EXERCICE SESSION: COSMIC RAYS AND MULTIMESSENGER ASTRONOMY

Radiative processes and neutrinos: lecture 3 to 5

EXERCISE 5 - THE INTERACTION RATE FOR A GIVEN CROSS SECTION

(i) Assume that a cosmic ray particle interacts with a mono-energetic beam of "target particles" that comes from a fixed direction, with a cross section $\sigma(s)$ which depends on the squared center of mass energy s (also known as the first Mandelstam-variable). Imagine the CR is at rest with respect to the observer frame (unprimed quantities), per definition of a cross section its interaction rate is given by:

$$R = j\sigma(s) \ , \tag{1}$$

where $\boldsymbol{j} = c\boldsymbol{\beta_b} n$ is the flux density of the beam of target particles whose velocity is $c\boldsymbol{\beta_b}$, corresponding to a target particle density n, and $j = |\boldsymbol{j}| = c\beta_b n$. Since the CR is moving with velocity $\boldsymbol{v} = c\boldsymbol{\beta}$, using Lorentz transformations, show that the interaction rate in the cosmic ray rest-frame (primed quantities) is given by,

$$R' = w(\beta, \beta_b, \cos\theta) n' \sigma(s) = v(\beta, \beta_b, \cos\theta) n \gamma \sigma(s) \ , \tag{2}$$

where θ is the angle between the CR velocity $\boldsymbol{\beta}$ and the direction of the target particle beam $\boldsymbol{\beta}_b$ and the *relative velocity* $w(\beta, \beta_b, \cos\theta)$ between the cosmic ray and the target particle. Use the fact that the components of the flux densities $(cn, \boldsymbol{j})$ and $(cn', \boldsymbol{j'})$, respectively, in these two frames (observer frame and CR frame) transform as a four-vector j_μ.

(ii) Use Eq. 2 to derive the general formula for the interaction length of a CR in a background of particles that is non-isotropic and non-monoenergetic in the observer frame,

$$l(E)^{-1} = \beta^{-1} \int d\epsilon \int_{-1}^{+1} 2\pi \, d\mu \, n(\epsilon, \mu) \, v(\beta, \beta_b, \mu) \sigma(s) \ , \tag{3}$$

where $\mu = \cos\theta$. Hints: In Eq. (2) substitute $n \to \frac{dn}{d\Omega d\epsilon} \equiv n(\epsilon, \mu)$ to define the differential interaction rate.

(iii) Compute the interaction length of relativistic 10GeV and 1EeV protons propagating in the interstellar hydrogen of average density ~ 1 proton cm^{-3}. Here we assume that the cross-section is given by $\sigma_{pp}^{\rm tot}(10\,{\rm GeV}) \approx 40$ mb and $\sigma_{pp}^{\rm tot}(1\,{\rm EeV}) \approx 130$ mb. Compare with the typical propagation length of cosmic rays in the galactic disk $\sim 4(E/1{\rm GeV})^{-1/2}$ Mpc. In the same way compute the interaction length of neutrinos with $\sigma_{p\nu}(10\,{\rm GeV}) \approx 10^{-36}{\rm cm}^2$ and $\sigma_{p\nu}(1\,{\rm EeV}) \approx 10^{-32}{\rm cm}^2$. Compare with the size of the observable Universe.

EXERCISE 6 - GALACTIC PEVATRONS

Recently, the H.E.S.S. collaboration reported evidence of a cosmic ray source in the Galactic center that is capable of accelerating cosmic rays up to 1 PeV ($= 10^{15}$ eV). The H.E.S.S. observatory is an Imaging Atmospheric Cherenkov Telescope (IACT) located in Namibia. They studied diffuse γ-ray emission in different locations along the Galactic disk in the vicinity of Sagittarius A*. The search regions are indicated in figure 1. Under the assumption that the diffuse gamma-ray emission in each search region is due to the decay of neutral pions, $\pi^0 \to \gamma + \gamma$, that are produced via cosmic ray interactions with molecular gas, one can infer the cosmic ray density in the Galactic center. This is shown in figure 2 in terms of the distance r from Sagittarius A*.

(i) In the lecture we derived the density of CRs for a source at distance r that emits a burst of $N_{\rm CR}$ cosmic rays. The solution can be extended to an emission spectrum $N(E)$ (number of CRs per energy) to give the local spectral energy density:

$$n(t, E, r) = \frac{N(E)}{(4\pi K(E) t)^{3/2}} \exp\left(-\frac{r^2}{4K(E)t}\right) \ . \tag{4}$$

Consider now the emission of this source over a long period $T \to \infty$ with a CR emission rate $Q(E) = dN(E)/dt$ (number of CRs per energy and time). Show that the radial distribution of the CR spectral density scales like $1/r$.

Hint: You can use MAPLE, MATHEMATICA, *etc.*, or simply the definition of the Gamma function with $\Gamma(1/2) = \sqrt{\pi}$:

$$\int_0^\infty \mathrm{d}x x^{\alpha-1} e^{-x} = \Gamma(\alpha) \,. \tag{5}$$

(ii) Use the best-fit $1/r$ result of the local CR energy density $w(r) = \int_{10\ \mathrm{TeV}}^\infty \mathrm{d}E E n(E,r)$ (red dashed line in figure 2) to estimate the source luminosity in erg/s:

$$L(\geq 10\ \mathrm{TeV}) = \int_{10\ \mathrm{TeV}}^\infty \mathrm{d}E E Q(E) \,, \tag{6}$$

assuming that the diffusion coefficient is constant in energy with $K \simeq 10^{30}\mathrm{cm}^2/\mathrm{s}$.

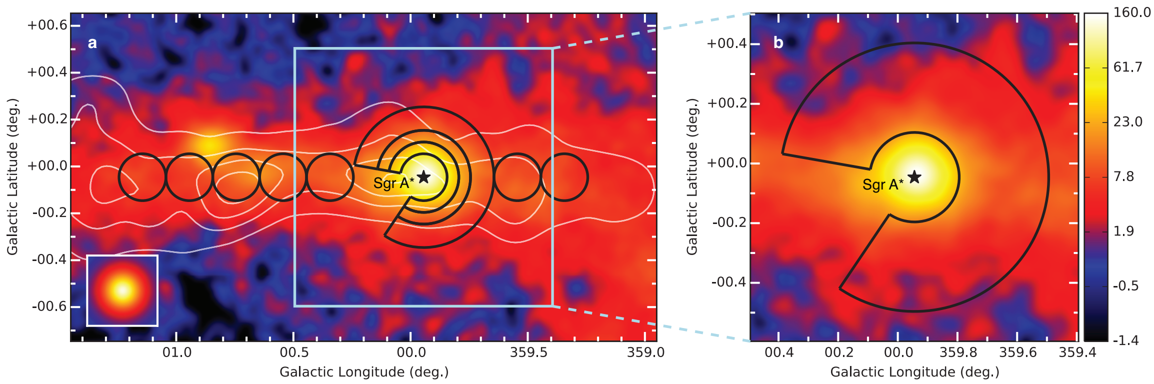

Figure 1: **VHE γ-ray image of the Galactic Centre region.** The colour scale indicates counts per $0.02^\circ \times 0.02^\circ$ pixel. *Left panel:* The black lines outline the regions used to calculate the CR energy density throughout the central molecular zone. A section of 66° is excluded from the annuli (see Methods). White contour lines indicate the density distribution of molecular gas, as traced by its CS line emission[30]. The inset shows the simulation of a point-like source. *Right panel:* Zoomed view of the inner $\sim$ 70 pc and the contour of the region used to extract the spectrum of the diffuse emission.

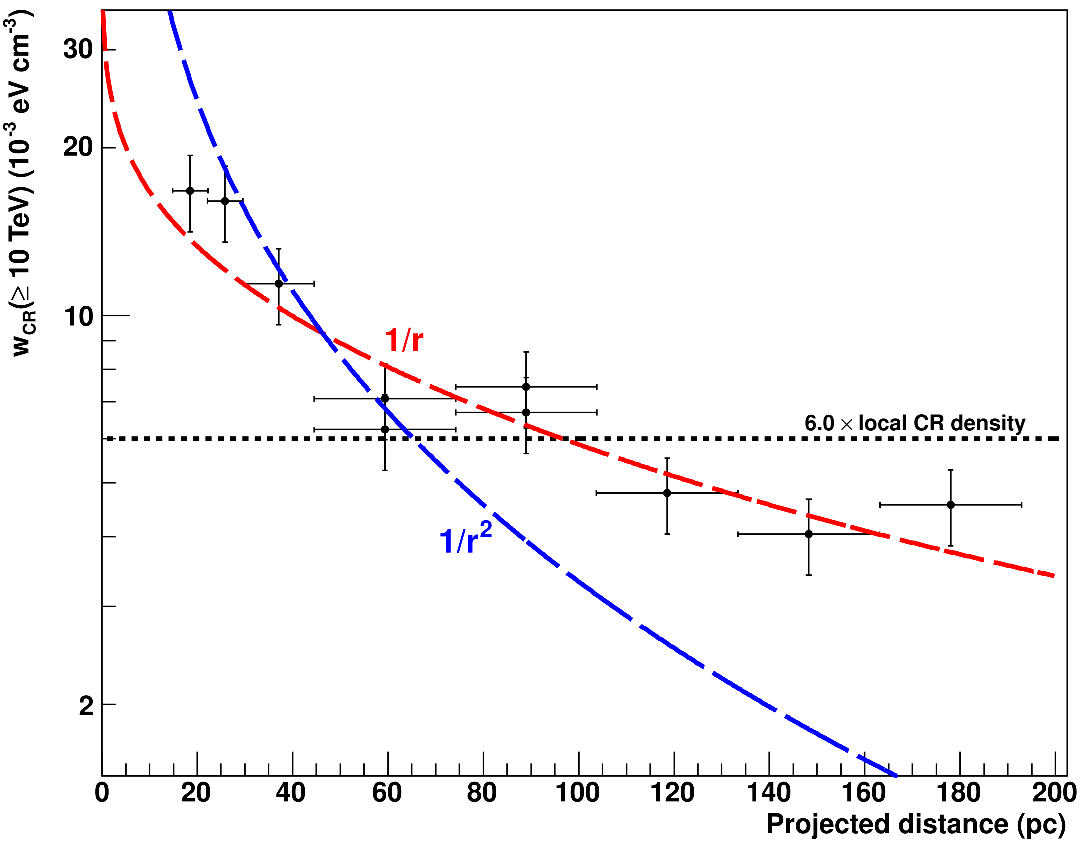

Figure 2: **Spatial distribution of the CR density versus projected distance from Sgr A*.** The vertical and horizontal error bars show the 1σ statistical plus systematical errors and the bin size, respectively. A fit to the data of a $1/r$ (red line, χ^2/d.o.f. = 11.8/9), $1/r^2$ (blue line, χ^2/d.o.f. = 73.2/9) and an homogeneous (black line, χ^2/d.o.f. = 61.2/9) CR density radial profiles integrated along the line of sight are shown. The best fit of a $1/r^\alpha$ profile to the data is found for $\alpha = 1.10 \pm 0.12$ (1σ).The $1/r$ radial profile is clearly preferred by the H.E.S.S. data.

(iii) What are the modelling assumptions leading to the $1/r^2$ dependence, the blue curve in Figure 2? Hint: You can check the original paper at https://arxiv.org/pdf/1603.07730.pdf and derive the corresponding behaviour using equation (7).

(iv) Cosmic ray pp interactions also lead to charge pions $\pi^+\pi^-$ which subsequently decay into leptons and neutrinos. Assuming the galactic center behaves as a neutrino point source, is it in the reach of the Icecube detector? Hint: You can check the gamma-ray luminosity and spectral shape in the original paper, you can also look for IceCube point source sensitivity in the direction of the Galactic center, use Fig.3 of https://arxiv.org/pdf/1910.08488.pdf

Exercise 7 - Multimessenger

(i) The same source as in Exercise 4 produces PeV gamma rays and neutrinos from pp interactions. Assuming that these particles are produced at the same time, what would be the time delay between a gamma-ray and a neutrino with mass 0.1eV observed at Earth?

(ii) Recently the resonant interaction of a cosmic electron anti-neutrino with an electron has been identified by IceCube. This reaction is know as *Glashow resonance* and implies the production of a W boson. Calculate the required energy of the electron anti-neutrinos to produce W bosons with electrons at rest. Check your computation with the recent IceCube publication (https://www.nature.com/articles/s41586-021-03256-1).