Particle Dark Matter Lecture 2

References:

Books: Kolb & Turner
"The Early Universe"
Addison - Wesley 1990

V. Mukhanov
"Physical Foundations of cosmology" Cambridge Univ. P. 2005

Reviews: Tongyan Lin
TASI Lectures on DM models & DD
arxiv: 1904.07915

Tracy Slatyer
TASI Lectures on ID of DM
arxiv 1710.05137

Summary: DM properties

1. dark: no light emission/absorption
   suppressed coupling to photons
2. decoupled from plasma/non-dissipative
3. non-baryonic: $\Omega_B < \Omega_{DM}$
4. cold enough
5. stable/very long-lived

Thermal big bang & thermal relics

We know: we have CMB with a thermal spectrum $\Rightarrow$ primordial plasma in equilibrium

$\Rightarrow$ key concept: local thermal equilibrium

$$\text{Thermal equilibrium} \begin{cases} \nearrow \text{Kinetic equilibrium (elastic scattering)} \\ \searrow \text{chemical equilibrium (inelastic scattering)} \end{cases}$$

Key quantity:

phase-space distribution $f(\vec{p}, \vec{x}, t)$

homogeneous & isotropic universe

$$\Rightarrow \quad f(\vec{p}, \vec{x}, t) = f(\vec{p}, t) + \delta f(\vec{p}, \vec{x}, t)$$

Then we have $\quad g = \#$ internal d.o.f.

$$n(t) = g \int d^3p \, f(\vec{p}, t)$$

$$\rho(t) = g \int d^3\vec{p}\; E(\vec{p})\, f(\vec{p})$$

$$P(t) = g \int d^3\vec{p}\; \frac{|\vec{p}|^2}{3E} f(\vec{p})$$

Relativistic particle: $E = |\vec{p}| + \frac{1}{2}\frac{m^2}{|\vec{p}|}$

$$P = \frac{1}{3}\rho \qquad w = \frac{1}{3}$$

Non-relativistic: $E = m + \frac{1}{2}\frac{|\vec{p}|^2}{m}$

$$\rho = mn$$

$$P = g \int d^3p\; \frac{|\vec{p}|^2}{3m} f(\vec{p}) \simeq 0$$

$$P = 0 \quad \text{i.e.} \quad w = 0$$

In thermal equilibrium we have

$$f^{eq}_{F/B}(\vec{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1}$$

μ = chemical potential, related to a conserved quantity particle/antiparticle $\pm\mu$

$$n_p - n_{\bar{p}} = \frac{g}{2\pi^2}\int_m^\infty dE\,(E^2-m^2)^{1/2}\,E\left(f^{eq}_p(E) - f^{eq}_{\bar{p}}\right)$$

$$= \frac{g}{2\pi^2}\int_m^\infty dE\,E\,(E^2-m^2)^{1/2}\,\frac{2\sinh\left(\frac{\mu}{T}\right)}{e^{E/T} + e^{-E/T} \pm 2\cosh\left(\frac{\mu}{T}\right)}$$

$$\xrightarrow[T \gg \mu, m]{} \frac{g}{6\pi^2}\,T^3\,\pi^2\,\frac{\mu}{T} + \dots$$

$$\xrightarrow[T \ll \mu, m]{} 2g\left(\frac{mT}{2\pi}\right)^{3/2} \sinh\left(\frac{\mu}{T}\right) e^{-m/T}$$

Density of relativistic spieces for $\mu = 0$

$$\rho_{rad} = \sum_B \frac{\pi^2}{30} g_i T_i^4 + \sum_F \frac{7}{8} \frac{\pi^2}{30} g_i T_i^4$$

$$= \frac{\pi^2}{30} g_* T^4 \quad \Big| \quad g_* = \sum_B g_i \left(\frac{T_i}{T}\right)^4 + g_i \frac{7}{8} \left(\frac{T_i}{T}\right)^4$$

$$g_* = \sum_B g_i \left(\frac{T_i}{T}\right)^4 + \sum_F \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

$$H^2 = \frac{8\pi G_N}{3} \rho = \frac{8\pi G_N}{3} \frac{\pi^2}{30} g_* T^4$$

$$H = \frac{\pi}{3} \left(\frac{g_*}{10}\right)^{1/2} \frac{T^2}{M_p} = \frac{1}{2t}$$

$$t \simeq 1.455\,\mathrm{s}\left(\frac{1\,\mathrm{MeV}}{T}\right)^2 \quad \text{fixed } g_*$$

Adiabatic expansion

$$\Rightarrow \quad S = s\,a^3 = \text{constant}$$

2nd law of thermodynamics

$$T\,dS = dE + p\,dV = d(\rho V) + p\,dV$$

$$= d\left[(\rho + p)V\right] - V\,dp$$

$$dp = \frac{\rho + p}{T}\,dT \quad \Rightarrow \quad \frac{S}{V} = \frac{\rho + p}{T}\cancel{V}$$

$$\Rightarrow \quad s = \frac{\rho + p}{T} = g_{*s} T^3$$

Relativistic: $s = \frac{4}{3}\frac{\rho}{T} \sim T^3$

Non-relativistic: $s = \underbrace{\frac{mn}{T}}_{\text{neglected}} \sim \frac{mg}{T}\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$

Condition for equilibrium?

$$\boxed{\Gamma \gg H} = \frac{\dot{a}}{a}$$

Adiabatic expansion $g_{*s} T^3 a^3 = \text{const}$

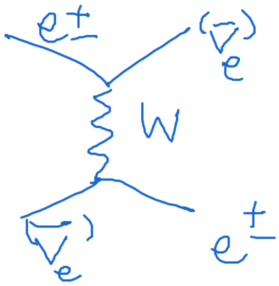
$g_{*s} = \text{const}$ $\quad T \sim \frac{1}{a}$ $\quad \left|\frac{\dot{T}}{T}\right| \simeq \frac{\dot{a}}{a}$

$$\dot{T} = -\frac{1}{a^2}\dot{a} = -T\frac{\dot{a}}{a}$$

Neutrino decoupling:

$$e^{\pm} + \overset{(-)}{\nu}_e \longleftrightarrow e^{\pm} + \overset{(-)}{\nu}_e$$

$$e^{+} + e^{-} \longleftrightarrow \nu_e + \overline{\nu}_e$$



Fermi theory: $\sigma_{e\nu} \sim \frac{\alpha_W^2}{M_Z^4} E_{cms}^2$

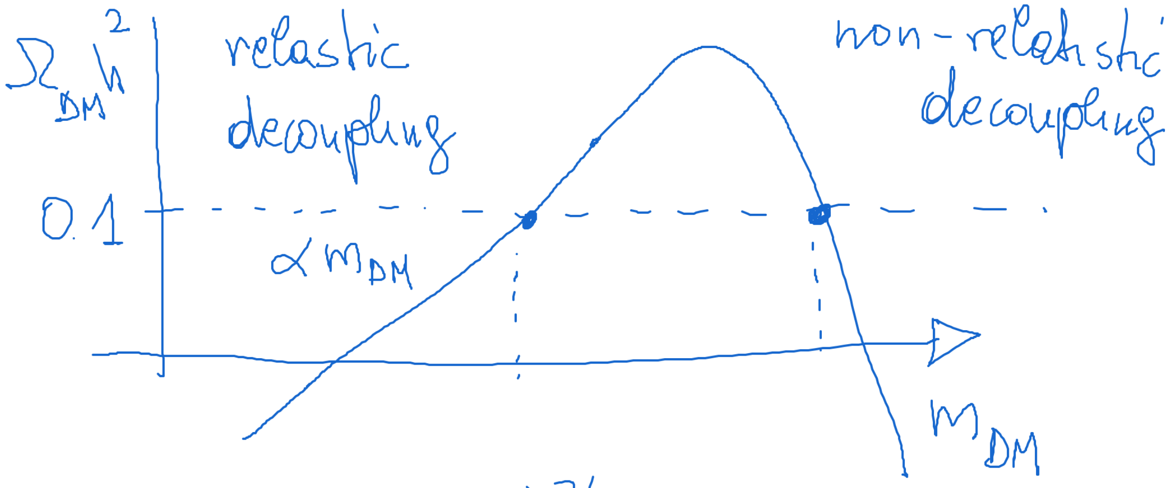
$$\langle \sigma_{e\nu} v \rangle \sim \frac{\alpha_W^2 T^2}{M_Z^4} \qquad \Gamma \sim \langle \sigma_{e\nu} \rangle n_e \propto \frac{\alpha_W^2 T^5}{M_Z^4}$$

$$\Gamma \sim H = \frac{\pi}{3} \sqrt{\frac{g_*}{10}} \frac{T^2}{\tilde{M}_P}$$

$$\frac{\Gamma}{H} = 1 = \frac{\alpha_W^2 T^3 \tilde{M}_P}{M_Z^4} \qquad T_\nu \sim \left(\frac{M_Z^4}{\alpha_W^2 \tilde{M}_P} \right)^{1/3}$$

$$\sim 3 \text{ MeV}$$

$$\Rightarrow \boxed{\Omega_\nu h^2 \sim \frac{\sum m_\nu}{94 \text{eV}}}$$



$$n^{NR} \sim \left(\frac{mT}{2\pi^2}\right)^{3/2} e^{-m/T}$$

Boltzmann equation in FRW

$$\hat{\mathcal{L}}[f] = C[f]$$

Liouville operator　　Collision integral

$$\hat{\mathcal{L}} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$$

$$\Gamma^0_{ii} = -H g_{ii} \qquad \Gamma^i_{i0} = \Gamma^i_{0i} = H$$

$$\hat{\mathcal{L}} = p^0 \partial_0 - \vec{p} \cdot \vec{\nabla} - \Gamma^0_{ii} p^i p^i \frac{\partial}{\partial p^0} - 2 \Gamma^i_{i0} p^0 p^i \frac{\partial}{\partial p^i} =$$

$$= p^0 \partial_0 - \vec{p} \cdot \vec{\nabla} - H \vec{p}^{\,2} \frac{\partial}{\partial p^0} - 2 H p^0 \vec{p} \cdot \frac{\partial}{\partial \vec{p}}$$

Simplify: $\vec{\nabla} f = 0$, & $f(|\vec{p}|, t)$

$$\Rightarrow \hat{\mathcal{L}} = E \frac{\partial}{\partial t} - H \vec{p}^{\,2} \frac{\partial}{\partial E}$$

$$\Rightarrow \frac{\partial f}{\partial t} - \frac{H \vec{p}^{\,2}}{E} \frac{\partial f}{\partial E} = \frac{1}{E} C[f]$$

Rewrite the equation for $n = g\int d^3p\, f(p,t)$

$$g\int d^3p \left[\frac{\partial f}{\partial t} - \frac{H\vec{p}^{\,2}}{E}\frac{\partial f}{\partial E}\right] = g\int d^3p\, \frac{C[f]}{E}$$

$$\Rightarrow \frac{dn}{dt} + 3Hn = g\int d^3p\, \frac{C[f]}{E}$$

$$\frac{dn}{dt} + 3Hn = g\int \frac{d^3p}{E}\, C\left[f(p,t)\right]$$

→ decays: $DM \rightarrow i + j$

(assume DM stable, so neglect)

→ $2 \rightarrow 2$ scatterings

$DM + \overline{DM} \rightarrow i + j$

$$d\Pi = g\frac{d^3p}{(2\pi)^3 2E}$$

$$C[f_{DM}] = -\int d\Pi_{DM}\, d\Pi_{\overline{DM}}\, d\Pi_i\, d\Pi_j\, \delta^4(p_{DM}+p_{\overline{DM}}-p_i-p_j)$$
$$\Big[|\mathcal{M}(DM+\overline{DM}\to ij)|^2 f_{DM} f_{\overline{DM}} (1\pm f_i)(1\pm f_j)$$
$$-|\mathcal{M}(ij\to DM+\overline{DM})|^2 f_i f_j (1\pm f_{DM})(1\pm f_{\overline{DM}})\Big]$$

Simplifications:

1. CP conservation $|\mathcal{M}(DM+\overline{DM}\to ij)|^2 =$
$$= |\mathcal{M}(ij\to DM+\overline{DM})|^2 = |\mathcal{M}|^2$$

2. i, j in equilibrium: $f_i f_j = f_{DM}^{eq} f_{\overline{DM}}^{eq}$

Maxwell-Boltzmann $f \propto e^{-E/T}$

$$\frac{dn}{dt} + 3Hn = -\int d\Pi_{DM}\, d\Pi_{\overline{DM}} \left(f_{DM} f_{\overline{DM}} - f^{eq}_{DM} f^{eq}_{\overline{DM}} \right)$$

$$\times \underbrace{\int d\Pi_i\, d\Pi_j\, |M|^2\, \delta(\ldots)}_{\sigma(DM\,\overline{DM} \to ij)\, v}$$

$$\frac{dn_{DM}}{dt} + 3Hn_{DM} = -\langle \sigma(DM + \overline{DM} \to ij) v \rangle \times$$

$$\times \left(n_{DM} n_{\overline{DM}} - n^{eq}_{DM} n^{eq}_{\overline{DM}} \right)$$