EXERCICES SOLUTIONS: COSMIC RAYS AND MULTIMESSENGER ASTRONOMY

Radiative processes and neutrinos: lecture 3 to 5

EXERCISE 5 - THE INTERACTION RATE FOR A GIVEN CROSS SECTION

(i) We define the rate in the CR rest frame $R' = j'\sigma(s)$. We proceed by applying a Lorentz boost to the four-current of target particles $(n, \boldsymbol{j}) = (n, \mu\beta_b n, \sqrt{1-\mu^2}\beta_b)$ to move to the CR rest frame, we obtain:

$$\begin{aligned} j' &= \frac{\left(\beta^2 + \beta_b^2 + (\mu^2-1)\beta^2\beta_b^2 - 2\beta_b\beta\mu\right)^{1/2}}{(1-\beta\beta_b\mu)} n' && (1) \\ &= \left(\beta^2 + \beta_b^2 + (\mu^2-1)\beta^2\beta_b^2 - 2\beta_b\beta\mu\right)^{1/2} \gamma\, n && (2) \\ &= v(\beta, \beta_b, \mu)\gamma\, n && (3) \end{aligned}$$

with $\mu = \cos(\theta)$.

(ii) Substituting $n \to \frac{dn}{d\Omega d\epsilon} \equiv n(\epsilon, \mu)$ we define the differential rate per unit of solid angle and energy of target particle:

$$\begin{aligned} \frac{dR'}{d\Omega d\epsilon} &= \frac{dN_{int}}{d\Omega d\epsilon dt'} && (4) \\ &= v(\beta, \beta_b, \mu)\gamma\, n(\epsilon, \mu)\ \sigma(s), && (5) \end{aligned}$$

with dN_{int} the number of interactions within the proper time (in the CR rest frame) dt'. In the observer frame, the differential interaction rate is thus:

$$\begin{aligned} \frac{dN_{int}}{d\Omega d\epsilon dt} &= \frac{dN_{int}}{d\Omega d\epsilon dt'}\frac{dt'}{dt} && (6) \\ &= v(\beta, \beta_b, \mu)\, n(\epsilon, \mu)\ \sigma(s). && (7) \end{aligned}$$

Then the total interaction rate is given by:

$$\Gamma = \int d\Omega \int d\epsilon\, n(\epsilon, \mu) v(\beta, \beta_b, \mu)\, \sigma(s)\ . \tag{8}$$

Thus the interaction length is given by:

$$l(E) = \frac{\beta}{\Gamma(E)}\ . \tag{9}$$

(iii) With fixed proton targets: $n(\epsilon) = n_0\, \delta(\epsilon - m_p)$ and $\beta_b = 0$. This leads to the simple relation:

$$l = \frac{1}{\sigma n_0}\ . \tag{10}$$

For CR protons we obtain $l(E = 10\text{GeV}) \approx 8\,\text{Mpc}$ and $l(E = 1\text{EeV}) \approx 2.5\,\text{Mpc}$. For neutrinos we obtain $l(E = 10\text{GeV}) \approx 3 \times 10^8$ Gpc and $l(E = 1\text{EeV}) \approx 3 \times 10^4$ Gpc

EXERCISE 6 - GALACTIC PEVATRONS

(i) The local CR spectral density becomes

$$n(E, r) = \int_0^\infty \mathrm{d}t \frac{Q(E)}{(4\pi K t)^{3/2}} \exp\left(-\frac{r^2}{4Kt}\right) \tag{11}$$

Substituting $x = r^2/4Kt$:

$$n(E,r) = \frac{Q(E)}{4\pi^{3/2}Kr}\int_0^\infty \mathrm{d}x \frac{e^{-x}}{\sqrt{x}} = \frac{Q(E)}{4\pi Kr} \tag{12}$$

(ii) The figure shows that for CR energy density at, say, $r = 100$ pc is $w_{\rm CR} \simeq 6\times10^{-3}$ eV/cm^3. The luminosity above 10 TeV is then:

$$L(\geq 10\,\mathrm{TeV}) = \int_{10\ \mathrm{TeV}}^{\infty} \mathrm{d}E E Q(E) = 4\pi Kr \int_{10\ \mathrm{TeV}}^{\infty} \mathrm{d}E E n(E) = 4\pi K r w_{\rm CR} \simeq 2\times10^{49}\frac{\mathrm{eV}}{\mathrm{s}} \simeq 4\times10^{37}\frac{\mathrm{erg}}{\mathrm{s}} \tag{13}$$

(iii) The $1/r^2$ behaviour is reproduced by the convection equation in steady state. In spherical coordinates it writes:

$$\nabla\cdot(\mathbf{V}n) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V n\right) = Q(E)\frac{\delta(r)}{r^2} \tag{14}$$

By integrating this equation over a sphere of radius r we get:

$$n(r) = \frac{Q(E)}{Vr^2}\ . \tag{15}$$

(iv) If the gammas mainly come from hadronic interactions, then one would expect a neutrino flux at the same level as the gamma one. Given that the gamma spectrum is close to E^{-3} and that the declination of the Galactic center is $-29°$, the point source sensitivity of IceCube (5σ discovery potential) is $4\times10^{-9}\,\mathrm{TeV^{-1}s^{-1}cm^{-2}}$ at 1TeV. This sensitivity does not reach the potential neutrino flux from the galactic center, of the order of $2\times10^{-11}\,\mathrm{TeV^{-1}s^{-1}cm^{-2}}$.

EXERCISE 7 - MULTIMESSENGER

(i) The time delay between the photon and the neutrino is given by:

$$\Delta t = \frac{d}{c}(\beta^{-1}-1) \tag{16}$$

$$\approx \frac{d}{2\,c}\frac{m_\nu^2}{E^2}\ . \tag{17}$$

For d =10 Mpc, $m_\nu = 0.1\,\mathrm{eV}$, and $E = 1\,\mathrm{PeV}$ this gives a time difference of ~ 5 fs.

(ii) To produce a W boson at resonance, the center of mass energy should be close to the W mass M_W. Thus, the energy of the neutrino E_ν should be such that:

$$(E_\nu + m_e)^2 - (E_\nu^2 - m_\nu^2) = M_W^2\ . \tag{18}$$

Neglecting m_ν and m_e with respect to E_ν, we obtain that $E_\nu = M_W^2/2m_e \approx 6.3\,\mathrm{PeV}$.