Evolution of Mass-Mixing Parameters in Matter with Neutrino Non-Standard Interactions (NSIs)

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Evolution of oscillation parameters in matter

Motivation

- complicated analytical expression of oscillation probability in presence of matter (also with NSIs).
- understanding the features of the oscillation parameters will make easier for the analysis of existing and upcoming oscillation data .

- Effective Hamiltonian for neutrino propagation in presence NSIs:

$$\mathrm{H_f} = \Delta_{31}\left[\mathrm{U}\begin{pmatrix}0 & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & 1\end{pmatrix}\mathrm{U}^\dagger + \hat{A}\begin{pmatrix}1 & \varepsilon_{e\mu} & \varepsilon_{e\tau}\\ \varepsilon_{e\mu} & \beta & \varepsilon_{\mu\tau}\\ \varepsilon_{e\tau} & \varepsilon_{\mu\tau} & \gamma\end{pmatrix}\right],$$

$\beta = \varepsilon_{\mu\mu} - \varepsilon_{ee}$, $\gamma = \varepsilon_{\tau\tau} - \varepsilon_{ee}$,

$\hat{A} = \frac{2EV_{CC}}{\Delta m^2_{31}}, \alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$

NSI parameters	2σ bounds
$\varepsilon_{e\mu}$	$[-0.372, +0.301]$
$\varepsilon_{e\tau}$	$[-1.657, +0.732]$
$\varepsilon_{\mu\tau}$	$[-0.076, +0.058]$
β $(\varepsilon_{\mu\mu} - \varepsilon_{ee})$	$[-2.861, +0.144]$
γ $(\varepsilon_{\tau\tau} - \varepsilon_{ee})$	$[-2.892, +0.836]$

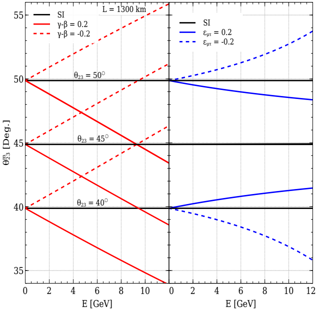
- Diagonalization by successive rotation:

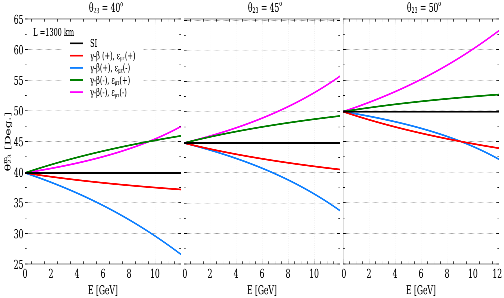
$$\tilde{U} = R_{23}(\theta^m_{23})R_{13}(\theta^m_{13})R_{12}(\theta^m_{12}).$$

Running of θ_{23}^m

- Rotation in (2,3) block:

$$\tan 2\theta_{23}^m \simeq \frac{(c_{13}^2 - \alpha c_{12}^2 + \alpha s_{12}^2 s_{13}^2)\sin 2\theta_{23} - \alpha \sin 2\theta_{12} s_{13}\cos 2\theta_{23} + 2\varepsilon_{\mu\tau}\hat{A}}{(c_{13}^2 - \alpha c_{12}^2 + \alpha s_{12}^2 s_{13}^2)\cos 2\theta_{23} + \alpha s_{13}\sin 2\theta_{12}\sin 2\theta_{23} + (\gamma-\beta)\hat{A}}$$





- Value of the oscillation parameters:

θ_{23}	θ_{13}	θ_{12}	δ	$\Delta m_{21}^2[\mathrm{eV}^2]$	$\Delta m_{31}^2[\mathrm{eV}^2]$
40°, 45°, 50°	8.5°	33°	0	7.5×10^{-5}	2.44×10^{-3}

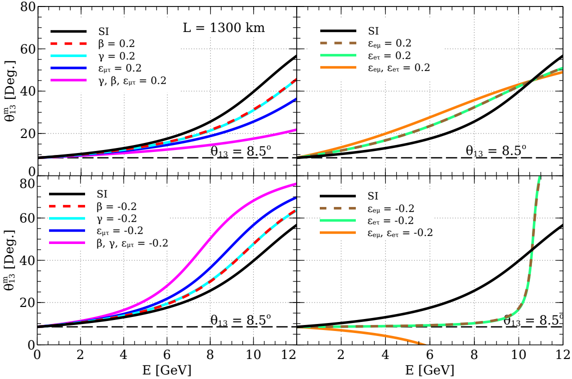
Running of θ_{13}^m

- Rotation in (1,3) block:

$$\tan 2\theta_{13}^m \simeq \frac{\sin 2\theta_{13}(1-\alpha {s_{12}}^2)(c_{23}^m + s_{23}^m) - \alpha \sin 2\theta_{12} c_{13}(c_{23}^m - s_{23}^m) + 2\sqrt{2}(\varepsilon_{e\mu} s_{23}^m + \varepsilon_{e\tau} c_{23}^m)\hat{A}}{\sqrt{2}(\lambda_3 - \hat{A} - \alpha s_{12}^2 c_{13}^2 - s_{13}^2)}$$

$$\lambda_3 \simeq \frac{1}{2}[{c_{13}}^2 + \alpha {c_{12}}^2 + \alpha s_{12}^2 s_{13}^2 + (\beta+\gamma)\hat{A} + \frac{(\gamma-\beta)\hat{A} + \alpha \sin 2\theta_{12} s_{13}}{\cos 2\theta_{23}^m}]$$

- β, γ, $\varepsilon_{\mu\tau}$ with +ve (-ve) strength suppress (enhance) running of θ_{13}^m
- $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ do not show any effect at the region near θ_{13} - resonance.



Running of θ_{12}^m

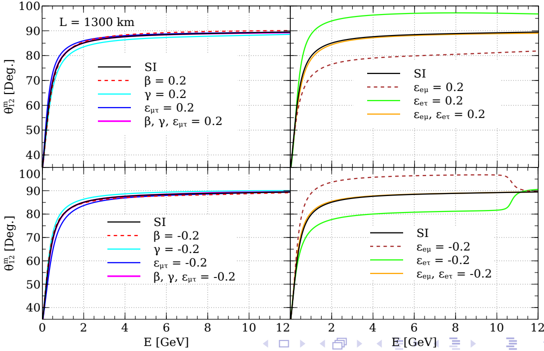
- Rotation in (1,2) block:

$$\tan 2\theta_{12}^m \simeq \frac{c_{13}^m\left[\alpha \sin 2\theta_{12} c_{13}(c_{23}^m + s_{23}^m) + \sin 2\theta_{13}(1-\alpha s_{12}^2)(c_{23}^m - s_{23}^m) + 2\sqrt{2}(\varepsilon_{e\mu} c_{23}^m - \varepsilon_{e\tau} s_{23}^m)\hat{A}\right]}{\sqrt{2}(\lambda_2 - \lambda_1)}$$

$$\lambda_2 \simeq \frac{1}{2}\left[\alpha c_{12}^2 + c_{13}^2 + \alpha s_{12}^2 s_{13}^2 + (\beta+\gamma)\hat{A} - \frac{(\gamma-\beta)\hat{A} + \alpha \sin 2\theta_{12} s_{13}}{\cos 2\theta_{23}^m}\right]$$

$$\lambda_1 \simeq \frac{1}{2}\left[\lambda_3 + \hat{A} + s_{13}^2 + \alpha s_{12}^2 c_{13}^2\right]\left[1 - \frac{1}{\cos 2\theta_{23}^m}\right].$$

- θ_{12}^m saturates to $\pi/2$ at low energy in SI case and NSIs from $\mu - \tau$ sector.
- $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ shift the value of saturation point up or down.

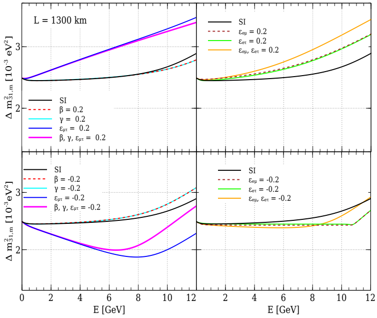


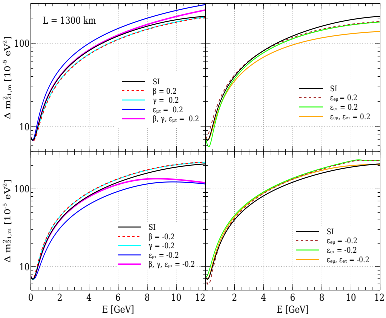
Running of mass-squared differences

$$\frac{(m_{3,m})^2}{2E} \simeq \frac{\Delta_{31}}{2}\left[\lambda_3 + \hat{A} + s_{13}^2 + \alpha s_{12}^2 c_{13}^2 + \frac{\lambda_3 - \hat{A} - s_{13}^2 - \alpha s_{12}^2 c_{13}^2}{\cos 2\theta_{13}^m}\right]$$

$$\frac{(m_{2,m})^2}{2E} \simeq \frac{\Delta_{31}}{2}\left[\lambda_1 + \lambda_2 - \frac{\lambda_1 - \lambda_2}{\cos 2\theta_{12}^m}\right], \qquad \frac{(m_{1,m})^2}{2E} \simeq \frac{\Delta_{31}}{2}\left[\lambda_1 + \lambda_2 + \frac{\lambda_1 - \lambda_2}{\cos 2\theta_{12}^m}\right]$$

- Modified mass-squared differences:
  $\Delta m_{31,m}^2 = m_{3,m}^2 - m_{1,m}^2$, $\Delta m_{21,m}^2 = m_{2,m}^2 - m_{1,m}^2$

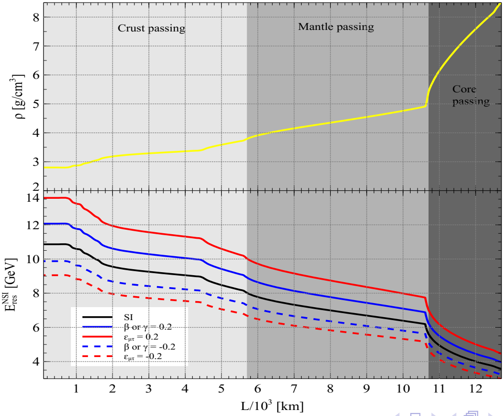




θ_{13} - resonance in presence of NSIs

- At resonance $\theta_{13}^m = 45^\circ$
- Resonance energy:

$$E_{\text{res}}^{\text{NSI}} \simeq \underbrace{\frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2V_{CC}}}_{E_{res}^{SI}} \left[\frac{1 - (\alpha s_{12}^2 c_{13}^2 / \cos 2\theta_{13})}{1 - \frac{1}{2}(\beta + \gamma + 2\varepsilon_{\mu\tau})} \right]$$



- With the approximation: $\theta_{12}^m \to \frac{\pi}{2}$

$$P_{\mu e}^m \simeq \underbrace{\sin^2\theta_{23}^m}_{T_1}\,\underbrace{\sin^2 2\theta_{13}^m}_{T_2}\,\underbrace{\sin^2\left[\frac{1.27\times\Delta m^2_{32,m}L}{E}\right]}_{T_3}.$$

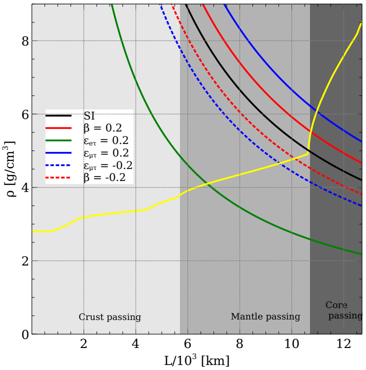
- $T_2 \to$ maximum at $E = E_{res}$,
  $T_3 \to$ maximum at $E = E_{max} \simeq \frac{1.27\times\Delta m^2_{32,m}L[km]}{(2n+1)\pi/2}$ with $n = 0, 1, 2...$
  $P_{\mu e}^m$ is maximum at $E_{res} = E_{max}$
- Baseline length corresponds to the maximum $\nu_\mu \to \nu_e$ appearance probability:

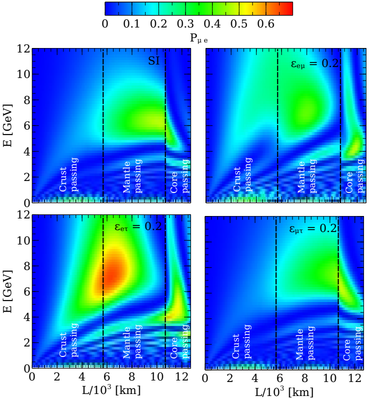
$$(\rho L)^{max}_{\rm NSI} \simeq \frac{\overbrace{(2n+1)\pi\times 5.18\times 10^3/\tan 2\theta_{13}}^{\text{SI case}}}{1-\underbrace{\left\{(\beta+\gamma+2\varepsilon_{\mu\tau})\left(\frac{c_{23}^m+s_{23}^m}{2\sqrt{2}}\right)\right\}+\left\{2(\varepsilon_{e\mu}s_{23}^m+\varepsilon_{e\tau}c_{23}^m)/\tan 2\theta_{13}\right\}}_{\text{Correction due to NSI}}}\,\text{km}\left[\frac{\rm g}{\rm cm^3}\right]$$

Imapact of NSIs in $\nu_\mu \to \nu_e$ appearance channel

- Assuming $\theta_{23}^m \approx 45^\circ$ for simplified analysis:

$$(\rho L)^{max}_{NSI} \simeq (\rho L)^{max}_{SI} \times \frac{1}{1 - \{(\beta + \gamma + 2\varepsilon_{\mu\tau})\}/2 + \{\sqrt{2}(\varepsilon_{e\mu} + \varepsilon_{e\tau})/\tan 2\theta_{13}\}} \text{km} \left[\frac{\text{g}}{\text{cm}^3}\right].$$



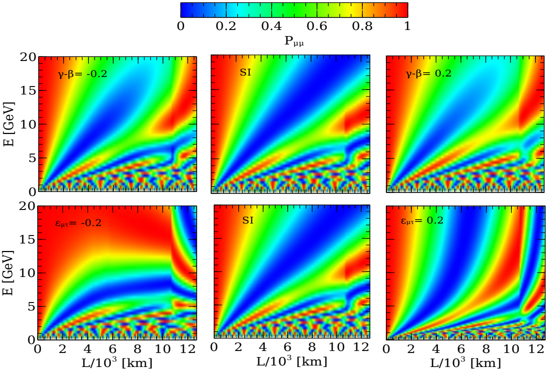


Impact of NSIs of $\nu_\mu \to \nu_\mu$ survival channel

- With the approximation $\Delta m^2_{21} L/4E << \Delta m^2_{31} L/4E$ and $\theta_{13} \to 0$

$$P^m_{\mu\mu} \simeq 1 - \sin^2 2\theta^m_{23} \sin^2 \left[\frac{\Delta m^2_{31,m} L}{4E}\right].$$

$$P^m_{\mu\mu} \approx 1 - \left[1 - \frac{(\gamma-\beta)^2 \hat{A}^2}{(1+2\varepsilon_{\mu\tau}\hat{A})^2}\right] \times \sin^2 \left[\left\{1 + 2\varepsilon_{\mu\tau}\hat{A} + \frac{1}{2}\frac{(\gamma-\beta)^2\hat{A}^2}{(1+2\varepsilon_{\mu\tau}\hat{A})}\right\}\frac{\Delta m^2_{31} L}{4E}\right]$$



Summary

- Running of θ_{23} in matter depends only on the NSIs parameters from the $\mu - \tau$ sector. In the absence of any NSIs, θ_{23} in matter is equal to θ_{23} in vacuum.
- All the six NSI parameters have impact on θ_{13}^m running. NSI parameters from $\mu - \tau$ sector suppress (enhance) the running when the parameter hs a positive (negative) strength.
- θ_{12}^m saturates to $\pi/2$ at very low energy in SI case as well as in presence of NSIs.
- At the near resonance region $\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$ does not have any impact on θ_{13} running.
- Baseline length required for maximum $\nu_\mu \to \nu_e$ transition probability is reduced significantly in presene of $\varepsilon_{e\mu}$ or $\varepsilon_{e\tau}$ with positive strength.
- NSI parameters from $\mu - \tau$ sector has most significant impact on the $\nu_\mu \to \nu_\mu$ survival channel. Sign of the diagonal NSI parameter does not affect the oscillation probability while sign of off-diagonal parameter $\varepsilon_{\mu\tau}$ has significant effect.

Thank You