The Cabibbo Angle: Past, Present and Future

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- The Present: the Precision Era and its implications
- The Future: what lies beyond?
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Introduction

- Having worked on flavour physics since I graduated, I can certainly say the Cabibbo angle was the foundation of almost all my activity
- Yet, having graduated 30 years after the angle was formulated, I somewhat took it for granted, so let me quickly go through its genesis with Cabibbo's slides

Photo by M. Bona

The Past: the Puppi Triangle

Universality of Weak Interactions

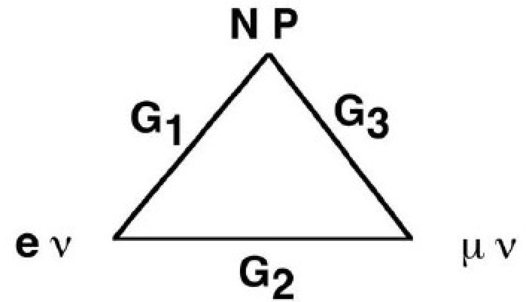
1 - $N \to P + e + \bar{\nu}$ $\quad G_1$

2 - $\mu \to e + \nu + \bar{\nu}$ $\quad G_2$

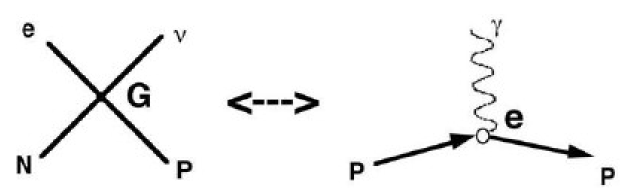
3 - $\mu^- + P \to N + \nu$ $\quad G_3$

In ~ 1950 $\quad G_1 \approx G_2 \approx G_3$

The Puppi Triangle



Suggestive in view of Fermi's idea:



N. Cabibbo

Angle

Charged current hadronic matrix element fixed by isospin symmetry

The Past: beyond the Puppi Triangle

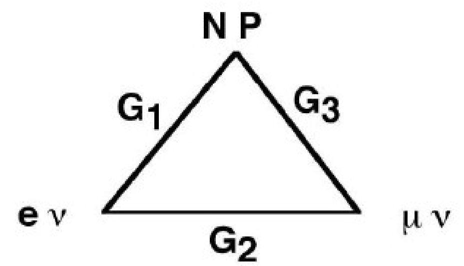
Universality of Weak Interactions 1962-63

$G_1 \approx G_2 \approx G_3$

Suggestive, but true?

$G_{\text{beta decay}} \approx 0.96\ G_{\mu\ \text{decay}}$

(Significative Difference)



And for strange particle decays....

4 - $\Lambda \rightarrow P + e + \bar{\nu}$ $\quad G_4$

$G_4 \approx 0.2\ G_{\text{m decay}}$

Does universality fail?

(*) *Note added in proof.* – Should this discrepancy be real, it would probably indicate a total or partial failure of the conserved vector current idea. It might also mean, however, that the current is conserved but with $G/G_\mu < 1$. Such a situation is consistent with universality if we consider the vector current for $\Delta S = 0$ and $\Delta S = 1$ together to be something like:

$$GV_\alpha + GV_\alpha^{(\Delta S=1)} = G_\mu \bar{p}\gamma_\alpha(n + \varepsilon\Lambda)(1+\varepsilon^2)^{-\frac{1}{2}} + \dots,$$

and likewise for the axial vector current. If $(1+\varepsilon^2)^{-\frac{1}{2}} = 0.97$, then $\varepsilon^2 = .06$, which is of the right order of magnitude for explaining the low rate of β decay of the Λ particle. There is, of course, a renormalization factor for that decay, so we cannot be sure that the low rate really fits in with such a picture.

Gell-Mann & Lévy '60

Universality of Weak Interactions 1962-63

Towards a solution:

1) Gell-Mann's SU(3) symmetry and its application to weak transitions. (N.C. + R. Gatto 1962)

2) High statistics (for that time) bubble chamber experiments. (V. Soergel, Filthut, P. Franzini, G. Snow, etc.)

	N		P	
Σ^-		Σ^0, Λ		Σ^+
	Ξ^-		Ξ^0	

Universality restored

5

Universality and weak mixing

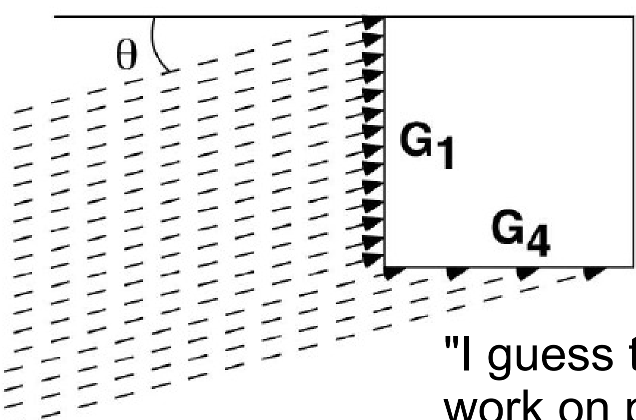
$N \to P + e^- + \nu$ $\quad G_1 \approx 0.96\, G_{\mu\text{-decay}}$

$\Lambda \to P + e^- + \nu$ $\quad G_4 \approx 0.2\, G_{\mu\text{-decay}}$

Broken Universality?
no, shared intensity

$G_1 = \cos\theta\, G_{\mu\text{-decay}}$

$G_4 = \sin\theta\, G_{\mu\text{-decay}}$



$\theta \approx 0.2$ (today 0.221)

Profound intuition:
strong interaction isospin → SU(3), BUT weak interactions still organized in weak isospin doublets!

"I guess there was in my mind a sort of mental interference between my work on photons and crystals, which had to do with polarization, and my work on hyperon decays. It was a kind of cross fertilization."

Fermi news '99

From the Cabibbo angle to CKM

- Then came the GIM mechanism, paving the road to a two-generation gauge model of weak interactions with a scale of few hundred GeV, and then to the Standard Model.
- The 2x2 mixing matrix parameterized by the Cabibbo angle was promoted to the CKM matrix, parameterized by three angles and one CP-violating phase

Photo by M. Bona

The CKM Matrix

- In the three-generation SM, generation (flavour) mixing happens in charged weak currents through the Cabibbo-Kobayashi-Maskawa matrix:

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

- Both flavour mixing and CP violation in weak interactions are ruled by the CKM matrix:
  - Strong correlations between FV and CPV observables
  - Very sensitive to New Physics

CKM à la Wolfenstein

- The CKM matrix has a hierarchical structure, with the Cabibbo angle as the basic building block ($\sin\theta_{12} \equiv \lambda$):

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Can be made unitary to arbitrary order in λ
- Hierarchical structure in "powers of Cabibbo"

The origin of the Cabibbo angle

- Phenomenologically, one has

  $\tan^2 \theta_c \sim m_d/m_s$

  suggesting a dynamical origin of the Cabibbo angle, since it can be obtained from a Yukawa structure of the form

$$m_s \begin{pmatrix} 0 & \lambda \\ -\lambda & 1 \end{pmatrix} \Rightarrow m_d \sim \lambda^2 m_s$$

  possibly originating from an $O(\lambda)$ breaking of a flavour symmetry

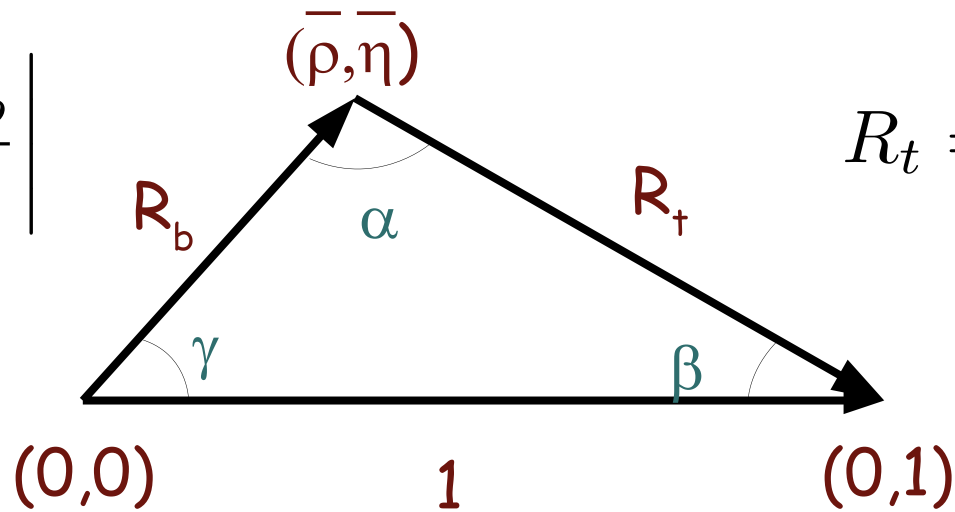
- Many very interesting steps in this direction (U(1), U(2), discrete flavour symmetries), no compelling flavour theory yet
- Understanding the origin of the Cabibbo angle becomes crucial to build phenomenologically acceptable NP models close to the EW scale

Unitarity Triangle(s)

- CKM Unitarity ⇔ Triangular relations a.k.a. Unitarity Triangles:

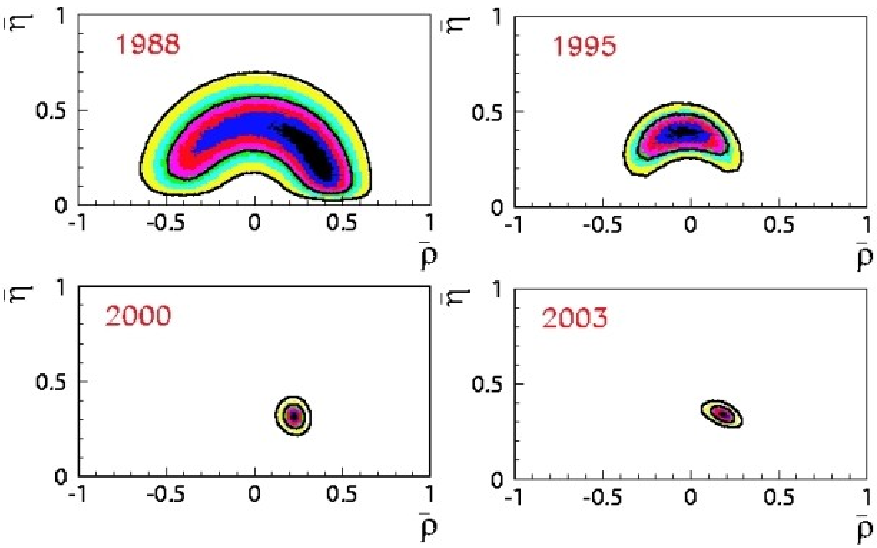
$$\left(VV^\dagger\right)_{i,j\neq i} = \sum_k V_{ik}V^*_{jk} = 0$$

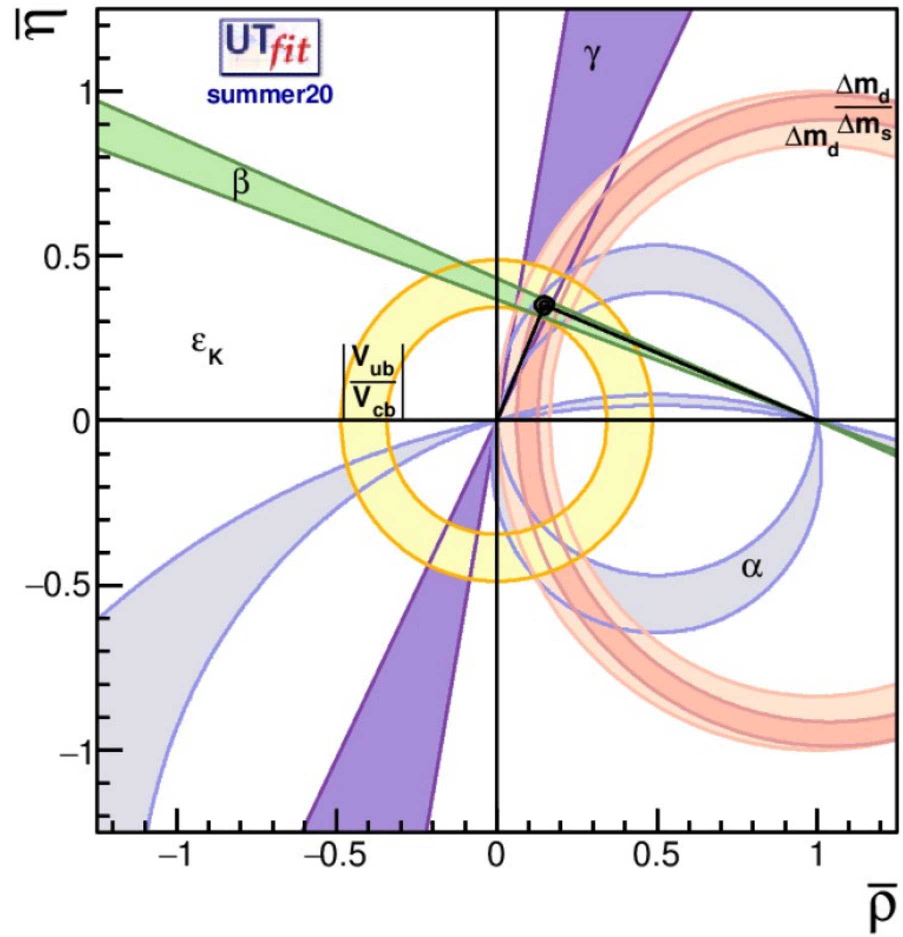
$$R_b = \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| \qquad R_t = \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right|$$



$$\alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) \quad \beta = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) \quad \gamma = \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right)$$

From the past to the present



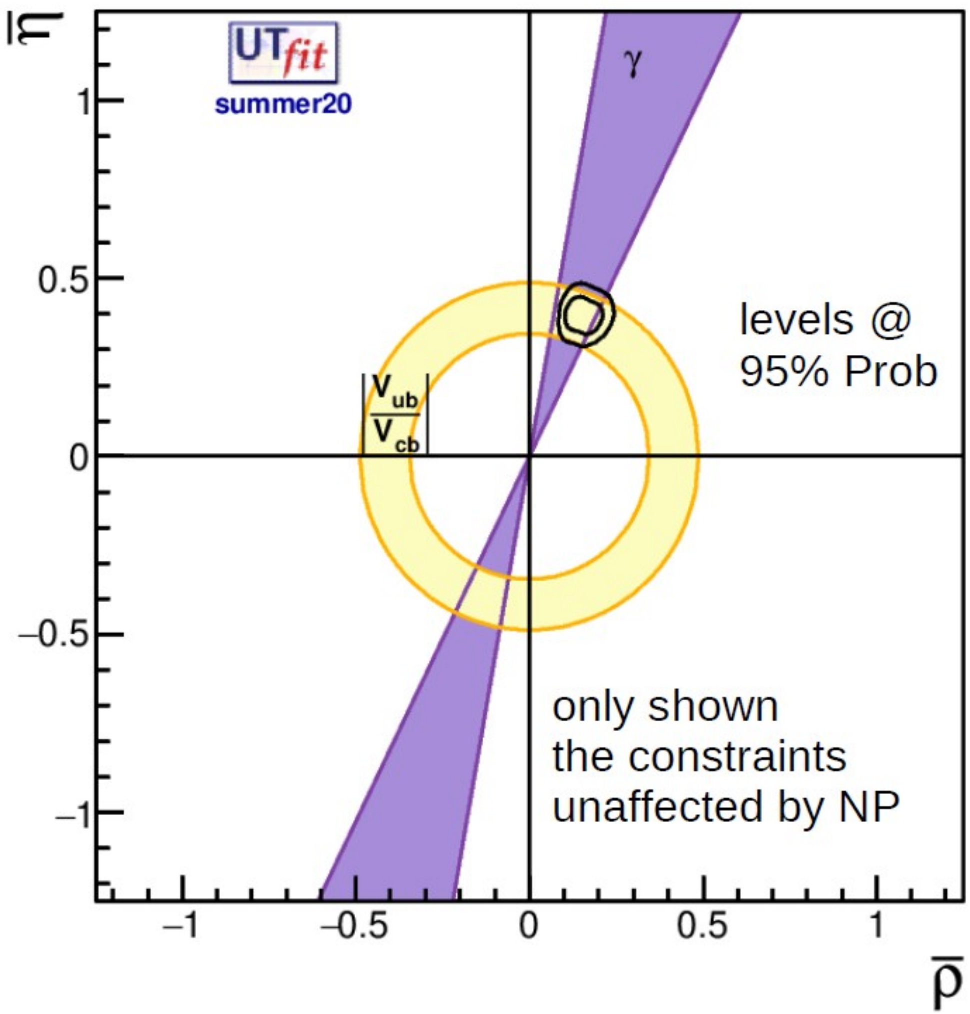


Impressive progress due to NLO calculations, LQCD calculations, HQET and of course experimental improvements. Rome strongly involved in all that.

The Quest for New Physics

- The UT is overconstrained: generalize to NP
- Working hypothesis: neglect NP contributions to tree-level decays, search for NP in loop-mediated processes
- Derive constraints on NP contributions to meson-antimeson mixing
- Translate into bounds on the NP scale for a given NP coupling and flavour structure

NP analysis results



$\bar{\rho} = 0.158 \pm 0.037$

$\bar{\eta} = 0.397 \pm 0.037$

SM is

$\bar{\rho} = 0.149 \pm 0.013$

$\bar{\eta} = 0.350 \pm 0.013$

testing the new-physics scale

M. Bona *et al.* (UTfit)
JHEP 0803:049,2008
arXiv:0707.0636

R G E

At the high scale

new physics enters according to its specific features

At the low scale

use OPE to write the most general effective Hamiltonian.
the operators have different chiralities than the SM
NP effects are in the Wilson Coefficients C

$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq}$$

$$\begin{aligned}
Q_1^{q_i q_j} &= \bar{q}_{jL}^{\alpha}\gamma_\mu q_{iL}^{\alpha}\bar{q}_{jL}^{\beta}\gamma^\mu q_{iL}^{\beta} , \\
Q_2^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha}\bar{q}_{jR}^{\beta} q_{iL}^{\beta} , \\
Q_3^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta}\bar{q}_{jR}^{\beta} q_{iL}^{\alpha} , \\
Q_4^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha}\bar{q}_{jL}^{\beta} q_{iR}^{\beta} , \\
Q_5^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta}\bar{q}_{jL}^{\beta} q_{iR}^{\alpha} .
\end{aligned}$$

$$C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$$

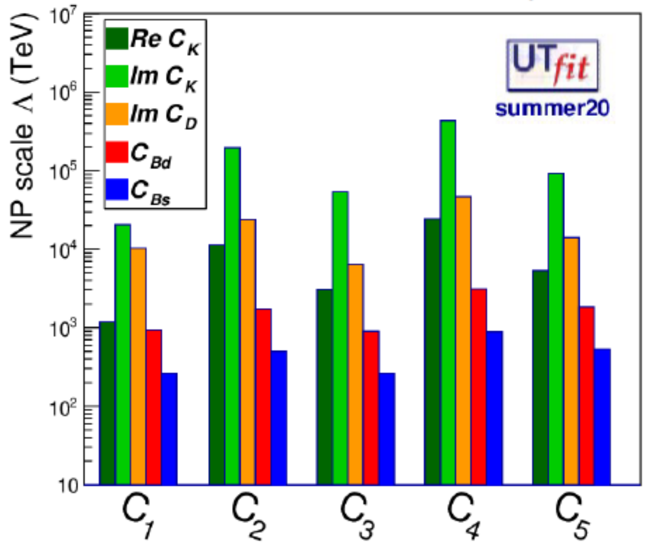
F_i: function of the NP flavour couplings

L_i: loop factor (in NP models with no tree-level FCNC)

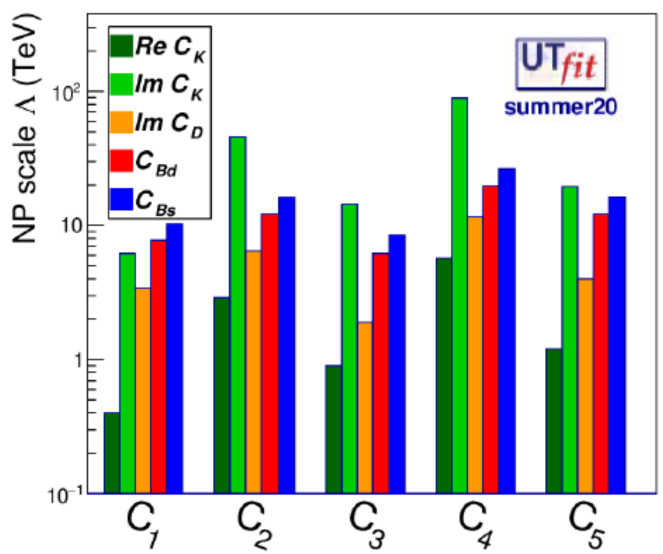
Λ: NP scale (typical mass of new particles mediating ΔF=2 processes)

results from the Wilson coefficients

Generic: $C(\Lambda) = \alpha/\Lambda^2$,
F_i~1, arbitrary phase
α ~ 1 for strongly coupled NP



NMFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$,
F_i~$|F_{SM}|$, arbitrary phase



$\Lambda > 4.3\ 10^5$ TeV

Lower bounds on NP scale (at 95% prob.)

$\Lambda > 89$ TeV

$\alpha \sim \alpha_W$ in case of loop coupling through weak interactions
$\Lambda > 1.3\ 10^4$ TeV

$\alpha \sim \alpha_W$ in case of loop coupling through weak interactions
$\Lambda > 2.7$ TeV

for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_W (~ 0.03).

Photo by M. Bona

The Cabibbo angle today

- Very precise measurements of leptonic and semileptonic decays determine

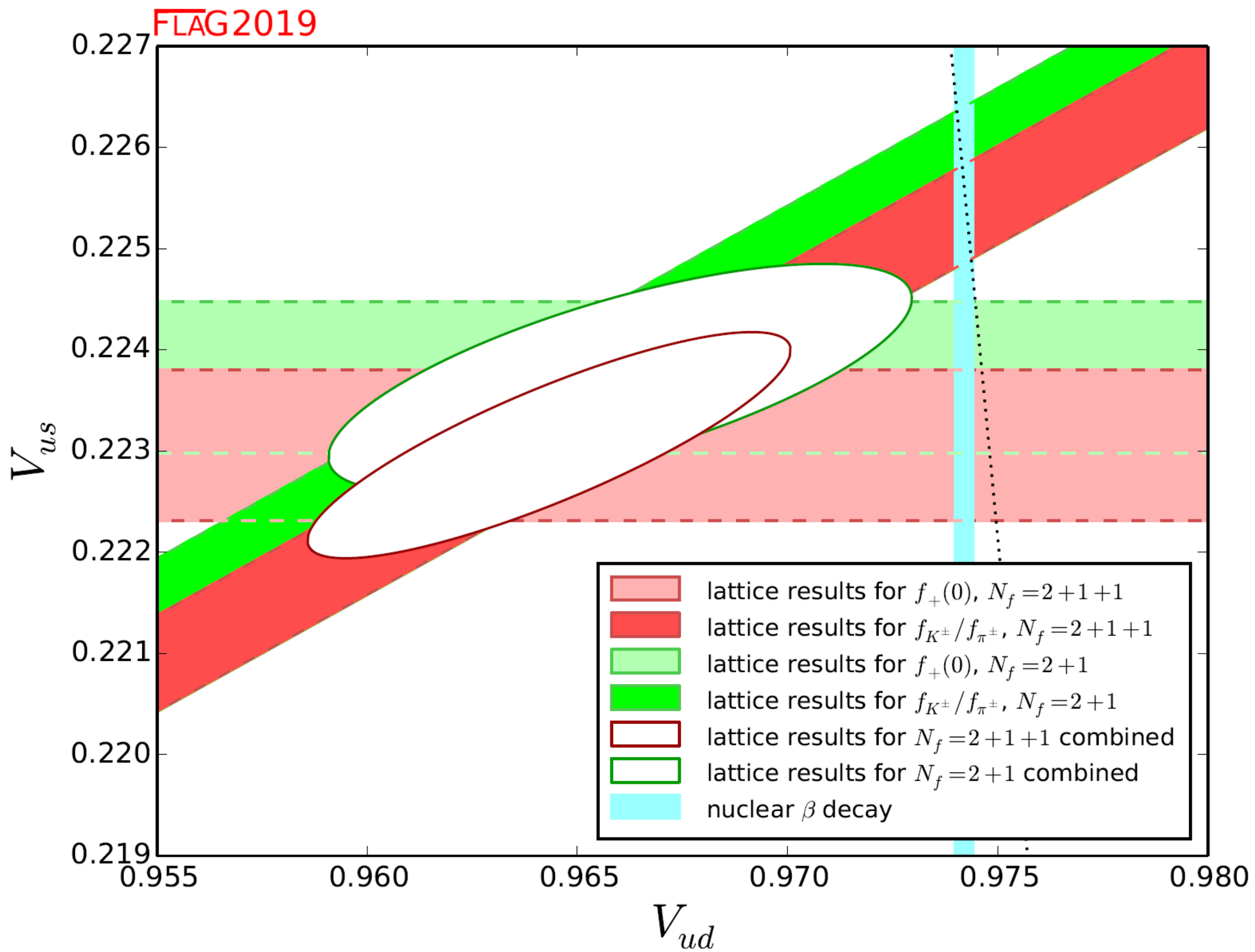
$$|V_{us}|f_+(0) = 0.2165(4)\,, \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4)$$

giving access to $|V_{ud}|$ and to $|V_{us}|$ through lattice QCD estimates of f_π, f_K and $f_+(0)$

- Nuclear β decays give access to

$$|V_{ud}| = 0.97420(21)$$

The Cabibbo angle today



From today to tomorrow

- Current LQCD precision on form factors and decay constants is sub-percent: need control of strong isospin breaking and of QED corrections
- Lattice calculations of weak interactions with QED corrections will be the future of the determination of the Cabibbo angle (and of precision flavour physics in general)

QED Corrections to Hadronic Processes in Lattice QCD

N.Carrasco,[1] V.Lubicz,[1] G.Martinelli,[2] C.T.Sachrajda,[3]
N.Tantalo,[4,5] C.Tarantino,[1] and M.Testa[6]

- Three steps, with increasing levels of difficulty:
  - Renormalizing the Lagrangian and computing masses (IR finite)
  - Computing leptonic decays (IR divergent amplitudes)
  - Computing semileptonic decays (IR divergent amplitudes, Maiani-Testa problem in Euclidean)

Lubicz et al. '16; Giusti et al. '17; Di Carlo et al. '19; Desiderio et al. '20; Frezzotti et al. '20

Leptonic decays

- The IR finite quantity is

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1(\Delta E)$$

- Idea: soft photon does not resolve the structure of the hadron, so use PT in pointlike approximation, writing

$$\Gamma(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\mathrm{pt}}) + \lim_{V\to\infty}(\Gamma_0^{\mathrm{pt}} + \Gamma_1(\Delta E))$$

- Both terms on the rhs are now IR finite and can be computed separately

Leptonic decays II

- Compute $\Gamma_0^{\rm pt} + \Gamma_1(\Delta E)$ in perturbation theory directly in infinite volume
- Compute Γ_0 with a simulation including the virtual photon at fixed momentum, subtract $\Gamma_0^{\rm pt}$, sum over momenta and take infinite volume limit
- For heavy mesons, a non-perturbative determination of $\Gamma_1(\Delta E)$ is needed

Semileptonic decays

Following the procedure used for leptonic decays we write

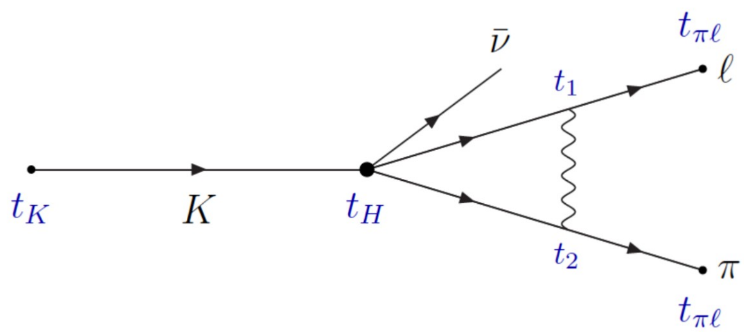
$$\frac{d^2\Gamma}{dq^2ds} = \lim_{V\to\infty}\left(\frac{d^2\Gamma_0}{dq^2ds} - \frac{d^2\Gamma_0^{pt}}{dq^2ds}\right) + \lim_{V\to\infty}\left(\frac{d^2\Gamma_0^{pt}}{dq^2ds} + \frac{d^2\Gamma_1^{pt}(\Delta E)}{dq^2ds}\right)$$

where the two terms are now infrared safe. For soft photons with $\Delta E/\Lambda_{QCD} \ll 1$ we can compute the real emission using the eikonal approximation and the virtual one using a simple effective model with suitable defined form factors.

The analogy however is not completely true: analytic continuation from Minkowsky and 1/L corrections are different

Slide from G. Martinelli, Tenerife '19

Semileptonic decays II



In general, depending on the volume and the pion-lepton invariant mass s, there are unphysical contributions from lighter intermediate $\pi\, l\, (\gamma)$ states, which grow exponentially with the temporal integration region and must be subtracted (as in the Lellouch-Luscher formula for $K\pi\pi$ decays). In fact this is a general feature in the calculation of long distance effects.

For semileptonic decays of heavy mesons however, for much of phase space there are too many lighter intermediate states to handle (and above the inelastic threshold). This is analogous to the fact that e.g. B $\pi\pi$ and B πK decays amplitudes cannot be calculated whereas K $\pi\pi$ amplitudes can.

Slide from G. Martinelli, Tenerife ‘19

Semileptonic decays & V_{ud}

- A recent paper has reanalyzed radiative corrections to superallowed nuclear β decays, obtaining a value for $|V_{ud}|$ corresponding to a ~ 4-5σ violation of CKM unitarity Seng et al. '18
- The recent progress on QED corrections to semileptonic decays opens up the road to a lattice calculation of nuclear β decay, crucial to clarify the situation

Conclusions

- The Cabibbo angle has paved the road to the Standard Model, and is a crucial ingredient in flavour phenomenology
- Current precision requires the inclusion of QED corrections, pioneered by the Rome group, to improve tests of CKM unitarity
- Understanding the origin of the Cabibbo angle and of the flavour hierarchy remains a crucial theoretical problem

Conclusions II

- So far the LHC has left us in a state of big confusion, with many open theoretical problems but no unified answer
- To move forward we probably need to take a giant leap as Nicola did for the angle
- We sorely miss his intuition and his depth!


VIETATO FUMARE