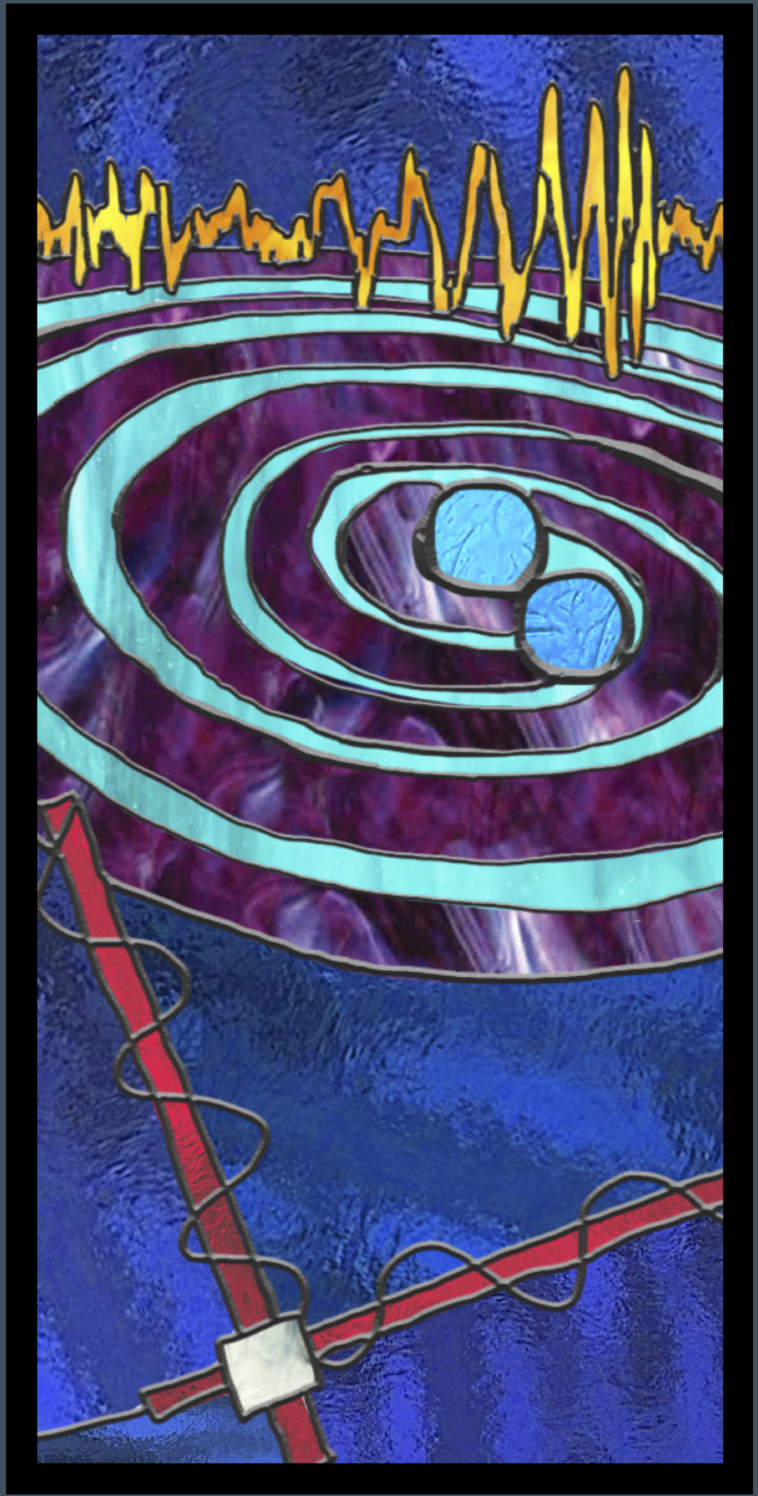

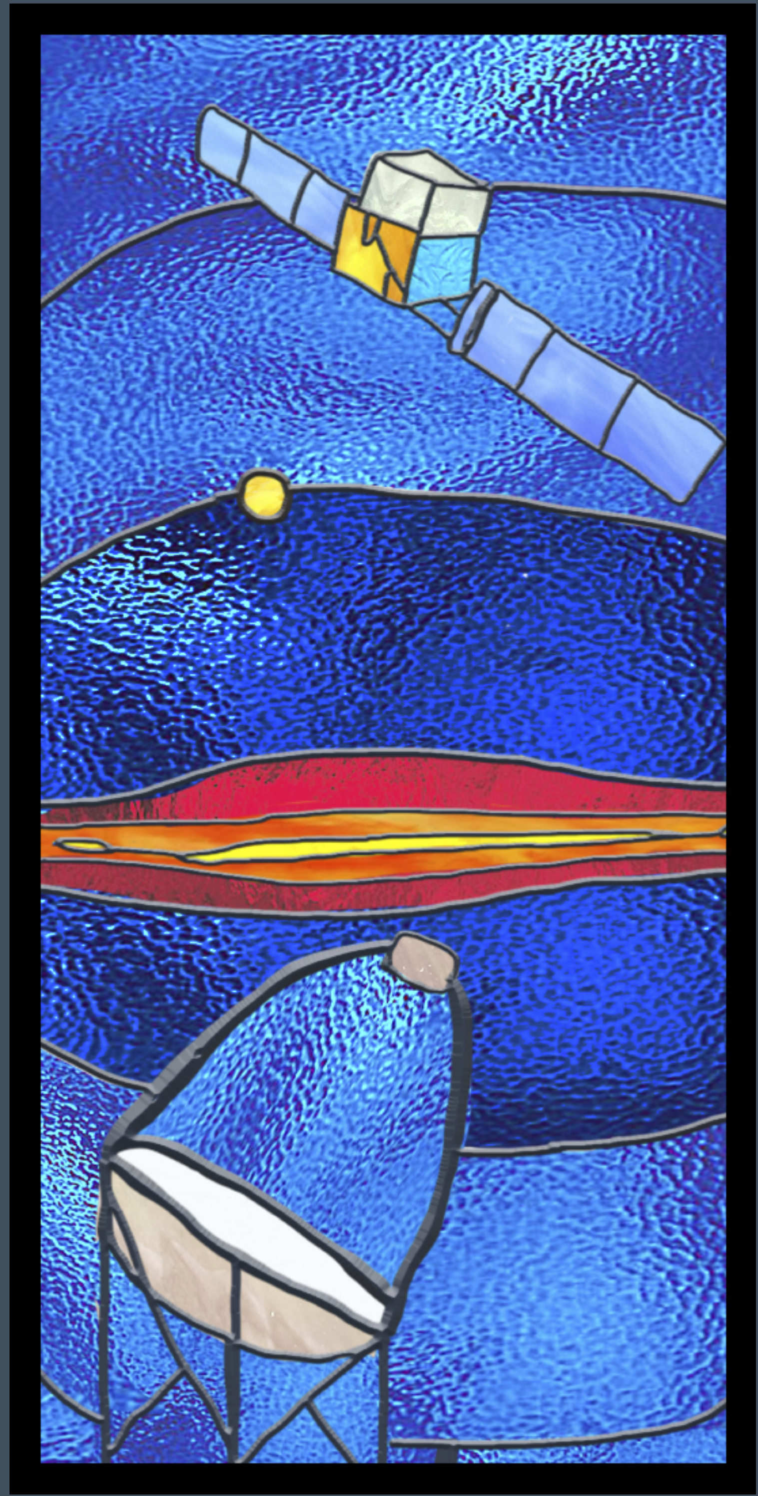

Cosmic Rays

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Phase-Space Density
&
Transport Equations

Liouville theorem : $\frac{d}{dt}(f) = 0$ ← phase-space density

$\Downarrow$

Vlasov equation : $\frac{\partial}{\partial t} f + \frac{p_a}{p_0}\frac{\partial}{\partial r_a} f + F_a \frac{\partial}{\partial p_a} f = 0$ (V.E)

$\Downarrow$

use: $f = \bar{f} + \delta f$ with $\langle f \rangle = \bar{f}$ & $F_a = \bar{F}_a + \delta F_a$ with $\langle F_a \rangle = \bar{F}_a$

ensemble-averaged V.E. : $\frac{\partial}{\partial t}\bar{f} + \frac{p_a}{p_0}\frac{\partial}{\partial r_a}\bar{f} + \bar{F}_a \frac{\partial}{\partial p_a}\bar{f} = \underbrace{-\left\langle \delta F_a \frac{\partial}{\partial p_a} \delta f \right\rangle}_{\left(\frac{\delta f}{\delta t}\right)_{coll}}$

$\langle$V.E.$\rangle$

$\langle \dots \rangle$ averages over magnetic ensemble

V.E. $- \langle$V.E.$\rangle$

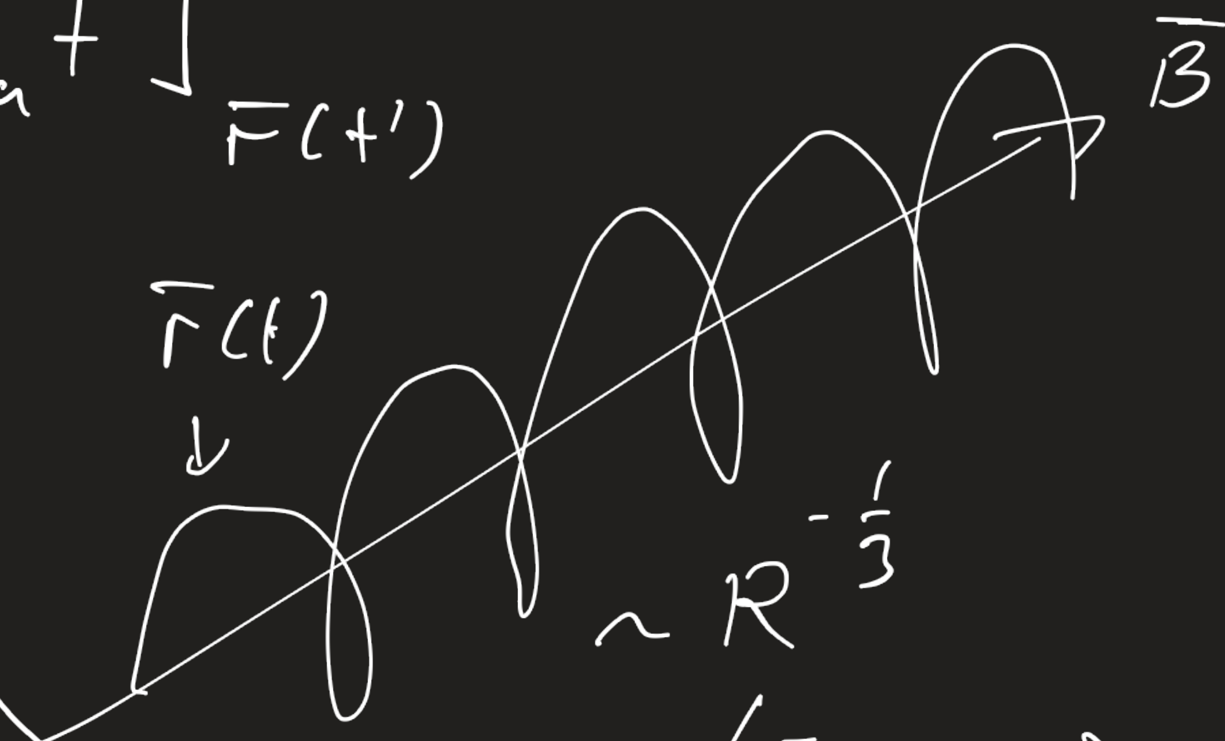
$$\frac{\partial}{\partial t}\delta f + \frac{p_a}{p_0}\frac{\partial}{\partial r_a}\delta f + \bar{F}_a \frac{\partial}{\partial p_a}\delta f \simeq -\delta F_a \frac{\partial}{\partial p_a}\bar{f} + \cancel{\mathcal{O}(\delta F_a, \delta f)}$$

$$= \frac{d}{dt}\delta f(t, \bar{r}(t), \bar{p}(t))$$

$$\delta f(t) = \delta f(-\infty) - \int_{-\infty}^{t} dt' \left[\delta F_a \frac{\partial}{\partial p_a} \bar{f}\right]_{\bar{r}(t')}$$

$$\left(\frac{\delta f}{\delta t}\right)_{coll} = \left\langle -\delta F_a \frac{\partial}{\partial p_a} \delta f \right\rangle$$

$$= \left\langle \delta F_a \frac{\partial}{\partial p_a} \left[\int_{-\infty}^{t} dt' \left[\delta F_b \frac{\partial}{\partial p_b} \bar{f}\right]_{\bar{r}(t')} \right] \right\rangle$$



$$= -\nu\left(\bar{f} - \phi\right)$$

$$\phi = \frac{1}{4\pi}\int d\hat{p}\, \bar{f}$$

Qualitatively:

$$\delta \tilde{B}_a = \frac{1}{(2\pi)^3}\int d^3r\, e^{i\vec{k}\vec{r}}\, \delta B_a(\vec{r})$$

$$\left\langle \delta \tilde{B}_a(\vec{k})\, \delta \tilde{B}_b(\vec{k}') \right\rangle = \delta(\vec{k}-\vec{k}')\, g(k)$$

$$\times \left(\delta_{ab} - \frac{k_a k_b}{k^2} + i\sigma(k)\, \varepsilon_{abc} \frac{k_c}{k}\right)$$

two real-valued functions

$$V_{\delta B} = \frac{\langle \delta B^2 \rangle}{2} = \int dk\, \boxed{k^2 g(k)} \quad k^{-\frac{11}{3}}$$

$$k = |\vec{k}|$$

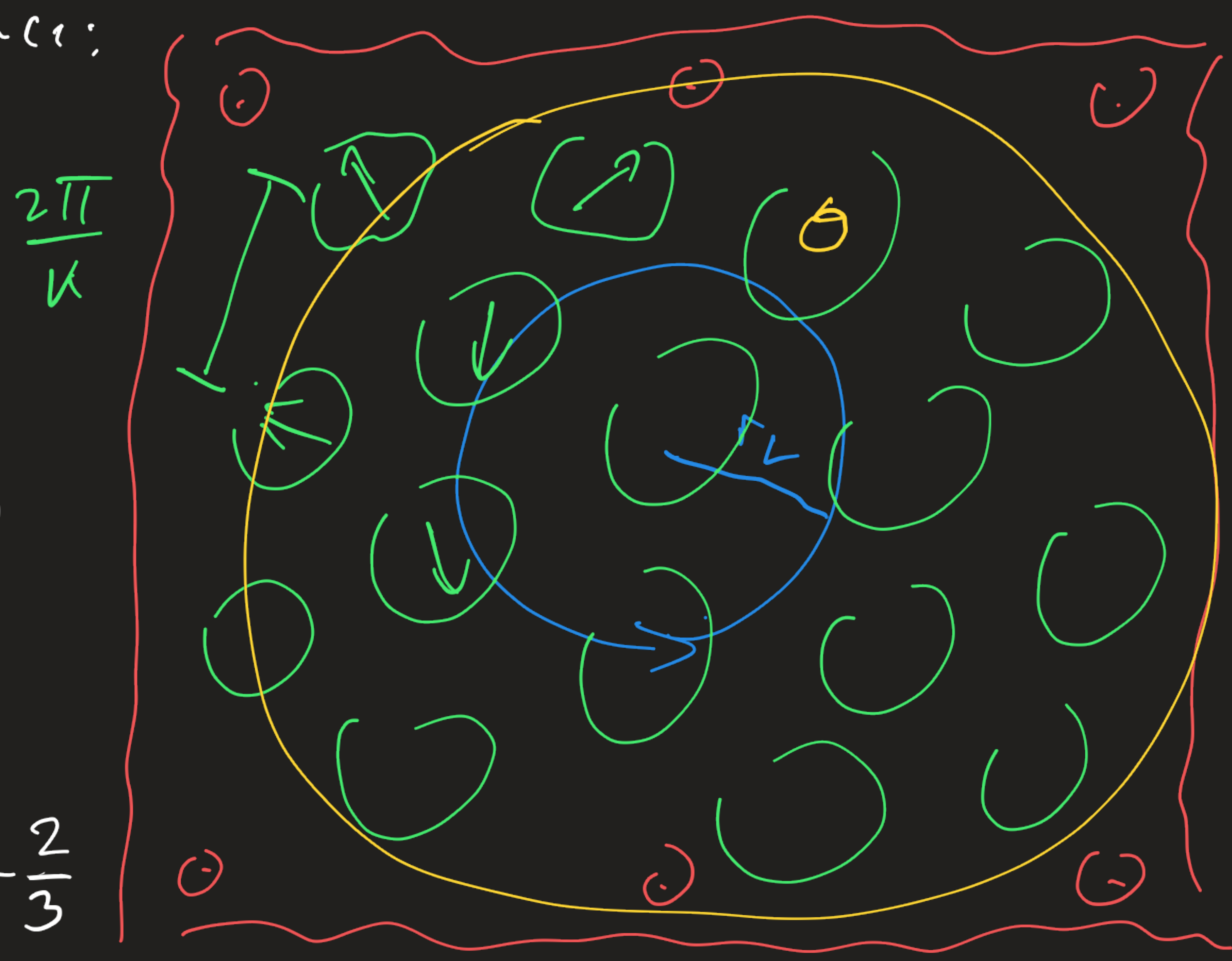
$\Rightarrow$ power of mag. turbulence

$$P(k) = k^2 \cdot g(k)$$

$k_{min} = \frac{2\pi}{r_{max}}$ $\quad k_{res} = \frac{2\pi}{r_L}$ $\quad k_{max} = \frac{2\pi}{r_{min}}$

Kolmogorov turbulence:

$$\alpha = \frac{5}{3}$$



Estimate: $\quad r_L \sim R$

$$k \cdot P(k) \sim k^{-\alpha+1} \sim k^{-\frac{2}{3}}$$

$$\nu \sim \frac{1}{r_L} \cdot \frac{k \cdot P(k)}{U_B} \Rightarrow R^{-\frac{1}{3}}$$

spherical harmonic expansion of $\langle V.E. \rangle$:

(monopole) (dipole)

$$\phi = \frac{1}{4\pi} \int d\hat{p}\, \bar{f} \qquad \Phi_a = \frac{1}{4\pi} \int d\hat{p}\, \hat{p}_a\, \bar{f}$$

$$\left\{ \dot{\vec{p}} = \vec{p} \times \underset{\sim}{\vec{\Omega}} \right.$$

$$(\mathrm{I}) \Rightarrow \partial_t \phi + \beta \frac{\partial}{\partial r_a} \Phi_a = 0$$

$$(\mathrm{II}) \Rightarrow \partial_t \Phi_a + \frac{\beta}{3} \frac{\partial}{\partial r_a} \phi + \varepsilon_{abc} \Omega_b \Phi_c = -\nu \Phi_a$$

Diffusion approximation :

① $\partial_t \Phi_a \simeq 0$

② $(K^{-1})_{ab} = \frac{3}{\beta^2} \left(\nu \delta_{ab} - \varepsilon_{abc} \Omega_c \right)$

inverse of the diffusion tensor K

$$(\mathrm{II}) \Rightarrow \frac{\partial}{\partial r_a} \phi = -(K^{-1})_{ab} \Phi_b \cdot \beta \Rightarrow \boxed{\Phi_a = -\frac{1}{\beta} K_{ab} \frac{\partial}{\partial r_b} \phi}$$

insert back into (1)

$$\Rightarrow \partial_t \phi = \frac{\partial}{\partial r_a}\left(K_{ab} \frac{\partial}{\partial r_b} \phi \right)$$

diffusion equation

diffusion tensor

$$K_{ab} = \frac{\ell^2}{3\nu_{\parallel}} \hat{\Omega}_a \hat{\Omega}_b + \frac{\ell^2}{3\nu_{\perp}} \left(\delta_{ab} - \hat{\Omega}_a \hat{\Omega}_b \right) + \frac{\ell^2}{3\nu_a} \varepsilon_{abc} \hat{\Omega}_c$$

$$\nu_{\parallel} = \nu \qquad \nu_{\parallel} + \frac{\Omega^2}{\nu_{\parallel}} \qquad \Omega + \frac{\nu_{\parallel}^2}{\Omega}$$

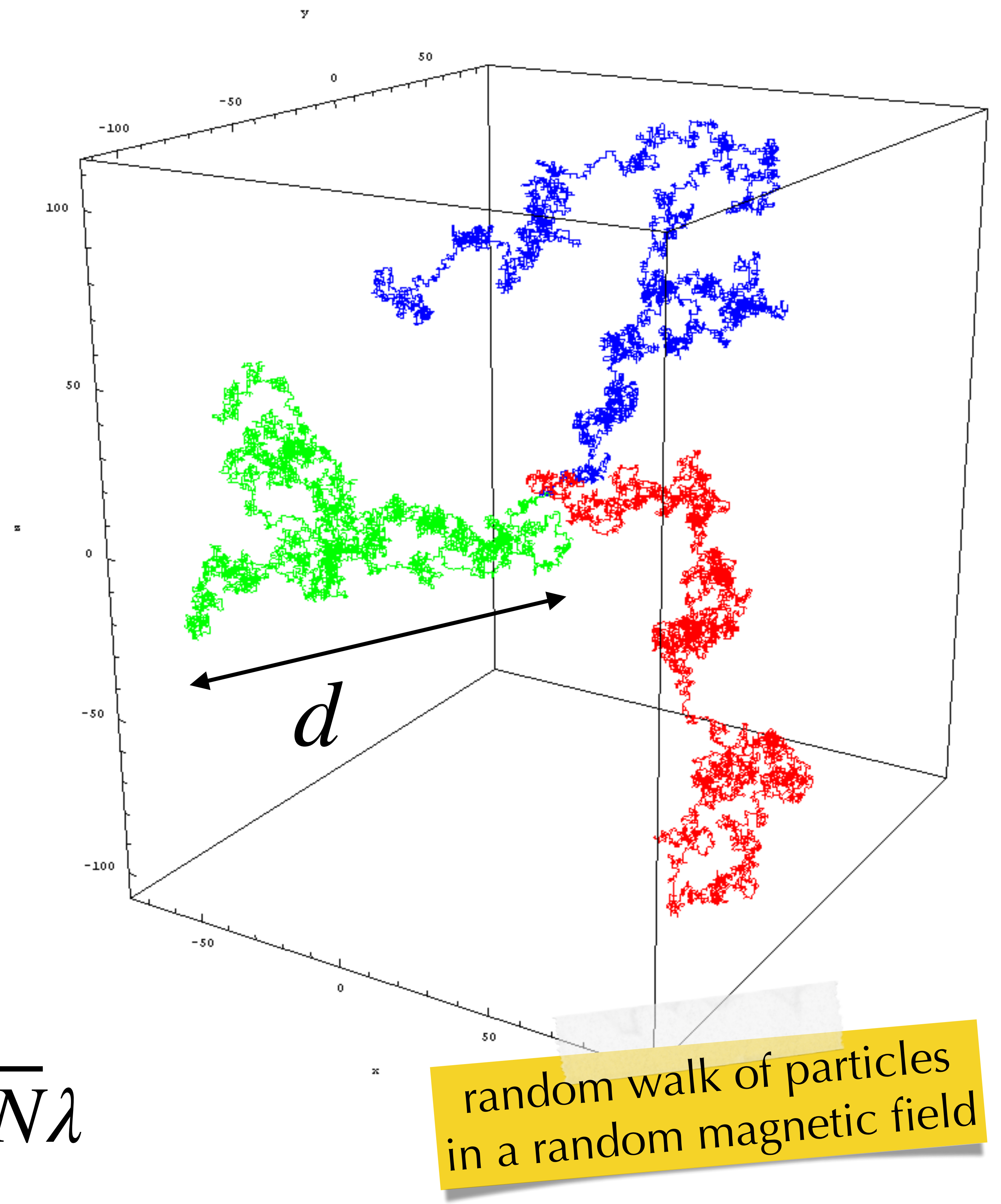
$$\hat{\Omega} = \hat{e}_3$$

$$K = \begin{pmatrix} \frac{\ell^2}{3\nu_{\perp}} & \frac{\ell^2}{3\nu_a} & 0 \\ -\frac{\ell^2}{3\nu_a} & \frac{\ell^2}{3\nu_{\perp}} & 0 \\ 0 & 0 & \frac{\ell^2}{3\nu_{\parallel}} \end{pmatrix}$$

Diffusion, Convection & Anisotropy

Cosmic Ray Diffusion

- Galactic and extragalactic magnetic fields have a random component (no preferred direction).
- Effectively, after some **characteristic distance** λ, a CR will be scattered into a random direction.
- Cosmic ray propagation follows a random walk.
- After N encounters the CR will have travelled an **average distance**: $d = \sqrt{N}\lambda$



diffusion equation: $\partial_t \phi + \vec{\nabla}(\kappa \vec{\nabla} \phi) = Q(t, \vec{r}, p)$

Green's function:

$$G(t, \vec{r}; t_s, \vec{r}_s) = \frac{1}{(4\pi \Delta t)^{3/2}} \frac{q(p)}{\sqrt{\det K}} \exp\left(-\frac{\Delta\vec{r}^T \kappa \Delta\vec{r}}{4\Delta t}\right)$$

$$\Delta\vec{r} = \vec{r} - \vec{r}_s \qquad \Delta t = t - t_s$$

$$\partial_t G + \vec{\nabla}(\kappa \vec{\nabla} G) = \delta(t - t_s)\,\delta(\vec{r} - \vec{r}_s)\,\boxed{q(p)}$$

simple setup: $\vec{r}_s = 0 \quad t_s = 0$ isotropic diffusion.

$$K_{ab} \sim \kappa_{iso} \cdot \delta_{ab}$$

$$\phi(t, \vec{r}, p) = \int d\vec{r}_s \int dt_s \, G(t, \vec{r}; \vec{r}_s, t_s) \qquad \delta(t_s)\,\delta(\vec{r}_s)$$

$$= \frac{1}{(4\pi t \kappa_{iso})^{3/2}} e^{-\frac{r^2}{4t\kappa_{iso}}}$$

Gaussian distribution: $\frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{r^2}{2\sigma^2}}$

$$\Rightarrow \sigma^2 = 2 \cdot \kappa_{iso} \cdot t$$

$$\langle r^2 \rangle = 3 \cdot \sigma^2 = 6 \cdot \kappa_{iso} \cdot t$$

$$\Rightarrow \lambda_{diff} = (6 \cdot \kappa_{iso} \cdot t)^{\frac{1}{2}} \sim \sqrt{N} \cdot \lambda$$

$$\kappa_{iso}(R) = 10^{28} \frac{cm^2}{s} \left(\frac{R}{GV}\right)^{\frac{1}{3}}$$

$$\lambda_{diff} \sim 1\,\mathrm{kpc} \qquad t = \frac{(1\,\mathrm{kpc})^2}{6 \cdot \kappa_{iso}} \sim 5\,\mathrm{Myr}$$

V.E. : $\frac{\partial}{\partial t} f + v_a \frac{\partial}{\partial r_a} f + F_a \frac{\partial}{\partial p_a} = 0$

$\frac{p_a}{p_0}$ $\quad p_0 = m\gamma$

substitution:

$$t \to t^*$$
$$\vec r \to \vec r^*$$
$$\vec v \to \vec v^* + \vec U(\vec r^*)$$

$$f^*(t^*, \vec r^*, \vec p^*) \equiv f(t^*, \vec r^*, \vec p^* - m\gamma \vec U(\vec r^*))$$

V.E. for f^*:

$$\frac{\partial}{\partial t^*} f^* + (v_a^* + U_a)\frac{\partial}{\partial r_a^*} f^* + (v_a^* + U_a)\frac{\partial p_b^*}{\partial r_a^*}\frac{\partial f^*}{\partial p_b^*} + F_a \frac{\partial}{\partial p_c^*} = 0$$

$$\frac{\partial p_b^*}{\partial r_a^*} = -m\gamma \frac{\partial U_b}{\partial r_a^*}$$

$$\underbrace{U_a \frac{\partial}{\partial r_a^*} f}_{\text{convection}} - p_a^* \frac{\partial U_b}{\partial r_a^*}\frac{\partial f^*}{\partial p_b^*} \Rightarrow \text{two new terms}$$

$$(*) \Rightarrow \quad v_a \frac{\partial}{\partial r_a^*}\phi^* - \underbrace{\frac{1}{4\pi}\int d\hat{p}\,\hat{p}_a\hat{p}_b}_{\frac{1}{3}\delta_{ab}} \frac{\partial v_b}{\partial r_a^*}\frac{1}{p^*}\frac{\partial \phi^*}{\partial p^*}$$

$$\partial_{t^*}\phi^* + \underbrace{v_a \frac{\partial}{\partial r_a^*}\phi^*}_{\text{convection}} - \underbrace{\frac{1}{3}(\vec{\nabla}\vec{v})\boxed{p^*\frac{\partial \phi^*}{\partial p^*}}}_{\text{adiabatic energy loss}} - \underbrace{\frac{\partial}{\partial r_a^*}\left(\kappa_{ab}\frac{\partial}{\partial r_b}\phi^*\right)}_{\text{diffusion}} = 0$$

spectral density:

$$n = \frac{4\pi}{\beta}\cdot p^2 \cdot \phi$$

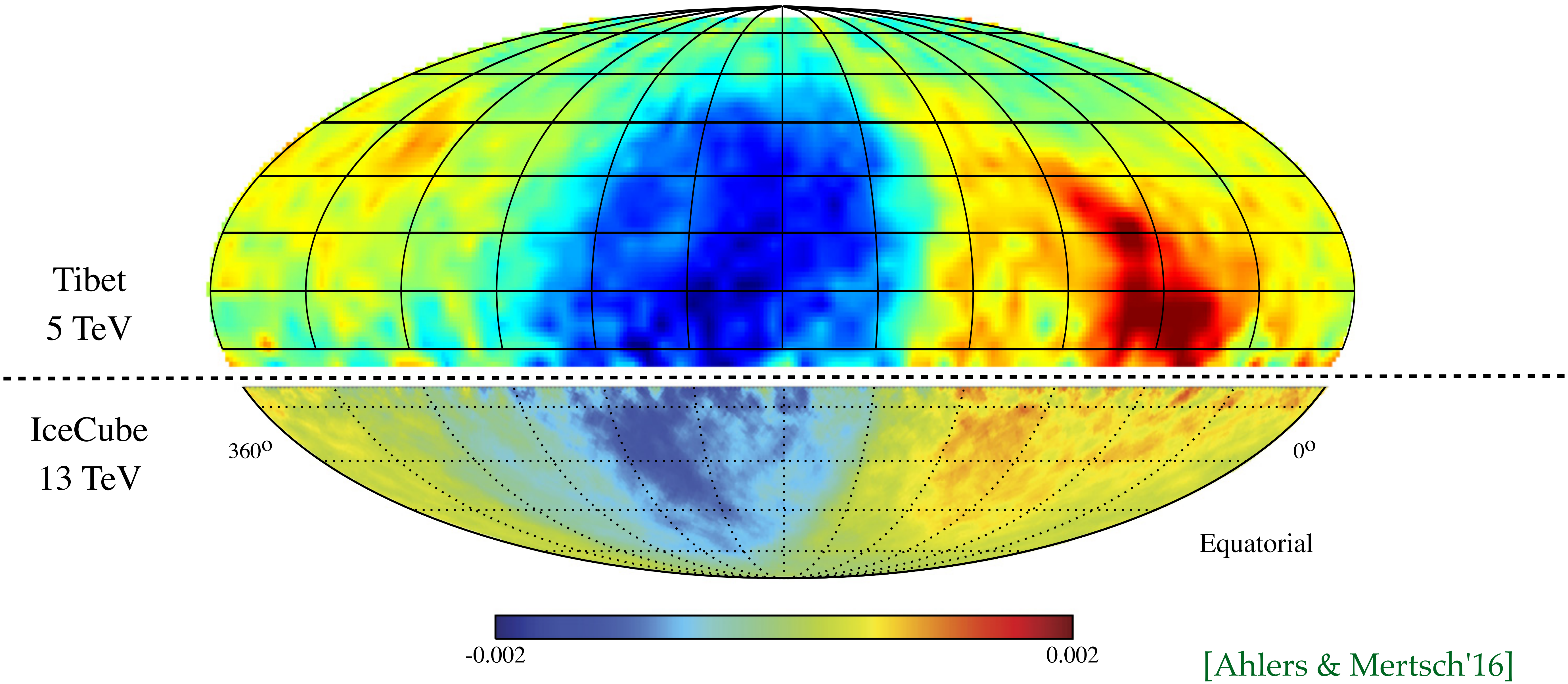
$$p\frac{\partial \psi}{\partial p} = \frac{1}{p^2}\frac{\beta}{4\pi}\left[-3n + \frac{\partial}{\partial p}(pn)\right]$$

$$\frac{\beta}{4\pi}\frac{1}{p^2}\left[\underbrace{u_a\frac{\partial}{\partial r_a}n + \vec{\nabla}\vec{U}n}_{\vec{\nabla}(\vec{U}\cdot n)} - \frac{1}{3}(\vec{\nabla}\vec{U})\frac{\partial}{\partial p}(pn)\right]$$

<u>final</u> <u>expression</u>:

$$\partial_t n + \vec{\nabla}(\vec{U}n) - \frac{1}{3}(\vec{\nabla}U)\frac{\partial}{\partial p}(p\cdot n) - \frac{\partial}{\partial r_a}\left(K_{ab}\frac{\partial}{\partial r_b}n\right) = 0$$

Cosmic Ray Anisotropy

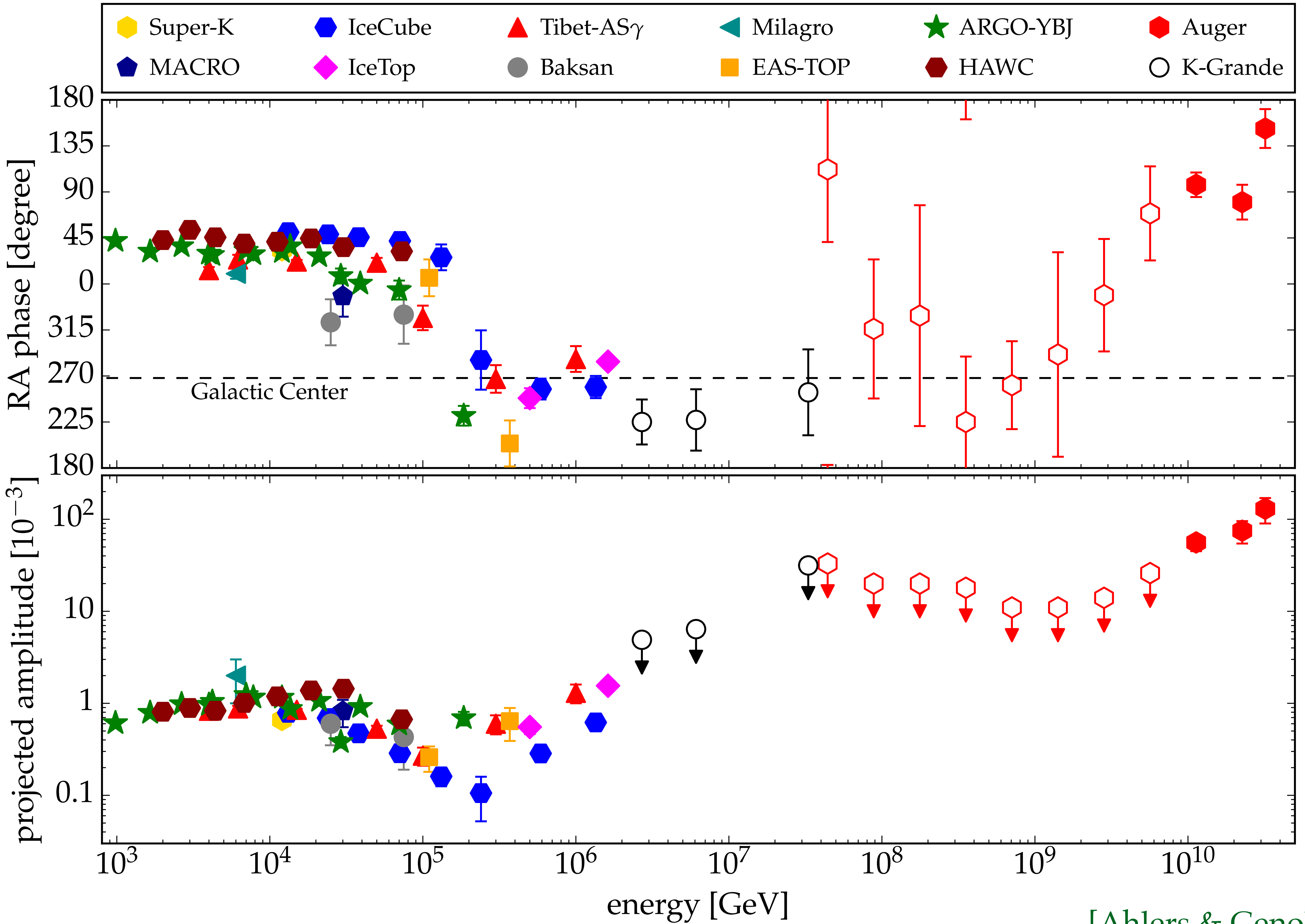


[Ahlers & Mertsch'16]

Cosmic ray anisotropies up to the level of one-per-mille are observed at various energies.

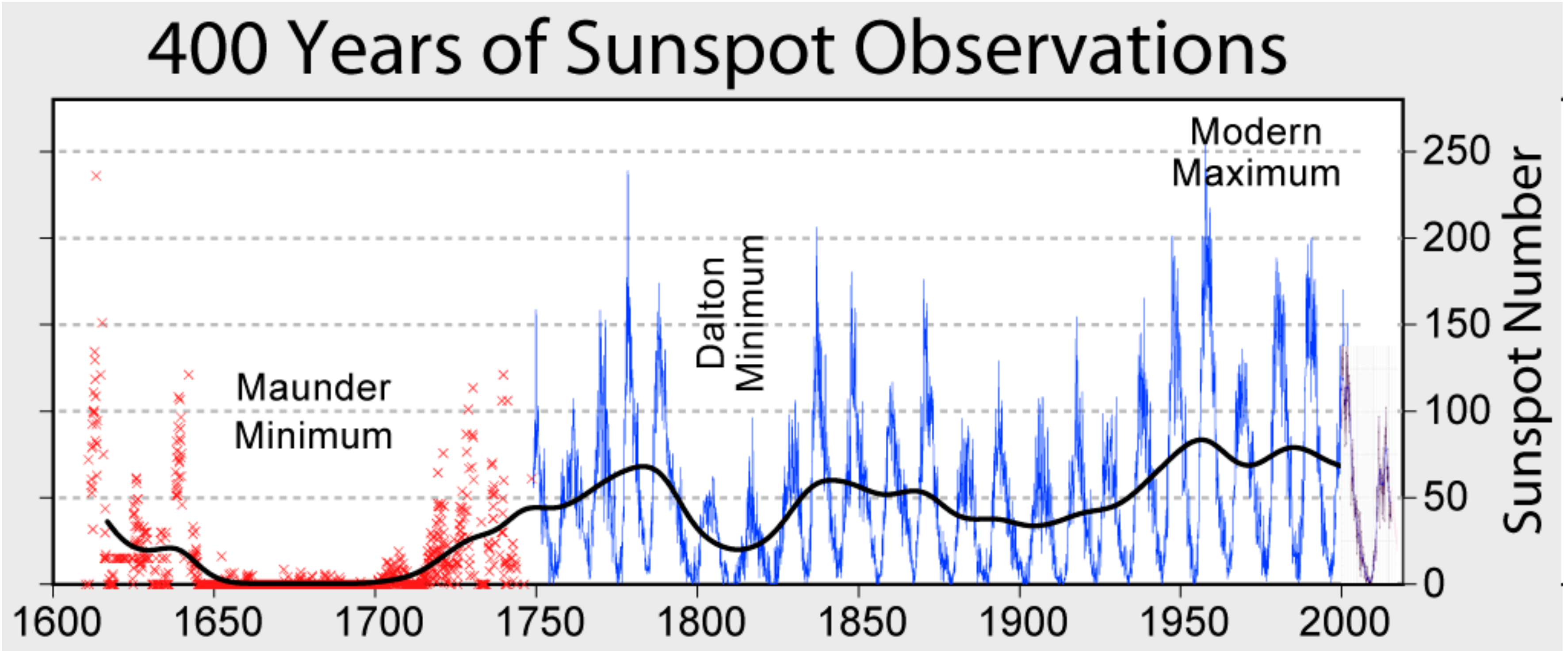
(Super-Kamikande, Milargo, ARGO-YBJ, EAS-TOP, Tibet AS-γ, Icecube, HAWC)

Dipole Anisotropy



[Ahlers & Genolini'21]

Solar Magnetic Field



solar maximum with sunspots and flares (outflow)

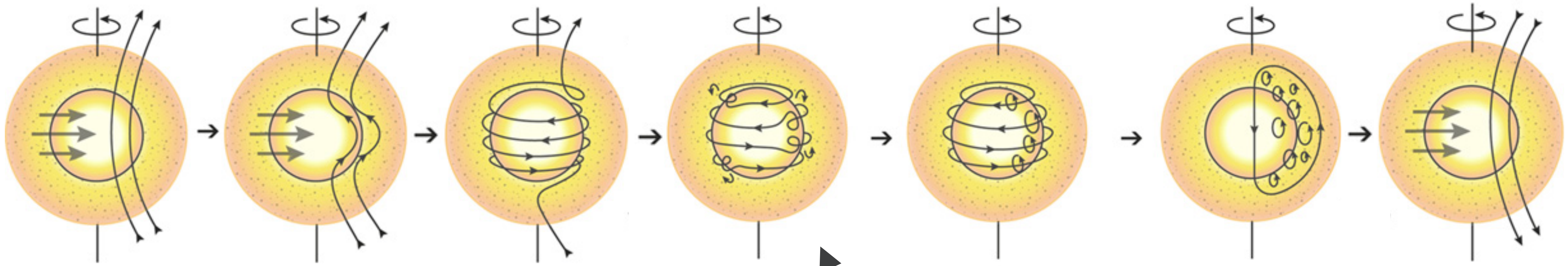
Solar cycle with period ~22year

solar minimum

Ω-effect

α-effect

next solar minimum ~11 years



poloidal

toroidal

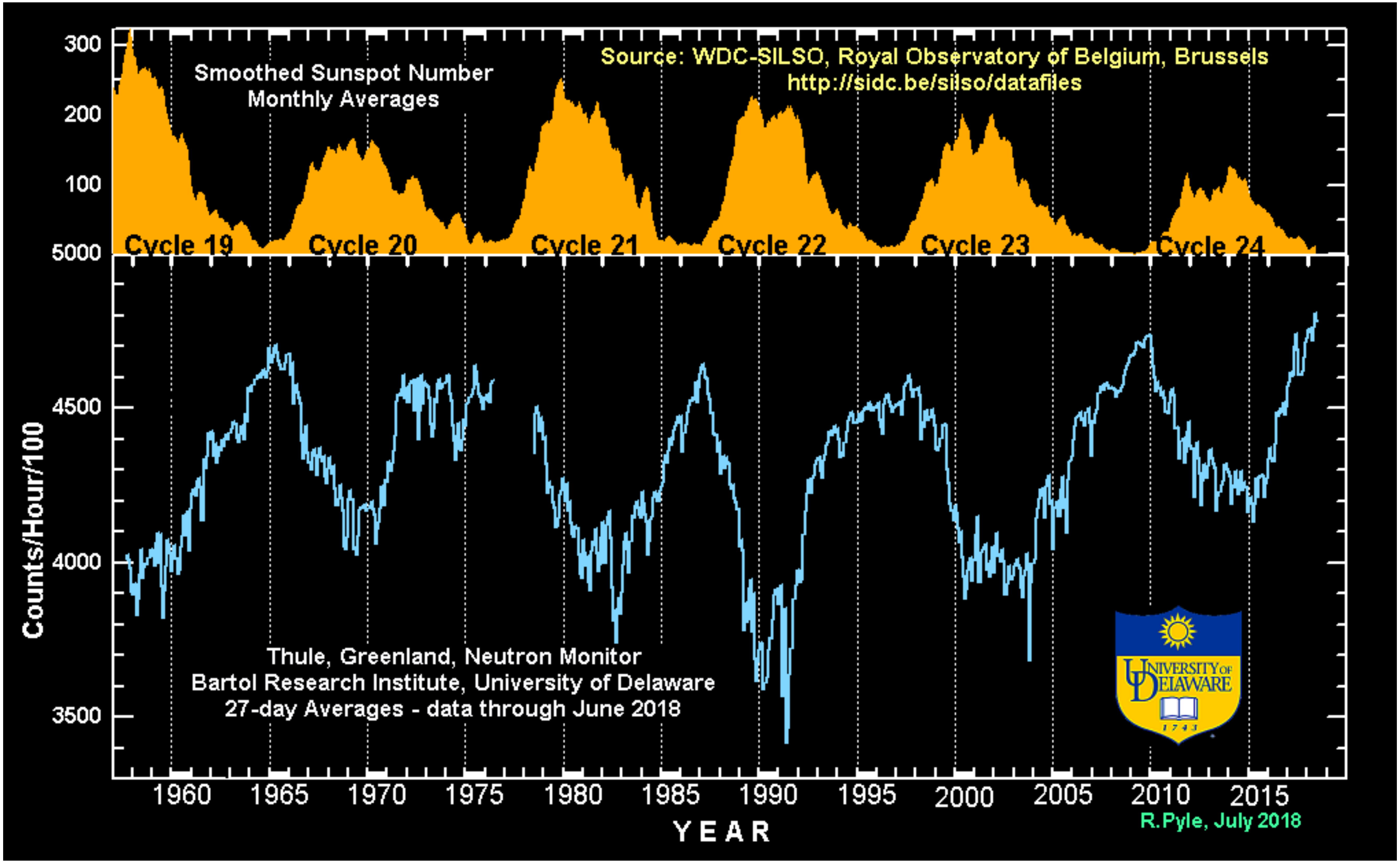
coriolis force twists field lines

field reversal

poloidal

[Sanchez, Fournier & Aubert, ApJ 2013]

Solar Cycle



Smoothed Sunspot Number
Monthly Averages
Source: WDC-SILSO, Royal Observatory of Belgium, Brussels
http://sidc.be/silso/datafiles
300
200
100
5000
Cycle 19
Cycle 20
Cycle 21
Cycle 22
Cycle 23
Cycle 24
4500
4000
3500
Counts/Hour/100
Thule, Greenland, Neutron Monitor
Bartol Research Institute, University of Delaware
27-day Averages - data through June 2018
University of Delaware
1743
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
YEAR
R.Pyle, July 2018

Solar Potential

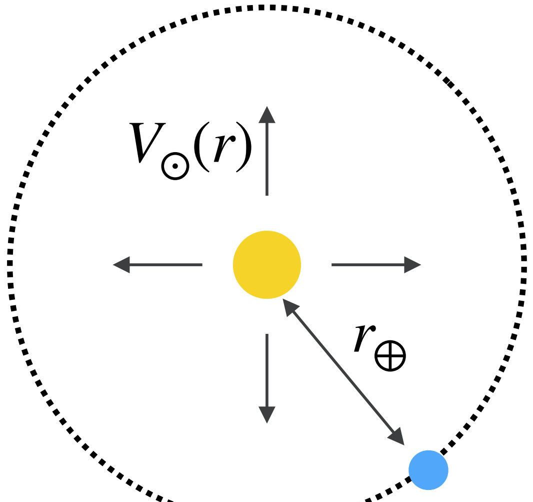
- CR phase-space diffusion f in spherically symmetric solar wind (see exercise):

$$\frac{V_\odot(r)}{3}\frac{\partial}{\partial p\partial r}\left(p^3 f\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \underbrace{\left(\frac{p^3 V_\odot(r)}{3}\frac{\partial f}{\partial p} + p^2 K_\odot(r,p)\frac{\partial f}{\partial r}\right)}_{\text{radial current density } S_r}\right]$$

➜ **force-field approximation**: $S_r \simeq 0$ [Gleeson & Axford'68;Gleeson & Urch'73]

- local solution ($r_\oplus = 1\mathrm{AU}$) related to distribution beyond heliosphere:

$$f(r_\oplus, p(r_\oplus)) = \lim_{R\to\infty} f(R, p(R))$$



- $p(r)$ solution of **characteristic equation**:

$$\frac{\partial p}{\partial r} = \frac{V_\odot(r)}{3}\frac{p}{K_\odot(r,p)}$$

➜ assume **Bohm diffusion** in heliosphere: $K_\odot(r,p) \simeq K_\odot(r,p_0)(p/p_0)$

$$cp(r_\oplus) = cp(\infty) - |Z|e\mathcal{V}_\odot \qquad \text{with} \qquad e\mathcal{V}_\odot = \underbrace{\frac{cp_0}{3}\int_{r_\oplus}^{\infty} \mathrm{d}r' \frac{V_\odot(r')}{K_\odot(r',p_0)}}_{\text{effective "solar potential"}} \lesssim 1\,\mathrm{GeV}$$

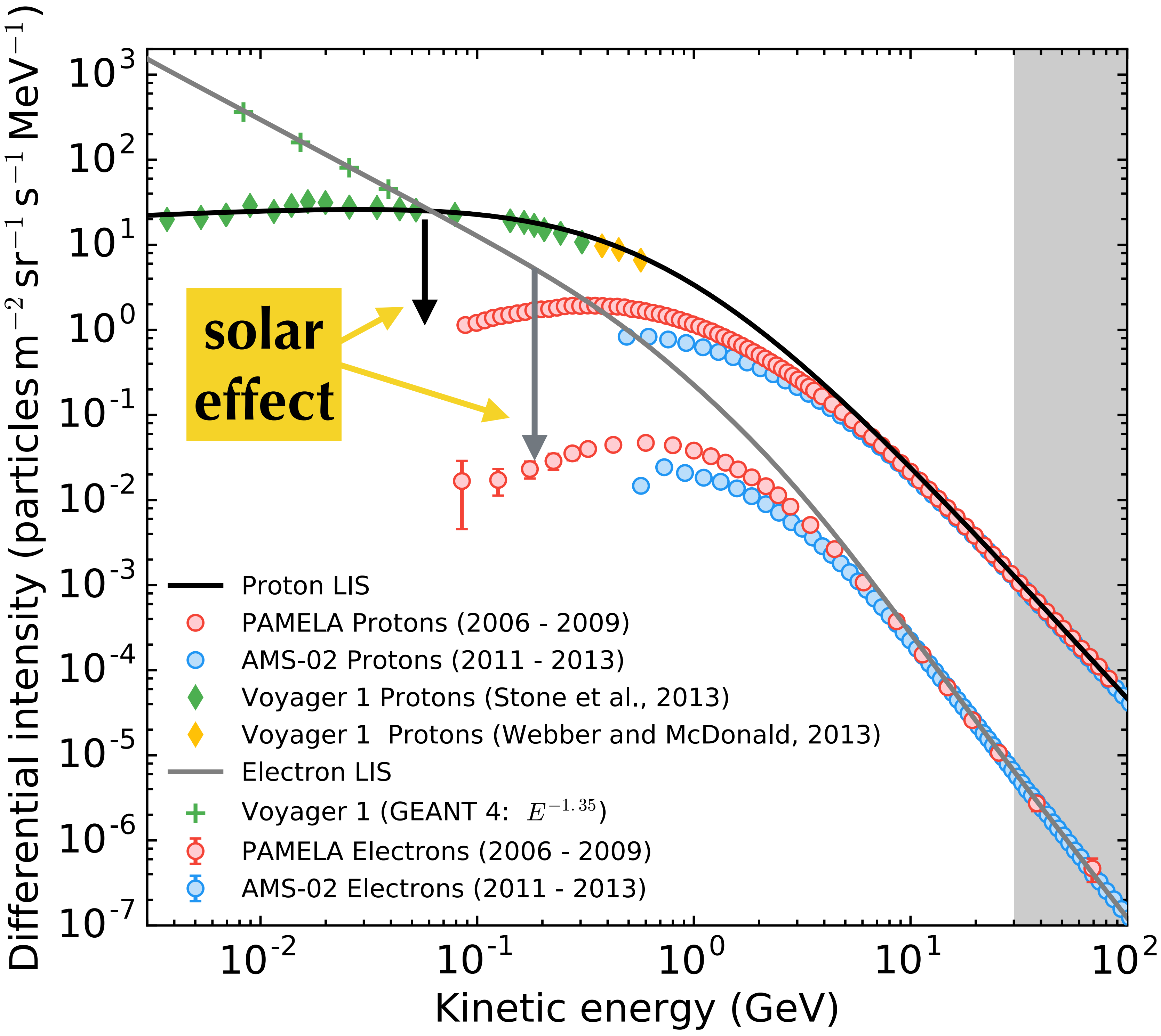
Solar Modulation

- Voyager satellite observes proton & electron spectra in local interstellar medium (LIS): **no solar effect**

PAMELA 2006-2009
solar minimum

AMS-02 2011-2013
solar maximum

- Effect can be treated via a *force field approximation* corresponding to a **solar potential** (*see exercise*).



[Potgieter & Vos, A&A 2017]