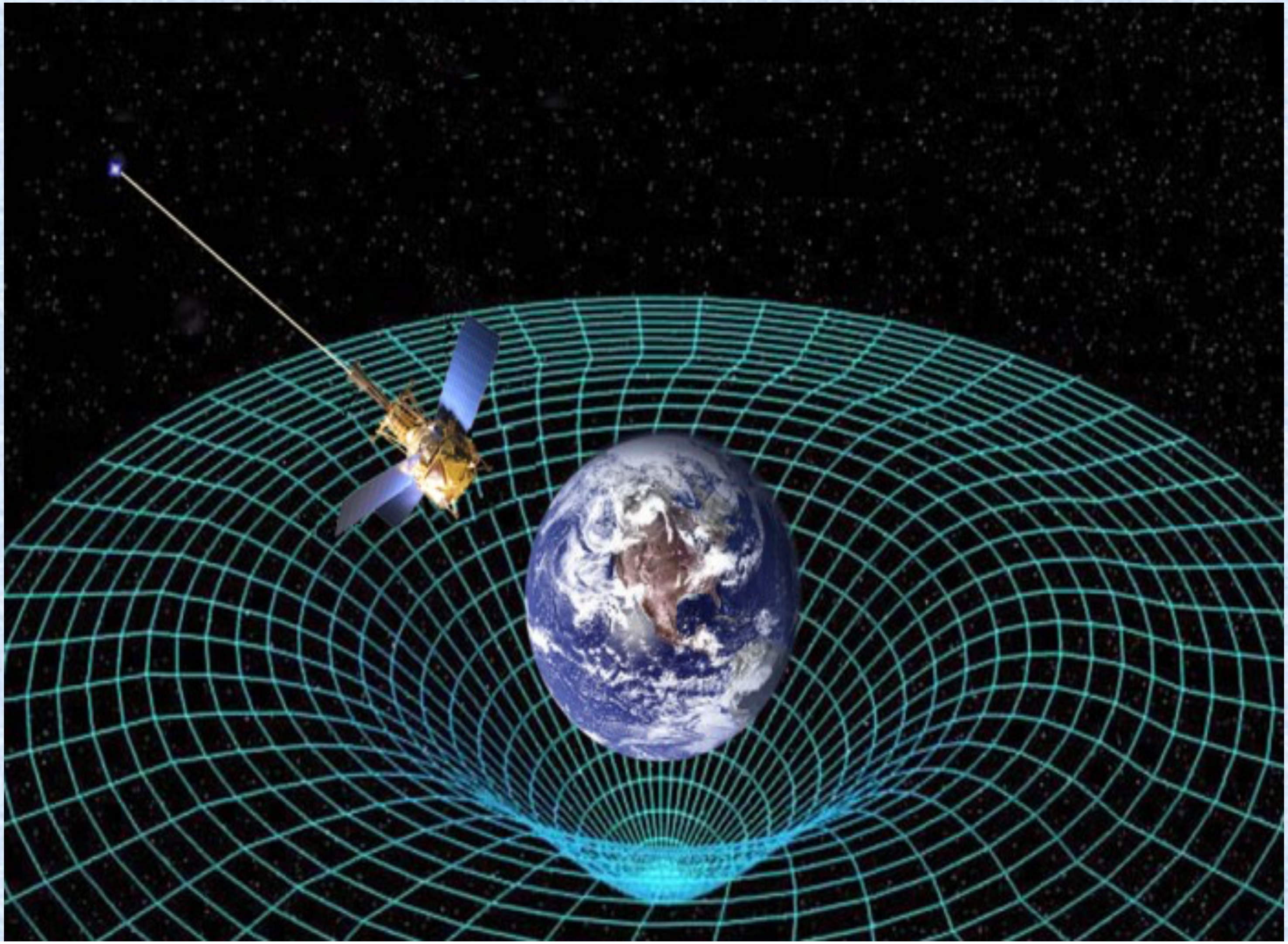
Probing Modified Theories of Gravity by Experimental Observations

Salvatore Capozziello ⟺ *INFN Conference. November, 12th 2020*



Università degli Studi di Napoli Federico II

Outline

- *Shortcomings of General Relativity*
- *Modified Theories of Gravity*
- *Spherically Symmetric Background*
- *The Weak-Field Limit*
- **Constraining Modified Gravity by Experiments**

General Constraints

LARES, GPB

GINGER

- *Conclusions and Perspectives*

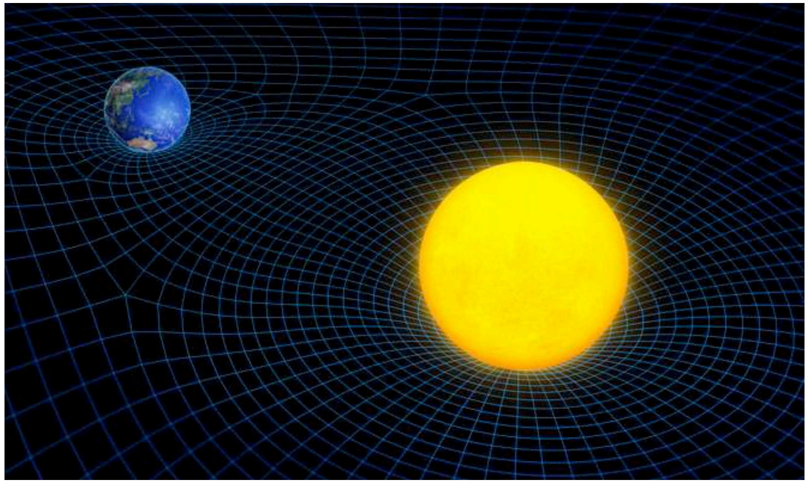
General Relativity: foundations and predictions

General Relativity

Describe the gravitational interaction through the spacetime curvature

↓

First theory to successfully pass the Solar System Tests



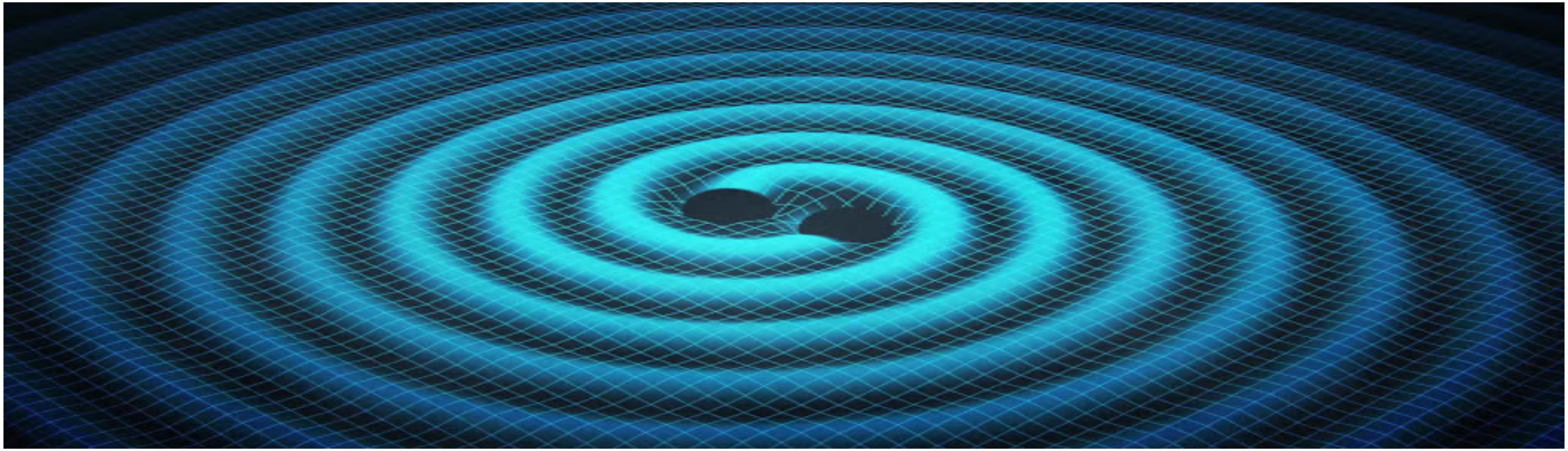
In a static and spherically Symmetric background

Schwarzschild Solution

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2\theta d\phi^2$$

- **Black Holes**

- **Gravitational Waves**



• *Lense Thirring Effect*

This effect predicted by GR can be obtained starting from a Kerr-like metric

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

↓

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

↓

$$ds^2 = \left(1 - \frac{r_S}{r}\right)dt^2 - \frac{1}{1 - \frac{r_S}{r}\frac{J^2}{M^2 r^2}}dr^2 - r^2 d\theta^2 - \left(r^2 + \frac{J^2}{M^2} + \frac{r_S J^2}{M^2 r}\right)d\phi^2 - \frac{2r_S J}{M r}dt\,d\phi$$

Angular Momentum

Correction to the precession of a gyroscope near a large rotating mass, due to the dragging of the spacetime!

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4Mr^3}J$$

General Relativity: shortcomings

Shortcomings of GR

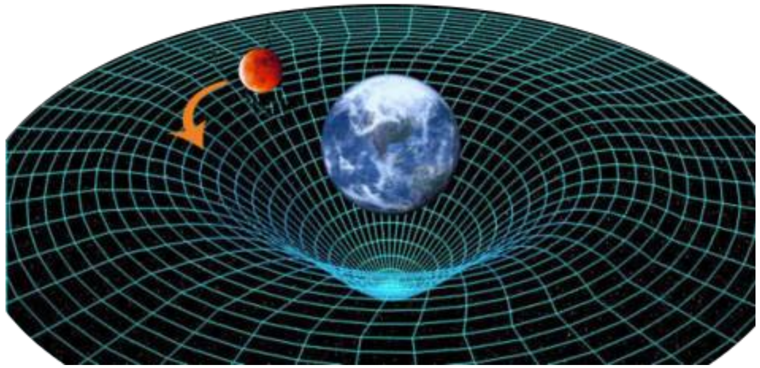
Large Scales

- Universe accelerated expansion
- Inflation
- Galaxy Rotation Curve
- Mass-Radius Diagram of some Neuton Stars
- Fine-tuning cosmological parameters

Small Scales

- Renormalizability
- GR cannot be quantized
- GR cannot be treated under the same standard of other interactions
- Discrepancy between theoretical and experimental value of Λ
- Classical spacetime singularities

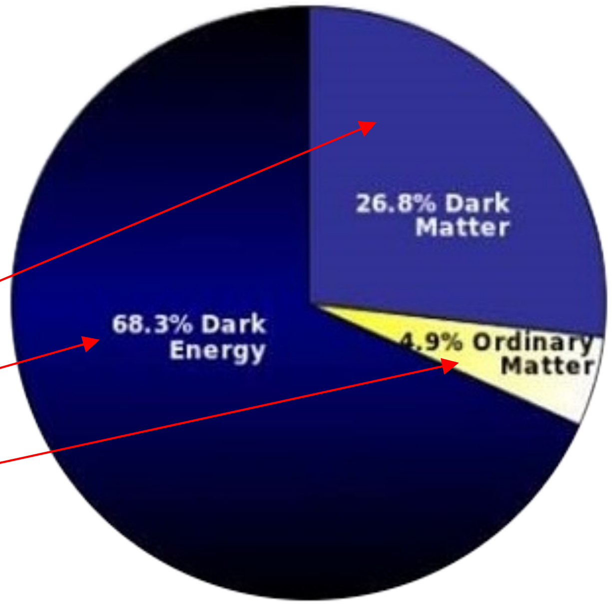
No theory is capable of solving these problems at once so far



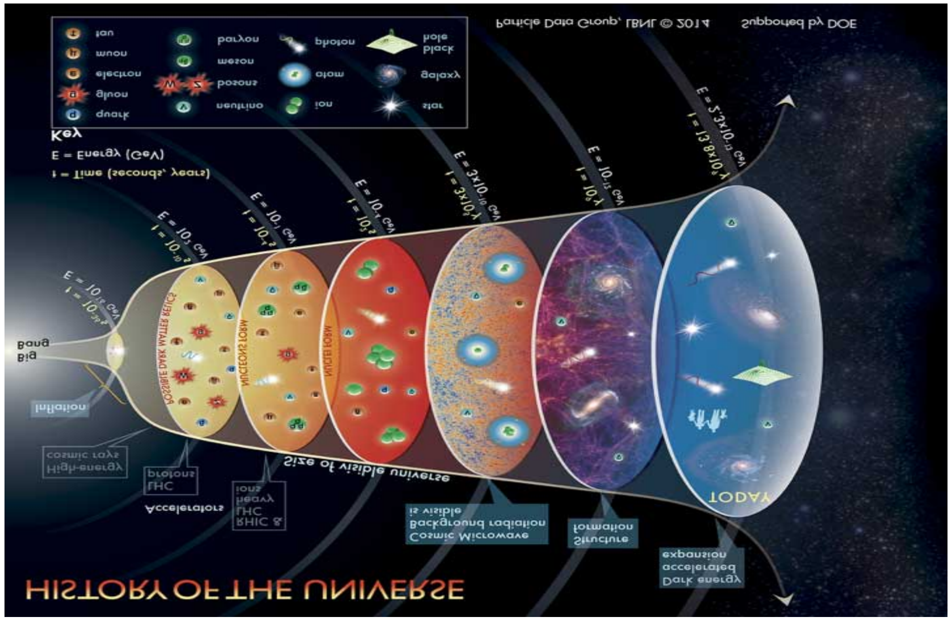
Cosmological Level

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Needs extra scalar fields to predict inflation



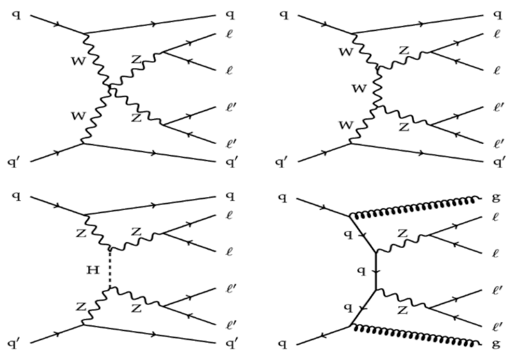
Alternative Theories of Gravity

Classification

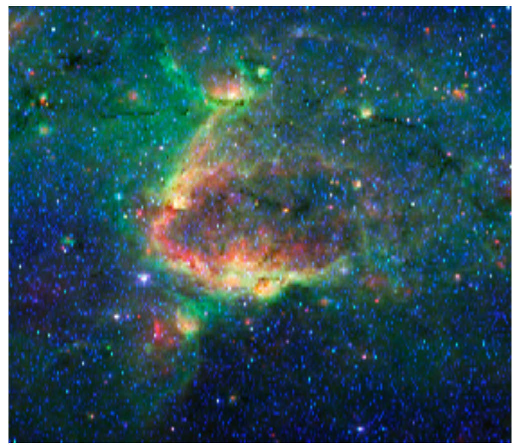
- Extended action → $f(R)$
- Modified Action → $f(T)$
- Coupling To Scalar fields → φR

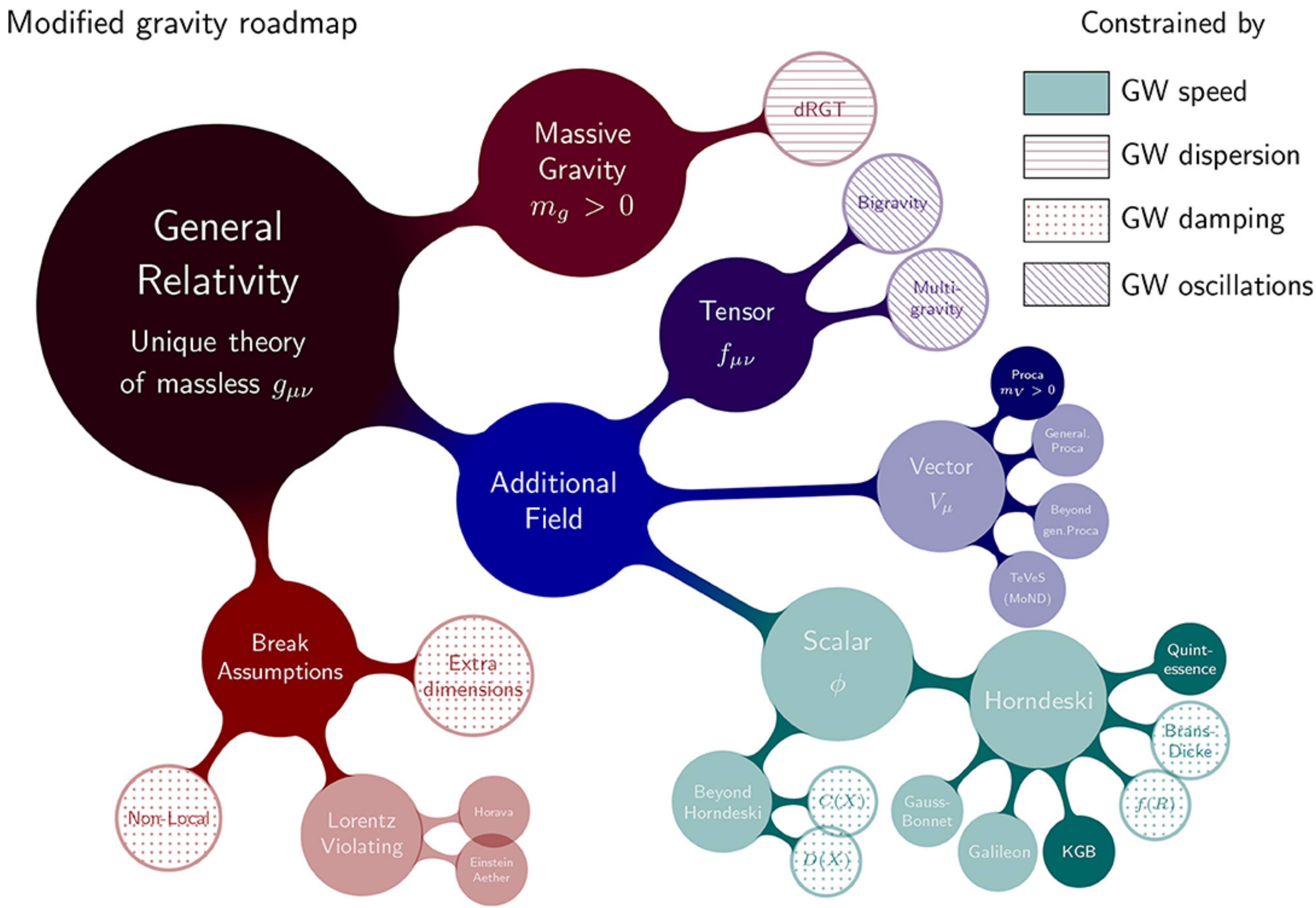
Motivations

- Could account for UV and IR quantum corrections ⟹



- Could reproduce both UV and IR cosmic evolution ⟹

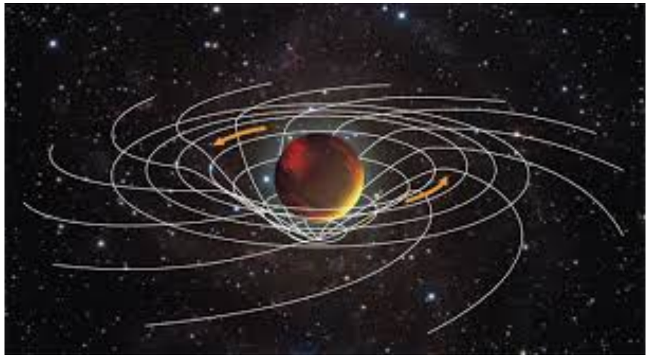




Modified gravity roadmap
Constrained by
GW speed
GW dispersion
GW damping
GW oscillations
General Relativity
Unique theory of massless $g_{\mu\nu}$
Massive Gravity
$m_g > 0$
dRGT
Tensor
$f_{\mu\nu}$
Bigravity
Multi-gravity
Additional Field
Proca
$m_V > 0$
General. Proca
Vector
V_μ
Beyond gen.Proca
TeVeS (MoND)
Scalar
ϕ
Horndeski
Quint-essence
Brans-Dicke
$f(R)$
KGB
Galileon
Gauss-Bonnet
Beyond Horndeski
$C(X)$
$D(X)$
Break Assumptions
Extra dimensions
Non-Local
Lorentz Violating
Horava
Einstein Aether

Is it possible to find out probes and test-beds for ETGs?

- *Geodesic motions around compact objects e.g- SgrA**
- **Lense-Thirring effect** →



- *Exact torsion-balance experiments*
- *Microgravity experiments from atomic physics*
- *Violation of Equivalence Principle (effective masses related to further gravitational degrees of freedom)*

Case Studies

Horava-Lifshits Gravity

$$S = \int d^3x\, dt \sqrt{-g} \left\{ \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^4} \left(\nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk} \nabla^j R^{ik} - \frac{1}{8} \nabla_i R \nabla^i R \right) \right\}$$

$$ds^2 = N^2 dt^2 - g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \qquad K^2 = g_{ij} K^{ij}$$

General Scalar-Tensor Gravity

$$S = \int \sqrt{-g} \left[f\left(R, R_{\alpha\beta} R^{\alpha\beta}, \phi \right) + \omega(\phi) \nabla_\alpha \phi \nabla^\alpha \phi \right] d^4x,$$

$$Y \equiv R^{\mu\nu} R_{\mu\nu}$$

$\omega(\phi)$ → Kinetic term: general function of ϕ

Both provide Schwarzschild solution as a particular limit

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2 \theta d\phi^2$$

General Scalar-Tensor Theory

$$S = \int \sqrt{-g}\left[f\left(R, R_{\alpha\beta}R^{\alpha\beta}, \phi\right) + \omega(\phi)\nabla_\alpha\phi\nabla^\alpha\phi\right] d^4x$$

Field equations

$$f_R R_{\mu\nu} - \frac{f + \omega(\phi)\nabla^\alpha\phi\nabla_\alpha\phi}{2} g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\Box f_R + 2 f_Y R^\alpha_\mu R_{\alpha\nu}$$
$$-2 f_Y\left(\nabla_\alpha\nabla_\nu R^\alpha_\mu + \nabla_\alpha\nabla_\mu R^\alpha_\nu\right) + \Box\left(f_Y R_{\mu\nu}\right) + g_{\mu\nu}\nabla_\beta\nabla_\alpha\left(f_Y R^{\alpha\beta}\right) + \omega(\phi)\nabla_\mu\phi\nabla_\nu\phi = 0$$

Klein-Gordon equation

$$2\omega(\phi)\Box\phi + \omega_\phi(\phi)\nabla_\alpha\phi\nabla^\alpha\phi - f_\phi = 0$$

Properties:

- **Explain late and early time evolution without DM and DE**
- **Fit the experimental observations at the astrophysical level**

Horava-Lifshitz Theory

Properties:

- **Theory of quantum gravity capable of solving the small-scale shortcomings of GR**
- **Lorentz-Invariance emerges at large distances**
- **Saccessfully passes the Solar System Tests**

***One possible* spherically symmetric solution:**

$$g_{00} = (g_{11})^{-1} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}$$

$\omega \longrightarrow$ Constant

Schwarzschild solution: $4M/\omega r^3 \ll 1$

However**.....**

Exact spherically symmetric solutions in ETGs are very rare

Weak Field Limit

General description of Weak-Field limit

Motivations:

Often exact solutions in ETGs cannot be found analytically

The typical values of the Newtonian gravitational potential Φ are larger than 10^{-5} in the Solar System (in geometrized units, Φ is dimensionless).

Scheme:

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}(\partial_\sigma \partial_\mu h^\sigma_\nu + \partial_\sigma \partial_\nu h^\sigma_\mu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu} \Box h)$$

Some restults provided by PN limit in ETGs

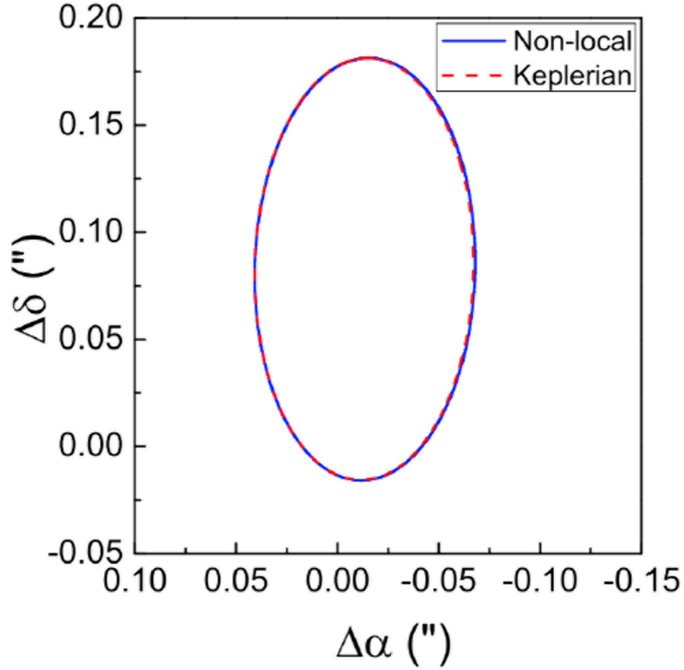
$$g_{00} \sim \mathcal{O}(6),\, g_{0i} \sim \mathcal{O}(5) \text{ and } g_{ij} \sim \mathcal{O}(4)$$

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} \left\{ R[1 + f(\phi)] + \varepsilon(r,t)(\Box\phi - R) \right\} d^4x$$

$$\phi \equiv \Box^{-1} R$$

Non-Local Gravity: after constraining the free parameters, it fits the Keplerian orbit better than GR

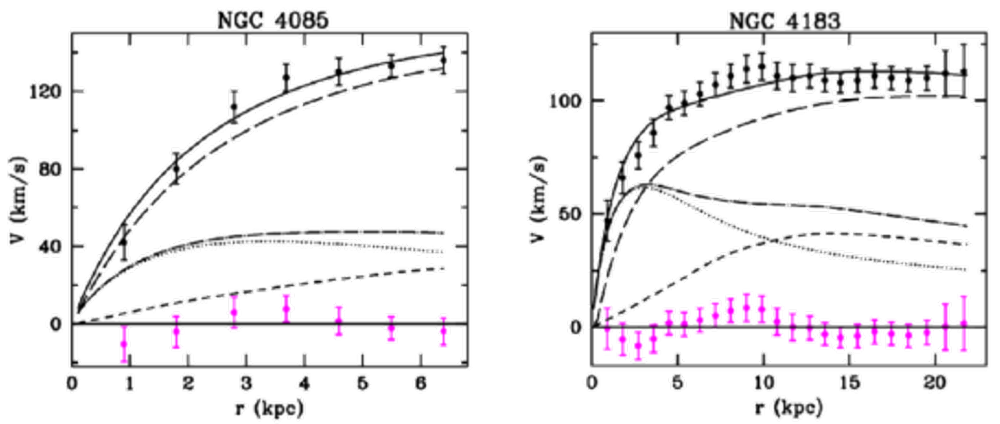
K. F. Dialektopoulos, D. Borka, S. Capozziello, V. Borka Jovanovic and P. Jovanovic Phys. Rev. D ***99*** *(2019) no.4, 044053 doi:10.1103/PhysRevD.99.044053*



$$g_{00} \sim \mathcal{O}(6),\, g_{0i} \sim \mathcal{O}(5) \text{ and } g_{ij} \sim \mathcal{O}(4)$$

$$S = \frac{c^4}{16\pi G} \int \sqrt{-g} R^k \, d^4x$$

Galaxy rotation curve for specific values of free parameters. Solid line is the best fit line of the total circular velocity, the dotted and the dashed lines refer to Newtonian contributions of star and gaseous components respectively respectively. The non-Newtonian contribution is labeled by the long-dashed line.



C. F. Martins and P. Salucci. Mon. Not. Roy. Astron. Soc ***381****(2007), 1103-1108 doi:10.1111/j.1365-2966.2007.12273.x*

First case: $f\left(R, R^{\mu\nu}R_{\mu\nu}, \varphi\right)$ *gravity*

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix} = \begin{pmatrix} 1 + 2\phi + 2\Xi & 2A_i \\ 2A_i & -\delta_{ij} + 2\Psi\delta_{ij} \end{pmatrix}$$

- *Three potentials arise: two scalar potentials and one vector potential*
- *Φ, Ψ are proportional to the power c^{-2} (Newtonian limit) while A_i is proportional to c^{-3} and Ξ to c^{-4} (post-Newtonian limit)*

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

Kerr spacetime

$$g_{00} \equiv \mathcal{A}(t,r,\theta)$$
$$g_{0i} = \mathcal{E}(t,r,\theta)$$
$$g_{ij}\delta^{ij} = \mathcal{B}(t,r,\theta) + \mathcal{C}(t,r,\theta) + \mathcal{D}(t,r,\theta)$$

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

By means of the decomposition of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$\begin{aligned} h_{00} &\sim \mathcal{O}(2) \\ h_{0i} &\sim \mathcal{O}(3) \\ h_{ij} &\sim \mathcal{O}(2), \end{aligned}$$

The function f, up to the c^{-4} order, can be developed as:

$$f(R, R_{\alpha\beta}R^{\alpha\beta}, \phi) = f_R(0,0,\phi^{(0)})R + \frac{f_{RR}(0,0,\phi^{(0)})}{2}R^2 + \frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2}(\phi - \phi^{(0)})^2 + f_{R\phi}(0,0,\phi^{(0)})R\phi + f_Y(0,0,\phi^{(0)})R_{\alpha\beta}R^{\alpha\beta},$$

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

Result:

- *Form of the vector potential* →

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1 - (1 + m_Y|\mathbf{x}|)\, e^{-m_Y|\mathbf{x}|}\right] \hat{\mathbf{x}} \times \mathbf{J}$$

- *Form of the scalar potential* →

$$\phi(r) = -\frac{GM}{r}\left[1 + g(\xi,\eta)e^{-m_R\tilde{k}_R r} + [1/3 - g(\xi,\eta)]e^{-m_R\tilde{k}_\phi r} - \frac{4}{3}e^{-m_Y r}\right]$$

with the definitions:

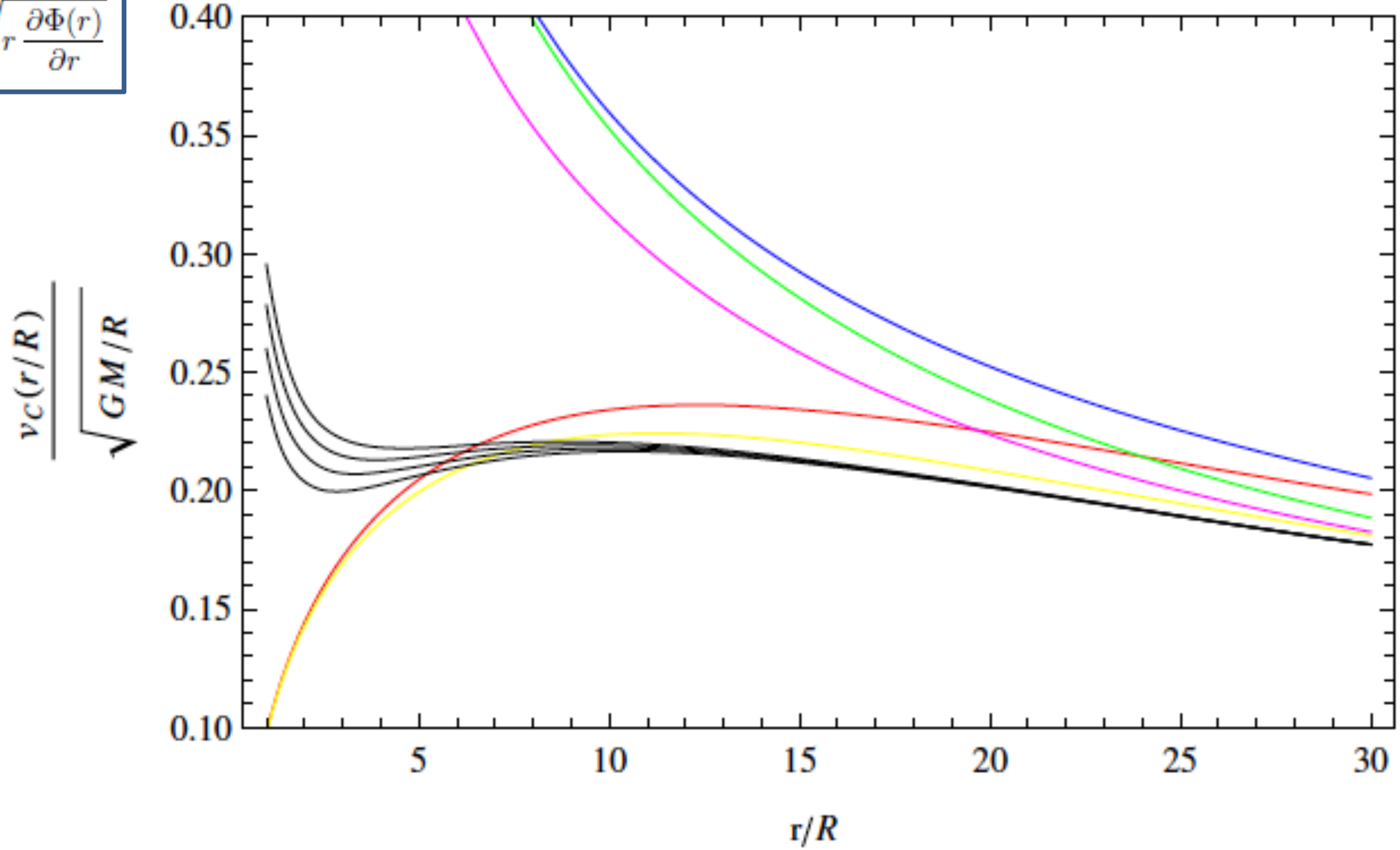
$$m_R^2 = -\frac{1}{3f_{RR}\left(0,0,\phi^{(0)}\right) + 2f_Y\left(0,0,\phi^{(0)}\right)}$$

$$m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)} \qquad \eta = \frac{m_\phi}{m_R}$$

$$m_\phi^2 = -\frac{f_{\phi\phi}\left(0,0,\phi^{(0)}\right)}{2\omega\left(\phi^{(0)}\right)} \qquad g(\xi,\eta) = \frac{1 - \eta^2 + \xi + \sqrt{\eta^4 + (\xi-1)^2 - 2\eta^2(\xi+1)}}{6\sqrt{\eta^4 + (\xi-1)^2 - 2\eta^2(\xi+1)}}$$

$$\xi = \frac{3f_{R\phi}\left(0,0,\phi^{(0)}\right)^2}{2\omega\left(\phi^{(0)}\right)} \qquad \tilde{k}^2_{R,\phi} = \frac{1 - \xi + \eta^2 \pm \sqrt{(1 - \xi + \eta^2)^2 - 4\eta^2}}{2}$$

$$v_c(r) = \sqrt{r \frac{\partial \Phi(r)}{\partial r}}$$



The circular velocity of a ball source of mass M and radius R, with the potentials of Table I. We indicate case A by a green line, case B by a yellow line, case D by a red line, case C by a blue line, and the GR case by a magenta line. The black lines correspond to the Sanders model for $-0.95<a<-0.92$. The values of free parameters are $\omega(\phi^{(0)})$.. . $-1/2$, $\Xi = -5$, $\eta=.3$, $m_Y = 1.5 * m_R$, $m_S=1.5 * m_R$, $m_R =.1* R^{-1}$.

Lense-Thirring precession in $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

$$\mathbf{\Omega}_{\rm LT}^{\rm (EG)} = \frac{1}{2}(\epsilon^{ijk}\partial_i A_k)(\epsilon_{\ell nk}\partial^\ell A^k) = \frac{G}{r^3}\sqrt{\left(\epsilon_{\ell km}\partial^m \epsilon^{ijk} J_i x_j\right)^2} = -e^{-m_Y r}\left(1 + m_Y r + m_Y^2 r^2\right)\mathbf{\Omega}_{\rm LT}^{\rm (GR)}$$

where

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1 - (1 + m_Y|\mathbf{x}|)\, e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}} \times \mathbf{J} \quad Y \equiv R^{\mu\nu}R_{\mu\nu}$$

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4Mr^3} J; \quad m_Y^2 = \frac{1}{f_Y\left(0, 0, \phi^{(0)}\right)}$$

For $f_Y \to 0$ i.e. $m_Y \to \infty$, we obtain the same outcome for the gravitational potential of $f(R, \phi)$-theory

Experimental constraints

Experimental constrains: GP-B

$$\Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + {m_Y}^2 r^2)\Omega_{\rm LT}^{\rm (GR)} \quad \textit{and} \quad \Omega_{\rm LT}^{\rm (GR)} = \frac{G}{2r^3}\mathbf{J}$$

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)}$$

The Gravity Probe B (GP-B) four gyroscopes aboard an Earth-orbiting satellite allowed to measure the frame-dragging effect with an error of about 19%

Effect	Measured (mas/y)	Predicted (mas/y)
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

$$\left|\frac{\Omega^{LT}_{obs} - \Omega^{LT}_{GR}}{\Omega^{LT}_{GR}}\right| = 0.05$$

The changes in the direction of spin gyroscopes, contained in the satellite orbiting at h = 650 km of altitude and crossing directly over the poles, have been measured with extreme precision

Experimental constrains: GP-B

Results:

1) $(1 + m_Y r^* + m_Y{}^2 r^{*2})\, e^{-m_Y r^*} \lesssim \frac{\delta|\mathbf{\Omega}_{\rm LT}|}{|\mathbf{\Omega}_{\rm LT}^{\rm (GR)}|} \simeq 0.19$

2) $m_Y \geq 7.3 \times 10^{-7} m^{-1}$

$$m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)}$$

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)}$$

$$\Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + m_Y{}^2 r^2)\Omega_{\rm LT}^{\rm (GR)}$$

Experimental constrains: LARES

The Laser Relativity Satellite (LARES) mission of the Italian Space Agency is designed to test the frame dragging and the Lense-Thirring effect, to within 1% of the value predicted in the framework of GR

The body of this satellite has a diameter of about 36.4 cm and weights about 400 kg

It was inserted in an orbit with 1450 km of perigee, an inclination of 69.5 ± 1 degrees and eccentricity 9.54×10^{-4}

It allows to obtain a stronger constraint for m_Y:

$$(1 + m_Y r^* + m_Y{}^2 r^{*2}) e^{-m_Y r^*} \lesssim \frac{\delta|\Omega_{\rm LT}|}{|\Omega_{\rm LT}^{\rm (GR)}|} \simeq 0.01$$

From which we obtain $m_Y \geq 1.2 \times 10^{-6} m^{-1}$

LARES and GP-B

Summing up, using data from the Gravity Probe B and LARES missions, we obtain constraints on m_Y.

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)} \qquad \Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + {m_Y}^2 r^2)\Omega_{\rm LT}^{\rm (GR)} \qquad m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)}$$

GP-B

$$m_Y \geq 7.3 \times 10^{-7} m^{-1}$$

LARES

$$m_Y > 1.2 \times 10^{-6} \mathrm{m}^{-1}$$

This result shows that space-based experiments can be used to test extensively parameters of fundamental theories

Perspective:

Put a further limit to the mass by GINGER

GINGER results: the case of Horava-Lifshitz Gravity

Application of PN limit to *Horava-Lifshitz Gravity*

$$S=\int d^3x\,dt\,\sqrt{-g}\left\{\frac{2}{\kappa^2}\left(K_{ij}K^{ij}-\lambda K^2\right)-\frac{\kappa^2}{2w^4}\left(\nabla_i R_{jk}\nabla^i R^{jk}-\nabla_i R_{jk}\nabla^j R^{ik}-\frac{1}{8}\nabla_i R\nabla^i R\right)\right\}$$

$$K_{ij}=\frac{1}{2N}\left(\dot{g}_{ij}-\nabla_i N_j-\nabla_j N_i\right)\qquad K^2=g_{ij}K^{ij}$$

Linearization of the metric tensor

$$g_{\mu\nu}\sim\begin{pmatrix}1+g_{00}^{(2)}+g_{00}^{(4)}+\dots & g_{0i}^{(3)}+\dots\\ g_{0i}^{(3)}+\dots & -\delta_{ij}+g_{ij}^{(2)}+\dots\end{pmatrix}$$

With similar computations as the previous case, the ratio between the Horava-Lifshitz and General Relativity Gyroscopic precession is

$$\frac{\Omega_{HL}^{G}}{\Omega_{GR}^{G}}=\frac{1}{3}\left(1+2\frac{G}{G_N}a_1-2\frac{a_2}{a_1}\right)$$

with

a_1, a_2 constants *to be constrained*

Ω_{HL}^{G} → *Gyroscopic precession*

G → effective gravitational constant

$$\frac{\Omega^G_{HL}}{\Omega^G_{GR}} = \frac{1}{3}\left(1 + 2\frac{G}{G_N}a_1 - 2\frac{a_2}{a_1}\right)$$

Importance of constraining a_1, a_2

It has been shown that, in order for the matter coupling to be consistent with solar system tests, the gauge field and the Newtonian potential must be coupled to matter in a specific way, but there are no indication on how to obtain the precise prescription from the action principle. Recently such a prescription has been generalised and a scalar-tensor extension of the theory has been developed to allow the needed coupling to emerge in the IR without spoiling the power-counting renormalizability of the theory.

Matter action

$$S_M = \int dtd^3x\tilde{N}\sqrt{\tilde{g}}\;\mathcal{L}_M\left(\tilde{N}, \tilde{N}_i, \tilde{g}_{ij}; \psi_n\right)$$

Lapse function

Scalar Potential

$$\begin{aligned}\tilde{N} &= (1 - a_1\sigma)N,\\ \tilde{N}^i &= N^i + Ng^{ij}\nabla_j\phi,\\ \tilde{g}_{ij} &= (1 - a_2\sigma)^2 g_{ij},\end{aligned}$$

Vector

$$\sigma = \frac{A - \mathcal{A}}{N}, \quad \text{with} \quad \mathcal{A} = -\dot{\phi} + N^i\nabla_i\phi + \frac{1}{2}N\nabla^i\phi\nabla_i\phi.$$

a_1, a_2 are then related to the potentials and can be constrained by GINGER measure as….

Terrestial experiment: GINGER

GINGER measures the difference in frequence of light of two beams circulating in a laser cavity in opposite directions. This translates into a time difference between the right-handed beam propagation time and the left-handed one

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}}\, ds^i$$

The difference in time can be linked to the Sagnac frequence $\boldsymbol{\Omega}_S$*, measured by GINGER*

Perimeter

Laser wavelength

$$c\delta\tau = N(\lambda_+ - \lambda_-) = Nc\left(\frac{f_- - f_+}{f^2}\right) = \frac{P\lambda}{c}\,\delta f \equiv \frac{P\lambda}{c}\,\Omega_S$$

Wavelength difference

Splitting in terms of frequence between the two beams

GIGNER in Horava-Lifshitz Gravity

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}}\, ds^i \Longleftrightarrow \Omega_S = -\frac{2c^2\sqrt{g_{00}}}{P\lambda} \oint \frac{g_{0i}}{g_{00}}\, ds^i$$

In Horava-Lifshitz gravity, it is

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Sagnac term

Lense-Thirring term

- A → Area encircled by the light beams
- α → Angle between the local radial direction and the normal to the plane of the array-laser ring
- θ → Colatitude of the laboratory
- Ω_E → Rotation rate of the Earth as measured in the local reference frame
- I_E → Momentum of Inertia
- P → Perimeter
- λ → Laser wavelength

Horava-Lifshitz vs General Relativity

General Relativity

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \quad \left[\cos(\theta+\alpha) - \frac{G_N M}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

$G = G_N$

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Horava-Lifshitz Gravity

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \quad \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Perspectives:

Free parameters to constrain by GINGER

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E\left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}\,a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

GINGER measure

Advantages to use GINGER

- The actual precision of GINGERINO is 1/1000 in the geodesic term, 1/100 in the LT term
- GINGER experiment should overcome such uncertainty providing a precision of 1/1000 in the LT term
- The presence of two rings yields a dynamic measure of the angle **α**

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E\left[\cos(\theta+\alpha) - \underbrace{\left(1+\frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta}_{\text{Geodesic Term}} - \underbrace{\frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)}_{\text{LT Term}}\right]$$

Notice that:

- *While the measure of the LT term can constrain the value of G, from the measure of the geodesic term we can get the value of a_1 and a_2*
- *The precision of GINGERINO is 10^{-15} rad/s, which corresponds to a precision of 1.4 $\cdot\, 10^{-9}$ with respect to the dominant term.*

Conclusions

Conclusions

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

- *In the context of ETGs, we have studied the linearized field equations in the limit of weak gravitational fields and small velocities generated by rotating gravitational sources, aimed at constraining the free parameters, which can be seen as effective masses (or lengths).*

- *The precession of spin of a gyroscope orbiting around a rotating gravitational source can be studied.*

- *Gravitational field gives rise, according to GR predictions, to geodesic and Lense-Thirring processions, the latter being strictly related to the off-diagonal terms of the metric tensor generated by the rotation of the source (Kerr metric)*

- *The gravitational field generated by the Earth can be tested by Gravity Probe B and LARES satellites. These experiments tested the geodesic and Lense-Thirring spin precessions with high precision.*

- *The corrections on the precession induced by scalar, tensor and curvature corrections can be measured and confronted with data.*

Conclusions

In $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity, GP-B and LARES satellites provide

$$m_Y \geq 7.3 \times 10^{-7} m^{-1}$$

$$m_Y > 1.2 \times 10^{-6} \mathrm{m}^{-1}$$

Perspective: constraint on m_y by GINGER

Perspective: constraints on a_1, a_2 by GINGER

In Horava-Lifshitz gravity, the weak-field limit provide

$$c\,\delta\tau = \frac{4A\Omega_E}{c}\left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\theta\sin\alpha - \frac{GI_E}{c^2R^3}(2\cos\theta\cos\alpha + \sin\theta\sin\alpha)\right]$$