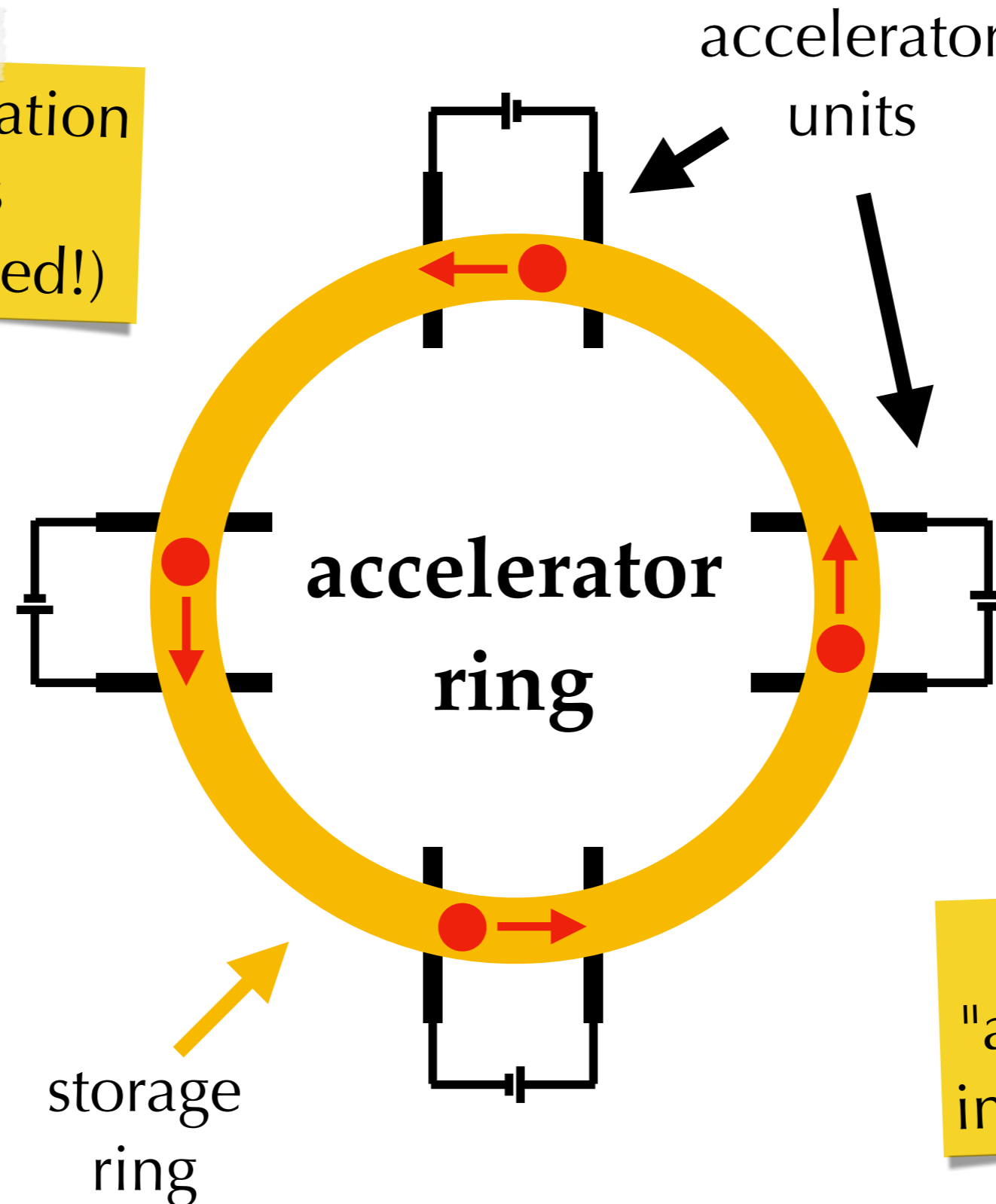


[Batista *et al.*'19]

Fermi Acceleration

Cosmic Ray Acceleration

Particle acceleration
in colliders
(wildly simplified!)



What are the
"accelerator units"
in cosmic sources?

Fermi Acceleration

PHYSICAL REVIEW

VOLUME 75, NUMBER 8

APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

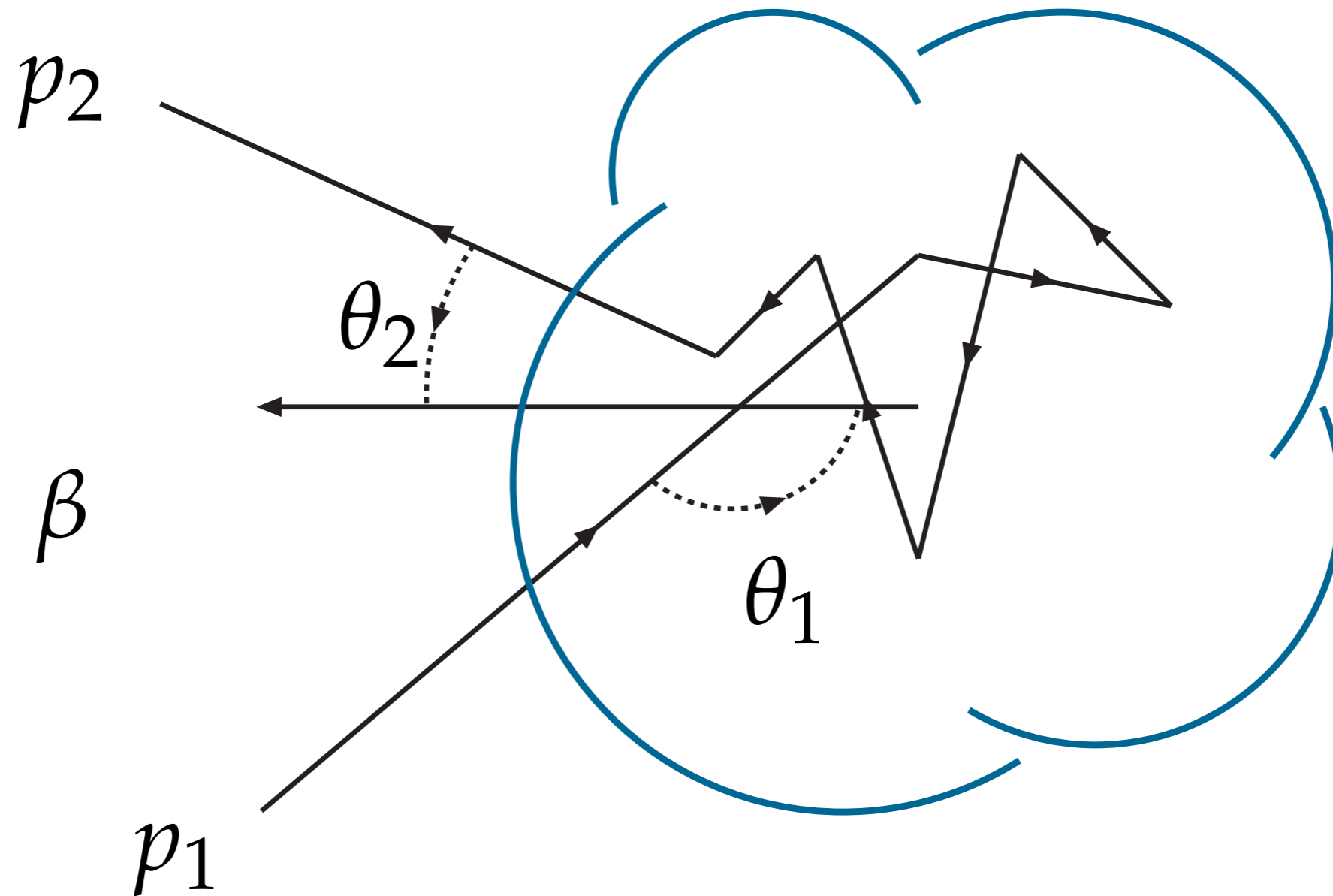
I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller¹ has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.² The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

(...)

I propose in the present note to discuss a hypothesis on the origin of cosmic rays which attempts to meet in part this objection, and according to which cosmic rays originate and are accelerated primarily in the interstellar space, although they are assumed to be prevented by magnetic fields from leaving the boundaries of the galaxy. The main process of acceleration is due to the interaction of cosmic particles with wandering magnetic fields which, according to Alfvén, occupy the interstellar spaces.

Fermi Acceleration (2nd order)



* initially $p_1 = (E_1, \vec{p}_1)$

* after (1) enters cloud: $E_1' = \gamma \cdot E_1 (1 - \beta \cdot \cos \theta_1)$

* on exit of cloud: $E_2 = \gamma \cdot E_2' (1 + \beta \cos \theta_2')$

* isotropization: $E_2' = E_1'$

* energy budget:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \gamma^2 (1 + \beta \langle \cos \theta_2' \rangle) (1 - \beta \langle \cos \theta_1 \rangle) - 1$$

$= 0$ $= -\frac{\beta}{3}$

* collision rate on enter: $R(\theta_1) \sim \frac{1}{2} (1 - \beta \cos \theta_1)$

* " " " " on exit: $R(\theta_2') \sim \frac{1}{2}$

$$\langle \cos \theta_1 \rangle = \int_{-1}^1 d \cos \theta_1 R(\theta_1) \cdot \cos \theta_1 = -\frac{\beta}{3}$$

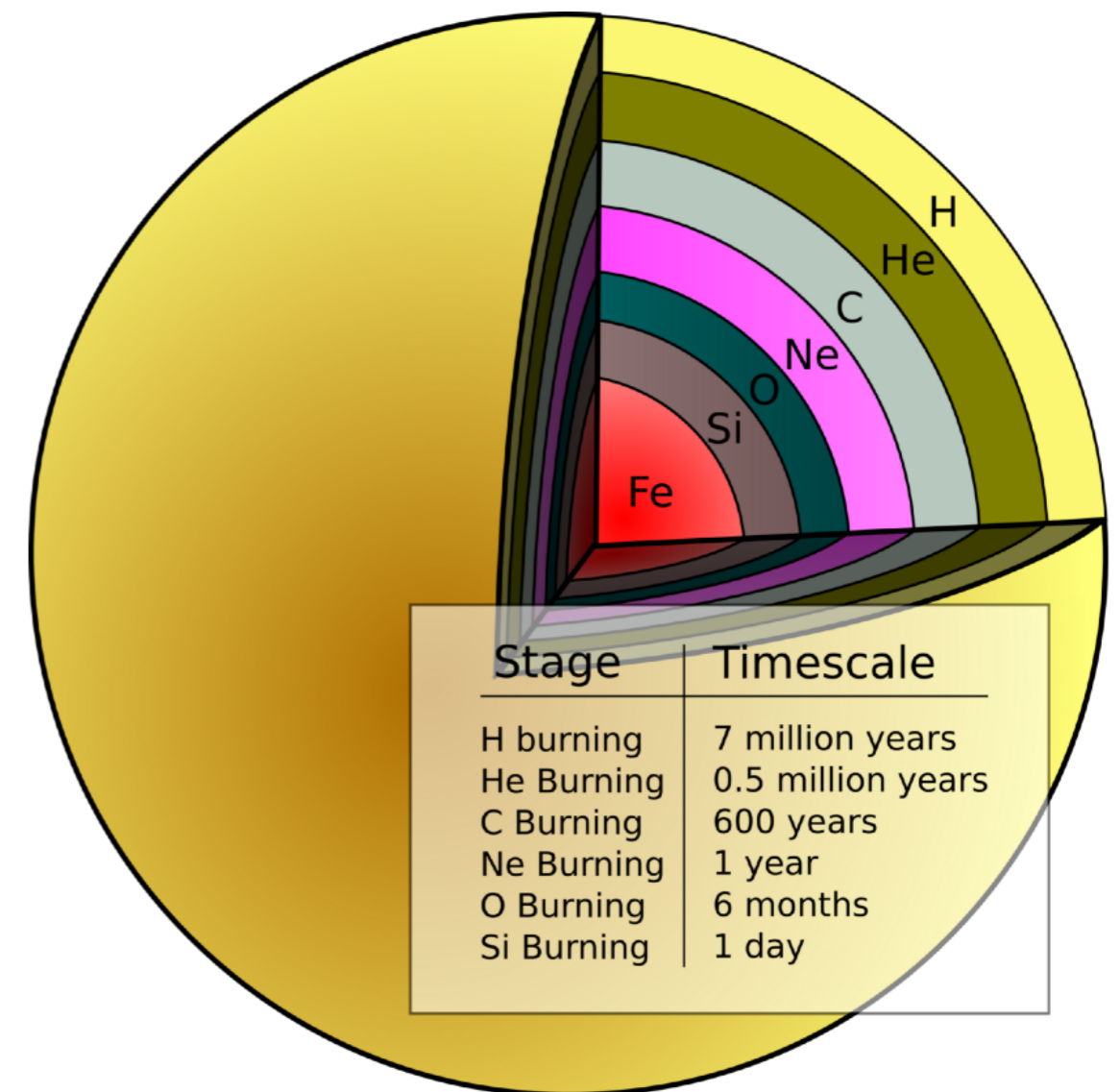
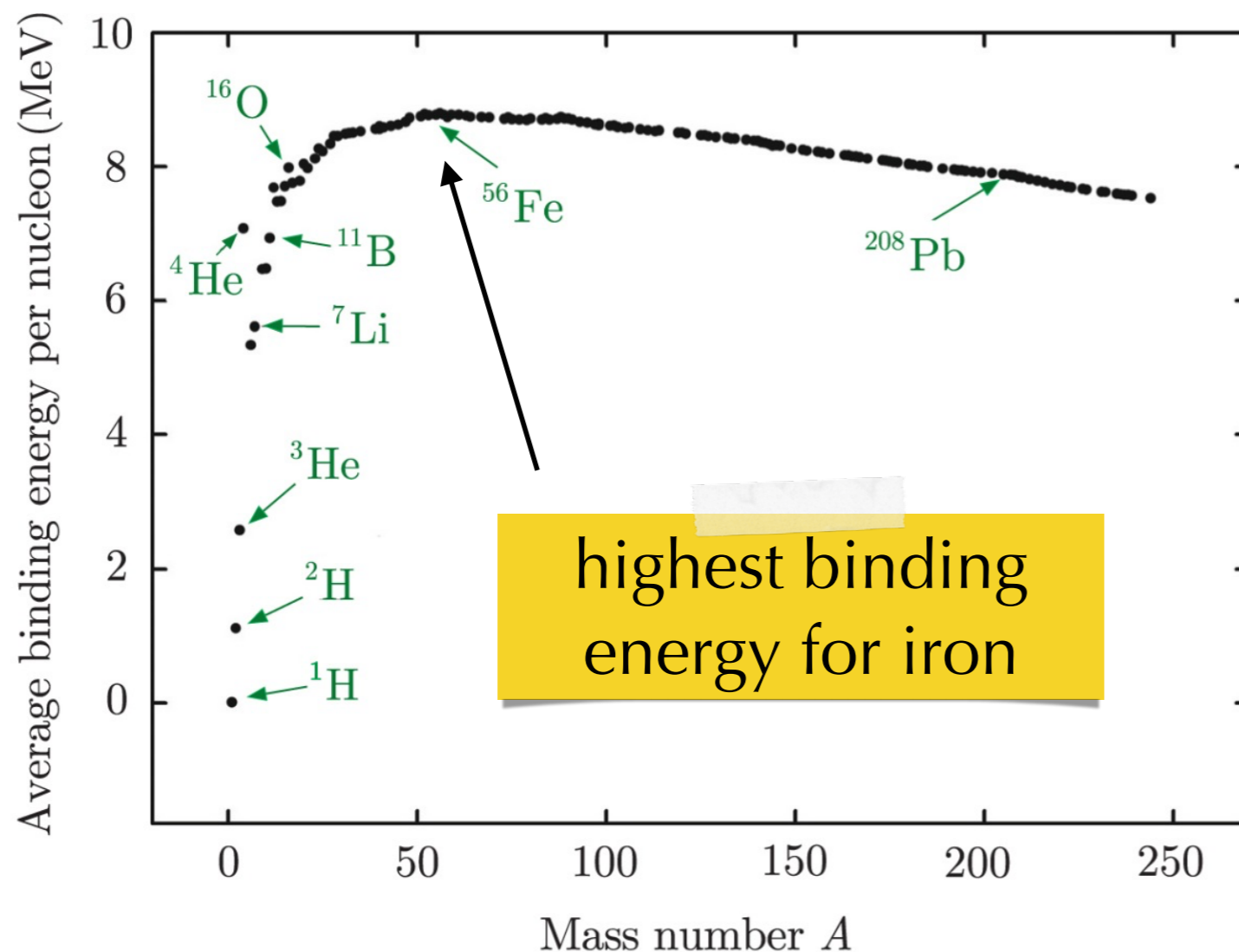
$$\langle \cos \theta_2' \rangle = \int_{-1}^1 d \cos \theta_2' \frac{1}{2} \cos \theta_2' = 0$$

$$\frac{\Delta E}{E_1} = \gamma^2 (1+0) \left(1 + \frac{1}{3} \beta^2\right) - 1 = \frac{4}{3} \beta^2 + \mathcal{O}(\beta^3)$$

\Rightarrow 2nd order Fermi acceleration: $\frac{\Delta E}{E} \sim \beta^2$ ⁽²⁾

Stellar Nucleosynthesis

- Stars are kept in hydrostatic equilibrium, balancing the gravitational force by thermal pressure.
- The star is powered by nuclear fusion processes, that build up layers of heavy elements up to the **iron group**. This is the element with the **highest nuclear binding energy**:



Chandrasekhar mass

$$M_{cl} \sim 1.4 M_{\odot}$$

$$\Delta \phi_{grav} = \frac{3}{5} G_N \cdot M_{cl}^2 \left(\frac{1}{R_{PNS}} - \frac{1}{R_0} \right)$$

$$R_{PNS} \sim 13 \text{ km}$$

$$\approx 3 \times 10^{53} \text{ erg}$$

$$e + p \rightarrow n + \nu_e$$

↑
(carry 99%)

$$E_{kin} = 1\% \Delta \phi_{grav}$$

$$\Rightarrow v_{ej} = \sqrt{\frac{2 E_{kin}}{M_{ej}}} \sim \frac{c}{100}$$

① ballistic expansion

$$R(t) \sim t$$

$$t < 100 - 1000 \text{ yrs}$$

⇒ ② Sedov-Taylor phase

$$R(t) \sim t^{2/5}$$

$$t < 10^5 \text{ yrs}$$

③ "snow-plow" phase

$$R(t) \sim t^{1/4}$$

$$t < 1 - 10 \text{ Myrs}$$

④ merger with ISM

$$\frac{\Delta E}{E_1} = \gamma^2 (1 + \beta \langle \cos \theta_2' \rangle) (1 - \beta \langle \cos \theta_1 \rangle) - 1$$

$$R(\theta_1) \sim \begin{cases} -2 \cos \theta_1 \\ 0 \end{cases}$$

$$\begin{aligned} \theta_1 &> \frac{\pi}{2} \\ \theta_1 &< \frac{\pi}{2} \end{aligned}$$

$$R(\theta_2') \sim \begin{cases} 0 \\ 2 \cos \theta_2' \end{cases}$$

$$\begin{aligned} \theta_2' &> \frac{\pi}{2} \\ \theta_2' &< \frac{\pi}{2} \end{aligned}$$

$$\langle \cos \theta_1 \rangle = -\frac{2}{3}$$

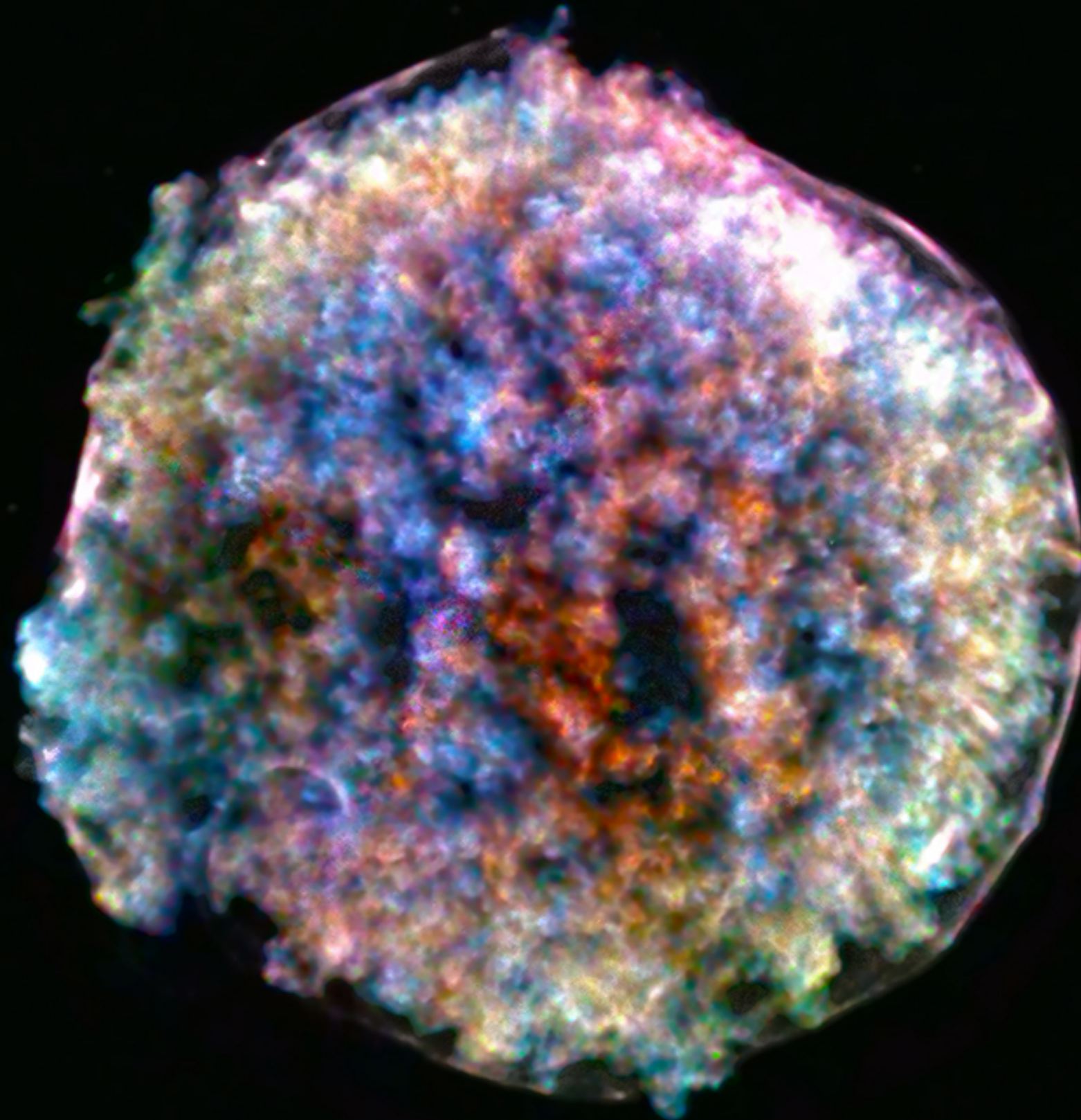
$$\langle \cos \theta_2' \rangle = \frac{2}{3}$$

$$\frac{\Delta E}{E_1} = \gamma^2 \left(1 + \frac{2}{3}\beta\right) \left(1 + \frac{2}{3}\beta\right) - 1 \approx \frac{4}{3} \cdot \beta + O(\beta^2)$$

1st order F.E.

$$\frac{\Delta E}{E} \sim \beta$$

Supernova Remnants



Tycho's Supernova Remnant (SN 1572)

Fermi Acceleration (1st order)

