Radiation effects for the next generation of synchrotron radiation facilities

Progress Report

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Requirement of high brightness electron bunches: $B = \frac{Q}{\mathcal{V}_{6D}}$

- Free Electron Laser
- Inverse Thomson/Compton scattering

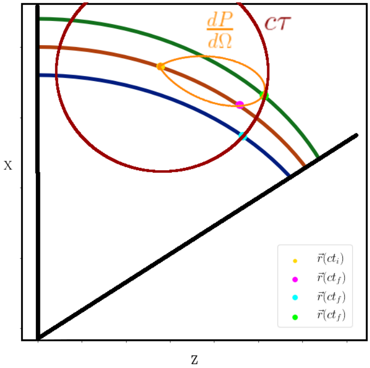
Acceleration methods	
LINAC	Plasma
MV/m	GV/m
Bunch compression	Energy spread

Lienard Wiechert Potentials

$$c\tau = ct - ct_{ret}$$
$$= |r_o(ct) - r_s(ct_{ret})|$$

Electric field

$$\vec{E}(ct,\vec{r}) = q\left(\frac{\hat{n}-\vec{\beta}}{\gamma^2(1-\hat{n}\vec{\beta})^3 c\tau^2} + \frac{\hat{n}\times\left((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right)}{(1-\hat{n}\vec{\beta})^3 c\tau}\right)_{ct_{ret}}$$



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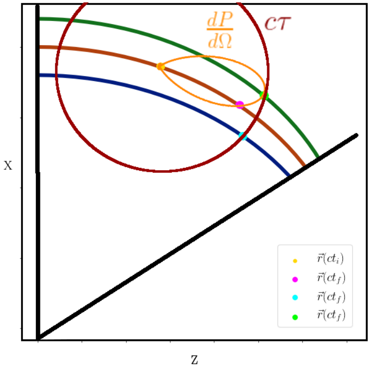

Power radiated per unit solid angle

$$\frac{dP}{d\Omega} = \frac{q^2 c}{4\pi}\frac{\left|\hat{n}\times\left((\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right)\right|^2}{\left(1-\hat{n}\cdot\vec{\beta}\right)^5}$$

Lienard Wiechert Potentials

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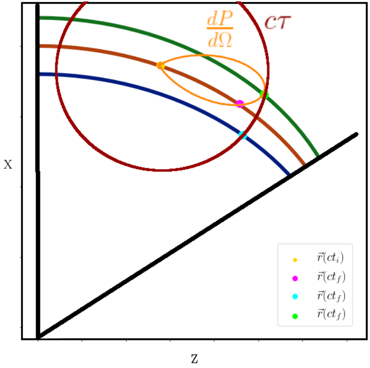

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Power for instantaneous circular motion

$$P = \frac{2q^2 c}{3}\left(\frac{\beta^2}{R}\right)^2\gamma^4$$

Existing schemes

Without retarded time
- Coulomb / Poisson solvers
- Emittance based schemes

With retarded time
- analytical: 1D CSR[1,2] - expanded into 2D[3,4]
- Numerical PIC (with Maxwell solver)
- Numerical Maxwell-Vlasov based
- Lienard Wiechert Potentials[5]

[1] Y. S. Derbenev et al TESLA-FEL 95-05 (1995)

[2] Saldin et al Nucl. Instrum. Methods Phys. Res., Sect. A398, 373 (1997)

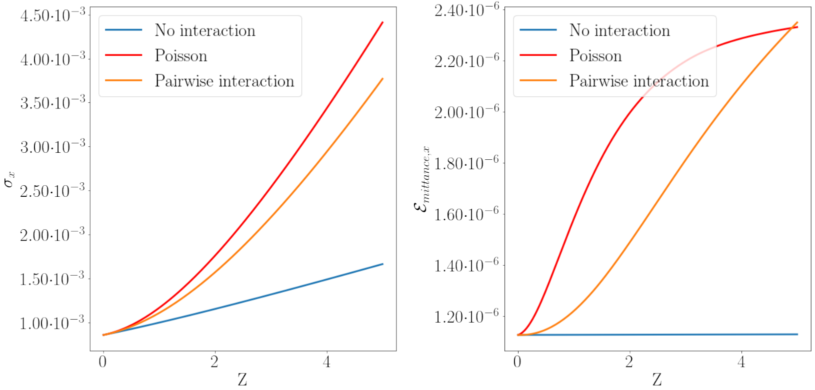
[3] A. D. Brynes New J. Phys. 20:073035 (2018)

[4] W. Lou et al, Phys. Rev. Accel. Beams, 23:054404 (2020)

[5] T. Shintake Nucl. Inst. Meth. A,507, 89 (2003)

GPT simulations

- Poisson: calculation in rest frame electron bunch, CPU $\mathcal{O}(N)$
- Pairwise: rest frame per pair of electrons, CPU $\mathcal{O}(N^2)$

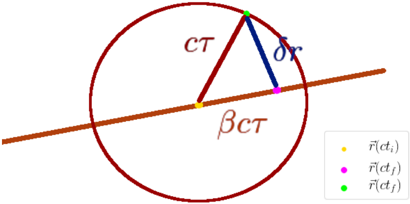


$$\vec{E} = q\left(\frac{\hat{n}-\vec{\beta}}{\gamma^2(1-\hat{n}\vec{\beta})^3 c\tau^2}\right)_{ct_{ret}}$$

$$c\tau^2 - \left(\vec{r}_s(ct_{ret}) - \vec{r}_o(ct)\right)^2 = 0$$

$$\vec{r}_s(ct_{ret}) = r_s(ct) - \int_{c\tau} dct\vec{\beta}$$

$$\vec{r}_s(ct) - \vec{r}_o(ct) = \delta\vec{r}$$



$$c\tau = \gamma^2\left(-\delta\vec{r}\cdot\vec{\beta} + \sqrt{\left(\delta\vec{r}\cdot\vec{\beta}\right)^2 + \left(\frac{\delta r}{\gamma}\right)^2}\right)$$

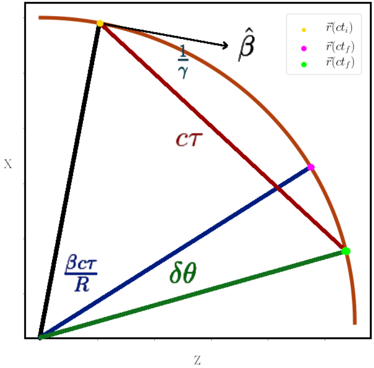
Analytical model 1D:

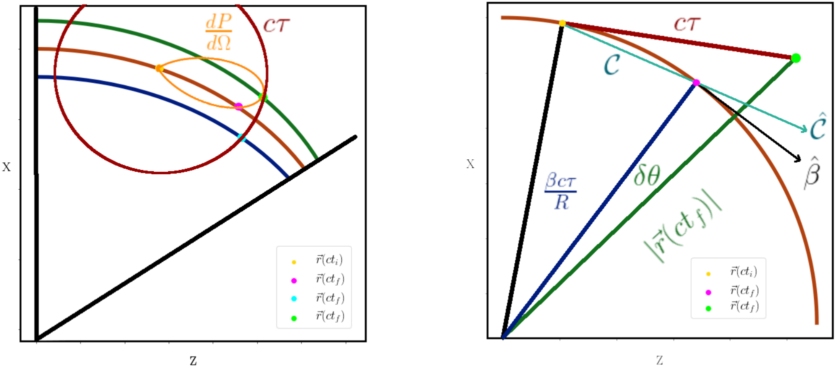
Y. S. Derbenev et al
TESLA-FEL 95-05 (1995)

$$\sigma_x \left(\frac{1}{R\sigma_z^2}\right)^{\frac{1}{3}} \ll 1$$

Saldin et al Nucl. Instrum. Methods Phys. Res., Sect. A398, 373 (1997)

$$\frac{R}{\gamma^3} \ll L_{\text{bunch}} \ll \frac{R\phi_B}{24}$$

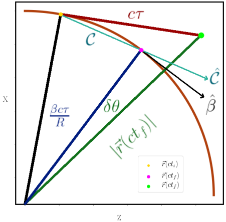




$$c\tau^2 - \left(\delta\vec{r} - \int_{c\tau} dct\vec{\beta}\right)^2 = 0$$

Chord: $\mathcal{C} = \int_{c\tau} dct\vec{\beta} = 2R\sin\left(\frac{\beta c\tau}{2R}\right)$. Very difficult approach.

For circular motion $|r_s(ct_{ret})| = R$



$$c\tau^2 - (\vec{r}_s(ct_{ret}) - \vec{r}_o(ct))^2 = 0$$
$$c\tau^2 - R^2 - \vec{r}_o(ct)^2 + 2R\,|\vec{r}_o(ct)|\cos(\alpha) = 0$$

where

$$\alpha = \frac{\beta c\tau}{R} + \arccos(\hat{r}_s(ct)\cdot\hat{r}_o(ct)) = \frac{\beta c\tau}{R} + \delta\theta$$

$$\cos(x+a) \approx cos(a) - \sin(a)x - \frac{\cos(a)}{2}x^2 + \frac{\sin(a)}{6}x^3 + \frac{cos(a)}{24}x^4 + \dots$$

Left to solve

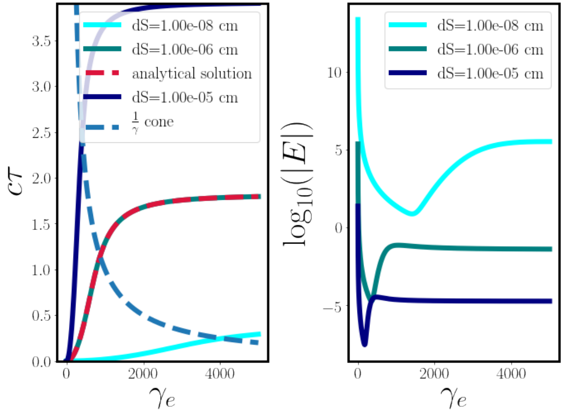
$$c\tau^2 - \delta r^2 + 2R\left|\vec{r}_o(ct)\right|\left[-\sin(\vartheta)\frac{\beta}{R}c\tau - \frac{\cos(\vartheta)}{2}\left(\frac{\beta}{R}\right)^2 c\tau^2 + \frac{\sin(\vartheta)}{6}\left(\frac{\beta}{R}\right)^3 c\tau^3 + \frac{\cos(\vartheta)}{24}\left(\frac{\beta}{R}\right)^4 c\tau^4\right] = 0$$

For r_o in front of $r_s(ct)$, i.e. $\arccos(\hat{r}_s(ct)\cdot\hat{r}_o(ct)) > 0$, we can assume $\delta r \ll c\tau$.

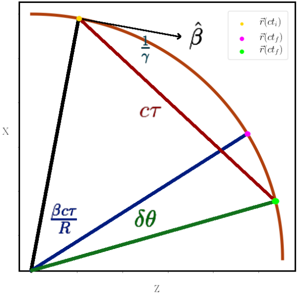
The equation can than be suppressed and solved using Cardano's formula: $x^3 + px + q = 0$

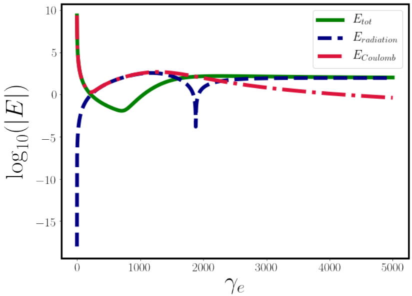
$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

$c\tau_{21}$: r2 is the source and is behind r1
$R = 5.00$ [m]
$dS = R\delta\theta$



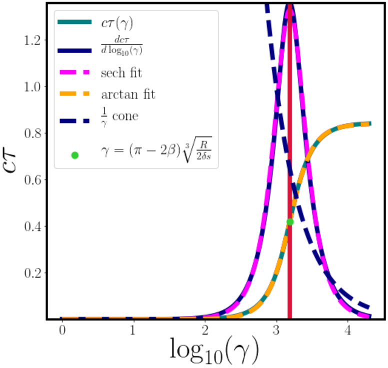
$dS = R\delta\theta = 1 \cdot 10^{-7}$ cm





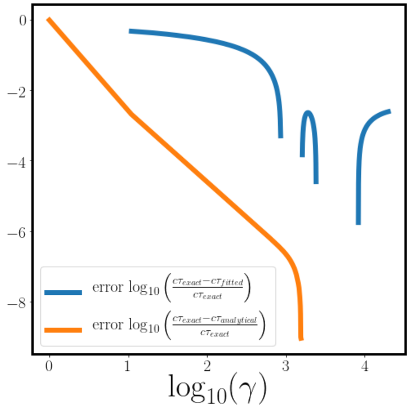
The point where the electron is at the edge of the γ-cone:

$\gamma = \sqrt[3]{\frac{1}{\delta\theta}} = 1710$



$$\frac{dc\tau}{d\gamma} = \frac{2A}{\exp[-B*(\log_{10}(\gamma)-C)]+\exp[B*(\log_{10}(\gamma)-C)]}$$

$$c\tau(\gamma) = \frac{2A}{B}\left[\arctan\left(\tanh\left(\frac{B(\log_{10}(\gamma)-C)}{2}\right)\right) + \arctan\left(\tanh\left(\frac{BC}{2}\right)\right)\right]$$



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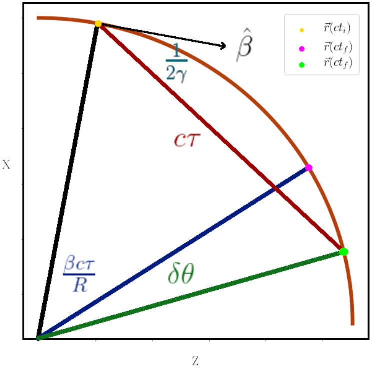
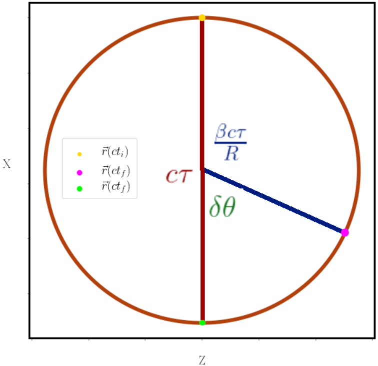
Where does $\gamma = (\pi - 2\beta)\sqrt[3]{\frac{1}{2\delta\theta}}$ come from?

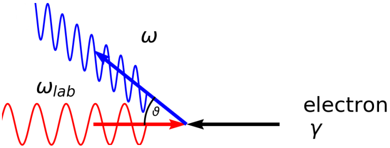
$$\gamma = \sqrt[3]{\frac{1}{2\delta\theta}}$$



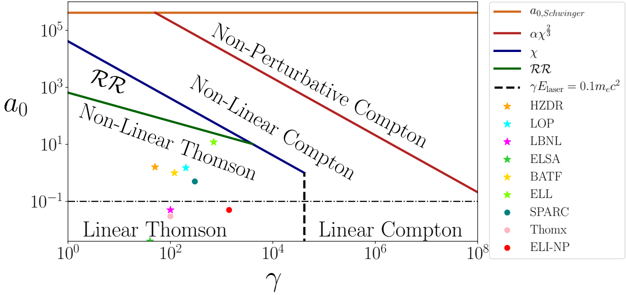
$$\delta\theta_{\max} = \pi - 2\beta$$



Inverse Thomson scattering



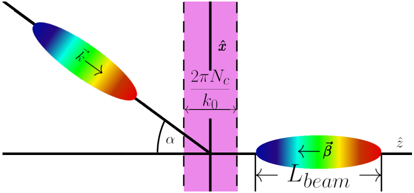
$$\omega = \frac{2^2\gamma^2}{1+(a)^2+\gamma^2\vartheta^2}\omega_l$$

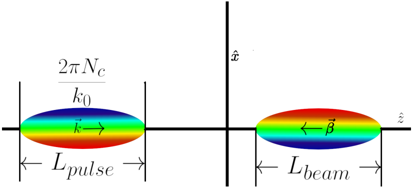


$$\frac{\sigma_\omega}{\omega} \cong \sqrt{\left(\Theta + \frac{\sigma_\epsilon}{\sigma_{W_{bunch}}}\right)^2 + \left(2\frac{\sigma_\gamma}{\gamma}\right)^2 + \left(\frac{\sigma_{\omega_l}}{\omega_l}\right)^2}$$

$$\omega = \frac{2^2\gamma^2}{1+(a)^2+\gamma^2\vartheta^2}\omega_l$$

$$\gamma^2(X)\omega_l(X) = \text{const}$$





Transverse chirp proposed by Vittoria Petrillo

Manuscript submitted to Phys. Rev. ST Accel. Beams, ZR10126

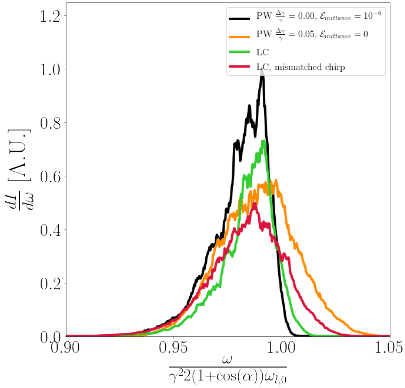
Linear energy spread & linear chirped laser pulse

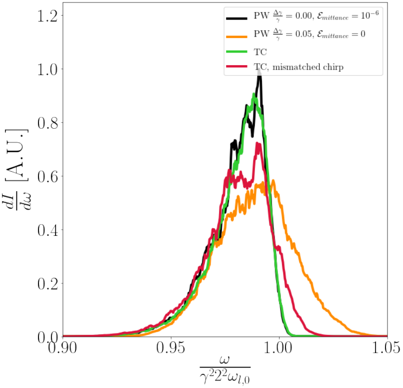
$\frac{\Delta\omega_l}{\omega_{l,0}} = 0.14$

$\vartheta_{\max} = \frac{1}{10\gamma}$

matched: $\frac{\Delta\gamma}{\gamma} = 0.05$

mismatched: $\frac{\Delta\gamma}{\gamma} = 0.07$



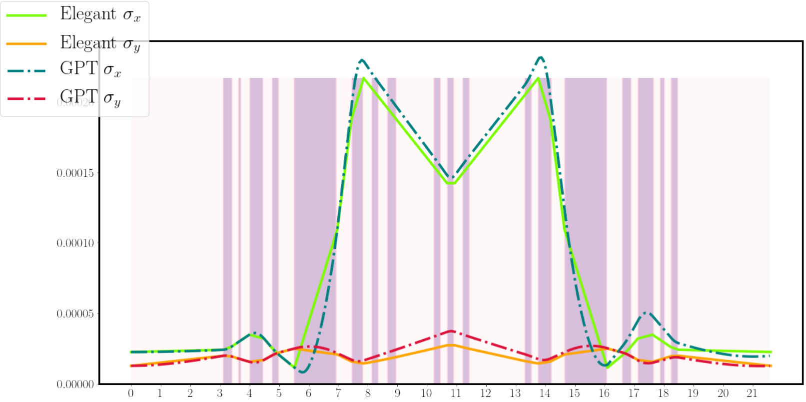


Charged Particle Interaction

- Further study of the 2D analytical solution of circular motion and expand to 3D
- Calculate change in trajectories due to particle interactions

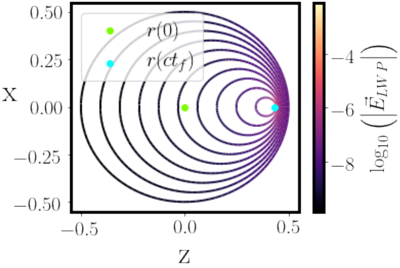
Inverse Thomson scattering

- Energy compensation: higher order frequency modulation
- Carrier Envelope Phase dependence



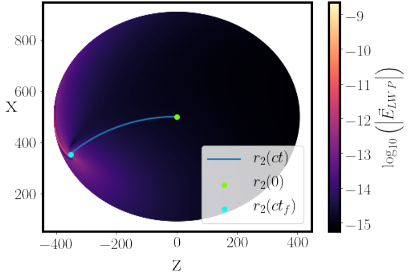
Elegant σ_x
Elegant σ_y
GPT σ_x
GPT σ_y
0.00020
0.00015
0.00010
0.00005
0.00000
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

$$\vec{E}(ct,\vec{r}) = \sum_{n}^{N} \vec{E}_{LW}\left(\vec{r}(ct[n]), \vec{\beta}(ct[n]), c\tau = (ct_f - ct[n]), \vartheta, \varphi\right)$$

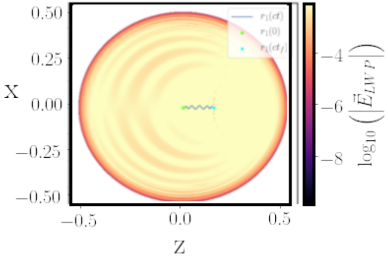


Similar to T. Shintake Nucl. Inst. Meth. A,507, 89 (2003)

$$\vec{E}(ct,\vec{r}) = \sum_{n=0}^{N-1} \vec{E}_{LW}\left(\vec{r}(ct[n]), \vec{\beta}(ct[n]), c\tau = (ct[N]-ct[n]), \vartheta, \varphi\right)$$



Dipole



Thomson scattering

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \omega \sum_{i=0}^{N_e} \int_{-\infty}^{\infty} dt \ \hat{n} \times \hat{n} \times \vec{\beta}_i \exp\left[i \ \frac{\omega}{c} \left(ct - \hat{n} \cdot \vec{r}_i \right) \right] \right|^2$$

$$a^\mu = a_0 \begin{pmatrix} 0 \\ \cos(\eta) \\ -\sin(\eta) \\ 0 \end{pmatrix} \Psi(\vec{r}) \mathcal{E}(\zeta)$$

Longitudinal Chirp: Geometry and