Workshop sulla Gravitazione Sperimentale: misure laser, fisica fondamentale e applicazioni in INFN-CSN2
12 e 13 Novembre 2020

Theoretical background of the LARASE and SaToR-G Experiments and the LARASE results in the field of Gravitation

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On behalf of the SaToR-G Experiment

Outline

- Introduction to SaToR-G high level goals
- Einstein Equivalence Principle and Metric theories of gravity
- General Relativity over time
- What to measure in the weak field and slow motion limit of GR
- Results of the LARASE experiment: Part I

Introduction to SaToR-G high level goals

- Started in 2020, **SaToR-G** (**Sa**tellites **T**ests **of** **R**elativistic **G**ravity) will expand the activities carried on by the **LA**ser **RA**nged **S**atellites **E**xperiment (**LARASE**, 2013-2019), investigating possible experimental signatures of deviation from General Relativity (**GR**)

- Similarly to **LARASE**, **SaToR-G** is dedicated to measurements of the gravitational interaction in the Weak-Field and Slow-Motion (**WFSM**) limit of **GR** by means of laser tracking to geodetic passive satellites orbiting around the Earth

- **SaToR-G** exploits the improvement of the dynamical model of the two **LAGEOS** and **LARES** satellites performed by **LARASE**. These satellites represent the proof-masses of the experiment

- While for **LARASE** the main scientific target was a reliable and robust measurement of the Lense-Thirring effect, **SaToR-G** **focuses on verifying the gravitational interaction beyond the predictions of GR, looking for possible effects connected with new physics, and foreseen by different alternative theories of gravitation**

Einstein Equivalence Principle and Metric theories of gravity

Weak Equivalence Principle (WEP)

- two different bodies fall with the same acceleration: Universality of the Free Fall (**UFF**)
- the inertial mass is proportional to the gravitational (passive) mass
- the trajectory of a freely falling "test" body is independent of its internal structure and composition
- in every local and non-rotating falling frame, the trajectory of a freely falling test body is a straight line, in agreement with special relativity

Einstein Equivalence Principle (EEP)

- **WEP**
- Local Lorentz Invariance (**LLI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed
- Local Position Invariance (**LPI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed

Clifford M. Will, ***Theory and Experiment in Gravitational Physics***. Cambridge University Press, Ed. 1981 and Ed. 2018

Einstein Equivalence Principle and Metric theories of gravity

Metric theories

- GR is a metric theory of gravity and all metric theories obey the EEP
- Indeed, the experimental results supporting the EEP supports the conclusion that the only theories of gravity that have a hope of being viable are metric theories, or possibly theories that are metric apart from very weak or short-range non-metric couplings (as in string theory):

1. there exist a symmetric metric
2. tests masses follow geodesics of the metric
3. in Local Lorentz Frames, the non-gravitational laws of physics are those of Special Relativity

$$g_{\alpha\beta} = g_{\beta\alpha}$$

$$det(g_{\alpha\beta}) \neq 0$$

$$ds^2 = g_{\alpha\beta} dx^\alpha \, dx^\beta$$

$$G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta}$$

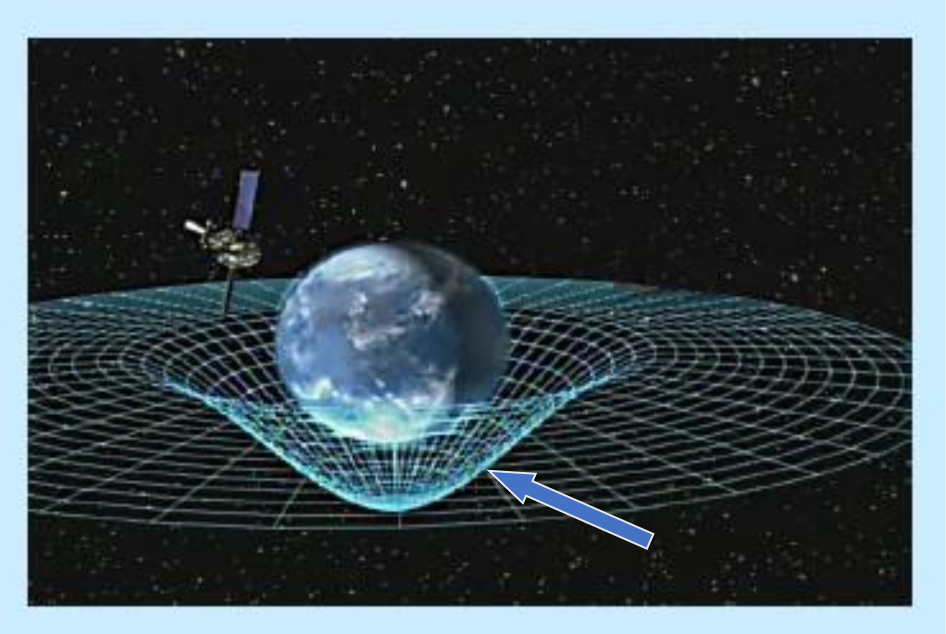
Einstein Equivalence Principle and Metric theories of gravity

Metric theories

- Metric theories different from **GR** provide additional fields (Scalars, Vectors, Tensors, …) beside the metric tensor $g_{\alpha\beta}$, that act as "new" gravitational fields
- The role of these gravitational fields is to "mediate" how the matter and the non-gravitational fields generate the gravitational fields and produce the metric

In Metric theories different from GR:

- the spacetime geometry tells mass-energy how to move as in GR
- but mass-energy tells spacetime geometry how to curve in a different way from GR
- and the metric alone acts back on the mass in agreement with EEP



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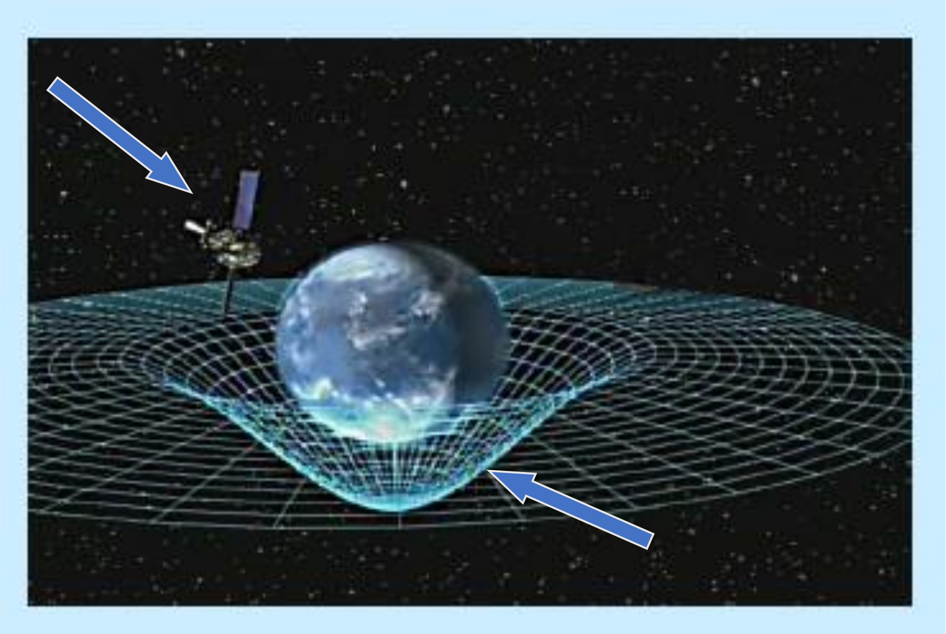

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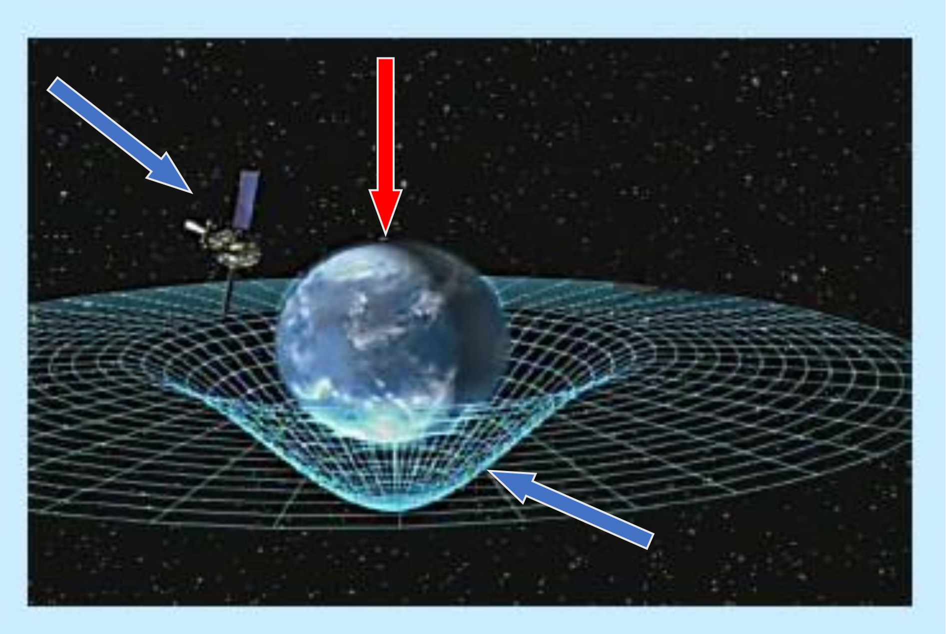

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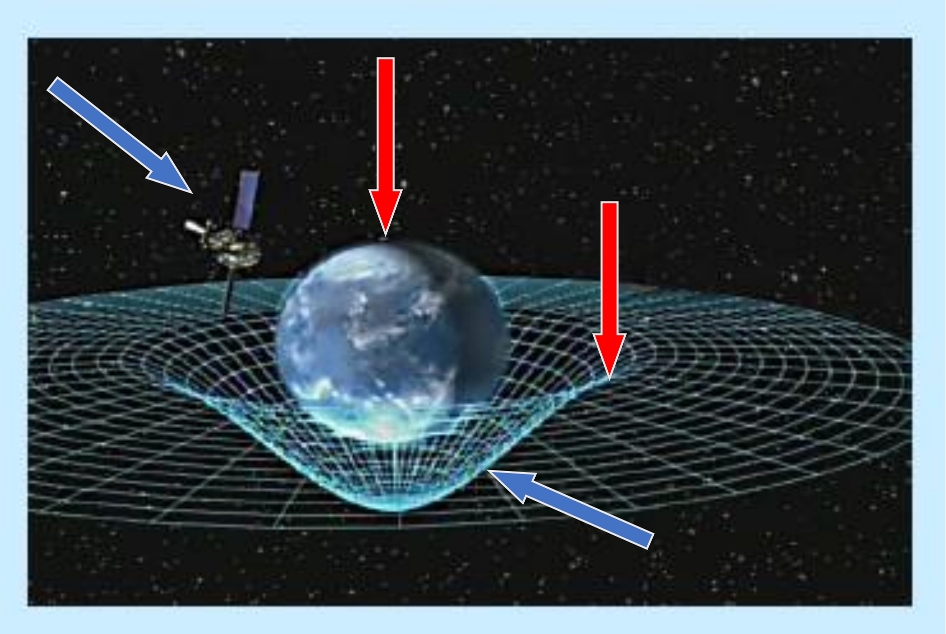

Einstein Equivalence Principle and Metric theories of gravity

In practice, in the other Metric theories of gravity,

the field equations and the spacetime metric are different with respect to GR

Metric theories

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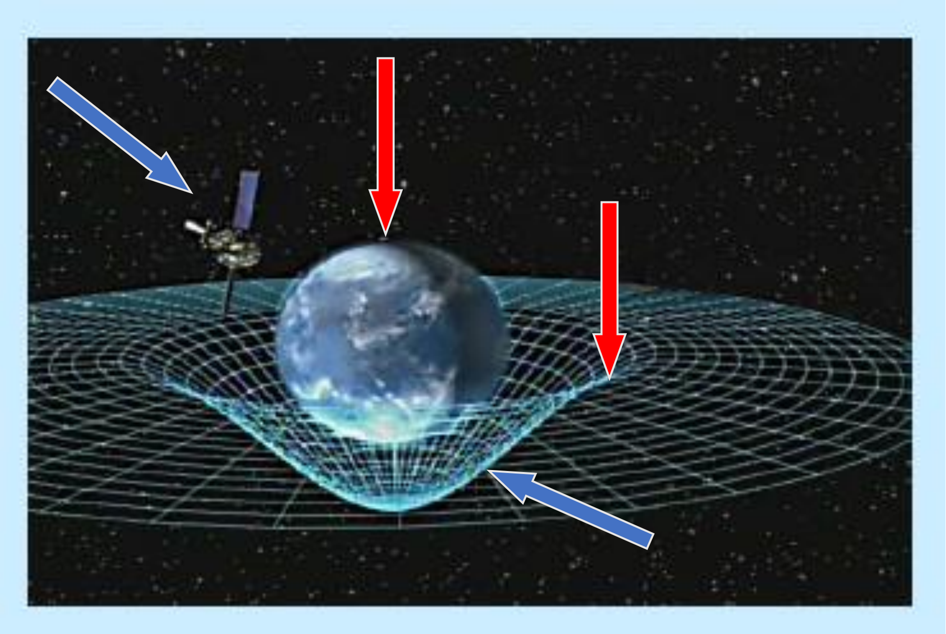

Einstein Equivalence Principle and Metric theories of gravity

Metric theories and the Strong Equivalence Principle (SEP)

A very fundamental question is:

- What is the nature of gravity in different Metric theories?
  - ✓ A way to answer to this very important question is to investigate the ''dynamical character'' of the theory

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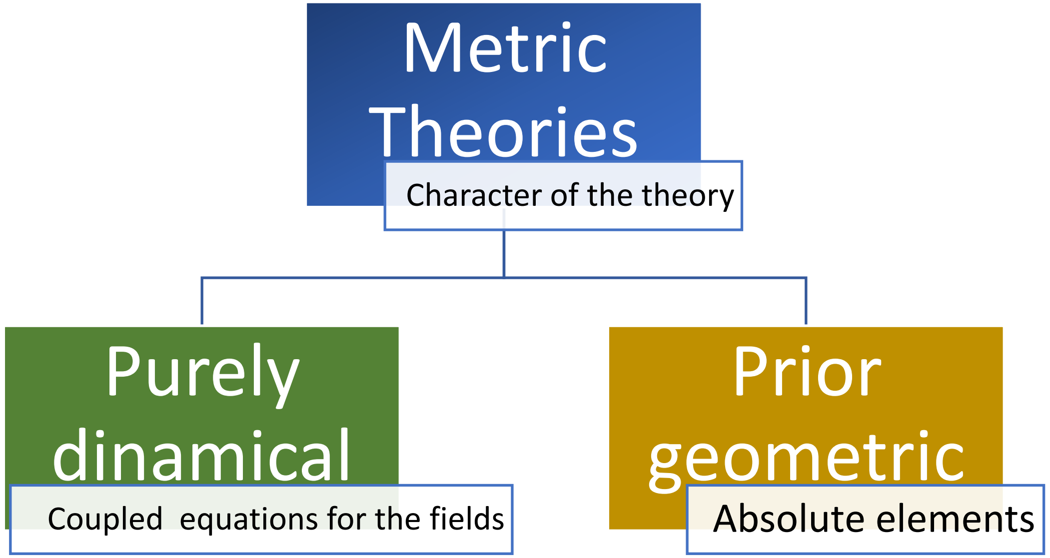

Einstein Equivalence Principle and Metric theories of gravity

Metric theories and the Strong Equivalence Principle (SEP)

A very fundamental question is:

- What is the nature of gravity in different Metric theories?
  1. A way to answer to this very important question is to investigate the ''dynamical character'' of the theory
  2. A second important aspect is to introduce gravity itself in the experiment
     - ✓ That is the inclusion of bodies with self-gravitational interactions as well as experiments that involve gravitational forces
     - ✓ **This leads to the so-called Strong Equivalence Principle, satisfied by GR but not by the other Metric theories of gravity**

Einstein Equivalence Principle and Metric theories of gravity

Strong Equivalence Principle (SEP)

- **WEP** is valid for self-gravitating bodies as well as for test bodies:
  - ❑ Gravitational Weak Equivalence Principle (**GWEP**)
- Local Lorentz Invariance (**LLI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed
- Local Position Invariance (**LPI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed

Einstein Equivalence Principle and Metric theories of gravity

Tests of the WEP

Clifford M. Will, ***Theory and Experiment in Gravitational Physics***.
Cambridge University Press, Ed. 2018

$$\eta = \frac{\Delta a}{a}$$

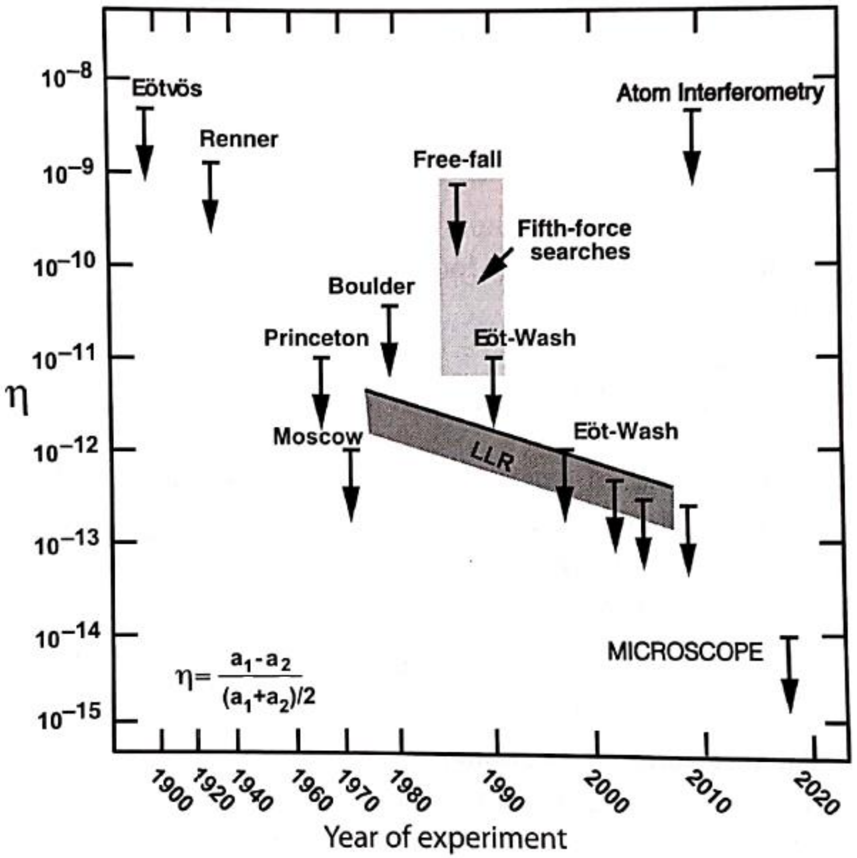

Fig. 2.2 Selected tests of the Weak Equivalence Principle, showing bounds on the Eötvös ratio η. The light grey region represents many experiments originally performed to search for a fifth force, including the "free-fall" and the initial Eöt-Wash experiments. The dark grey band shows evolving bounds on η for gravitating bodies from lunar laser ranging (LLR) (see Section 8.1 for discussion).

Einstein Equivalence Principle and Metric theories of gravity

Tests of the SEP

Is there a different contribution of gravitational binding energy (self-energy) to its gravitational (passive) mass and its inertial mass?

- If so, this is known as the **Nordvedt Effect** and is directly related to possible **SEP** violations (massive bodies)

$$m_g = m_i + \eta_N E_g = m_i \left(1 + \eta_N \frac{E_g}{m_i}\right)$$

$$\eta_N = (4\beta - \gamma - 3) - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2$$

$$\vec{a} = -\frac{GM_\odot}{r^2}\frac{m_g}{m_i}\hat{r} = -\frac{GM_\odot}{r^2}\left(1 + \eta_N \frac{E_g}{m_i}\right)\hat{r}$$

$$\begin{cases} \dfrac{E_g}{m_i} \simeq 4.6 \times 10^{-10} \; Earth \\ \dfrac{E_g}{m_i} \simeq 0.2 \times 10^{-10} \; Moon \end{cases}$$

If $\eta_N \neq 0$, the Earth and the Moon must fall in the field of the Sun with a little bit different acceleration

$$\delta\vec{a} = -\frac{GM_\odot}{r^2}\eta_N \left(\left.\frac{E_g}{m_i}\right|_\oplus - \left.\frac{E_g}{m_i}\right|_M\right)\hat{r}$$

$$|\boldsymbol{\eta_N}| = (\mathbf{4.4} \pm \mathbf{4.5}) \times \mathbf{10^{-4}}$$

From Lunar Laser Ranging (LLR) measurements

Einstein Equivalence Principle and Metric theories of gravity

C.M. Will Living Rev. Relativity, 17, (2014), 4

The parametrized post-Newtonian (PPN) formalism

Post-Newtonian formalism or **PPN** formalism details the parameters in which different theories of gravity, under **WFSM** conditions, can differ from Newtonian gravity

Metric

$$
\begin{aligned}
g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\
& + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} \\
& + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3), \\
g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\
& - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}), \\
g_{ij} = & (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).
\end{aligned}
$$

Stress-Energy Tensor

$$
\begin{aligned}
T^{00} &= \rho(1 + \Pi + v^2 + 2U), \\
T^{0i} &= \rho v^i \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right), \\
T^{ij} &= \rho v^i v^j \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right) + p\delta^{ij}(1 - 2\gamma U).
\end{aligned}
$$

Metric Potentials

$$
\begin{aligned}
U &= \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
U_{ij} &= \int \frac{\rho'(x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3} d^3x', \\
\Phi_W &= \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|}\right) d^3x'\, d^3x'', \\
\mathcal{A} &= \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3} d^3x', \\
\Phi_1 &= \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
\Phi_2 &= \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
\Phi_3 &= \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
\Phi_4 &= \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
V_i &= \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|} d^3x', \\
W_i &= \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'.
\end{aligned}
$$

Einstein Equivalence Principle and Metric theories of gravity

The parametrized post-Newtonian (PPN) formalism

C.M. Will Living Rev. Relativity, 17, (2014), 4

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Theory	Arbitrary functions or constants	Cosmic matching parameters	PPN parameters				
			γ	β	ξ	α_1	α_2
General relativity	none	none	1	1	0	0	0
Scalar–tensor							
Brans–Dicke	ω_{BD}	ϕ_0	$\frac{1+\omega_{BD}}{2+\omega_{BD}}$	1	0	0	0
General, $f(R)$	$A(\varphi), V(\varphi)$	φ_0	$\frac{1+\omega}{2+\omega}$	$1+\frac{\lambda}{4+2\omega}$	0	0	0
Vector–tensor							
Unconstrained	$\omega, c_1, c_2, c_3, c_4$	u	γ'	β'	0	α_1'	α_2'
Einstein-Æther	c_1, c_2, c_3, c_4	none	1	1	0	α_1'	α_2'
Khronometric	$\alpha_k, \beta_k, \lambda_k$	none	1	1	0	α_1'	α_2'
Tensor–Vector–Scalar	k, c_1, c_2, c_3, c_4	ϕ_0	1	1	0	α_1'	α_2'

General Relativity over time

- The history of General Relativity (GR), together with the history of the so-called Alternative Theories of Gravitation (ATG), can be roughly divided into three main periods:
  - **1915 $\rightarrow$ 1960**
  - **1960 $\rightarrow$ 1980**
  - **1980 $\rightarrow$ Today**

Clifford M. Will, ***Theory and Experiment in Gravitational Physics***. Cambridge University Press, Ed. 1981 and Ed. 2018

General Relativity over time

- **Classical tests of GR**:
  - Gravitational redshift
  - Deflection of light
  - Precession of the perihelion
- **Several difficulties with GR**:
  - Lack of an effective experimental support
  - Curved spacetime: concepts and consequences
  - Mach's Principle

- **ATG**:
  - Whitehead (1922)
  - Birkhoff (1943)
  - Belifante/Swihart (1957)

General Relativity over time

- **Dicke framework + PPN framework (Will & Nordtvedt)**:
  - Schiff (1960)
  - Dicke (1960)
  - Bertotti (1962)
  - Nordtvedt & Will (1968–1972)

- **New theories with respect to GR**:
  - New effects to be predicted
  - Differences with GR
  - PPN parameters ≠ from those of GR

- **ATG**:
  - Brans-Dicke (1960)
  - Will-Nordtvedt (1972)
  - Rosen (1973)

General Relativity over time

Schiff (1960)

- L.I. Schiff, On Experimental Tests of the General Theory of Relativity. *American Journal of Physics*, Vol. 28, Issue 4, pp. 340-343 (1960).

On Experimental Tests of the General Theory of Relativity*

L. I. Schiff
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
(Received October 6, 1959)

This paper explores the extent to which the three "crucial tests" support the full structure of the general theory of relativity, and do not merely verify the equivalence principle and the special theory of relativity, which are well established by other experimental evidence. It is shown how the first-order changes in the periods of identically constructed clocks and the lengths of identically constructed measuring rods can be found without using general relativity, and how the red shift and the deflection of light can be computed from them. Only the planetary orbit precession provides a real test of general relativity. Terrestrial or satellite experiments that would go beyond supplying corroborative evidence for the equivalence principle and special relativity would be extremely difficult to perform, and would, for example, require a frequency standard with an accuracy somewhat better than one part in 10^{18}.

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2. Deflection of light
3. Orbit precession

Only the planetary orbit precession provides a real test of general relativity

General Relativity over time

The Dicke's Framework (1960)

1. Spacetime is a 4-dimensional differentiable manifold, with each point in the manifold corresponding to a physical event. The manifold need not a priori have either a metric or an affine connection
   - The hope is that experiment will force us to conclude that it has both
2. The equations of gravity and the mathematical entities in them are to be expressed in a form that is independent of the particular coordinates used, i.e., in covariant form

Dicke imposes two constraints:

1. Gravity must be associated with one or more fields of tensor character: scalars, vectors and tensors of various ranks
2. The dynamical equations that govern gravity must be derivable from an invariant action principle

General Relativity over time

The Dicke's Framework (1960)

From Dicke's Framework, theorists have been able to formulate a set of criteria that any theory of gravitation should satisfy if it is to be viable:

1. It must be complete
2. It must be self-consistent
3. It must be relativistic
4. It must have the correct Newtonian limit

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THEORETICAL FRAMEWORKS FOR TESTING RELATIVISTIC GRAVITY. I. FOUNDATIONS*

KIP S. THORNE AND CLIFFORD M. WILL†
California Institute of Technology, Pasadena, California
Received 1970 August 24

ABSTRACT

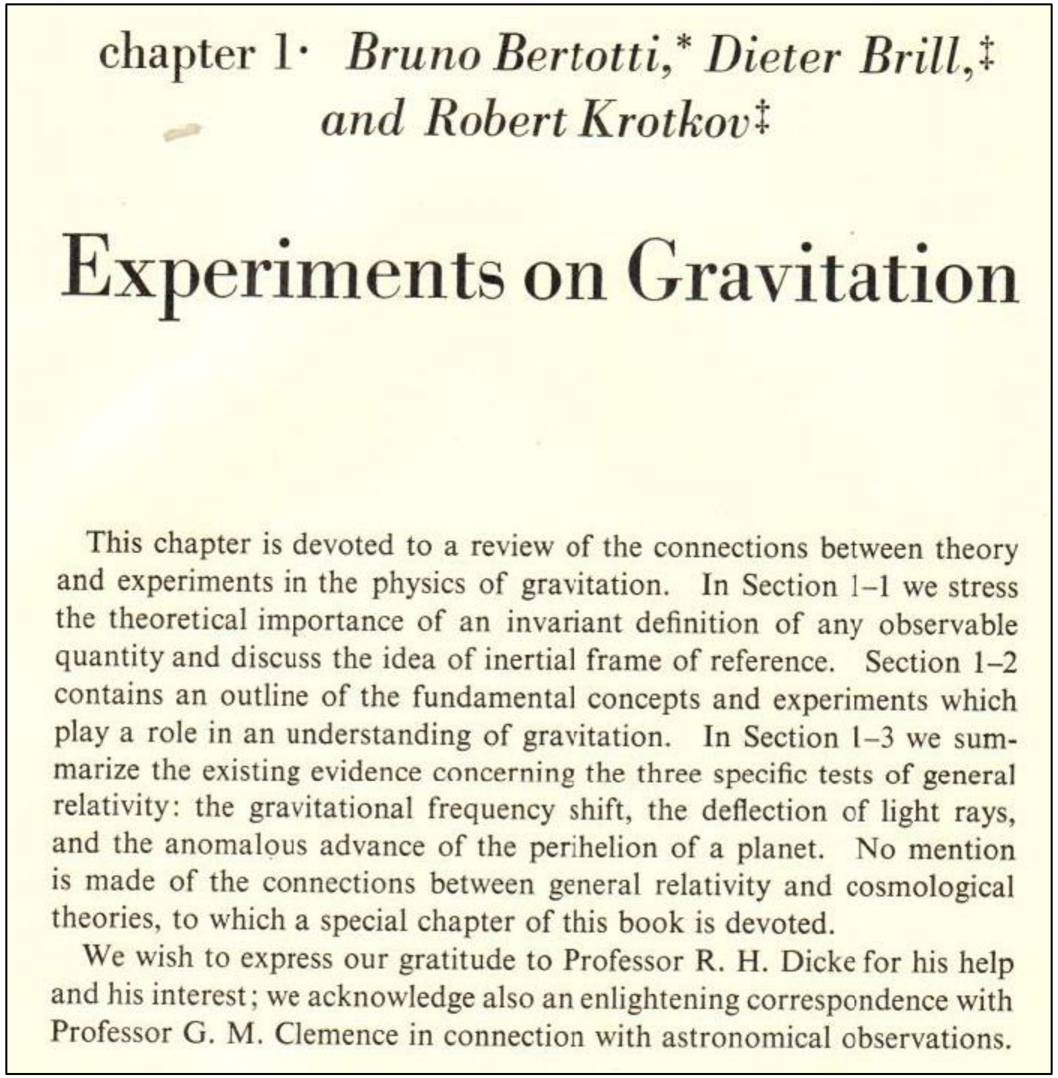
This is the first in a series of theoretical papers which will discuss the experimental foundations of general relativity This paper reviews, modifies, and compares two very different theoretical frameworks, within which one devises and analyzes tests of gravity. The *Dicke framework* assumes almost nothing about the nature of gravity; and it uses a variety of experiments to delineate the gross features of the gravitational interaction. Two of its tentative conclusions (the presence of a metric, and the "gravitational response equation," $\nabla \cdot T = 0$, for stressed matter) become the postulates of the *Parametrized Post-Newtonian framework*. The PPN framework encompasses most, if not all, of the theories of gravity that are currently compatible with experiment. Future papers in this series will develop the PPN framework in detail, and will use it to analyze a variety of relativistic gravitational effects that should be detectable in the solar system during the coming decade.

General Relativity over time

Bertotti (1962)

- B. Bertotti, D. Brill and R. Krotkov, Gravitation: Experiments on gravitation, An introduction to current research, ed. L. Witten, J. Wiley and Sons Inc., New York, 1-48, 1962.

This is a review of experiments in gravitation, and it proves how thin and feeble was (at that time) the experimental evidence supporting GR



chapter 1 · *Bruno Bertotti,* Dieter Brill,‡ and Robert Krotkov‡*

Experiments on Gravitation

This chapter is devoted to a review of the connections between theory and experiments in the physics of gravitation. In Section 1–1 we stress the theoretical importance of an invariant definition of any observable quantity and discuss the idea of inertial frame of reference. Section 1–2 contains an outline of the fundamental concepts and experiments which play a role in an understanding of gravitation. In Section 1–3 we summarize the existing evidence concerning the three specific tests of general relativity: the gravitational frequency shift, the deflection of light rays, and the anomalous advance of the perihelion of a planet. No mention is made of the connections between general relativity and cosmological theories, to which a special chapter of this book is devoted.

We wish to express our gratitude to Professor R. H. Dicke for his help and his interest; we acknowledge also an enlightening correspondence with Professor G. M. Clemence in connection with astronomical observations.

General Relativity over time

Bertotti (2003)

- B. Bertotti, L. Iess, & P. Tortora: A test of general relativity using radio links with the CASSINI spacecraft. *Nature*, 425, 374-376, 2003

The most precise measurement of the PPN parameter γ:

$$\boldsymbol{\gamma - 1 = (2.1 \pm 2.2) \times 10^{-5}}$$

letters to nature

A test of general relativity using radio links with the Cassini spacecraft

B. Bertotti[1], L. Iess[2] & P. Tortora[3]

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[2] *Dipartimento di Ingegneria Aerospaziale ed Astronautica, Università di Roma "La Sapienza", Via Eudossiana 16, I-00184, Roma, Italy*
[3] *II Facoltà di Ingegneria, Università di Bologna, Via Fontanelle 40, I-47100, Forlì, Italy*

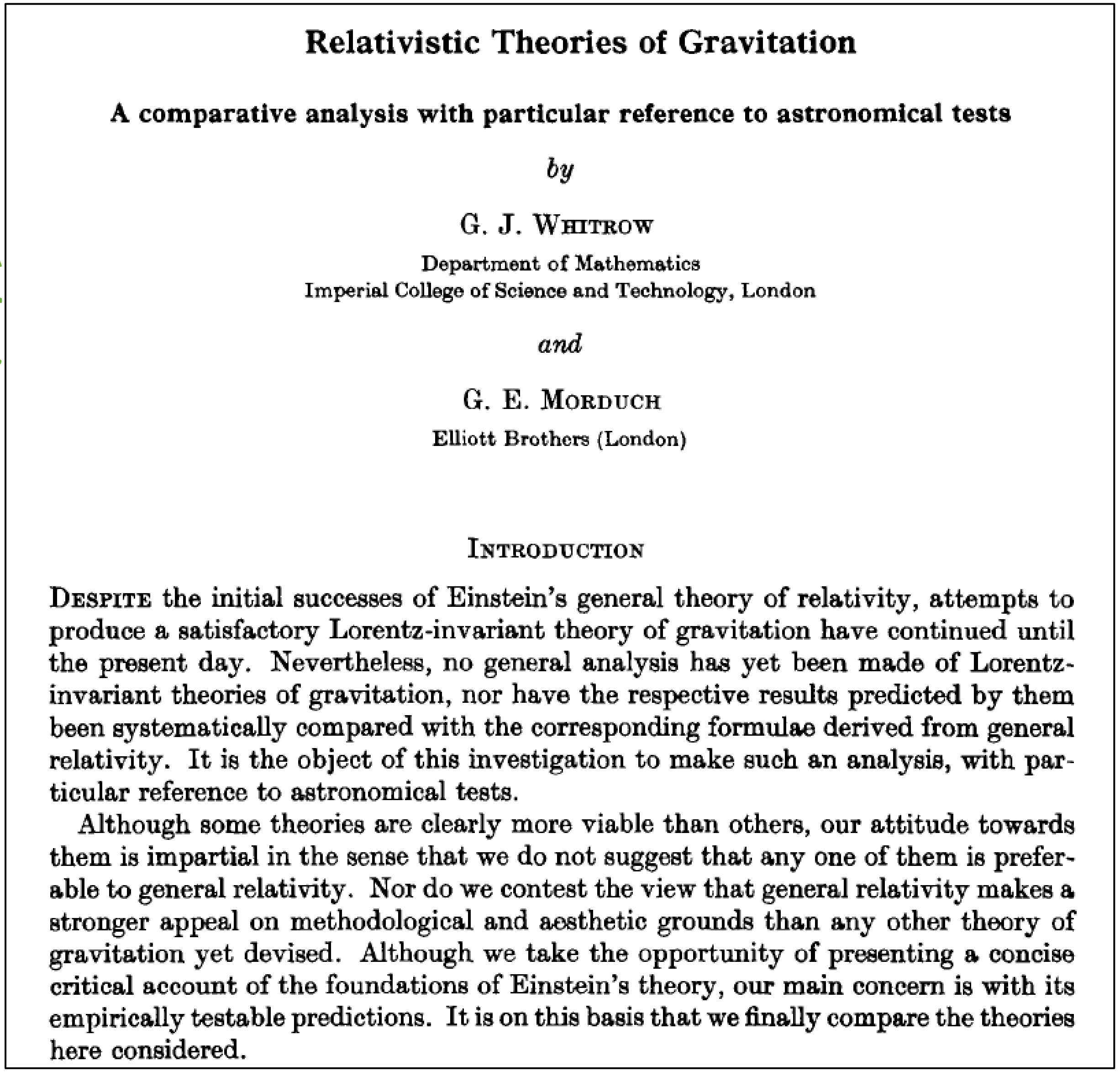
According to general relativity, photons are deflected and delayed by the curvature of space-time produced by any mass[1–3]. The bending and delay are proportional to $\gamma + 1$, where the parameter γ is unity in general relativity but zero in the newtonian model of gravity. The quantity $\gamma - 1$ measures the degree to which gravity is not a purely geometric effect and is affected by other fields; such fields may have strongly influenced the early Universe, but would have now weakened so as to produce tiny—but still detectable—effects. Several experiments have confirmed to an accuracy of ~0.1% the predictions for the deflection[4,5] and delay[6] of photons produced by the Sun. Here we report a measurement of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun. Our result, $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$, agrees with the predictions of standard general relativity with a sensitivity that approaches the level at which, theoretically, deviations are expected in some cosmological models[7,8].

General Relativity over time

Whitrow and Morduch (1965)

- G.J. Whitrow and G.E. Morduch, Relativistic Theories of Gravitation, A comparative analysis with particular reference to astronomical tests, *Vistas in Astronomy* 6, 1-67, 1965

This is a review of nominally viable (at that time) ATG



Relativistic Theories of Gravitation

A comparative analysis with particular reference to astronomical tests

by

G. J. Whitrow
Department of Mathematics
Imperial College of Science and Technology, London

and

G. E. Morduch
Elliott Brothers (London)

Introduction

Despite the initial successes of Einstein's general theory of relativity, attempts to produce a satisfactory Lorentz-invariant theory of gravitation have continued until the present day. Nevertheless, no general analysis has yet been made of Lorentz-invariant theories of gravitation, nor have the respective results predicted by them been systematically compared with the corresponding formulae derived from general relativity. It is the object of this investigation to make such an analysis, with particular reference to astronomical tests.

Although some theories are clearly more viable than others, our attitude towards them is impartial in the sense that we do not suggest that any one of them is preferable to general relativity. Nor do we contest the view that general relativity makes a stronger appeal on methodological and aesthetic grounds than any other theory of gravitation yet devised. Although we take the opportunity of presenting a concise critical account of the foundations of Einstein's theory, our main concern is with its empirically testable predictions. It is on this basis that we finally compare the theories here considered.

General Relativity over time

The Brans-Dicke ATG (1961)

The Brans-Dicke's theory arises from Dicke's idea to turn Mach's principle (as well as Dirac's Large Number Hypothesis) into a gravity theory, since GR was unsatisfactory from this point of view

- C. Brans and R.H. Dicke, Mach's principle and a relativistic theory of gravitation, *Phys. Rev*. 124, 925-935, 1961
- R.H. Dicke, Mach's principle and invariance under transformations of units, *Phys. Rev*. 125, 2163-2167, 1962
- R.H. Dicke, The Theoretical Significance of Experimental Relativity, Blackie and Son Ltd. London and Glasgow, 1964
- R.H. Dicke, Scalar-tensor gravitation and the cosmic fireball, *Astrophys. J*. 152, 1-24, 1968

P. Jordan, Zum gegenwärtigen Stand der Diracschen kosmologischen Hypothesen, *Zeitschrift für Physik* 157, 112-121, 1959

General Relativity over time

The Brans-Dicke ATG (1961)

PHYSICAL REVIEW VOLUME 124, NUMBER 3 NOVEMBER 1, 1961

Mach's Principle and a Relativistic Theory of Gravitation*

C. BRANS† AND R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received June 23, 1961)

The role of Mach's principle in physics is discussed in relation to the equivalence principle. The difficulties encountered in attempting to incorporate Mach's principle into general relativity are discussed. A modified relativistic theory of gravitation, apparently compatible with Mach's principle, is developed.

INTRODUCTION

IT is interesting that only two ideas concerning the nature of space have dominated our thinking since the time of Descartes. According to one of these pictures, space is an absolute physical structure with properties of its own. This picture can be traced from Descartes vortices[1] through the absolute space of Newton,[2] to the ether theories of the 19th century. The contrary view that the geometrical and inertial properties of space are meaningless for an empty space, that the physical properties of space have their origin in the matter contained therein, and that the only meaningful motion of a particle is motion relative to other matter in the universe has never found its complete expression in a physical theory. This picture is

small mass, its effect on the metric is minor and can be considered in the weak-field approximation. The observer would, according to general relativity, observe normal behavior of his apparatus in accordance with the usual laws of physics. However, also according to general relativity, the experimenter could set his laboratory rotating by leaning out a window and firing his 22-caliber rifle tangentially. Thereafter the delicate gyroscope in the laboratory would continue to point in a direction nearly fixed relative to the direction of motion of the rapidly receding bullet. The gyroscope would rotate relative to the walls of the laboratory. Thus, from the point of view of Mach, the tiny, almost massless, very distant bullet seems to be more important that the massive, nearby walls of the laboratory in determining

General Relativity over time

The Brans-Dicke ATG (1961)

The Brans-Dicke's theory played a primary role in the development of an intense experimental activity to verify the gravitational interaction during the late 1960s and throughout the 1970s

$$S = \frac{1}{16\pi G}\int\left(\phi R - \frac{\omega}{\phi} g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}\right)\sqrt{-g}\,d^4x + S_{ng}$$

$$S_{GR} = \frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x + S_{ng}$$

$$G_{\alpha\beta} = \frac{8\pi G}{\phi}T_{\alpha\beta} + \frac{\omega}{\phi^2}\left(\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\phi_{,\mu}\phi^{,\mu}\right) + \frac{1}{\phi}\left(\phi_{;\alpha\beta} - g_{\alpha\beta}\Box_g\phi\right)$$

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

$$\Box_g\phi = \frac{8\pi G}{3+2\omega}T$$

Where ϕ is a scalar field and ω represents the dimensionless Dicke's coupling constant: it is tested by the experiments

$$\omega \to \infty \;\Rightarrow\; Brans - Dicke \;\to\; General\ Relativity$$

General Relativity over time

- **Beyond Einstein's theory of GR**:
  - String theory and its extensions
  - Lorentz Symmetry Violations
  - 5^{th} force
  - Rotation curves of Galaxies
  - Acceleration of the Universe
- **New theories with respect to GR?**
  - Not exactly or not only
  - To extend GR into different regimes beyond those where it had been "well" tested so far:
    - cosmological scale
    - Strong fields
  - Due to several motivations:
    - Particle physics
    - Quantum gravity
    - Cosmology

- **ATG**:
  - Scalar-Tensor theories
  - Vector-Tensor theories
  - Tensor-Vector-Scalar theories

General Relativity over time

String Theory and its extensions (late 1960s $\rightarrow$ about 2000)

- The point-like particle of particle physics is replaced by a string characterized by several vibrational states
- One of these vibrational states corresponds to the graviton, i.e. to the particle that mediates the gravitational interaction
- The theory of strings, since evolve and interact according to the rules of quantum mechanics, automatically describes quantum gravity

String Theory

- In the small string-coupling of the theory, String Theory predicts a relativistic theory very close to GR …
  - It is Brans-Dicke theory with $\omega = -1$, but …
    - The scalar field ϕ would acquire a large mass (via spontaneous symmetry breaking), with $\mu \propto 1/\lambda$, and its effect would be exponentially suppressed on any macroscopic scale
    - This would restore a theory of gravity very close to GR with an high level of accuracy

General Relativity over time

Lorentz Symmetry Violations

- We restrict to the gravitational interaction only. In this regard, there are some aspects to take into consideration:
  1. Possible evidence of new physics "beyond" Einstein, such as apparent, or "effective" violations of Lorentz invariance might result from certain models of quantum gravity

$$\mathcal{L}_{\mathcal{P}} = \sqrt{\frac{hG}{2\pi c^3}} \cong 1.6 \times 10^{-33}\ cm$$

General Relativity over time

Lorentz Symmetry Violations

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    1. Possible evidence of new physics “beyond” Einstein, such as apparent, or “effective” violations of Lorentz invariance might result from certain models of quantum gravity
    2. Metric theories of gravity (different from GR) may be responsible of violations of the LLI
        - These are theories that are not compatible with SEP
        - Due to the boundary values of the auxiliary fields that can act back on local gravitational dynamics

General Relativity over time

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In these cases (especially if vectors or tensors fields are present), we could have both violations of:

- **LPI (i.e. preferred location effects: G = G(r) and/or G = G(t)), see Brans-Dicke theory and its generalizations**
- **LLI (i.e. preferred frame effects)**

a scalar field is invariant under these transformations, so Brans-Dicke satisfies LLI

General Relativity over time

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     - These are theories that are not compatible with SEP
     - Due to the boundary values of the auxiliary fields that can act back on local gravitational dynamics
  3. Non-Metric theories of gravity may be responsible of violations of the LLI
     - These are theories that are not compatible with SEP
     - These theories are characterized by a coupling of the additional dynamical fields with matter

General Relativity over time

Lorentz Symmetry Violations

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     - Due to the boundary values of the auxiliary fields that can act back on local gravitational dynamics
  3. Non-Metric theories of gravity may be responsible of violations of the LLI
     - These are theories that are not compatible with SEP
     - These theories are characterized by a coupling of the additional dynamical fields with matter
  4. Superstring theory
     - The additional fields, such as dilatons and moduli, can couple directly to stress-energy in a way that can result in violations (see Damour et al., 2002: PRL 89, PRD 66)

General Relativity over time

5th Force

- In the mid-1980s the following work aroused much interest in the scientific community:
- Fischbach, E., D. Sudarsky, A. Szafer, C. Talmadge and S.H. Aronson. **Reanalysis of the Eötvös Experiment**. *Physical Review Letters* 56: 3-6, 1986

General Relativity over time

PHYSICAL REVIEW
LETTERS

VOLUME 56 | 6 JANUARY 1986 | NUMBER 1

Reanalysis of the Eötvös Experiment

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Daniel Sudarsky, Aaron Szafer, and Carrick Talmadge
Physics Department, Purdue University, West Lafayette, Indiana 47907

and

S. H. Aronson
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 7 November 1985)

We have carefully reexamined the results of the experiment of Eötvös, Pekár, and Fekete, which compared the accelerations of various materials to the Earth. We find that the Eötvös-Pekár-Fekete data are sensitive to the composition of the materials used, and that their results support the existence of an intermediate-range coupling to baryon number or hypercharge.

PACS numbers: 04.90.+e

General Relativity over time

5th Force

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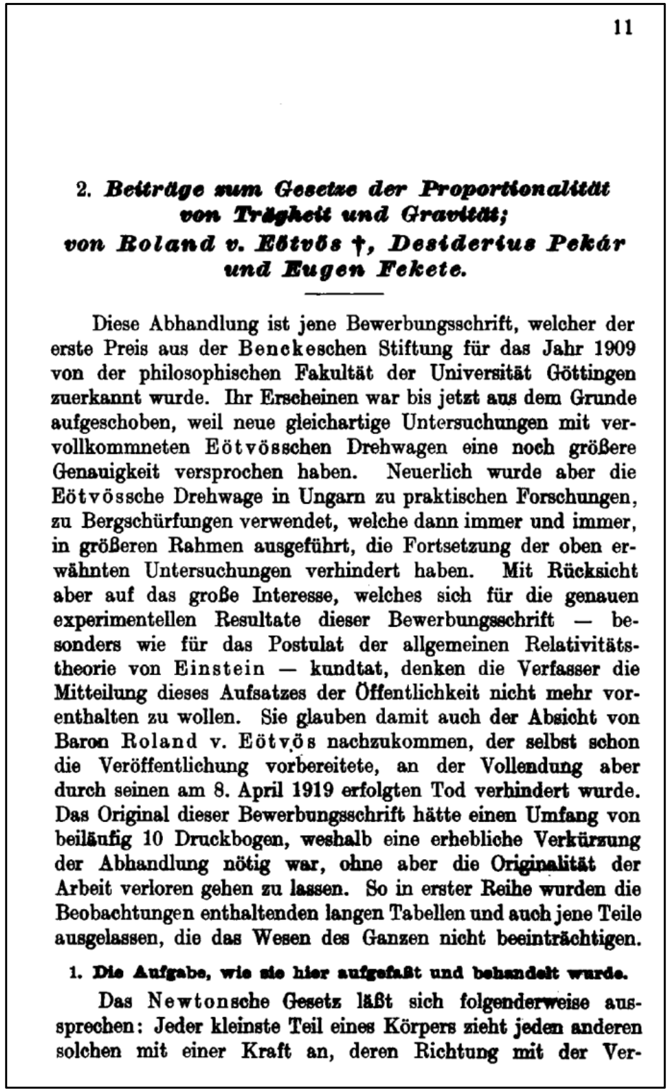
- Fischbach, E., D. Sudarsky, A. Szafer, C. Talmadge and S.H. Aronson. **Reanalysis of the Eötvös Experiment**. *Physical Review Letters* 56: 3-6, 1986
- Eötvös, R.V., D. Pekár and E. Fekete. Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität. *Annalen der Physik* (Leipzig) 68: 11-66, 1922
- **WEP test:** whether the behaviour of objects in a gravitational field was the same regardless of their different chemical composition:

$$\eta = \frac{\Delta a}{a} \cong 10^{-9}$$

General Relativity over time

5th Force

- Front page from: Eötvös, Pekár, Fekete (EPF): *Annalen der Physik* (Leipzig) 68: 11-66, 1922



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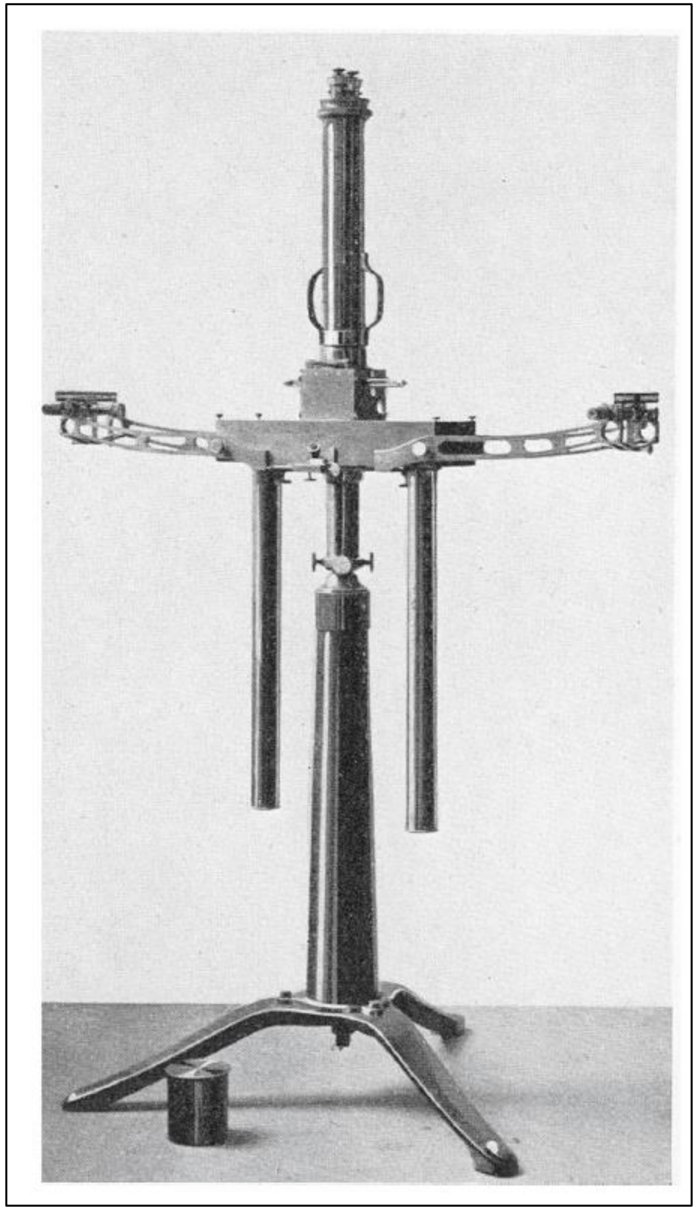
2. ***Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität;***
von Roland v. Eötvös †, Desiderius Pekár und Eugen Fekete.

Diese Abhandlung ist jene Bewerbungsschrift, welcher der erste Preis aus der Beneckeschen Stiftung für das Jahr 1909 von der philosophischen Fakultät der Universität Göttingen zuerkannt wurde. Ihr Erscheinen war bis jetzt aus dem Grunde aufgeschoben, weil neue gleichartige Untersuchungen mit vervollkommneten Eötvösschen Drehwagen eine noch größere Genauigkeit versprochen haben. Neuerlich wurde aber die Eötvössche Drehwage in Ungarn zu praktischen Forschungen, zu Bergschürfungen verwendet, welche dann immer und immer, in größeren Rahmen ausgeführt, die Fortsetzung der oben erwähnten Untersuchungen verhindert haben. Mit Rücksicht aber auf das große Interesse, welches sich für die genauen experimentellen Resultate dieser Bewerbungsschrift — besonders wie für das Postulat der allgemeinen Relativitätstheorie von Einstein — kundtat, denken die Verfasser die Mitteilung dieses Aufsatzes der Öffentlichkeit nicht mehr vorenthalten zu wollen. Sie glauben damit auch der Absicht von Baron Roland v. Eötvös nachzukommen, der selbst schon die Veröffentlichung vorbereitete, an der Vollendung aber durch seinen am 8. April 1919 erfolgten Tod verhindert wurde. Das Original dieser Bewerbungsschrift hätte einen Umfang von beiläufig 10 Druckbogen, weshalb eine erhebliche Verkürzung der Abhandlung nötig war, ohne aber die Originalität der Arbeit verloren gehen zu lassen. So in erster Reihe wurden die Beobachtungen enthaltenden langen Tabellen und auch jene Teile ausgelassen, die das Wesen des Ganzen nicht beeinträchtigen.

1. Die Aufgabe, wie sie hier aufgefaßt und behandelt wurde.

Das Newtonsche Gesetz läßt sich folgenderweise aussprechen: Jeder kleinste Teil eines Körpers zieht jeden anderen solchen mit einer Kraft an, deren Richtung mit der Ver-

Torsion Balance



General Relativity over time

5^{th} Force. Following Fischbach words:

- *However, the result of our reanalysis of the EPF paper was that the EPF data were in fact “. . . sensitive to the composition of the materials used”, in contrast to what EPF themselves had claimed.*
- *If the EPF data and our reanalysis of them were both correct, then one implication of our paper would be that EPF had discovered a new “fifth force” in nature*

Eur. Phys. J. H **40**, 385–467 (2015)
DOI: 10.1140/epjh/e2015-60044-5

THE EUROPEAN PHYSICAL JOURNAL H

Personal recollection

The fifth force: A personal history

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Abstract. On January 6, 1986, a paper written by our group appeared in *Physical Review Letters* entitled “Reanalysis of the Eötvös Experiment”. In that Letter we reanalyzed a well-known 1922 paper by Eötvös, Pekár, and Fekete (EPF) which compared the accelerations of samples of different composition to the Earth. Our surprising conclusion was that “Although the Eötvös experiment has been universally interpreted as having given null results, we find in fact that this is not the case”. Two days later a front page story appeared in the *New York Times* under the headline “Hints of 5th Force in Universe Challenge Galileo’s Findings”, and so was born the concept of a “fifth force”. In this personal history I review the pre-history which motivated our paper, and discuss details of our reanalysis of the EPF paper that have not been presented previously. Our work led to illuminating correspondence with Robert Dicke and Richard Feynman which are presented here for the first time. I also discuss an interesting meeting with T.D. Lee, one of whose papers with C.N. Yang provided part of the theoretical motivation for our work. Although there is almost no support from the many experiments motivated by the EPF data for a fifth force with properties similar to those that we hypothesized in our original paper, interest in the EPF experiment continues for reasons I outline in the Epilogue.

- This generally refers to a gravity-like long-range force (its effects extend over macroscopic distances) co-existing with gravity, presumably arising from the exchange of any of the ultra-light quanta whose existence is predicted by various unification theories such as supersymmetry
- Depending on the specific characteristics of this hypothesized force, it could manifest itself in various experiments as an apparent deviation from the predictions of Newtonian gravity

General Relativity over time

5th Force: Experimental and Theoretical support

- Experimental:
  - R. Colella, A. W. Overhauser, and S. A. Werner, Observation of Gravitationally Induced Quantum Interference. *Phys. Rev. Lett*. 34, 1472, 1975

- Theoretical:
  - Fujii, Y., Dilaton and possible non-Newtonian gravity. Nature (Physical Science), 234: 5-7, 1971
  - Fujii, Y., Scale invariance and gravity of hadrons. Annals of Physics (New York) 69: 494-521, 1972
  - Fujii, Y., Scalar-tensor theory of gravitation and spontaneous breakdown of scale invariance. Physical Review D 9: 874-876, 1974
  - Fujii, Y., Spontaneously broken scale invariance and gravitation. General Relativity and Gravitation 6: 29-34, 1975
  - Fujii, Y., Composition independence of the possible finite-range gravitational force. General Relativity and Gravitation 13:1147-1155, 1981

General Relativity over time

5th Force: Experimental and Theoretical support

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  - R. Colella, A. W. Overhauser, and S. A. Werner, Observation of Gravitationally Induced Quantum Interference. *Phys. Rev. Lett*. 34, 1472, 1975

VOLUME 34, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1975

Observation of Gravitationally Induced Quantum Interference*

R. Colella and A. W. Overhauser
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and

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Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121
(Received 14 April 1975)

We have used a neutron interferometer to observe the quantum-mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field.

General Relativity over time

5th Force: Experimental and Theoretical support

- Theoretical:

NATURE PHYSICAL SCIENCE VOL. 234 NOVEMBER 1 1971

Dilaton and Possible Non-Newtonian Gravity

YASUNORI FUJII

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A model is proposed which allows a dilaton to show up in a possible non-Newtonian part of the gravitational force. By examining the available observational facts it can be shown that the force-range of the additional force, if it exists, will be either between 10 m and 1 km or smaller than ~1 cm.

A DILATON—a Nambu-Goldstone boson of dilatation invariance[1-6]—will, if it exists, couple to the graviton, because the dilaton dominates the energy-momentum tensor which is supposed to be a source of the graviton. The fact that the dilaton is a scalar particle does not prevent it from coupling to the graviton, which is described by a symmetric tensor field, but is not a genuine spin-2 particle because of its masslessness. As a consequence the dilaton may affect the gravitational force between two masses.

If the dilaton mass is of the order of hadronic masses, any modifications will occur only within the distances of the order of fm. The dilaton mass could be, on the other hand, of the order of $\kappa \sim [G\alpha'^{-2}]^{1/2}$ which is a typical combination of two fundamental constants in the gravitational and strong interactions. (G is the Newtonian gravity constant, α' is the universal slope of Regge trajectories. I use the unit system with $c=\hbar=1$.) Possible non-Newtonian behaviour will then occur

We have an order of magnitude estimate of the constant F_θ[7,9]

$$F_\theta \sim \alpha'^{-1} \tag{1}$$

The θ-graviton mixing problem is then resolved to give a gravity potential

$$V(r) = -\frac{3}{4} G \frac{1}{r} \left[1 + \frac{1}{3} \left(\cos \kappa r - \frac{1 - t_\theta/2\kappa^2}{\sqrt{-D}} \sin \kappa r\right) e^{-\kappa\sqrt{-D}\,r}\right] \tag{2}$$

where $\kappa^2 = (3/8)GF_\theta^2$, and $-D = t_\theta/\kappa^2 - 1$ with the restriction $t_\theta > \kappa^2$. From (1), with $G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2} from the Cavendish experiment, we obtain $\kappa \sim 10^{-20}\, m_{N'}$ or $\kappa^{-1} \sim 10^5$ cm $= 1$ km.

If the "bare" dilaton mass squared (t_θ) vanishes, that is, dilatation invariance is strict, there is no change in the gravitational interaction. If t_θ is of the order of a hadronic mass squared, then $\kappa\sqrt{-D} \sim \sqrt{t_\theta}$ in the exponent in (2), because $t_\theta \gg \kappa^2$. The finite-range part vanishes for any macroscopic distance. On the other hand, t_θ may be of the same order of, but still larger than, κ^2. We obtain $\kappa\sqrt{-D} \sim \kappa$, because $-D \sim 1$. The force-range is of the order of $\kappa^{-1} \sim$ km. We have then an entirely new situation.

Consider the Cavendish experiment with the distance $r \sim 10$ cm. The potential (2) becomes

$$V(r) \sim -G\frac{1}{r}$$

General Relativity over time

5th Force: Experimental and Theoretical support

- Theoretical:

PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

Scalar-tensor theory of gravitation and spontaneous breakdown of scale invariance

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(Received 13 March 1973)

A version of Brans-Dicke theory of massive scalar and massless tensor fields is given. A connection to a spontaneously broken scale invariance is shown.

Recently O'Hanlon,[1] Acharya and Hogan[2] have shown that a generally covariant theory of gravitation can accommodate a *massive* scalar field in addition to the massless tensor field.[3] This seems to shed a new light on the scalar-tensor theory of gravitation[4] by showing a close connection with another intriguing hypothesis in the theory of quantized fields: the spontaneous breaking of scale invariance.[5]

while the latter manifests itself in the non-Newtonian part of the static gravitational potential, as suggested previously in a different approach.[9] The present model provides us with a better understanding of why the force range of this unusual part is expected to be most likely a macroscopic distance roughly of the order of $(Gm_N^4)^{-1/2} \sim 10^5$ cm, where G is the Newtonian gravitational constant while m_N is the nucleon mass ($c = \hbar = 1$ throughout).

General Relativity over time

5th Force: Experimental and Theoretical support

- Theoretical:

VOLUME 89, NUMBER 8 PHYSICAL REVIEW LETTERS 19 AUGUST 2002

Runaway Dilaton and Equivalence Principle Violations

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and Laboratoire de Physique Théorique, Université Paris-Sud, 91405 Orsay, France
(Received 30 April 2002; published 5 August 2002)

In a recently proposed scenario, where the dilaton decouples while cosmologically attracted towards infinite bare string coupling, its residual interactions can be related to the amplitude of density fluctuations generated during inflation, and are large enough to be detectable through a modest improvement on present tests of free-fall universality. Provided it has significant couplings to either dark matter or dark energy, a runaway dilaton can also induce time variations of the natural "constants" within the reach of near-future experiments.

DOI: 10.1103/PhysRevLett.89.081601 PACS numbers: 11.25.Mj, 04.80.Cc, 98.80.Cq

PHYSICAL REVIEW D **66**, 046007 (2002)

Violations of the equivalence principle in a dilaton-runaway scenario

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[2]*Dipartimento di Fisica, Università di Milano Bicocca, Piazza delle Scienze 3, I-20126 Milan, Italy*
[3]*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[4]*Laboratoire de Physique Théorique, Université Paris Sud, 91405 Orsay, France*
(Received 13 May 2002; published 28 August 2002)

We explore a version of the cosmological dilaton-fixing and decoupling mechanism in which the dilaton dependence of the low-energy effective action is extremized for infinitely large values of the bare string coupling $g_s^2 = e^{\phi}$. We study the efficiency with which the dilaton ϕ runs away toward its "fixed point" at infinity during a primordial inflationary stage, and thereby approximately decouples from matter. The residual dilaton couplings are found to be related to the amplitude of the density fluctuations generated during inflation. For the simplest inflationary potential $V(\chi) = \frac{1}{2} m_\chi^2(\phi)\,\chi^2$, the residual dilaton couplings are shown to predict violations of the universality of gravitational acceleration near the $\Delta a/a \sim 10^{-12}$ level. This suggests that a modest improvement in the precision of equivalence principle tests might be able to detect the effect of such a runaway dilaton. Under some assumptions about the coupling of the dilaton to dark matter and/or dark energy, the expected time variation of natural "constants" (in particular of the fine-structure constant) might also be large enough to be within reach of improved experimental or observational data.

DOI: 10.1103/PhysRevD.66.046007 PACS number(s): 11.25.−w, 04.80.Cc, 98.80.Cq

General Relativity over time

5^{th} Force: How it would manifest?

- Yukawa-like potential parameterized by the strength α and a characteristic range λ:

$$V(r) = -G_\infty \frac{M_1 M_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

- It corresponds to a violation of the $1/r^2$ law for the gravitational interaction:

$$\vec{F}(r) = -\vec{\nabla} V(r) = -G_\infty \left[1 + \alpha\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda}\right] \frac{M_1 M_2}{r^2}\hat{r}$$

- It may or may not envisage a violation of the EEP depending on the nature of the strength α

General Relativity over time

5^{th} Force: How it would manifest?

1. the deviations from the usual 1/r law for the gravitational potential lead to new weak interactions between macroscopic objects
2. The interesting point is that these supplementary interactions may be either consistent with Einstein Equivalence Principle or not
3. In this second case, non–metric phenomena will be produced with tiny, but significant, consequences in the gravitational experiments
4. The characteristic of such very weak interactions, which are predicted by several theories, is to produce deviations for masses separations ranging through several orders of magnitude, starting from the sub–millimeter level up to the astronomical scale

General Relativity over time

Summarizing

- A Yukawa-like parameterization seems general
  - at the lowest order interaction and in the non-relativistic limit, independently of a:
    - **Scalar** field with the exchange of a spin-0 light boson
    - **Vector** field with the exchange of a spin-1 light boson
    - **Tensor** field with the exchange of a spin-2 light boson

$$V_{yuk} = -\alpha \frac{G_\infty M_1}{r} e^{-r/\lambda}$$

$$\alpha = \frac{1}{G_\infty}\left(\frac{K_1}{M_1} \cdot \frac{K_2}{M_2}\right)$$

$$\lambda = \frac{h}{\mu c}$$

M_1 = Mass of the primary source;

M_2 = Mass of the secondary source;

G_∞ = Newtonian gravitational constant;

r = Distance;

α = Strength of the interaction; K_1,K_2 = Coupling strengths;

λ = Range of the interaction; μ = Mass of the light-boson;

h = Planck constant; c = Speed of light

General Relativity over time

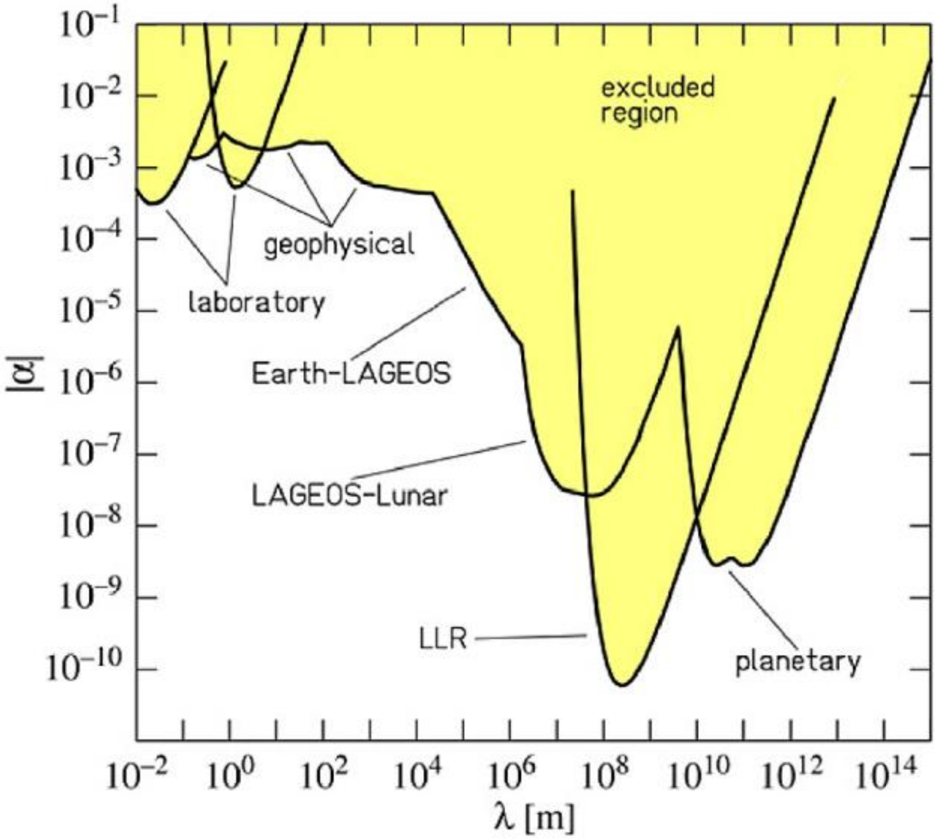

Fig. 11. Limits on the fifth force strength $|\alpha|$ for $\lambda \geq 1$ cm from laboratory, geo[...] and astronomical measurements [Adelberger 2009].

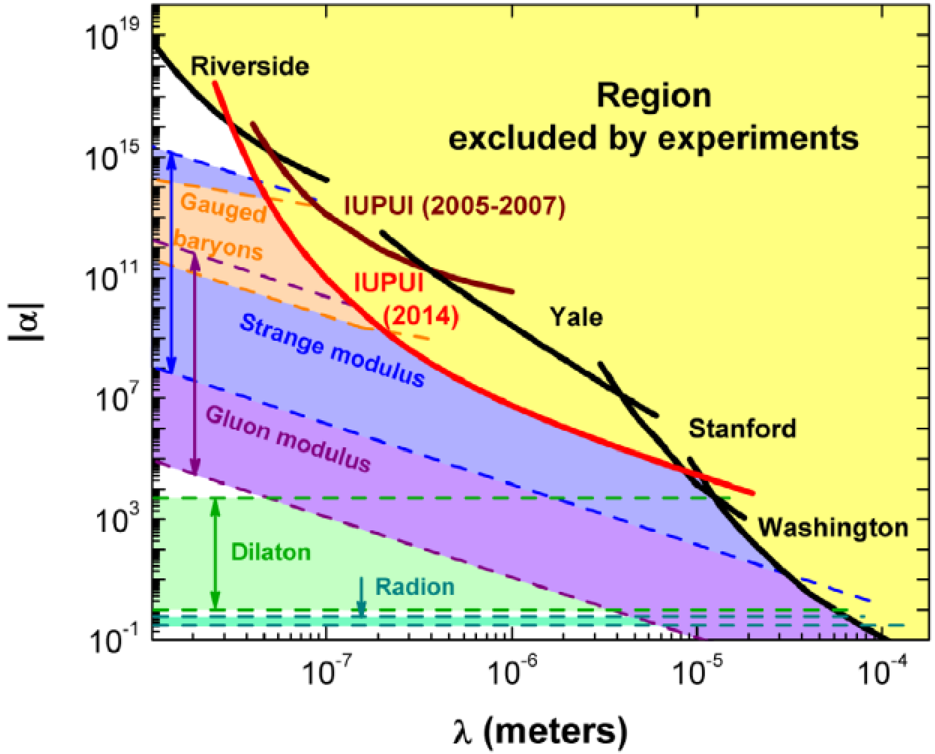

Fig. 12. Limits on the fifth force strength $|\alpha|$ for $\lambda \leq 0.1$ mm from short-distance force experiments along with predicted strengths from various theories [Chen 2014]. "IUPUI" labels constraints coming from experiments with Ricardo Decca and Daniel López utilizing "iso-electronic" effect experiments.

scale distances between 10^{-4} m — 10^{15} m have been tested during the last 35 years with null results for a possible violation of NISL and for the WEP

What to measure in the weak field and slow motion limit of GR

What to measure and to test with laser-ranged satellites:

1. **The validity of the Equivalence Principle**
2. **The validity of the geometric structure and of the equation of motion of geodesics**
3. **The validity of Einstein's field equations**

What to measure in the weak field and slow motion limit of GR

What to measure and to test with laser-ranged satellites:

1. **The validity of the Equivalence Principle**

   1.1 Direct test

   - WEP from UFF

   1.2 Indirect test

   - EEP, WEP and SEP from the effects on the orbit

What to measure in the weak field and slow motion limit of GR

What to measure and to test with laser-ranged satellites:

1. **The validity of the Equivalence Principle**

   1.1 Direct test

   - WEP from UFF

- A.M. Nobili, G.L. Comandi, D. Bramanti, Suresh Doravari, D.M. Lucchesi, F. Maccarrone. Limitations to testing the equivalence principle with satellite laser ranging. *Gen. Relativity and Grav.*, DOI 10.1007/s10714-007-0560-x, 2007
- I. Ciufolini, R. Matzner, A. Paolozzi, E.C. Pavlis, G. Sindoni, J. Ries, V. Gurzadyan, R. Koenig. Satellite Laser-Ranging as a Probe of Fundamental Physics. *Scientific Reports Nature*, doi.org/10.1038/s41598-019-52183-9, 2019

$$\eta = \frac{\Delta a}{a} \cong \frac{\Delta(GM_{\oplus})}{GM_{\oplus}} \cong 2 \cdot 10^{-9}$$

What to measure in the weak field and slow motion limit of GR

What to measure and to test with laser-ranged satellites:

2. **The validity of the geometric structure and of the equation of motion of geodesics**

   2.1 Space curvature

   - De Sitter precession and Lense-Thirring precession

   2.2 Space curvature + non-linearity of the gravitational interaction

   - Schwarzschild precession (argument of pericenter)

$$\vec{\Omega}_{dS} \cong -\frac{3}{2}\vec{V}_E \wedge \left(\frac{GM_S}{c^2 R_{ES}^3}\right)\vec{X}_{ES}$$

$$\Delta\dot{\Omega}_{dS} = \left|\vec{\Omega}_{dS}\right| \cos\varepsilon$$

$$\frac{3}{2} = \left(\frac{1}{2} + \gamma\right)$$

$$\langle\dot{\Omega}_{LT}\rangle_{sec} = \frac{2G}{c^2 a^3}\frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\langle\dot{\omega}_{LT}\rangle_{sec} = -\frac{6G}{c^2 a^3}\frac{J_\oplus}{(1-e^2)^{3/2}}\cos i$$

$$\frac{1+\gamma}{2} \qquad \mu$$

$$\langle\dot{\omega}_{Schw}\rangle_{sec} = \frac{3}{c^2 a^{5/2}}\frac{GM_\oplus{}^{3/2}}{(1-e^2)}$$

$$\frac{2+2\gamma-\beta}{3}$$

What to measure in the weak field and slow motion limit of GR

What to measure and to test with laser-ranged satellites:

3. The validity of Einstein’s field equations

3.1 Indirect test

- Schwarzschild precession (argument of pericenter)
- de Sitter precession
- Lense-Thirring precession

Results of the LARASE experiment

IOP Publishing — Classical and Quantum Gravity

Class. Quantum Grav. **32** (2015) 155012 (50pp) — doi:10.1088/0264-9381/32/15/155012

Testing the gravitational interaction in the field of the Earth via satellite laser ranging and the Laser Ranged Satellites Experiment (LARASE)

D M Lucchesi[1,2,3], L Anselmo[2], M Bassan[3,4], C Pardini[2], R Peron[1,3], G Pucacco[3,4] and M Visco[1,3]

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Results of the LARASE experiment

- During the **LARASE** experiment in the period 2013–2019 various activities were developed in order to reach final measurements in the field of fundamental physics that were
  - not only precise,
  - but also accurate and robust in the evaluation of the systematic sources of error

Results of the LARASE experiment

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  - not only precise,
  - but also accurate and robust in the evaluation of the systematic sources of error
- Prerequisite for the final gravitation measurements, is a precise orbit determination (**POD**) of the satellites involved in our analyses

Results of the LARASE experiment

- During the LARASE experiment in the period 2013–2019 various activities were developed in order to reach final measurements in the field of fundamental physics that were
  - not only precise,
  - but also accurate and robust in the evaluation of the systematic sources of error
- Prerequisite for the final gravitation measurements, is a precise orbit determination (**POD**)of the satellites involved in our analyses
- This is achieved by minimizing a cost function ***Q*** consisting of the square of the residuals of the observed distance of the satellite from an on-ground tracking station with the corresponding distance obtained from a dynamic model of the satellite's orbit

Results of the LARASE experiment

Table 2. Models currently used, within the LARASE research program, for the analysis of the orbit of the two LAGEOS and LARES satellites. The models are grouped in gravitational perturbations, non-gravitational perturbations and reference frames realizations.

Model For	Model Type	Reference
Geopotential (static)	EIGEN-GRACE02S/GGM05S	[84,90,91]
Geopotential (time-varying, tides)	Ray GOT99.2	[92]
Geopotential (time-varying, non tidal)	IERS Conventions 2010	[89]
Third–body	JPL DE-403	[93]
Relativistic corrections	Parameterized post-Newtonian	[88,94]
Direct solar radiation pressure	Cannonball	[46]
Earth albedo	Knocke-Rubincam	[63]
Earth-Yarkovsky	Rubincam	[56,64,65]
Neutral drag	JR-71/MSIS-86	[50,51]
Spin	LASSOS	[42]
Stations position	ITRF2008	[95]
Ocean loading	Schernek and GOT99.2 tides	[46,92]
Earth Rotation Parameters	IERS EOP C04	[96]
Nutation	IAU 2000	[97]
Precession	IAU 2000	[98]

Results of the LARASE experiment

$R_i = O_i - C_i = O_i - C(\vec{x}(t_i), t_i, \vec{\beta}) = \sum_j \frac{\partial C_i}{\partial P_j}\delta P_j + \delta O_i$

Orbits:

$\frac{d}{dt}\vec{x} = f(\vec{x}, t, \vec{\alpha})$ Differential equation

$\begin{cases} \vec{x} \in \mathbb{R}^{\ell} \\ \vec{\alpha} \in \mathbb{R}^{m} \end{cases}$

State vector (position and velocity, …)

Models dynamic parameters (C_{20}, Cr, …)

$\vec{x}(t_0 = \vec{x}_0 \in \mathbb{R}^{\ell})$ Initial condition at a given epoch: $\ell = 6 + \dots$

$\vec{x} = \vec{x}(t, \vec{x}_0, \vec{\alpha})$ General solution for the orbits (*integral flow*)

Observations:

$C = C(\vec{x}, t, \vec{\beta})$ Observation function, $\vec{\beta} \in \mathbb{R}^{n}$ kinematic parameters

$$R_i = O_i - C_i = O_i - C(\vec{x}(t_i), t_i, \vec{\beta}) = \sum_j \frac{\partial C_i}{\partial P_j}\delta P_j + \delta O_i$$

$$Q(\vec{R}) = \frac{1}{q}\vec{R}^T\vec{R} = \frac{1}{q}\sum_{i=1}^{q} R_i^2$$

Results of the LARASE experiment

Orbits:

$$\frac{d}{dt}\vec{x} = f(\vec{x}, t, \vec{\alpha})$$ Differential equation

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State vector (position and velocity, …)

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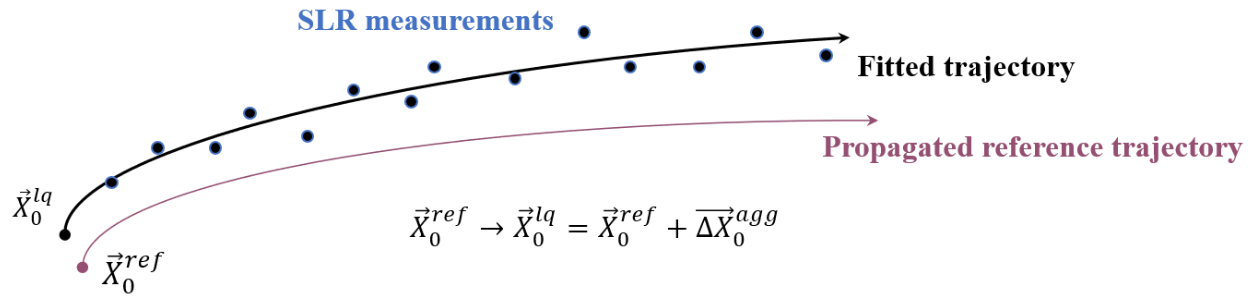

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$$R_i = O_i - C_i = O_i - C(\vec{x}(t_i), t_i, \vec{\beta}) = \sum_j \frac{\partial C_i}{\partial P_j} \delta P_j + \delta O_i$$

$$Q(\vec{R}) = \frac{1}{q} \vec{R}^T \vec{R} = \frac{1}{q} \sum_{i=1}^{q} R_i^2$$

Results of the LARASE experiment

- Therefore, in order to achieve precise and accurate measurements for the gravitational interaction in the **WFSM** limit of **GR**, we developed more refined and reliable models to account for the main
  - **non-gravitational and**
  - **gravitational perturbations**
- These are part of our results in modeling efforts, and will be discussed in the presentation of this afternoon and of tomorrow morning (13^{th} of November)
- Let us focus on the main results we have achieved in the measurements regarding the gravitational interaction

Results of the LARASE experiment

The main results of **LARASE** are:

1. The measurement of the **GR total precession** of **LAGEOS II** argument of pericenter
2. The measurement of the **GR Lense-Thirring precession** of the combined right ascensions of the ascending node (**RAAN**) of the satellites **LAGEOS**, **LAGEOS II** and **LARES**

These precessions are related respectively with the Earth's:

1. **Gravitoelectric field:**
   - produced by masses, and analogous to the electric field produced by charges
2. **Gravitomagnetic field**
   - Produced by mass-currents, analogous to the magnetic field produced by electric currents

Results of the LARASE experiment

Linearised theory in the WFSM limit of GR

$$G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{cases} \bar{h}^{\alpha\beta}{}_{,\beta} = 0 \\ g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \\ \Delta \bar{h}_{\alpha\beta} = 16\pi \frac{G}{c^4} T_{\alpha\beta} \end{cases}$$

Gauge conditions

Metric tensor

Field equations

$h_{\alpha\beta}$ represents the correction due to the curvature of spacetime

$$\begin{cases} \bar{h}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h \\ h \equiv h^{\alpha}_{\alpha} = \eta^{\alpha\beta} h_{\alpha\beta} \end{cases}$$

$$\Delta \bar{h}_{\alpha\beta} = 16\pi \frac{G}{c^4} T_{\alpha\beta}$$

Weak fields means that $|h_{\alpha\beta}| \ll 1$; in the solar system $\Rightarrow$

$$|h_{\alpha\beta}| \cong \left|\frac{\Phi}{c^2}\right| \leq 10^{-6}$$

Φ represents the Newtonian or Gravitoelettric potential

$$\Phi = -\frac{GM_{\odot}}{R_{\odot}}$$

$$\begin{cases} \bar{h}^{00} = 4\frac{\Phi}{c^2} \\ \bar{h}^{0l} = -2\frac{A^l}{c^2} \\ \bar{h}^{ij} = O(c^{-4}) \end{cases}$$

Represents the solution far from the source: (M,J)

Gravitomagnetic potential:

$$A^l = \frac{G}{c}\frac{J^n x^k}{r^3}\varepsilon^l_{nk}$$

J represenst the total angular momentum (spin) of the source

Results of the LARASE experiment

$$G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta}$$

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$$A^l = \frac{G}{c}\frac{J^n x^k}{r^3}\varepsilon^l_{nk}$$

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Gravitoelectromagnetic fields

$$\begin{aligned} \vec{\nabla}\cdot\vec{E}_G &= -4\pi G\rho \\ \vec{\nabla}\cdot\vec{B}_G &= 0 \\ \vec{\nabla}\wedge\vec{E}_G &= -\frac{1}{2c}\frac{\partial}{\partial t}\vec{B}_G \\ \vec{\nabla}\wedge\vec{B}_G &= \frac{2}{c}\frac{\partial}{\partial t}\vec{E}_G - \frac{8\pi G}{c}\vec{j} \\ \vec{E}_G &= -\vec{\nabla}\Phi - \frac{1}{2c}\frac{\partial}{\partial t}\vec{A} \\ \vec{B}_G &= \vec{\nabla}\wedge\vec{A} \\ \frac{1}{c}\frac{\partial}{\partial t}\Phi + \frac{1}{2}\vec{\nabla}\cdot\vec{A} &= 0 \end{aligned}$$

ρ = mass-charge density

j = mass-current density

- H. Thirring, Über die formale Analogie zwischen den elektromagnetischen Grundgleichungen und den Einsteinschen Gravitationsgleichungen erster Näherung, *Phys. Z.* 19, 204, 1918
- I. Ciufolini and. J.A. Wheeler, Gravitation and Inertia. Princeton Univ. Press, 1995

Results of the LARASE experiment

Measurement of **LAGEOS II** argument of pericenter **GR** precession

Results of the LARASE experiment

PHYSICAL REVIEW D **89**, 082002 (2014)

LAGEOS II pericenter general relativistic precession (1993–2005): Error budget and constraints in gravitational physics

David M. Lucchesi*

Istituto di Astrofisica e Planetologia Spaziali, Istituto Nazionale di Astrofisica, (IAPS/INAF), Via del Fosso del Cavaliere 100, 00133 Roma, Italy, Istituto di Scienza e Tecnologie dell'Informazione, Consiglio Nazionale delle Ricerche, (ISTI/CNR), Via G. Moruzzi 1, 56124 Pisa, Italy, and Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

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(Received 16 April 2013; published 7 April 2014)

The aim of this paper is to extend, clarify, and deepen the results of our previous work [D. M. Lucchesi and R. Peron, Phys. Rev. Lett. **105**, 231103 (2010)], related to the precise measurement of LAGEOS (LAser GEOdynamics Satellite) II pericenter shift. A 13-year time span of LAGEOS satellites' laser tracking data has been considered, obtaining a very precise orbit and correspondingly residuals time series from which to extract the relevant signals. A thorough description is provided of the data analysis strategy and the dynamical models employed, along with a detailed discussion of the known sources of error in the experiment, both statistical and systematic. From this analysis, a confirmation of the predictions of Einstein's general relativity, as well as strong bounds on alternative theories of gravitation, clearly emerge. In particular, taking conservatively into account the stricter error bound due to systematic effects, general relativity has been confirmed in the Earth's field at the 98% level (meaning the measurement of a suitable combination of β and γ PPN parameters in weak-field conditions). This bound has been used to constrain possible deviations from the inverse-square law parameterized by a Yukawa-like new long range interaction with strength $|\alpha| \lesssim 1 \times 10^{-10}$ at a characteristic range $\lambda \simeq 1$ Earth radius, a possible nonsymmetric gravitation theory with the interaction parameter $\mathcal{C}_{\oplus \mathrm{LAGEOS\ II}} \lesssim (9 \times 10^{-2}\ \mathrm{km})^4$, and a possible spacetime torsion with a characteristic parameter combination $|2t_2 + t_3| \lesssim 7 \times 10^{-2}$. Conversely, if we consider the results obtained from our best fit of the LAGEOS II orbit, the constraints in fundamental physics improve by at least 2 orders of magnitude.

DOI: 10.1103/PhysRevD.89.082002 PACS numbers: 04.80.Cc, 91.10.Sp, 95.10.Eg, 95.40.+s

1. Measurement of **LAGEOS II** argument of pericenter **GR** precession

This represents the extension and completion of a previous work published on *Phys. Rev. Lett.* in 2010

Results of the LARASE experiment

Selected for a Viewpoint in *Physics*
PRL **105,** 231103 (2010) PHYSICAL REVIEW LETTERS week ending 3 DECEMBER 2010

Accurate Measurement in the Field of the Earth of the General-Relativistic Precession of the LAGEOS II Pericenter and New Constraints on Non-Newtonian Gravity

David M. Lucchesi[1,2] and Roberto Peron[1]
[1]*Istituto di Fisica dello Spazio Interplanetario, Istituto Nazionale di Astrofisica, IFSI/INAF, Via del Fosso del Cavaliere 100, 00133 Roma, Italy*
[2]*Istituto di Scienza e Tecnologie dell'Informazione, Consiglio Nazionale delle Ricerche, ISTI/CNR, Via G. Moruzzi 1, 56124 Pisa, Italy*
(Received 18 July 2010; published 29 November 2010)

The pericenter shift of a binary system represents a suitable observable to test for possible deviations from the Newtonian inverse-square law in favor of new weak interactions between macroscopic objects. We analyzed 13 years of tracking data of the LAGEOS satellites with GEODYN II software but with no models for general relativity. From the fit of LAGEOS II pericenter residuals we have been able to obtain a 99.8% agreement with the predictions of Einstein's theory. This result may be considered as a 99.8% measurement in the field of the Earth of the combination of the γ and β parameters of general relativity, and it may be used to constrain possible deviations from the inverse-square law in favor of new weak interactions parametrized by a Yukawa-like potential with strength α and range λ. We obtained $|\alpha| \lesssim 1 \times 10^{-11}$, a huge improvement at a range of about 1 Earth radius.

DOI: 10.1103/PhysRevLett.105.231103 PACS numbers: 04.80.Cc, 91.10.Sp, 95.10.Eg, 95.40.+s

Physics *Physics* **3**, 100 (2010)

Viewpoint

Via satellite

David Rubincam
Planetary Geodynamics Laboratory NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA
Published November 29, 2010

More than a decade's worth of data collected from the LAGEOS II satellite is offering a new way to test general relativity.

Subject Areas: **Gravitation**

A Viewpoint on:
Accurate Measurement in the Field of the Earth of the General-Relativistic Precession of the LAGEOS II Pericenter and New Constraints on Non-Newtonian Gravity
David M. Lucchesi and Roberto Peron
Phys. Rev. Lett. **105**, 231103 (2010) – Published November 29, 2010

Results of the LARASE experiment

The expected GR precession vs. classical precession:

$$\langle \dot{\omega}_{Schw} \rangle_{sec} = \frac{3}{c^2 a^{5/2}} \frac{GM_{\oplus}^{3/2}}{(1-e^2)}$$

$$\langle \dot{\omega}_{LT} \rangle_{sec} = -\frac{6G}{c^2 a^3} \frac{J_{\oplus}}{(1-e^2)^{3/2}} \cos i$$

$$U = -\frac{GM_{\oplus}}{r} \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(\frac{R_{\oplus}}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)\left(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right),$$

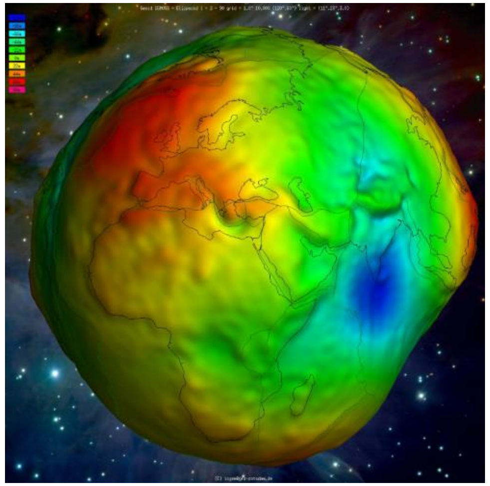
$$\langle \dot{\omega}_{class} \rangle_{sec} = \frac{3}{2} n \left(\frac{R_{\oplus}}{a}\right)^2 \frac{1}{(1-e^2)^2}\left\{\cos i + \left(1 - \frac{3}{2}\sin^2 i\right)\right\}\left[-\sqrt{5}\bar{C}_{20}\right] + \cdots$$

PHYSICAL REVIEW D **89,** 082002 (2014)

TABLE I. Rate (mas/yr) and orbital shift (over 14 days) of the different types of secular relativistic precession on the arguments of pericenter of LAGEOS II and LAGEOS, and their sum (1 mas/yr = 1 milli-arc second per year).

	Precession	Rate (mas/yr)	Shift (m)
LAGEOS II	$\Delta\dot{\omega}^{\rm Schw}$	3351.95	7.61
	$\Delta\dot{\omega}^{\rm LT}$	−57.00	-1.29×10^{-1}
	Total	3294.95	7.48
LAGEOS	$\Delta\dot{\omega}^{\rm Schw}$	3278.77	7.44
	$\Delta\dot{\omega}^{\rm LT}$	32.00	0.72×10^{-1}
	Total	3310.77	7.51

$$\langle \dot{\omega}_{class} \rangle_{sec} = \begin{cases} -2.8 \times 10^8 \ mas/yr & LAGEOS \\ 5.7 \times 10^8 \ mas/yr & LAGEOS\ II \end{cases}$$



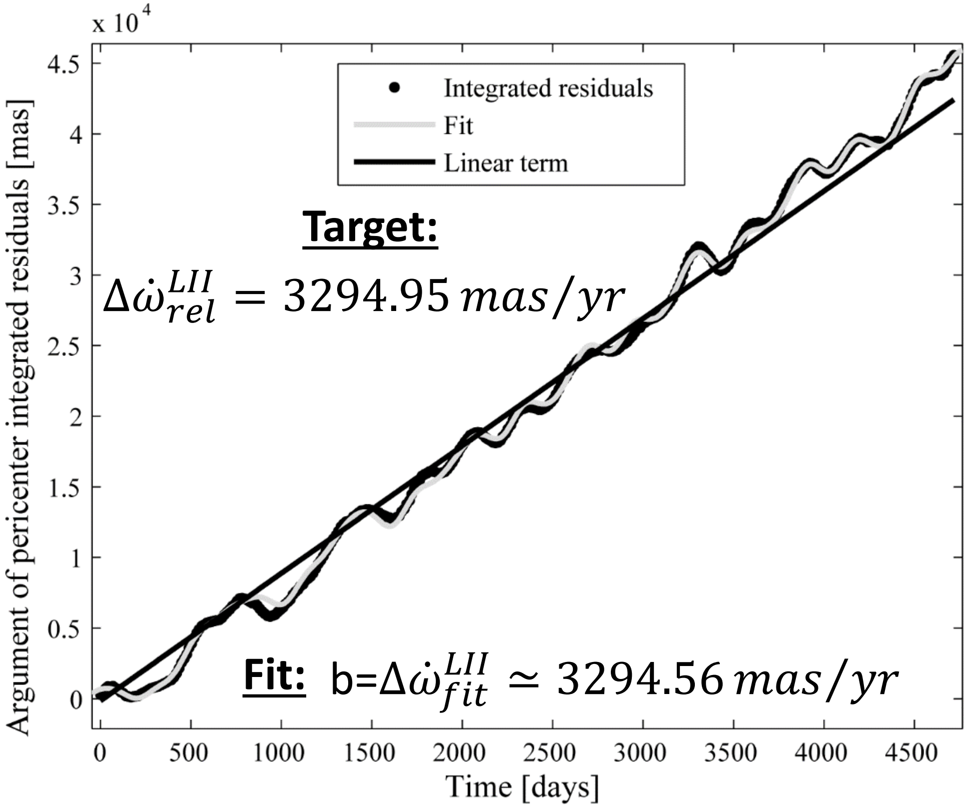
$\dot{\omega}_{GR} \cong 3300 \ mas/yr$ **The GR precession is about 5 orders of magnitude smaller!**

Results of the LARASE experiment

Post data reduction analysis: 13-yr analysis of the **LAGEOS II** orbit (FIT)

Fit to the pericenter residuals:

$$\Delta\omega^{FIT} = a + b\cdot t + c\left(t - t_0\right)^2 + \sum_{i=1}^{n} D_i \sin\left(\frac{2\cdot\pi}{P_i}\cdot t + \Phi_i\right)$$



We obtained b ≅ 3294.6 mas/yr, very close to the prediction of **GR**

The discrepancy is just **0.01%**

From a sensitivity analysis, with constraints on some of the parameters that enter into the least squares fit, we obtained an upper bound of **0.2%**

$$\Delta\dot{\omega} = \Delta\dot{\omega}_{GP} + \Delta\dot{\omega}_{NGP} + \varepsilon\cdot\Delta\dot{\omega}_{GR}$$

$$\varepsilon = 1 - (0.12 \pm 2.10)\cdot 10^{-3} \pm 2.5\cdot 10^{-2}$$

Results of the LARASE experiment

The overall error budget

DAVID M. LUCCHESI AND ROBERTO PERON — PHYSICAL REVIEW D **89,** 082002 (2014)

TABLE XVII. Error budget of the LAGEOS II pericenter general relativity shift. Top: summary of the errors from the data reduction and the *a posteriori* best fit (see Sections VI and VII). Middle: summary of the systematic errors from the gravitational perturbations (see Section VIII). Bottom: summary of the systematic errors from the nongravitational perturbations (see Section IX).

Statistical errors		
Residuals	Mean	Standard deviation
Range	9.67 cm	3.88 cm
Pericenter	4.57 mas	1.87 mas
Adjusted $\mathcal{R}_a^2$	0.998	
Reduced χ_ν^2 test	0.14	
	$\epsilon_\omega^{\rm sta} - 1 = (-0.12 \pm 2.10) \times 10^{-3}$	
Systematic errors: gravitational perturbations		
Error source	Error value (% $\Delta\dot{\omega}_{\rm II}^{\rm rel}$)	Total not correlated (% $\Delta\dot{\omega}_{\rm II}^{\rm rel}$)
Even zonal harmonics	2.45	
Odd zonal harmonics	4.10×10^{-2}	
Tides (solid + ocean)	2.48×10^{-2}	2.46
Secular trends (ℓ = even)	3.30×10^{-2}	
Seasonal-like effects	0.24	
Systematic errors: nongravitational perturbations		
Error source	Error value (% $\Delta\dot{\omega}_{\rm II}^{\rm rel}$)	Total not correlated (% $\Delta\dot{\omega}_{\rm II}^{\rm rel}$)
Direct solar radiation	0.50	
Earth's albedo	0.39	
Thermal thrusts	0.09	0.64
Drag (neutral + charged)	negligible	
Total not correlated		2.54
	$\epsilon_\omega^{\rm sys} - 1 = \pm 2.54 \times 10^{-2}$	

Results of the LARASE experiment

The overall error budget

DAVID M. LUCCHESI AND ROBERTO PERON — PHYSICAL REVIEW D **89**, 082002 (2014)

TABLE XVII. Error budget of the LAGEOS II pericenter general relativity shift. Top: summary of the errors from the data reduction and the *a posteriori* best fit (see Sections VI and VII). Middle: summary of the systematic errors from the gravitational perturbations (see Section VIII). Bottom: summary of the systematic errors from the nongravitational perturbations (see Section IX).

Statistical errors		
Residuals	Mean	Standard deviation
Range	9.67 cm	3.88 cm
Pericenter	4.57 mas	1.87 mas
Adjusted $\mathcal{R}_a^2$	0.998	
Reduced χ_ν^2 test	0.14	
	$\epsilon_\omega^{sta} - 1 = (-0.12 \pm 2.10) \times 10^{-3}$	
Systematic errors: gravitational perturbations		
Error source	Error value (% $\Delta\dot{\omega}_{II}^{rel}$)	Total not correlated (% $\Delta\dot{\omega}_{II}^{rel}$)
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Results of the LARASE experiment

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TABLE XVIII. Summary of the results obtained in the present work; together with the measurement error budget, the constraints on fundamental physics are listed and compared with the literature.

Parameter	Values and uncertainties (this study)	Uncertainties (literature)	Remarks
$\epsilon_\omega - 1$	$-1.2 \times 10^{-4} \pm 2.10 \times 10^{-3} \pm 2.54 \times 10^{-2}$	$\cdots$	Error budget of the perigee precession measurement in the field of the Earth
$\frac{\lvert 2+2\gamma-\beta \rvert}{3} - 1$	$-1.2 \times 10^{-4} \pm 2.10 \times 10^{-3} \pm 2.54 \times 10^{-2}$	$\pm(1.0 \times 10^{-3}) \pm (2 \times 10^{-2})$[a]	Constraint on the combination of PPN parameters
$\lvert \alpha \rvert$	$\lesssim \lvert 0.5 \pm 8.0 \pm 101 \rvert \times 10^{-12}$	$\pm 1 \times 10^{-8}$[b]	Constraint on a possible (Yukawa-like) NLRI
$\mathcal{C}_{\oplus \mathrm{LAGEOSII}}$	$\leq (0.003\ \mathrm{km})^4 \pm (0.036\ \mathrm{km})^4 \pm (0.092\ \mathrm{km})^4$	$\pm(0.16\ \mathrm{km})^4$[c]; $\pm(0.087\ \mathrm{km})^4$[d]	Constraint on a possible NSGT
$\lvert 2t_2 + t_3 \rvert$	$\lesssim 3.5 \times 10^{-4} \pm 6.2 \times 10^{-3} \pm 7.49 \times 10^{-2}$	3×10^{-3}[e]	Constraint on torsion

[a]From the preliminary estimate of the systematic errors of [166] for the perihelion precession of Mercury.
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[d]From [7] and based on the analysis of the systematic errors only.
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Results of the LARASE experiment

LARASE
LASERRANGEDSATELLITESEXPERIMENT

Summary of the constraints obtained

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$\|\alpha\|$	$\lesssim\|0.5\pm8.0\pm101\|\times10^{-12}$	$\pm1\times10^{-8b}$	Constraint on a possible (Yukawa-like) NLRI
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[166] I.I. Shapiro, in General Relativity and Gravitation, 1989, edited by N. Ashby, D. F. Bartlett, and W. Wyss (Cambridge University Press, Cambridge, 1990), p. 313.

Combination of PPN Parameters

$$\langle\dot{\omega}_{Schw}\rangle_{sec} = \left(\frac{2+2\gamma-\beta}{3}\right)\frac{3}{c^2a^{5/2}}\frac{GM_\oplus{}^{3/2}}{(1-e^2)}$$

$$\frac{|2+2\gamma-\beta|}{3} - 1 = -1.2\cdot10^{-4}\pm2.10\cdot10^{-3}\pm2.5\cdot10^{-2}$$

This result can be compared with the measurement by Shapiro and collaborators of Mercury's perihelion advance, determined by the radar ranging technique based on the measurement of the echo delay between the Earth and Mercury in the period between 1966 and 1990

Results of the LARASE experiment

LARASE
LASERRANGEDSATELLITESEXPERIMENT

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Violation of $1/r^2$ law: Yukawa-like potential

- Fujii; Fischbach; Damour

$$V(r) = -G_\infty \frac{M_1 M_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$\vec{F}(r) = -\vec{\nabla} V(r) = -G_\infty \left[1 + \alpha\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda}\right] \frac{M_1 M_2}{r^2}\hat{r}$$

$$\alpha = \frac{1}{G_\infty}\left(\frac{K_1}{M_1}\cdot\frac{K_2}{M_2}\right) \qquad \lambda = \frac{h}{\mu c}$$

As we have described, this type of parameterization, at the lowest interaction order and in the non-relativistic limit, is compatible with many metric theories of gravitation and with modern theories of physics regardless of the additional fields they consider: Scalar, Tensor and Vector fields

D.M. Lucchesi, *Phys. Lett. A* 318, 234, 2003; D.M. Lucchesi, Adv. *Space Res.* 47, 1232 (2011)

Results of the LARASE experiment

Violation of $1/r^2$ law: Yukawa-like potential

$$\Re = -\frac{G_\infty M_\oplus}{a^2}\left(\frac{a}{r}\right)^2 \alpha\left(1+\frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}$$

In order to retain the long period and secular effects we need to average Gauss equations over one cycle of a fast variable, like *M* or *f* :

GAUSS equations

$$\begin{aligned}
\dot{a} &= e\frac{2}{n\sqrt{1-e^2}}\Re\sin f\\
\dot{e} &= \frac{\sqrt{1-e^2}}{na}\Re\sin f\\
\dot{I} &= 0\\
\dot{\Omega} &= 0\\
\dot{\omega} &= -\frac{\sqrt{1-e^2}}{nae}\Re\cos f\\
\dot{M} &= n+\frac{1}{na}\Re\left(\frac{\cos u}{e(1-e^2)}-\sqrt{1-e^2}\sin f\sin u+2\frac{(1-e^2)}{(1+e\cos f)}\right)
\end{aligned}$$

$$\boldsymbol{r = \frac{a(1-e^2)}{1+e\cos f}}$$

$$\cos f = -e+\frac{2(1-e)}{e}\sum_{k=1}^{\infty}J_k(ke)\cos kM$$

$$\sin f = 2\sqrt{1-e^2}\sum_{k=1}^{\infty}\frac{1}{k}J_k'(ke)\sin kM$$

$$\cos u = \frac{e+\cos f}{1+e\cos f}$$

$$\sin u = \sqrt{1-e^2}\frac{\sin f}{1+e\cos f}$$

$$dM = \left(\frac{r}{a}\right)^2\frac{df}{\sqrt{1-e^2}}$$

$$\begin{aligned}
\langle\dot{a}\rangle_{2\pi} &= 0\\
\langle\dot{e}\rangle_{2\pi} &= 0\\
\langle\dot{\omega}\rangle_{2\pi} &\neq 0\\
\langle\dot{M}\rangle_{2\pi} &\neq 0
\end{aligned}$$

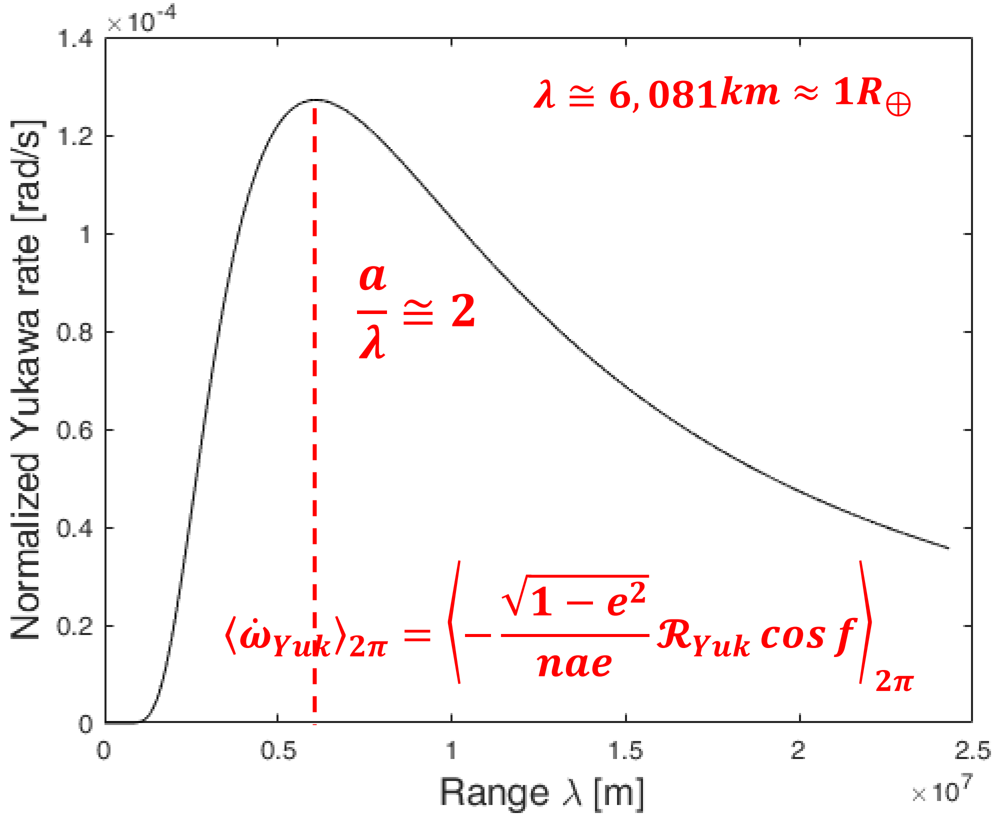
We have secular effects only on the satellite perigee $\boldsymbol{\omega}$ and mean anomaly ***M***

n = Satellite mean motion of the unperturbed two–body problem

$$n^2a^3 = G_\infty(M_\oplus + m_s) \cong G_\infty M_\oplus$$

Results of the LARASE experiment

Violation of $1/r^2$ law: Yukawa-like potential



$$\vec{\mathcal{R}}_{Yuk} = -\alpha \frac{G_\infty M_\oplus}{r^2}\left(1+\frac{r}{\lambda}\right) e^{-r/\lambda}\hat{r}$$

$$|\alpha| \cong \left|(0.5 \pm 8)\cdot 10^{-12} \pm 101 \cdot 10^{-12}\right|$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{2\pi}^{Peak} \cong 1.27394 \cdot 10^{-4} \cdot \alpha\, rad/s$$

Results of the LARASE experiment

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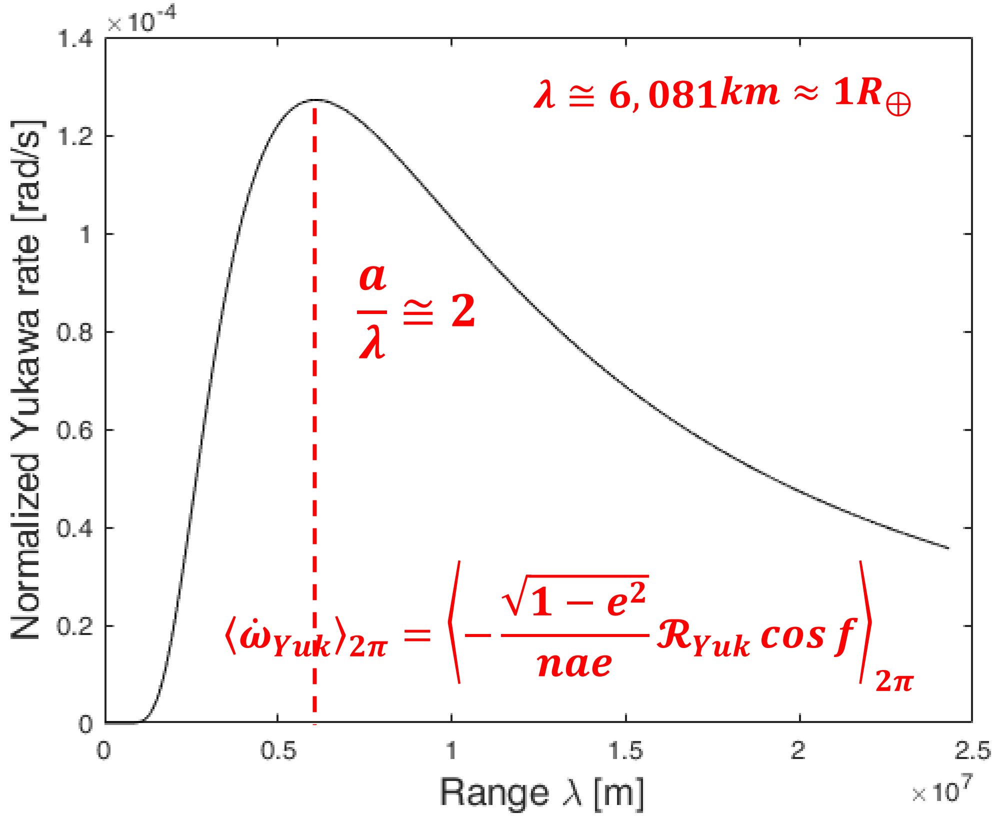

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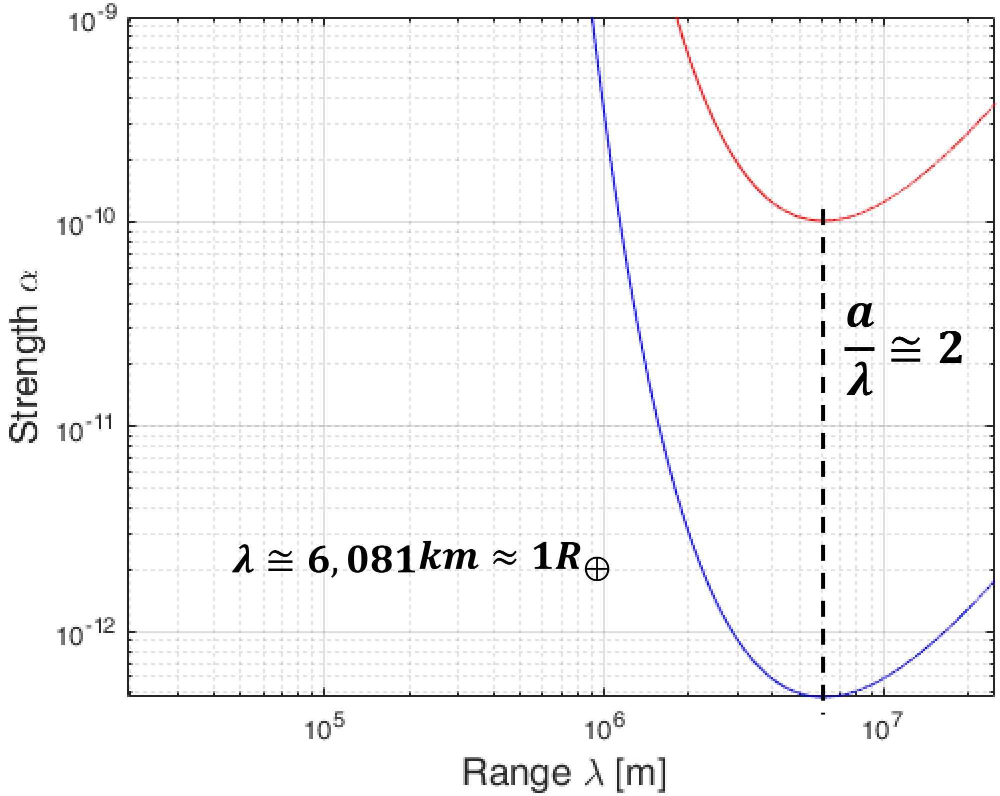


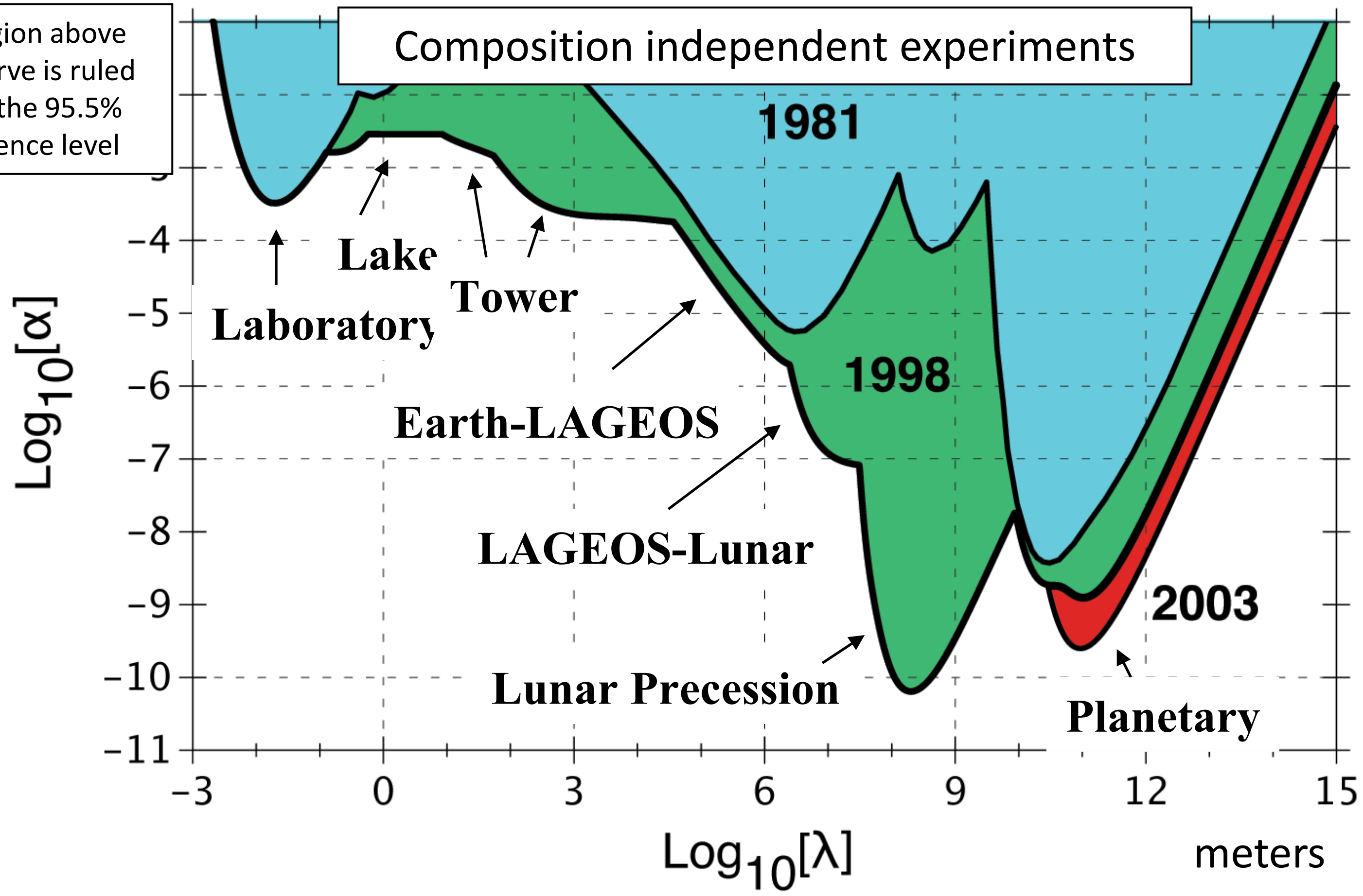
Reference: Coy, Fischbach, Hellings, Standish, & Talmadge (2003)

Results of the LARASE experiment

Constraints on a long-range force: Yukawa like interaction

$$|\alpha| \cong |(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}|$$

The region above each curve is ruled out at the 95.5% confidence level



Previous limits with **LAGEOS's**:

$|\alpha| < 10^{-5} \div 10^{-8}$

Reference: Coy, Fischbach, Hellings, Standish, & Talmadge (2003)

Results of the LARASE experiment

Constraints on a long-range force: Yukawa like interaction

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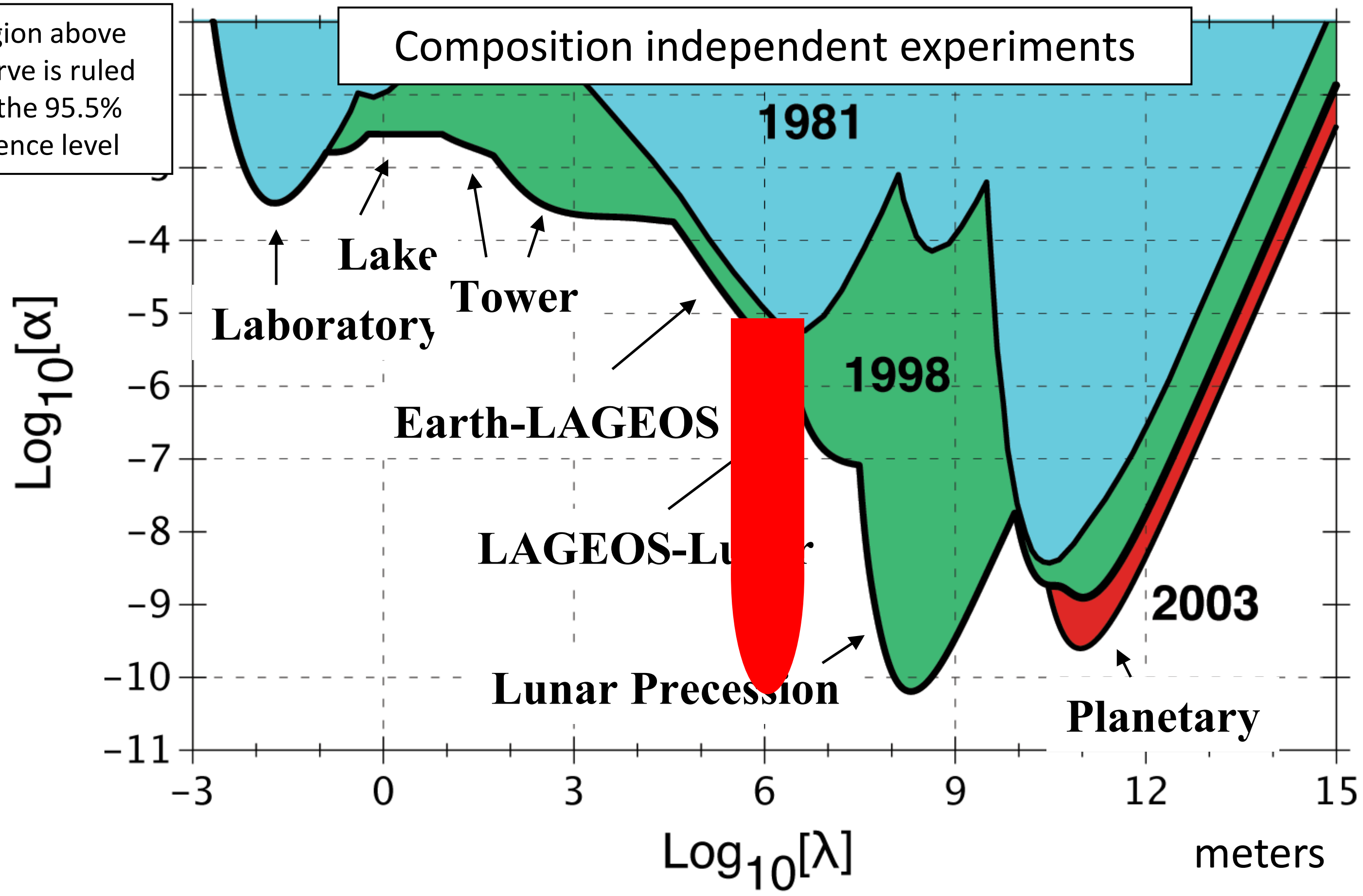

Previous limits with **LAGEOS's**:

$|\alpha| < 10^{-5} \div 10^{-8}$

$$|\alpha| \cong 1 \cdot 10^{-10}$$

Reference: Coy, Fischbach, Hellings, Standish, & Talmadge (2003)

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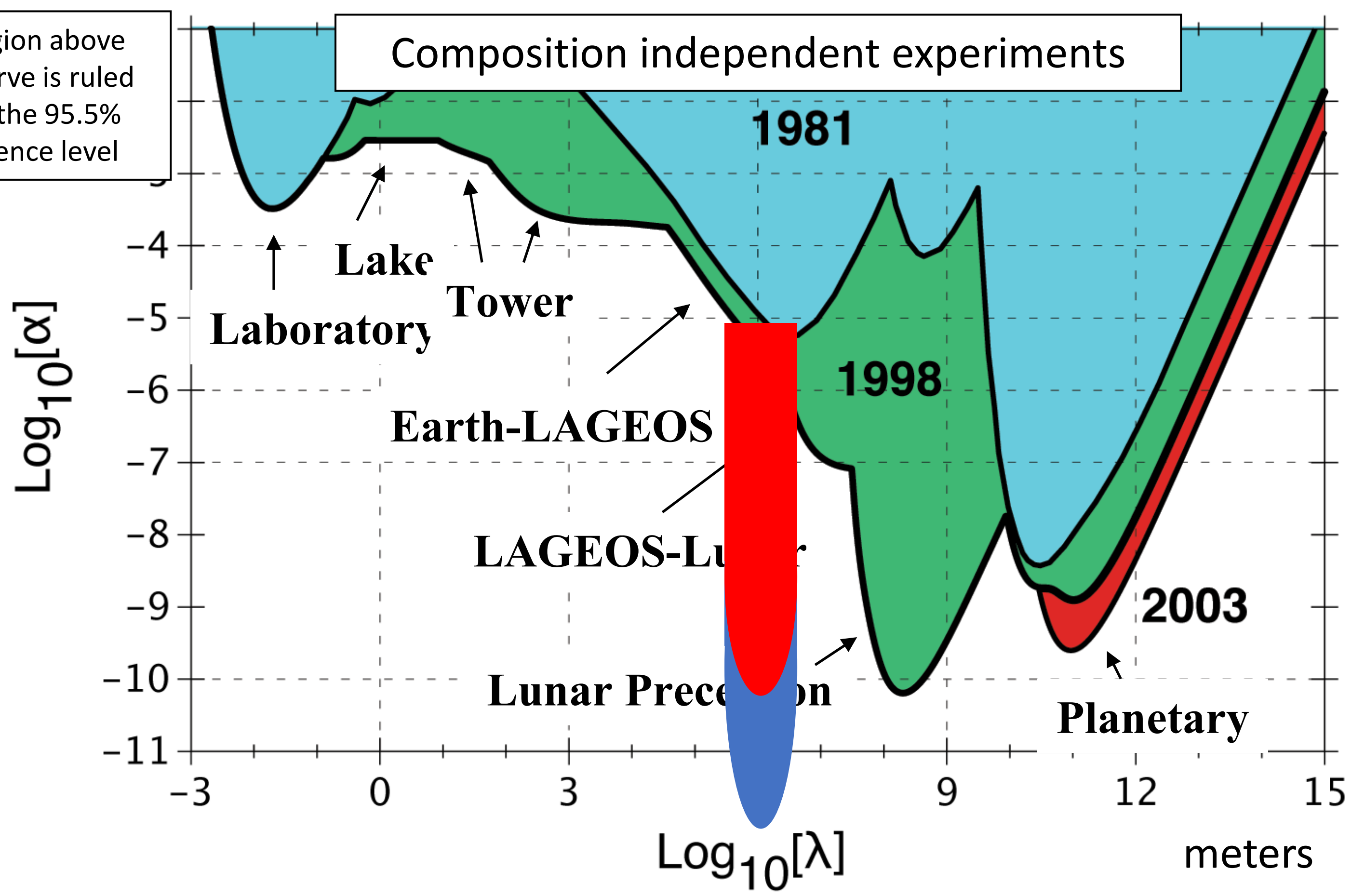

Previous limits with **LAGEOS's**:

$|\alpha| < 10^{-5} \div 10^{-8}$

$|\alpha| \cong 1 \cdot 10^{-10}$

$|\alpha| \cong 5 \cdot 10^{-13}$

Results of the LARASE experiment

Further possible Constraints of a long-range force

1. On the mass of the graviton
2. On the spatial variation of G

$$\mu = \frac{h}{\lambda c} \quad \xrightarrow{\lambda \cong 6{,}081 km} \quad \mu = 2 \cdot 10^{-13} \; eV/c^2$$

Results of the LARASE experiment

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$$\vec{F}(r) = -\vec{\nabla} V(r) = -G_\infty \left[1 + \alpha\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda}\right] \frac{M_1 M_2}{r^2} \hat{r}$$

$$G(r) = G_\infty \left[1 + \alpha\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda}\right] \qquad G(r \ll \lambda) = G_{Lab} \cong G_\infty[1 + \alpha] \quad \Longrightarrow \quad \frac{\Delta G}{G} = \frac{G_{Lab} - G_\infty}{G_\infty} \cong \alpha$$

$$|\alpha| \cong |(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}|$$

However, in Celestial mechanics we deal with *GM* and not with *G*

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Moffat Non-Symmetric Theory of Gravitation

- J.W. Moffat, *Phys. Rev. D* 19, 3554, 1979
- J.W. Moffat and E. Woolgar, *Phys. Rev. D* 37, 918, 1988

$$\Delta\dot{\omega}^{\mathrm{Mof}} = \frac{3(GM_\oplus)^{3/2}}{c^2 a^{5/2}(1-e^2)}\left[\mathcal{C}_{BS}\frac{c^4(1+e^2/4)}{(GM_\oplus a(1-e^2))^2}\right],$$

$$\mathcal{C}_{\oplus Lag2} = (M_\oplus - m_{Lag2})\left(\frac{\ell_\oplus^2}{M_\oplus} - \frac{\ell_{Lag2}^2}{m_{Lag2}}\right)(\ell_\oplus^2 - \ell_{Lag2}^2)$$

$$\mathcal{C}_{\oplus \mathrm{LAGEOS\,II}} \lesssim (0.003\ \mathrm{km})^4 \pm (0.036\ \mathrm{km})^4 \pm (0.092\ \mathrm{km})^4.$$

Among the various features of this theory, we are interested in the one which specifies that a given body B has an associated NSGT charge ℓ_B^2 (in addition to its mass) which arises from the coupling of the nonmetric with a vector current

[5] I. Ciufolini and R. Matzner, *Int. J. Mod. Phys. A* 07, 843, 1992; [7] D.M. Lucchesi, *Phys. Lett. A* 318, 234, 2003

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Torsion

- F.W. Hehl, P. von der Heyde, G.D. Kerlick, and J.M. Nester, *Rev. Mod. Phys*. 48, 393, 1976
- R.T. Hammond, *Rep. Prog. Phys*. 65, 599, 2002
- Y. Mao, M. Tegmark, A.H. Guth, and S. Cabi, *Phys. Rev. D* 76, 104029, 2007

$$\Delta\dot{\omega}_{\mathrm{torsion}} = \epsilon_{\mathrm{Schw}}\Delta\dot{\omega}^{\mathrm{Schw}}\left(\frac{2t_2+t_3}{3}\right) + \Delta\dot{\omega}^{\mathrm{LT}}_{\mathrm{torsion}},$$

$$|2t_2+t_3| \lesssim 3.5\times10^{-4}\pm6.2\times10^{-3}\pm7.49\times10^{-2},$$

A generalization of Einstein's GR may be obtained when a Riemann-Cartan spacetime is considered. In this case a nonvanishing torsional tensor is present because of nonsymmetric connection coefficients $\Gamma^{\gamma}_{\alpha\beta}$

[168] R. March, G. Bellettini, R. Tauraso, and S. Dell'Agnello, *Phys. Rev. D* 83, 104008, 2011

Results of the LARASE experiment

Part II

Measurement of the **Lense-Thirring** precession
on the orbits of the two **LAGEOS** and **LARES** satellites

Results of the LARASE experiment

The measurement of the **Lense-Thirring** precession has been the primary goal of **LARASE**, and this was explicitly requested by Prof. R. Battiston, President of the **INFN-CSN2** on Astroparticle Physics in 2013

As already underlined, this was mainly pursued:

- by improving the reliability of the dynamic model used in the **POD**
- and following **IERS** Conventions **2010**, **IAU** **2000** Resolutions, and **ILRS** Recommendations

Results of the LARASE experiment

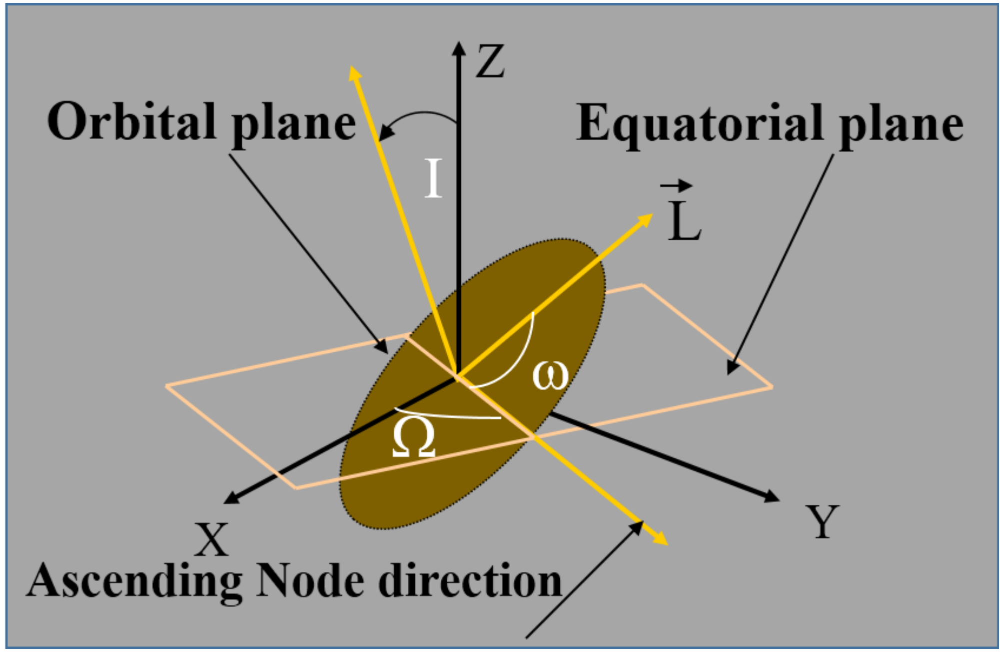
The **Lense-Thirring** effect consists of a precession of the orbit of a satellite around a primary produced by its rotation, i.e. by its angular momentum (mass-currents)

This precession produces a secular effect in two orbital elements:

- the right ascension of the ascending node
- the argument of pericenter

$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{sec} = -\frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$



Results of the LARASE experiment

On the **Lense-Thirring** effect and the importance of an accurate measurement of the **Gravitomagnetic Field**

$$\dot{\vec{\Omega}}_{GM} = -\frac{1}{2c}\vec{B}_{GM} = \frac{G}{c^2 r^3}\left[3(\vec{S}\cdot\hat{r})\hat{r} - \vec{S}\right]$$

An accurate and reliable measurement of the gravitomagnetic field of the Earth is not only important *per se*, as a further and robust test of the **GR** predictions in the **WFSM** limit. There are at least three main issues that, for their importance, require a much more precise and accurate measurement of gravitomagnetism, even in weak-field conditions:

- Intrinsic gravitomagnetism
- Strong fields and compact objects
- Mach's Principle

Results of the LARASE experiment

Gravity Probe B (GPB)

GPB, after 40 years of effort and $ 700 million satellite project, was launched on April 19, 2004 from Vandenberg Air Force Base (CA/USA) with a Delta II rocket

Photos: (Left) Russ Underwood, Lockheed Martin Corporation; (Right) Boeing Corporation

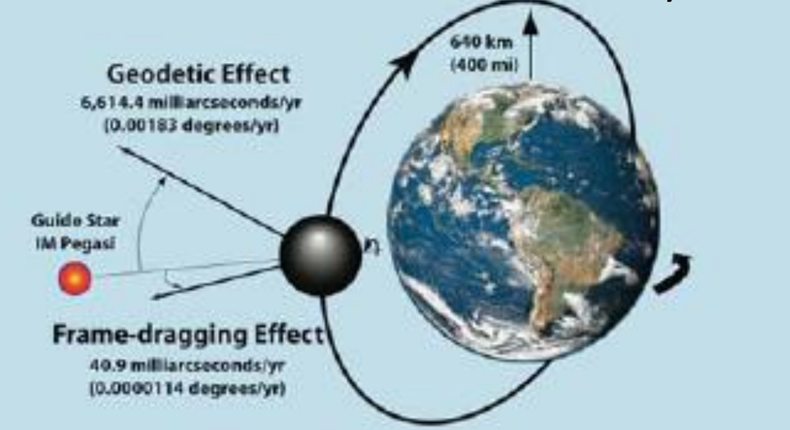
http://einstein.stanford.edu PI Prof. Francis Everitt

The two primary goals of GPB were:

1. The measurement of the frame–dragging effect with an accuracy of about **0.3%**;
2. The measurement of the de Sitter effect with an accuracy of about **0.002%**;

For 18 months of nominal duration

The readout error was $\leq$ 0.016 mas/yr



de Sitter Lense–Thirring

$$\dot{\vec{\Omega}}_{rel.} = \frac{3}{2}\frac{M_\oplus}{r^2}(\hat{r}\times\vec{v}) + \frac{-\vec{J}_\oplus + 3\hat{r}(\vec{J}_\oplus \bullet \hat{r})}{r^3}$$

de Sitter	Lense–Thirring
6,614.4 mas/yr	40.9 mas/yr
$\approx 2\cdot 10^{-5}$	$\approx 3\cdot 10^{-3}$

Comparable with the CASSINI measurement (2002) of γ ($\delta\gamma \cong 2\cdot 10^{-5}$, Bertotti et al. 2003, Letters to Nature).

Results of the LARASE experiment

- **Schiff (1960)**

PHYSICAL REVIEW LETTERS

VOLUME 4 MARCH 1, 1960 NUMBER 5

POSSIBLE NEW EXPERIMENTAL TEST OF GENERAL RELATIVITY THEORY*

L. I. Schiff
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
(Received February 11, 1960)

In a paper now in process of publication,[1] it is argued that only the planetary orbit precession provides real support for the full structure of the general theory of relativity. The other two of the three "crucial tests," the gravitational red shift and deflection of light, can be inferred correctly from the equivalence principle and the special theory of relativity, both of which are well established by other experimental evidence. It is also pointed out that a terrestrial or satellite experiment that would really test general relativity theory would have either to use particles of finite rest mass in such a way that the equation of motion can be confirmed beyond the Newtonian approximation, or to verify the second-order deviations of the metric tensor from its Minkowski form.

vector measured by a co-moving observer, $\vec{S}^0$, obeys the equation

$$d\vec{S}^0/dt = \vec{\Omega}\times\vec{S}^0, \qquad (2)$$

where

$$\vec{\Omega} = (\vec{F}\times\vec{v})/2mc^2 + (3GM/2c^2r^3)(\vec{r}\times\vec{v}) + (GI/c^2r^3)[3(\vec{\omega}\cdot\vec{r})\vec{r}/r^2 - \vec{\omega}]; \qquad (3)$$

$I = 2MR^2/5$ is the moment of inertia of the earth of radius R, assumed to be homogeneous, and $\vec{\omega}$ is its angular velocity vector. The first term on the right side of (3) is the Thomas precession,[3] which is a special relativity effect. The other two are the lowest order effects of general relativity; the second term arises whether or not the earth is rotating, and the third term is

Results of the LARASE experiment

Selected for a Viewpoint in *Physics*
PRL **106,** 221101 (2011) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2011

Gravity Probe B: Final Results of a Space Experiment to Test General Relativity

C. W. F. Everitt,[1,*] D. B. DeBra,[1] B. W. Parkinson,[1] J. P. Turneaure,[1] J. W. Conklin,[1] M. I. Heifetz,[1] G. M. Keiser,[1] A. S. Silbergleit,[1] T. Holmes,[1] J. Kolodziejczak,[2] M. Al-Meshari,[3] J. C. Mester,[1] B. Muhlfelder,[1] V. G. Solomonik,[1] K. Stahl,[1] P. W. Worden, Jr.,[1] W. Bencze,[1] S. Buchman,[1] B. Clarke,[1] A. Al-Jadaan,[3] H. Al-Jibreen,[3] J. Li,[1] J. A. Lipa,[1] J. M. Lockhart,[1] B. Al-Suwaidan,[3] M. Taber,[1] and S. Wang[1]

[1]*HEPL, Stanford University, Stanford, California 94305-4085, USA*
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[3]*King Abdulaziz City for Science and Technology, Riyadh, Saudi Arabia*
(Received 1 April 2011; published 31 May 2011)

Gravity Probe B, launched 20 April 2004, is a space experiment testing two fundamental predictions of Einstein's theory of general relativity (GR), the geodetic and frame-dragging effects, by means of cryogenic gyroscopes in Earth orbit. Data collection started 28 August 2004 and ended 14 August 2005. Analysis of the data from all four gyroscopes results in a geodetic drift rate of -6601.8 ± 18.3 mas/yr and a frame-dragging drift rate of -37.2 ± 7.2 mas/yr, to be compared with the GR predictions of -6606.1 mas/yr and -39.2 mas/yr, respectively ("mas" is milliarcsecond; 1 mas $= 4.848 \times 10^{-9}$ rad).

DOI: 10.1103/PhysRevLett.106.221101 PACS numbers: 04.80.Cc

Measurement of de Sitter precession
0.3%

Measurement of Gravitomagnetism
19%

… both measures are far from the initial objectives …

Results of the LARASE experiment

$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{sec} = -\frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$

LARASE
LASERRANGEDSATELLITESEXPERIMENT

The **Lense-Thirring** precession is very small compared to classical orbit precessions due to deviations from the spherical symmetry for the Earth's mass distribution, or with the same relativistic **Schwarzschild** precession produced by the mass of the primary (≈ 3350 mas/yr for **LAGEOS**)

TABLE I. Mean orbital elements of LAGEOS, LAGEOS II and LARES.

Element	Unit	Simbol	LAGEOS	LAGEOS II	LARES
semi-major axis	[km]	a	12 270.00	12 162.07	7 820.31
eccentricity		e	0.004433	0.013798	0.001196
inclination	[deg]	i	109.84	52.66	69.49

$$V(r,\varphi,\lambda) = -\frac{GM_\oplus}{r}\left[1+\sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{R_\oplus}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda)\right]$$

TABLE II. Rate in milli-arc-sec per year (mas/yr) for the secular Lense-Thirring precession on the right ascension of the ascending node and on the argument of pericenter of LAGEOS, LAGEOS II and LARES satellites.

Rate in the element	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}_{L-T}$	30.67	31.50	118.48
$\dot{\omega}_{L-T}$	31.23	−57.31	−334.68

30 mas ≅ 1.8 m in 1-year

$$\left\langle \dot{\Omega}_{class} \right\rangle_{sec} = -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos i}{(1-e^2)^2} \left\{-\sqrt{5}\bar{C}_{2,0}\right\} + \cdots$$

$$\dot{\Omega}^{Obser}_{Lageos} \approx +126\, deg/yr \qquad \dot{\Omega}^{Obser}_{LageosII} \approx -231\, deg/yr$$

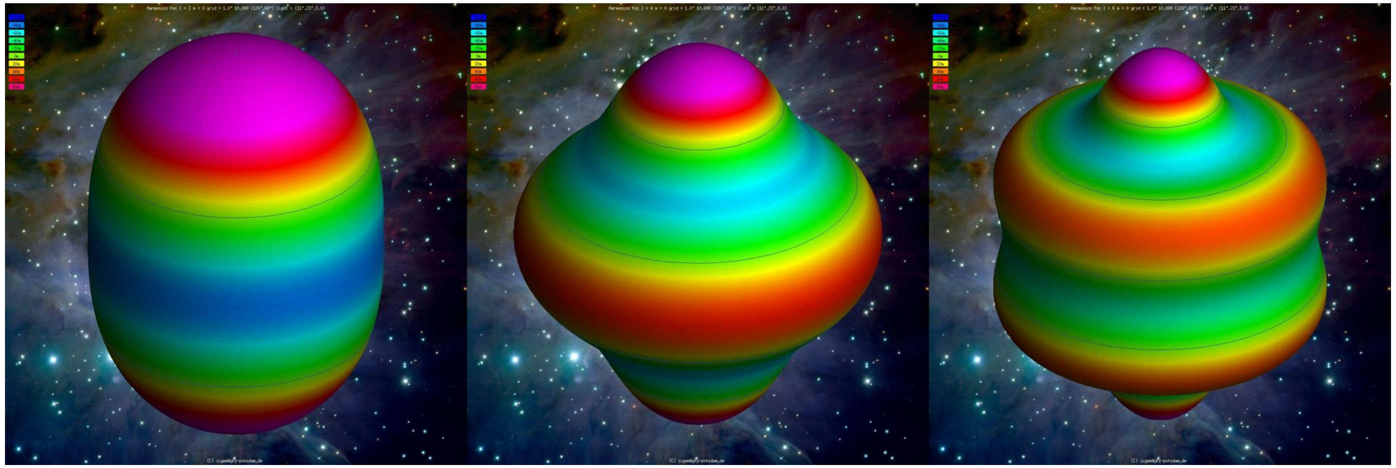
$$\begin{cases} G \cong 6.670 \cdot 10^{-8} cm^3 s^{-2} g^{-1} \\ J_\oplus \cong 5.861 \cdot 10^{40} cm^2 g s^{-1} \\ c \cong 2.99792458 \cdot 10^{10}\, cm/s \end{cases}$$

$$\dot{\Omega}^{Obser}_{Lares} \approx -624\, deg/yr$$

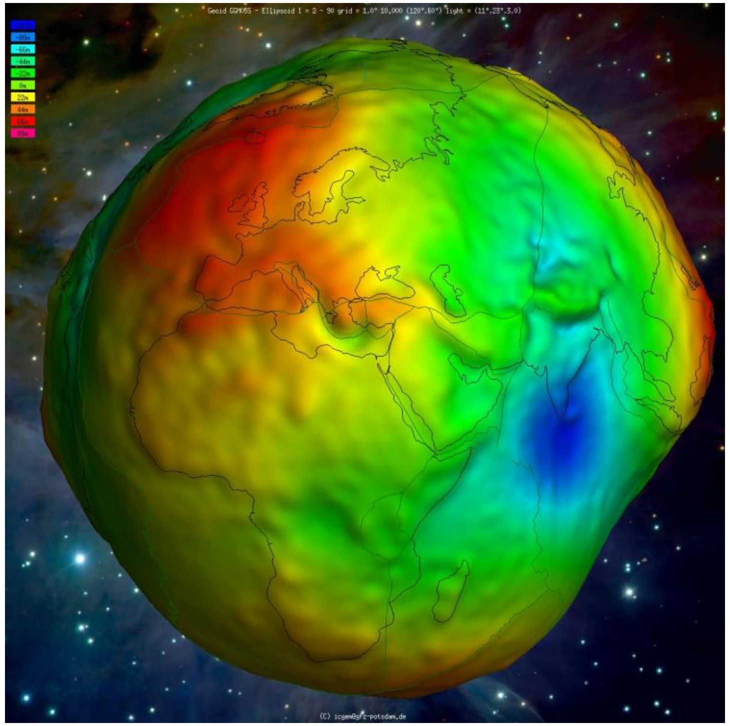
Results of the LARASE experiment

Therefore, the correct modelling of the even zonal harmonics ($\ell = \text{even}$, $m = 0$) represents the main challenge in this kind of measurements, since they have the same signature of the relativistic effect but much larger amplitudes

2,0 4,0 6,0



$$\langle \dot{\Omega}_{class} \rangle_{sec} = -\frac{3}{2} n \left(\frac{R_{\oplus}}{a}\right)^2 \frac{\cos i}{(1-e^2)^2} \{-\sqrt{5}\bar{C}_{2,0}\} + \cdots$$



Results of the LARASE experiment

By solving a linear system of three equations in three unknowns, we can solve for the relativistic precession while reducing the impact in the measurement of the non perfect knowledge of the Earth's gravitational field:

$$\begin{cases} \dot{\Omega}_2^{L1}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L1}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L1}\mu + \cdots = \delta\dot{\Omega}_{res}^{L1} \\ \dot{\Omega}_2^{L2}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L2}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L2}\mu + \cdots = \delta\dot{\Omega}_{res}^{L2} \\ \dot{\Omega}_2^{LR}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{LR}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{LR}\mu + \cdots = \delta\dot{\Omega}_{res}^{LR} \end{cases}$$

$$\dot{\Omega}_{GR}^{comb} = 50.17\ mas/yr$$

$$\mu = \frac{\dot{\Omega}^{comb}}{\dot{\Omega}_{GR}^{comb}} = \begin{cases} 1 & \text{• In General Relativity} \\ 0 & \text{• In Newtonian physics} \end{cases}$$

$$k_1 \cong 0.345$$
$$k_2 \cong 0.073$$

$$\dot{\Omega}^{comb} = \delta\dot{\Omega}_{res}^{L1} + k_1\delta\dot{\Omega}_{res}^{L2} + k_2\delta\dot{\Omega}_{res}^{LR}$$

- LT effect observable
- k_1 and k_2 are such that to cancel the unmodelled effects/errors of two even zonal harmonics (order *m*=0) of the Earth's gravitational field: quadrupole and octupole coefficients

$$\langle\delta\dot{\Omega}_{class}\rangle_{sec} = -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2\frac{\cos i}{(1-e^2)^2}\{-\sqrt{5}\delta\bar{C}_{2,0}\} + \cdots$$

Results of the LARASE experiment

By solving a linear system of three equations in three unknowns, we can solve for the relativistic precession while reducing the impact in the measurement of the non perfect knowledge of the Earth's gravitational field:

$$\begin{cases} \dot{\Omega}_2^{L1}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L1}\delta\bar{C}_{4,0} + \boxed{\dot{\Omega}_{LT}^{L1}\mu} + \cdots = \boxed{\delta\dot{\Omega}_{res}^{L1}} \\ \dot{\Omega}_2^{L2}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L2}\delta\bar{C}_{4,0} + \boxed{\dot{\Omega}_{LT}^{L2}\mu} + \cdots = \boxed{\delta\dot{\Omega}_{res}^{L2}} \\ \dot{\Omega}_2^{LR}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{LR}\delta\bar{C}_{4,0} + \boxed{\dot{\Omega}_{LT}^{LR}\mu} + \cdots = \boxed{\delta\dot{\Omega}_{res}^{LR}} \end{cases}$$

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$$k_1 \cong 0.345$$
$$k_2 \cong 0.073$$

1. Ciufolini, I.; Lucchesi, D.; Vespe, F.; Mandiello, A. Measurement of dragging of inertial frames and gravitomagnetic field using laser-ranged satellites. *Nuovo Cim*. A, 109, 575–590, 1996
2. Ciufolini, I. On a new method to measure the gravitomagnetic field using two orbiting satellites. *Nuovo Cim*. A, 109, 1709–1720, 1996
3. Lucchesi, D.M.; Balmino, G. The LAGEOS satellites orbital residuals determination and the Lense Thirring effect measurement. *Plan. Space Sci*., 54, 581–593, 2006

Results of the LARASE experiment

LARASE
LASER RANGED SATELLITES EXPERIMENT

IL NUOVO CIMENTO VOL. 109 A, N. 12 Dicembre 1996

On a new method to measure the gravitomagnetic field using two orbiting satellites

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Dipartimento Aerospaziale, Università di Roma «La Sapienza» - Roma, Italy

(ricevuto il 20 Settembre 1996; approvato il 15 Novembre 1996)

Summary. — We describe a new method to obtain the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct detection of the gravitomagnetic field. This method is based on the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II. By this new approach one achieves a measurement of the gravitomagnetic field with accuracy of about 25%, or less, of the Lense-Thirring effect in general relativity.

PACS 11.90 – Other topics in general field and particle theory.
PACS 04.80.Cc – Experimental test of gravitational theories.

IL NUOVO CIMENTO VOL. 109 A, N. 5 Maggio 1996

Measurement of dragging of inertial frames and gravitomagnetic field using laser-ranged satellites

I. CIUFOLINI [1], D. LUCCHESI [2], F. VESPE [3] and A. MANDIELLO [4]
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[2] *Dipartimento di Matematica, Università di Pisa - Pisa, Italy*
[3] *ASI-CGS - Matera, Italy*
[4] *IFSI-CNR - Frascati (Roma), Italy*

(ricevuto il 28 Febbraio 1996; approvato il 3 Aprile 1996)

Summary. — By analysing the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II, using the program GEODYN, we have obtained the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct experimental evidence for the gravitomagnetic field. The accuracy of our measurement is of about 30%.

PACS 11.90 – Other topics in general field and particle theory.
PACS 04.80.Cc – Experimental tests of gravitational theories.

Results of the LARASE experiment

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The LAGEOS satellites orbital residuals determination and the Lense–Thirring effect measurement

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[b] *Istituto di Scienza e Tecnologie della Informazione (ISTI/CNR), Via Moruzzi No. 1, 56124 Pisa, Italy*
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Received 6 December 2004; received in revised form 28 February 2006; accepted 9 March 2006

Abstract

The method applied since 1996 for the analysis of the orbital residuals of the LAGEOS satellites in order to measure the Lense–Thirring effect has been the subject of the present work. This method, based on the difference between the orbital elements of consecutive arcs, is explained and analysed also from the analytical point of view. It is proved that this "difference method" works well for the determination of the secular effects, as in the case of the relativistic precession induced by the Earth's angular momentum, but also very useful for the determination and study of the long-term periodic effects. Indeed, the only limitation in the determination of the periodic effects is the possibility of the reduction of their amplitude by a factor which depends from the periodicity of the given perturbation and from the orbital arc length chosen for the satellite during the data analysis. In the case of the Yarkovsky–Schach effect, the main non-gravitational perturbation seen in the LAGEOS satellites orbital residuals, in particular in its perigee rate and eccentricity vector excitation residuals, we show that the "difference method" is quite good also for the determination of the long-period perturbations induced by this subtle non-conservative force.
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Keywords: LAGEOS satellites; Orbital residuals determination; Secular and long-period perturbations; Gravitational and non-gravitational perturbations; Yarkovsky–Schach effect; General relativity; Lense–Thirring effect

Results of the LARASE experiment

On the modelling of the even zonal harmonics

Results of the LARASE experiment

LARASE
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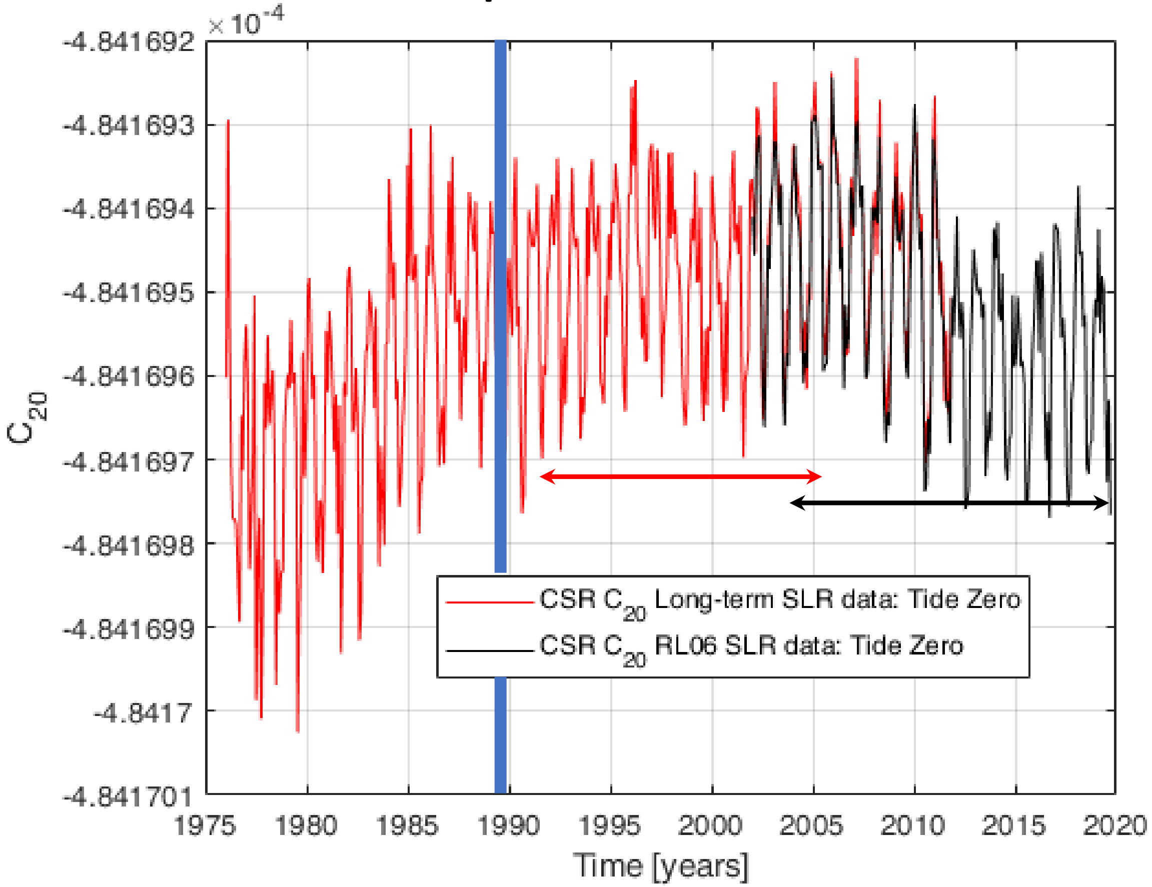
A non-exhaustive list of references

- Yoder C.; Williams J.; Dickey J.; Schutz B.; Eanes R.; Tapley B. Secular variations of Earth's gravitational harmonic J2 coefficient from Lageos and nontidal acceleration of Earth rotation. Nature 303, pp. 757-762, **1983**
- Cheng, M.K.; Shum, C.K.; Tapley, B.D. Determination of long-term changes in the Earth's gravity field from satellite laser ranging observations. *J. Geophys. Res*., 102, 22377, **1997**
- Devoti, R.; Luceri, V.; Sciarretta, C.; Bianco, G.; Di Donato, G.; Vermeersen, L.L.A.; Sabadini, R. The SLR secular gravity variations and their impact on the inference of mantle rheology and lithospheric thickness. *Geophys. Res. Lett*., 28, 555–858, **2001**
- Cox, C.M.; Chao, B.F. Detection of a Large-Scale Mass Redistribution in the Terrestrial System Since 1998. *Science*, 297, 831–833, **2002**
- Cheng, M.; Tapley, B.D.; Ries, J.C. Deceleration in the Earth's oblateness. *J. Geophys. Res. Solid Earth*, 118, 740–747, **2013**
- Chen, J.L.; Wilson, C.R.; Ries, J.C.; Tapley, B.D. Rapid ice melting drives Earth's pole to the east. *Geophys. Res. Lett*., 40, 2625–2630, **2013**
- Cheng, M.; Ries, J.C. Decadal variation in Earth's oblateness (J2) from satellite laser ranging data. *Geophys. J. Int*., 212, 1218–1224, **2018**

Results of the LARASE experiment

Quadrupole coefficient: SLR data



$$C_{2,0} = C_{2,0}(t_0) + \dot{C}_{2,0}(t - t_0)$$

Results of the LARASE experiment

LARASE
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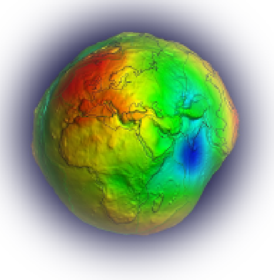

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Documentation

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Global Gravity Field Models

We kindly ask the authors of the models to check the links to the original websites of the models from time to time. Please let us know if something has changed.

The table can be interactively re-sorted by clicking on the column header fields (Nr, Model, Year, Degree, Data, Reference).
In the data column, the datasets used in the development of the models are summarized, where **A** is for altimetry, **S** is for satellite (e.g., GRACE, GOCE, LAGEOS), **G** for ground data (e.g., terrestrial, shipborne and airborne measurements) and **T** is for topography.

The links calculate and show in the last columns of the table directly invoke the *Calculation Service* and *Visualization page* for the selected model.
For models with a registered **doi** ("digital object identifier") the last column contains the symbol ✓, which directly opens the page on "http://dx.doi.org/".
If you click on the reference, the complete list of references can be seen.

Nr	Model	Year	Degree	Data	References	Download	Calculate	Show	DOI
176	XGM2019e_2159	2019	2190 5540 760	A, G, S(GOCO06s), T	Zingerle, P. et al, 2019	gfc zip gfc zip gfc zip	Calculate	Show	✓
175	GO_CONS_GCF_2_TIM_R6e	2019	300	G (Polar), S(Goce)	Zingerle, P. et al, 2019	gfc zip	Calculate	Show	✓
174	ITSG-Grace2018s	2019	200	S(Grace)	Mayer-Gürr, T. et al, 2018	gfc zip	Calculate	Show	✓
173	EIGEN-GRGS.RL04.MEAN-FIELD	2019	300	S	Lemoine et al, 2019	gfc zip	Calculate	Show	✓
172	GOCO06s	2019	300	S	Kvas et al., 2019	gfc zip	Calculate	Show	✓
171	GO_CONS_GCF_2_TIM_R6	2019	300	S(Goce)	Brockmann, J. M. et al, 2014	gfc zip	Calculate	Show	✓
170	GO_CONS_GCF_2_DIR_R6	2019	300	S	Bruinsma, S. L. et al, 2014	gfc zip	Calculate	Show	✓
169	IGGT_R1C	2018	240	G, S(Goce), S(Grace)	Lu, B. et al., 2019	gfc zip	Calculate	Show	✓
168	Tongji-Grace02k	2018	180	S(Grace)	Chen, Q. et al, 2018	gfc zip	Calculate	Show	
167	SGG-UGM-1	2018	2159	EGM2008, S(Goce)	Liang, W. et al., 2018 & Xu, X. et al. (2017)	gfc zip	Calculate	Show	✓
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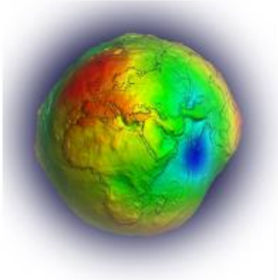
165	IGGT_R1	2017	240	S(Goce)	Lu, B. et al, 2017	gfc zip	Calculate	Show	✓
164	IfE_GOCE05s	2017	250	S	Wu, H. et al, 2017	gfc zip	Calculate	Show	✓
163	GO_CONS_GCF_2_SPW_R5	2017	330	S(Goce)	Gatti, A. et al, 2016	gfc zip	Calculate	Show	✓

icgem@gfz-potsdam.de

Static Models

Results of the LARASE experiment

Temporal Models



ICGEM Home

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Gravity Field Solutions for dedicated Time Periods

The following gravity field time series are presently available:

GRACE and Grace-FO solutions from the Science Data System centers CSR, GFZ and JPL — collapse all

- CSR				Center for Space Research at University of Texas, Austin
CSR Release 05		monthly		UTCSR Level-2 Processing Standards Document, Rev 4.0 May 29, 2012
CSR Release 06	DOI	monthly		UTCSR Level-2 Processing Standards Document, Rev 5.0 April 18, 2018
CSR Release 06 (GFO)	DOI	monthly		UTCSR Level-2 Processing Standards Document, V 1.1 June 6, 2019
- GFZ				Helmholtz Centre Potsdam German Research Centre for Geosciences
GFZ Release 05		monthly	weekly	GFZ GRACE Level-2 Processing, Revised Edition, January 2013
GFZ Release 06	DOI	monthly		GFZ GRACE Level-2 Processing Standards Document for Level-2 Products, Rev. 1.0, October 26, 2018
GFZ Release 06 (GFO)	DOI	monthly		GFZ GRACE Level-2 Processing Standards Document for Level-2 Products, Rev. 1.0, June 3, 2019
- JPL				Jet Propulsion Laboratory
JPL Release 05		monthly		JPL Level-2 Processing Standards Document, Release 05.1 November 3, 2014
JPL Release 06	DOI	monthly		JPL Level-2 Processing Standards Document, Release 06.0 June 1, 2018
JPL Release 06 (GFO)	DOI	monthly		JPL Level-2 Processing Standards Document, v 1.0 May 28, 2019

The processing standards to generate the GRACE Level-2 products of CSR, GFZ and JPL are also available in the Document Section of the GRACE archives at GFZ ISDC or JPL PO.DAAC

COST-G (International Combination Service for Time-variable Gravity Field) — collapse all

GRACE	DOI	monthly
Swarm	DOI	monthly

GRACE / CHAMP solutions from other groups — expand all

+ AIUB — Astronomical Institute University Bern

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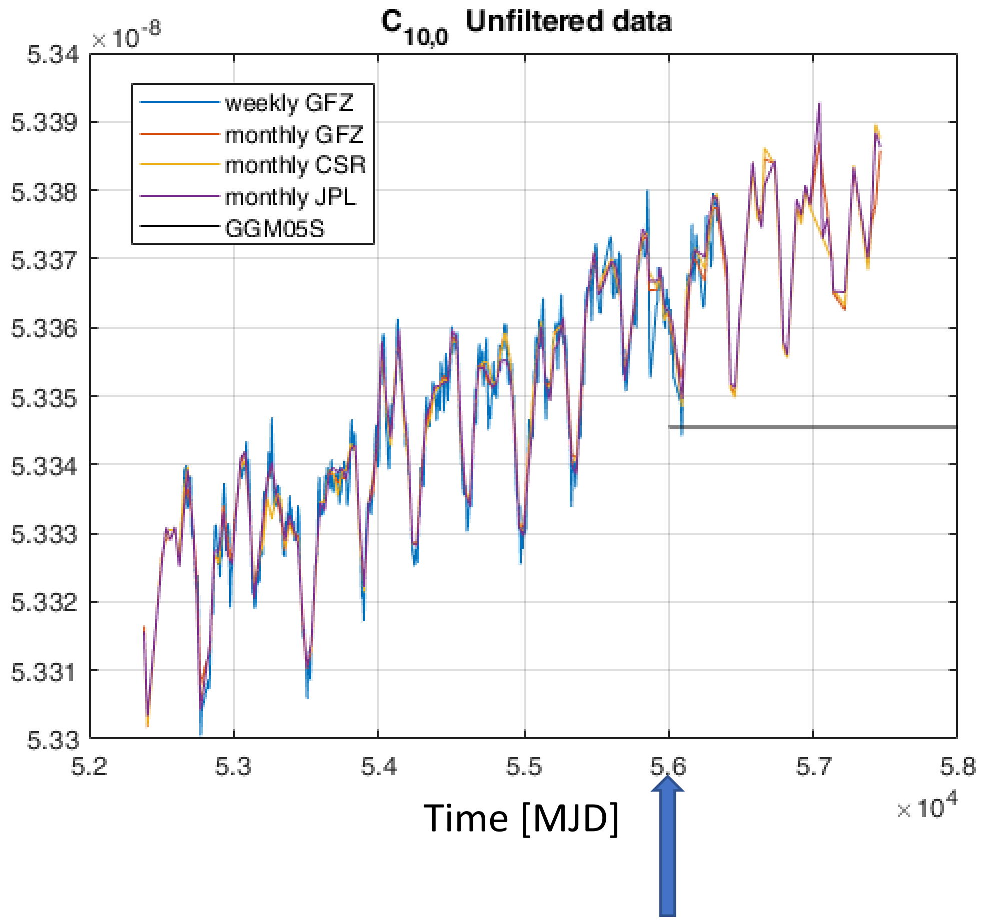
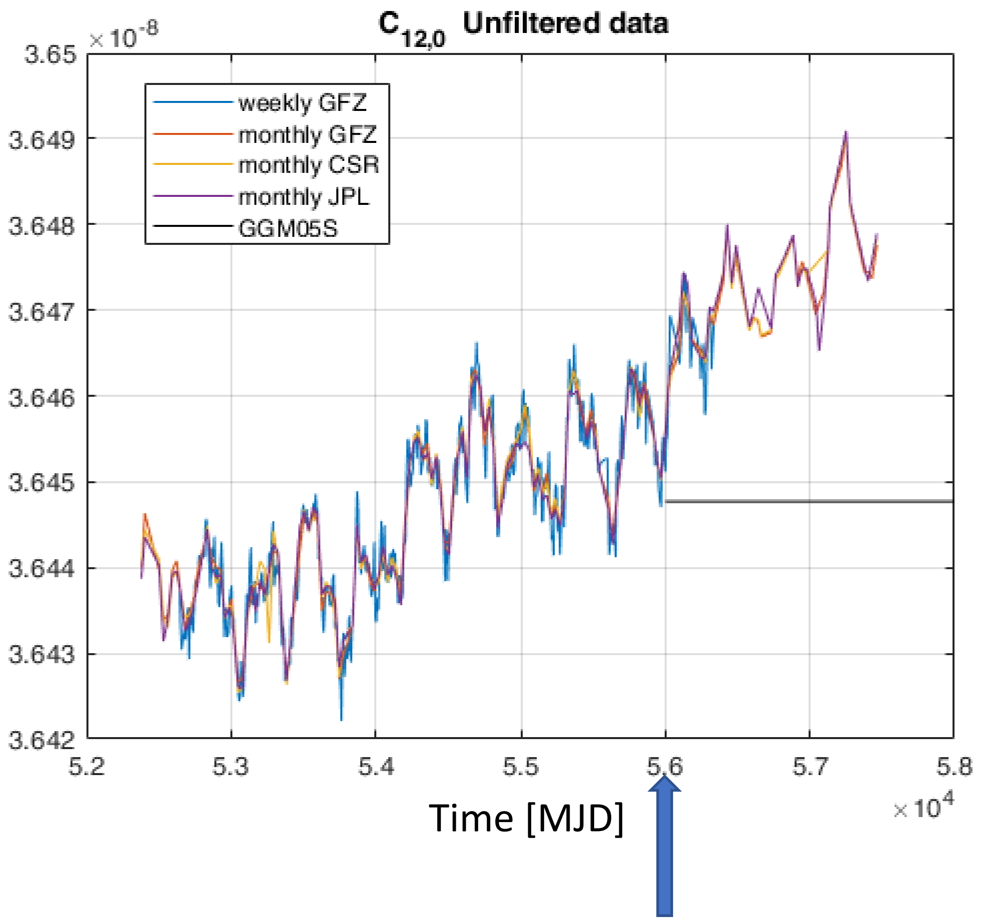
Results of the LARASE experiment

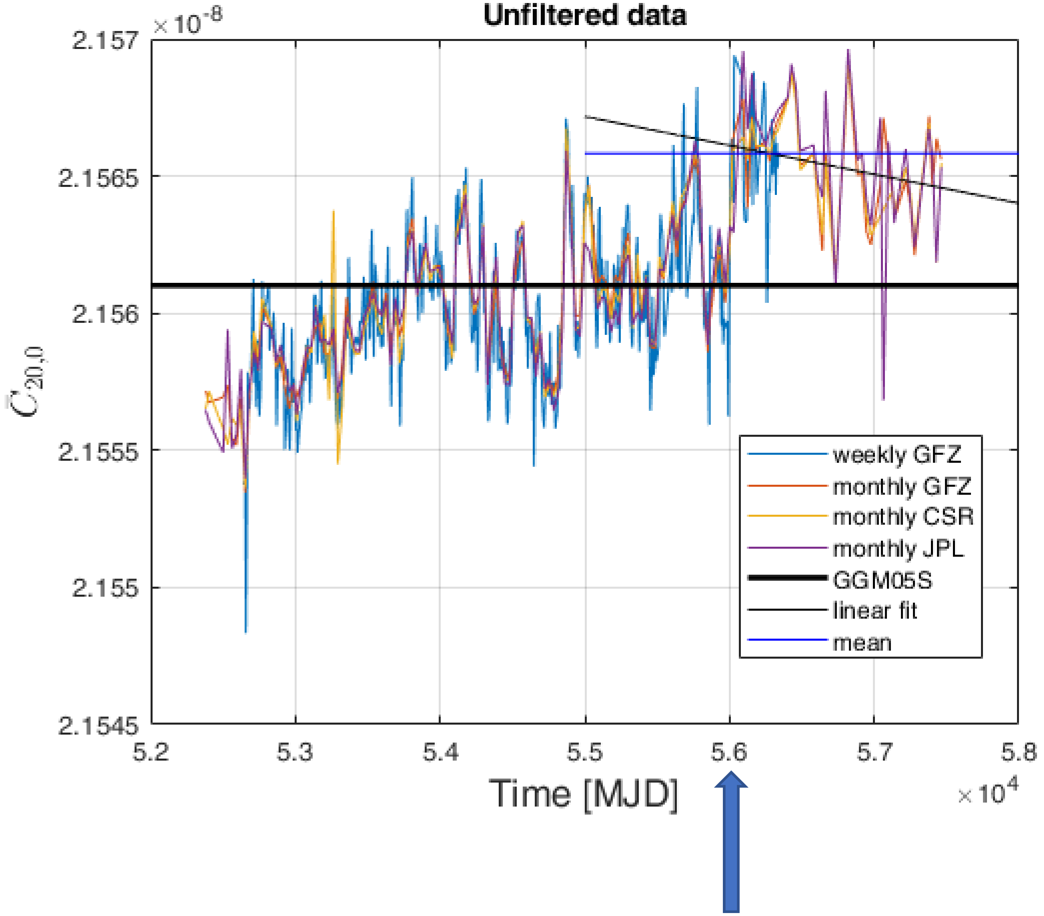
From **GRACE** Temporal Solutions





Results of the LARASE experiment

From **GRACE** Temporal Solutions



Linear fit to better capture evolution over time

$$\bar{C}_{\ell,0}(t) = \bar{C}_{\ell,0}(t_0) + \dot{\bar{C}}_{\ell,0}(t - t_0)$$

Results of the LARASE experiment

The measurement of the Lense-Thirring effect

Results of the LARASE experiment

Starting from December 2017 and until spring 2019 we carried out an intense analysis activity:

- for different models of static gravitational field
- and from **GRACE's** monthly solutions from three different analysis centers
  - ❑ zonal harmonics
  - ❑ but not only

For each of these analysis we performed a **POD** over a time of about 6.5 years (over 7-day arcs), processing a considerable number of **SLR** observations in the form of Normal Points, for an average of about 1344 (**LAGEOS**), 1207 (**LAGEOS II**) and 1487 (**LARES**) normal points

Results of the LARASE experiment

- We considered several models for the background gravitational field of the Earth
  - **This allows to highlight possible systematics among the different models**
- For the first **10/15** even zonal harmonics we considered their explicit time dependency following the monthly solutions from **GRACE** measurements
  - **This has reduced the systematic error of the background gravitational field**
- Together with the relativistic **Lense-Thirring** precession we estimated also some of the low-degree even zonal harmonics (ℓ = even and m = 0) of the background gravitational field
  - **This allows to estimate the direct correlation between the relativistic Lense-Thirring precession with the coefficients of the gravitational field**

Results of the LARASE experiment

- The relativistic **Lense-Thirring** precession has been measured both in the residuals of the rates of the combined nodes and in their integration
  - **This is the first time that the measurement has been performed on the rate of the combined observables**
- The measurement has been obtained both via linear fits and non-linear fits
  - **This is also the first time that a reliable measurement of the Lense-Thirring precession has been obtained by means of a simple linear fit**

Results of the LARASE experiment

- The data reduction of the satellites orbit has been done with **GEODYN II** (NASA/GSFC) on a time span of about 6.5 years (2359 days) from **MJD 56023**, i.e. April 6th 2012, and we computed the effects on the orbit elements of **LAGEOS**, **LAGESOS II** and **LARES**:
  - **Background gravity model: GGM05S + other fields from GRACE**
  - **Arc length of 7 days**
  - **No empirical accelerations**
  - **Thermal effects (Yarkovsky Schach and Rubincam) not modelled**
  - **General relativity modelled with the exception of the Lense-Thirring effect**

1. **EIGEN-GRACE02S (2004)**
2. **GGM05S (2014): official field of the ILRS**
3. **ITU_GRACE16 (2016)**
4. **Tonji-Grace02s (2017)**

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\langle \dot{\Omega}_{LT} \rangle_{sec}$	30.67	31.50	118.48

Results of the LARASE experiment

Table 2. Models currently used, within the LARASE research program, for the analysis of the orbit of the two LAGEOS and LARES satellites. The models are grouped in gravitational perturbations, non-gravitational perturbations and reference frames realizations.

Model For	Model Type	Reference
Geopotential (static)	EIGEN-GRACE02S/GGM05S	[84,90,91]
Geopotential (time-varying, tides)	Ray GOT99.2	[92]
Geopotential (time-varying, non tidal)	IERS Conventions 2010	[89]
Third–body	JPL DE-403	[93]
Relativistic corrections	Parameterized post-Newtonian	[88,94]
Direct solar radiation pressure	Cannonball	[46]
Earth albedo	Knocke-Rubincam	[63]
Earth-Yarkovsky	Rubincam	[56,64,65]
Neutral drag	JR-71/MSIS-86	[50,51]
Spin	LASSOS	[42]
Stations position	ITRF2008	[95]
Ocean loading	Schernek and GOT99.2 tides	[46,92]
Earth Rotation Parameters	IERS EOP C04	[96]
Nutation	IAU 2000	[97]
Precession	IAU 2000	[98]

Results of the LARASE experiment

universe

MDPI

Article

A 1% Measurement of the Gravitomagnetic Field of the Earth with Laser-Tracked Satellites

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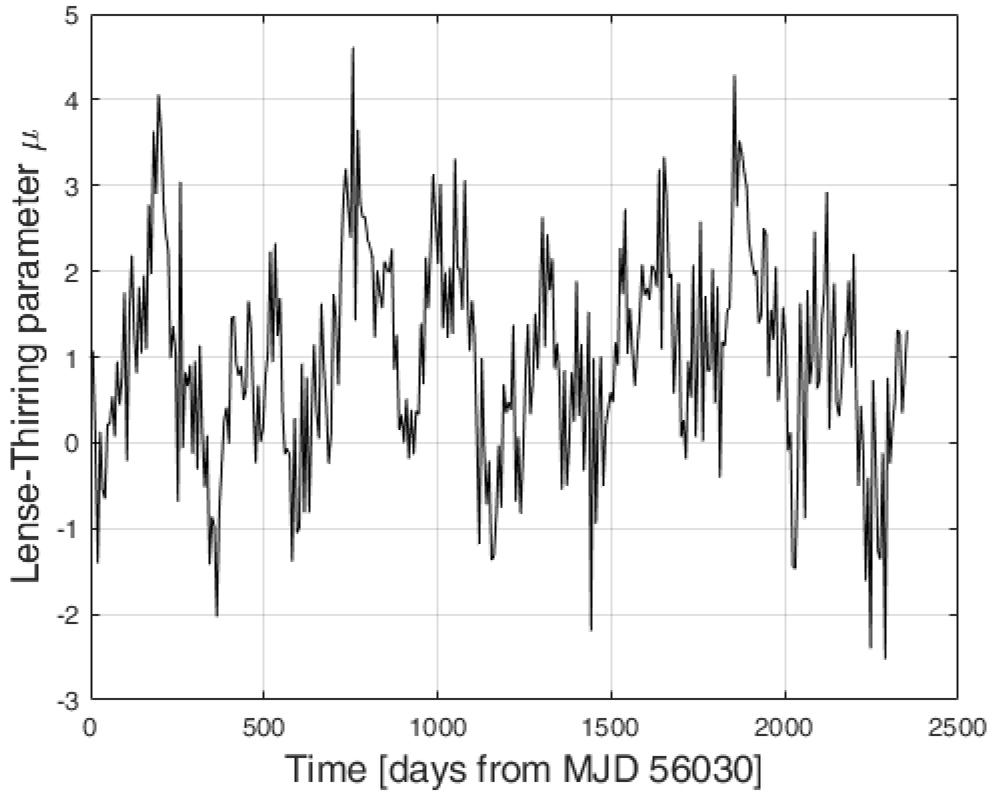
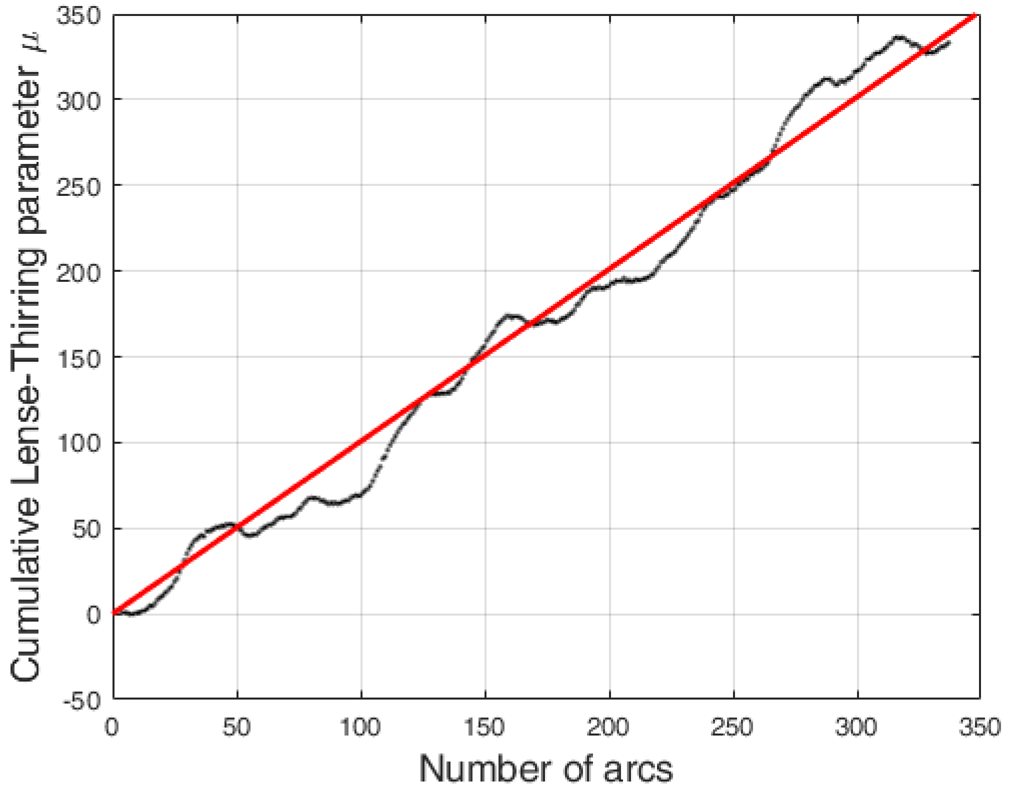
check for updates

Results of the LARASE experiment

Results for μ from the linear system



Cumulative sum for μ



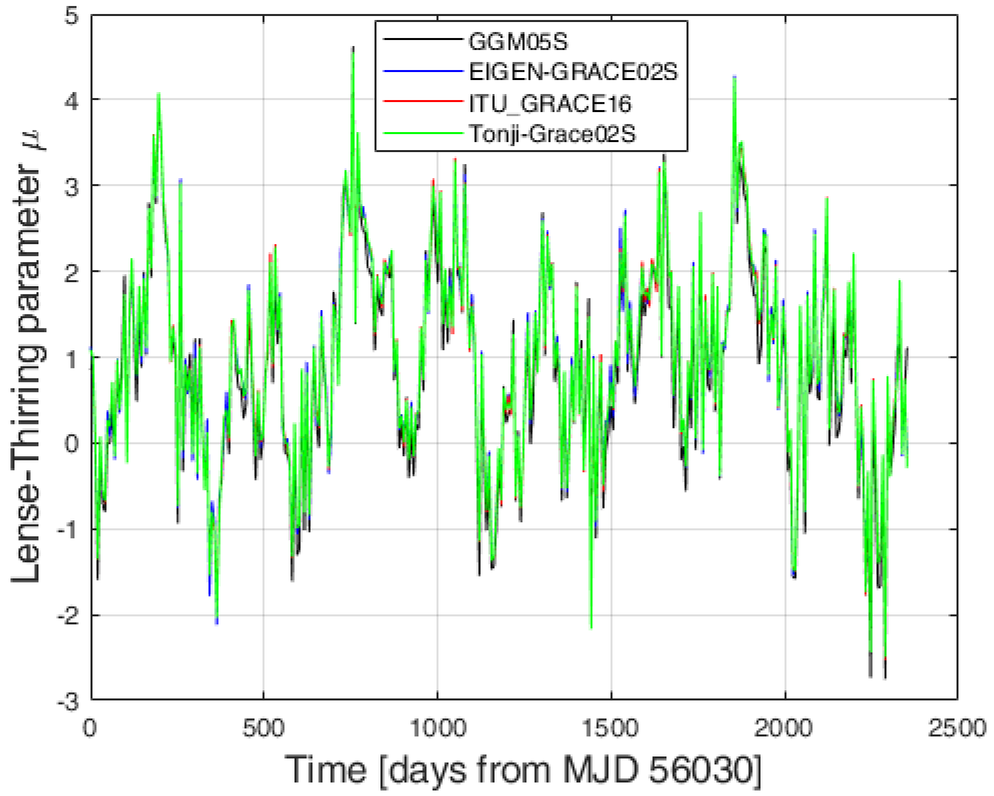
Gussian-like distribution

$K \cong +3.097 \quad S \cong -8.4\times10^{-3}$

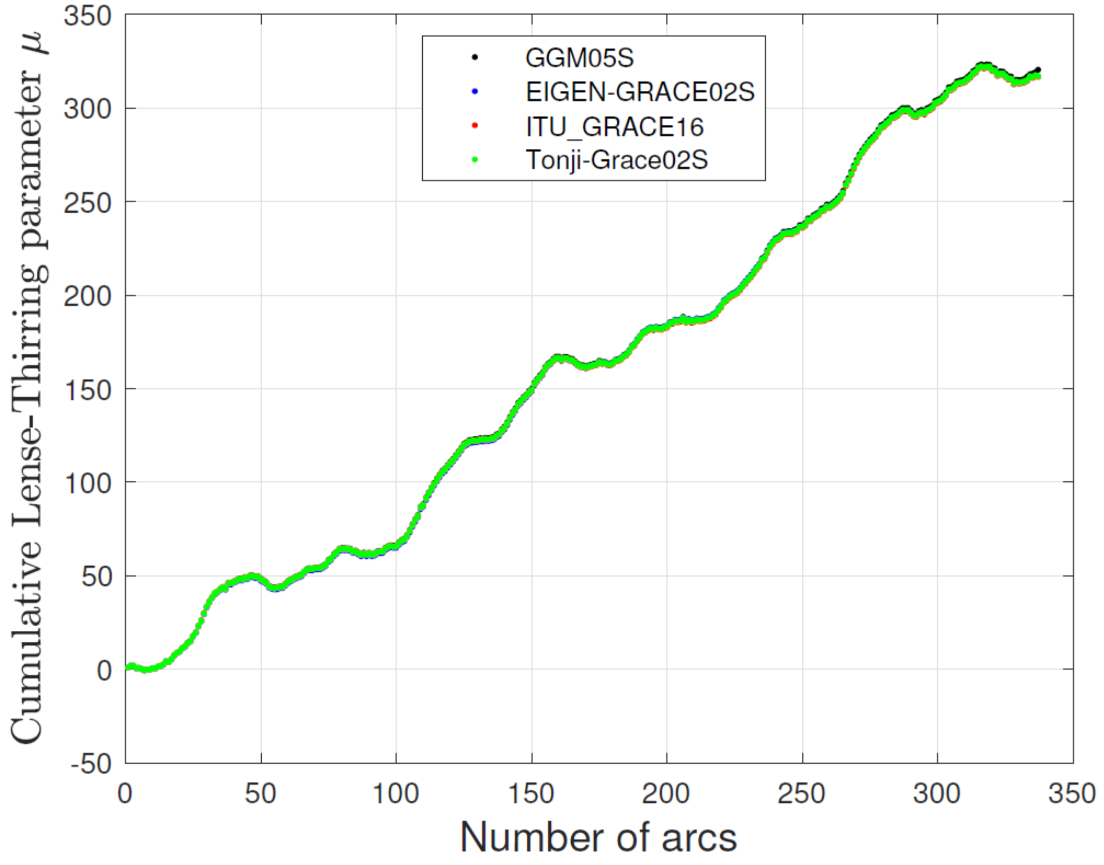
GGM05S model

Results of the LARASE experiment

Results for μ from the linear system

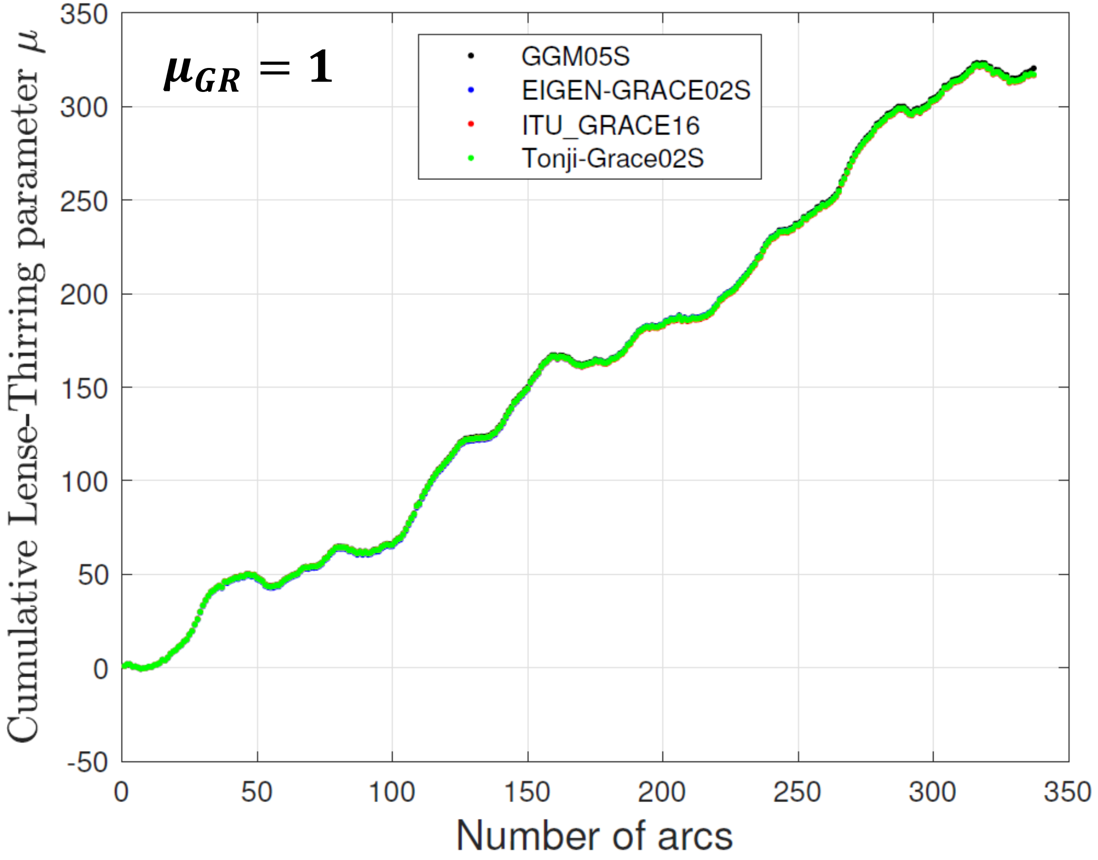


Cumulative sum for μ



Results of the LARASE experiment

Lense-Thirring effect measurement: frame dragging



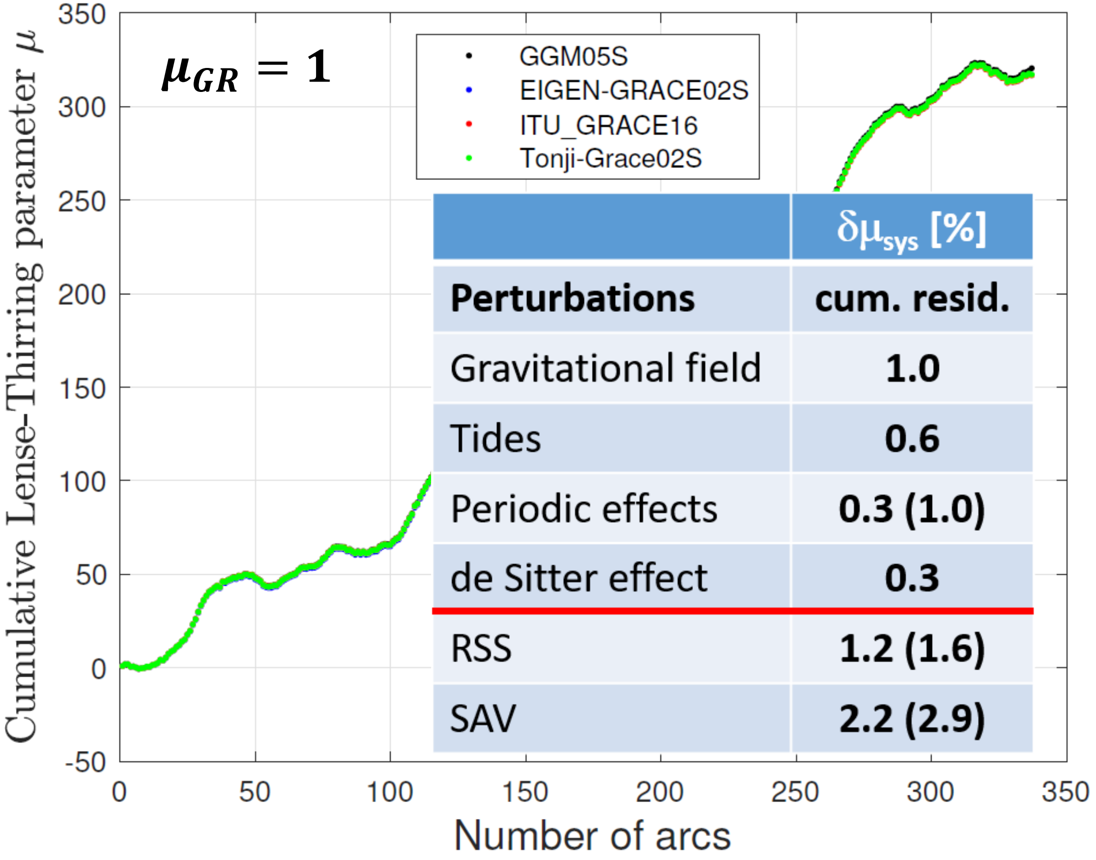
Errors @ 95% CL

Model	$\mu \pm \delta\mu$	$\mu - 1$
GGM05S	1.0053 ± 0.0074	$+0.0053$
EIGEN-GRACE02S	1.0002 ± 0.0074	$+0.0002$
ITU_GRACE16	0.9996 ± 0.0074	-0.0004
Tonji-Grace02s	1.0008 ± 0.0074	$+0.0008$

$$\mu_{meas} - 1 = 1.5 \times 10^{-3} \pm 7.4 \times 10^{-3}$$

Results of the LARASE experiment

Lense-Thirring effect measurement: frame dragging



	$\delta\mu_{sys}$ [%]
Perturbations	**cum. resid.**
Gravitational field	**1.0**
Tides	**0.6**
Periodic effects	**0.3 (1.0)**
de Sitter effect	**0.3**
RSS	**1.2 (1.6)**
SAV	**2.2 (2.9)**

Errors @ 95% CL

Model	$\mu \pm \delta\mu$	$\mu - 1$
GGM05S	1.0053 ± 0.0074	$+0.0053$
EIGEN-GRACE02S	1.0002 ± 0.0074	$+0.0002$
ITU_GRACE16	0.9996 ± 0.0074	-0.0004
Tonji-Grace02s	1.0008 ± 0.0074	$+0.0008$

$$\mu_{meas} - 1 = 1.5 \times 10^{-3} \pm 7.4 \times 10^{-3} \pm 16 \times 10^{-3}$$

Estimation of the systematic errors

Results of the LARASE experiment

The detailed description of the error budget, with the exception of the tidal effects, is the subject of a forthcoming paper

Celestial Mechanics and Dynamical Astronomy (2018) 130:66
https://doi.org/10.1007/s10569-018-9861-5

ORIGINAL ARTICLE

CrossMark

Tidal effects on the LAGEOS–LARES satellites and the LARASE program

Giuseppe Pucacco[1,2] · David M. Lucchesi[2,3,4]

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© Springer Nature B.V. 2018

Solid and Ocean tides

Results of the LARASE experiment

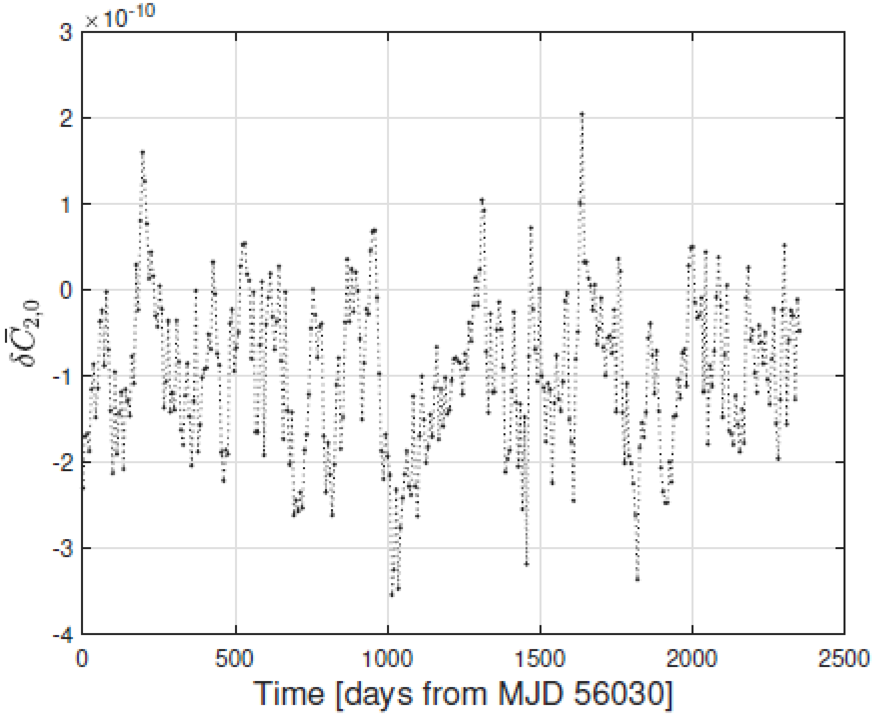
With this precise and accurate measurement of the **GR's Lense-Thirring** precession

- new constraints on alternative theories of gravitation will soon be derived (in preparation)

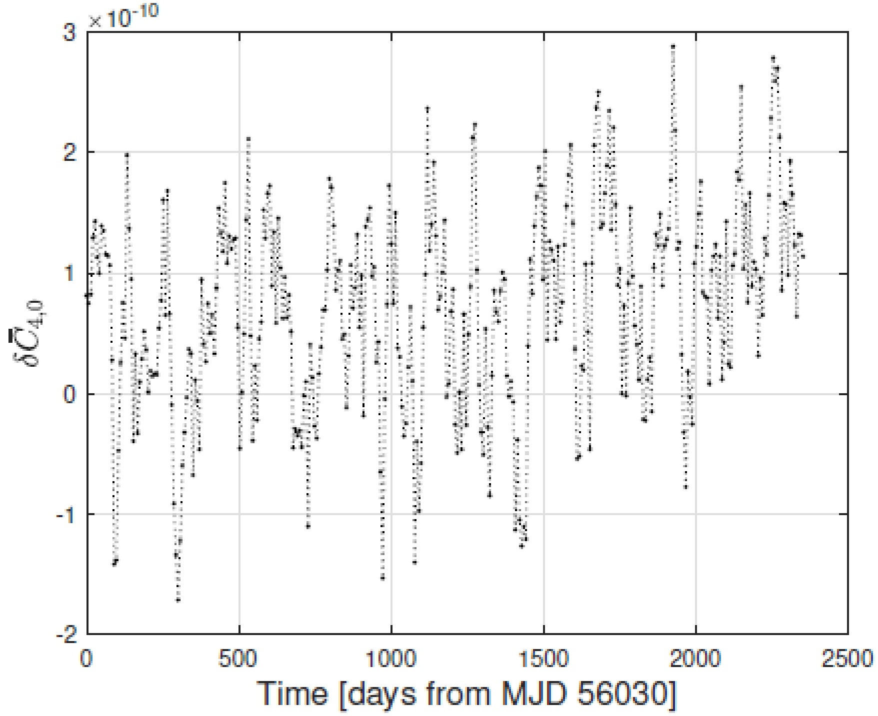
Results of the LARASE experiment

Results from the linear system: μ, $\delta\bar{C}_{2,0}$, $\delta\bar{C}_{4,0}$

$$
\begin{cases}
\dot{\Omega}_2^{L1}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L1}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L1}\mu = \delta\dot{\Omega}_{res}^{L1} \\
\dot{\Omega}_2^{L2}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{L2}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{L2}\mu = \delta\dot{\Omega}_{res}^{L2} \\
\dot{\Omega}_2^{LR}\delta\bar{C}_{2,0} + \dot{\Omega}_4^{LR}\delta\bar{C}_{4,0} + \dot{\Omega}_{LT}^{LR}\mu = \delta\dot{\Omega}_{res}^{LR}
\end{cases}
$$



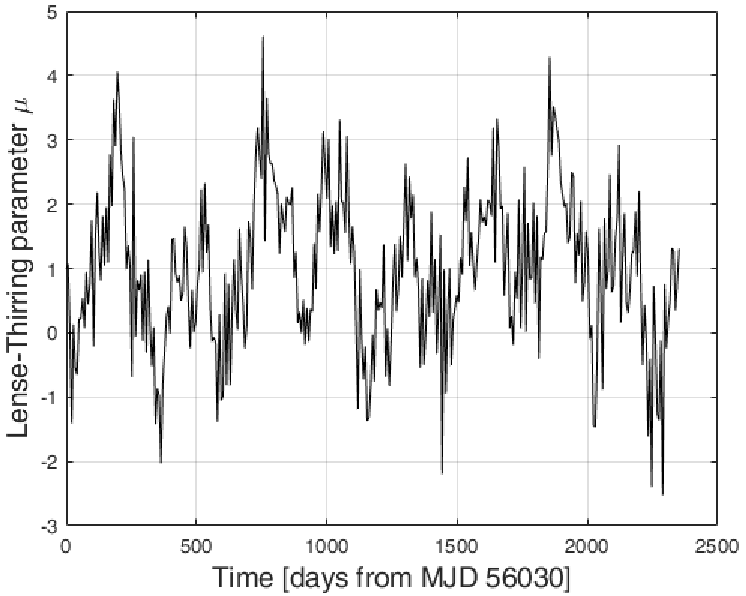
(a) *Corrections to the quadrupole.*



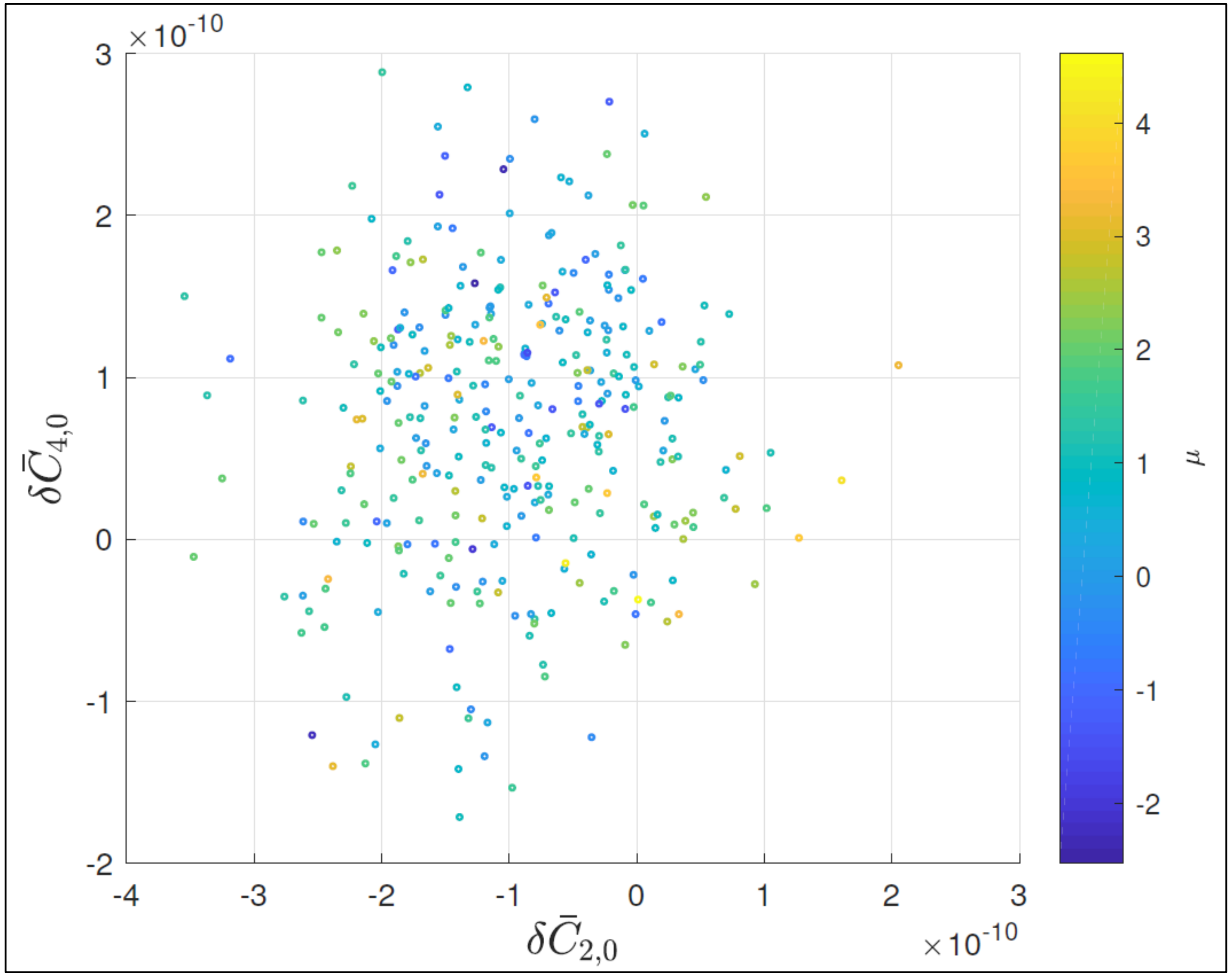
(b) *Corrections to the octupole.*

Results of the LARASE experiment

Results from the linear system: μ, $\delta\bar{C}_{2,0}$, $\delta\bar{C}_{4,0}$

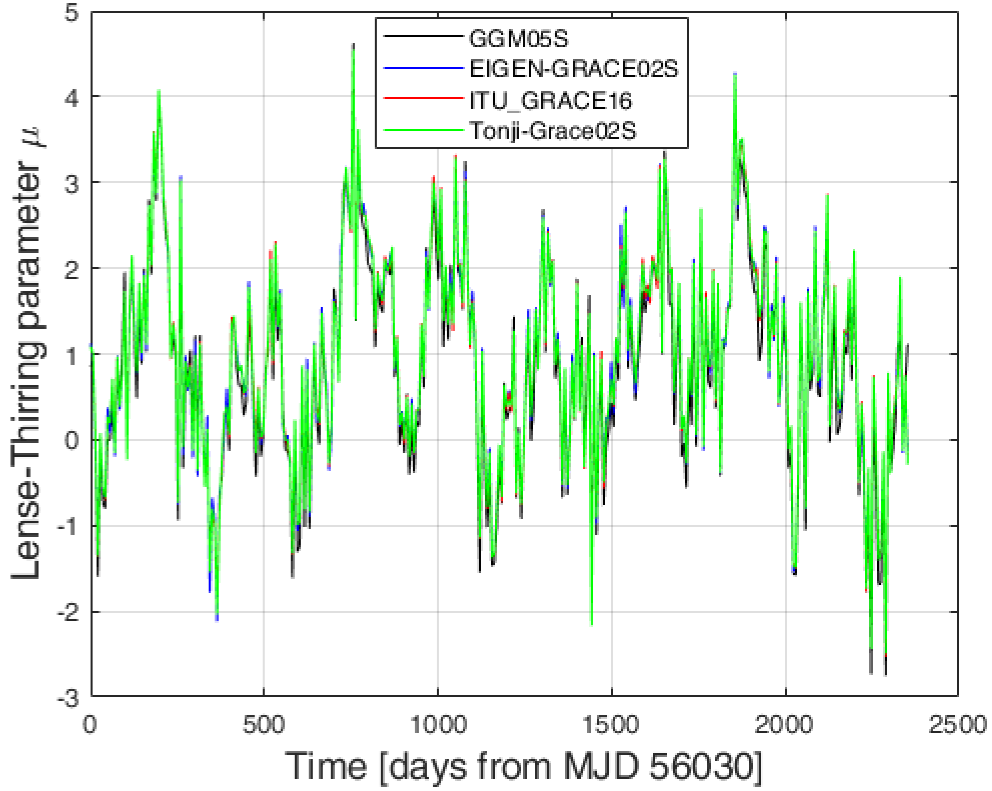


GGM05S model



Results of the LARASE experiment

Results for the Lense-Thirring effect from the residuals in μ



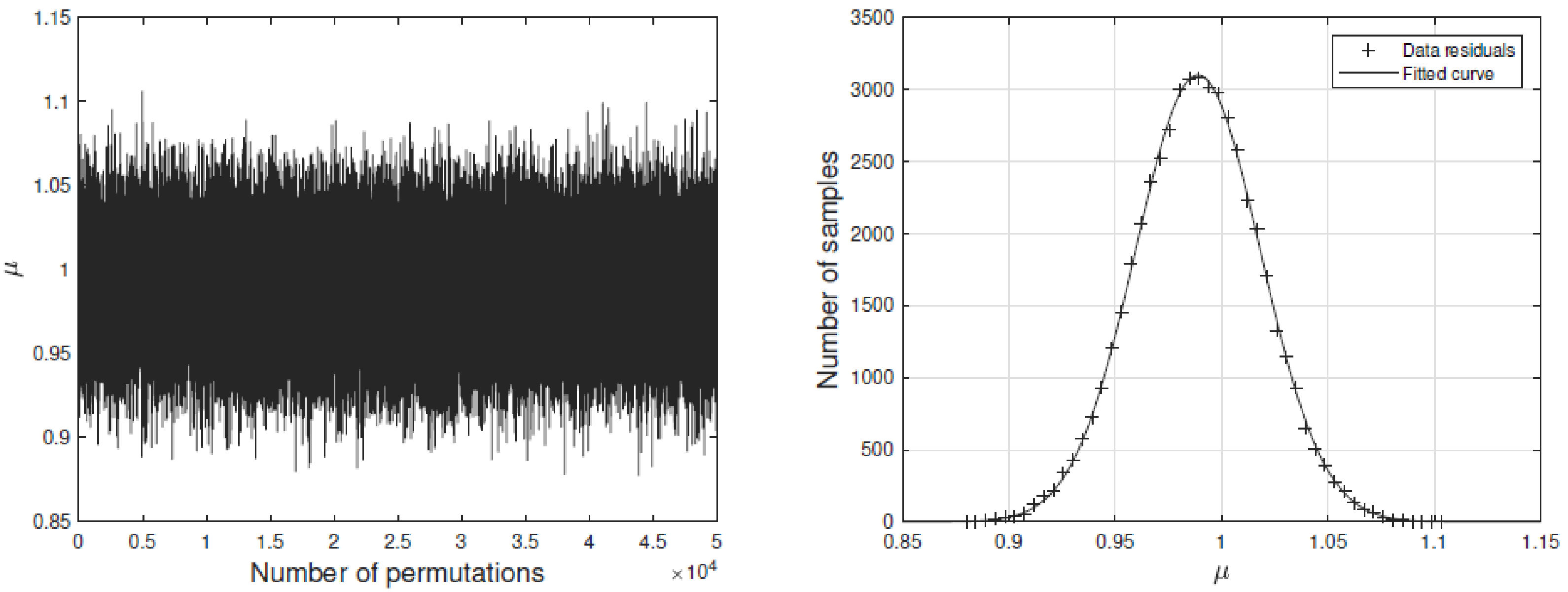
GGM05S model $\mu_{meas} = 0.99 \pm 0.13$

Other models $\mu_{meas} = 0.98 \pm 0.13$

S.D.=1.20

Results of the LARASE experiment

A statistical approach to the measurement of μ



(a) *Random permutations.* (b) *Gaussian fit.*

Figure 7. (a) Realizations for μ after 50,000 random permutations of the 337 residuals of Figure 3. (b) Distribution of the results for the Lense-Thirring parameter μ after the 50,000 permutations. For each realization, μ is obtained by means of the slope of the best linear fit like in Figure 4. The GGM05S model was considered as the background gravitational field of the Earth.

Results of the LARASE experiment

A statistical approach to the measurement of μ

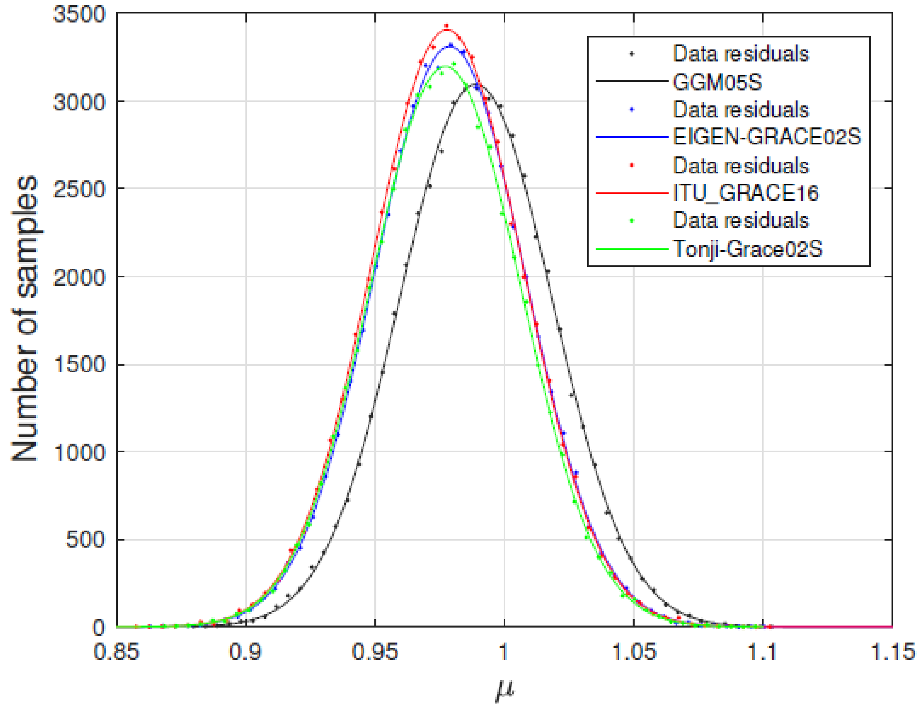


Figure 8. Distribution of the results for the Lense-Thirring parameter μ after 50,000 random permutations. The Gaussian fit for each of the gravitational fields considered in our analyses are shown.

Model	$\bar{\mu} \pm \sigma_\mu$
GGM05S	0.9889 ± 0.0294
EIGEN-GRACE02S	0.9778 ± 0.0292
ITU_GRACE16	0.9772 ± 0.0291
Tonji-Grace02s	0.9788 ± 0.0292

Results of the LARASE experiment

LARASE
LASER RANGED SATELLITES EXPERIMENT

Many thanks for your kind attention