

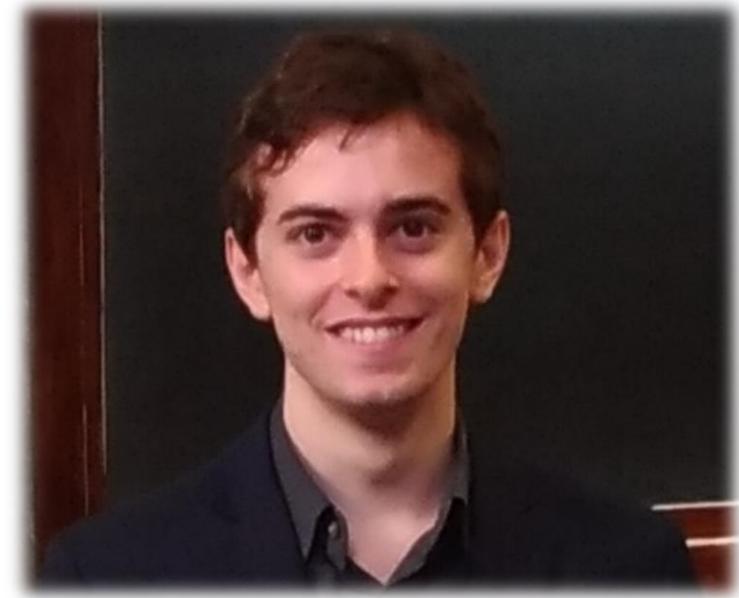
# Weak Measurements for Reutilizing Entanglement

Giulio Foletto

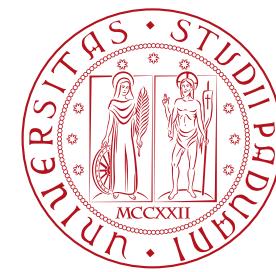
Webinar «Is Quantum Theory Exact? Exploring Quantum Boundaries»

# About me

- PhD Student at QuantumFuture, led by Prof. Villoresi and Prof. Vallone
- The group focuses on experimental quantum communication and technologies



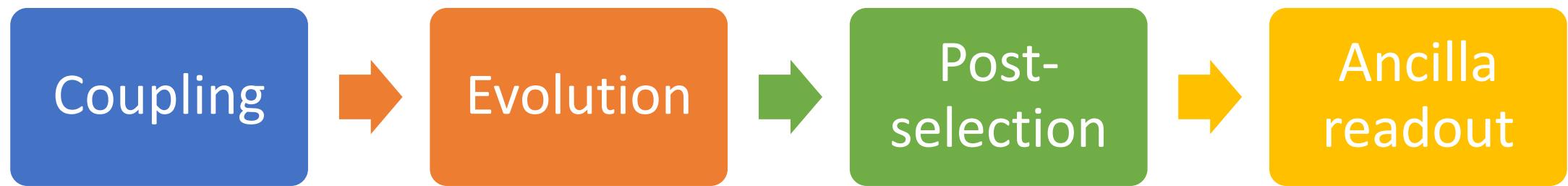
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# Weak values



$$|\psi\rangle \otimes |0_A\rangle$$

$$U = e^{-ig S \otimes P_A}$$

$$\vdash$$

$$\approx \mathbb{I} - ig S \otimes P_A$$

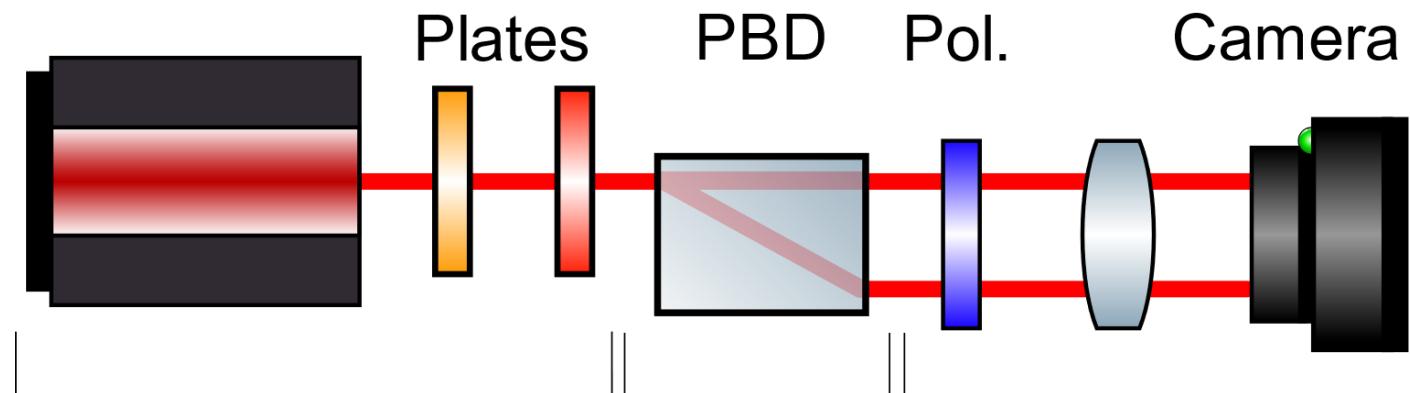
$$\Pi_\phi(U|\psi\rangle \otimes |0_A\rangle)$$

$$\begin{aligned} &\Pi_\phi \otimes O_A \\ &(U|\psi\rangle \otimes |0_A\rangle) \end{aligned}$$

$$|0_A\rangle - i \frac{g}{\hbar} \frac{\langle \phi | S | \psi \rangle}{\langle \phi | \psi \rangle} P_A |0_A\rangle$$

# Weak values

$$|1_A\rangle \approx |0_A\rangle - i\frac{g}{\hbar}\langle\hat{S}^W\rangle_\psi^\phi P_A|0_A\rangle$$
$$\Re(\langle\hat{S}^W\rangle_\psi^\phi) \approx \frac{1}{g} \langle\hat{X}_A\rangle_{1_A}$$
$$\Im(\langle\hat{S}^W\rangle_\psi^\phi) \approx \frac{2\Delta_x^2}{g\hbar} \langle\hat{P}_A\rangle_{1_A}$$



Preparation

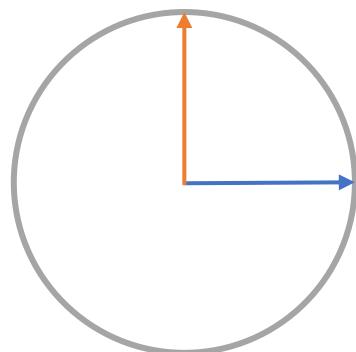
Coupling

Post-selection  
and readout

# Weak Measurements

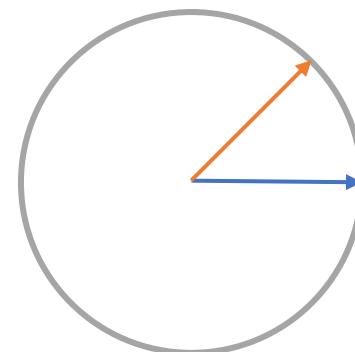


$$|\psi\rangle \otimes |0_A\rangle$$



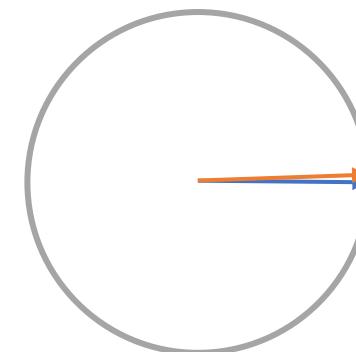
Projective

$$U = e^{-ig S \otimes P_A}$$



Generic weak

$$\text{Tr} [O_A(U|\psi\rangle \otimes |0_A\rangle)]$$



Weak approximation

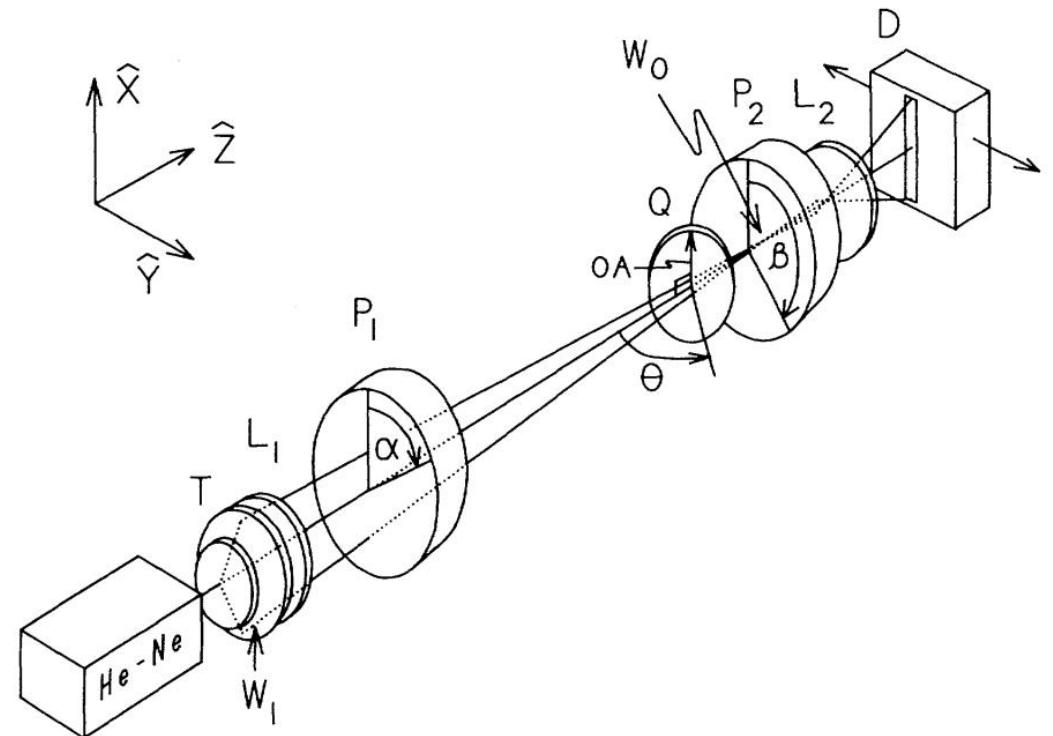
# Applications of WVs and WMs

Amplification of small quantities through post selection

$$\Re(\langle \hat{S}^W \rangle_{\psi}^{\phi}) \approx \frac{1}{g} \langle \hat{X}_A \rangle_{1_A} \longrightarrow g \approx \frac{\langle \hat{X}_A \rangle_{1_A}}{\Re(\langle \hat{S}^W \rangle_{\psi}^{\phi})}$$

Examples:

- First measurement of a weak value [Ritchie *et al.*, Phys. Rev. Lett. **66**, 1107, 1991]
- Spin Hall effect of light [Hosten and Kwiat, Science **319**, 787, 2008]
- Angular deflection [Dixon *et al.* Phys. Rev. Lett. **102**, 1, 2009]
- Imbert-Fedorov effect [Jayaswal *et al.*, Opt. Lett. **39**, 2266, 2014]

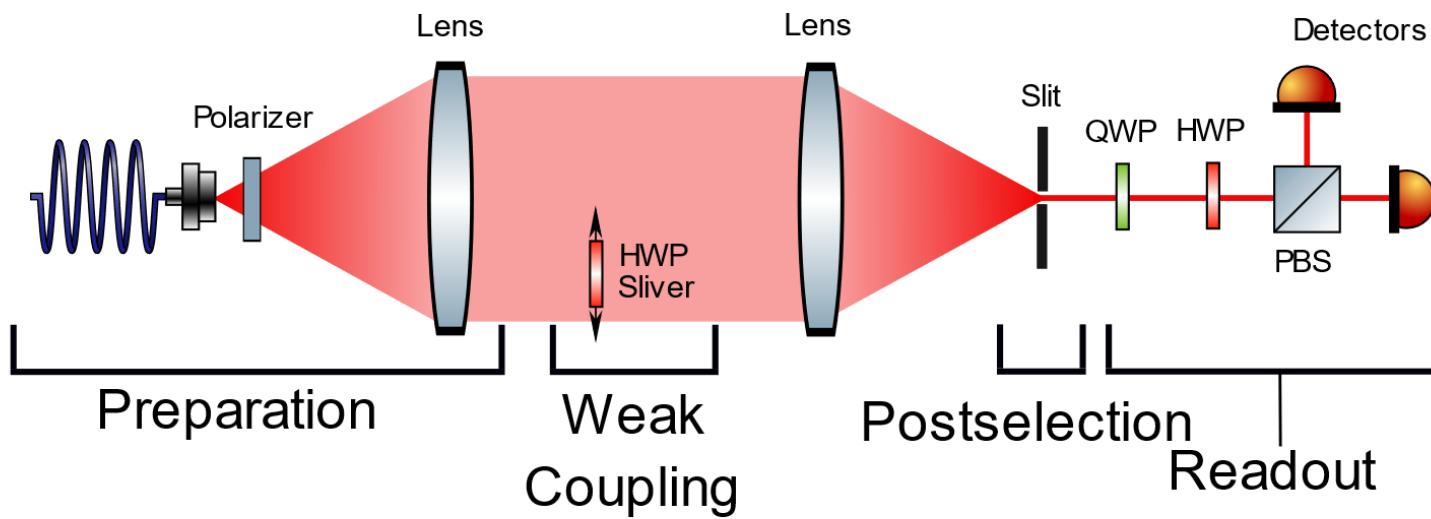


Ritchie *et al.*, Phys. Rev. Lett. **66**, 1107, 1991

# Applications of WVs and WMs

## Quantum state reconstruction

$$\langle \Pi_x^W \rangle_{\psi}^{p_0} = \frac{\langle p_0 | x \rangle \langle x | \psi \rangle}{\langle p_0 | \psi \rangle} \propto \langle x | \psi \rangle$$

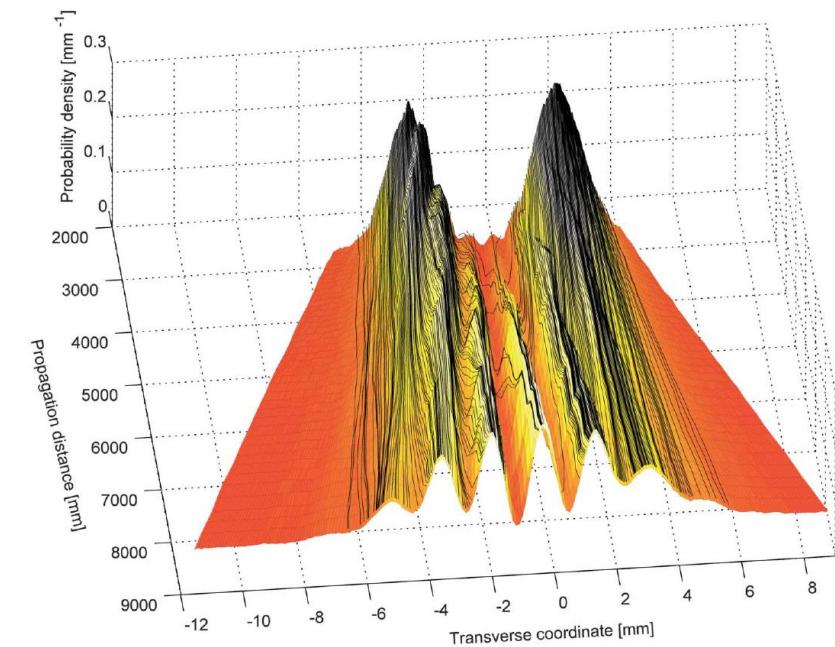
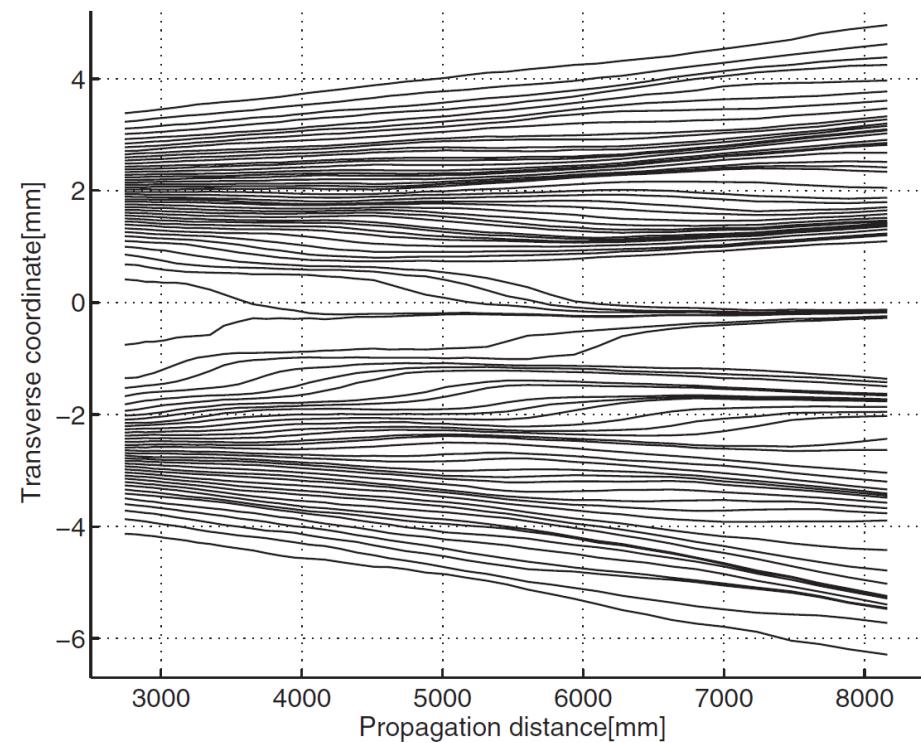


Works with:

- Pure states [Lundeen *et al.*, Nature **474**, 188, 2011]
- Mixed states [Thekkadath *et al.*, Phys. Rev. Lett. **117**, 120401, 2016]
- Strong measurements [Calderaro *et al.*, Phys. Rev. Lett. **121**, 230501, 2018]

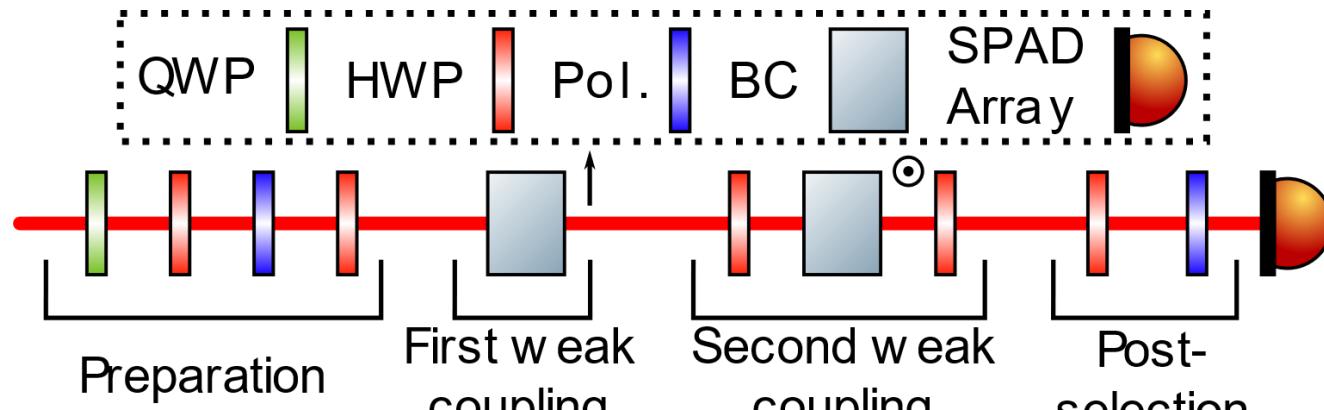
# Applications of WVs and WMs

Path and interference in two-slit interferometer

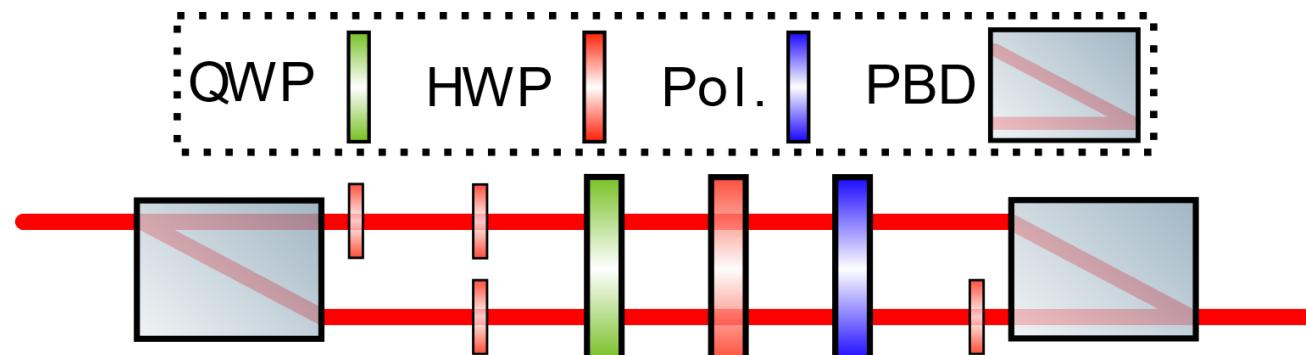


# Applications of WVs and WMs

Measurement of products of incompatible observables



Piacentini *et al.*, Phys. Rev. Lett. **117**, 170402, 2016

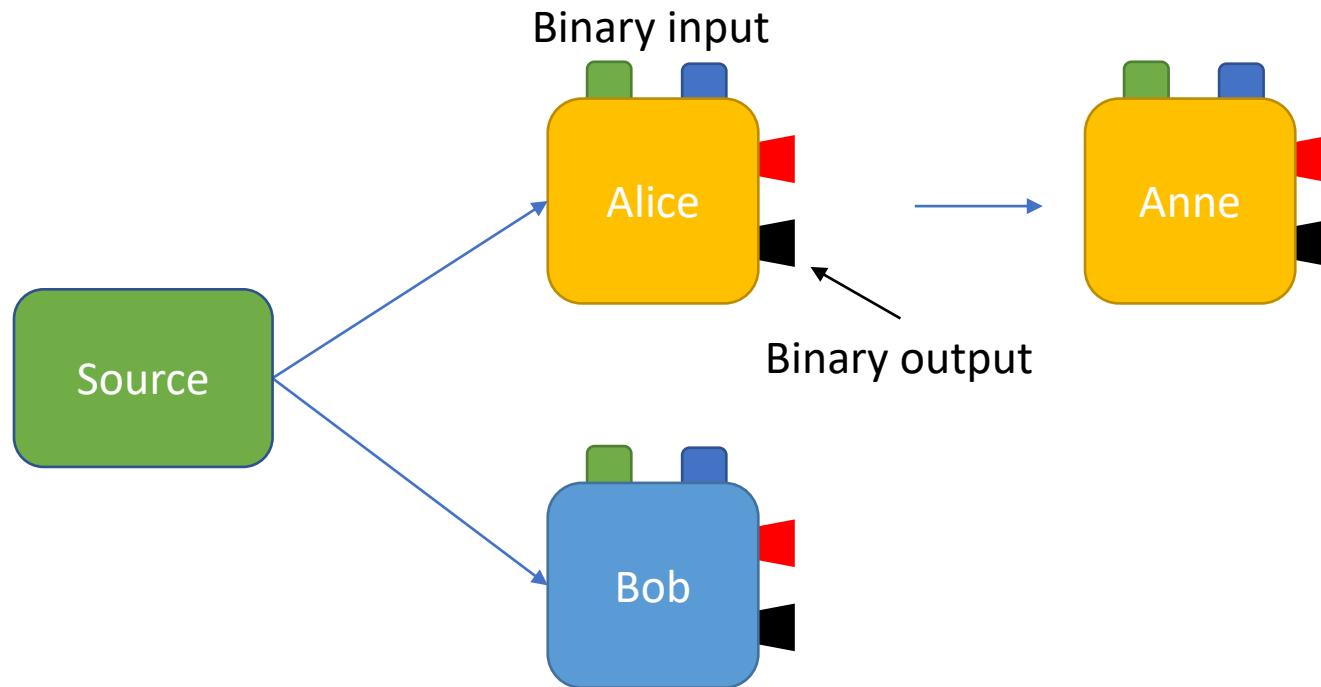


Chen *et al.*, Opt. Expr. Lett. **27**, 6089, 2019

# Entanglement

- The characteristic trait of quantum mechanics [Schrödinger, Math. Proc. Cambridge. Ph. Soc. **31**, 555, 1935]
- A resource:
  - Cryptography [Ekert, Phys. Rev. Lett. **67**, 661, 1991]
  - Teleportation [Bennett *et al.*, Phys. Rev. Lett. **70**, 1895, 1993]
  - Metrology [Giovannetti *et al.*, Science **306**, 1330, 2004]
  - Device-independent protocols [Pironio *et al.*, New J. Phys. **18**, 100202, 2016]
- Certification of entanglement does not require assumptions
- Entanglement is fragile

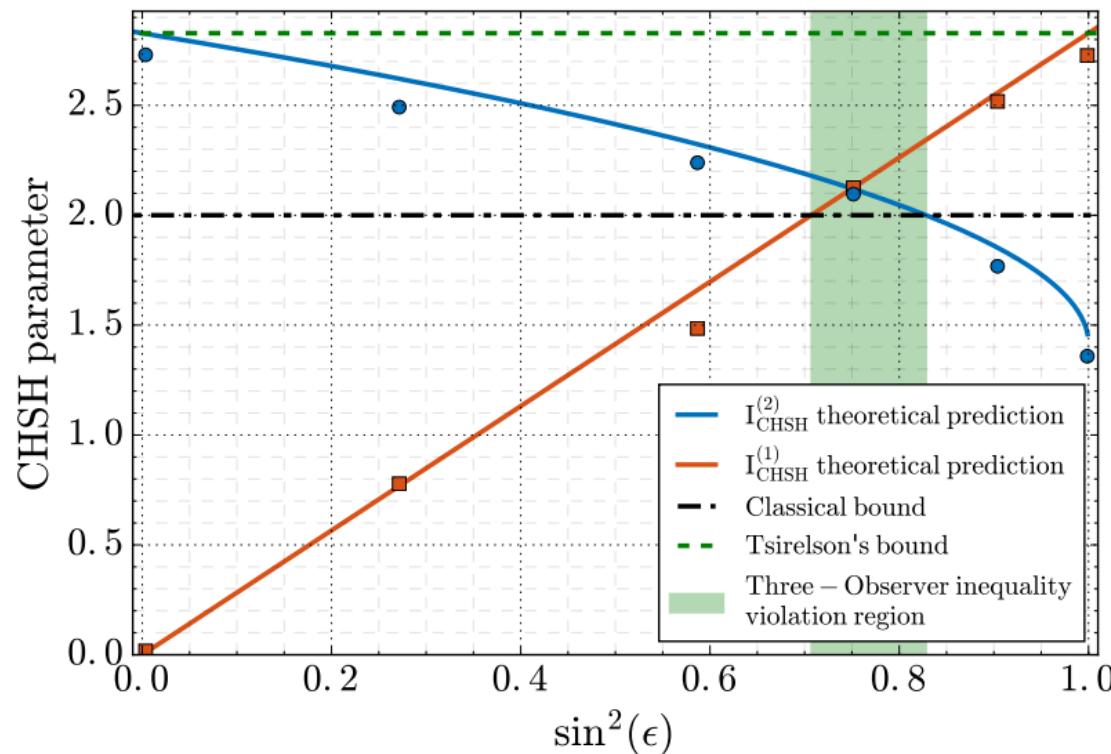
# Double violation of the CHSH inequality



$$S_{CHSH} = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle > 2$$

Weak measurements do not destroy entanglement, and at the same time can certify its presence

# Double violation of the CHSH inequality



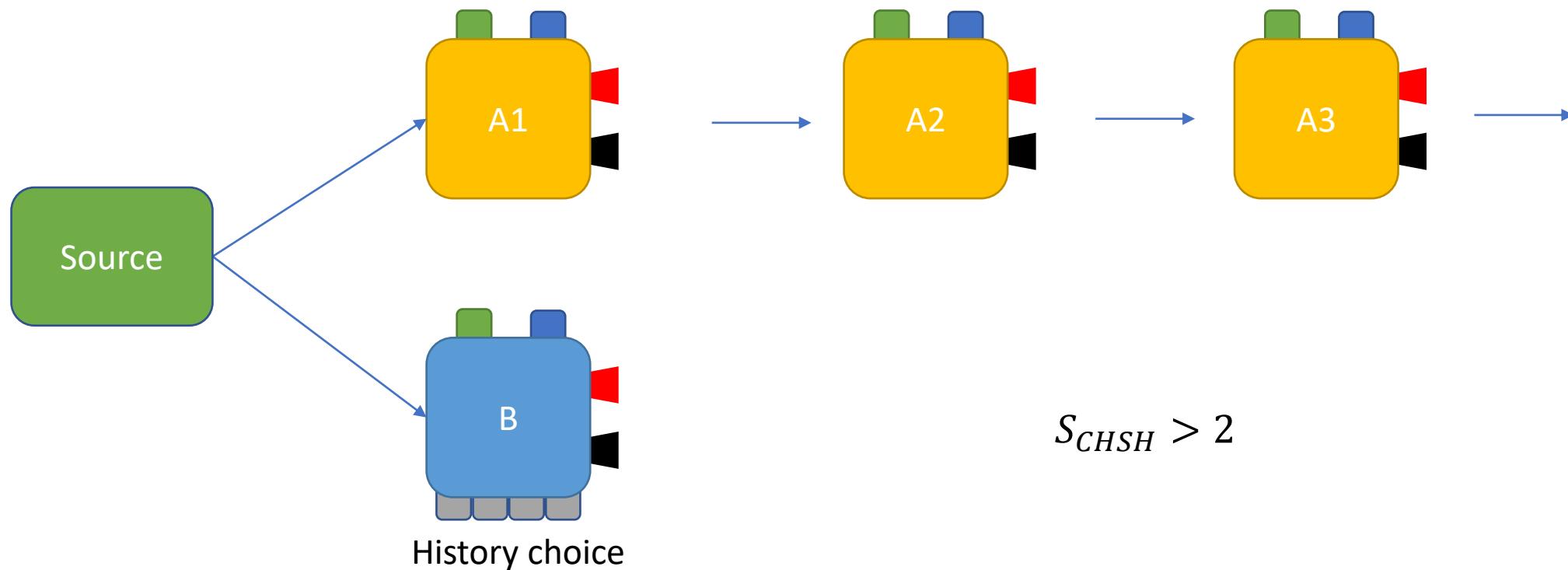
Schiavon *et al.*, Quantum Sci. And Tech. **2**, 015010, 2017

It has been observed experimentally  
[Schiavon *et al.*, Quantum Sci. And Tech.  
**2**, 015010, 2017]

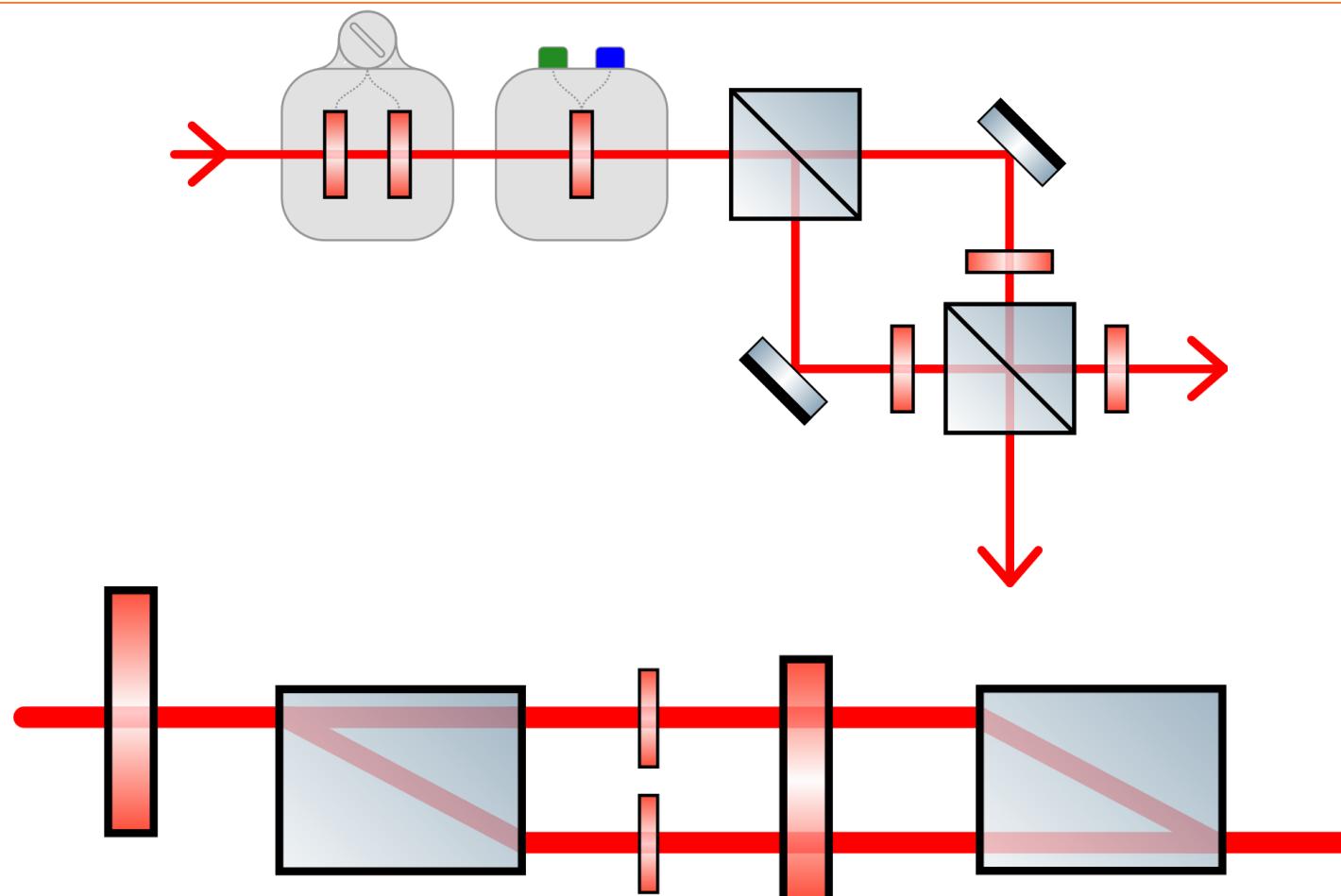
Two violations is a hard limit [Mal *et al.*,  
Mathematics **4**, 48, 2016], unless

- Measurements have different strengths [Brown and Colbeck, Phys. Rev. Lett. **125**, 090401, 2020]
- Measurements are adapted to previous outcomes [Tavakoli and Cabello, Phys. Rev. A **97**, 032131, 2018]

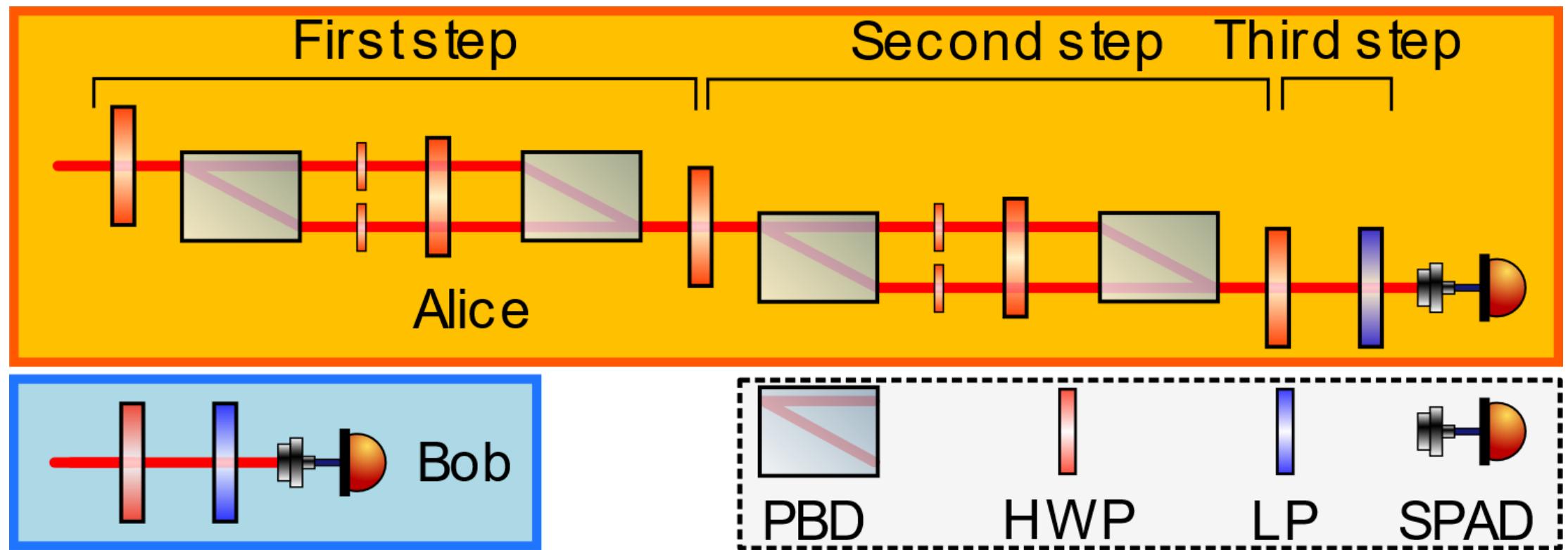
# Multiple violations of the CHSH inequality



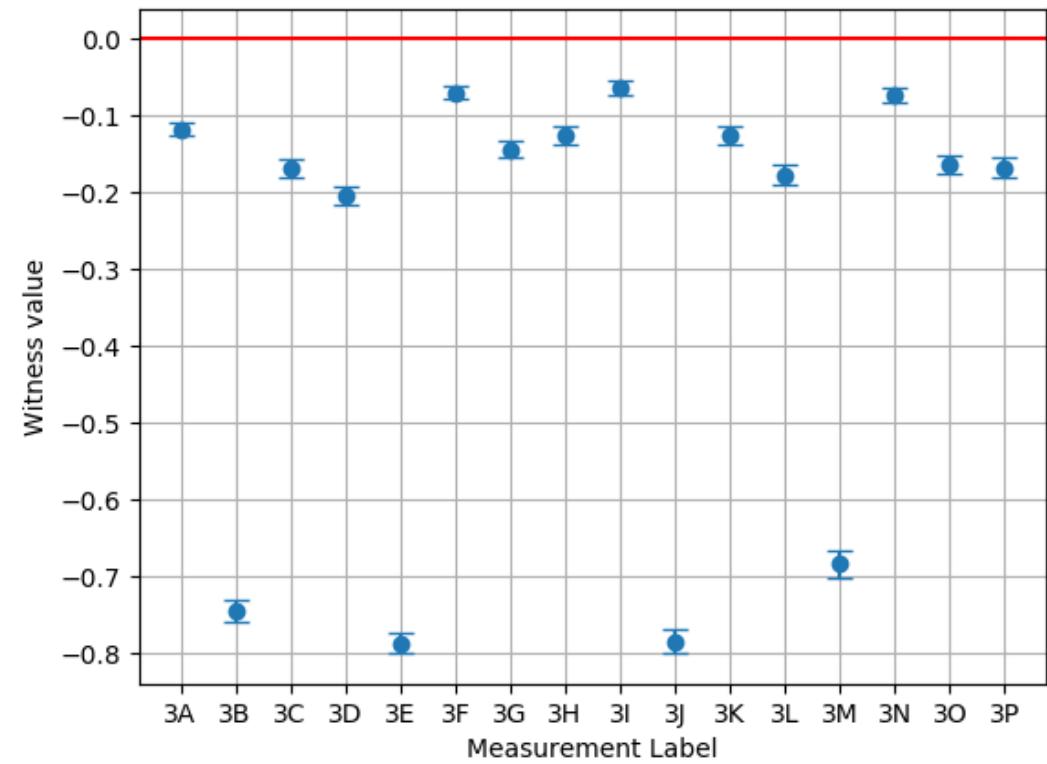
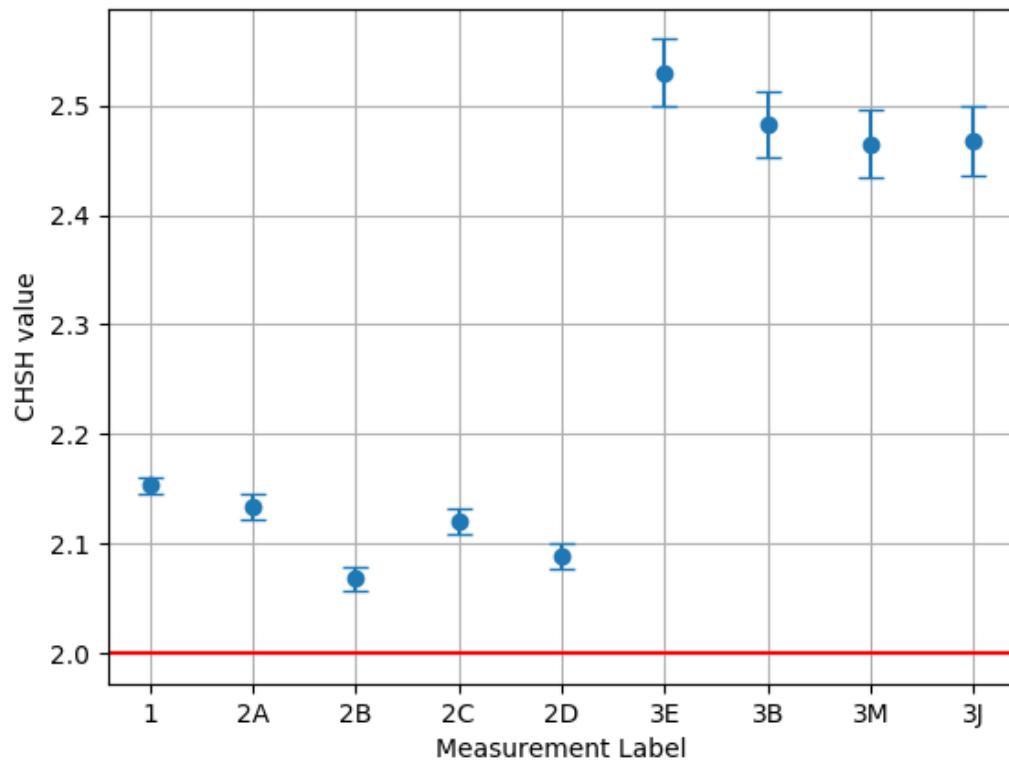
# Experimental Scheme



# Experimental Scheme



# Results

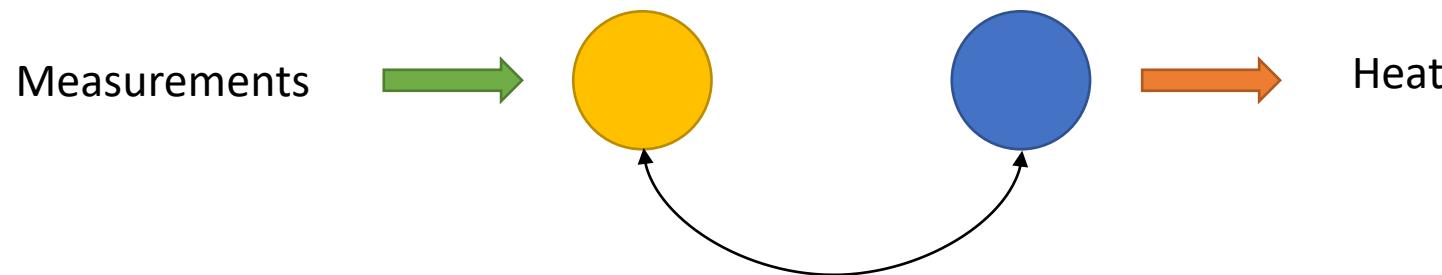


# Applications: Fundamental tests

1. System – centric interpretation
2. Measurement choice is independent of the system
3. Limited memory
4. Landauer's principle



Emission of heat



# Applications: Fundamental tests

- $\neg 2 \rightarrow$  No free will
  - $\neg 3 \rightarrow$  Finite systems can store infinite information
  - $\neg 4 \rightarrow$  Contradiction with experimental evidence
  - 0 Temperature  $\rightarrow$  Contradiction with the 3° law of thermodynamics
- 
- $\neg 1 \rightarrow$  Quantum probabilities depend on the observer

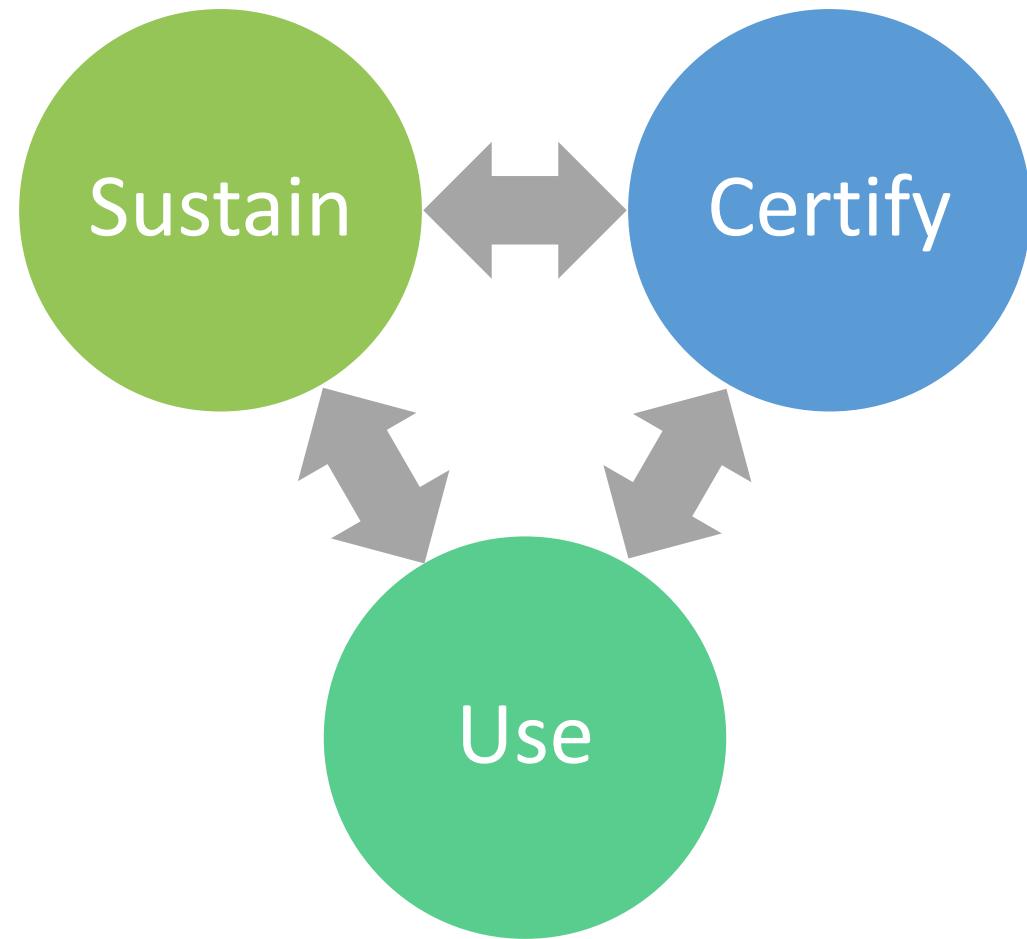
# Applications: Randomness extraction

- Device-independent quantum randomness requires entanglement
- Entanglement can be certified with the violation of a Bell inequality
- Measurement outcomes can be used to generate random bits
- This can be done sequentially: many bits per entangled pair

Pironio *et al.*, Nature **464**, 1021, 2010

Curchod *et al.*, Phys. Rev. A **95**, 020102(R), 2017

# Applications



# Conclusions

- Weak measurements have been relevant in experiments of foundational and applied physics
- They can be used to repeatedly certify entanglement
- We have tested this experimentally with an optical setup
- Repeated use of entanglement can improve device independent protocols or distinguish interpretations of quantum theory

# Bibliography

- Aharonov *et al.*, Phys. Rev. Lett. **60**, 1351, 1988
- Ritchie *et al.*, Phys. Rev. Lett. **66**, 1107, 1991
- Hosten and Kwiat, Science **319**, 787, 2008
- Dixon *et al.* Phys. Rev. Lett. **102**, 1, 2009
- Jayawal *et al.*, Opt. Lett. **39**, 2266, 2014
- Lundeen *et al.*, Nature **474**, 188, 2011
- Thekkadath *et al.*, Phys. Rev. Lett. **117**, 120401, 2016
- Calderaro *et al.*, Phys. Rev. Lett **121**, 230501, 2018
- Kocsis *et al.*, Science **332**, 1170, 2011
- Piacentini *et al.*, Phys. Rev. Lett. **117**, 170402, 2016
- Chen *et al.*, Opt. Expr. Lett. **27**, 6089, 2019
- Schrödinger, Math. Proc. Cambridge. Ph. Soc. **31**, 555, 1935
- Ekert, Phys. Rev. Lett. **67**, 661, 1991
- Bennett *et al.*, Phys. Rev. Lett. **70**, 1895, 1993
- Giovannetti *et al.*, Science **306**, 1330, 2004
- Pironio *et al.*, New J. Phys. **18**, 100202, 2016
- Silva *et al.*, Phys. Rev. Lett. **114**, 250401, 2015
- Schiavon *et al.*, Quantum Sci. And Tech. **2**, 015010, 2017
- Mal *et al.*, Mathematics **4**, 48, 2016
- Brown and Colbeck, Phys. Rev. Lett. **125**, 090401, 2020
- Tavakoli and Cabello, Phys. Rev. A **97**, 032131, 2018
- Cabello *et al.*, Phys. Rev. A **94**, 052127, 2016
- Pironio *et al.*, Nature **464**, 1021, 2010
- Curchod *et al.*, Phys. Rev. A **95**, 020102(R), 2017
- Foletto *et al.*, Phys. Rev. Appl. **13**, 044008, 2020