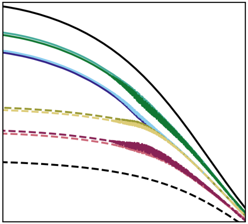
Breaking isotropy in the early Universe with neutrino oscillations



Rasmus S. L. Hansen

NBIA and DARK at the Niels Bohr Institute

in collaboration with Shashank Shalgar and Irene Tamborra

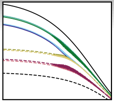
based on 2012.03948

XIX International Workshop on Neutrino Telescopes,
Padova/virtual, February 22, 2021

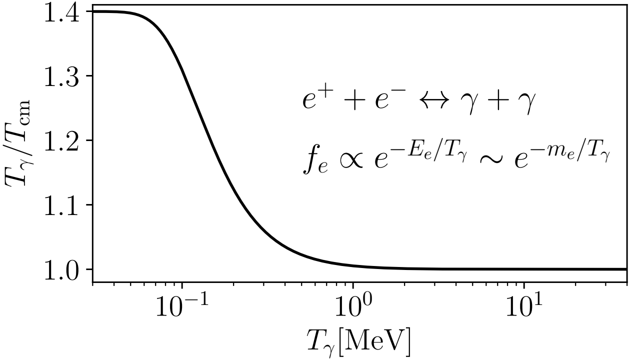
INTERACTIONS

Co-financed by the Connecting Europe
Facility of the European Union

Sapere Aude



Expansion of the Universe



Friedmann equation:

$$H = \sqrt{\frac{8\pi\varrho}{3}}\frac{1}{m_{\rm Pl}}\ ,$$

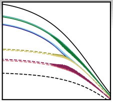
Continuity equation:

$$\dot\rho = -3H(\rho + P),$$

Neutrino oscillations

Equation of motion:

$$\frac{d\rho(\mathbf{p},\mathbf{x})}{dt} = -i\,[\mathcal{H}(\rho,\mathbf{p},\mathbf{x}),\rho(\mathbf{p},\mathbf{x})] + \mathcal{C}(\rho,\mathbf{p},\mathbf{x}) \ ,$$



Neutrino oscillations

Equation of motion:

$$\frac{d\rho(\mathbf{p},\mathbf{x})}{dt} = -i\left[\mathcal{H}(\rho,\mathbf{p},\mathbf{x}),\rho(\mathbf{p},\mathbf{x})\right] + \mathcal{C}(\rho,\mathbf{p},\mathbf{x}) \ ,$$

Hamiltonian:

$$\mathcal{H}(\rho,\mathbf{p},\mathbf{x}) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p} + \sqrt{2}G_{\mathrm{F}}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}(\rho(\mathbf{p}',\mathbf{x}) - \bar{\rho}^*(\mathbf{p}',\mathbf{x}))(1-\mathbf{v}'\cdot\mathbf{v})$$

vacuum term — asymmetric neutrino term

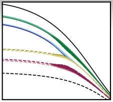
$$-\ \frac{8\sqrt{2}G_{\mathrm{F}}p}{4}\ \frac{\mathcal{E}_l + \frac{1}{3}\mathcal{P}_l}{m_W^2}$$

symmetric matter term

Homogeneous universe model with two angle bins

Assumptions:

- Universe is homogeneous.
- Angular dependence approximated with two angle bins.
- Two neutrino oscillation framework.
- Relaxation time like approximation for the collision term.



Linear stability analysis

μ - Neutrino term.

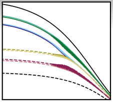
ω_λ - Matter and vacuum term.

$s = \rho_{ee} - \rho_{xx}$ - Difference between ν_e and ν_x densities.

Eigenvalues:

$$\Omega_1^\pm = \frac{1}{2}\left(3\mu(s-\bar{s}) \pm \sqrt{-4\omega_\lambda\mu(s+\bar{s}) + 4\omega_\lambda^2 + \mu^2(s-\bar{s})^2}\right),$$

$$\Omega_2^\pm = \frac{1}{2}\left(\mu(s-\bar{s}) \pm \sqrt{4\omega_\lambda\mu(s+\bar{s}) + 4\omega_\lambda^2 + \mu^2(s-\bar{s})^2}\right).$$



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Eigenvectors:

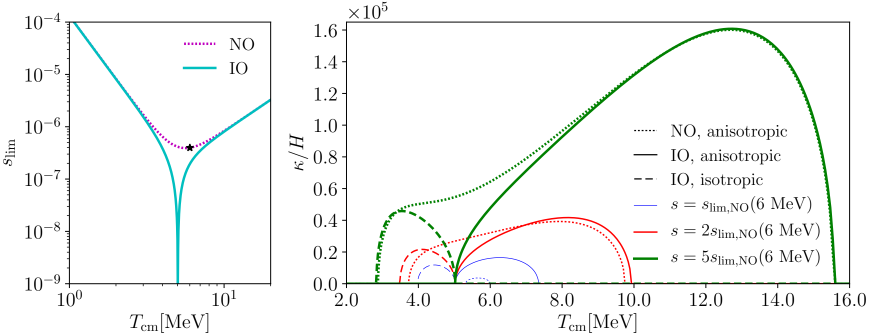
$$w_1^{\pm T} = \left(-\frac{-\mu(s-2\bar{s})-\omega_\lambda+\Omega_1^\pm}{\mu\bar{s}}, \quad -1, \quad +\frac{-\mu(s-2\bar{s})-\omega_\lambda+\Omega_1^\pm}{\mu\bar{s}}, \quad +1\right)^T,$$

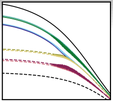
$$w_2^{\pm T} = \left(+\frac{\mu s+\omega_\lambda-\Omega_2^\pm}{\mu\bar{s}}, \quad +1, \quad +\frac{\mu s+\omega_\lambda-\Omega_2^\pm}{\mu\bar{s}}, \quad +1\right)^T.$$

In the basis: $(\epsilon_R, \bar{\epsilon}_R, \epsilon_L, \bar{\epsilon}_L)^T$.

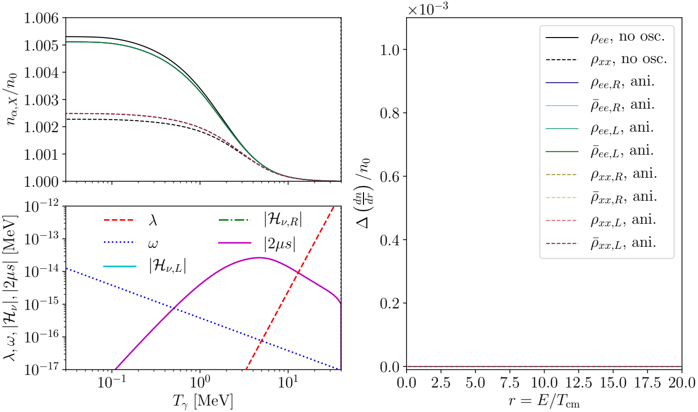
Linear stability analysis

$$s_{\rm lim} = \left| \frac{56\pi^4}{270\zeta(3) m_W^2} \langle p \rangle \, T_{\rm cm} + \frac{\sqrt{2}\pi^2 \Delta m^2}{6\zeta(3) G_{\rm F}} \frac{1}{\langle p \rangle \, T_{\rm cm}^3} \right| .$$





Isotropic initial conditions, NO



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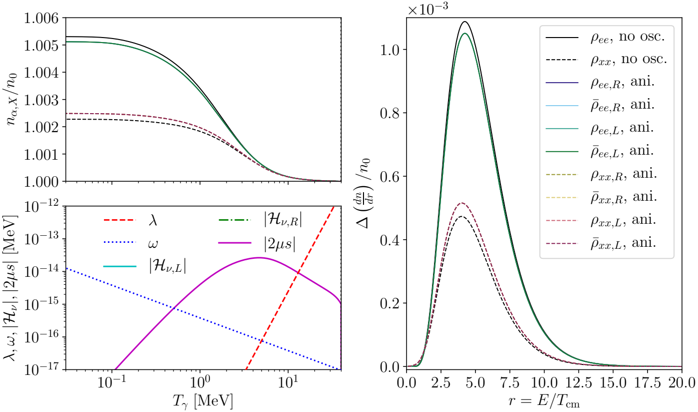

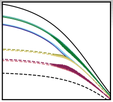

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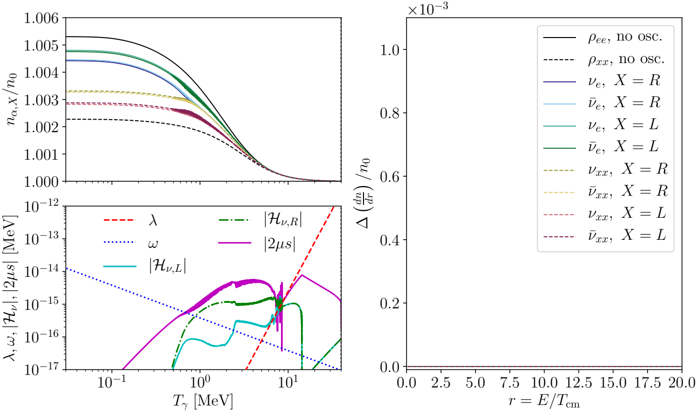

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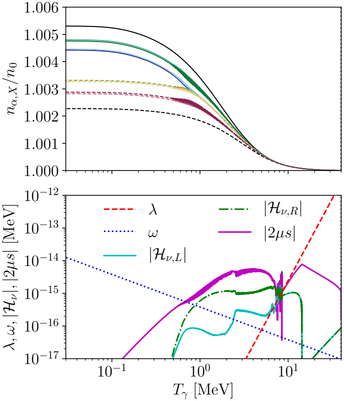
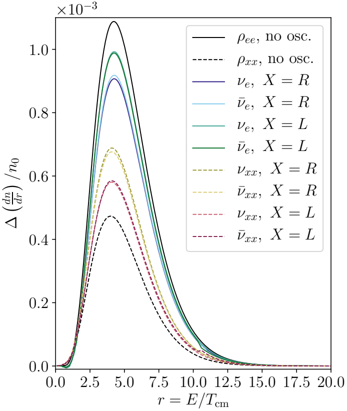




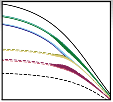
Change in $N_{\rm eff}$

$$
\begin{aligned}
N_{\rm eff} &\equiv \frac{\rho_\nu}{7/8\rho_\gamma}\left(\frac{11}{7}\right)^3 \\
&\approx \left(\frac{\int dr\ r^3(\rho_{ee}+\bar{\rho}_{ee}+\rho_{xx}+\bar{\rho}_{xx})}{2\int drr^3 f_0}+1\right)\frac{(11/7)^3}{(T_\gamma/T_{\rm cm})^4}\ .
\end{aligned}
$$

For no neutrino oscillations, $N_{\rm eff} = 3.04596$.

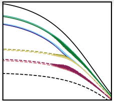
The cases with oscillations give:

	NO, iso	NO, ani	NO, ani ($\mu_{\rm ini} = 10^{-9}$)	IO, iso/ani
$\Delta N_{\rm eff}$	0.9×10^{-4}	5.0×10^{-4}	4.9×10^{-4}	5.8×10^{-4}



Summary

- Neutrino-neutrino forward scattering cannot be neglected for neutrino oscillations in the early Universe even when $n_{\nu_\alpha} \approx n_{\bar{\nu}_\alpha}$.
- Our model with two angle bins show substantial anisotropy for NO, and we find a correction to N_{eff} comparable to higher order QED corrections.
- A small neutrino-antineutrino asymmetry is amplified by many orders of magnitude through non-linear neutrino flavor evolution.



Homogeneous universe model with two angle bins

$$\frac{\partial \rho_R(p)}{\partial t} - Hp\frac{\partial \rho_R(p)}{\partial p} = -i[\mathcal{H}_R(\rho_R, \rho_L, p), \rho_R] + \mathcal{C}_R(\rho_R, \rho_L, p) \ ,$$
$$\frac{\partial \rho_L(p)}{\partial t} - Hp\frac{\partial \rho_L(p)}{\partial p} = -i[\mathcal{H}_L(\rho_R, \rho_L, p), \rho_L] + \mathcal{C}_L(\rho_R, \rho_L, p) \ ,$$

Hamiltonian:

$$\mathcal{H}_R(\rho_R, \rho_L, p) = \frac{\mathcal{U}\mathcal{M}^2\mathcal{U}^\dagger}{2p} + \sqrt{2}G_{\mathrm{F}} \int \frac{dp'}{2\pi^2}(\rho_L(p') - \bar{\rho}_L^*(p')) - \frac{8\sqrt{2}G_{\mathrm{F}}\,p}{3}\frac{\mathcal{E}_l}{m_W^2} \ ,$$

Collision term - approximations

Divide into four different types of reactions each with a rate:

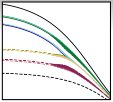
1. Scattering with electrons and positrons, $\Gamma_{s,\alpha}$
2. Annihilations to electrons and positrons, $\Gamma_{a,\alpha}$
3. Neutrino-neutrino scatterings, $\Gamma_{\nu\nu}$
4. Neutrino-antineutrino collisions, $\Gamma_{\nu\bar{\nu}}$

Equilibrium distributions are assumed when calculating the rates.

Functional form: One equilibrium distribution in each gain term.

$$f(T_{\mathrm{eq}}, \mu) = \frac{1}{\exp(p/T_{\mathrm{eq}} - \mu/T_{\mathrm{eq}}) + 1} \,,$$

All other distribution functions are represented by normalized energy densities, $u_{\alpha\beta}$.



Collision term - approximations

Divide into four different types of reactions each with a rate:

1. Scattering with electrons and positrons, $\Gamma_{s,\alpha}$, $T_{\rm eq} = T_\gamma$, $\mu = \pi_\alpha$
2. Annihilations to electrons and positrons, $\Gamma_{a,\alpha}$, $T_{\rm eq} = T_\gamma$, $\mu = \mu_\alpha$
3. Neutrino-neutrino scatterings, $\Gamma_{\nu\nu}$, $T_{\rm eq} = T_{\nu_\alpha}$, $\mu = \pi_{\nu_\alpha}$
4. Neutrino-antineutrino collisions, $\Gamma_{\nu\bar{\nu}}$, $T_{\rm eq} = T_{\nu_\alpha}$, $\mu = \pi_{\nu_\alpha}$

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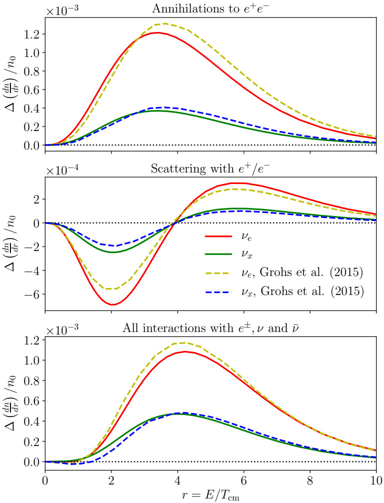
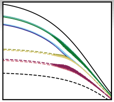
Collision term

For electron neutrinos:

$$
\begin{aligned}
\mathcal{C}_{ee} = \Gamma_{a,e}\left[\left(\frac{T_\gamma}{T_{\rm cm}}\right)^4 f(T_\gamma,\mu_e) - \rho_{ee}\right] \\
+ \Gamma_{s,e}\left[f(T_\gamma,\pi_e) - \rho_{ee}\right] \\
- \Gamma_G \mathrm{Re}\left(\bar{u}_{ex}\rho^*_{ex}\right) \\
+ \Gamma_{\nu\nu}(2u_{ee} + u_{xx})\left(f_{\nu_e} - \rho_{ee}\right) \\
+ \Gamma_{\nu\bar{\nu}}(4\bar{u}_{ee} + \bar{u}_{xx})\left(f_{\nu_e} - \rho_{ee}\right) \\
+ \Gamma_{\nu\bar{\nu}}\left(\bar{u}_{xx} f_{\nu_x} - \bar{u}_{ee}\rho_{ee}\right) \\
+ \mathrm{Re}\left[\left(\Gamma_{\nu\nu}u^*_{ex} + 4\Gamma_{\nu\bar{\nu}}\bar{u}^*_{ex}\right)\left(f_{ex} - \rho_{ex}\right)\right],
\end{aligned}
$$





Collision term

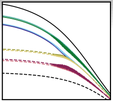
For the off-diagonal:

$$f_{ex} = (a_x + b_x p) f_0 + i(a_y + b_y p) f_0 \ ,$$

a_x, b_x, a_y and b_y conserve the first and second moments of ρ_{ex}.

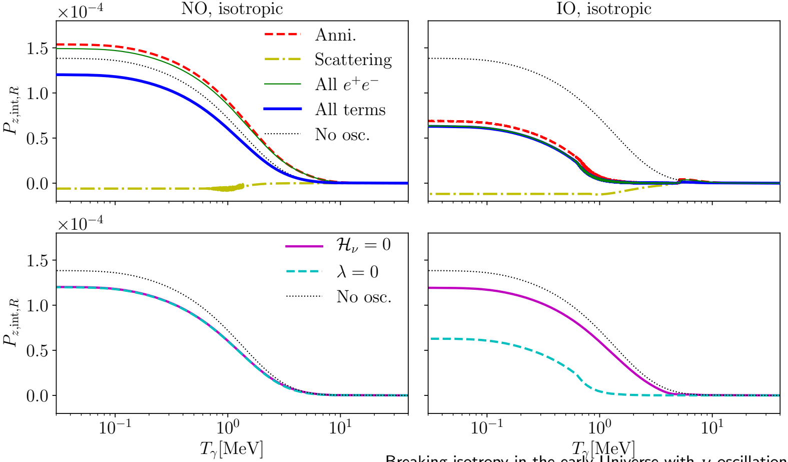
Off-diagonal:

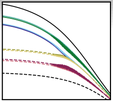
$$\begin{aligned}\mathcal{C}_{ex} = &-D\rho_{ex} + d\ f_{ex} - C\,\bar{u}_{ex}(\rho_{ee} + \rho_{xx}) \\ &+ \Gamma_{\nu\bar{\nu}}(\bar{u}_{ee} + \bar{u}_{xx})(2f_{ex} - 3\rho_{ex}) + \Gamma_{\nu\bar{\nu}}\bar{u}_{ex}((f_{\nu_e} + f_{\nu_x}) - 2(\rho_{ee} + \rho_{xx})) \\ &+ \frac{3}{2}\Gamma_{\nu\nu}(u_{ee} + u_{xx})(f_{ex} - \rho_{ex}) + \frac{1}{2}\Gamma_{\nu\nu}u_{ex}((f_{\nu_e} + f_{\nu_x}) - (\rho_{ee} + \rho_{xx}))\end{aligned}$$



Effects of collisions and potentials

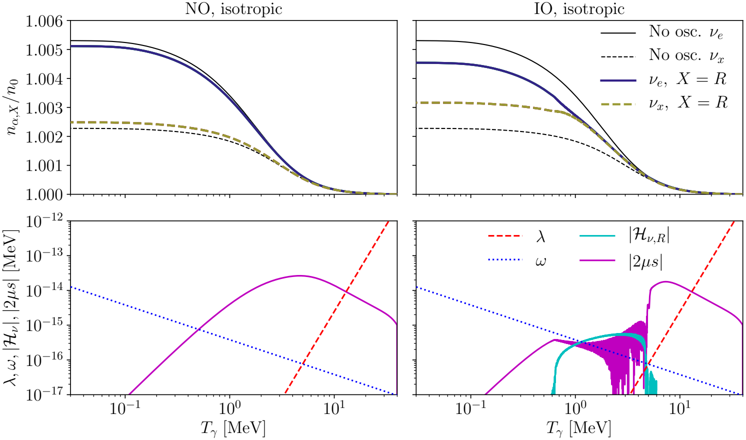
$$P_{z,\mathrm{int},\mathrm{X}} = \int \frac{dr}{4\pi^2} P_{z,X}(r) \qquad \text{for} \quad X \in \{R, L\}$$

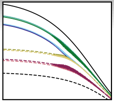




Isotropic initial conditions

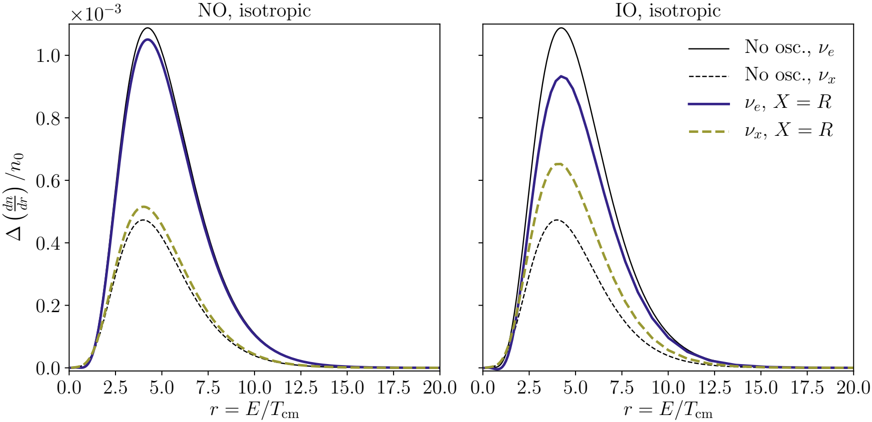
$$n_\alpha = \int \frac{dp}{2\pi^2} p^2 \rho_{\alpha\alpha}, \qquad n_0 = \int \frac{dp}{2\pi^2} p^2 f_0(p)$$





Isotropic initial conditions - spectrum

$$\Delta\left(\frac{dn}{dr}\right) = \frac{T_{\rm cm}}{2\pi^2}p^2(\rho_{\alpha\alpha} - f_0), \qquad n_0 = \int \frac{dp}{2\pi^2}p^2 f_0(p)$$



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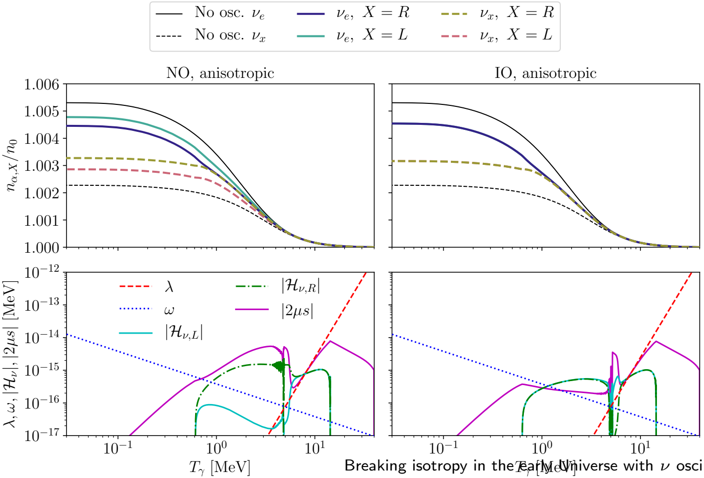

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