Abstract

In the constrained sequential dominance (CSD), tri-bimaximal mixing (TBM) pattern in the neutrino sector has been explained, by proposing a certain Yukawa coupling structure for the right-handed neutrinos of the model. However, from the current experimental data it is known that the values of neutrino mixing angles are deviated from the TBM values. In order to explain this neutrino mixing, we first propose a phenomenological model where we consider Yukawa couplings which are modified from that of CSD. Essentially, we add small complex parameters to the Yukawa couplings of CSD. Using these modified Yukawa couplings, we demonstrate that neutrino mixing angles can deviate from their TBM values. We also construct a model, based on a flavor symmetry, in order to justify the modified form of Yukawa couplings of our work.

Neutrino Mixing by Modifying the Yukawa Coupling Structure of Constrained Sequential Dominance

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March 10, 2021

Plan of the Talk

- Introduction to Sequential Dominance and Constrained Sequential Dominance (CSD),
- Our model and deviation from CSD.
- Numerical Analysis.
- Our flavor model.

Sequential Dominance and CSD

Assuming the charged lepton mass matrix to be diagonal, we add three right handed neutrinos ν_R^{atm}, ν_R^{sol} and ν_R^{dec} to the Standard Model. Yukawa Lagrangian for neutrino mass is

$$\mathcal{L}^{Yuk} = (\tfrac{H_u}{v_u})(d\overline{L}_e + e\overline{L}_\mu + f\overline{L}_\tau)\nu_R^{atm} + (\tfrac{H_u}{v_u})(a\overline{L}_e + b\overline{L}_\mu + c\overline{L}_\tau)\nu_R^{sol} \\ +(\tfrac{H_u}{v_u})(a'\overline{L}_e + b'\overline{L}_\mu + c'\overline{L}_\tau)\nu_R^{dec} + H.c. \tag{1}$$

Majorana Lagrangian is given by

$$\mathcal{L}_\nu^{\mathcal{M}} = M_{sol}\overline{\nu_R^{sol}}(\nu_R^{sol})^c + M_{atm}\overline{\nu_R^{atm}}(\nu_R^{atm})^c + M_{dec}\overline{\nu_R^{dec}}(\nu_R^{dec})^c. \tag{2}$$

So,

$$M_R = \begin{pmatrix} M_{atm} & 0 & 0 \\ 0 & M_{sol} & 0 \\ 0 & 0 & M_{dec} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}, \tag{3}$$

$$m^\nu = m_D M_R^{-1} m_D^T. \tag{4}$$

Sequential Dominance and CSD

$$M_{\rm atm} \ll M_{\rm sol} \ll M_{\rm dec}, \quad \frac{(e,f)^2}{M_{\rm atm}} \gg \frac{(a,b,c)^2}{M_{\rm sol}} \gg \frac{(a',b',c')^2}{M_{\rm dec}}. \tag{5}$$

- Third column of m_D and m_R can be decoupled with the above mentioned conditions.
- Three right handed neutrino model $\implies$ Two right handed neutrino model.
- m_D and M_R take the form

$$M_R = \begin{pmatrix} M_{atm} & 0 \\ 0 & M_{sol} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a \\ e & b \\ f & c \end{pmatrix}$$

$$d = 0, \quad e = f, \quad , a = b = -c. \tag{6}$$

After performing above mentioned decoupling and above condition, Dirac and Majorana mass matrices take the form

$$m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{atm} & 0 \\ 0 & M_{sol} \end{pmatrix}. \tag{7}$$

Putting this m_D and M_R in m_ν of Eq.(4), we find

$$U_{TBM}^T m_\nu U_{TBM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{atm}} \end{pmatrix}, U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{8}$$

Our Model and Deviation from CSD

We consider a phenomenological model where we modified the Dirac mass matrix as

$$m'_D = m_D + \Delta m_D, \quad m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad \Delta m_D = \begin{pmatrix} e\epsilon_1 & a\epsilon_4 \\ e\epsilon_2 & a\epsilon_5 \\ e\epsilon_3 & a\epsilon_6 \end{pmatrix}, \tag{9}$$

Here ϵ_i, i=1..6 are complex parameters. Hence the seesaw formula for active neutrinos

$$m^s_\nu = m'_D M_R^{-1} (m'_D)^T \tag{10}$$

Our Model and Deviation from CSD

- since we are in a basis where charged leptons are diagonalized, this form of mass matrix should be diagonalized by PMNS matrix which in terms of PDG convention is

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix} \tag{11}$$

  Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.
- In order to simplify our calculations we parameterize s_{12} and s_{23} as

$$s_{12} = \frac{1}{\sqrt{3}}(1+r), \quad s_{23} = \frac{1}{\sqrt{2}}(1+s) \tag{12}$$

- The ranges for r and s can be found as $(-8.8\times 10^{-2}, 2.5\times 10^{-2})$ and $(-8.2\times 10^{-2}, 0.13)$. The ranges for s_{13} is also around 0.15. So, if we allow non-zero ϵ_i in our model, we can get non-zero r, s, s_{13}.

Our Model and Deviation from CSD

- The relation for diagonalization of the seesaw mass matrix is

$$m_\nu^d = U_{PMNS}^T m_\nu^s U_{PMNS} = diag(m_1, m_2, m_3) \tag{13}$$

- Expanding m_ν^s and U_{PMNS} in power series of ϵ_i, r, s, s_{13} which are small, one can see that m_ν^d need not be in diagonal form. Therefore we demand off-diagonal terms to be zero giving ϵ_i in terms of r, s, s_{13}. Now from the diagonal terms of m_ν^d we can get the three neutrino masses in terms of the model parameters.

Our Model and Deviation from CSD

In limit where ϵ_i, r, s, s_{13} tend to zero, we get the leading order expressions

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}}, \quad m_3 = \frac{2e^2}{M_{atm}}. \tag{14}$$

- m_1 is zero at leading order. m_1 will also zero at sub-leading orders. This is a consequence of the fact that two right handed neutrino model is proposed.
- So, we can have only normal mass hierarchy. Then m_2 and m_3 can be fitted into $\sqrt{\Delta m^2_{sol}}$ and $\sqrt{\Delta m^2_{atm}}$.
- We can fit

$$\frac{a^2}{M_{sol}} \sim \sqrt{\Delta m^2_{sol}}, \quad \frac{e^2}{M_{atm}} \sim \sqrt{\Delta m^2_{atm}}. \tag{15}$$

- It is noticed $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \sim s_{13}$.

We re-express our diagonalization formula

$$\frac{1}{\sqrt{\Delta m^2_{atm}}} m^d_\nu \equiv \frac{1}{\sqrt{\Delta m^2_{atm}}} (U^T_{PMNS} m^s_\nu U_{PMNS}) = diag(\frac{m_1}{\sqrt{\Delta m^2_{atm}}}, \frac{m_2}{\sqrt{\Delta m^2_{atm}}}, \frac{m_3}{\sqrt{\Delta m^2_{atm}}}). \tag{16}$$

So, $\frac{1}{\sqrt{\Delta m^2_{atm}}} m^d_\nu$ can be expanded in power series of ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$.

First Order Correction

- Up to first order in ϵ_i, m_ν^s can be expanded as

$$m_\nu^s = m^s_{\nu(0)} + m^s_{\nu(1)}, \tag{17}$$
$$m^s_{\nu(0)} = m_D M_R^{-1} m_D^T, \quad m^s_{\nu(1)} = m_D M_R^{-1} (\Delta m_D)^T + \Delta m_D M_R^{-1} m_D^T$$

- Similarly, up to first order in r, s and s_{13}, the expansion for $U_{\rm PMNS}$ is

$$U_{\rm PMNS} = U_{\rm TBM} + \Delta U, \tag{18}$$
$$\Delta U = \begin{pmatrix} -\frac{r}{\sqrt{6}} & \frac{r}{\sqrt{3}} & e^{-i\delta_{\rm CP}} s_{13} \\ \frac{-r+s}{\sqrt{6}} - \frac{e^{i\delta_{\rm CP}} s_{13}}{\sqrt{3}} & -\frac{r+2s+\sqrt{2}e^{i\delta_{\rm CP}} s_{13}}{2\sqrt{3}} & \frac{s}{\sqrt{2}} \\ \frac{r+s}{\sqrt{6}} - \frac{e^{i\delta_{\rm CP}} s_{13}}{\sqrt{3}} & \frac{r-2s-\sqrt{2}e^{i\delta_{\rm CP}} s_{13}}{2\sqrt{3}} & -\frac{s}{\sqrt{2}} \end{pmatrix}$$

First Order Correction

Terms up to first order in $\frac{1}{\sqrt{\Delta m^2_{\rm atm}}} m^d_\nu$ are given below.

$$\frac{1}{\sqrt{\Delta m^2_{\rm atm}}} m^d_\nu = \frac{1}{\sqrt{\Delta m^2_{\rm atm}}} \left(m^d_{\nu(0)} + m^d_{\nu(1)} \right),$$

$$m^d_{\nu(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{\rm sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{\rm atm}} \end{pmatrix}, \quad m^d_{\nu(1)} = \begin{pmatrix} x'_{11} & x'_{12} & x'_{13} \\ x'_{12} & x'_{22} & x'_{23} \\ x'_{13} & x'_{23} & x'_{33} \end{pmatrix},$$

$$x'_{11} = 0, \quad x'_{12} = 0,$$

$$x'_{13} = \frac{e^2}{\sqrt{6} M_{\rm atm}} [\sqrt{2}(2\epsilon_1 - \epsilon_2 + \epsilon_3 + 2s) - 4e^{i\delta_{\rm CP}} s_{13}], \quad x'_{22} = 0,$$

$$x'_{23} = \frac{e^2}{\sqrt{3} M_{\rm atm}} [\sqrt{2}(\epsilon_1 + \epsilon_2 - \epsilon_3 - 2s) - 2e^{i\delta_{\rm CP}} s_{13}],$$

$$x'_{33} = \frac{2e^2}{M_{\rm atm}}.(\epsilon_2 + \epsilon_3) \tag{19}$$

First Order Correction

- Now, equating the diagonal elements on both sides of the diagonalization formula we get the expressions for the three neutrino masses, which are

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{\rm sol}}, \quad m_3 = \frac{2e^2}{M_{\rm atm}} + \frac{2e^2(\epsilon_2 + \epsilon_3)}{M_{\rm atm}} \tag{20}$$

- From the above equations we can see that only m_3 get correction at the first order level.
- Now, from the off-diagonal elements, we get the following expressions.

$$\epsilon_1 = \sqrt{2}e^{i\delta_{\rm CP}} s_{13}, \quad \epsilon_2 - \epsilon_3 = 2s \tag{21}$$

- From the above two equations we can see that, in our model, $\sin\theta_{13}$ will be non-zero if we take $\epsilon_1 \neq 0$.

Second Order Correction

Expansion for m_ν^s and $U_{\rm PMNS}$, up to second order in ϵ_i, r, s and s_{13} are given below

$$m_\nu^s = m_{\nu(0)}^s + m_{\nu(1)}^s + m_{\nu(2)}^s, \quad m_{\nu(2)}^s = \Delta m_D M_R^{-1} (\Delta m_D)^T, \tag{22}$$

$$U_{PMNS} = U_{TBM} + \Delta U + \Delta^2 U, \tag{23}$$

$$\Delta^2 U = \begin{pmatrix} -\frac{2s_{13}^2+r^2}{2\sqrt{6}} & -\frac{s_{13}^2}{2\sqrt{3}} & 0 \\ \frac{2s_{13}e^{i\delta_{\rm CP}}(r-2s)+\sqrt{2}(2rs+s^2)}{4\sqrt{3}} & \frac{-r^2+2rs-2s^2-2\sqrt{2}s_{13}e^{i\delta_{\rm CP}}(r+s)}{4\sqrt{3}} & -\frac{s_{13}^2+s^2}{2\sqrt{2}} \\ \frac{s_{13}e^{i\delta_{\rm CP}}(r+2s)+\sqrt{2}rs}{2\sqrt{3}} & \frac{r^2-2\sqrt{2}s_{13}e^{i\delta_{\rm CP}}(r-s)+2rs}{4\sqrt{3}} & -\frac{s_{13}^2+s^2}{2\sqrt{2}} \end{pmatrix}.$$

- $\frac{1}{\sqrt{\Delta m_{\rm atm}^2}} m_\nu^d$ can be computed up to second order in ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m_{\rm sol}^2}{\Delta m_{\rm atm}^2}}$.

Second Order Correction

Now, after using Eq.(21), the second order terms in $\frac{1}{\sqrt{\Delta m^2_{\rm atm}}} m^d_\nu$ will be simplified. These are given below.

$$\frac{1}{\sqrt{\Delta m^2_{\rm atm}}} m^d_{\nu(2)} = \frac{1}{\sqrt{\Delta m^2_{\rm atm}}} \begin{pmatrix} x''_{11} & x''_{12} & x''_{13} \\ x''_{12} & x''_{22} & x''_{23} \\ x''_{13} & x''_{23} & x''_{33} \end{pmatrix},$$

$$\begin{aligned}
x''_{11} &= 0, \quad x''_{12} = \frac{a^2}{\sqrt{2} M_{\rm sol}}(2\epsilon_4 - \epsilon_5 + \epsilon_6 - 3r), \\
x''_{13} &= \frac{e^2}{\sqrt{3} M_{\rm atm}}[s(3s - 2\sqrt{2} e^{i\delta_{\rm CP}} s_{13}) + 2\epsilon_3(s - \sqrt{2} e^{i\delta_{\rm CP}} s_{13})], \\
x''_{22} &= \frac{2a^2}{M_{\rm sol}}(\epsilon_4 + \epsilon_5 - \epsilon_6), \quad x''_{33} = \frac{2e^2}{M_{\rm atm}}(\epsilon_3^2 + 2\epsilon_3 s + 2s^2 + s_{13}^2) \\
x''_{23} &= \frac{\sqrt{3} a^2}{2 M_{\rm sol}}[\sqrt{2}(\epsilon_5 + \epsilon_6 + 2s) + 2e^{-i\delta_{\rm CP}} s_{13}] \\
&\quad - \frac{e^2}{\sqrt{3} M_{\rm atm}}[2\epsilon_3(\sqrt{2} s + e^{i\delta_{\rm CP}} s_{13}) + s(3\sqrt{2} s + 2e^{i\delta_{\rm CP}} s_{13})], \quad (24)
\end{aligned}$$

Second Order Correction

Now, after equating the diagonal elements on both sides of Eq. (16), we get corrections up to second order to neutrino masses, which are given below.

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{\rm sol}} + \frac{2a^2}{M_{\rm sol}}(\epsilon_4 + \epsilon_5 - \epsilon_6),$$
$$m_3 = \frac{2e^2}{M_{\rm atm}} + \frac{4e^2}{M_{\rm atm}}(\epsilon_3 + s) + \frac{2e^2}{M_{\rm atm}}(s_{13}^2 + \epsilon_3^2 + 2\epsilon_3 s + 2s^2) \quad (25)$$

Second Order Correction

After demanding that the off-diagonal elements of $\frac{1}{\sqrt{\Delta m^2_{\rm atm}}} m_\nu^d$ should be zero, we get the following three relations.

$$
\begin{aligned}
&2\epsilon_4 - \epsilon_5 + \epsilon_6 = 3r, \\
&s(3s - 2\sqrt{2}e^{i\delta_{\rm CP}} s_{13}) + 2\epsilon_3(s - \sqrt{2}e^{i\delta_{\rm CP}} s_{13}) = 0, \\
&\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} e^{i\phi}[\sqrt{2}(\epsilon_5 + \epsilon_6 + 2s) + 2e^{-i\delta_{\rm CP}} s_{13}] \\
&-[2\epsilon_3(\sqrt{2}s + e^{i\delta_{\rm CP}} s_{13}) + s(3\sqrt{2}s + 2e^{i\delta_{\rm CP}} s_{13})] = 0. \qquad (26)
\end{aligned}
$$

Numerical Results

- the best fit values for the two mass-squared differences among the neutrinos, which are given below

$$\Delta m^2_{\rm sol} = 7.39 \times 10^{-5}\ {\rm eV}^2, \quad \Delta m^2_{\rm atm} = 2.525 \times 10^{-3}\ {\rm eV}^2. \qquad (27)$$

- In the analysis, we have varied the three neutrino mixing angles and the *CP* violating Dirac phase $\delta_{\rm CP}$ over the 3σ ranges. These ranges are

$$\sin^2\theta_{12} : 0.275 \rightarrow 0.350, \quad \sin^2\theta_{23} : 0.418 \rightarrow 0.627,$$
$$\sin^2\theta_{13} : 0.02045 \rightarrow 0.02439, \quad \delta_{\rm CP} : 125^{\rm o} \rightarrow 392^{\rm o}. \qquad (28)$$

- Since ϵ_i are complex, we have resolved them in to real and imaginary parts, whose expressions are given below.

$$\epsilon_i = Re(\epsilon_i) + iIm(\epsilon_i). \qquad (29)$$

Numerical Results

- In order to be compatible with neutrino oscillation observables, we have obtained the allowed ranges for $Re(\epsilon_i)$ and $Im(\epsilon_i)$.

$Re(\epsilon_1)$	$Im(\epsilon_1)$	$Re(\epsilon_2)$	$Im(\epsilon_2)$, $Im(\epsilon_3)$	$Re(\epsilon_3)$
$(-0.221, 0.221)$	$(-0.221, 0.182)$	$(-0.106, 0.225)$	$(-0.064, 0.064)$	$(-0.15, 0.095)$

ϕ	ϵ_4	$Re(\epsilon_5)$	$Im(\epsilon_5)$	$Re(\epsilon_6)$	$Im(\epsilon_6)$
0	0.1	$(-0.084, 0.462)$	$(-0.119, 0.101)$	$(-0.375, 0.168)$	$(-0.119, 0.101)$
0	-0.1	$(-0.282, 0.26)$	$(-0.119, 0.101)$	$(-0.175, 0.367)$	$(-0.119, 0.101)$
0	$0.1i$	$(-0.182, 0.362)$	$(-0.019, 0.199)$	$(-0.275, 0.267)$	$(-0.219, 0.001)$
0	$-0.1i$	$(-0.182, 0.362)$	$(-0.219, 0.001)$	$(-0.275, 0.267)$	$(-0.019, 0.199)$

Table: Allowed ranges for the real and imaginary parts of the ϵ_i parameters.

- The maximum values of $|\mathrm{Re}(\epsilon_5)|$ and $|\mathrm{Re}(\epsilon_6)|$can be around 0.4 depending on ϵ_4 and ϕ values.
- For this reason we have computed allowed values for neutrino mixing angles and δ_{CP} by restricting $|\mathrm{Re}(\epsilon_5)|$ and $|\mathrm{Re}(\epsilon_6)|$ to be less than 0.23 for the case of $\phi = 0$ and $\epsilon_4 = 0.1$.

Numerical Results

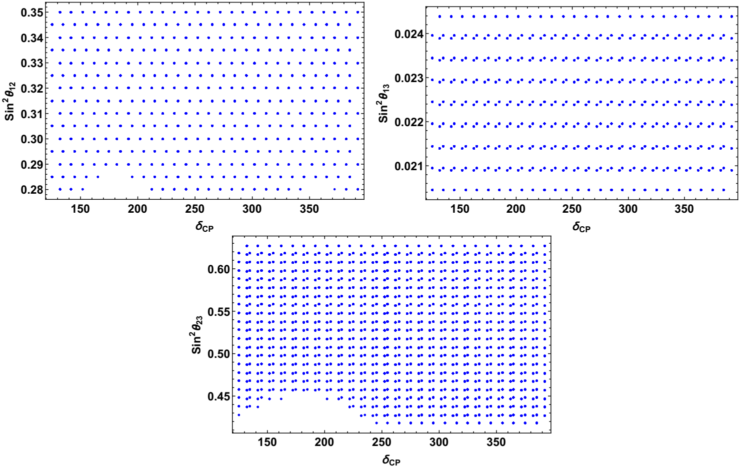

Figure: Allowed regions in neutrino mixing angles and $\delta_{\rm CP}$ by demanding $|Re(\epsilon_5)|$ and $|Re(\epsilon_6)|$ to be less than 0.23, for the case of $\phi = 0$ and $\epsilon_4 = 0.1$. $\delta_{\rm CP}$ is expressed in degrees.

A Model for the Dirac Mass matrix

Here we construct a model in order to justify our Dirac mass matrix and also to explain the smallness of ϵ_i.

	ϕ_a	ϕ_s	ϕ'_a	ϕ'_s	ξ	χ_a	χ_s	ν_R^{atm}	ν_R^{sol}	L	H
$SU(3)$	3	3	3	3	1	1	1	1	1	3	1
Z_3	ω	ω^2	ω	ω^2	1	ω^2	ω	ω^2	ω	1	1
Z'_3	ω^2	ω^2	ω	ω	ω	ω	ω	ω	ω	1	1

Table: Charge assignments of the relevant fields under the flavor symmetry $SU(3) \times Z_3 \times Z'_3$ are given. Here, $\omega = e^{2\pi i/3}$. For other details, see the text.

With these charge assignments, the leading terms in the Lagrangian are

$$\mathcal{L} = \frac{\phi_a}{M_P}\bar{L}\nu_R^{atm}H + \frac{\phi_s}{M_P}\bar{L}\nu_R^{sol}H + \frac{\xi}{M_P}\frac{\phi'_a}{M_P}\bar{L}\nu_R^{atm}H + \frac{\xi}{M_P}\frac{\phi'_s}{M_P}\bar{L}\nu_R^{sol}H + \frac{\chi_a}{2}\overline{(\nu_R^{atm})^c}\nu_R^{atm} + \frac{\chi_s}{2}\overline{(\nu_R^{sol})^c}\nu_R^{sol} + h.c. \tag{30}$$

Here, $M_P \sim 2 \times 10^{18}$ GeV is the reduced Planck scale, which is the cutoff scale for this model.

we can see that neutrinos acquire Dirac mass terms, once the following scalar fields acquire vevs: $\phi_a, \phi_s, \phi'_a, \phi'_s, \xi$.

- In order to explain the structure of Dirac mass matrix of CSD, we assume that these vevs to have the following pattern

$$\frac{\langle\phi_a\rangle}{M_P} = y_a \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{\langle\phi_s\rangle}{M_P} = y_s \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{31}$$

  Here, y_a, y_s are dimensionless quantities. These vevs will give leading order contribution to effective Yukawa couplings.

- The vevs of ϕ'_a, ϕ'_s, ξ give sub-leading contribution to Yukawa couplings for neutrinos. Here, we need not assume any pattern for the vevs of ϕ'_a, ϕ'_s. Hence, after writing $\frac{\langle\xi\rangle}{M_P} = \epsilon$, we can have

$$\frac{\langle\xi\rangle}{M_P}\frac{\langle\phi'_a\rangle}{M_P} = y_a \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \epsilon = y_a \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}, \quad \frac{\langle\xi\rangle}{M_P}\frac{\langle\phi'_s\rangle}{M_P} = y_s \begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix} \epsilon = y_s \begin{pmatrix} \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix}$$

  Here, y_i, y'_i, where $i = 1, \cdots, 3$, are $\mathcal{O}(1)$ parameters.

A Model For Dirac Mass Matrix

- $\langle\chi_a\rangle$ and $\langle\chi_s\rangle$ generate masses for right handed neutrinos, we should have $\langle\chi_a\rangle, \langle\chi_s\rangle \sim 1$ TeV.
- The vev of ξ is such that it explains the smalleness of ϵ_i parameters. For this we have $\frac{\langle\xi\rangle}{M_P} \sim 0.1$, we get $\langle\xi\rangle \sim 10^{17}$ GeV.
- By taking the masses of the active neutrinos to be $\mathcal{O}(0.1)$ eV, we found that $\langle\phi_a\rangle, \langle\phi_s\rangle, \langle\phi'_a\rangle, \langle\phi'_s\rangle \sim 10^{12}$ GeV.
- After noticing that there is a large hierarchy among all vevs, We can achieve this hierarchy, in this model, by appropriately fixing the relevant parameters in the scalar potential among all mentioned scalar fields.

Conclusion

- In this work we have attempted to explain the neutrino mixing in order to explain neutrino oscillation data.
- Here we have considered a model where we have modified the Yukawa couplings of CSD model by introducing small ϵ_i parameters.
- Thereafter we followed an approximation procedure in order to diagonalize the seesaw formula and we have computed expressions up to second order level to neutrino mass and mixing angles in terms of small ϵ_i parameters.
- Using these expressions we have demonstrated that neutrino mixing can deviate from TBM pattern by choosing ϵ_i parameters.
- Finally we have constructed a model in order to justify the neutrino Yukawa coupling structure of our model.

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