H2020 MSCA COFUND
G.A. 754496

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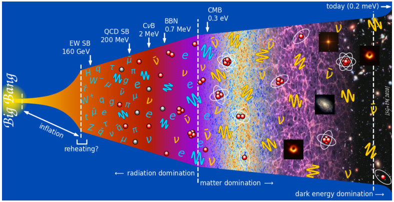
Neutrino thermalization in the early universe: precision calculations

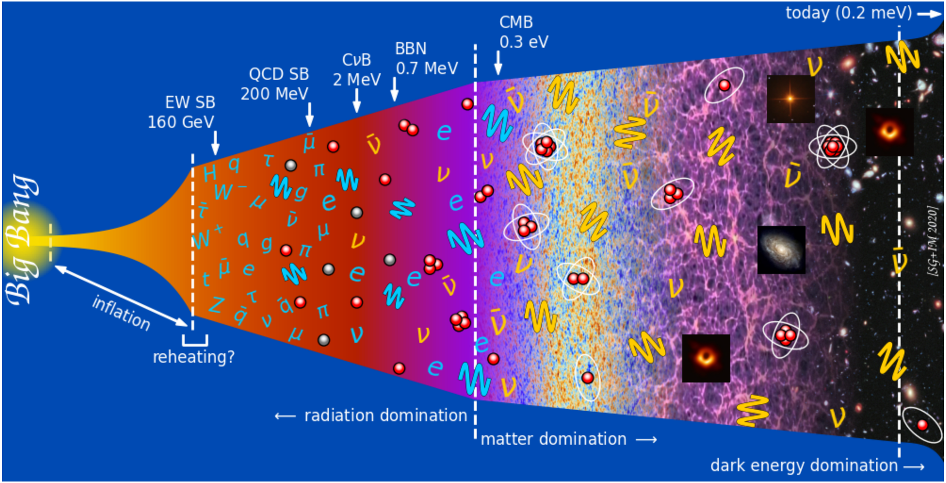
Mostly based on arxiv:2012.02726

XIX International Workshop on Neutrino Telescopes, online, 22/02/2021



today (0.2 meV)
CMB
0.3 eV
BBN
0.7 MeV
CνB
2 MeV
QCD SB
200 MeV
EW SB
160 GeV
Big Bang
inflation
reheating?
radiation domination
matter domination
dark energy domination
[SG+FM 2020]

History of the universe



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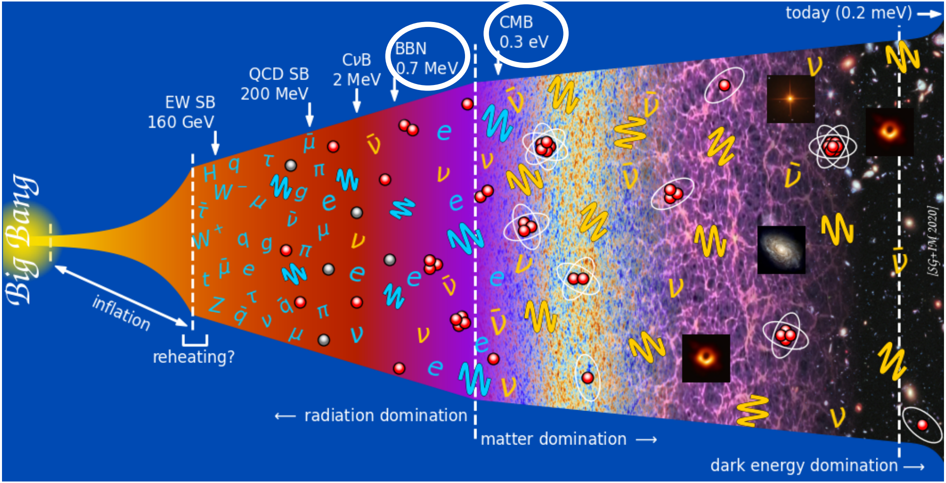

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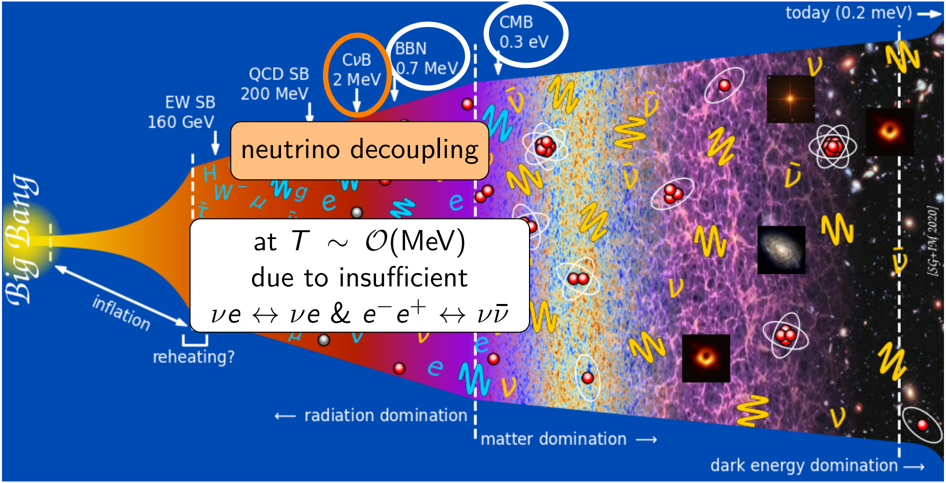
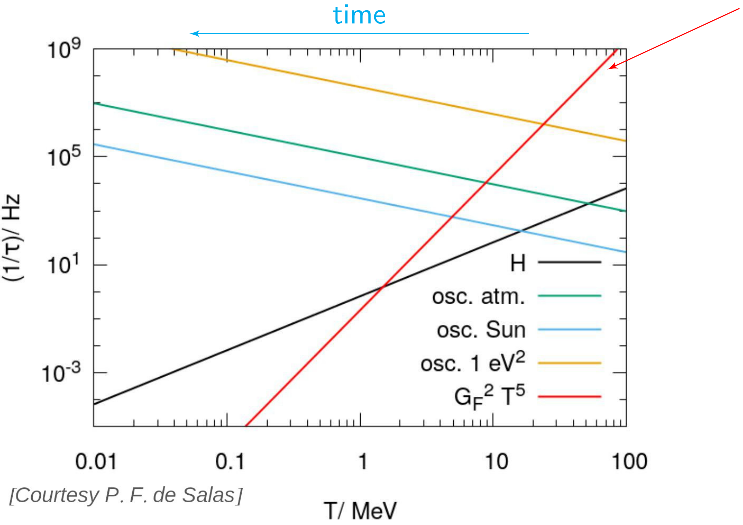


neutrino decoupling

at $T \sim \mathcal{O}(\mathrm{MeV})$
due to insufficient
$\nu e \leftrightarrow \nu e$ & $e^- e^+ \leftrightarrow \nu\bar{\nu}$

Neutrinos in the early Universe

before BBN: neutrinos coupled to plasma ($\nu_\alpha\bar{\nu}_\alpha \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$)



[Courtesy P. F. de Salas]

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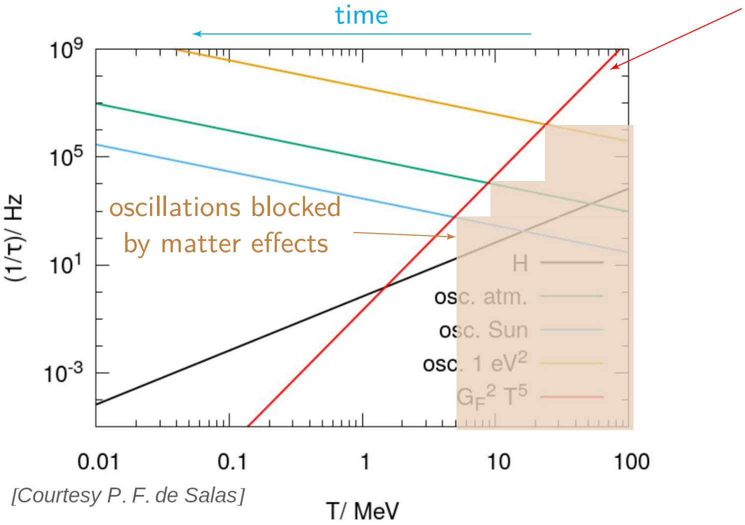

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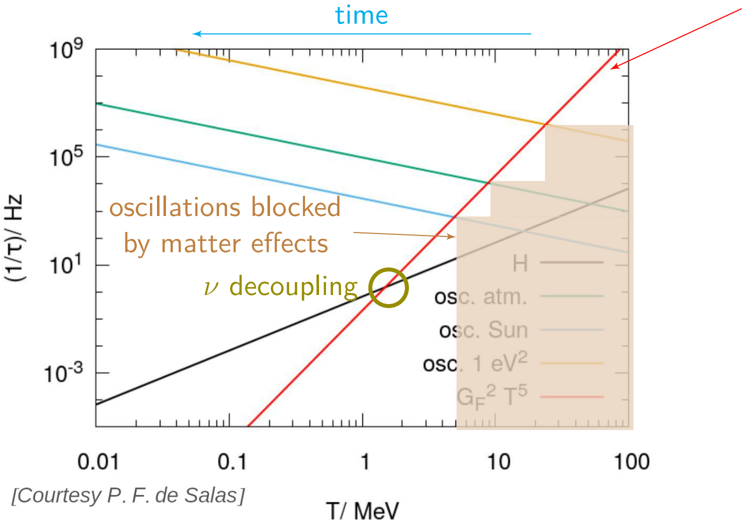

ν decouple mostly before $e^+e^- \to \gamma\gamma$ annihilation!

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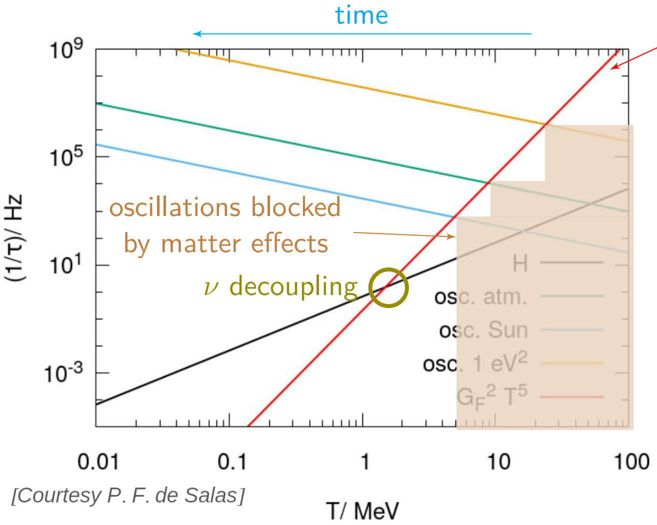

[Courtesy P. F. de Salas]

$T_\nu \simeq (4/11)^{1/3}\,T_\gamma$ after $e^+e^- \to \gamma\gamma$

f_ν: frozen Fermi-Dirac distribution

Today:

$T_{\nu,0} = 1.945$ K $\simeq 1.676 \times 10^{-4}$ eV

$\langle E_\nu\rangle \simeq 3.1\,T_{\nu,0} \simeq 5 \times 10^{-4}$ eV

$n_0 = n_{\nu,0} = n_{\bar{\nu},0} \simeq 56$ cm^{-3} per family

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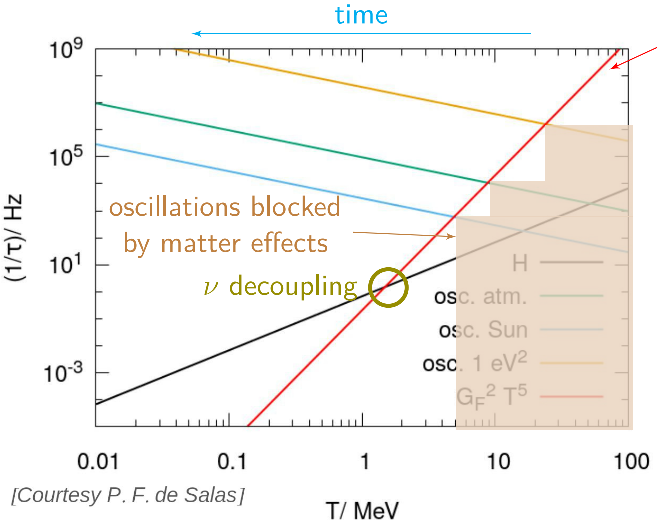

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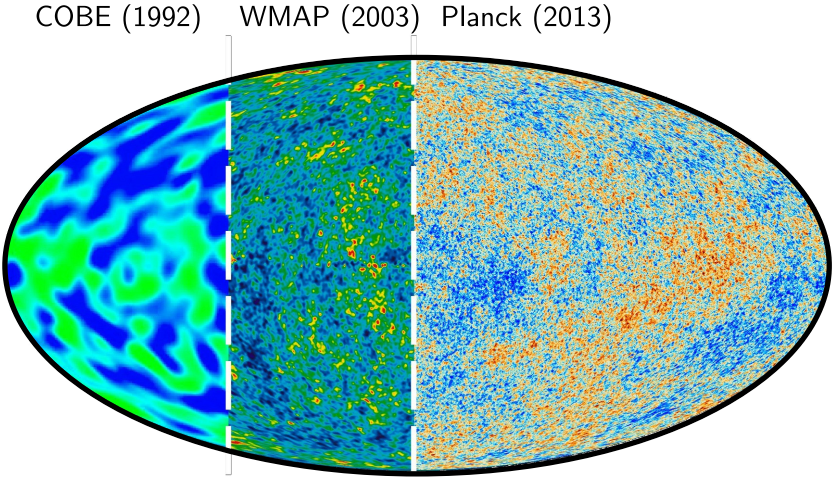
actually, the decoupling T is momentum dependent!

→ distortions to equilibrium f_ν!

The oldest picture of the Universe

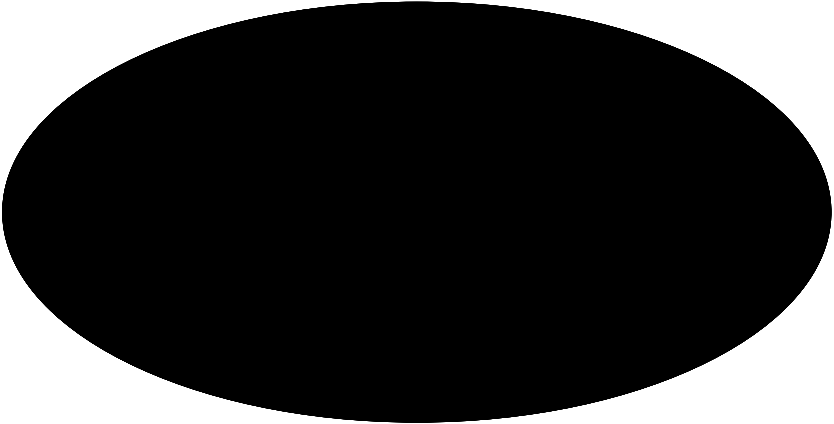
The Cosmic Microwave Background, generated at $t \simeq 4 \times 10^5$ years



The oldest picture of the Universe

The Cosmic Neutrino Background, generated at $t \simeq 1$ s

$\ldots \rightarrow 2019 \rightarrow \ldots$



Relic neutrinos in cosmology: $N_{\rm eff}$

Radiation energy density ρ_r in the early Universe:

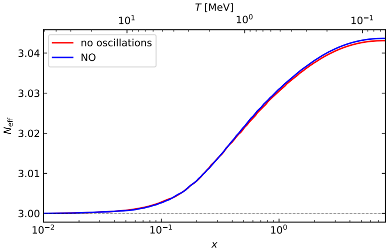
$$\rho_r = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\rho_\gamma = [1 + 0.2271 N_{\rm eff}]\,\rho_\gamma$$

ρ_γ photon energy density, $7/8$ is for fermions, $(4/11)^{4/3}$ due to photon reheating after neutrino decoupling

- $N_{\rm eff} \rightarrow$ all the radiation contribution not given by photons
- $N_{\rm eff} \simeq 1$ correspond to a single family of active neutrino, in equilibrium in the early Universe
- Active neutrinos:
  [Bennett, SG et al., 2020] [Froustey et al., 2020]: $N_{\rm eff} = 3.044$ See later!
  due to not instantaneous decoupling for the neutrinos
- $+$ Non Standard Interactions: $3.040 < N_{\rm eff} < 3.059$ [de Salas et al., 2016]

Observations: $N_{\rm eff} \simeq 3.0 \pm 0.2$ [Planck 2018]
Indirect probe of cosmic neutrino background!

$\gg 10\sigma$!



$N_{\mathrm{eff}} = 3.0440$

ν oscillations in the early universe

[Bennett, SG+, 2012.02726]

comoving coordinates: $a = 1/T \quad x \equiv m_e\, a \quad y \equiv p\, a \quad z \equiv T_\gamma\, a \quad w \equiv T_\nu\, a$

density matrix: $$\varrho(x,y) = \begin{pmatrix} \varrho_{ee} \equiv f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho_{\mu e} & \varrho_{\mu\mu} \equiv f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho_{\tau e} & \varrho_{\tau\mu} & \varrho_{\tau\tau} \equiv f_{\nu_\tau} \end{pmatrix}$$

$$\frac{\mathrm{d}\varrho(y,x)}{\mathrm{d}x} = \sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\rho_{\mathrm{T}}}} \left\{ -i\frac{x^2}{m_e^3}\left[\frac{\mathbb{M}_{\mathrm{F}}}{2y} - \frac{2\sqrt{2}G_{\mathrm{F}}\,y}{x^6/m_e^6}\left(\frac{\mathbb{E}_\ell+\mathbb{P}_\ell}{m_W^2} + \frac{4\mathbb{E}_\nu}{3m_Z^2}\right), \varrho\right] + \frac{m_e^3 G_{\mathrm{F}}^2}{(2\pi)^3 x^4 y^2}\mathcal{I}(\varrho) \right\}$$

m_{Pl} Planck mass – ρ_T total energy density – $m_{W,Z}$ mass of the W, Z bosons – G_{F} Fermi constant – $[.,.]$ commutator

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$$\mathbb{M}_{\rm F} = U\mathbb{M}U^\dagger$$

$$\mathbb{M} = \mathrm{diag}(m_1^2,\ldots,m_N^2)$$

$$U = R^{23}R^{13}R^{12}$$

e.g. $R^{13} = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$

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lepton densities — neutrino densities — (only for active neutrinos)

take into account matter effects in oscillations

ν oscillations in the early universe

[Bennett, SG+, 2012.02726]

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$\mathcal{I}(\varrho)$ collision integrals

take into account neutrino-electron scattering and pair annihilation, plus neutrino–neutrino interactions

2D integrals over the momentum, take most of the computation time

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from continuity equation

$$\dot{\rho} = -3H(\rho + P)$$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\sum_{\ell=e,\mu}\left[\frac{r_\ell^2}{r}J(r_\ell)\right] + G_1(r) - \frac{1}{2\pi^2 z^3}\int_0^\infty dy\, y^3 \sum_{\alpha=e}^{\tau} \frac{\mathrm{d}\varrho_{\alpha\alpha}}{\mathrm{d}x}}{\sum_{\ell=e,\mu}\left[r_\ell^2 J(r_\ell) + Y(r_\ell)\right] + G_2(r) + \frac{2\pi^2}{15}}$$

$r = x/z$, $r_\ell = m_\ell/m_e\, r$ $\qquad$ $J(r)$, $Y(r)$ from non-relativistic transition of $e^\pm$, $\mu^\pm$

$G_1(r)$ and $G_2(r)$ from electromagnetic corrections

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neutrino temperature w: same equation as z, but electrons always relativistic

initial conditions: $\varrho_{\alpha\alpha}$ = Fermi-Dirac at $x_{\mathrm{in}} \simeq 0.001$, with $w = z \simeq 1$

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$m_{\rm Pl}$ ... commutator

FORTran-Evolved PrimordIAl Neutrino Oscillations (FortEPiaNO)
https://bitbucket.org/ahep_cosmo/fortepiano_public

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will be public soon

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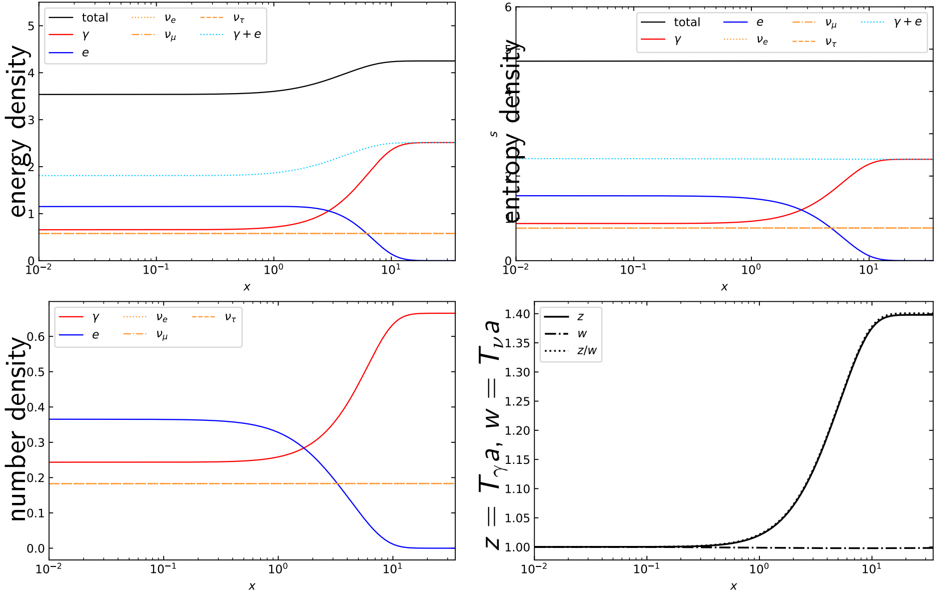
Energy, entropy, number temperatures



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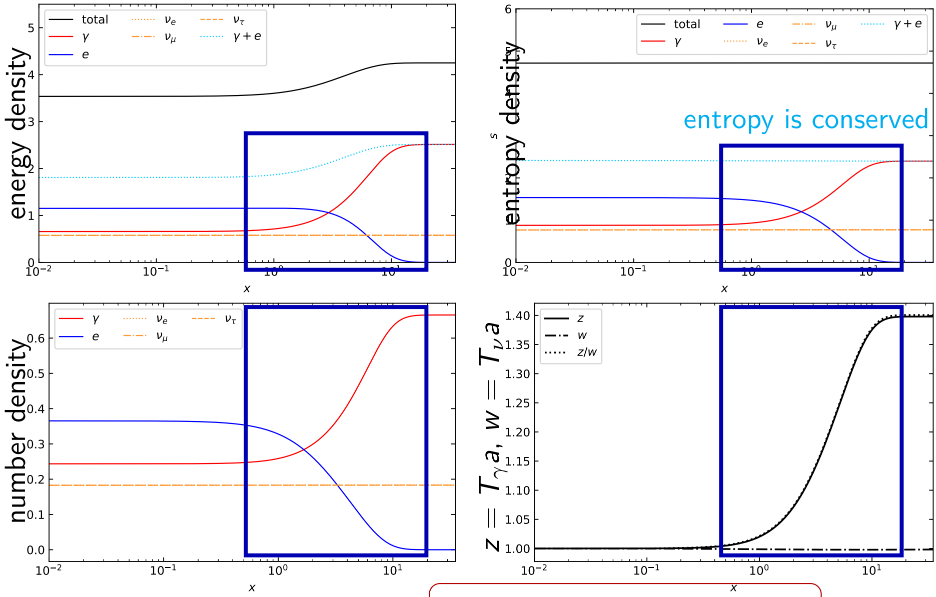
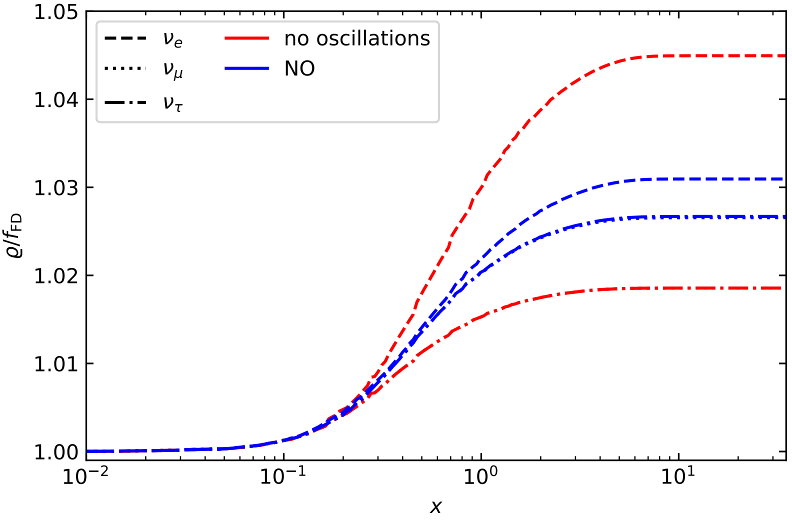


electrons annihilate → energy goes mostly to photons

Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, 2012.02726]

Distortion of the momentum distribution ($f_{\rm FD}$: Fermi-Dirac at equilibrium)



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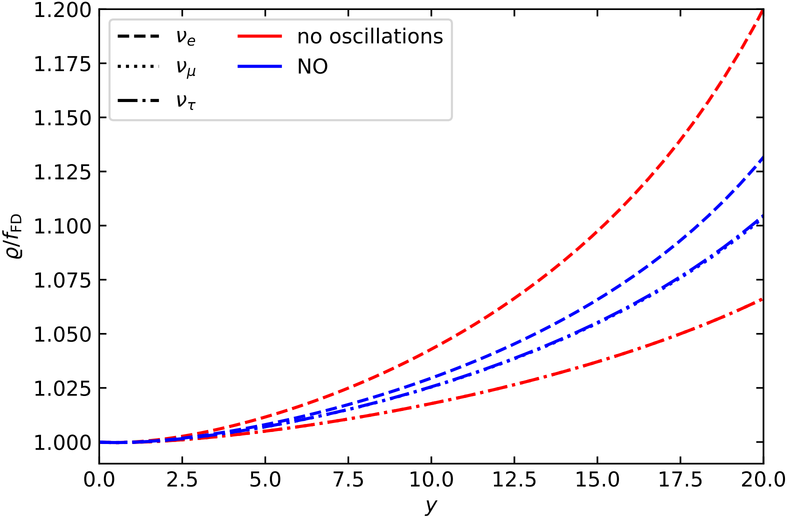
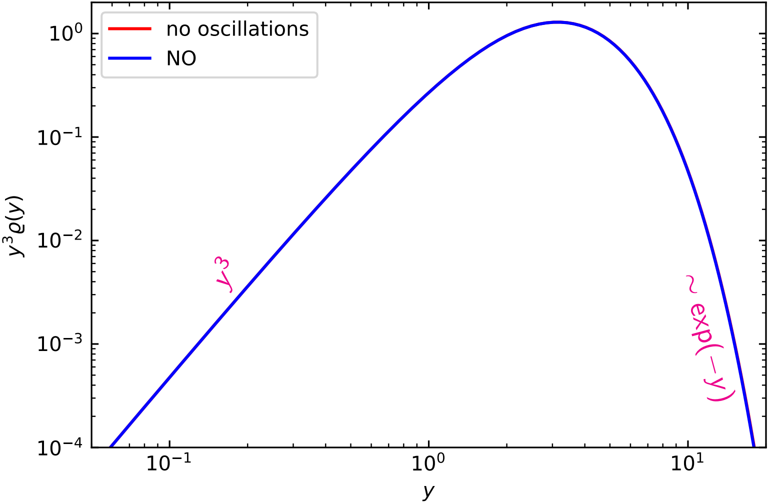


Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, 2012.02726]

$$N_{\rm eff}^{\rm final} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{1}{\rho_\gamma}\sum_i g_i \int \frac{d^3p}{(2\pi)^3} E(p)\, f_{\nu,i}(p)$$

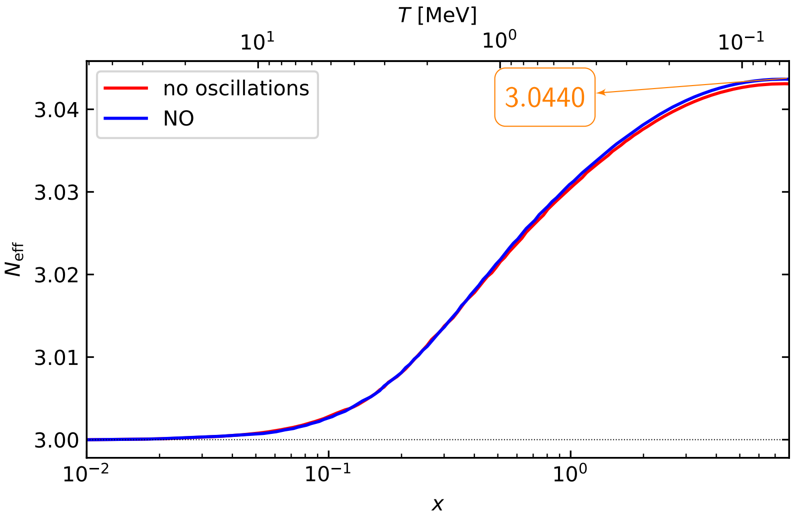
$(11/4)^{1/3} = (T_\gamma/T_\nu)^{\rm fin}$

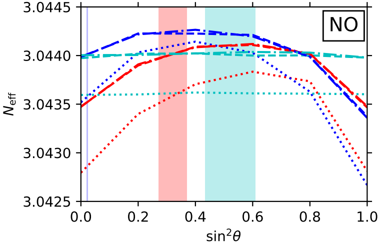
$\hookrightarrow \propto y^3 \varrho_{ii}(y)$



Neutrino momentum distribution and $N_{\rm eff}$ [Bennett, SG+, 2012.02726]

$$N_{\rm eff}^{\rm any\ time} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{\rho_\nu}{\rho_\gamma} = \frac{8}{7}\left(\frac{T_\gamma}{T_\nu}\right)^4 \frac{1}{\rho_\gamma}\sum_i g_i \int \frac{d^3p}{(2\pi)^3} E(p)\, f_{\nu,i}(p)$$





NO
3.0445
3.0440
3.0435
3.0430
3.0425
N_{eff}
0.0
0.2
0.4
0.6
0.8
1.0
$\sin^2\theta$

$\sim 10^{-4}$

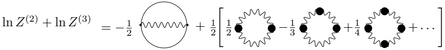
Finite temperature QED

[Bennett+, JCAP 2020]

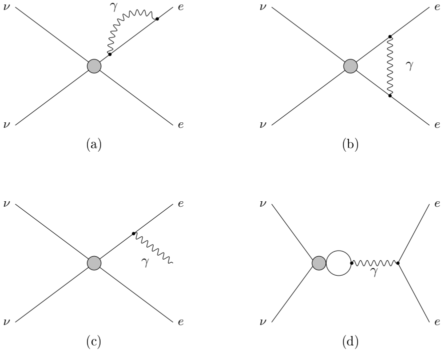
ν decoupling strongly depends on interactions occurring at $T \gtrsim 1$ MeV

finite temperature effects!

altered equation of state of QED plasma



corrections to weak rates



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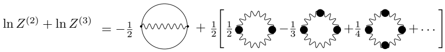
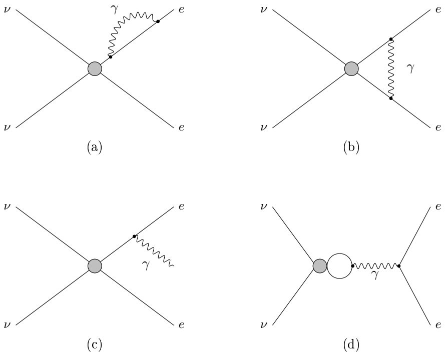


corrections to weak rates



Leading contribution $\mathcal{O}(e^2)$
gives $\delta N_{\rm eff} \sim 0.01$!
[Fornengo+, 1997]

Finite temperature QED

[Bennett+, JCAP 2020]

ν decoupling strongly depends on interactions occurring at $T \gtrsim 1$ MeV

finite temperature effects!

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)
Finite-temperature QED corrections		
$(2)\not{\ln}$	3.04361	3.04458
$(2)\not{\ln} + (2)\ln$	3.04358	3.04452
$(2)\not{\ln} + (3)$	3.04264	3.04361
$(2)\not{\ln} + (2)\ln + (3)$	3.04263	3.04360

[Bennett, SG+, 2020]

$\mathcal{O}(e^2) \sim 0.01$ and $\mathcal{O}(e^3) \sim -0.001$ are important!

Logarithmic term and following orders affect less than numerical parameters for configuring the y_i grid

Neutrino-neutrino interactions

[Bennett, SG+, 2012.02726]

Contribution to collision terms:

$$\mathcal{I}_{\nu\nu}[\varrho(y)] \propto G_F^2 \int \mathrm{d}y_2\, \mathrm{d}y_3\, \Pi_{\nu\nu}(y, y_2, y_3; x)\, F_{\nu\nu}(\varrho(y), \varrho(y_2), \varrho(y_3), \varrho(y_4))$$

$\Pi_{\nu\nu}(y, y_2, y_3; x)$: integrals of some combination of neutrino momenta

Critical function: $F_{\nu\nu}$!

it contains combinations such as $\varrho^{(1)}\varrho^{(3)}\varrho^{(2)}\varrho^{(4)}$ and permutations

it increases complexity of the code!

→ couples modes non-linearly

→ numerically more expensive (stronger dependence on y_i grid than νe terms)

Neutrino-neutrino interactions

Contribution to collision terms:

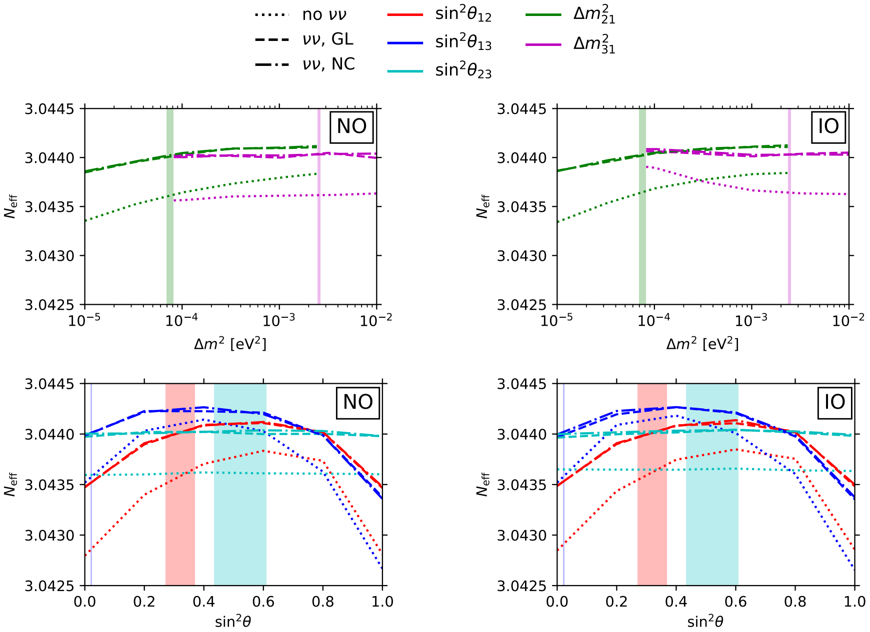
$$\mathcal{I}_{\nu\nu}[\varrho(y)] \propto G_F^2 \int \mathrm{d}y_2\, \mathrm{d}y_3\, \Pi_{\nu\nu}(y, y_2, y_3; x)\, F_{\nu\nu}(\varrho(y), \varrho(y_2), \varrho(y_3), \varrho(y_4))$$

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)
Benchmark A — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$		
Assuming: • $(2)\not{\ln} + (2)\ln + (3)+$ type (a) weak rates • Damping for $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$ • $N_y = 60$, $y_{\rm max} = 20$, NC linearly spaced y_i	**3.04263**	**3.04360**
— $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$		
Assuming: • $(2)\not{\ln} + (2)\ln + (3)+$ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\rm max} = 30$, NC linearly spaced y_i	**3.04341**	**3.04398**
Neutrino–neutrino collision integral - $y_{\rm max} = 20$		
Diagonal ϱ	3.04333	3.04416
Full ϱ, interpolate ϱ/FD only in diagonal	3.04334	3.04389
Full ϱ, interpolate ϱ/FD also in off-diagonal	3.04334	3.04389

$\mathcal{I}_{\nu\nu}[\varrho(y)]$ is important! $(4 \div 8) \times 10^{-4}$

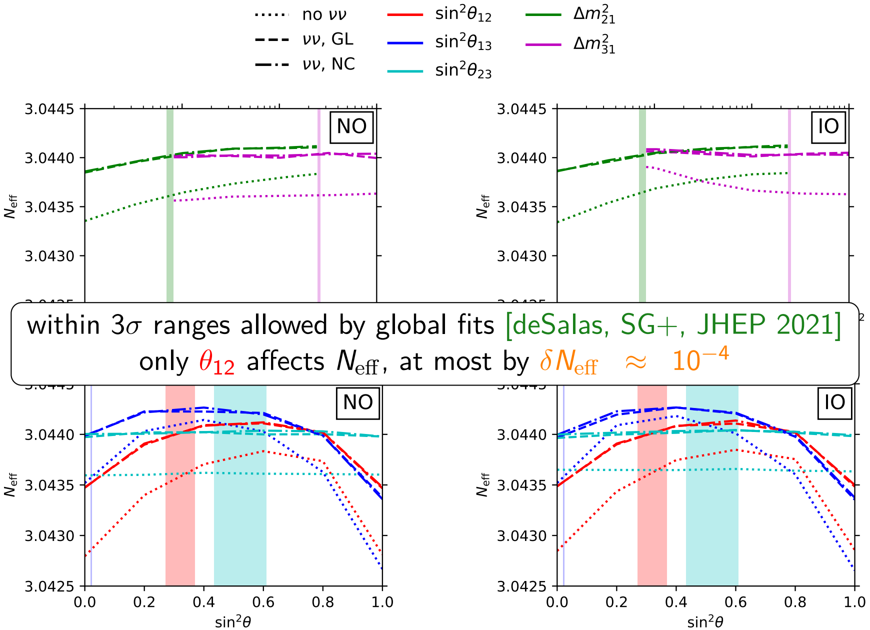
approximations may work

Effect of neutrino oscillations



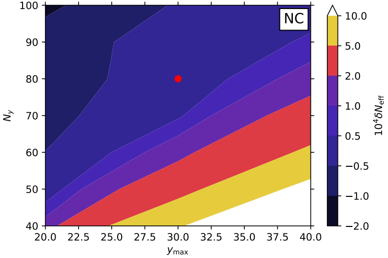
Effect of neutrino oscillations

[Bennett, SG+, 2012.02726]



within 3σ ranges allowed by global fits [deSalas, SG+, JHEP 2021]

only θ_{12} affects $N_{\rm eff}$, at most by $\delta N_{\rm eff} \approx 10^{-4}$

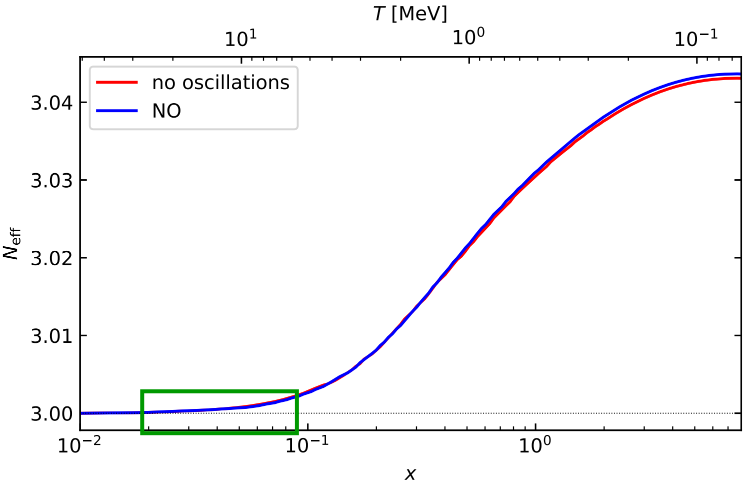


$\mathcal{O}(10^{-4})$

Initial time

assume neutrinos are in equilibrium until
some initial temperature $T_{\rm in} \rightarrow x_{\rm in} = m_e/T_{\rm in}$

Do the final results depend on $x_{\rm in}$?



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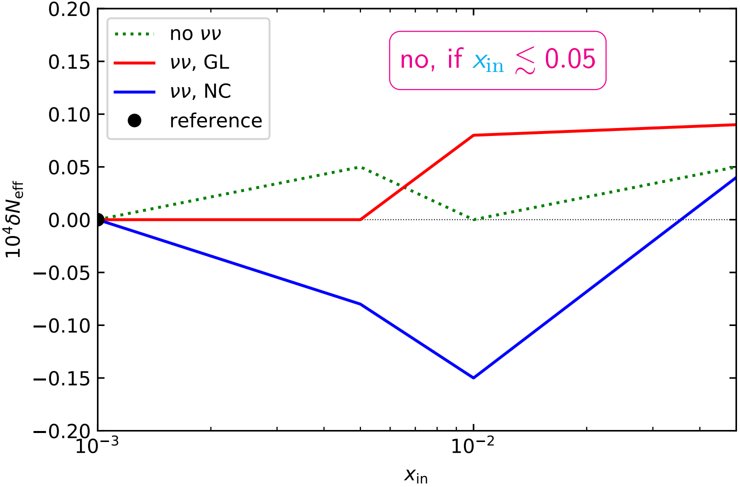


Sampling the y momenta

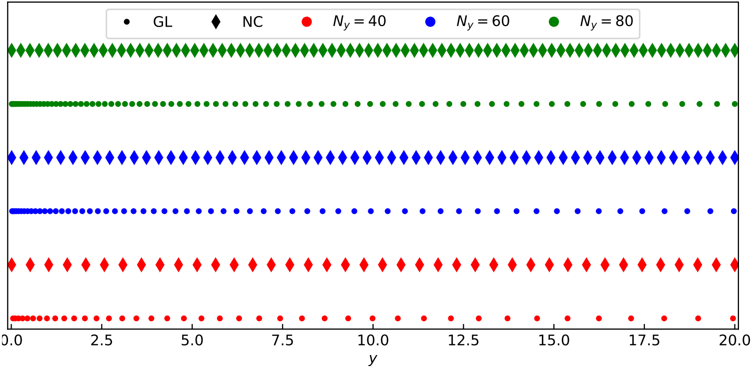
[Bennett, SG+, 2012.02726]

Discretize neutrino momenta to compute integrals and evolution

two sampling methods for y_i, with $i = 1, \ldots, N_y$:

linear spacing,
Newton-Cotes (NC) integration

Gauss-Laguerre (GL)
optimized for computing $\int_0^\infty dy\, f(y) e^{-y}$



Need to define range ($y_{\rm min} \le y \le y_{\rm max}$) and number of nodes N_y

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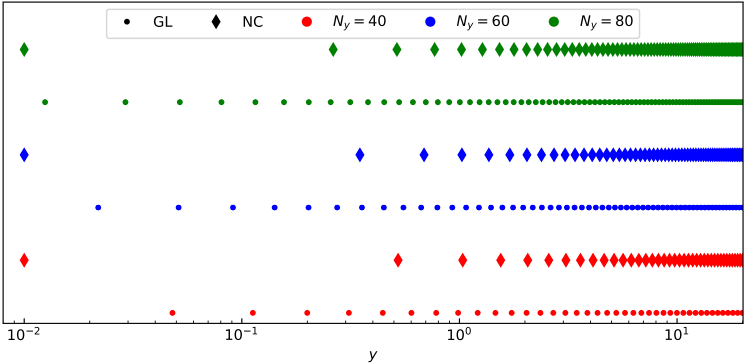


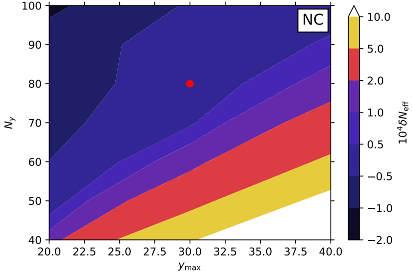
Need to define range ($y_{\min} \leq y \leq y_{\max}$) and number of nodes N_y

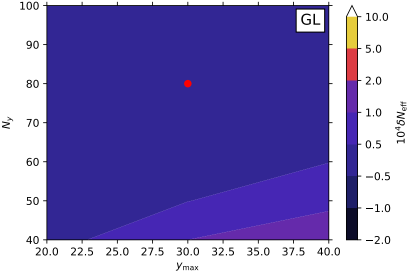
Sampling the y momenta

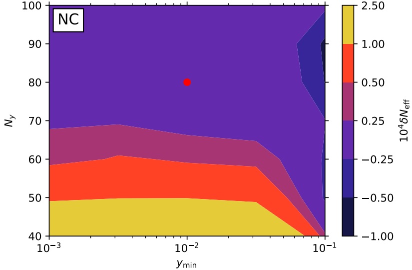
[Bennett, SG+, 2012.02726]

Discretize neutrino momenta to compute integrals and evolution

Results may depend on $y_{\rm min}$, $y_{\rm max}$, N_y







at same N_y,
GL results are more stable!

GL is more efficient

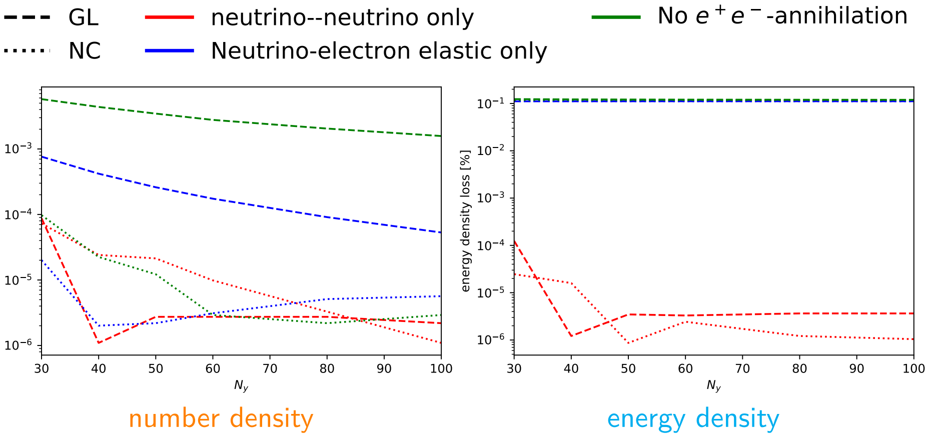
$\delta N_{\rm eff} \approx 10^{-4}$ from varying N_y, $y_{\rm max}$

Energy & number density conservation

[Bennett, SG+, 2012.02726]

Depending on the considered interactions, there can be conservations:

- $\overset{(-)}{\nu}\overset{(-)}{\nu}$ interactions conserve neutrino energy and number density
- $\overset{(-)}{\nu}e^{\pm}$ scattering conserves neutrino number density only;
- $\overset{(-)}{\nu}e^{\pm}$ annihilations break number and energy density conservation



Energy & number density conservation

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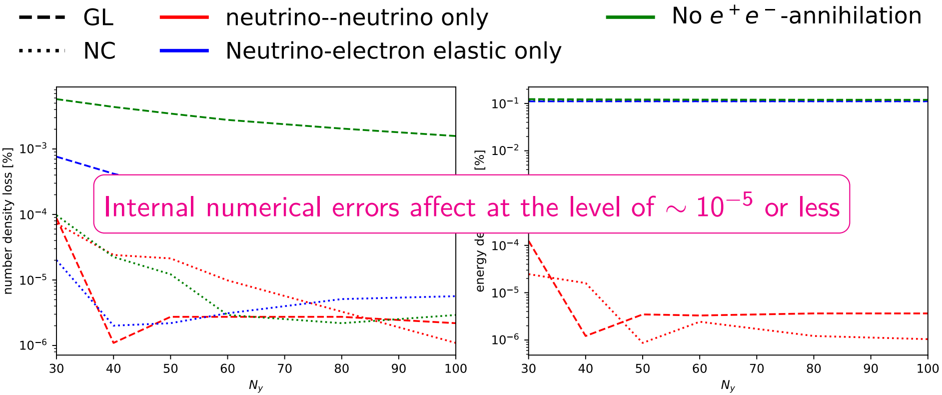

Internal numerical errors affect at the level of $\sim 10^{-5}$ or less

number density

energy density

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark B — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$			
Assuming: • (2)$\not{\rm ln}$ + (2) ln +(3)+ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\rm max} = 30$, NC linearly spaced y_i	**3.04341**	**3.04398**	**3.04399**
Alternative estimates			
Momentum grid			
$N_y = 80$, $y_{\rm max} = 30$, GL spacing of y_i	3.04334	3.04392	3.04392
$N_y = 80$, $y_{\rm max} = 20$, NC linearly spaced y_i	3.04334	3.04389	3.04391
$N_y = 80$, $y_{\rm max} = 20$, GL spacing of y_i	3.04334	3.04386	3.04393
Off-diagonal collision terms			
Damping terms, NC quadrature	3.04342	3.04408	
Damping terms, GL quadrature	3.04335	3.04399	
Neutrino–neutrino collision integral - $y_{\rm max} = 20$			
Diagonal ϱ	3.04333	3.04416	
Full ϱ, interpolate ϱ/FD only in diagonal	3.04334	3.04389	
Full ϱ, interpolate ϱ/FD also in off-diagonal	3.04334	3.04389	

$$N_{\rm eff} = 3.0440 \pm 0.0002$$

Recommended $N_{\rm eff}$ value

[Bennett, SG+, 2012.02726]

Benchmark A: no $\nu\nu$ collisions

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark A — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$			
Assuming: • (2)l̸n + (2) ln +(3)+ type (a) weak rates • Damping for $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$ • $N_y = 60$, $y_{\rm max} = 20$, NC linearly spaced y_i	**3.04263**	**3.04360**	**3.04361**
Alternative estimates			
Momentum grid			
$N_y = 40$, $y_{\rm max} = 20$, GL spacing of y_i nodes	3.04261	3.04355	3.04360
Integrals for off-diagonal $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$			
$N_y = 60$, $y_{\rm max} = 20$, NC linearly spaced y_i	3.04261	3.04357	3.04362
$N_y = 40$, $y_{\rm max} = 20$, GL spacing of y_i	3.04261	3.04357	3.04364
Finite-temperature QED corrections			
(2)l̸n	3.04361	3.04458	
(2)l̸n + (2) ln	3.04358	3.04452	
(2)l̸n + (3)	3.04264	3.04361	
(2)l̸n + (2) ln +(3)	3.04263	3.04360	

Recommended $N_{\rm eff}$ value

[Bennett, SG+, 2012.02726]

Benchmark B: full collision terms

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark B — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$			
Assuming: • (2)l̸n + (2) ln +(3)+ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\rm max} = 30$, NC linearly spaced y_i	**3.04341**	**3.04398**	**3.04399**
Alternative est			
Momentum grid			
$N_y = 80$, $y_{\rm max} = 30$, GL spacing of y_i	3.04334	3.04392	3.04392
$N_y = 80$, $y_{\rm max} = 20$, NC linearly spaced y_i	3.04334	3.04389	3.04391
$N_y = 80$, $y_{\rm max} = 20$, GL spacing of y_i	3.04334	3.04386	3.04393
Off-diagonal collision terms			
Damping terms, NC quadrature	3.04342	3.04408	
Damping terms, GL quadrature	3.04335	3.04399	
Neutrino–neutrino collision integral - $y_{\rm max} = 20$			
Diagonal ϱ	3.04333	3.04416	
Full ϱ, interpolate ϱ/FD only in diagonal	3.04334	3.04389	
Full ϱ, interpolate ϱ/FD also in off-diagonal	3.04334	3.04389	

$\nu\nu$ terms add $\sim (4 \div 8) \times 10^{-4}$

Recommended $N_{\rm eff}$ value

[Bennett, SG+, 2012.02726]

Benchmark B: full collision terms

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark B — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$			
Assuming: • (2)l̸n + (2) ln +(3)+ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\rm max} = 30$, NC linearly spaced y_i	**3.04341**	**3.04398**	**3.04399**
Alternative estimates			
$N_y = 80$, $y_{\rm max}$			3.04392
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Off-diagonal collision terms			
Damping t			
Damping t			
Neutrino–neutrino collision integral - $y_{\rm max} = 20$			
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Our recommended value (normal ordering):

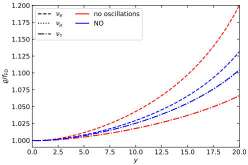
$$N_{\rm eff} = 3.0440 \pm 0.0002$$

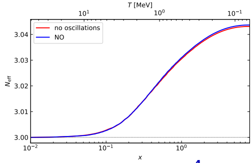
(numerical+physical uncertainty)

Full agreement with other results in literature
e.g. [Froustey+, JCAP 2020] & [Akita+, JCAP 2020]

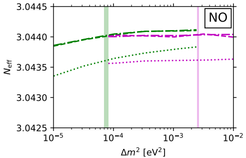
What do we learn on $N_{\rm eff}$?

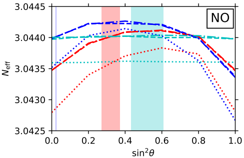
$\int$ Precision calculation $\rightarrow$ $N_{\rm eff} = 3.0440$



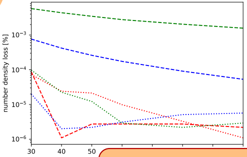


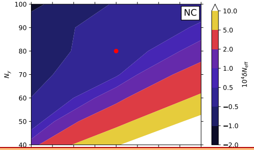
θ_{12} Physical uncertainties $\rightarrow$ $\simeq 10^{-4}$





$y_{\rm max}$ Numerical uncertainties $\rightarrow$ $\mathcal{O}(10^{-4})$





$\Rightarrow$ recommendation

	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark A — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$			
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	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)	$N_{\rm eff}^{\rm SM}$ (IO)
Benchmark B — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} \neq 0$			
Assuming: • (2)$\not\ln$ + (2) ln +(3)+ type (a) weak rates • Full $\mathcal{I}_{\nu e}[\varrho]$ and $\mathcal{I}_{\nu\nu}[\varrho]$ • $N_y = 80$, $y_{\rm max} = 30$, NC linearly spaced y_i	**3.04341**	**3.04398**	**3.04399**
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$$N_{\rm eff} = 3.0440 \pm 0.0002$$

Thanks for the attention!