Particle Dark Matter Lecture 3

Useful relations:

$$\Omega = \frac{\rho}{\rho_c} \qquad \rho_c = \frac{3H^2}{8\pi G_N} = 11{,}23\, h^2\, m_p/m^3$$

here $H_0 = h\ 100\, \mathrm{km/s/Mpc}$

m_p = proton mass ~ 1 GeV

$h \sim 0.67 - 0.72$

$$\Omega h^2 = \frac{\rho h^2}{\rho_c} = \frac{\rho}{11{,}23\, m_p/m^3}$$

independent of h!

Unitarity bound: $\sigma_{non\text{-}el,J} = \dfrac{4\pi(2J+1)}{(2s_p+1)^2}\dfrac{(1-\eta_J^2)}{p_i^2}$

i.e. maximal σ for $J = s_p = 0$

$$\sigma^{max}_{non\text{-}el} = \frac{4\pi}{p_i^2} = \frac{4\pi}{m_{DM}^2 v^2} \quad \text{i.e. } \sigma^{max} v \sim \frac{4\pi}{m_{DM}^2 v}$$

giving $\langle \sigma^{max} v\rangle_x \sim \dfrac{4\pi}{m_{DM}^2}\left\langle \dfrac{1}{v}\right\rangle_x \sim \dfrac{4\pi}{m_{DM}^2}\dfrac{x^{1/2}}{\sqrt{6}}$

So we have

$$\Omega_{DM} h^2 = \frac{m_{DM} Y_{DM} s(T_\gamma)}{\rho_c/h^2} = \frac{m_{DM}}{\rho_c/h^2} \frac{H(m_{DM}) s(T_{now})}{s(m_{DM})} \frac{\sqrt{6}\, m_{DM}^2}{4\pi} \frac{5}{2} x_f^{3/2}$$

$$= 0.04 \times 10^{-10} \left(\frac{m_{DM}}{GeV}\right)^2 x_f^{3/2}$$

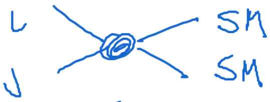
$$\Rightarrow m_{DM} < 10^5 \, GeV \, x_f^{-3/4} \left(\frac{\Omega_{DM} h^2}{0.04}\right)^{1/2}$$

$$\Rightarrow m_{DM} < 12 \, TeV \quad \text{for } x_f \sim 30$$

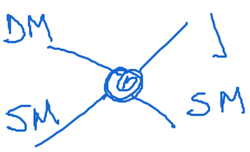
More complex picture:

* coannihilation: particle j decoupling at the same time as DM

Coupled Boltzmann eqs:



$$\frac{dn_i}{dt} + 3Hn_i = -\sum_j \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq})$$



$$-\sum_{j \neq i} \langle \sigma'_{ij} v_{ij} \rangle (n_i n_{SM} - n_i^{eq} n_{SM})$$

+ decays / inverse decays

But consider the sum: $n_J + n_{DM} = n$

$$\Rightarrow \frac{dn}{dt} + 3Hn = -\langle \sigma_{eff} v \rangle \left(n^2 - (n^{eq})^2 \right)$$

where $\langle \sigma_{eff} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \dfrac{n_i^{eq} n_j^{eq}}{(n^{eq})^2}$

assuming conversion in equilibrium

$DM + SM \longleftrightarrow J + SM$