Particle Dark Matter Lecture 4

Axion as Dark Matter

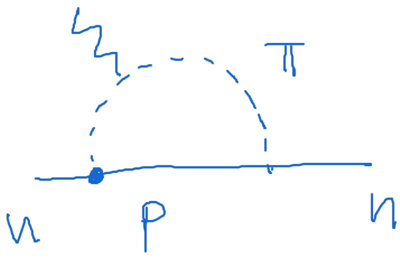
Axion $\Rightarrow$ CP problem in QCD

$$\mathcal{L}_\theta = \frac{g_3^2 \theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

violates T & CP

$$\tilde{G}^a_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^{a\,\rho\sigma}$$

Electric dipole of the neutron



$$d_n \sim e \frac{m_q}{m_n^2} \theta \sim 10^{-16} \theta \; e\,cm$$

$$d_n < 10^{-26} \; e\,cm$$

$$\Rightarrow \boxed{\theta < 10^{-10}}$$

In reality we have also CP violation connected to the quark masses

chiral $U(1)_A$ for quarks

$$\psi \to e^{i\alpha\gamma^5}\psi$$

$$m_q \bar{\psi}\psi \to m_q e^{2i\alpha} \bar{\psi}\psi + h.c.$$

$\Rightarrow$ Physically relevant parameter is a combination of strong & weak phases:

$$\bar{\Theta} = \Theta + \operatorname{Arg}\det(M_q)$$

This quantity has to be $< 10^{-10}$!

Peccei - Quinn solution to the strong CP problem: make $\bar{\theta}$ dynamical!

Add another anomalous global $U(1)_{PQ}$ & break it spontaneously

Goldstone boson φ massless with coupling $\mathcal{L}_{anom} = \xi \frac{\varphi}{f_a} \frac{g_3^2}{32\pi^2} G\tilde{G}$

$$\Rightarrow \mathcal{L}_{\theta + anom} = \frac{g^2}{32\pi^2} G\tilde{G} \left(\bar{\theta} + \xi \frac{\varphi}{f_a}\right)$$

where f_a = PQ scale, scale of $U(1)_{PQ}$ breaking

To cancel $\bar{\theta}$ need to have

$$\langle \varphi \rangle = -\bar{\theta} \frac{f_a}{\xi}$$

Instantons effects generate $\langle G\tilde{G}\rangle \neq 0$ & give a non-trivial potential for the axion, periodic :

$$V_{eff} = \Lambda^4_{QCD} \cos\left(\underbrace{\bar{\Theta} + \frac{3\varphi}{f_a}}_{\varphi/f_a}\right)$$

$$m_a \sim \frac{\Lambda^2_{QCD}}{f_a} \sim 6\,\mu eV \left(\frac{10^{12}\,GeV}{f_a}\right)$$

Model of invisible axion :

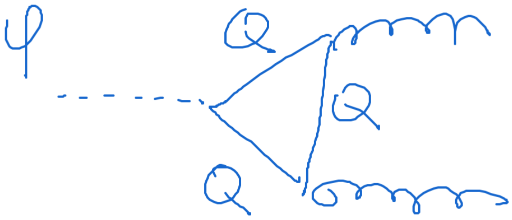
KSVZ (Kim - Shifman - Vainshtein - Zakharov)

heavy vectorial Q_L, Q_R quark

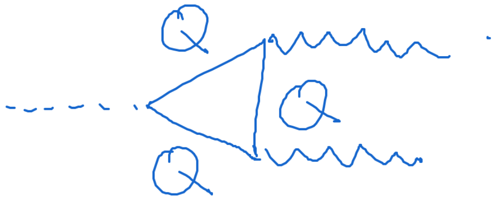
$\mathcal{L} = \lambda S \bar{Q} Q$ S is scalar breaking the PQ symmetry

$\langle S \rangle = f_a$

$$S = \frac{f_a}{\sqrt{2}}\left(1+\frac{\rho}{f_a}\right) e^{i\varphi/f_a}$$



$$\Rightarrow \frac{g^2}{32\pi^2}\frac{\varphi}{f_a} G \tilde{G}$$



$$\Rightarrow \frac{e^2}{32\pi^2}\frac{\varphi}{f_a} F \tilde{F}$$

Axion couplings: (pseudo-Goldstone boson)

→ anomalous couplings to gauge bosons

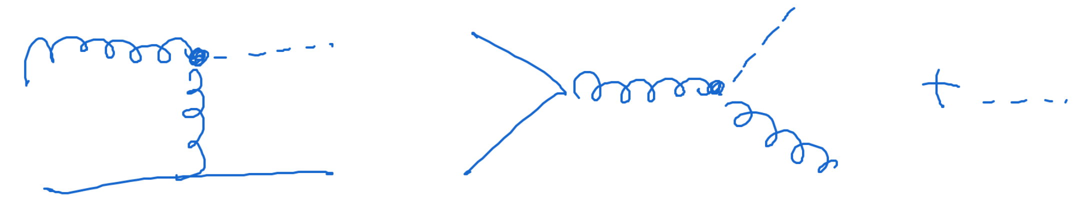
→ derivative couplings to all fields charged under PQ

↳ connected to mass couplings!

Cosmology of the axion

Axion as thermal relics?

Yes, it can be produced by scatterings:



$\Rightarrow$ FIMP mechanism

$$\frac{dn_a}{dt} + 3Hn_a = \Gamma\left(n_a^{eq} - n_a\right)$$

$$\Gamma = \sum_i n_i^{eq} \langle \sigma_i v \rangle \sim \langle \sigma v \rangle \frac{\zeta(3)}{\pi^2} T^3$$

$$\langle \sigma_i v \rangle \sim \frac{\alpha_s^3}{8\pi^2} \frac{1}{f_a^2} \qquad \alpha_s = \frac{g_3^2}{4\pi}$$

$$\Gamma \sim H(T) = \frac{T}{3}\sqrt{\frac{g_*}{10}}\,\frac{T^2}{\tilde{M}_P}$$

Equilibrium for $T > \frac{8\pi^2}{\alpha_s^3}\frac{f_a^2}{\tilde{M}_P} \dots$

$$\sim 5 \cdot 10^{11}\,\mathrm{GeV}\left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^2$$

$$Y_a \sim \frac{\zeta(3)}{\pi^2}\,\frac{45}{2\pi^2}\,\frac{g}{g_s(T_D)}$$

$$\Rightarrow \Omega_a h^2 \simeq \left(\frac{m_a}{0.2\,\mathrm{eV}}\right)\left(\frac{100}{g_s(T_D)}\right)$$

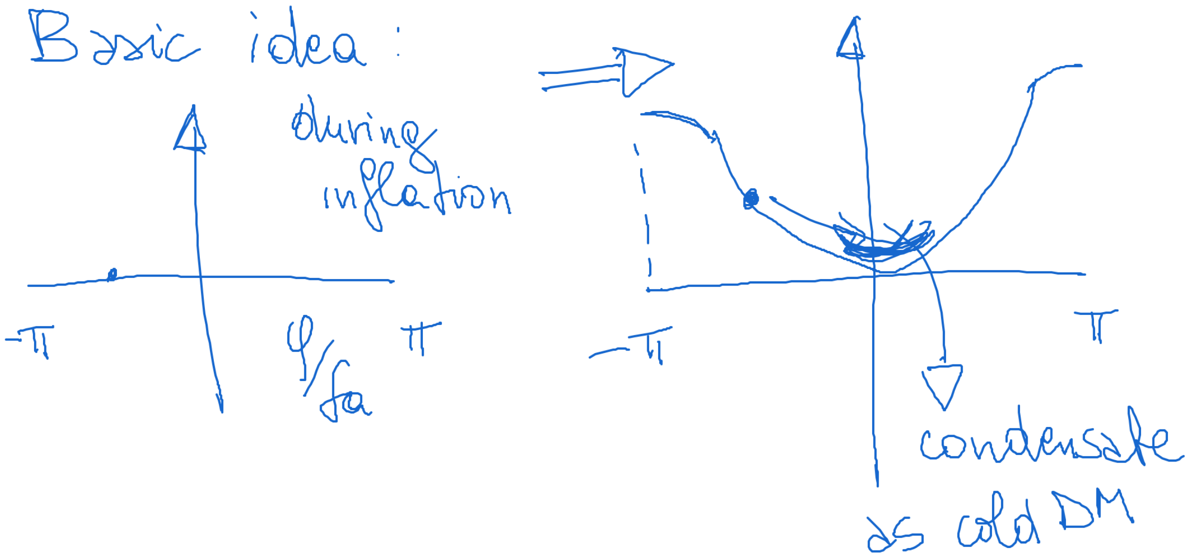
$\Rightarrow$ need a very small mass to match the DM energy density

$\Rightarrow$ HOT Dark Matter

not the correct structure formation history

Misalignment mechanism

Basic idea:



We can consider the field evolution in a FRW metric

$$\Rightarrow \Box\varphi + 3H\dot{\varphi} + V'(\varphi) = 0$$

During inflation: $V' = 0$, $H = H_I \simeq \text{const}$

$$\Rightarrow \ddot{\varphi} + 3H\dot{\varphi} = 0 \Rightarrow \dot{\varphi} = \dot{\varphi}_0\, e^{-3Ht}$$

for homogeneous part $\Rightarrow \varphi(t) = \text{const.}$

Now consider the behaviour after the QCD phase transition : $H \sim$ radiation dominance

for the zero-mode :

$$\ddot{\varphi} + 3H\dot{\varphi} + m_a^2(t)\, f_a \sin\left(\frac{\varphi(t)}{f_a}\right) = 0$$

$$\Rightarrow \ddot{\varphi} + 3H\dot{\varphi} + m_a^2(t)\,\varphi(t) = 0$$

Need the behaviour of $m_a^2(t)$

$$m_a^2(t) = \begin{cases} 0 & T > T_{QCD} \sim 1\,\mathrm{GeV} \\ 4 \cdot 10^{-9}\,\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)\left(\frac{\mathrm{GeV}}{T}\right)^4 & \\ m_a^2 = \frac{\Lambda_{QCD}^4}{f_a} & \end{cases}$$

$$\varphi(t) = a^{-3/2}(t)\,\chi(t)$$

$$\dot{\varphi}(t) = -\frac{3}{2}H(t)\,a^{-3/2}\chi(t) + a^{-3/2}(t)\,\dot{\chi}(t)$$

$$\ddot{\varphi}(t) = \frac{9}{4}H^2 a^{-3/2}\chi(t) - \frac{3}{2}\dot{H}\,a^{-3/2}\chi(t)$$

$$-3H(t)\,a^{-3/2}\dot{\chi}(t) + a^{-3/2}(t)\,\ddot{\chi}(t)$$

$$\ddot{\chi} + \omega^2(t)\chi = 0$$

$$\omega^2(t) = m_a^2(t) + \frac{9}{4}H^2 - \frac{3}{2}\dot{H}$$

$$\chi(t) \approx \frac{C}{\sqrt{m_a(t)}}\cos\left(\int^t dt'\,\omega(t')\right)$$

$$\varphi(t) = \frac{C}{\sqrt{m_a(t)}}\,a^{-3/2}\cos\left(\int^t dt'\,\omega(t')\right)$$

$$\rho_a = \frac{1}{2} m_a^2 \varphi^2(t) + \frac{1}{2} \dot{\varphi}^2 = C^2 m_a^2 a^{-3}$$

$\Rightarrow$ the energy density scales as a^{-3}

$$n_a \sim \frac{\rho_a}{m_a} \sim \frac{1}{2} m_a(t_1)\, \varphi^2(t_1) \Big|_{\substack{\text{at } t_1 \\ m_a \sim H}}$$

$\varphi^2(t_1) = \theta_i^2 f_a^2$ & number of particles is conserved!

$$\Omega_a h^2 = n_a(t_1) \left(\frac{a(t_1)}{a(t_{now})}\right)^3 m_a \frac{1}{\rho_c/h^2}$$

$$= \frac{1}{2} m_a(t_1)\, \theta_i^2 f_a^2 \left(\frac{a(t_1)}{a(t_{now})}\right)^2 \frac{m_a}{\rho_c/h^2}$$

$\Rightarrow t_1 = 2 \cdot 10^{-7}\,\text{sec} \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{1/3}$ $\quad m_a = H(t_1)$

$$\Omega_a h^2 \simeq 0.15\, \theta_i^2 \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6}$$