The Normal $\nu$ Mass Ordering is Exactly What We Need!

Shao-Feng Ge (葛韶锋)
(gesf@sjtu.edu.cn)

Tsung-Dao Lee Institute & School of Physics and Astronomy
Shanghai Jiao Tong University

2021-2-22

Dependence on Mass Ordering

$0\nu 2\beta$ Decay

The Normal $\nu$ Mass Ordering is Exactly What We Need!
Argument 1: Solar Octant
Degeneracy in MO, Solar Octant, CP & NSI

- Hamiltonian reverses sign under combined transformations

\[
\begin{align*}
\sin \theta_s & \leftrightarrow \cos \theta_s , \\
\delta_D & \rightarrow \pi - \delta_D , \\
\Delta m^2_a & \rightarrow -\Delta m^2_a + \Delta m^2_s , \\
\epsilon_{ee} & \rightarrow -2 - \epsilon_{ee},
\end{align*}
\]

together with a minus sign in the Dirac CP phase.

- Degeneracy
  - **Mass Ordering:** NO ↔ IO
  - **Solar Octant:** \( \theta_s \leftrightarrow \frac{\pi}{2} - \theta_s \)
  - **CP Octant:** \( \delta_D \leftrightarrow \pi - \delta_D \)

- Any unique way of distinguishing the solar octant!
Distinguishing Solar Octant in the $0\nu 2\beta$ Decay

\[ |m_{ee}| \geq 3.2 \text{ meV} @ m_1 = 5.3 \text{ meV} \quad \text{w/o JUNO} \]

\[ |m_{ee}| \geq 3.8 \text{ meV} @ m_1 = 4.5 \text{ meV} \quad \text{with JUNO} \]

also Choubey & Goswami [arXiv:1901.04313]
Argument 2

- Majorana CP Phases
Any chance of obtaining some information?

\[ \langle m \rangle_{ee} \equiv \vec{L}_1 + \vec{L}_2 + \vec{L}_3, \]

with

\[ \vec{L}_1 \equiv m_1 U^2_{e1} = m_1 c_r^2 s_s^2 e^{i\delta_{M1}}, \]

\[ \vec{L}_2 \equiv m_2 U^2_{e2} = \sqrt{m_1^2 + \Delta m^2_{1s} c_r s_s}, \]

\[ \vec{L}_3 \equiv m_3 U^2_{e3} = \sqrt{m_1^2 + \Delta m^2_{2s} s_r e^{i\delta_{M3}}}. \]


Rodejohann, arXiv:1106.1334
Determine 2 Majorana Phases Simultaneously

\[ |L_1 - L_3| \leq L_2 \leq L_1 + L_3. \]

\[
\cos \delta_{M1} = -\frac{L_1^2 + L_2^2 - L_3^2}{2L_1L_2} = -\frac{m_1^2 c_r^4 c_s^4 + m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_1 m_2 c_r^4 c_s^2 s_s^2},
\]

\[
\cos \delta_{M3} = \frac{L_1^2 - L_2^2 - L_3^2}{2L_2L_3} = \frac{m_1^2 c_r^4 c_s^4 - m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_2 m_3 c_r^2 s_r^2 s_s^2}.
\]
Uncertainties from $\langle m \rangle_{ee}$

$\Delta M_1 / \text{Degree}$

$\Delta M_3 / \text{Degree}$

$m_1 = 3\text{ meV}$

$m_1 = 4\text{ meV}$

$m_1 = 5\text{ meV}$

$m_1 = 6\text{ meV}$

also Xing, Zhao & Zhou [1504.05820]; Xing & Zhao [1612.08538]
There is no need to be scared of NO.
At $\mathcal{O}(\text{meV})$, $0\nu2\beta$ can distinguish solar octant;
At sub-meV scale, $0\nu2\beta$ can simultaneously determine the two Majorana CP phases.
Thank You!
The octant transformation, \( c_s \leftrightarrow s_s \) is equivalent to \( m_1 \leftrightarrow m_2 \),

\[
m_{ee} = c_r^2 c_s^2 m_1 e^{i\delta_{M1}} + c_r^2 s_s^2 m_2 + s_r^2 m_3 e^{i\tilde{\delta}_M}
\]

where \( \tilde{\delta}_{Mi} \equiv \delta_{Mi} - \delta_D \).

\[ c_s^2 m_1 \leftrightarrow s_s^2 m_2 \]
Probability of NO

\[ \sqrt{\sigma_{m_{ee}}^2} \text{ (meV)} \]

\[ \sigma_{\text{sum}} \text{ (meV)} \]

\[ P_{\text{NH}} \]

0.7
0.8
0.9
0.999

0.55

The Normal $\nu$ Mass Ordering is Exactly What We Need!
Uncertainties from Oscillation Parameters

\[
\Delta m_1^2 = (7.5 \pm 0.18) \times 10^{-5} \text{eV}^2
\]

\[
\Delta m_2^2 = (2.457 \pm 0.047) \times 10^{-3} \text{eV}^2
\]

\[
\theta_s = 33.48^\circ \pm 0.76^\circ
\]

\[
\theta_r = 8.50^\circ \pm 0.2^\circ
\]

see also SFG & Werner Rodejohann [arXiv:1507.05514]
Uncertainties from Oscillation Parameters

The Normal $\nu$ Mass Ordering is Exactly What We Need!
**Majorana Pyramid & Projected Uncertainty**

The Normal $\nu$ Mass Ordering is Exactly What We Need!
Extracting Majorana CP Phases from Nothing

- Null observation seems to be very unfortunate!
- But **not bad at all!**
- Vanishing $m_{ee}$ ⇒ Determine the 2 Majorana CP Phases Simultaneously!

\[
|m_{ee}| = 0 \Rightarrow R(m_{ee}) = I(m_{ee}) = 0 \\
|m_{ee}| < f \Rightarrow R(m_{ee}) < f \quad \& \quad I(m_{ee}) < f
\]

- Non-zero $m_{ee}$ can only determine a **single** degree of freedom

\[
|m_{ee}| = f
\]

**Missing Piece**
- Null observation of $0\nu2\beta \not\Rightarrow$ 2 Majorana CP phases;
- Neutrinos have to be Majorana type in the first place!
- Either assumption or independent measurement.