Theory of Fast Flavor Conversion of Supernova neutrinos

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CONTENT OF THE TALK :

- **Equations of motion and a bit of background in the field**

- **Complicacy and Challenges in the field**

- **Our Progress**

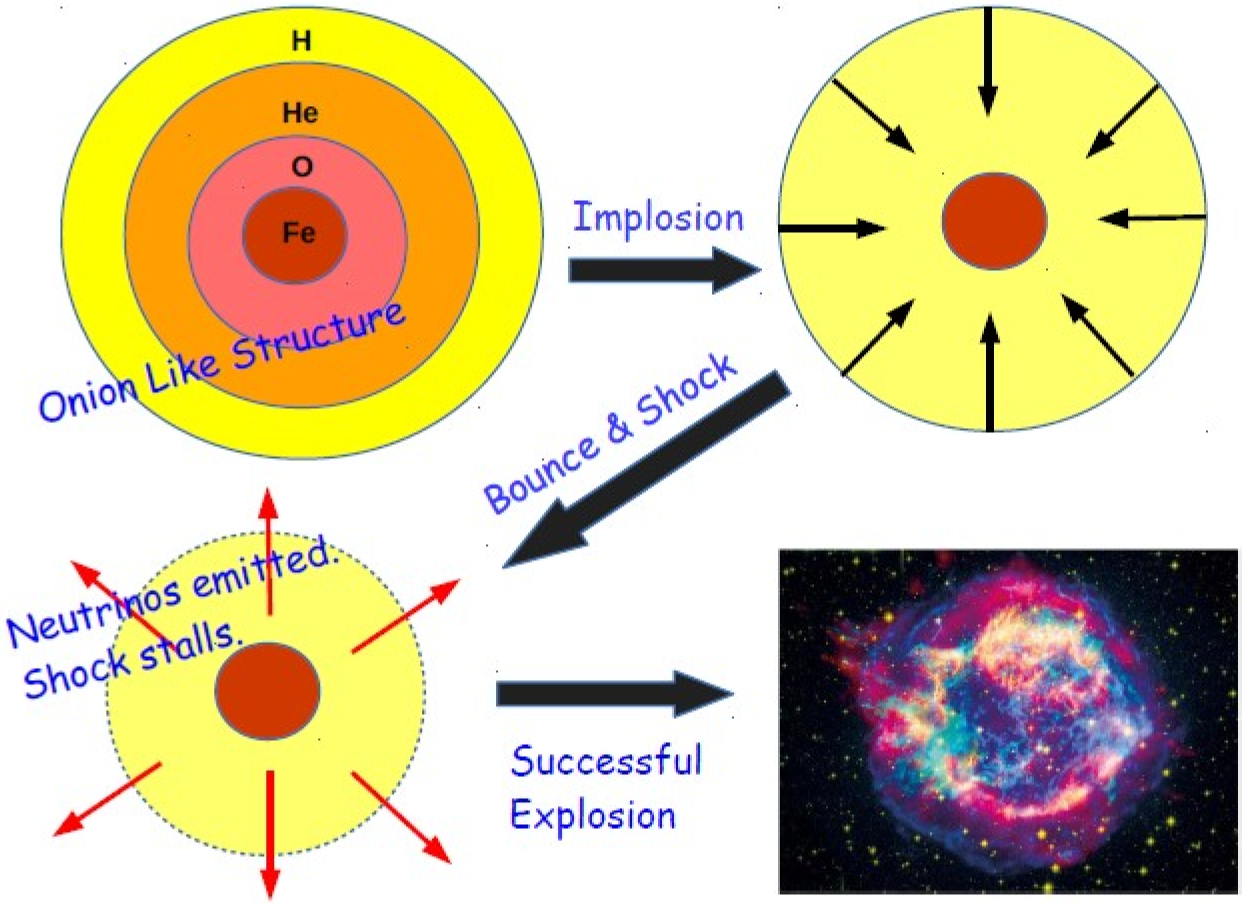
- **Theory of SN neutrino oscillations :**

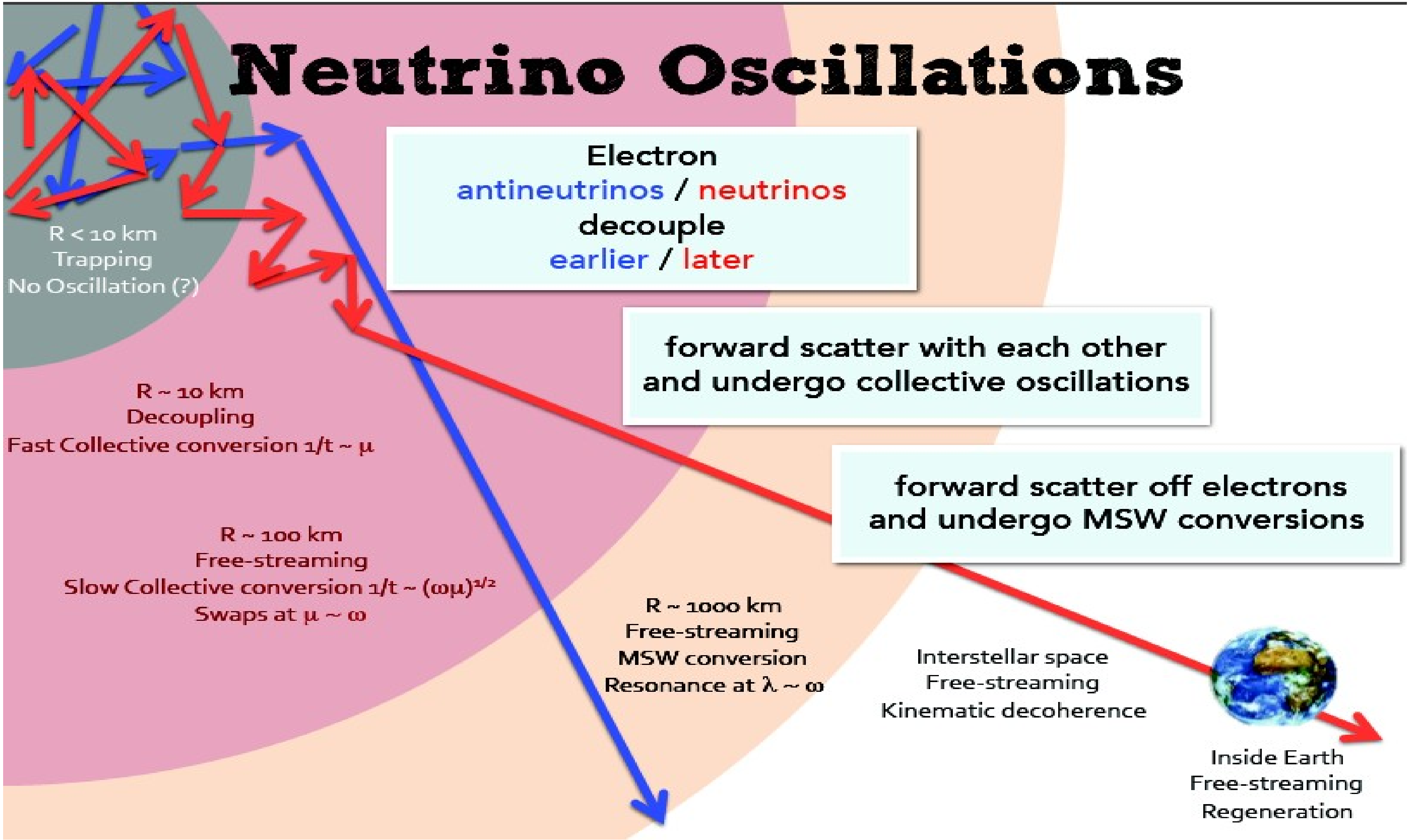
    1) Multipole diffusion
    2) Transverse relaxation

- **Main results**

- **Phenomenological Consequences and Conclusion**

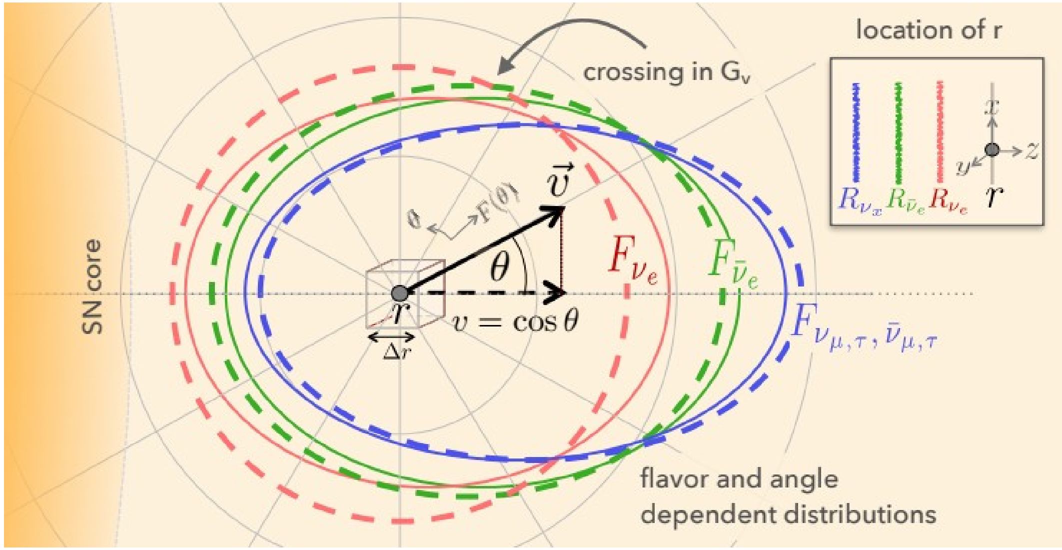
SN Explosion & Neutrinos :

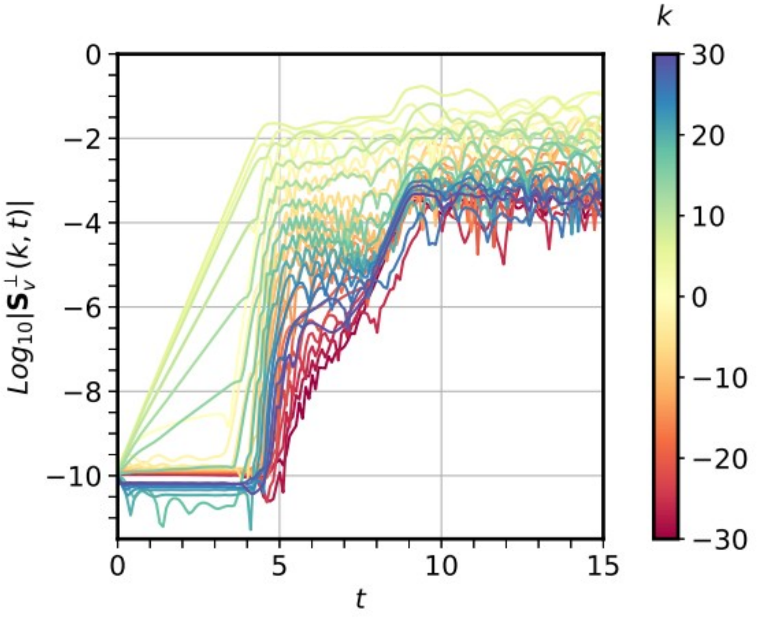



Neutrino Oscillations
Electron
antineutrinos / neutrinos
decouple
earlier / later
forward scatter with each other
and undergo collective oscillations
forward scatter off electrons
and undergo MSW conversions
R < 10 km
Trapping
No Oscillation (?)
R ~ 10 km
Decoupling
Fast Collective conversion 1/t ~ μ
R ~ 100 km
Free-streaming
Slow Collective conversion 1/t ~ (ωμ)^1/2
Swaps at μ ~ ω
R ~ 1000 km
Free-streaming
MSW conversion
Resonance at λ ~ ω
Interstellar space
Free-streaming
Kinematic decoherence
Inside Earth
Free-streaming
Regeneration

Fast Flavor Conversions : A review

Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Ray Sawyer (2005, 2015)





- Conversion occurs *very close* to the SN core (few km's) with *rate* $\propto$ *Neutrino number density*
- 10^5 times *faster* compared to neutrino oscillation in vacuum/ordinary matter
- Requires a *"zero" Crossing* in neutrino angular distribution and *independent of energy (and mass hierarchy*)

EQUATION OF MOTION :

- **Neutrino density matrix :** $\hat{\rho}[\vec{r}, E, \vec{p}, t] \equiv \hat{\rho}_{E,\vec{p}}$ $\quad \hat{\rho} = \begin{pmatrix} \langle\nu_e|\nu_e\rangle & \langle\nu_e|\nu_\mu\rangle & \langle\nu_e|\nu_\tau\rangle \\ \langle\nu_\mu|\nu_e\rangle & \langle\nu_\mu|\nu_\mu\rangle & \langle\nu_\mu|\nu_\tau\rangle \\ \langle\nu_\tau|\nu_e\rangle & \langle\nu_\tau|\nu_\mu\rangle & \langle\nu_\tau|\nu_\tau\rangle \end{pmatrix}$

$\langle\nu_i|\nu_i\rangle \longrightarrow$ Total flavor content

$\langle\nu_i|\nu_j\rangle \longrightarrow$ Amount of flavor conversion

- **Equations governing flavor evolution :**

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right)\hat{\rho}_{E,\vec{p}} = \hat{H}_{E,\vec{p}}\hat{\rho}_{E,\vec{p}} - \hat{\rho}_{E,\vec{p}}\hat{H}_{E,\vec{p}} \qquad \hat{H}_{E,\vec{p}} = H_E^{vac} + H^{mat} + H_{\vec{p}}^{self}$$

$$H_E^{vac} = \frac{\Delta m^2}{2E} \qquad H^{\rm mat} = \sqrt{2}G_F n_e \qquad H_{\vec{p}}^{\rm self} = \int d^3\vec{q}/(2\pi)^3\left(1 - \vec{v}_{\vec{q}}\cdot\vec{v}_{\vec{p}}\right)\left(\hat{\rho}_{E',\vec{q}} - \bar{\hat{\rho}}_{E',\vec{q}}\right)$$

- **Vacuum oscillation**
- **Dependence on energy**

- **MSW potential**
- **Dependence on electron number density**

- **Neutrino-Neutrino self interaction potential**
- **Depends on neutrino number density**
- **Responsible for collective effects (e.g : fast conversion)**

EQUATION OF MOTION (FFC) :

$g_{\vec{p}}$:
Neutrino angular distribution

We stick to two-flavor framework :

$$\hat{\rho}_{E,\vec{v}} = \begin{pmatrix} \langle \nu_e | \nu_e \rangle_{E,\vec{v}} & \langle \nu_e | \nu_x \rangle_{E,\vec{v}} \\ \langle \nu_x | \nu_e \rangle_{E,\vec{v}} & \langle \nu_x | \nu_x \rangle_{E,\vec{v}} \end{pmatrix} = \frac{Tr\left(\hat{\rho}_{E,\vec{v}}\right)}{2}\mathbb{I}_{2\times 2} + \frac{g_{E,\vec{v}}}{2}\vec{S}_{E,\vec{v}}\cdot\sigma$$

$$\hat{H}_{E,\vec{v}} = \frac{Tr\left(\hat{H}_{E,\vec{v}}\right)}{2}\mathbb{I}_{2\times 2} + \frac{1}{2}\vec{H}_{E,\vec{v}}\cdot\sigma$$

Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Dasgupta, Mirizzi, Sen (2017)
Bhattacharyya, Dasgupta (2020)

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right)\hat{\rho}_{E,\vec{v}} = \hat{H}_{E,\vec{v}}\hat{\rho}_{E,\vec{v}} - \hat{\rho}_{E,\vec{v}}\hat{H}_{E,\vec{v}} \longrightarrow \left(\partial_t + \vec{v}.\vec{\nabla}\right)\mathsf{S}_{\omega,\vec{v}} = \left(\mathsf{H}^{\rm vac}_{\omega} + \mathsf{H}^{\rm mat} + \mathsf{H}^{\rm self}_{\vec{v}}\right)\times\mathsf{S}_{\omega,\vec{v}}$$

$$\mathsf{H}^{\rm mat} = \sqrt{2}G_F\left(n_{e^-} - n_{e^+}\right)\left(0,0,1\right) \qquad \mathsf{H}^{\rm vac}_{\omega} = \omega\left(\sin 2\vartheta, 0, \cos 2\vartheta\right)$$

$$\mathsf{H}^{\rm self}_{\vec{v}} = \int d^3\vec{p}'_{\omega',\vec{v}'}/(2\pi)^3\, g_{\omega',\vec{v}'}\left(1 - \vec{v}\cdot\vec{v}'\right)\mathsf{S}_{\omega',\vec{v}'}$$

EQUATION OF MOTION (FFC) :

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$$\hat{H}_{E,\vec{v}} = \frac{Tr\left(\hat{H}_{E,\vec{v}}\right)}{2}\mathbb{I}_{2\times 2} + \frac{1}{2}\vec{H}_{E,\vec{v}}\cdot\sigma$$

$$\boxed{\left(\partial_t + \vec{v}.\vec{\nabla}\right)\mathsf{S}_{\vec{v}} = \mu_0 \int d^3v' G_{\vec{v}'}\left(1 - \vec{v}\cdot\vec{v}'\right)\mathsf{S}_{\vec{v}'} \times \mathsf{S}_{\vec{v}}}$$

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right)\hat{\rho}_{E,\vec{v}} = \hat{H}_{E,\vec{v}}\hat{\rho}_{E,\vec{v}} - \hat{\rho}_{E,\vec{v}}\hat{H}_{E,\vec{v}} \longrightarrow \left(\partial_t + \vec{v}.\vec{\nabla}\right)\mathsf{S}_{\omega,\vec{v}} = \left(\mathsf{H}^{\mathrm{vac}}_{\omega} + \mathsf{H}^{\mathrm{mat}} + \mathsf{H}^{\mathrm{self}}_{\vec{v}}\right) \times \mathsf{S}_{\omega,\vec{v}}$$

$$\mathsf{H}^{\mathrm{mat}} = \cancel{\sqrt{2}G_F}\left(n_{e^-} - n_{e^+}\right)\left(0,0,1\right) \qquad \mathsf{H}^{\mathrm{vac}}_{\omega} = \cancel{\omega}\left(\sin 2\vartheta, 0, \cos 2\vartheta\right)$$

“Zero crossing” : FFC

$$\mathsf{H}^{\mathrm{self}}_{\vec{v}} = \int d^3\vec{p}'_{\omega',\vec{v}'}/(2\pi)^3\, g_{\omega',\vec{v}'}\left(1 - \vec{v}\cdot\vec{v}'\right)\mathsf{S}_{\omega',\vec{v}'}$$

$$\boxed{\mathsf{S}_{\omega',\vec{v}'} \equiv \mathsf{S}_{\vec{v}'}} \qquad \boxed{G_{\vec{v}} = \int d\omega g_{\omega,\vec{v}}}$$

$\left|\mathsf{S}^{\perp}_{\vec{v}}\right|$ **: Flavor conversion** $\qquad$ $\mathsf{S}^{\parallel}_{\vec{v}}$ **: Total Flavor content**

Difficulties and Challenges :

What are the challenges ?	What did we do ?	
Huge Phase space dimensionality 3 sp. + 3 mom. + 1 time = 7 dim	Partially resolved 2 sp. + 2 mom. + 1 time = 5 dim.	
Large set of coupled nonlinear P.D.E's	Developed : a) Analytical techniques b) Numerical code	✓
Lack of numerical techniques to give accurate and precise result in the nonlinear regime	Developed our own code that gives accurate and precise answer even in the nonlinear regime	✓
Lack of analytical development / theory beyond the linear regime (Linear stability) which can predict the final outcome	Developed our own theory that can predict how, when and to what extent fast conversion can happen.	✓

OUR NUMERICAL RECIPE :

Discretize in space for each velocity

Method : Fast Fourier Transform

B.C : Periodic in space

Solver : scipy.fftpack.diff in python

A set of coupled nonlinear O.D.E

Method : Backward differentiation

Solver : Zvode solver in python

Solve O.D.E's in time

RESULTS : Irreversibility

Model :

1Time + 1 Sp. + 1 Mom.

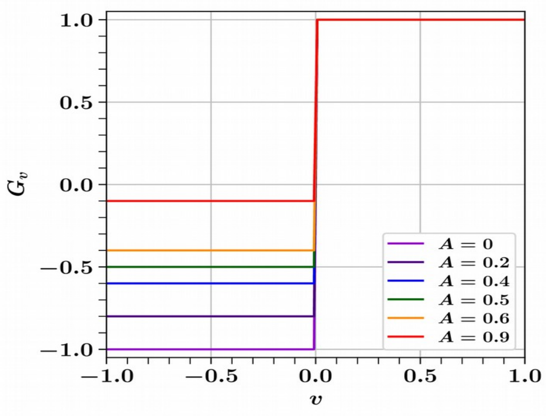
Initial Cond :

$$\mathsf{S}_v^{\|}\Big|^{\text{ini}} = +1$$

$$\left|\mathsf{S}_v^{\perp}\right|^{\text{ini}} = 10^{-6}\delta\left(z\right)$$

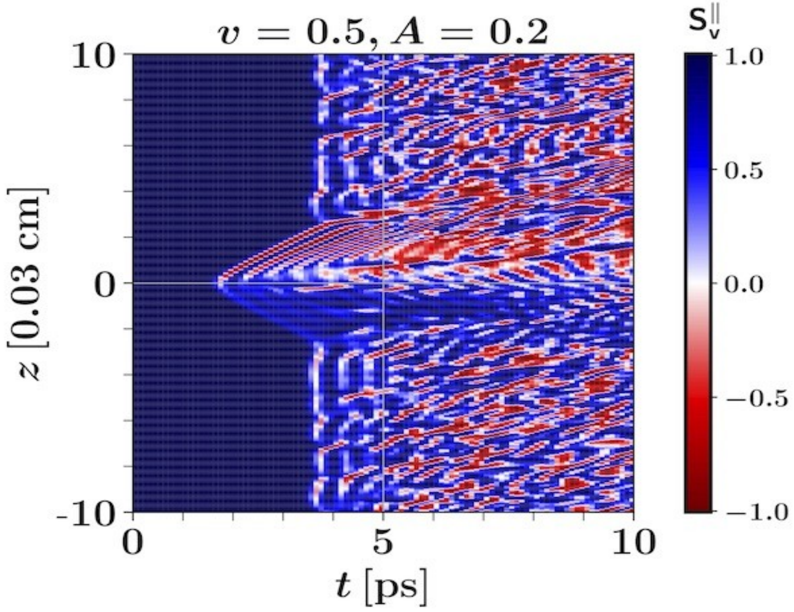
Angular dist. :

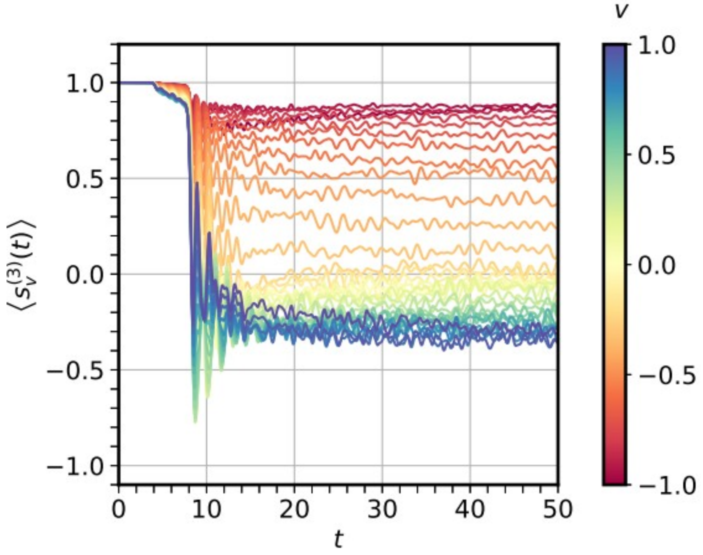
$G_v = 1, v > 0; A - 1, v < 0$



- **Irreversible and steady state behaviour in time**
- **length shrinks**

Bhattacharyya, Dasgupta (2020)
arXiv : 2005.00459





RESULTS : Multipole Diffusion

- In terms of multipole moments, $\mathrm{M}_n = \int_{-1}^{+1} dv\, G_v L_n \mathrm{S}_v$ and considering n as continuum we get :

$$\boxed{\partial_t \mathrm{M}_n - \mathrm{M}_0 \times \mathrm{M}_n = \partial_z \left(\mathrm{M}_n + \partial_n \mathrm{M}_n/(2n+1) + \partial_n^2 \mathrm{M}_n/2\right) - \mathrm{M}_1 \times \left(\mathrm{M}_n + \partial_n \mathrm{M}_n/(2n+1) + \partial_n^2 \mathrm{M}_n/2\right)}$$

- Further coarse graining over z , and using $2n + 1 \approx 2n$

$$\boxed{\partial_t \langle M_n \rangle = \frac{\langle M_1 \rangle}{2} \left(\partial_n^2 \langle M_n \rangle + \frac{1}{n} \partial_n \langle M_n \rangle\right)}$$

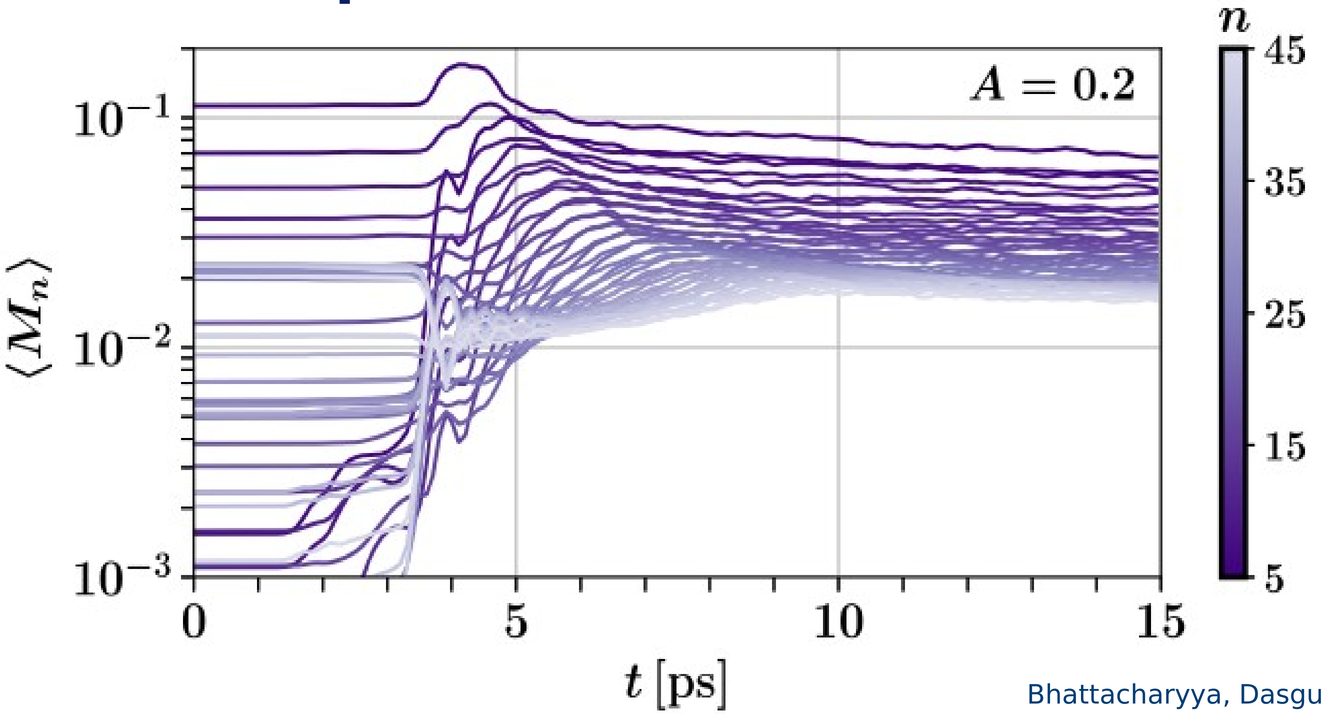
Diffusion in multipole space

- The above equation remains same under $n \to an\,,\ t \to a^2 t \longrightarrow \langle M_n(t) \rangle = f\left(\frac{n^2}{t}\right) = f(\xi)$

$$\boxed{2\frac{d^2}{d\xi^2} f(\xi) + \left(1/\langle M_1 \rangle + 2/\xi\right) \frac{d}{d\xi} f(\xi) = 0}$$

$$\boxed{\langle M_n(t) \rangle = c_1\, \mathrm{Ei}\left[-n^2/\left(2\langle M_1 \rangle t\right)\right] + c_2}$$

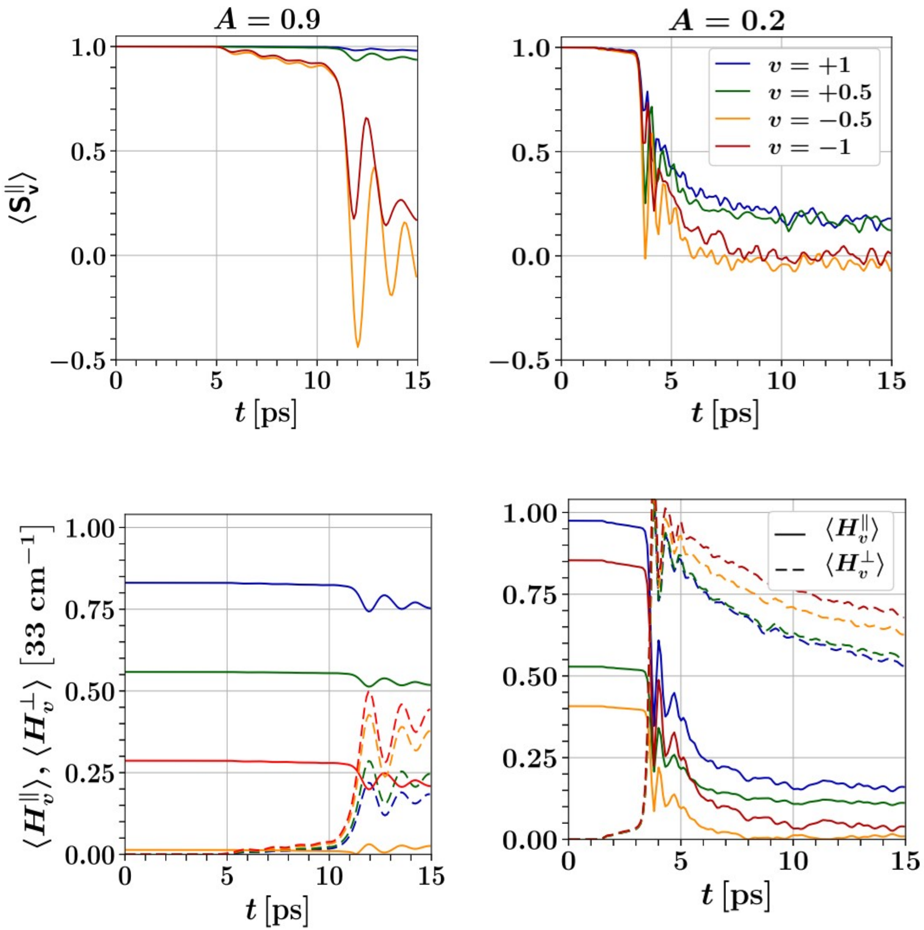
RESULTS : Multipole Diffusion

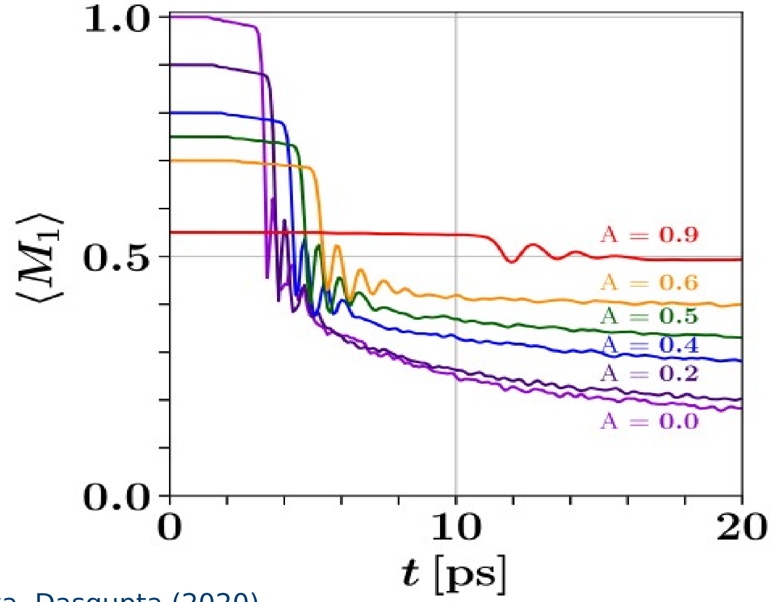


Bhattacharyya, Dasgupta (2020)
arXiv : 2009.03337

Power flow in multipole space from low to high n values and coarse-graining causes irreversibiltiy in time and also shrinking in the length for high n multipole moments

RESULTS : Transverse Relaxation





Bhattacharyya, Dasgupta (2020)
arXiv : 2009.03337

- **Modes for which $|H_v^{\perp}| \approx |H_v^{\parallel}|$, S_v crosses the transverse plane and gets depolarized**
- **Amount depends on lepton asymmetry and choice of v**

RESULTS : Flavor Depolarization

Depolarization factor :

$$\boxed{\boldsymbol{f_v^{\mathrm{D}} = \tfrac{1}{2}\left(1 - \langle \mathsf{S}_v\rangle^{\mathbf{fin}} / \langle \mathsf{S}_v\rangle^{\mathbf{ini}}\right)}}$$

$f_v^{\mathrm{D}} = 0.5 \longrightarrow$ Complete depolarization

$f_v^{\mathrm{D}} = 0 \longrightarrow$ No depolarization

Multipole expansion upto linear order :

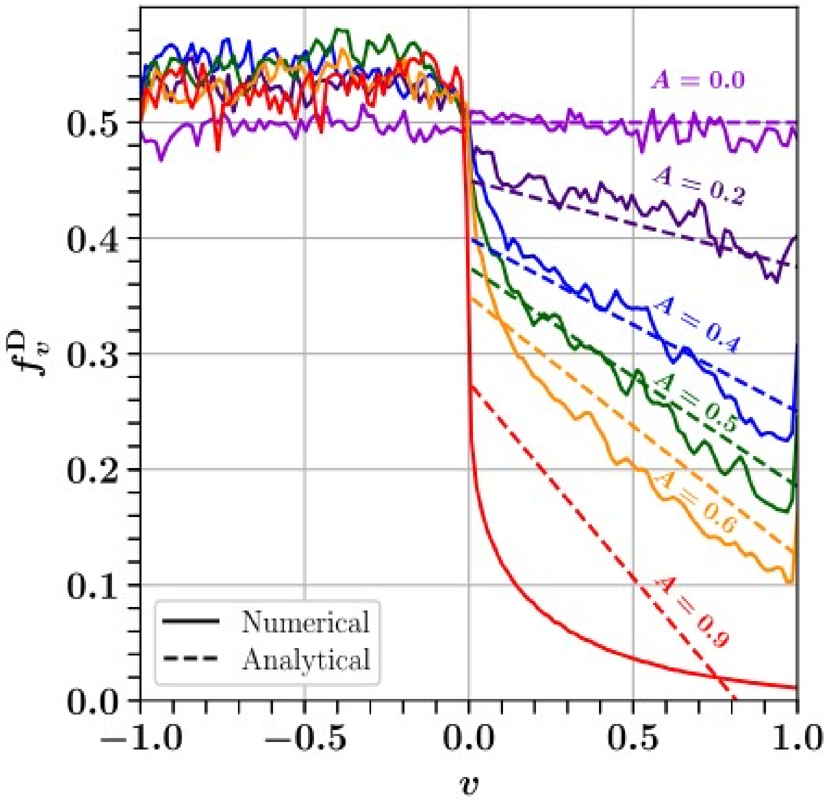
$$G_v \mathsf{S}_v^{\parallel}|_{fin} = \frac{\mathsf{M}_0^{fin}}{2} + \frac{3v\mathsf{M}_1^{fin}}{2} + O(v^2)$$

$$+$$

Lepton number conservation :

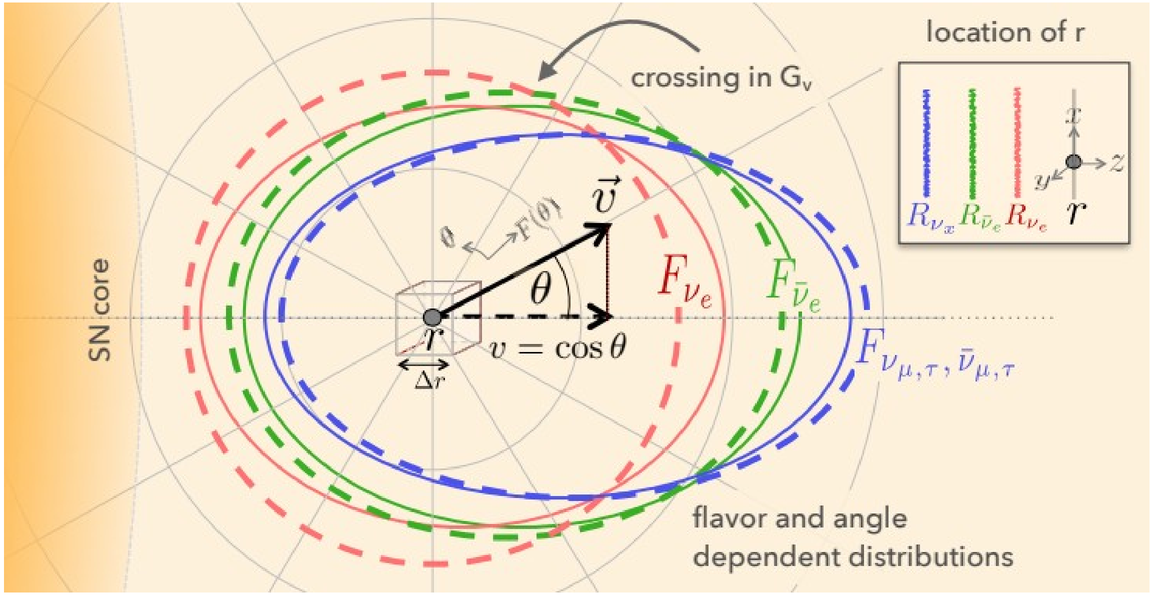
$$\langle \mathsf{M}_0^{ini} \rangle = \left\langle \mathsf{M}_0^{fin} \right\rangle = A \quad \left\langle \mathsf{M}_1^{fin} \right\rangle = \frac{A}{2}$$

$$\boxed{\boldsymbol{f_v^{\mathrm{D}} \approx 0.5, \text{if } v < 0}} \qquad \boxed{\boldsymbol{f_v^{\mathrm{D}} \approx \frac{1}{2} - \frac{A}{4} - \frac{3A}{8} v, \text{if } v > 0}}$$



CONCLUSION

- We have presented an ***analytical theory*** of fast neutrino flavor conversions in the ***nonlinear regime***.
- We showed fast conversions can bring different neutrino flavors ***close to each other*** (Flavor Depolarization) and ***irreversibility*** in the system.
- ***T2 relaxation*** and ***multipole diffusion*** governs such behaviour.
- We gave a strategy and a formula for computing the ***extent of flavor depolarization***



Phenomenological Consequences :

- Flavor depolarization can cause significant increse in ***neutrino heating rate*** and ***change the explosion scenario***.

- Including MSW conversions, propagation and eartheffects our formula will allow one to determine the ***final neutrino signal from a SN explosion***

$$F^{\text{fin}}_{\overset{(-)}{\nu}_e,\,\overset{(-)}{\nu}_\mu}[\vec{p}] = (1 - f^{\text{D}}_{\vec{p}})F^{\text{ini}}_{\overset{(-)}{\nu}_e,\,\overset{(-)}{\nu}_\mu}[\vec{p}] + f^{\text{D}}_{\vec{p}}F^{\text{ini}}_{\overset{(-)}{\nu}_\mu,\,\overset{(-)}{\nu}_e}[\vec{p}]$$

- This signal can be the first ever direct probe of testing the ***neutrino-neutrino self-interaction***.

- The final output of fast conversions can have implications even in the ***nucleosynthesis of elements***, astrophysics of ***binary neutron star mergers***, ***diffuse SN background*** and can be detected in future experiments.