Workshop sulla Gravitazione Sperimentale: misure laser, fisica fondamentale e applicazioni in INFN-CSN2

12 e 13 Novembre 2020

Results of the LARASE Experiment: Part IV SaToR-G: attività in corso

David M. Lucchesi

IAPS/INAF

INFN Tor Vergata

david.lucchesi@inaf.it

on behalf of the SaToR-G experiment

Summary

- Results of the LARASE Experiment: Part IV
- Current activities

Results of the LARASE Experiment: Part IV

Summary

- Thermal thrust effects
- Mass and moments of inertia
- The LASSOS Spin Model
- The LATOS thermal model
- Preliminary results
- Conclusions

LAser RAnged Satellites Experiment

LARASE
LASERRANGEDSATELLITESEXPERIMENT

INFN Istituto Nazionale di Fisica Nucleare

2013/2019

Satellites Tests of Relativistic - Gravity

SaToR-G

INFN Istituto Nazionale di Fisica Nucleare

Thermal thrust effects

An intricate role, among the complex non-gravitational perturbations, is played by the subtle thermal thrust effects that arise from the radiation emitted from the satellite surface as consequence of the non uniform distribution of its temperature

In the literature of the LAGEOS satellite this problem was addressed since the early 80s' of the past century to explain the (apparently) anomalous behavior of the along-track acceleration of the satellite, characterized by a complex pattern:

Rubincam, Afonso, Ries, Scharroo, Farinella, Metris, Vokrouhlicky, Slabinsky, Lucchesi, Andres, …

represents a non exhaustive list of the researchers that have successfully worked on this very important issue

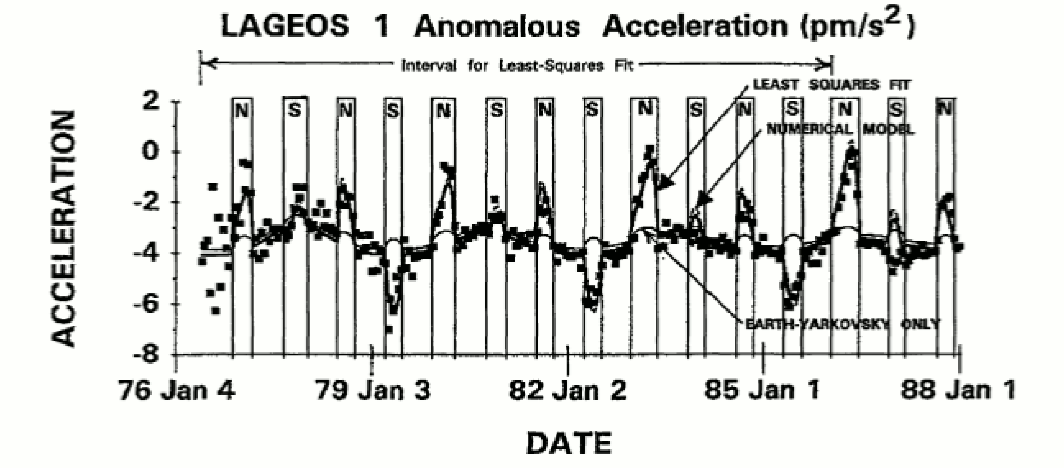


Figure 2. LAGEOS 1 anomalous acceleration: observed data points (squares) are based on 15 day fits to laser data by the Center for Space Research, University of Texas at Austin. The vertical bars mark eclipse seasons. N at top of bar denotes season when satellite travels northward through earth shadow; S denotes season with southward travel.

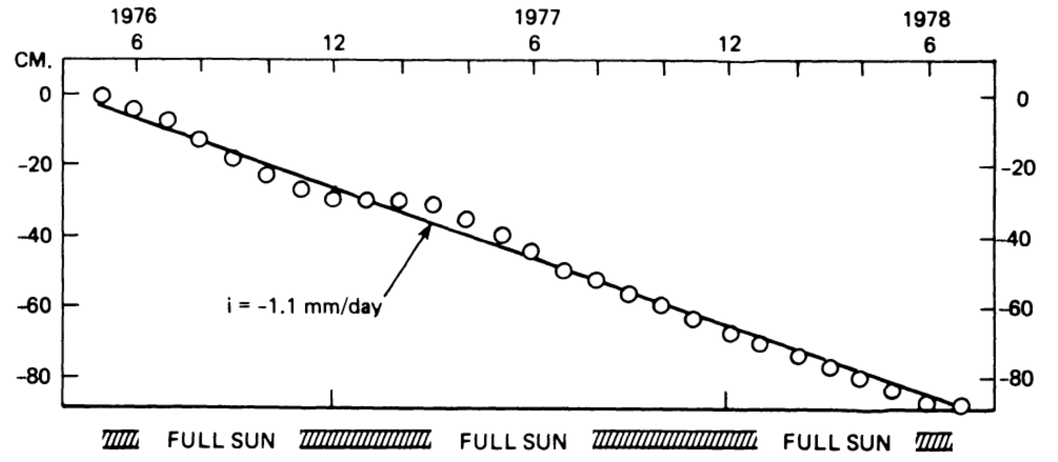
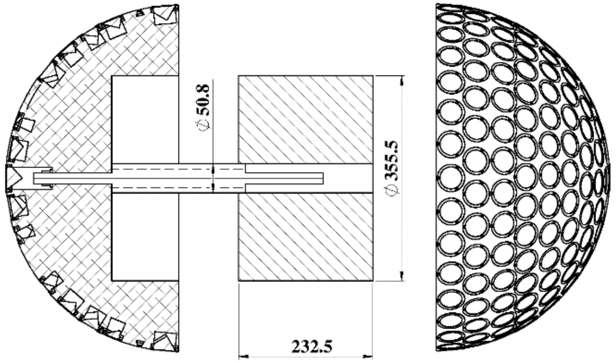


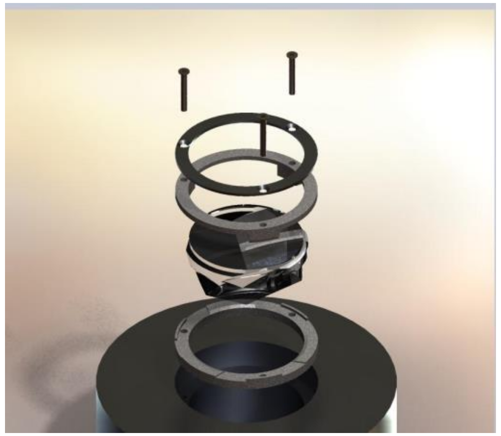
Fig. 1. The secular decrease in the semimajor axis of Lageos's orbit. The straight line fitted through the data points has a slope of -1.1 mm day^{-1}. When the orbit is in full sunlight and shadow is shown along the horizontal axis.

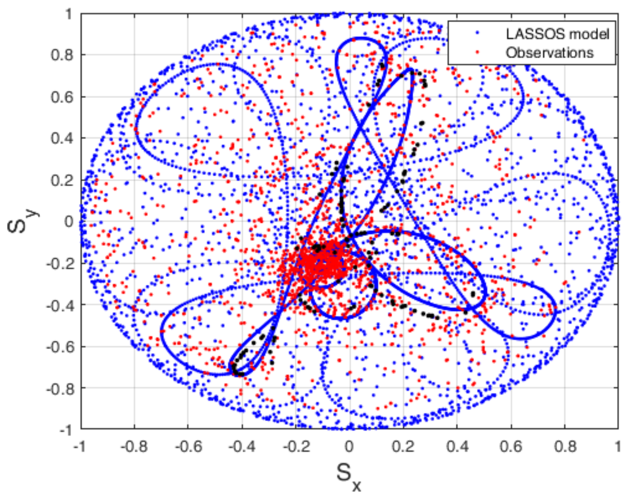
Thermal thrust effects

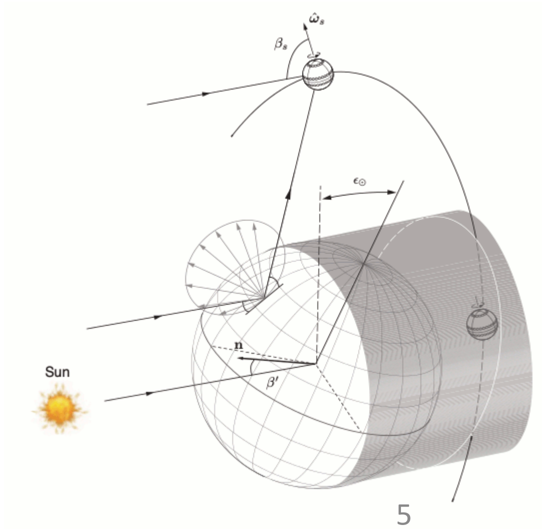
The dynamical problem to solve is quite complex and should account for the following main aspects:

- A deep physical characterization of the satellite
  - **emission and absorption coefficients, thermal conductivity, heat capacity, thermal inertia, …**
- Rotational dynamics of the satellite
  - **Spin orientation and rate**
- Radiation sources
  - **Sun and Earth**







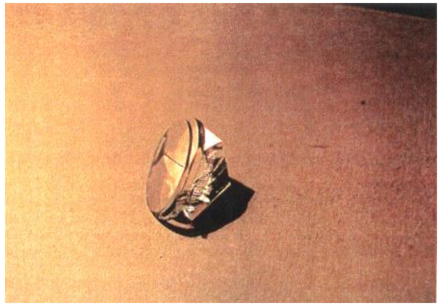


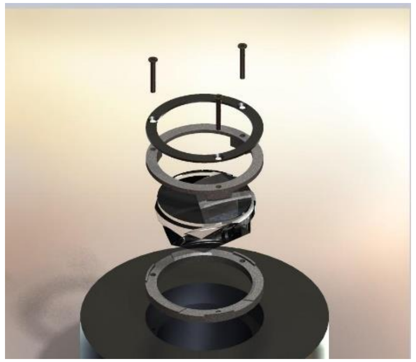
Thermal thrust effects

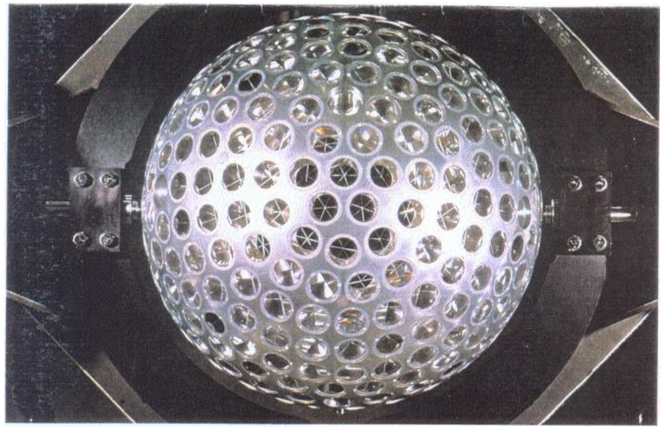
We have tackled the problem following the two approaches considered in the past in the literature (but with some differences):

- We developed a simplified thermal model of the satellite based on
  - **the energy balance equation on its surface**
  - **a linear approach for the distribution of the temperature with respect to its equilibrium (mean) temperature**

- A general thermal model based on
  - **a satellite (metallic structure) in thermal equilibrium**
  - **the CCRs rings are at the same temperature of the satellite**
  - **for each CCR the thermal exchange with the satellite is computed**







LAGEOS II

Thermal thrust effects

The main perturbations to be taken into account are:

- **The solar Yarkovsky-Schach effect**
  - **an anisotropic emission of thermal radiation that arises from the temperature gradients across the surface produced by the solar heating and the thermal inertia of the various parts (mainly from the CCRs)**
  - **it produces long-term effects when the thermal radiation is modulated by the eclipses**
- **The Earth Yarkovsky thermal (or Rubincam) effect**
  - **the temperature gradients responsible of the anisotropic emission of thermal radiation are produced by the Earth's infrared radiation**
  - **the bulk of the effect is due to the CCRs and their thermal inertia**
- **The Earth's Albedo**
  - **the temperature gradients responsible of the anisotropic emission of thermal radiation are produced by the Earth's visible radiation**
  - **the bulk of the effect is due to the CCRs and their thermal inertia**

Thermal thrust effects

Simplified (average) thermal model

Farinella P, Vokrouhlicky D., Thermal force effects on slowly rotating, spherical artificial satellites - I. Solar heating, Plan. Space Sci. 44, 12 (1996)

Characteristic amplitude: $A_{YS} \cong \frac{16}{9}\frac{A}{m}\frac{\varepsilon\sigma}{c}T_0^3\Delta T$

<u>Accelerations in body:</u>

With no eclipses

$$a_X = A_{YS}\frac{sinz_\odot}{1+\left(\omega_{spin}\tau\right)^2}$$

$$a_Y = A_{YS}\frac{sinz_\odot}{1+\left(\omega_{spin}\tau\right)^2}\omega_{spin}\tau$$

$$a_Z = A_{YS}cosz_\odot$$

With eclipses

$$a_X = A_{YS}\frac{sinz_\odot}{1+\left(\omega_{spin}\tau\right)^2}\Gamma_X$$

$$a_Y = A_{YS}\frac{sinz_\odot}{1+\left(\omega_{spin}\tau\right)^2}\Gamma_Y$$

$$a_Z = A_{YS}cosz_\odot\Gamma_Z$$

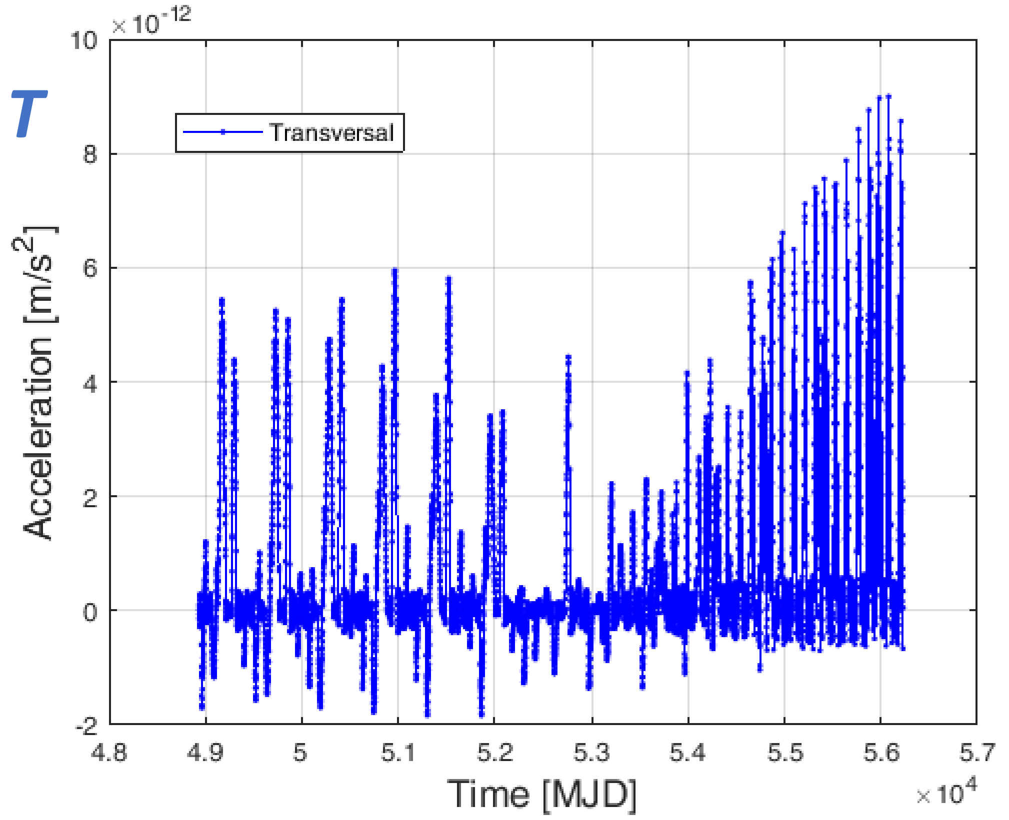
- We used:
  - $A_{YS} \cong -1.035\times10^{-10}$ m/s^2
  - $\tau \cong 2113$ s

Lucchesi D.M., Reassessment of the error modelling of the non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring derivation - Part II, Plan. Space Sci. 50 (2002)

Thermal thrust effects

Results for the simplified (averaged) model: LAGEOS II

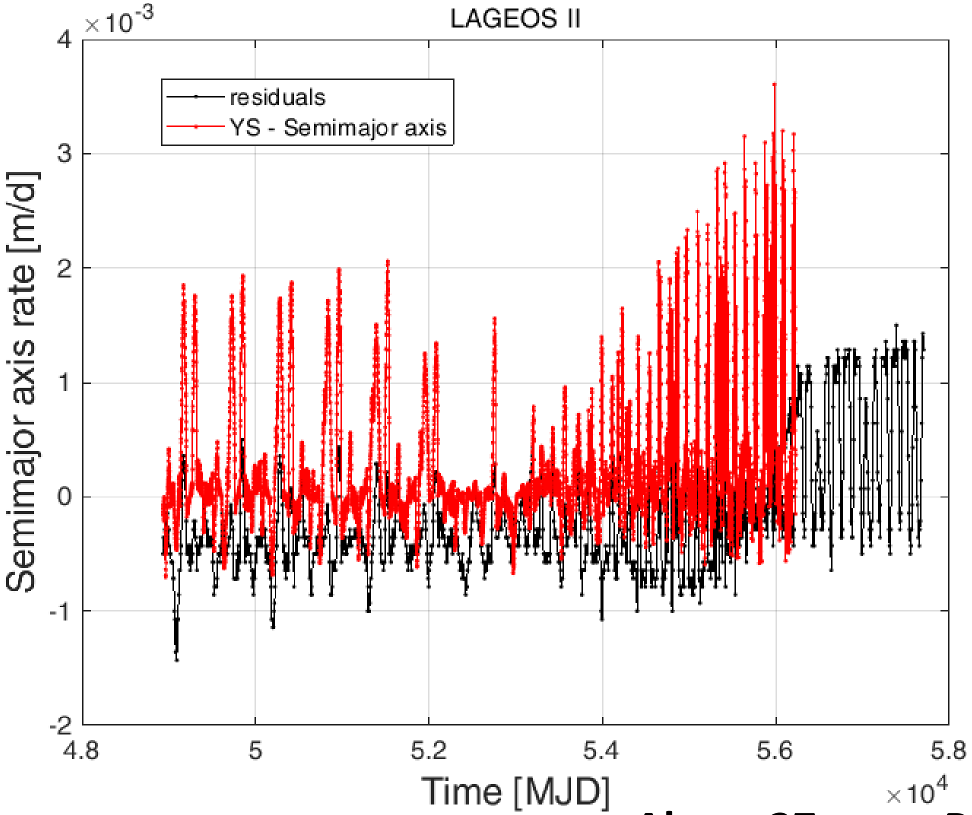
$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left[T + e(T\cos f + R\sin f)\right]$$

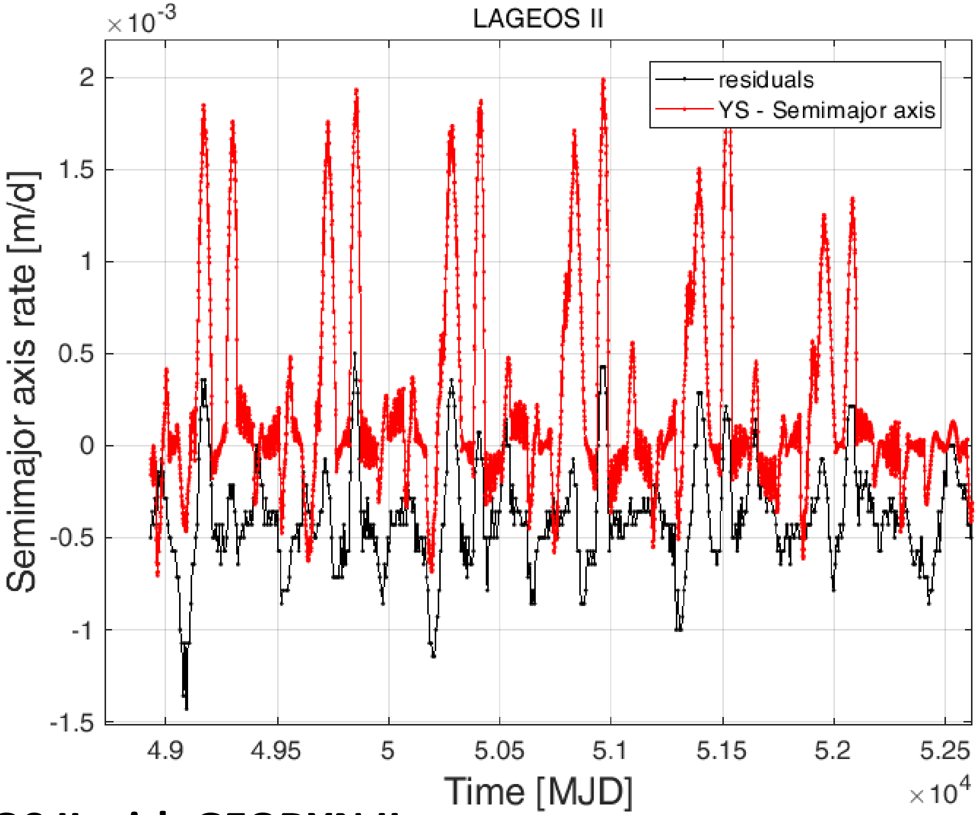


Thermal thrust effects

Results for the simplified (averaged) model: LAGEOS II

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}[T + e(T\cos f + R\sin f)]$$



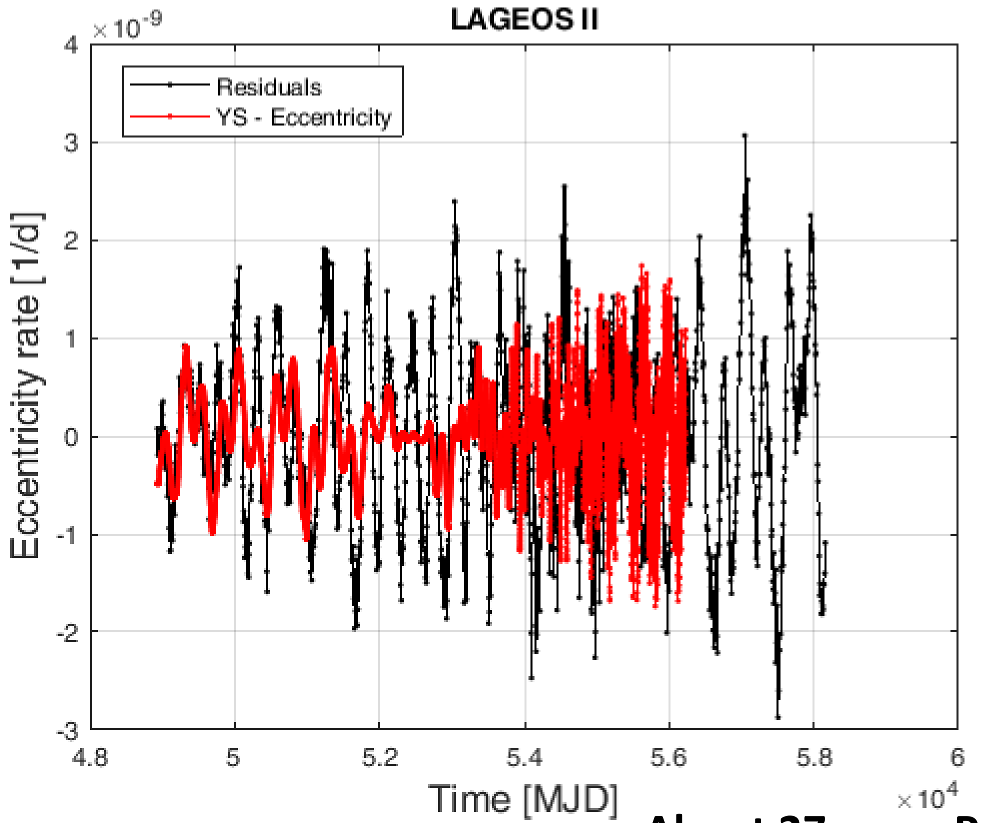


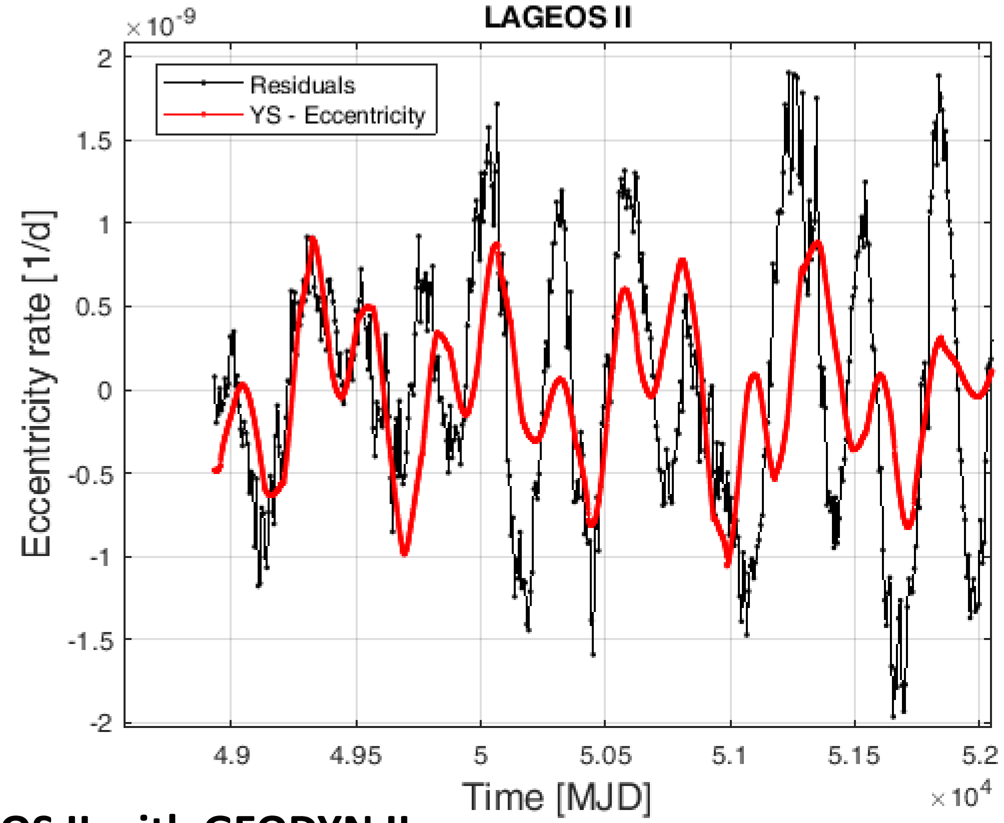
About 27 years POD of LAGEOS II with GEODYN II

Thermal thrust effects

Results for the simplified (averaged) model: LAGEOS II

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[R\sin f + T(\cos f + \cos u)\right]$$



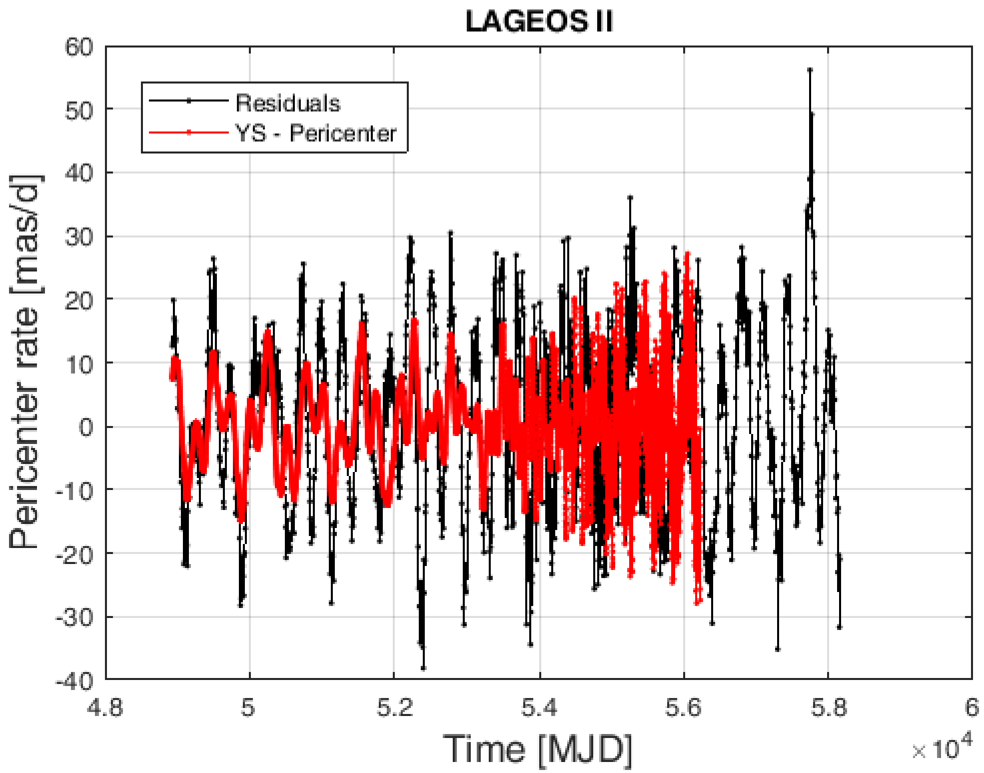


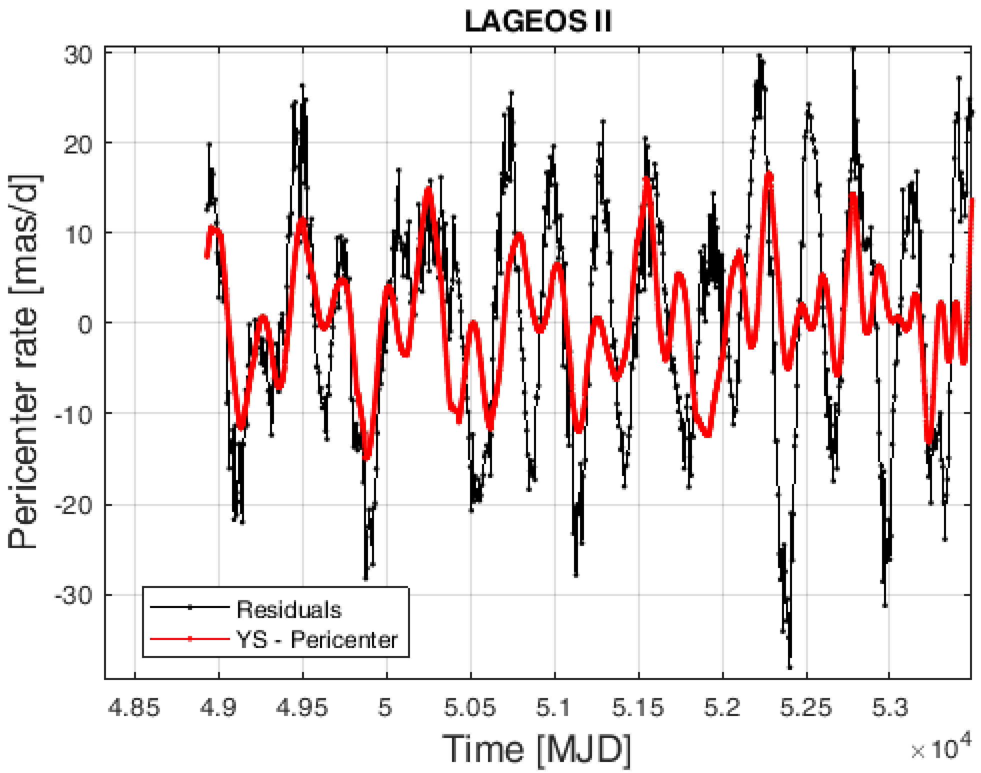
About 27 years POD of LAGEOS II with GEODYN II

Thermal thrust effects

Results for the simplified (averaged) model: LAGEOS II

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nea}\left[-R\cos f + T\left(\sin f + \frac{1}{\sqrt{1-e^2}}\sin u\right)\right] - \frac{W}{na^2\sqrt{1-e^2}}\frac{1}{\tan i} r\sin(\omega + f)$$





About 27 years POD of LAGEOS II with GEODYN II

Mass and moments of inertia

Among the main parameters necessary to correctly model the dynamical behavior of an artificial satellite we have to consider:

- **mass**
- **center of mass position**
- **moments of inertia**

If we look to the scientific literature and to the official documents (NASA, ASI, ALENIA) of LAGEOS, we can easily see a number of different values for these fundamental parameters, differences that we have mainly confined within the following categories:

1. **lack of complete measurements (hence of the flight model knowledge)**
2. **mistakes in information/popularization and its ''error propagation…''**
3. **material alloys and manufacturing tolerances**

Mass and moments of inertia

For instance, just to give an example, it is well known the controversy and the consequent very long debate about the material of the inner core of LAGEOS:

- **it is of BRASS (NASA-TN 1975; Cohen and Smith 1985) ?**
- **it is of BERYLLIUM and COPPER (Johnson et al. 1976) ?**

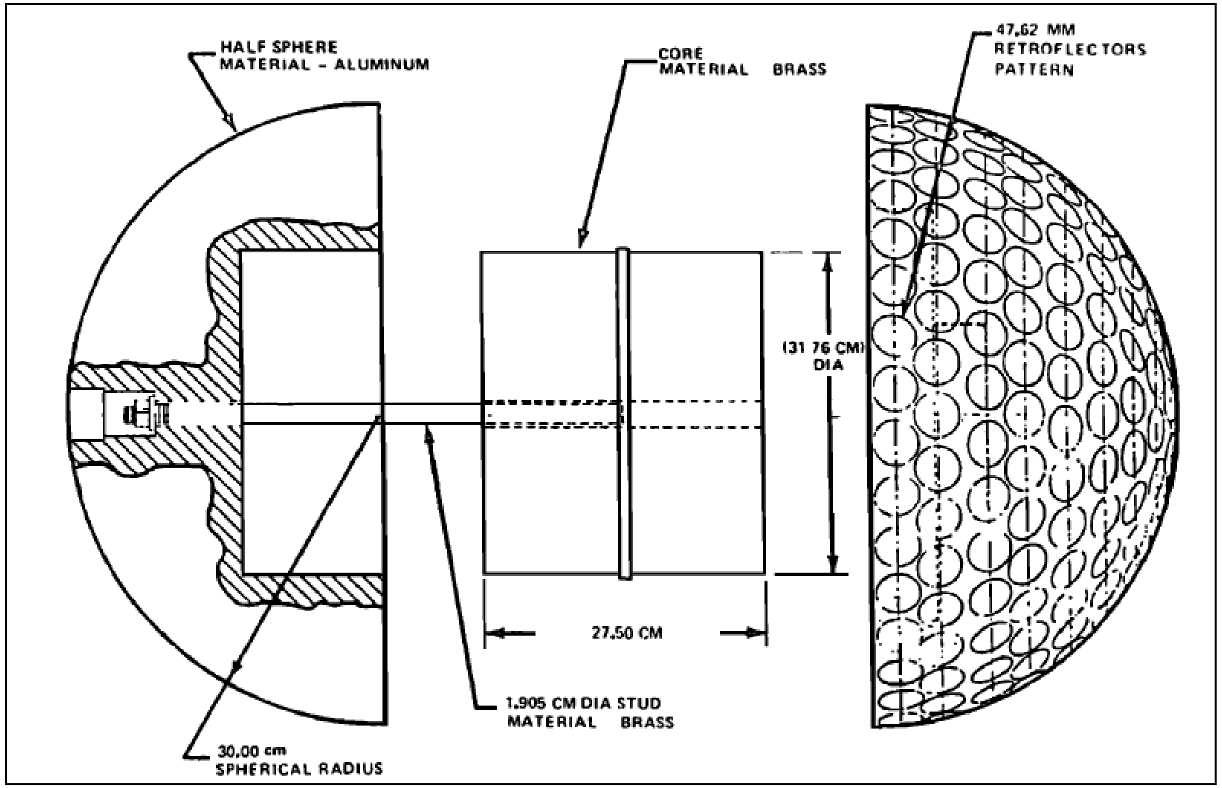
➢ Still in Slabinski 1997 (more than 20 years after LAGEOS launch), it was reported Be-Cu for the core

➢ Only 10 years later Andrés concluded that the core was probably made of Brass as that of LAGEOS II, but he wrongly concluded that the dimensions were slightly different

Slabinski, V.J., 1996. *A numerical solution for LAGEOS thermal thrust: the rapid-spin case*. Celestial Mech. Dyn. Astron. 66, 131–179.
Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.

Mass and moments of inertia

A second and significant example are the correct dimensions for the stud and the core of LAGEOS: for instance, those reported in Cohen and Smith 1985 are wrong



Mass and moments of inertia

- We reconstruct information about the structure, the material used and the moments of inertia of the two LAGEOS
- We built a 3D-CAD model of the satellites structure useful for finite element-based analysis
- We also solve for contradictions and overcome several misunderstanding present in the historical literature of the older LAGEOS (carefully re-analyzing the earlier technical documents)

LAGEOS

LARES

Mass and moments of inertia

Documents on LAGEOS

- NASA, 1975. LAGEOS Phase B Technical Report, NASA Technical Memorandum X-64915. Technical Report TMX-64915. Marshall Space Flight Center. Marshall Space Flight Center, Alabama 35812. February 1975
- Siry, J.W., 1975. The LAGEOS system. Technical Report TM-X-73072. NASA
- LAGEOS Press Kit, 1976. NASA (1976) Project LAGEOS Press Kit release 76/67. Technical Report 76/67. NASA. National Aeronautics and Space Administration, Washington, DC
- Fitzmaurice, M.W., Minott, P.O., Abshire, J.B., Rowe, H.E., 1977. Prelaunch Testing of the Laser Geodynamic Satellite. Technical Report TP-1062. NASA
- Wong, C., 1978. Watching the Earth move from space. Sky Telesc., 198–202

Documents on LAGEOS II

- Cogo, F., 1988. Weight discrepancy analysis between LAGEOS 1 and LAGEOS 2 satellites. Technical Report LG-TN-AI-035. Aeritalia
- Fontana, F., 1989. Physical properties of LAGEOS II satellite. Technical Report LG-TN-AI-037. Aeritalia
- Fontana, F., 1990. Physical properties of LAGEOS II satellite. Technical Report LG-TN-AI-037. Aeritalia
- Minott, P.O., Zagwodzki, T.W., Varghese, T., Seldon, M., 1993. Prelaunch Optical Characterization of the Laser Geodynamic Satellite (LAGEOS 2). Technical Report 3400. NASA Technical Paper 3400. National Aeronautics and Space Administration, Washington, DC

Mass and moments of inertia

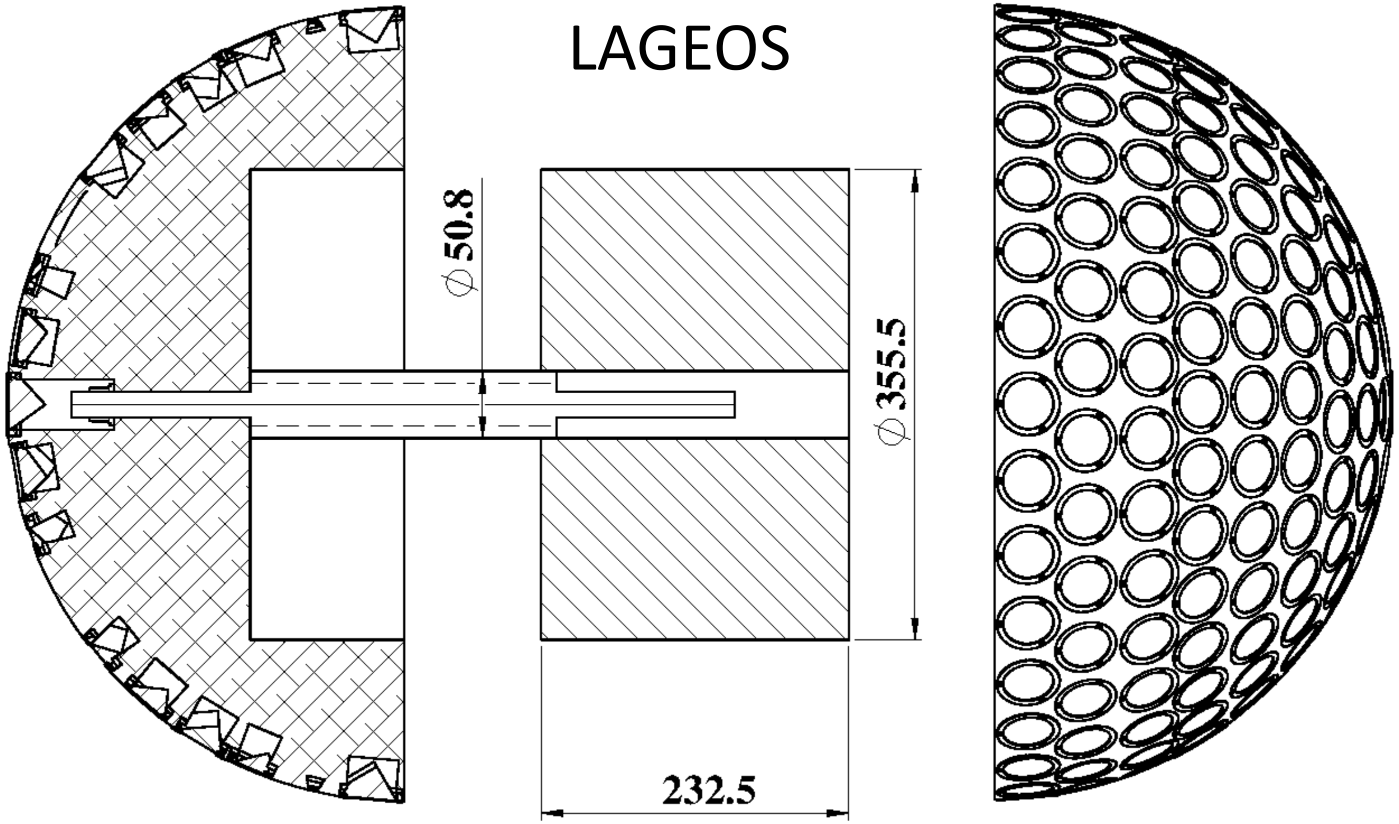
Table 1
Materials used for the construction of the two LAGEOS satellites (Cogo, 1988) and their nominal densities.

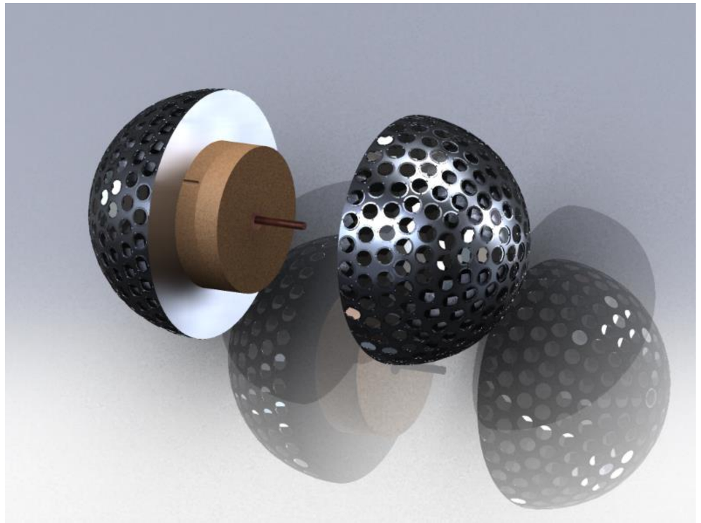
Satellite	Material density ρ_n (kg/m^3)		
	Hemispheres	Core	Stud
LAGEOS	AA6061 2700[a]	QQ-B-626 COMP.11 8440[a]	Cu-Be 8230[b]
LAGEOS II	AlMgSiCu UNI 6170 2740[c]	PCuZn39Pb2 UNI 5706 8280[c]	Cu-Be QQ-C-172 8250[c]

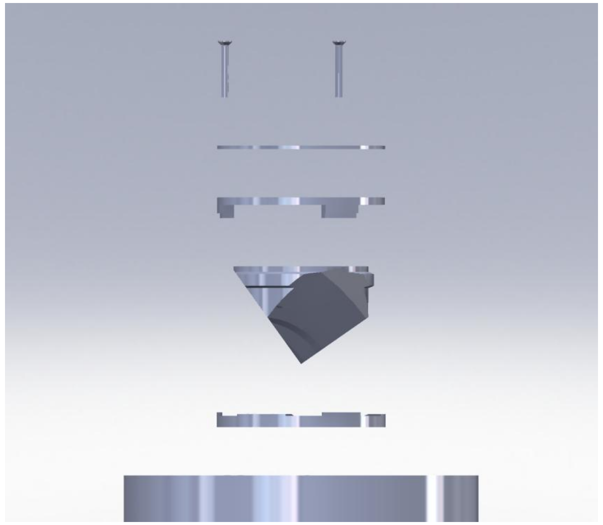
[a] ASM International Handbook Committee (1990).
[b] Bauccio (1993).
[c] It is the value calculated in Cogo (1988) starting from the measured averaged composition.

Mass and moments of inertia

LAGEOS







CCR

Mass and moments of inertia

$$\bar{\rho} = \rho_N \frac{M_m}{M_c}$$

Our conclusions on the materials and shape of the satellites have been validated by re-computing the mass and moments of inertia of the satellites and comparing the results with those reported in the official documentation in the case of the pre-flight configurations

Table 2
Comparison of masses and moments of inertia for the two LAGEOS satellites. In the notation we follow NASA (1975). The x axis coincides (nominally) with the principal axis of inertia (the angle between the symmetry axis and the principal axis orientation was bound to be below 0.02 radians). Practically, this axis coincides with the initial rotation axis of the satellites.

Satellite origin of value	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement				
Computed value in NASA (1975)	409.8	11.516	11.084	11.084
Measured value in NASA (1975)	406.965	–	–	–
Values computed in the present work using nominal density of Table 1	405.93	11.40	10.93	10.93
LAGEOS balance model				
Computed value in NASA (1975)	440.3	13.14	12.71	12.71
Measured value in NASA (1975)	440.0	13.11	12.69	12.71
Value computed in the present work using nominal density of Table 1	437.68	13.09	12.62	12.62
Values computed in the present work using normalized density	440.00	13.16	12.68	12.68
LAGEOS II flight arrangement				
Computed values in Fontana (1990)	–	11.45	11.00	11.00
Measured value in Fontana (1990), Fontana (1989) and Cogo (1988)	405.38	–	–	–
Values computed in the present work using nominal density of Table 1	404.97	11.44	10.99	10.99
LAGEOS II without CCRs				
Computed value in Fontana (1989)	386.59	10.39	9.95	9.95
Measured value in Fontana (1989)	387.20	9.67	9.37	9.15
Values computed in the present work using nominal density of Table 1	386.71	10.41	9.95	9.95
Values computed in the present work using normalized density	387.20	10.42	9.96	9.96

Mass and moments of inertia

Table 3
Mass and moments of inertia of LAGEOS and LAGEOS II to be used in the future. The masses are the one measured. The moments of inertia are those computed in the present work with normalized densities.

Satellite	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement	406.97	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II flight arrangement	405.38	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

This work was also extended to **LARES**:

Table 1. Principal moments of inertia of LAGEOS, LAGEOS II and LARES in their flight arrangement.

Satellite	Moments of Inertia (kg m^2)		
	I_{zz}	I_{xx}	I_{yy}
LAGEOS	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03
LARES	4.77 ± 0.03	4.77 ± 0.03	4.77 ± 0.03

- The two **LAGEOS** have almost the same oblateness of about 0.04
- **LARES** is practically spherical in shape, even if an oblateness as small as 0.002 is however possible

Mass and moments of inertia

Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research 57 (2016) 1928–1938

ELSEVIER

CrossMark

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program

Massimo Visco [a,b], David M. Lucchesi [a,b,c,*]

[a] *Istituto Nazionale di Astrofisica (INAF) – Istituto di Astrofisica e Planetologia Spaziali (IAPS), Via del Fosso del Cavaliere, 100, 00133 Roma, Italy*
[b] *Istituto Nazionale di Fisica Nucleare (INFN), Sez. Tor Vergata, Via della Ricerca Scientifica 1, 00133 Roma, Italy*
[c] *Istituto di Scienza e Tecnologie della Informazione (ISTI) – Consiglio Nazionale delle Ricerche (CNR), Via G. Moruzzi 1, 56124 Pisa, Italy*

Received 18 November 2015; received in revised form 8 February 2016; accepted 9 February 2016
Available online 15 February 2016

The LASSOS Spin Model

Spin Models

The rotational dynamics of a satellite represents a very important issue that deeply impacts the goodness of the orbit modeling

Indeed, the modeling of several disturbing effects (like the thermal thrust ones) depends on the knowledge of the spin period and orientation in the inertial space:

1. **Yarkovsky–Schach effect**
2. **Earth–Yarkovsky (Rubincam) effect**
3. **Asymmetric reflectivity from the satellite surface**

Their modelling will greatly improve the POD of the two LAGEOS satellites avoiding the current (and significant) use of empirical accelerations during the data reduction

The LASSOS Spin Model

Spin Models

A general theory is not easy to be developed because the modelling of the spin evolution depends on several factors:

- **spacecraft materials, its shape, details of the structure**
- **space environment: orbit altitude and inclination**
- **spin rate: kind of equations to be solved, resonances (rotational period, thermal inertia, orbit period)**

The LASSOS Spin Model

Past Spin Models

The best spin models developed in the past are:

1. **Bertotti and Iess (JGR 96 B2, 1991)**
2. **Habib et al. (PRD 50, 1994)**
3. **Farinella, Vokrouhlicky and Barlier (JGR 101, 1996); Vokrouhlicky (GRL 23, 1996)**
4. **Andrés, 1997 (PhD Thesis): LOSSAM model**

All of these studies, with the exception of Habib et al., attack and solve the problem of the evolution of the rotation of a satellite in a terrestrial inertial reference system, in the so-called rapid spin approximation and they introduce equations for the external torques that are averaged over time; their fit to the spin observations was good, especially in the case of the LOSSAM model for the LAGEOS II satellite. Habib et al. use a body-fixed reference system and non-averaged torques; their model does not fit so well the observations

The LASSOS Spin Model

We have deeply reviewed previous spin models, in particular we:

- **first built our own spin model in the rapid spin approximation**
- **adopted non-averaged torques in the equations to describe the slow spin approximation: we solved the problem of a metallic sphere rotating in an alternate magnetic field**
- **introduced in the equations all known possible torques (like in LOSSAM model)**
- **solved the equations in a body-fixed reference system in order to better describe the misalignment between the symmetry axis and the spin**
- **included in the equations the terms due to the transversal asymmetry**
- **carefully studied the satellites moments of inertia**

The LASSOS Spin Model

The model for the magnetic torque

Since we are working with conductive satellites moving and rotating in the Earth's magnetic field $\boldsymbol{B}$, a magnetic moment $\boldsymbol{m}$ will be induced in their body and, consequently, a torque $\boldsymbol{M}_{mag}$ will be applied:

$$\boldsymbol{M}_{mag} = \boldsymbol{m} \times \boldsymbol{B}$$

In previous works, **LAGEOS** was modeled as a conducting sphere rotating in a **static magnetic** field

- **The value of the constant magnetic field was computed averaging the magnetic field over the entire orbit of the satellite**

The LASSOS Spin Model

This solution, which is completely valid in a **quasi-stationary** field, can be suitably used as long as the rotation period of the satellite is much shorter than its orbital period as well as of the Earth's rotation period:

$$T_{rot} \ll T_{orb} \qquad T_{rot} \ll T_{\oplus}$$

but it could produce wrong results when is used in slow-spin conditions

In order to obtain a more general expression of the magnetic torque we faced the problem to find an easily integrable expression for the torque acting on a conducting sphere rotating in an **alternating magnetic** field

The LASSOS Spin Model

The involved torques

We consider in the case of the two LAGEOS satellites four torques:

1. The magnetic torque (eddy currents)

$\alpha(\omega) = \alpha' + j\alpha''$ Fourier transform of the magnetic polarizability per unit of volume V of the satellite

$\boldsymbol{\omega}_s^E$ Satellite spin angular velocity

$$\mathbf{M}_{\text{mag}}^E = V\sum_{i=0}^{8}\frac{|\mathbf{B_i}|^2}{2|\boldsymbol{\omega}_\mathbf{s}|}\{-A_i''[1+\cos(2\omega_i t+2\varphi_i)]+D_i'\sin(2\omega_i t+2\varphi_i)\}\boldsymbol{\omega}_\mathbf{s}^\mathbf{E}$$
$$+V\sum_{i=0}^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|^2}\{[\alpha'(\omega_i)-A_i'][1+\cos(2\omega_i t+2\varphi_i)]-[D_i''+\alpha''(\omega_i)]\sin(2\omega_i t+2\varphi_i)\}(\boldsymbol{\omega}_\mathbf{s}^\mathbf{E}\times\mathbf{B_i})$$
$$+V\sum_{i=0}^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|}\{A_i''[1+\cos(2\omega_i t+2\varphi_i)]-D_i'\sin(2\omega_i t+2\varphi_i)\}\mathbf{B_i},$$

where

$$A_i' = \frac{\alpha'(\omega_s^E-\omega_i)+\alpha'(\omega_s^E+\omega_i)}{2} \qquad D_i' = \frac{\alpha'(\omega_s^E-\omega_i)-\alpha'(\omega_s^E+\omega_i)}{2}$$
$$A_i'' = \frac{\alpha''(\omega_s^E-\omega_i)+\alpha''(\omega_s^E+\omega_i)}{2} \qquad D_i'' = \frac{\alpha''(\omega_s^E-\omega_i)-\alpha''(\omega_s^E+\omega_i)}{2}.$$

$$\omega_0 = 0$$
$$\omega_1 = \omega_2 = \omega_\oplus - 2n$$
$$\omega_3 = \omega_4 = \omega_\oplus + 2n$$
$$\omega_5 = \omega_6 = 2n$$
$$\omega_7 = \omega_8 = \omega_\oplus,$$

Angular velocities of the harmonic components of the magnetic field

The LASSOS Spin Model

The involved torques

We consider in the case of the two LAGEOS satellites four torques:

1. The magnetic torque (eddy currents)

$$\begin{aligned} M^E_{mag} = \; & V\sum_{i=1}^{9} \frac{|\boldsymbol{B_i}|^2}{2|\boldsymbol{\omega_s}|} \left\{A''_i\left[1+\cos(2\omega_i t+2\varphi_i)\right] - D'_i \sin(2\omega_i t+2\varphi_i)\right\} \boldsymbol{\omega_s} + \\ & V\sum_{i=1}^{9} \frac{\boldsymbol{B_i}\cdot\boldsymbol{\omega_s}}{2|\boldsymbol{\omega_s}|^2} \left\{\left[\alpha'(\omega_i) - A'_i\right]\left[1+\cos(2\omega_i t+2\varphi_i)\right] - \left[D''_i + \alpha''(\omega_i)\right]\sin(2\omega_i t+2\varphi_i)\right\} (\boldsymbol{\omega_s}\times\boldsymbol{B_i}) + \\ & V\sum_{i=1}^{9} \frac{\boldsymbol{B_i}\cdot\boldsymbol{\omega_s}}{2|\boldsymbol{\omega_s}|} \left\{-A''_i\left[1+\cos(2\omega_i t+2\varphi_i)\right] + D'_i \sin(2\omega_i t+2\varphi_i)\right\} \boldsymbol{B_i} \end{aligned}$$

2. The gravitational torque

$$M^b_{grav} = 3\omega^2_{\oplus}\left\{ \hat{\boldsymbol{s}}^{\boldsymbol{b}} \times \left[I_x(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{x}}^{\boldsymbol{b}})\hat{\boldsymbol{x}}^{\boldsymbol{b}} + I_y(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{y}}^{\boldsymbol{b}})\hat{\boldsymbol{y}}^{\boldsymbol{b}} + I_z\left(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{z}}^{\boldsymbol{b}}\right)\hat{\boldsymbol{z}}^{\boldsymbol{b}}\right]\right\}$$

3. The asymmetric reflectivity torque (C_R differences)

$$M^b_{ar} = \nu\,\frac{2}{3}\rho^3\,\frac{\Phi}{c}\,\Delta\rho\, C_R\,(\hat{\boldsymbol{z}}^{\boldsymbol{b}}\times\hat{\boldsymbol{s}}^{\boldsymbol{b}}_{\odot})\left|\hat{\boldsymbol{z}}^{\boldsymbol{b}}\times\hat{\boldsymbol{s}}^{\boldsymbol{b}}_{\odot}\right|$$

4. The CoM offset torque (with respect to the center of geometry)

$$M^b_{off} = \nu\pi\rho^2\,\frac{\Phi}{c}\,C_R\,\left(\boldsymbol{h}^{\boldsymbol{b}}\times\hat{\boldsymbol{s}}^{b}_{\odot}\right)$$

$$\frac{d\vec{L}}{dt} = \boldsymbol{M}_{mag} + \boldsymbol{M}_{grav} + \boldsymbol{M}_{ar} + \boldsymbol{M}_{offset}$$

Angular momentum evolution

The LASSOS Spin Model

$$I_x\dot{\omega}^b_{sx} - \omega^b_{sy}\omega^b_{sz}(I_y - I_z) = M_x$$
$$I_y\dot{\omega}^b_{sy} - \omega^b_{sx}\omega^b_{sz}(I_z - I_x) = M_y$$
$$I_z\dot{\omega}^b_{sz} - \omega^b_{sx}\omega^b_{sy}(I_x - I_y) = M_z$$

The Euler equations

We wrote the Euler equations in the body frame using the Euler angles with respect to the Earth Centered Inertial (ECI) reference frame:

$$\ddot{\theta} = \frac{M_x}{I_x}\cos\psi - \frac{M_y}{I_y}\sin\psi - \frac{I_z}{I_y}\dot{\phi}\dot{\psi}\sin\theta + \frac{I_y - I_z}{I_x}\dot{\phi}^2\frac{\sin(2\theta)}{2} + \frac{I_x - I_y}{I_x}\frac{\Lambda}{I_y}\left[\dot{\theta}(\dot{\psi} + \dot{\phi}\cos\theta)\frac{\sin(2\psi)}{2} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\sin^2\psi - \dot{\phi}\dot{\psi}\sin\theta\left(\frac{I_y - I_z}{\Lambda} - \sin^2\psi\right)\right] \quad (4)$$

$$\ddot{\phi} = \frac{M_y}{I_y}\frac{\cos\psi}{\sin\theta} + \frac{M_x}{I_x}\frac{\sin\psi}{\sin\theta} + \frac{I_z}{I_y}\frac{\dot{\psi}\dot{\theta}}{\sin\theta} - \frac{\Lambda}{I_x}\frac{\cos\theta}{\sin\theta}\dot{\phi}\dot{\theta} + \frac{I_x - I_y}{I_y}\frac{\Lambda}{I_x}\left[\frac{1}{\sin\theta}\left(\sin^2\psi - \frac{I_x}{\Lambda}\right)\dot{\psi}\dot{\theta} - \frac{\sin(2\psi)}{2}(\cos\theta\dot{\phi} + \dot{\psi})\dot{\phi} - \frac{\cos\theta}{\sin\theta}\dot{\phi}\dot{\theta}\cos^2\psi\right] \quad (5)$$

$$\ddot{\psi} = \frac{M_z}{I_z} - \frac{\cos(\theta)}{\sin(\theta)}\left(\frac{M_y}{I_y}\cos(\psi) + \frac{M_x}{I_x}\sin(\psi)\right) + \dot{\phi}\dot{\theta}\frac{1}{\sin\theta}\left(\frac{I_y - I_z}{I_x}\cos^2\theta + 1\right) - \frac{I_z}{I_y}\dot{\psi}\dot{\theta}\frac{\cos\theta}{\sin\theta} + (I_x - I_y)\left[\frac{1}{I_z}\dot{\phi}\dot{\theta}\frac{1}{\sin\theta}\left(\sin^2\theta\cos(2\psi) + \frac{\Lambda}{I_xI_y}\cos^2\psi\cos^2\theta\right) - \dot{\theta}^2\frac{1}{2I_z}\sin(2\psi) - \dot{\phi}^2\frac{1}{2I_z}\sin(2\psi)\left(\frac{\Lambda I_z}{I_xI_y}\cos^2\theta - \sin^2\theta\right) - \dot{\psi}\dot{\theta}\frac{1}{I_y}\frac{\cos\theta}{\sin(\theta)}\left(\frac{\Lambda}{I_x}\sin^2\psi - 1\right) + \frac{\Lambda}{2I_xI_y}\dot{\phi}\dot{\psi}\cos\theta\sin(2\psi)\right], \quad (6)$$

$$\Lambda = I_x + I_y - I_z$$

The LASSOS Spin Model

TABLE I. Mechanical parameters used in the equations: moments of inertia **I**, ray R and offset **h** of the satellites.

	LAGEOS	*LAGEOS* II	*LARES*
I_x[kg m^2]	10.96 ± 0.03	11.00 ± 0.03	4.76 ± 0.03
I_y[kg m^2]	10.96 ± 0.03	11.00 ± 0.03	4.76 ± 0.03
I_z[kg m^2]	11.42 ± 0.03	11.45 ± 0.03	4.77 ± 0.03
R[cm]	30.0	30.0	18.2
h_x[cm]	0.000	0.000	0.000
h_y[cm]	0.000	0.000	0.000
h_z[cm]	0.040	0.055	0.000

TABLE II. Electromechanical parameters used in the equations: dimensionless magnetic factors β' and β'', electrical conductivity σ and the relative magnetic permeability μ_r.

	LAGEOS	*LAGEOS* II	*LARES*
β'	$< 10^{-2}$	$< 10^{-2}$	1
β''	0.22	0.23	1
$\sigma[s]$	2.37×10^{17}	2.38×10^{17}	5.1×10^{16}
$\mu_r - 1$	2.2×10^{-5}	2.2×10^{-5}	3.3×10^{-7}

TABLE III. Optical parameters used in the equations: radiation coefficient C_R and reflectivity difference between the hemispheres $\Delta\rho$ of the satellites.

	LAGEOS	*LAGEOS* II	*LARES*
C_R	1.13	1.12	1.07
$\Delta\rho$	0.013	0.012	0

TABLE IV. Spin initial conditions: reference epoch in Modified Julian Date (MJD), rotational period P_s, right ascension RA and declination dec.

	LAGEOS	*LAGEOS* II	*LARES*
Epoch [MJD]	42913.5	48918	55970
P_s [s]	0.48	0.81	11.8
RA [degree]	150	230	186.5
dec [degree]	−68	−81.8	−73

The LASSOS Spin Model

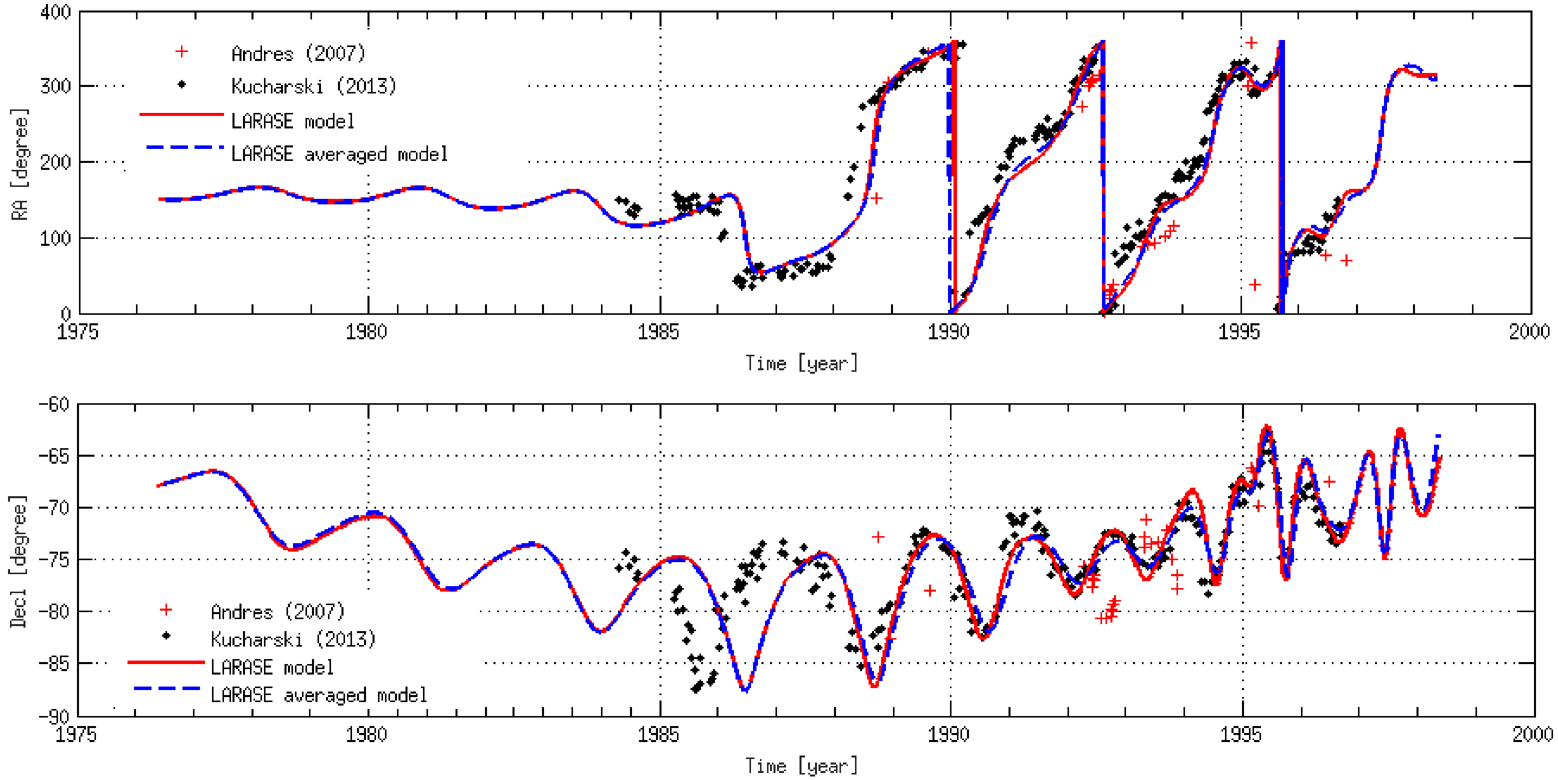
LASSOS Spin Model: results for LAGEOS

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Spin Orientation: α, δ

Andrés de la Fuente, J.I., 2007. ***Enhanced Modelling of LAGEOS Non-Gravitational Perturbations*** (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.
Kucharski, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. ***Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data***. Adv. Space Res. 52, 1332–1338.



The LASSOS Spin Model

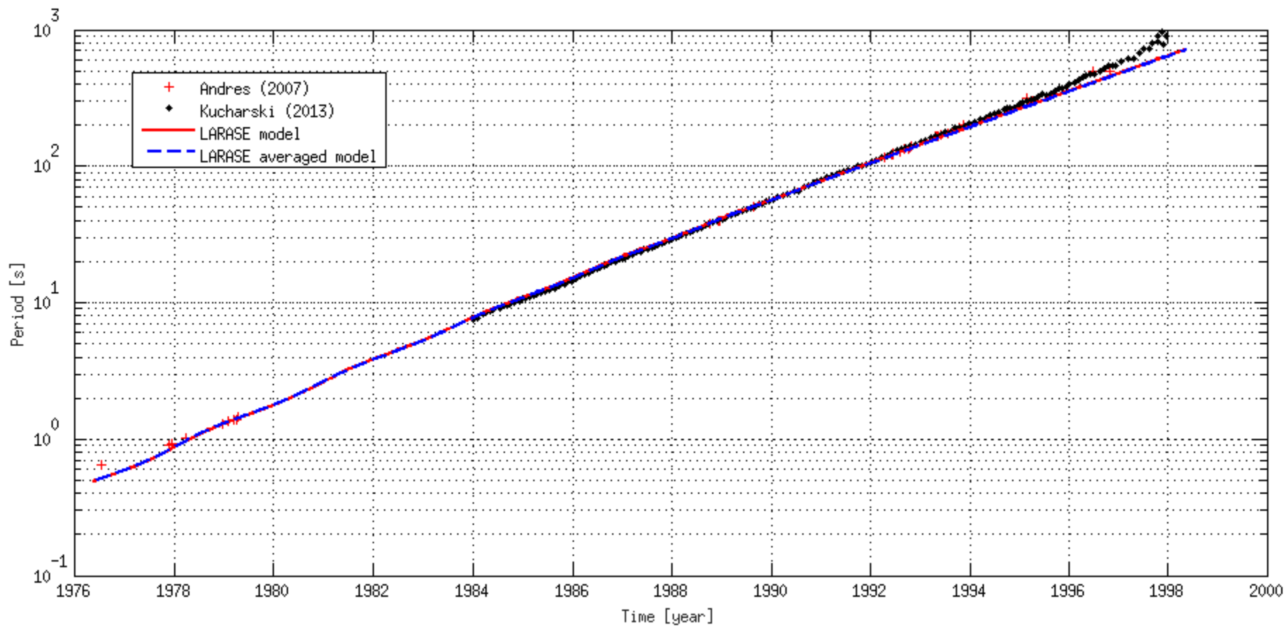
LASSOS Spin Model: results for LAGEOS

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Andrés de la Fuente, J.I., 2007. ***Enhanced Modelling of LAGEOS Non-Gravitational Perturbations*** (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.
Kucharski, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. ***Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data***. Adv. Space Res. 52, 1332–1338.

Rotational Period: P



The LASSOS Spin Model

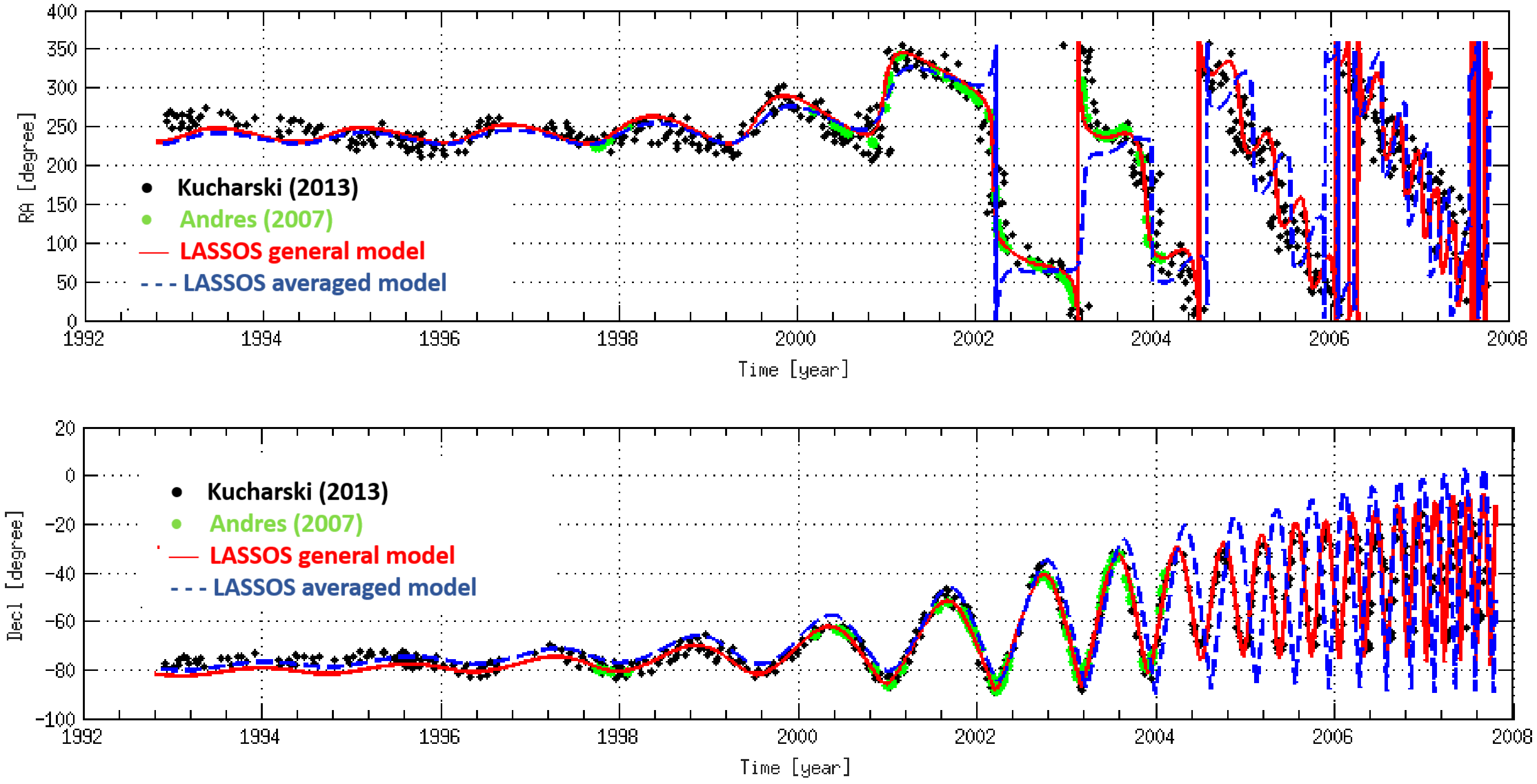
LASSOS Spin Model: results for LAGEOS II

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Spin Orientation: α, δ

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.
Kucharski, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. *Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data*. Adv. Space Res. 52, 1332–1338.



The LASSOS Spin Model

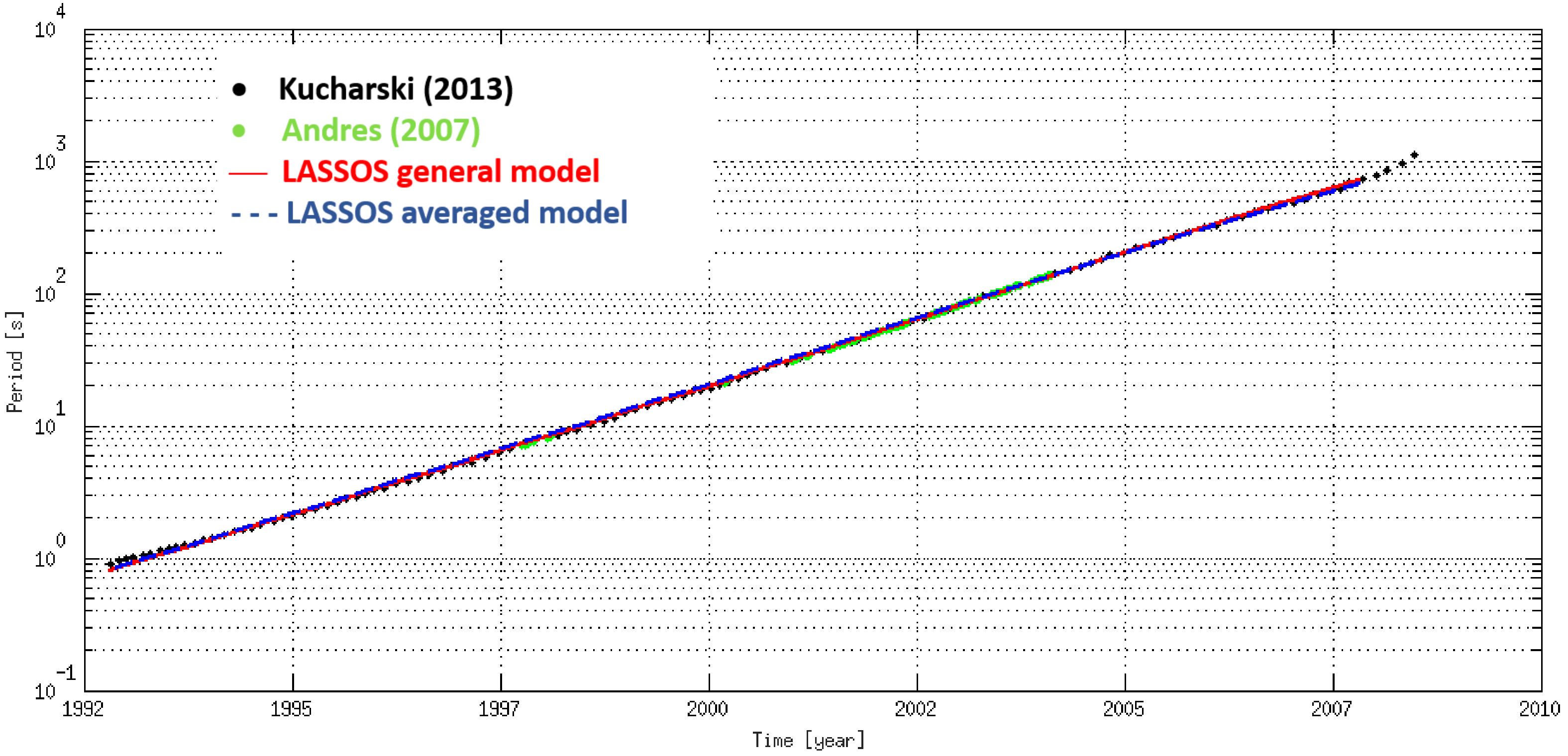
LASSOS Spin Model: results for LAGEOS II

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.
Kucharski, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. *Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data*. Adv. Space Res. 52, 1332–1338.

Rotational Period: P

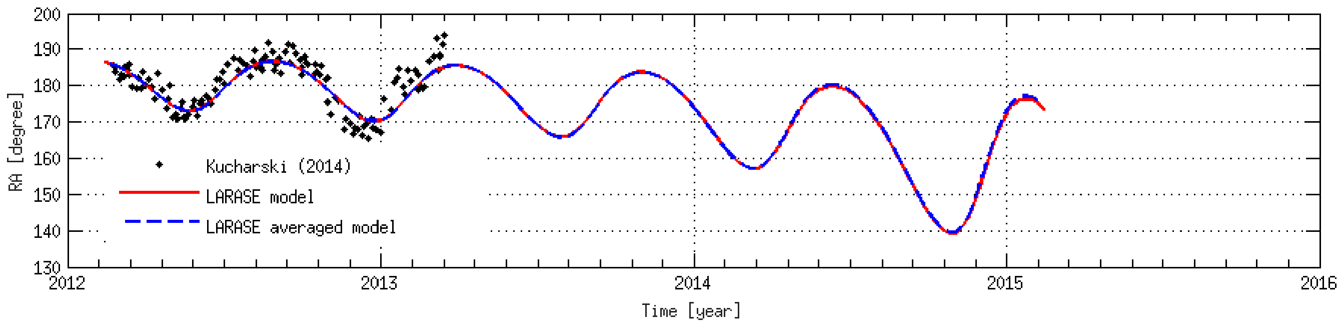


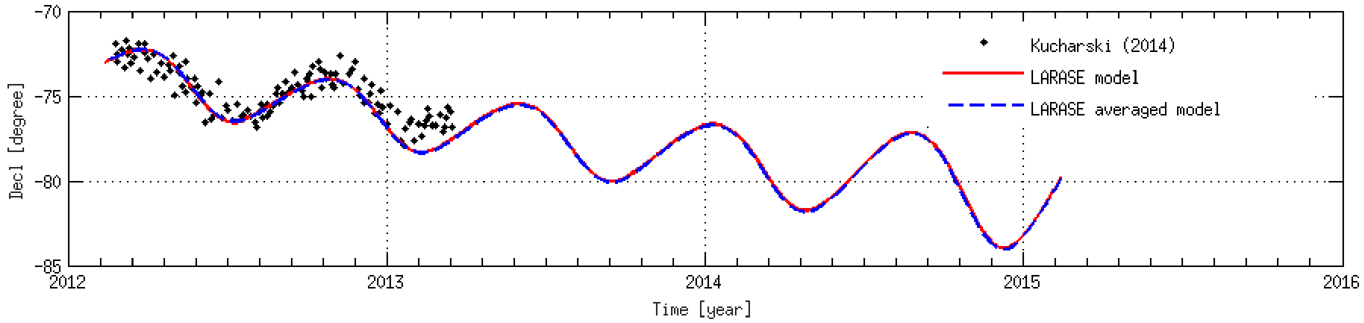
The LASSOS Spin Model

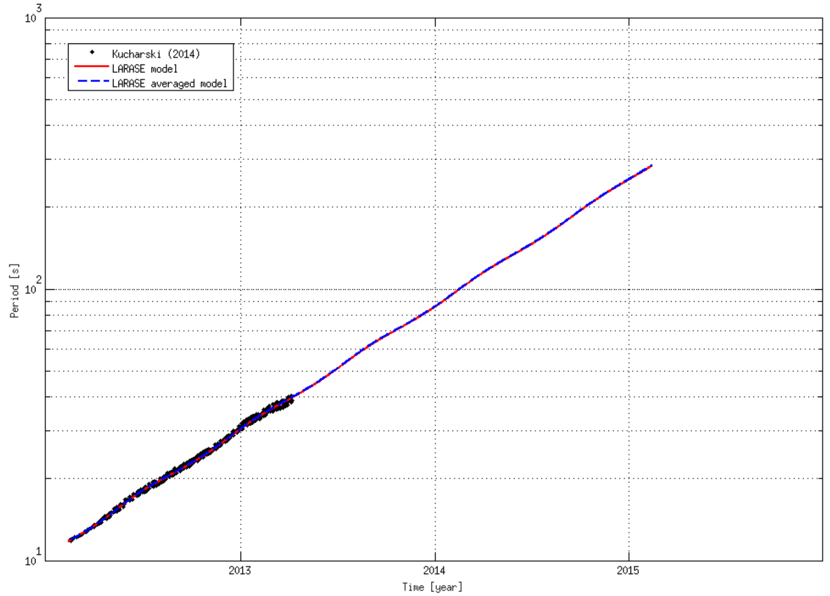
LASSOS Spin Model: results for LARES

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)







Kucharski et al., IEEE Geos. Rem. Sens. Lett. 11, 2014

T = P_{orb} = 115 min. after ≅ 5.9 years

$T(s) \cong 11.8 \cdot e^{D/341}$ D [days]

- The spin evolution is almost due to the magnetic torque
- The gravitational torque is almost null, we fit the data with an oblateness of about: $\frac{C-A}{C} < 10^{-4}$

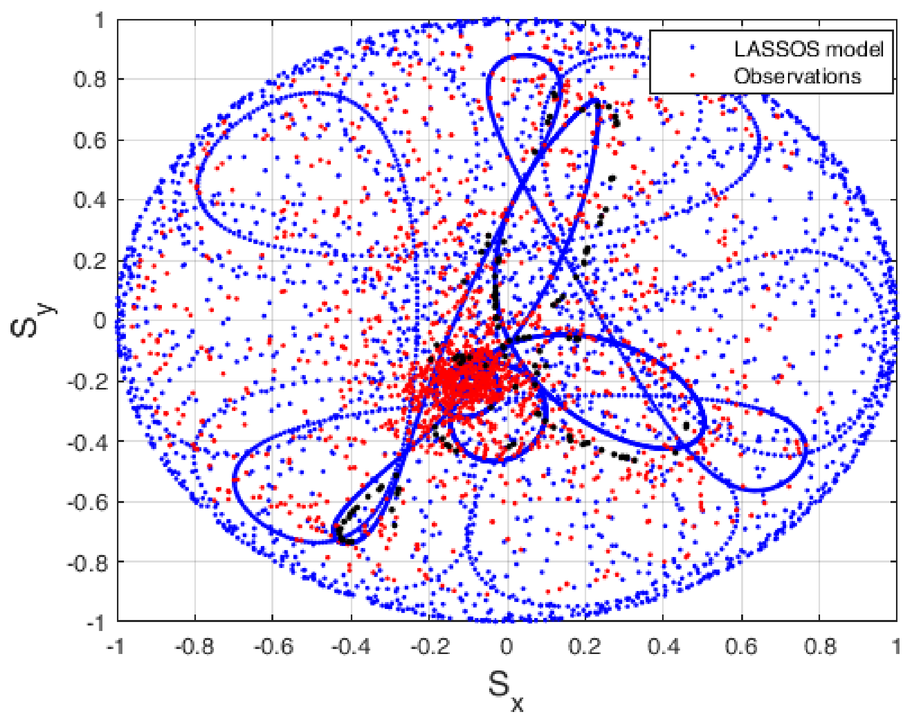
The LASSOS Spin Model

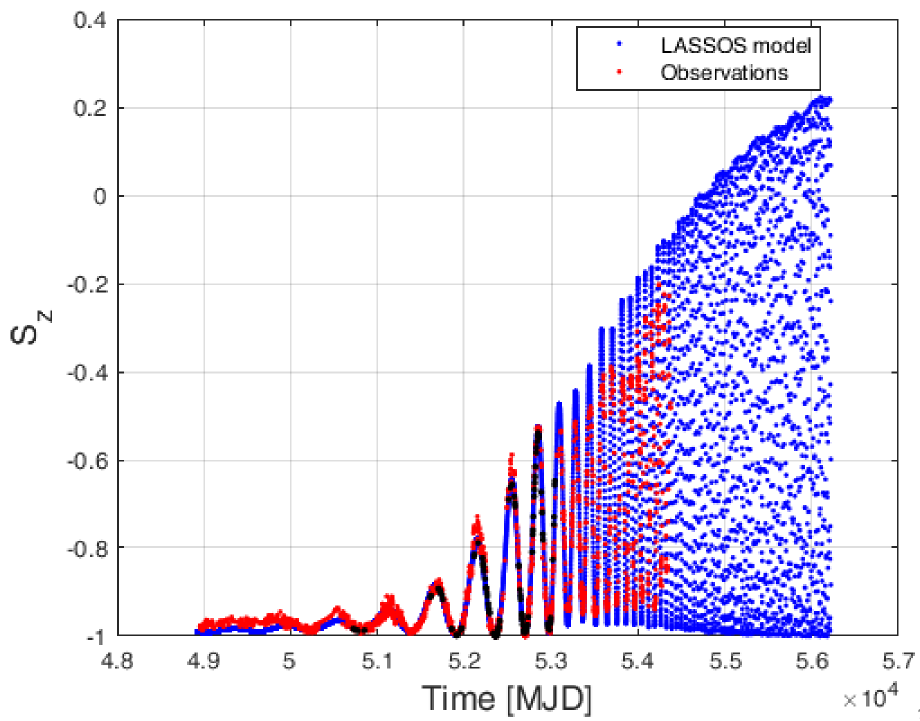
LASSOS Spin Model: results for LAGEOS II

Blue = LARASE model for the rapid-spin
Red = LARASE general model

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Cartesian components in J2000





The LASSOS Spin Model

PHYSICAL REVIEW D **98,** 044034 (2018)

Comprehensive model for the spin evolution of the *LAGEOS* and *LARES* satellites

Massimo Visco[1,2] and David M. Lucchesi[1,2,3]

[1]*Istituto Nazionale di Astrofisica (INAF), Istituto di Astrofisica e Planetologia Spaziali (IAPS), Via del Fosso del Cavaliere, 100, 00133 Roma, Italy*

[2]*Istituto Nazionale di Fisica Nucleare (INFN), Sez. Tor Vergata, Via della Ricerca Scientifica 1, 00133 Roma, Italy*

[3]*Istituto di Scienza e Tecnologie della Informazione (ISTI), Consiglio Nazionale delle Ricerche (CNR), via G. Moruzzi 1, 56124 Pisa, Italy*

(Received 1 June 2018; published 21 August 2018)

The LATOS thermal model

We have developed **LATOS** a new thermal model for LAGEOS satellites

LArase Thermal mOdel Solutions (LATOS)

Motivation:

Necessity of improved models for the NGP

- **Thermal drag/thrust effects (Yarkovsky effect, Yarkovsky-Schach effect)**
- **Albedo**
- **Asymmetric reflectivity (LAGEOS, LAGEOS II)**

Previous models:

Rubincam, D.P., 1987. *LAGEOS orbit decay due to infrared radiation from Earth*. J. Geophys. Res. 92, 1287–1294.

Rubincam, D.P., 1988. *Yarkovsky thermal drag on LAGEOS*. J. Geophys. Res. 93, 13805–13810.

Rubincam, D.P., 1990. *Drag on the LAGEOS satellite*. J. Geophys. Res. 95, 4881–4886.

Farinella, P., **Nobili**, A.M., **Barlier**, F., **Mignard**, F., 1990. *Effects of thermal thrust on the node and inclination of LAGEOS*. Astron. Astrophys. 234, 546–554.

Farinella, P., **Vokrouhlicky**, D., 1996. *Thermal force effects on slowly rotating, spherical artificial satellites-I. Solar Heating*. Plan. Space Sci. 44, 1551–1561.

Vokrouhlicky, D., **Farinella**, P., 1996. *Thermal force effects on slowly rotating, spherical artificial satellites-II. Earth infrared heating*. Plan. Space Sci. 45, 419–425.

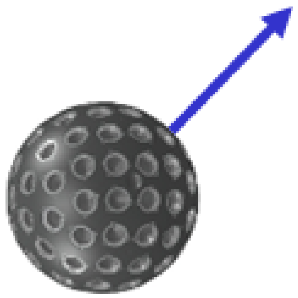
Slabinski, V.J., 1996. *A numerical solution for LAGEOS thermal thrust: the rapid-spin case*. Celestial Mech. Dyn. Astron. 66, 131–179.

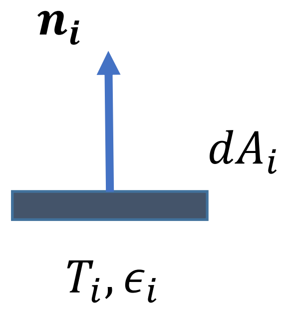
Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.

The LATOS thermal model

The thermal thrust force:

$$\mathbf{dF_T} = -\frac{2}{3}\frac{\epsilon\sigma T^4 dA}{c}\mathbf{n}$$





The force, normal to each surface element *dA*, depends from the temperature *T* and emissivity *ε* of the part considered

It is necessary to know the temperature distribution inside the satellite and the satellite position with respect to the external heat sources (Sun and Earth)

The LATOS thermal model

The thermal equations:

$$\frac{dT_i}{dt}C_i = \left(\Sigma_k P_{abs\ k} - P_{em\ i}\right) + \Sigma_j R_{i,j}\left(T_i^4 - T_j^4\right) + \Sigma_j C_{i,j}\left(T_i - T_j\right)$$

Thermal capacity

Difference between the total Power absorbed and emitted

Heat exchanged between the different elements of the satellite due to radiation and conduction

The input to the system of differential equations are:

- **Attitude of the satellite (from LASSOS model)**
- **Thermal and optical parameters of the satellite (from technical documentation and tests) that contribute to the different constants in the system**

The LATOS thermal model

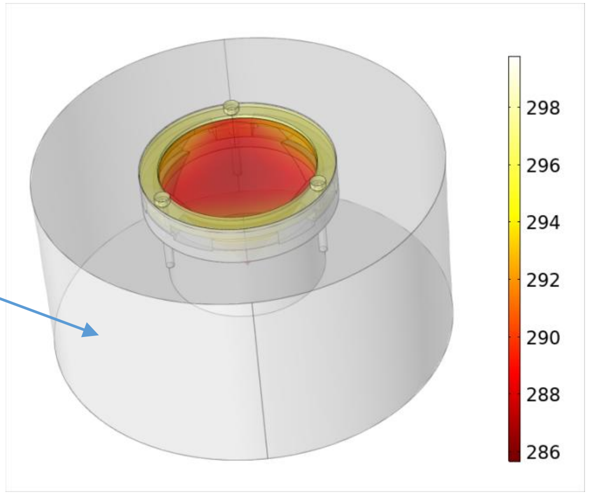
- The satellite is divided into several parts which are assumed to have no thermal gradient within them. For the two **LAGEOS**: the **CCRs**, the two **hemispheres** and the **core**. The **rings** that block the **CCRs** are considered isothermal to the hemispheres

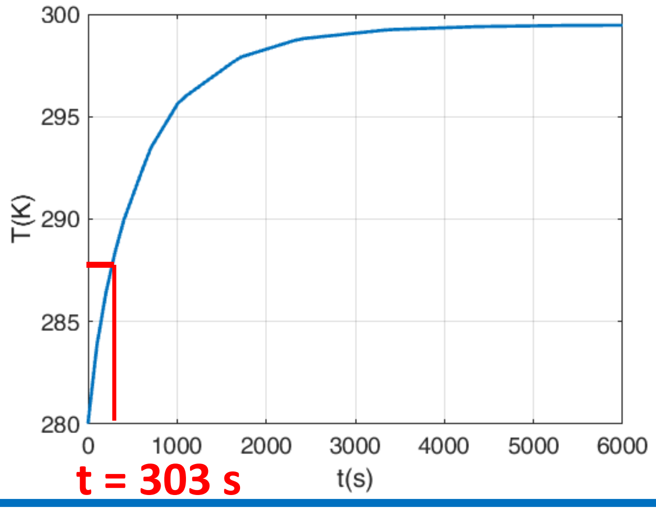
- The conduction constant between the **CCRs** and the hemisphere in which they are inserted was numerically calculated using a **FEM** model

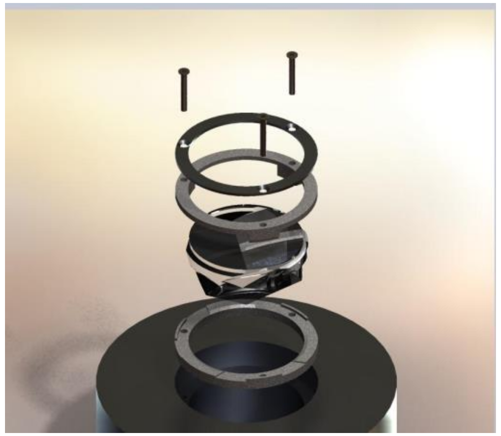
Coupling of a CCR with the structure

T = 300 K

t = 303 s







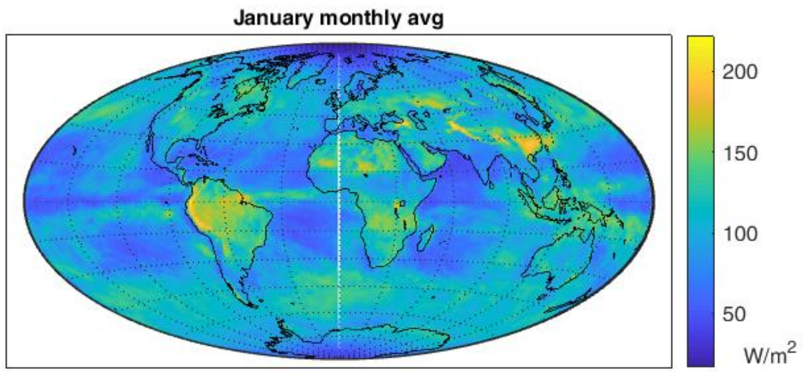
The LATOS thermal model

Clouds and Earth's Radiant Energy System (CERES)

We considered three external heat sources:

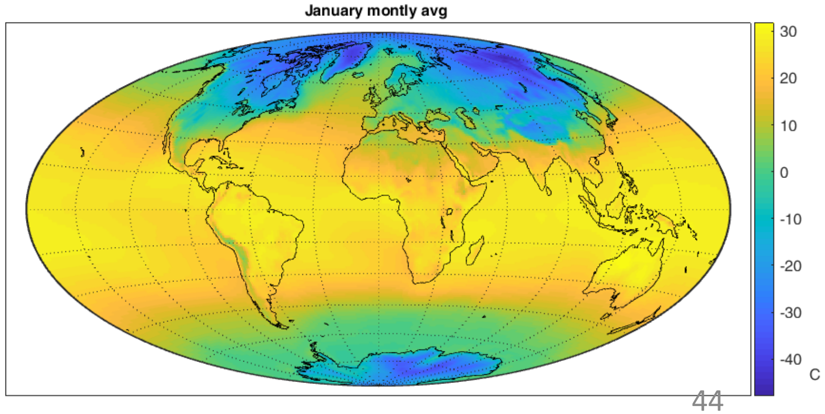
- **The direct Sun radiation – using the standard value of $\phi_{\odot}$= 1360.8 $\frac{W}{m^2}$ at 1 A.U.**
- **The Sun radiation reflected from Earth (Albedo)**

  We use CERES monthly averaged SW radiation data at the top of the atmosphere taking into account night-day alternance, satellite attitude and orbital position. The grid is 1°x1° Latitude-Longitude



- **The infrared radiation from the Earth**

  We take into account the temperature of the different parts of the Earth using the monthly averaged data from Land + Ocean 1°x1° Latitude-Longitude grid from Berkeley Earth Organization and from CERES. Attitude and orbit of the satellite are considered



Preliminary results

- We developed two versions of the model (**LATOS**), an averaged one, usable for fast-spin conditions, and a general one, not averaged, to be used when the spin is slow with respect to the orbital period

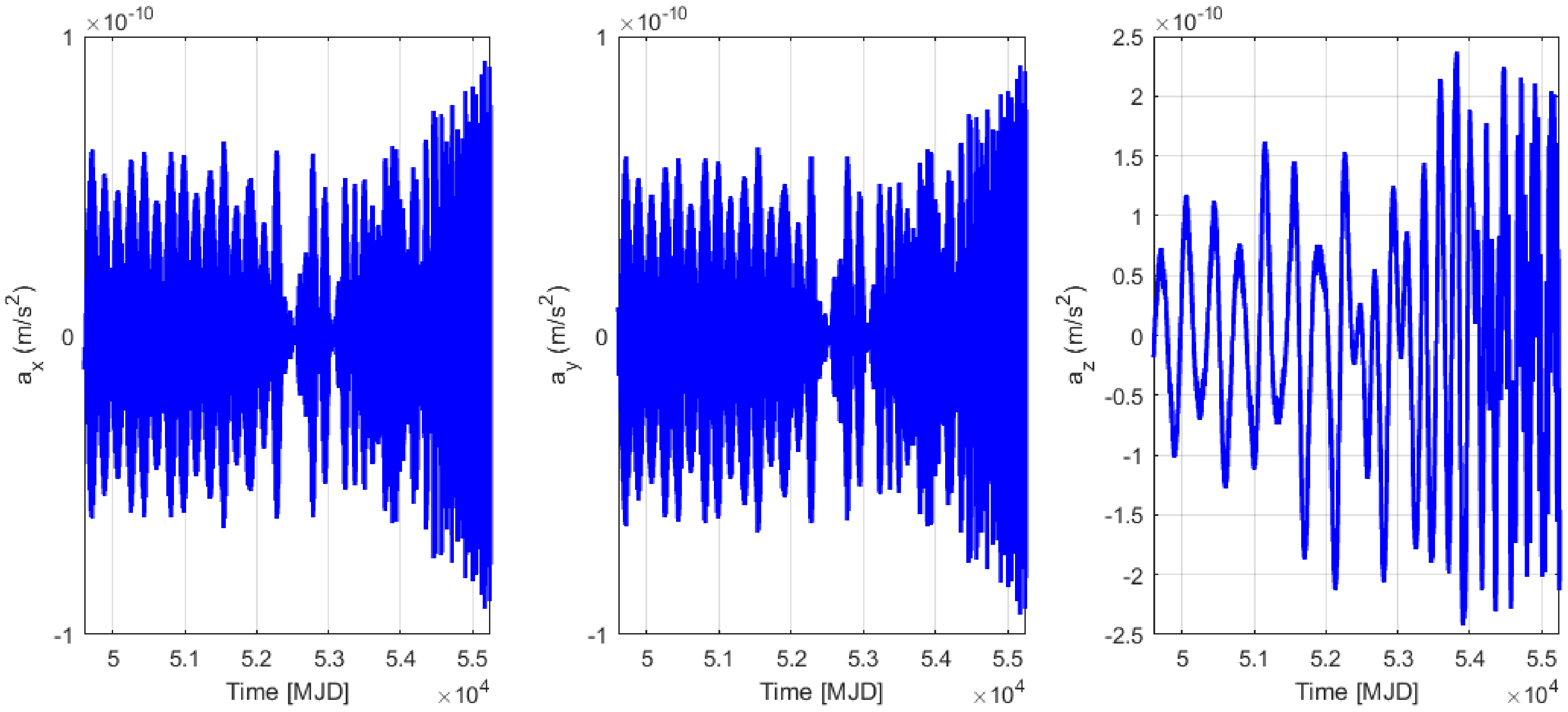
- By integrating the thermal equations we get the temperature distribution in the satellite and from this distribution we calculate the thermal thrust accelerations

- We then calculated the effects of the thermal accelerations (via **Gauss** equations) on the rate of the Keplerian elements. The results can be compared with the corresponding rate residuals from a **precise orbit determination** (**POD**)

LArase Thermal mOdel Solutions (LATOS)

Preliminary results

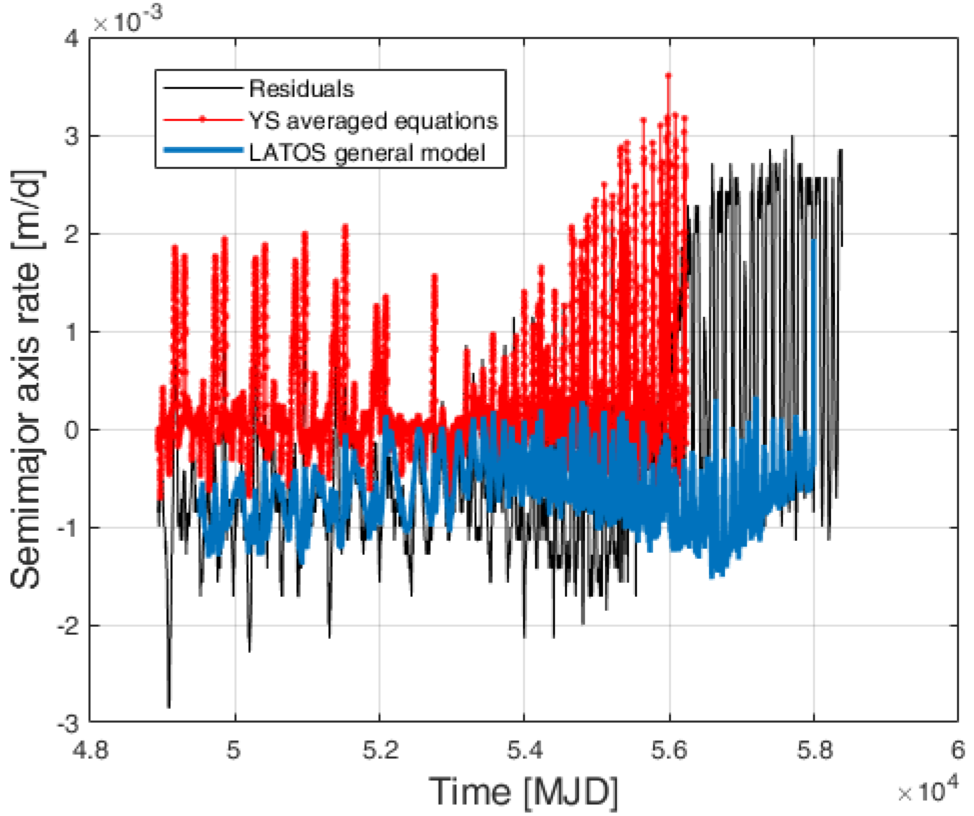
LAGEOS II: accelerations in Gauss reference frame



Radial **Transverse** **Out-of-plane**

Preliminary results

LAGEOS II

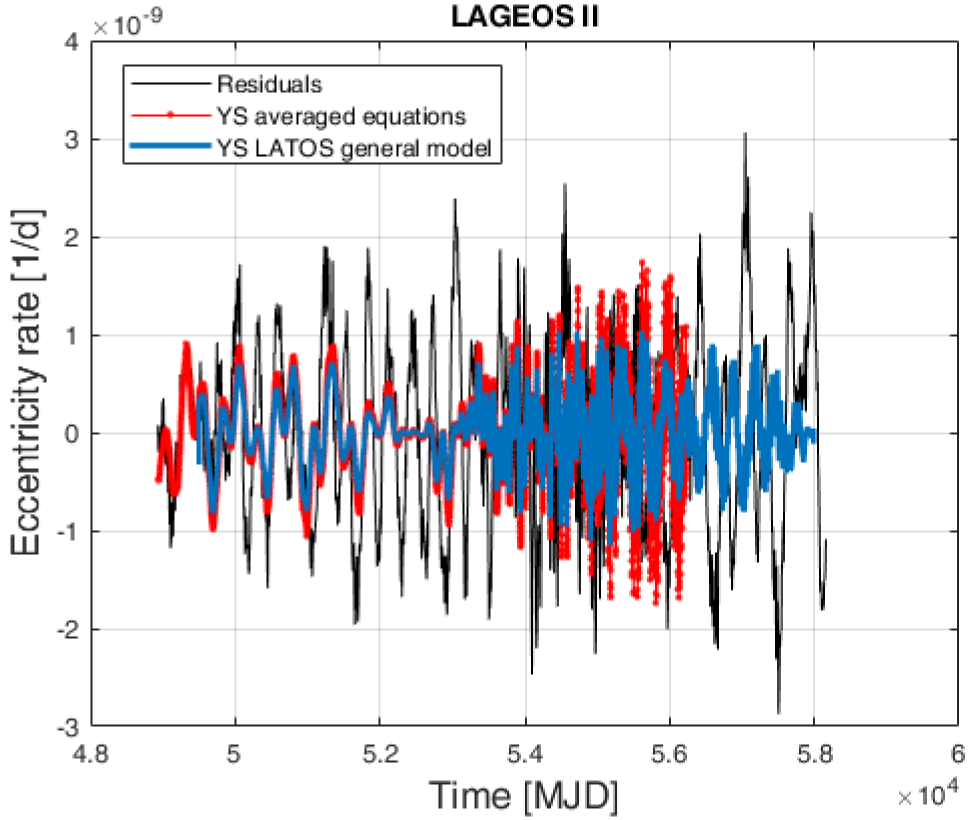


$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}[T + e(T\cos f + R\sin f)]$$

About 27 years POD of LAGEOS II with GEODYN II

Preliminary results

LAGEOS II

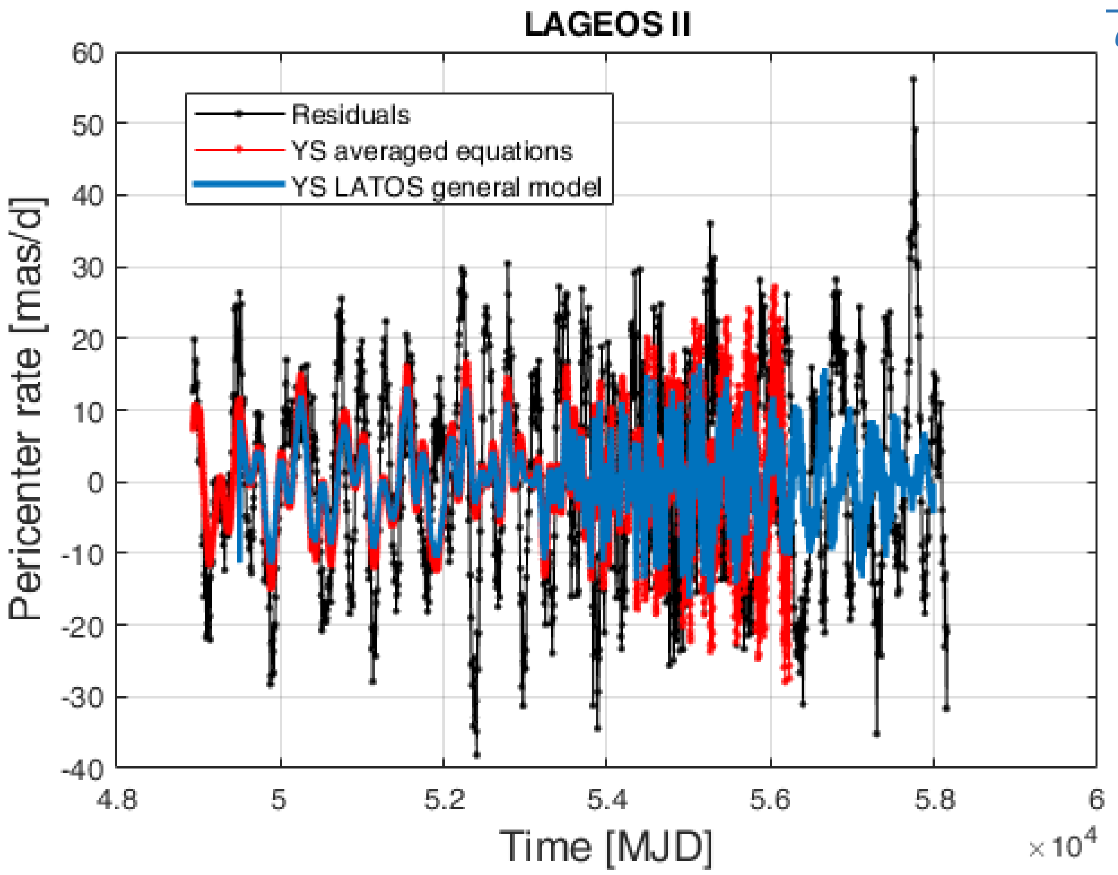


$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[R\sin f + T(\cos f + \cos u)\right]$$

About 27 years POD of LAGEOS II with GEODYN II

Preliminary results

LAGEOS II

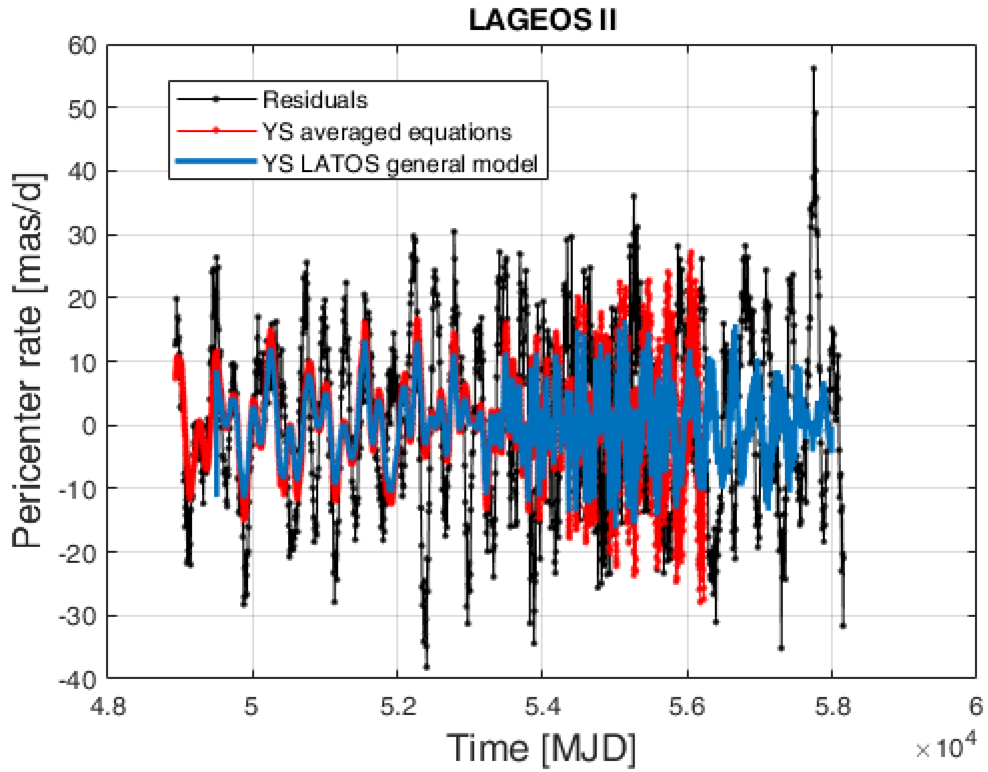


$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\left[-R\cos f + T\left(\sin f + \frac{\sin u}{\sqrt{1-e^2}}\right)\right] - \frac{W}{H\sin i} r\sin(\omega + f)\cos i$$

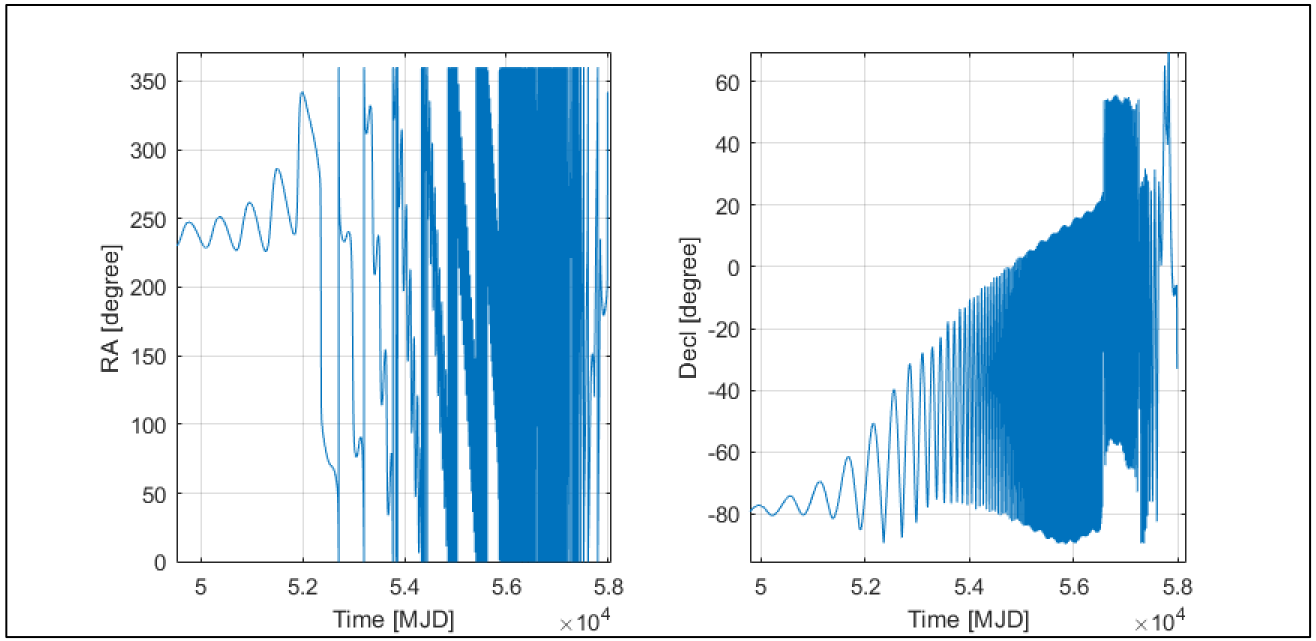
Being able to clean up this parameter has a particular importance for us: it contains a secular effect from **General Relativity**, due to the **Gravitoelectric** field (M) and to the **Gravitomagnetic** field (J)

About 27 years POD of LAGEOS II with GEODYN II

Preliminary results



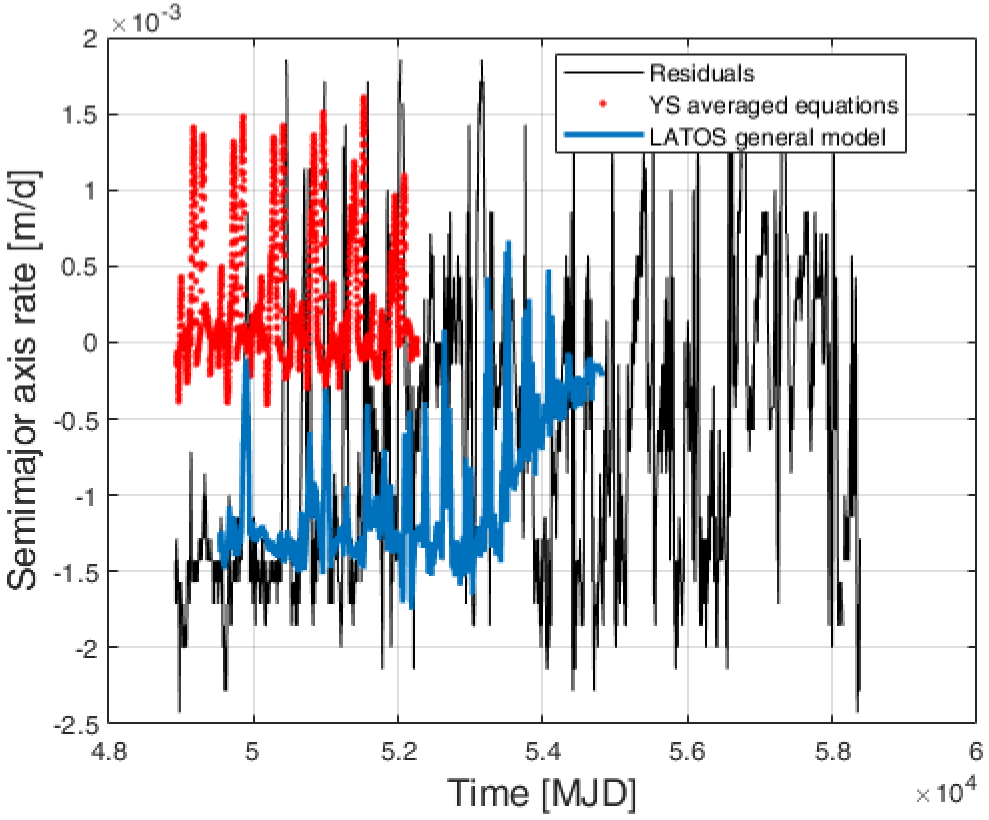
Spin evolution



Preliminary results

LAGEOS

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}[T + e(T\cos f + R\sin f)]$$

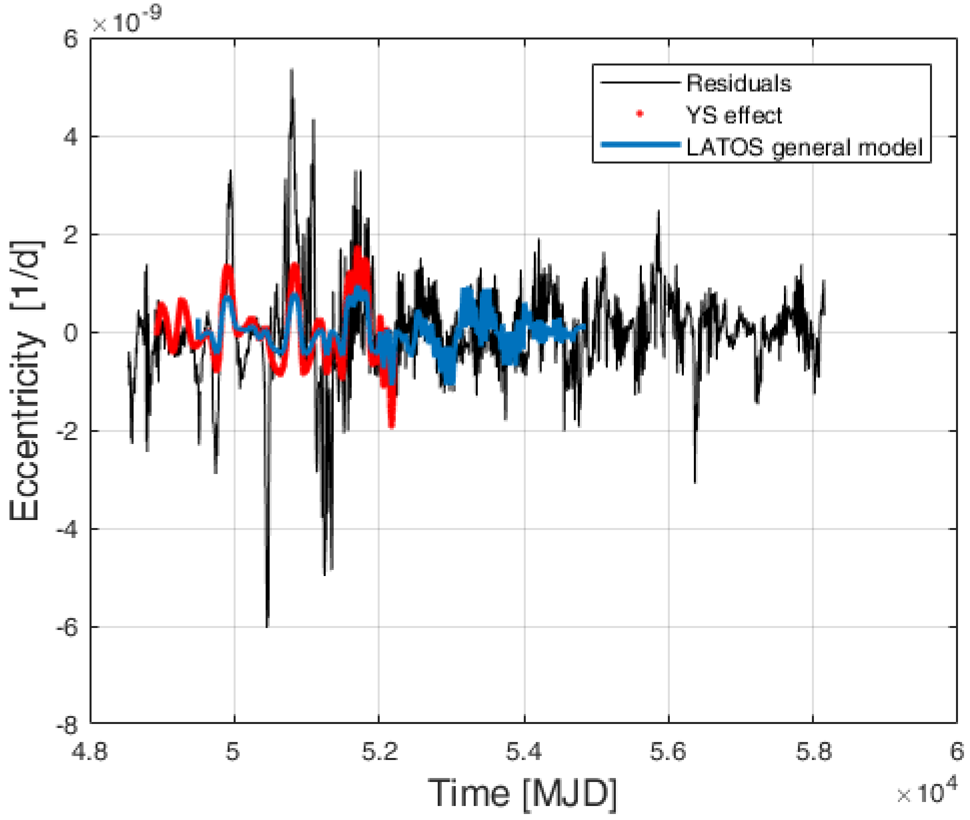


About 27 years POD of LAGEOS with GEODYN II

Preliminary results

LAGEOS

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[R\sin f + T(\cos f + \cos u)\right]$$

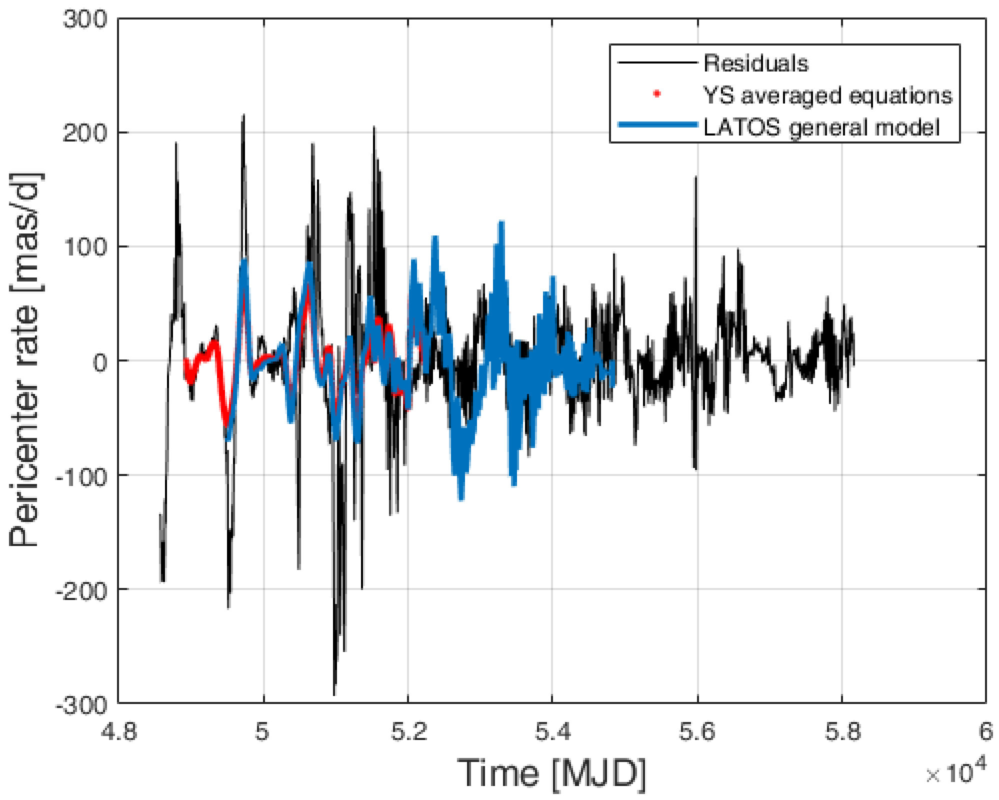


About 27 years POD of LAGEOS with GEODYN II

Preliminary results

LAGEOS

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\left[-R\cos f + T\left(\sin f + \frac{\sin u}{\sqrt{1-e^2}}\right)\right] - \frac{W}{H\sin i} r\sin(\omega + f)\cos i$$



About 27 years POD of LAGEOS with GEODYN II

Conclusions

Within the many activities of **LARASE** we developed:

1. a 3D cad of the satellites for a correct handling of their structure
   - Masses and moments of inertia have been computed
2. a new general spin model (**LASSOS**) for the two **LAGEOS** and **LARES** satellites
   - The results for the spin are in very good agreement with the available observations (orientation and rate)
   - The spin model can be used in synergy with kHz laser observations to capture the current attitude of the satellites (LAGEOS and LAGEOS II)
3. a new general model (**LATOS**) to account for the thermal thrust acceleration acting on **LAGEOS II** and **LAGEOS**
   - preliminary results for the orbital effects due to the thermal thrust accelerations on the two satellites are in good agreement with the orbital residuals
   - Thermal accelerations determined from a reliable model may reduce the use of empirical accelerations in the satellites' **POD**, with possible improvements in
     - **Geophysical products**
     - **Fundamental physics measurements**

Current activities

1. GEODYN II software update
2. NGP models improvements
3. Models for the Gravitational field of the Earth
4. Relativistic effects due to the nonlinearity of the gravitational interaction
5. Constraints to alternative theories of gravitation
6. Measurement of the GR shift of the argument of pericenter

GEODYN II software update

- update of the **Normal Points Data Base** and their conversion in **TDF**
- update of control scripts (Bash): **GEODYN** and ancillary
- update of the post-processing scripts (Python)
- creation of new arcs and new **PODs** (routine)
- creation of interface files
- **ITRF2014**:
    - identification of the reference files
    - generation of a new routine for coordinate conversion
    - preparation of the files import procedure, their pre-processing and consequent generation of **GEODYN** cards (Python)
    - validation with specific **PODs** series

NGP models improvements

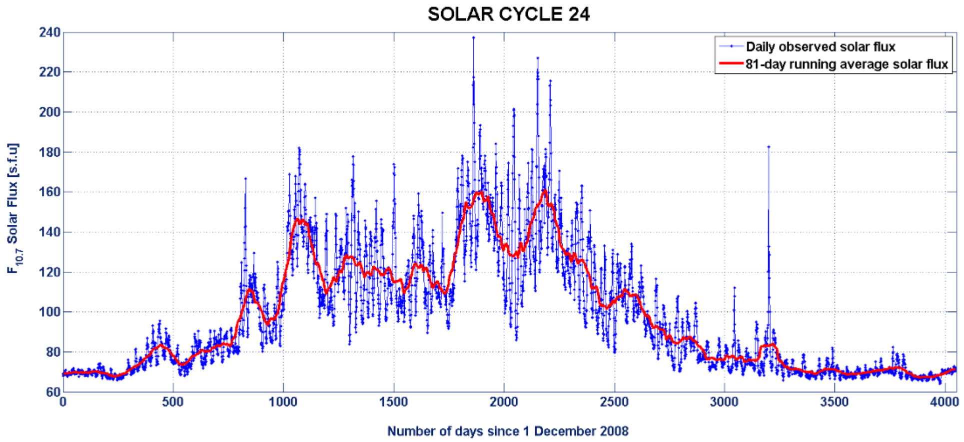
- Thermal model for **LARES**
  - Several differences with respect to **LAGEOS**

LAGEOS

LARES

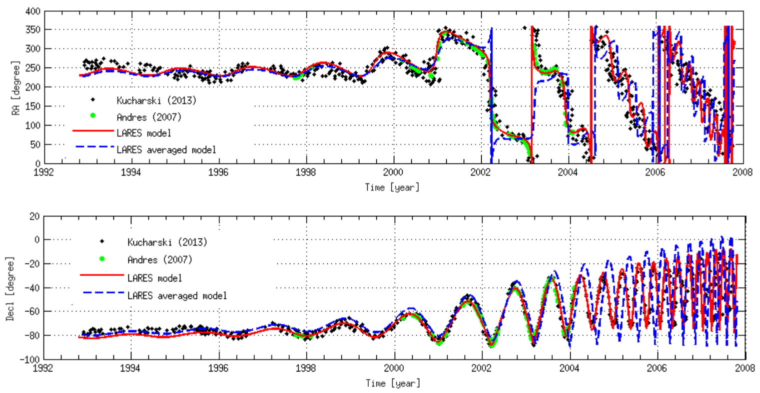
NGP models improvements

- Thermal model for **LARES**
  - Several differences with respect to **LAGEOS**
- Neutral drag:
  - Extension of the analysis of the decay of **LARES** semi-major axis to the full solar cycle #24
  - Comparison of the effects on **LARES** and **Ajisai**



NGP models improvements

- Thermal model for **LARES**
  - Several differences with respect to **LAGEOS**
- Neutral drag:
  - Extension of the analysis of the decay of **LARES** semi-major axis to the full solar cycle #24
  - Comparison of the effects on **LARES** and **Ajisai**
- Spin model for dissemination
  - General model
  - Averaged model



Models for the Gravitational field of the Earth

As we have shown during the presentation concerning the measurement on the **Lense-Thirring** effect, a correct modeling of the gravitational field (**GF**) of the Earth (in particular of the even zonal harmonics) that takes into account the time dependence of the different coefficients is fundamental to obtain accurate measurements:

- **GF** for the joint analyses with **LAGEOS** and **LAGEOS II**
  - Starting from November 1992
- **GF** for the joint analyses of the two **LAGEOS** with **LARES**
  - Starting from February 2012

Relativistic effects due to the nonlinearity of the gravitational interaction

The precision of the measurements obtained on the relativistic precessions requires taking into account, in certain circumstances, effects related to the nonlinearity of the gravitational interaction

Constraints to alternative theories of gravitation

The recent precise and accurate measurement of the **Lense-Thirring** effect will allow us to constraint the predictions of some alternative theories of gravitation

Measurement of the GR shift of the argument of pericenter

A new measurement of the overall **GR** precession of the argument of pericenter of **LAGEOS II** and, possibly, of that of the older **LAGEOS**

LARASE: publications

- Lucchesi, D.M., Peron, R., The LAGEOS II pericenter general relativistic precession (1993–2005): error budget and constraints in gravitational physics, ***Phys. Rev. D*** 89, 082002, doi:10.1103/PhysRevD.89.082002, **2014**
- Lucchesi, D.M., R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan and G. Pucacco, Fundamental physics in the field of the Earth with the LAser RAnged Satellites Experiment (LARASE). 2nd Metrology for Aerospace (MetroAeroSpace), 2015 IEEE, **2015**
- Lucchesi, D.M., L. Anselmo, M. Bassan, C. Pardini, R. Peron, G. Pucacco and M. Visco, Testing the gravitational interaction in the field of the Earth via Satellite Laser Ranging and the LAser RAnged Satellites Experiment (LARASE). ***Class. Quantum Grav***. 32, 155012, **2015**
- Visco, M., Lucchesi, D.M., Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program. ***Adv. Space Res***., Volume 57, Issue 9, 1 May 2016, Pages 1928-1938, http://dx.doi.org/10.1016/j.asr.2016.02.006, **2016**
- Lucchesi, D.M., C. Magnafico, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, G. Pucacco and R. Stanga, Measurements of General Relativity precessions in the field of the Earth with laser-ranged satellites and the LARASE program. 3rd Metrology for Aerospace (MetroAeroSpace), 2016 IEEE, Pages 522-529, DOI: 10.1109/MetroAeroSpace.2016.7573270, **2016**
- Lucchesi, D.M., C. Magnafico, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, G. Pucacco and R. Stanga, The LARASE research program. State of the art on Modelling and Measurements of General Relativity effects in the field of the Earth: a preliminary measurement of the Lense-Thirring effect. 4th International Workshop on Metrology for Aerospace (MetroAeroSpace), 2017 IEEE, Pages 131-145, DOI: 10.1109/MetroAeroSpace.2017.7999552, **2017**

LARASE: publications

- Pardini, C., Anselmo, L., Lucchesi, D.M. and Peron, R., On the secular decay of the LARES semi-major axis. ***Acta Astronautica*** 140, 469–477, DOI: 10.1016/j.actaastro.2017.09.012, **2017**
- Visco, M., Lucchesi, D.M., Comprehensive model for the spin of the LAGEOS and LARES satellites. ***Phys. Rev. D*** 98, 044034, DOI: 10.1103/PhysRevD.98.044034, **2018**
- Pucacco, G., Lucchesi, D.M., Tidal effects on the LAGEOS-LARES satellites and the LARASE program. ***Celest. Mech. And Dyn. Astron***. 130:66, doi.org/10.1007/s10569-018-9861-5, **2018**
- Pardini, C., Anselmo, L., Lucchesi, D.M., Peron, R., Neutral Atmosphere Drag at the altitude of LARES and AJISAI. IAC-18-C1.1.12, **2018**
- Lucchesi, D.M., L. Anselmo, M. Bassan, C. Magnafico, C. Pardini, R. Peron, G. Pucacco, M. Visco, General Relativity Measurements in the Field of Earth with Laser-Ranged Satellites: State of the Art and Perspectives. ***Universe***, 5, 141, doi:10.3390/universe5060141, **2019**
- Lucchesi, D.M., , M. Visco, R. Peron, M. Bassan, G. Pucacco, C. Pardini, L. Anselmo, C. Magnafico, An improved measurement of the Lense-Thirring precession on the orbits of laser-ranged satellites with an accuracy approachin the 1% level. ***arXiv:1910.01941v1 [gr-qc]***, 4 Oct **2019**
- Lucchesi, D.M., , M. Visco, R. Peron, M. Bassan, G. Pucacco, C. Pardini, L. Anselmo, C. Magnafico, A 1% Measurement of the Gravitomagnetic Field of the Earth with Laser-Tracked Satellites. ***Universe***, 6, 139, doi:10.3390/universe6090139, **2020**
- Pardini, C., Anselmo, L., Lucchesi, D.M. and Peron, R., M. Bassan, C. Magnafico, G. Pucacco, M. Visco, Sounding the Atmosphere Density at the Altitude of LARES and Ajisai During Solar Cycle 24. Transactions of the Japan Society for Aeronautical and Space Sciences, **2020**

Many thanks for your kind attention