Radiative decays of charged leptons as constraints of leptonic unitarity

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Based on works done in collaboration with Zhi-zhong Xing and Shun Zhou:

- **Z.-Z. Xing & DZ, EPJC 80 (2020) 12, 1134, e-Print: 2009.09717**
- **DZ & S. Zhou, e-Print: 2102.04954**

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Motivation

Type-I seesaw mechanism: P. Minkowski, 1977; T. Yanagida, 1979; M. Gell-Mann et al, 1979; S. L. Glashow, 1980; R. N. Mohapatra, G. Senjanovic, 1980

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i} \partial\!\!\!/ N_{\rm R} - \left(\frac{1}{2} \overline{N^{\rm c}_{\rm R}} M N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + {\rm h.c.} \right)$$

- **Tiny neutrino masses** $M_\nu \equiv -M_{\rm D} M^{-1} M_{\rm D}^{\rm T}$ **(seesaw formula)**
- **Matter-antimatter asymmetry in the Universe via leptogenesis**

M. Fukugita, T. Yanagida, 1986

The charged-current interaction:

$$\mathcal{L}_{\rm cc} = \frac{g_2}{\sqrt{2}} \overline{\ell_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.} \qquad UU^\dagger + RR^\dagger = I$$

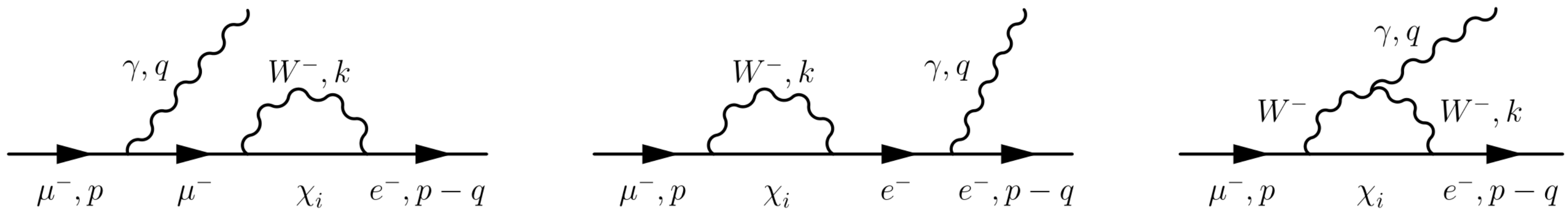
U: **PMNS matrix non-unitarity** — R: **light-heavy mixing**

How to constrain the size of the unitarity violation?

- **Neutrino oscillation experiments** Z.-J. Hu et al, 2020; S.A.R. Ellis et al, 2020a; 2020b; …
- **Charged lepton flavor violation processes, e.g., the radiative decays of charged leptons (simplicity and precision measurements)** S. Antusch et al, 2006; S. Antusch, O. Fischer, 2014; R. Alonso et al, 2013; E. Fernandez-Martinez et al, 2016; Z.-Z. Xing, DZ, 2020
- …

Radiative decays of charged leptons

Feynman diagrams in unitary gauge : S. T. Petcov, 1977; S. M. Bilenky et al, 1977; T. P. Cheng, L. F. Li, 1977; W. J. Marciano, A. I. Sanda,1977; B. W. Lee et al, 1977; B. W. Lee et al, 1977; …



(a) (b) (c)

Corresponding amplitudes: T. P. Cheng, L. F. Li, 1980; A. Ilakovac, A. Pilaftsis, 1995

$$\mathrm{i}\mathcal{M}_{\mathrm{a}} = -\frac{1}{2} e g^2 \epsilon_\rho^*(q) \sum_{i=1}^{6} \mathcal{U}_{ei}\mathcal{U}_{\mu i}^* \, \mu^{\frac{3}{2}\varepsilon} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \bar{u}(p-q)\,\gamma^\mu P_{\mathrm{L}} \frac{\not{p}-\not{q}-\not{k}+\lambda_i}{(p-q-k)^2-\lambda_i^2}\gamma^\nu P_{\mathrm{L}} \times \frac{\not{p}-\not{q}+m_\mu}{(p-q)^2-m_\mu^2}\gamma^\rho u(p)\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}\right)\frac{1}{k^2-M_W^2},$$

$$\mathrm{i}\mathcal{M}_{\mathrm{b}} = -\frac{1}{2} e g^2 \epsilon_\rho^*(q) \sum_{i=1}^{6} \mathcal{U}_{ei}\mathcal{U}_{\mu i}^* \, \mu^{\frac{3}{2}\varepsilon} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \bar{u}(p-q)\,\gamma^\rho \frac{\not{p}+m_e}{p^2-m_e^2}\gamma^\mu P_{\mathrm{L}} \times \frac{\not{p}-\not{k}+\lambda_i}{(p-k)^2-\lambda_i^2}\gamma^\nu P_{\mathrm{L}} u(p)\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{M_W^2}\right)\frac{1}{k^2-M_W^2},$$

$$\mathrm{i}\mathcal{M}_{\mathrm{c}} = \frac{1}{2} e g^2 \epsilon_\rho^*(q) \sum_{i=1}^{6} \mathcal{U}_{ei}\mathcal{U}_{\mu i}^* \, \mu^{\frac{3}{2}\varepsilon} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \bar{u}(p-q)\,\gamma^\mu P_{\mathrm{L}} \frac{\not{p}-\not{q}-\not{k}+\lambda_i}{(p-q-k)^2-\lambda_i^2}\gamma^\nu P_{\mathrm{L}} u(p) \times \left(g_{\mu\sigma}-\frac{k_\mu k_\sigma}{M_W^2}\right)\frac{1}{k^2-M_W^2}\left[-g^{\sigma\lambda}(q+2k)^\rho + g^{\lambda\rho}(2q+k)^\sigma + g^{\rho\sigma}(k-q)^\lambda\right] \times \left[g_{\lambda\nu}-\frac{(q+k)_\lambda (q+k)_\nu}{M_W^2}\right]\frac{1}{(q+k)^2-M_W^2},$$

Dimensional regularization

$$D = 4 - \varepsilon$$

On-shell conditions

$$q^2 = 0$$

$$p^2 = m_\mu^2$$

$$p \cdot q = \frac{m_\mu^2 - m_e^2}{2}$$

Physical polarizations

$$\epsilon(q) \cdot q = 0$$

Divergences are cancelled out

Radiative decays of charged leptons

Decay rate:

$$\Gamma\left(\mu^{-}\to e^{-}+\gamma\right)=\frac{1}{2m_{\mu}}\cdot\frac{1}{8\pi}\left(1-\frac{m_e^2}{m_\mu^2}\right)\cdot\frac{1}{2}\sum|\mathcal{M}|^2=\frac{\alpha_{\rm em}G_{\rm F}^2m_\mu^5}{128\pi^4}\left(1+\frac{m_e^2}{m_\mu^2}\right)\left(1-\frac{m_e^2}{m_\mu^2}\right)^3\left|\sum_{i=1}^{6}\mathcal{U}_{ei}\mathcal{U}_{\mu i}^{*}G_\gamma\left(x_i\right)\right|^2$$

$$G_\gamma\left(x_i\right)=-\frac{5}{6}-\frac{2x_i^3+5x_i^2-x_i}{4\left(1-x_i\right)^3}-\frac{3x_i^3}{2\left(1-x_i\right)^4}\ln x_i \qquad x_i\equiv\frac{\lambda_i^2}{M_W^2} \qquad \lambda_i=\begin{cases}m_i & i=1,2,3\\ M_{i-3} & i=4,5,6\end{cases}$$

To eliminate irrelevant parameters, one can define a dimensionless ratio

$$\xi(\beta^-\to\alpha^-+\gamma)\equiv\frac{\Gamma(\beta^-\to\alpha^-+\gamma)}{\Gamma(\beta^-\to\alpha^-+\overline{\nu}_\alpha+\nu_\beta)}\simeq\frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3}U_{\alpha i}U_{\beta i}^{*}G_\gamma\left(\frac{m_i^2}{M_W^2}\right)+\sum_{i=1}^{3}R_{\alpha i}R_{\beta i}^{*}G_\gamma\left(\frac{M_i^2}{M_W^2}\right)\right|^2$$

$$(\alpha,\beta)=\begin{cases}(e,\mu)\\(e,\tau)\\(\mu,\tau)\end{cases}\simeq\frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3}U_{\alpha i}U_{\beta i}^{*}\right|^2 \qquad m_i\ll M_W,\quad M_i\gg M_W$$

Considering the current experimental upper bounds for dimensionless ratios

$$\xi\left(\mu^-\to e^-+\gamma\right)<4.20\times10^{-13}$$
$$\xi\left(\tau^-\to e^-+\gamma\right)<1.85\times10^{-7}$$
$$\xi\left(\tau^-\to \mu^-+\gamma\right)<2.53\times10^{-7}$$

P. A. Zyla et al, 2020

$$\left|\sum_{i=1}^{3}U_{ei}U_{\mu i}^{*}\right|=\left|\sum_{i=1}^{n}R_{ei}R_{\mu i}^{*}\right|<2.20\times10^{-5}$$
$$\left|\sum_{i=1}^{3}U_{ei}U_{\tau i}^{*}\right|=\left|\sum_{i=1}^{n}R_{ei}R_{\tau i}^{*}\right|<1.46\times10^{-2}$$
$$\left|\sum_{i=1}^{3}U_{\mu i}U_{\tau i}^{*}\right|=\left|\sum_{i=1}^{n}R_{\mu i}R_{\tau i}^{*}\right|<1.70\times10^{-2}$$

Compared to MUV scheme

Minimal unitarity violation (MUV) scheme: S. Antusch et al, 2006

$$\mathcal{L}_{eff} = \mathcal{L}_{\rm SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)}$$

where $\mathcal{L}^{(5)} = C^{d=5}_{\alpha\beta} \overline{\ell_{\alpha \rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\beta \rm L} + \text{h.c.}$, $\mathcal{L}^{(6)} = C^{d=6}_{\alpha\beta} \left(\overline{\ell_{\alpha \rm L}} \widetilde{H}\right) {\rm i} \partial\!\!\!/ \left(\widetilde{H}^\dagger \ell_{\beta \rm L}\right)$

unique Weinberg operator,
generates neutrino masses
S. Weinberg, 1979

correction to neutrino kinetic term,
violates unitarity of PMNS matrix

Result in the MUV scheme: $\xi(\beta^- \to \alpha^- + \gamma) \simeq \frac{25\alpha_{\rm em}}{24\pi} \left|\sum_{i=1}^{3} U_{\alpha i} U^*_{\beta i}\right|^2$ S. Antusch et al, 2006; S. Antusch, O. Fischer, 2014

Result in the type-I seesaw mechanism: $\xi(\beta^- \to \alpha^- + \gamma) \simeq \frac{3\alpha_{\rm em}}{8\pi} \left|\sum_{i=1}^{3} U_{\alpha i} U^*_{\beta i}\right|^2$

Different by a factor 25/9

How to understand this difference?

- As a practically useful framework, MUV scheme contains one dim-5 operator and one dim-6 operator, which are both at tree level after heavy neutrinos are integrated out in the type-I seesaw model.
- The radiative decays of charged leptons are one-loop transitions in present case, to which there are operators at one-loop level contributing in addition to these two tree-level operators via one-loop diagrams.

Seesaw Effective Field Theory (SEFT)

After integrating out heavy neutrinos, one can obtain seesaw effective field theory:

$$\mathcal{L}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \boxed{\mathcal{L}^{(6)}_{\rm loop}}$$

one-loop matching

DZ, S. Zhou, 2021

where

$$\mathcal{L}^{(6)}_{\rm loop} = \frac{\left(Y_\nu M^{-2} Y_\nu^\dagger Y_l\right)_{\alpha\beta}}{24\left(4\pi\right)^2}\left[g_1\left(\overline{\ell_{\alpha \rm L}}\sigma_{\mu\nu}E_{\beta \rm R}\right)HB^{\mu\nu} + 5g_2\left(\overline{\ell_{\alpha \rm L}}\sigma_{\mu\nu}E_{\beta \rm R}\right)\tau^I H W^{I\mu\nu}\right] + {\rm h.c.}$$

After electroweak spontaneous symmetry breaking, normalizing the kinetic term and diagonalizing the mass term of neutrinos, the relevant terms for radiative decays of charged leptons are

$$\mathcal{L} = \underbrace{\left(\frac{g_2}{\sqrt{2}}\overline{l_{\rm L}}\gamma^\mu U \nu_{\rm L} W^-_\mu + {\rm h.c.}\right)}_{\text{tree level}} \underbrace{- \frac{e g_2^2}{12\left(4\pi\right)^2 M_W^2}\overline{l_{\rm L}}\sigma_{\mu\nu}RR^\dagger M_l l_{\rm R} F^{\mu\nu} + {\rm h.c.}}_{\text{one-loop level}}$$

tree level **one-loop level**

Result in the seesaw effective field theory:

absent in the MUV scheme

$$\xi(\beta^- \to \alpha^- + \gamma) \simeq \frac{3\alpha_{\rm em}}{2\pi}\left| -\frac{5}{6}\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i} \boxed{-\frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}}\right|^2 = \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2$$

Consistent with that in the full type-I seesaw mechanism

$$m_i \ll M_W\,, \quad M_i \gg M_W$$

It is necessary to consider one-loop matching for SEFT when considering some one-loop transitions in effective field theory

Summary

- Unitarity violation of the PMNS matrix as signals of new physics is important and becomes possible to be observed in the near future.

- The unitarity of the PMNS mixing matrix is violated in the type-I seesaw mechanism.

- The radiative decays of charged leptons can be used to constrain the unitarity violation in the type-I seesaw mechanism.

- When considering some one-loop transitions in the effective field theory, one needs to take into account the one-loop matching.

THANKS FOR YOUR ATTENTION