

Particle Dark Matter Lecture 1

Dark Matter - Plan

1. Introduction to DM & cosmology
- 2-3. WIMP mechanism & pheno
- 4-5 Axion, ALPs as DM / FIMPs

History of DM

1605.04.909

Review by
Bertone & Hooper

- * First evidence from excess velocity in MW
factor 2 discrepancy: Kapteyn, Oort, Jeans..
- * 1933 Fritz Zwicky: galaxies in the Coma cluster

Basic evidence on DM from virial theorem

$$2 \langle T \rangle = - \langle V \rangle$$

$$\langle v^2 \rangle = \frac{3}{5} G_N \frac{M(R)}{R}$$

$$\sigma_N^2 = \frac{1}{3} \langle v^2 \rangle \approx \frac{1}{5} G_N \frac{M(R)}{R}$$

\Rightarrow factor 400 discrepancy!

Nowadays more precise measurements based on the gas in the cluster

Assume hydrostatic equilibrium & spherical symmetry:

$$\frac{dP}{dr} = -\alpha(r) g \quad P \propto g T$$

$$\frac{dP}{dr} = \frac{P}{r} \frac{d \ln P}{d \ln r} = \frac{P}{r} \left(\frac{d \ln g}{d \ln r} + \frac{d \ln T}{d \ln r} \right) =$$

$$\Rightarrow \left(\frac{d\ln \rho}{d\ln r} + \frac{d\ln T}{d\ln r} \right) = - \frac{r}{P} g \alpha(r) =$$

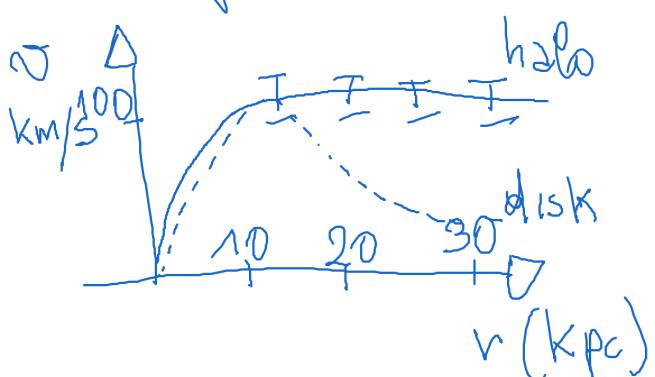
$$= - \frac{r}{T} \left(\frac{\mu m_p}{K} \right) \alpha(r)$$

$$\alpha(r) \approx \frac{G_N M(r)}{r^2} \Rightarrow kT \approx 1.5 \text{ keV} \frac{M}{10^{14} M_\odot}$$

\Rightarrow factor 5 hotter! $\times \left(\frac{1 \text{ Mpc}}{r} \right)$

1970: rotational curves of galaxies
Vera Rubin

\Rightarrow flat curves



Luminous matter
only at small r!

$$\frac{m v^2}{r^2} = G_N \frac{m M(r)}{r^3}$$

need $M(r) \propto r$

$$\Rightarrow M(r) = 4\pi \int_0^r dr' \rho_{DM}(r') (r')^2 \sim r$$

$$\Rightarrow \rho_{DM}(r) \propto \frac{\rho_0}{(r')^2} \text{ approximately}$$

$$\rho_{DM}(r') \sim \frac{\rho_0}{r^\gamma (a + r^\beta)^\alpha} \quad \begin{matrix} \alpha, \beta, \gamma \text{ constant} \\ \text{exponent} \end{matrix}$$

All the evidence so far based on gravitational interaction & it is present at different length scales.

If DM is a particle possibly it has also other interactions than gravity.

Introduction to cosmology & global evidence

Cosmological principle: the universe is homogeneous & isotropic on the large scale

\Rightarrow Friedman-Robertson-Walker-Le Maître metric:

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

Consider conformal time $d\eta = \frac{dt}{a(t)}$

$$ds^2 = a^2(\eta) \left[d\eta^2 - \frac{dr^2}{1-kr^2} - r^2 d\Omega^2 \right]$$

conformal to Minkowski metric for $k=0$

Light rays travel as in Minkowski for conformal $d\eta = \pm \frac{dr}{\sqrt{1-kr^2}}$

The evolution of the scale factor is determined by Einstein's equation:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G_N T^{\mu\nu}$$

\Rightarrow Friedmann's equations $T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu}$

$$\left\{ \begin{array}{l} H^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3} \rho \\ \ddot{a}/a = - \frac{4\pi G_N}{3} (3\rho + p) \end{array} \right. \quad H = \frac{\dot{a}}{a}$$

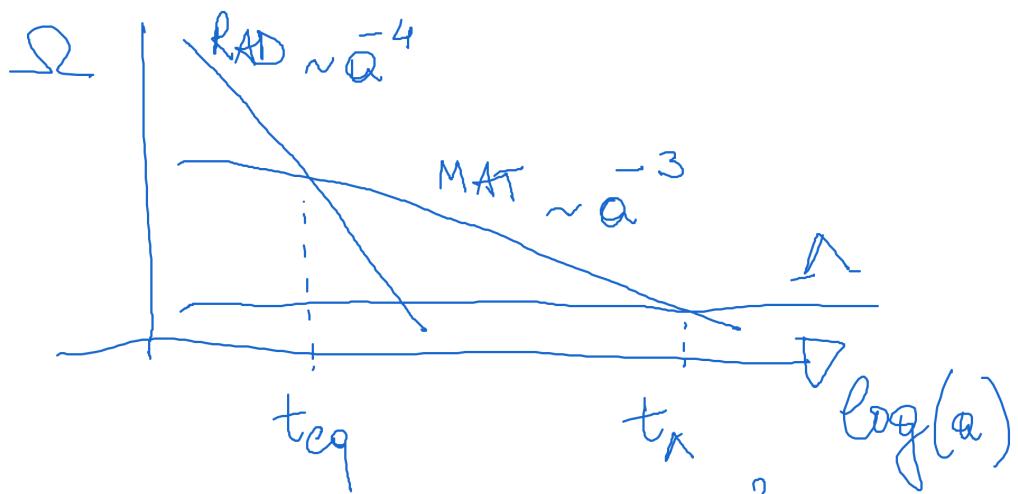
The energy-momentum is covariantly conserved

$$\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} + 3H(\rho + p) = 0$$

Equation of state: $p = w\rho$ $w = \text{const.}$

$$\Rightarrow \dot{\rho} + 3H(1+w)\rho = 0 \Rightarrow \rho \sim a^{-3(1+w)}$$

$$\dot{\rho} + 3(1+w) \frac{\dot{a}}{a} = 0$$



$$\Omega_i = \frac{S_i}{S_0} \quad S_0 = \frac{3H^2}{8\pi G_N}$$

Add cosmological perturbations:

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}(\vec{x}, t) \quad \rho = \rho_0 + \delta\rho$$

FRWL $\ll g_{\mu\nu}^{(0)}$ $C_2^2 = \left(\frac{\partial p}{\partial \rho}\right)_S$

Consider the first order in perturbations

$$\delta = \frac{\delta g}{g_0} \Rightarrow \boxed{\ddot{\delta} + 2H\dot{\delta} - \frac{C_S}{a^2} \vec{\nabla}^2 \delta - 4\pi G_N \rho_0 \delta = 0}$$

Go to Fourier space $\delta(x) = \int \frac{d^3 k}{(2\pi)^3} \delta_{\vec{k}} e^{i\vec{k}\vec{x}}$

$\vec{k} \cdot \vec{x}$ comoving momentum

$$\Rightarrow \ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} + \left(\frac{c_s^2 \vec{k}^2}{a^2} - 4\pi G_N \rho_0 \right) \delta = 0$$

Jean's length: $c_s k_J = a(t) \sqrt{4\pi G_N \rho_0}$

So for $\vec{k}^2 \ll \vec{k}_J^2$ one has a growing & one decreasing solution

$$\delta_{\vec{k}} = A_{\vec{k}} H + B_{\vec{k}} H \int \frac{dt}{a^2 H^2}$$

Matter: $a(t) \sim t^{3/2}$ $aH \sim t^{-1/3}$

$$\delta_{\vec{k}} = A'_{\vec{k}} t^{-1} + B_{\vec{k}} t^{2/3}$$

\Rightarrow Growth goes like $a(t)$ in MD

$$\frac{\delta_k(t)}{\delta_k(t_{eq})} \simeq \frac{a(t)}{a(t_{eq})} \quad t = \text{today} \rightarrow 1 + z_{eq}$$

At $t = t_{\text{CMB}}$ we have $\delta \sim 10^{-4}$

& it has time to grow to $\delta \sim 1$.