

Hadron Resonance Gas model and Lattice QCD

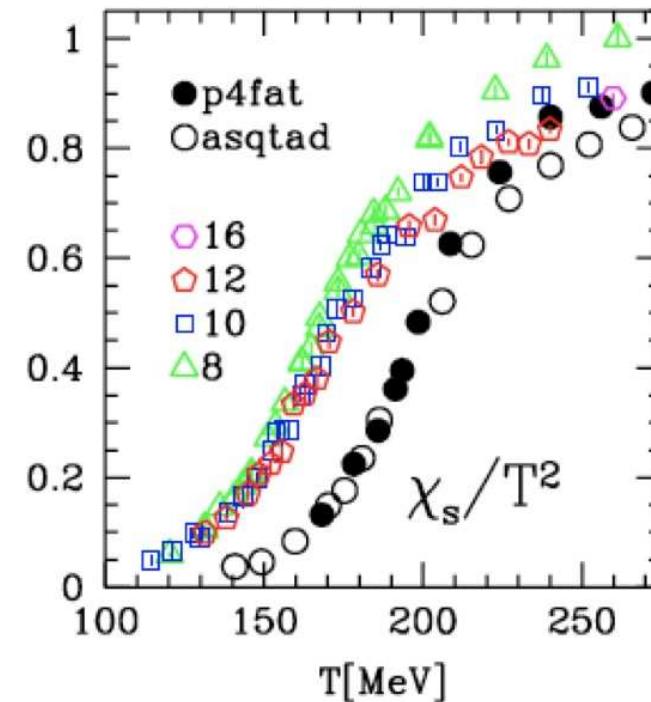
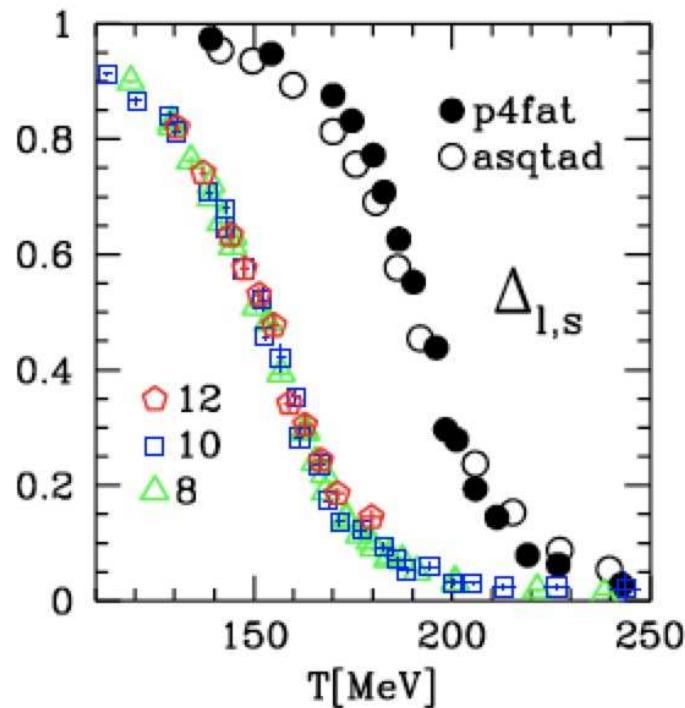
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Motivation



Hot QCD Collaboration:

- Chiral and deconfinement transition at $T_c = 190$ MeV

Budapest-Wuppertal Collaboration:

- Chiral transition at $T_c = 146$ MeV
- Deconfinement transition at $T_c = 170$ MeV

Possible reasons for discrepancy

- ◆ "Non-lattice artefact/formulation" related reasons

- ➡ Bug in the code
- ➡ Vastly underestimated systematic errors

- ◆ "Lattice artefact/formulation" related reasons

- ➡ Difference in pion mass (135 MeV vs. 220 MeV)
- ➡ Not small enough lattice spacings
- ➡ Staggered formulation does not describe QCD

Purpose of our analysis

Use the Hadron Resonance Model

in order to identify

the origin of the discrepancy

In particular:

⇒ Discretization effects

⇒ Effects due to heavy pions

What happens below T_c ?

- ❖ At low T and $\mu = 0$, QCD thermodynamics is dominated by pions
- ❖ The interaction between pions is suppressed
 - ➡ chiral perturbation theory: pion contribution to the thermodynamic potential
 - ➡ the energy density of pions from 3-loop ChPT differs only less than 15% from the ideal gas value

P. Gerber and H. Leutwyler (1989)
- ❖ as T increases, heavier hadrons start to contribute
- ❖ for $T \geq 150$ MeV heavy states dominate the energy density
- ❖ their mutual interactions are proportional to $n_i n_k \sim \exp[-(M_i + M_k)/T]$: they are suppressed
 - ➡ the virial expansion can be used to calculate the effect of the interaction

Why HRG?

- ❖ In the **virial expansion**, the partition function can be split into a **non-interacting** piece and a piece which includes **all interactions**
- ❖ **virial expansion** and experimental information on **scattering phase shift**
Prakash and Venugopalan (1992)
 - ➡ interplay between **attractive** and **repulsive** interaction

Interacting hadronic matter

can be well approximated by

a **non-interacting** gas of **resonances**

Partition function of HRG model

- ❖ The pressure can be written as

$$\begin{aligned}
 p^{HRG}/T^4 = & \frac{1}{VT^3} \sum_{i \in \text{mesons}} \ln \mathcal{Z}_{m_i}^M(T, V, \mu_{X^a}) \\
 & + \frac{1}{VT^3} \sum_{i \in \text{baryons}} \ln \mathcal{Z}_{m_i}^B(T, V, \mu_{X^a}) ,
 \end{aligned}$$

where

$$\ln \mathcal{Z}_{m_i}^{M/B} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp \left(\left(\sum_a X_i^a \mu_{X^a} \right) / T \right) .$$

X^a : all possible conserved charges, including the baryon number B , electric charge Q , strangeness S .

Characteristics of hadronic species included in the calculation

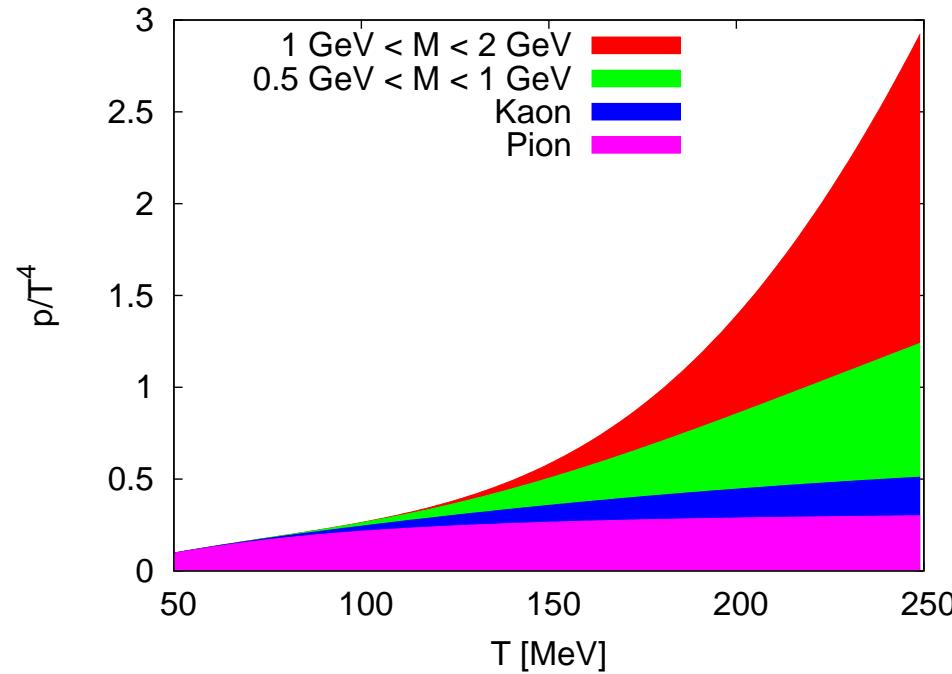
hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
π	0.140	3	0	0	1	N (1535)	1.530	4	1	0	1/2
K	0.496	2	0	1	1/2	π_1 (1600)	1.596	9	0	0	1
\bar{K}	0.496	2	0	-1	1/2	Δ (1600)	1.600	16	1	0	3/2
η	0.543	1	0	0	0	Λ (1600)	1.600	2	1	-1	0
ρ	0.776	9	0	0	1	Δ (1620)	1.630	8	1	0	3/2
ω	0.782	3	0	0	0	η_2 (1645)	1.617	5	0	0	0
K^*	0.892	6	0	1	1/2	N (1650)	1.655	4	1	0	1/2
\bar{K}^*	0.892	6	0	-1	1/2	ω (1650)	1.670	3	0	0	0
N	0.939	4	1	0	1/2	Σ (1660)	1.660	6	1	-1	1
η'	0.958	1	0	0	0	Λ (1670)	1.670	2	1	-1	0
f_0	0.980	1	0	0	0	Σ (1670)	1.670	2	1	-1	1
a_0	0.980	3	0	0	1	ω_3 (1670)	1.667	7	0	0	0
ϕ	1.020	3	0	0	0	π_2 (1670)	1.672	15	0	0	1
Λ	1.116	2	1	-1	0	Ω^-	1.672	4	1	-3	0
h_1	1.170	3	0	0	1	N (1675)	1.675	12	1	0	1/2
Σ	1.189	6	1	-1	1	ϕ (1680)	1.680	3	0	0	0
a_1	1.230	9	0	0	1	K^* (1680)	1.717	6	0	1	1/2
b_1	1.230	9	0	0	1	\bar{K}^* (1680)	1.717	6	0	-1	1/2
Δ	1.232	16	1	0	3/2	N (1680)	1.685	12	1	0	1/2
f_2	1.270	5	0	0	0	ρ_3 (1690)	1.688	21	0	0	1
K_1	1.273	6	0	1	1/2	Λ (1690)	1.690	4	1	-1	0
\bar{K}_1	1.273	6	0	-1	1/2	Ξ (1690)	1.690	8	1	-2	1/2
f_1	1.285	3	0	0	1	ρ (1700)	1.720	9	0	0	1
η (1295)	1.295	1	0	0	0	N (1700)	1.700	8	1	0	1/2
π (1300)	1.300	3	0	0	1	Δ (1700)	1.700	16	1	0	3/2

Characteristics of hadronic species included in the calculation

hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
Ξ	1.315	4	1	-2	1/2	N (1710)	1.710	4	1	0	1/2
a_2	1.318	15	0	0	1	f_0 (1710)	1.714	1	0	0	0
f_0 (1370)	1.370	1	0	0	1	N (1720)	1.720	8	1	0	1/2
Σ (1385)	1.385	12	1	-1	1	Σ (1750)	1.750	6	1	-1	1
K_1 (1400)	1.400	6	0	1	1/2	K_2 (1770)	1.773	10	0	1	1/2
\bar{K}_1 (1400)	1.400	6	0	-1	1/2	\bar{K}_2 (1770)	1.773	10	0	-1	1/2
η (1405)	1.405	1	0	0	0	Σ (1775)	1.775	18	1	-1	1
Λ (1405)	1.406	2	1	-1	0	K_3^* (1780)	1.776	14	0	1	1/2
K^* (1410)	1.414	6	0	1	1/2	\bar{K}_3^* (1780)	1.776	14	0	-1	1/2
\bar{K}^* (1410)	1.414	6	0	-1	1/2	π (1800)	1.812	3	0	0	1
f_1 (1420)	1.420	3	0	0	1	Λ (1800)	1.800	2	1	-1	0
ω (1420)	1.420	3	0	0	0	Λ (1810)	1.810	2	1	-1	0
K_0^*	1.425	2	0	1	1/2	K_2 (1820)	1.816	10	0	1	1/2
\bar{K}_0^*	1.425	2	0	-1	1/2	\bar{K}_2 (1820)	1.816	10	0	-1	1/2
K_2^*	1.430	10	0	1	1/2	Λ (1820)	1.820	6	1	-1	0
\bar{K}_2^*	1.430	10	0	-1	1/2	Ξ (1820)	1.823	8	1	-2	1/2
N (1440)	1.440	4	1	0	1/2	Λ (1830)	1.830	6	1	-1	0
ρ (1450)	1.465	9	0	0	1	ϕ_3 (1850)	1.854	7	0	0	0
a_0 (1450)	1.472	3	0	0	1	π_2 (1880)	1.895	15	0	0	1
η (1475)	1.476	1	0	0	0	Λ (1890)	1.890	4	1	-1	0
f_0 (1500)	1.505	1	0	0	0	Δ (1905)	1.890	24	1	0	3/2
Λ (1520)	1.520	4	1	-1	0	Δ (1910)	1.910	8	1	0	3/2
N (1520)	1.520	8	1	0	1/2	Δ (1920)	1.920	16	1	0	3/2
f'_2 (1525)	1.525	5	0	0	0	Δ (1930)	1.930	24	1	0	3/2
Ξ (1530)	1.533	8	1	-2	1/2	Δ (1950)	1.930	32	1	0	3/2

How many resonances do we include?

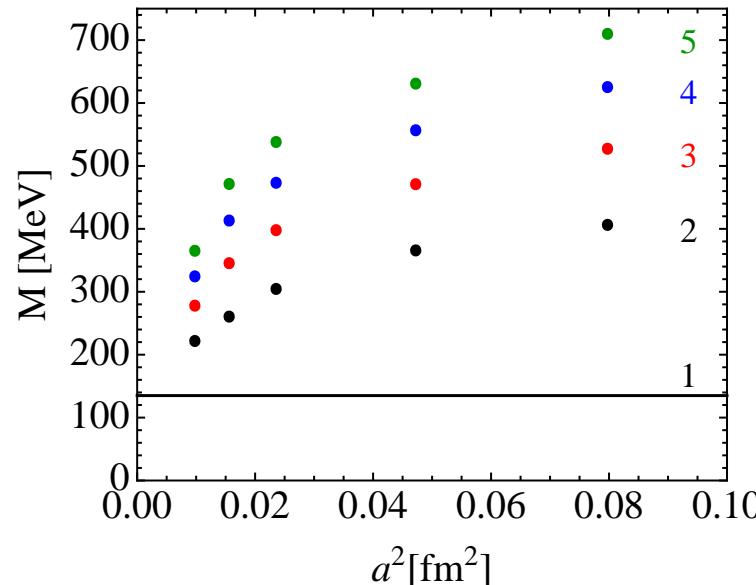
- ◆ With different mass cutoffs we can separate the contributions of different particles



- ◆ No visible difference between cuts at 2 GeV and 2.5 GeV in our temperature regime
- ◆ We include all resonances with $M \leq 2.5 \text{ GeV}$
 - $\simeq 170$ different masses \leftrightarrow 1500 resonances

Pseudo-scalar mesons in staggered formulation

- ◆ Staggered formulation: **four degenerate quark flavors** in the continuum limit
- ◆ **Rooting procedure**: replace fermion determinant in the partition function by its **fourth root**
- ◆ At **finite lattice spacing** the four flavors are not degenerate
 - ➡ each pion is split into 16
 - ➡ the sixteen pseudo-scalar mesons have **unequal masses**
 - ➡ **only one** of them has vanishing mass in the chiral limit

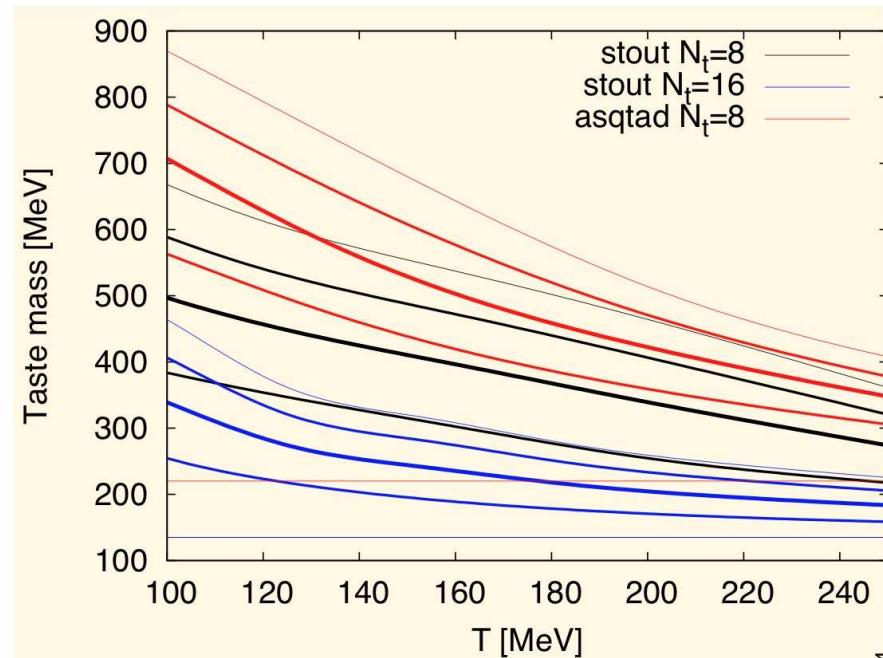


Pseudo-scalar mesons in staggered formulation: comparison between stout and asqtad

- ◆ The lattice-spacing dependence of the masses can be translated into a temperature dependence:

$$T = \frac{1}{N_t a}$$

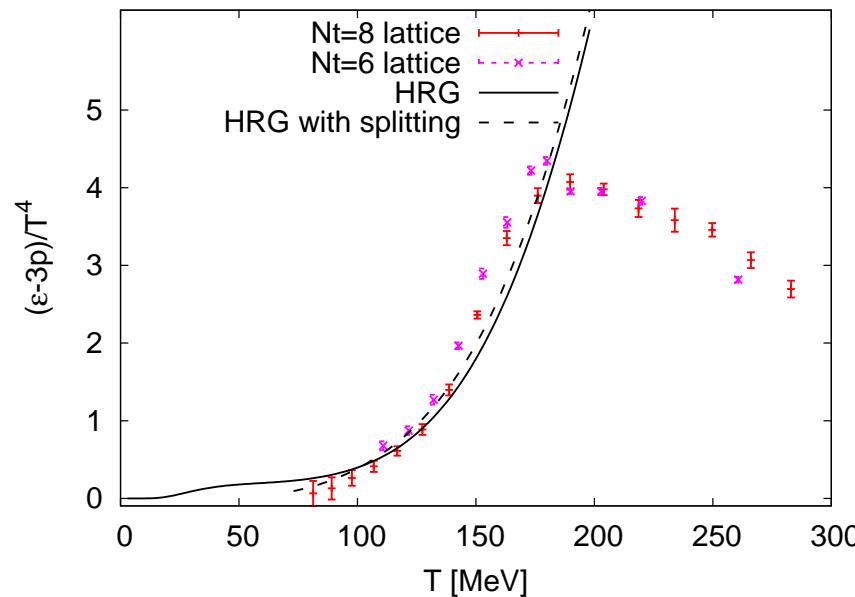
- ◆ The mass splitting for asqtad action is larger than the one for stout action



- ◆ The lighter pion for asqtad action is ~ 100 MeV heavier than the one for stout action

Results: trace anomaly

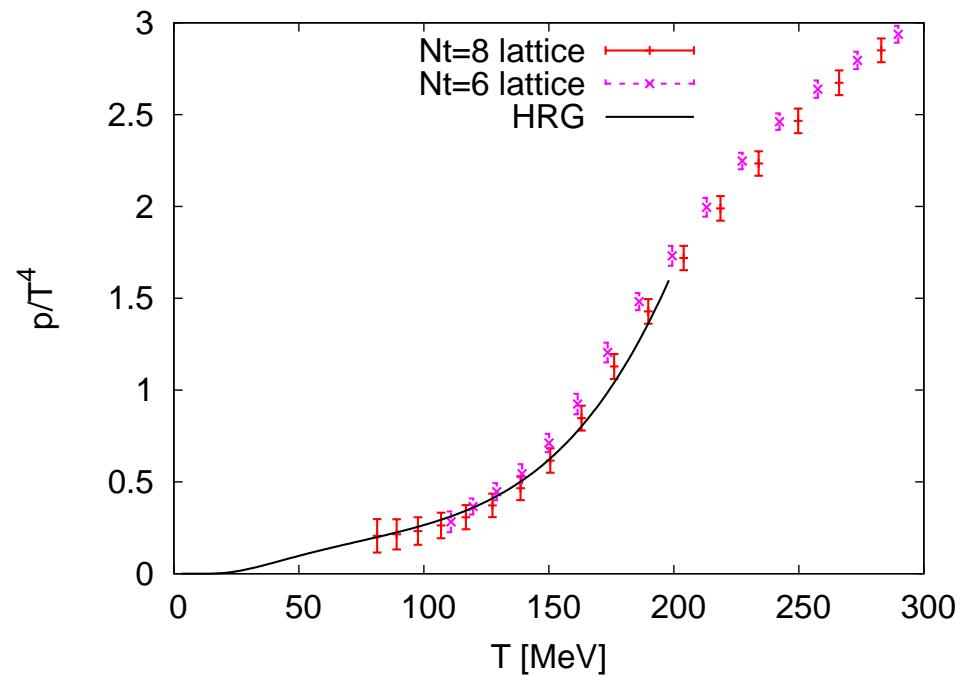
$$\frac{\theta(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \left(p/T^4 \right)$$



- ❖ No significant difference between curves with and without splitting
- ❖ Very good agreement between HRG and lattice results
- ❖ No lattice data available for $T \leq 80$ MeV
 - ➡ Use HRG result to set the integration constant for the pressure

Results: pressure

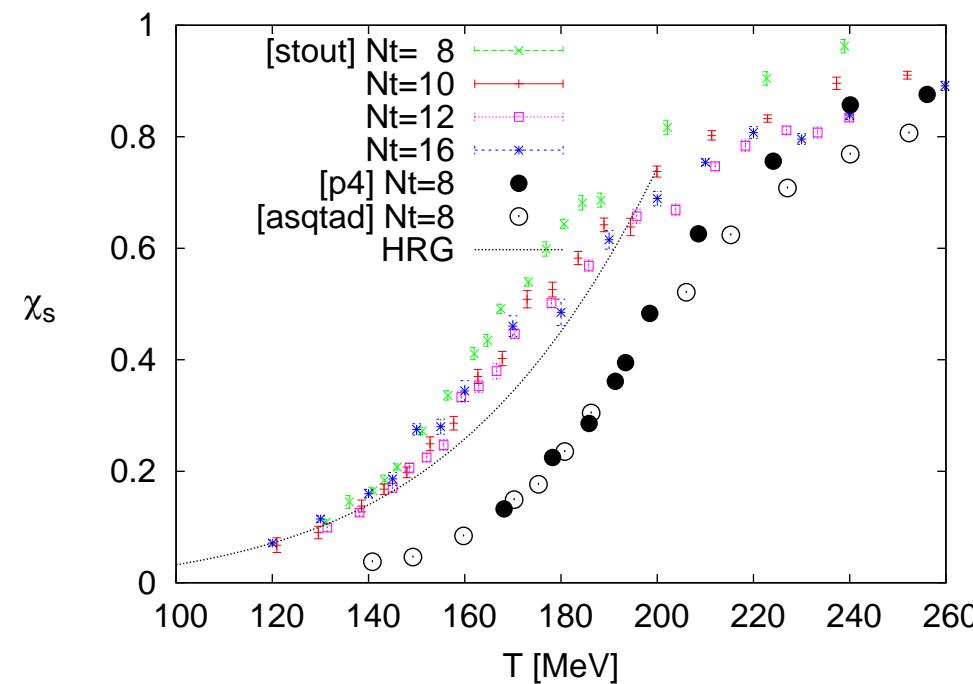
$$\frac{p(T)}{T^4} = \frac{p(T_0)}{T_0^4} + \int_{T_0}^T dT' \frac{1}{T'^5} \theta(T')$$



- ◆ Very good agreement between HRG and lattice results

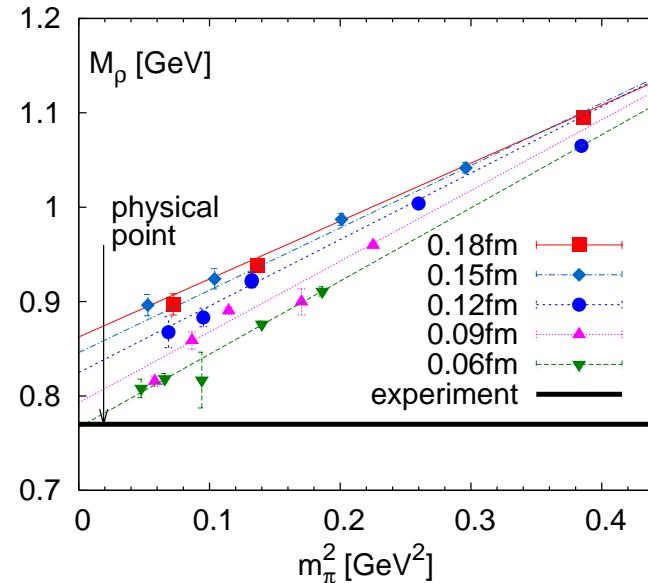
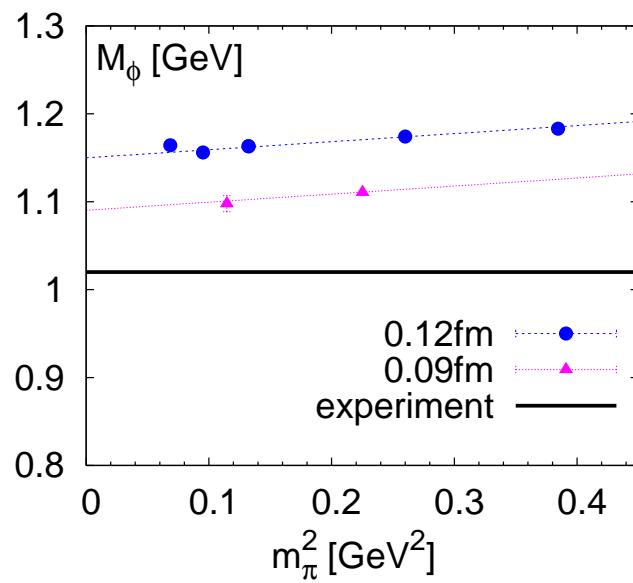
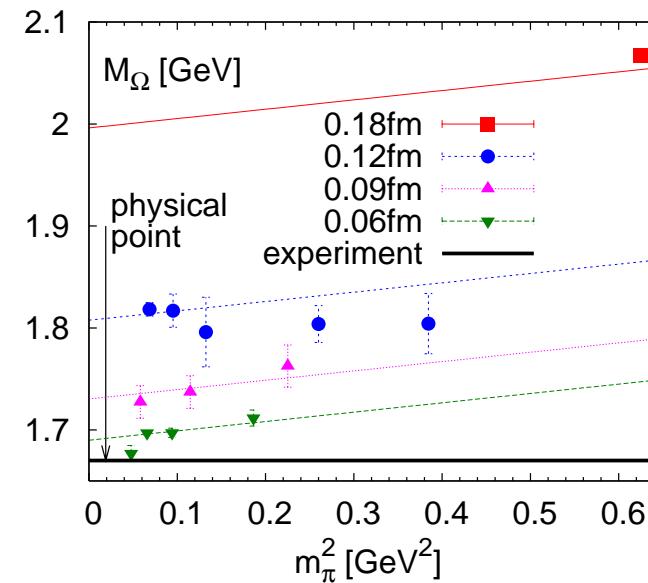
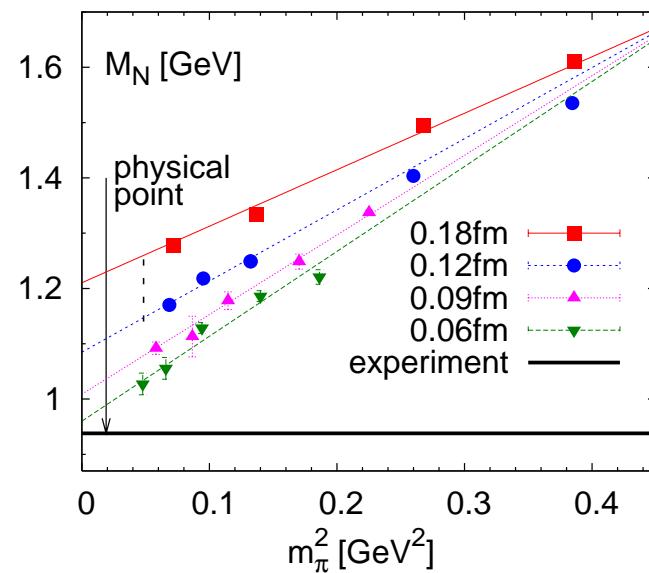
Results: strangeness fluctuations

$$\chi_n^S = T^n \frac{\partial^n p(T, \mu_B, \mu_S, \mu_I)}{\partial \mu_S^n} |_{\mu_X=0}$$



- ❖ HRG results in **good agreement** with stout action
- ❖ asqtad and p4 results show **similar shape** but **shift in temperature**

Discretization effects



Hadron masses

❖ Non-strange baryons and mesons:

$$r_1 m = r_1 m_0 + \frac{a_1(r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x}, \quad x = \left(\frac{a}{r_1}\right)^2$$

❖ Strange baryons and mesons:

$$r_1 \cdot m_\Lambda(a, m_\pi) = m_\Lambda^{phys} + \frac{2}{3} \frac{a_1(r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Lambda^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

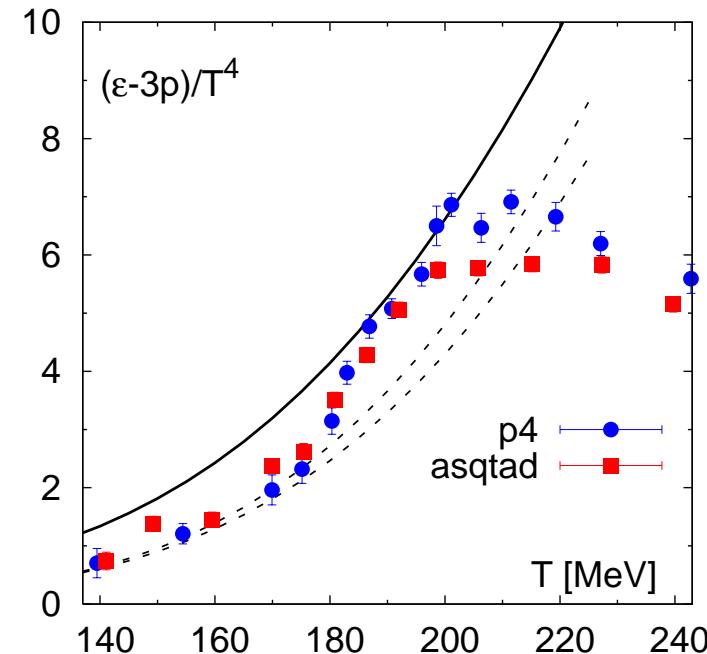
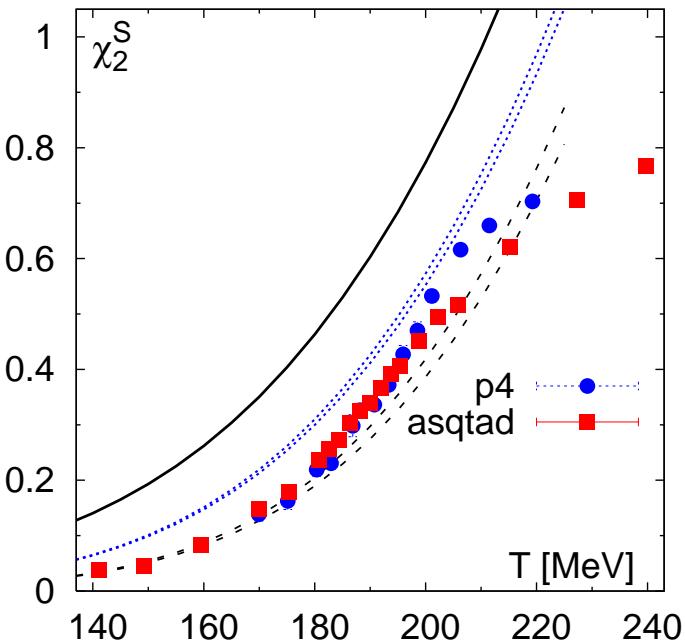
$$r_1 \cdot m_\Sigma(a, m_\pi) = m_\Sigma^{phys} + \frac{1}{3} \frac{a_1(r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Sigma^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

$$r_1 \cdot m_\Xi(a, m_\pi) = m_\Xi^{phys} + \frac{1}{3} \frac{a_1(r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Xi^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right)$$

$$r_1 m_\Omega(a, m_\pi) = r_1 m_\Omega^{phys} + a_1(r_1 m_\pi)^2 - a_1(r_1 m_\pi^{phys})^2 + b_1 x + (m_\Omega^{phys} - m_\Delta^{phys}) \cdot 1.02 x$$

❖ Assumption: all resonances behave as their fundamental states

HRG results with asqtad input



- ❖ Lattice results and dashed curves: $N_t = 8$
- ❖ good agreement between HRG and asqtad results once the splitting and lattice spacing dependence of hadron masses are taken into account
- ❖ discretization effects shift the curves towards higher temperatures

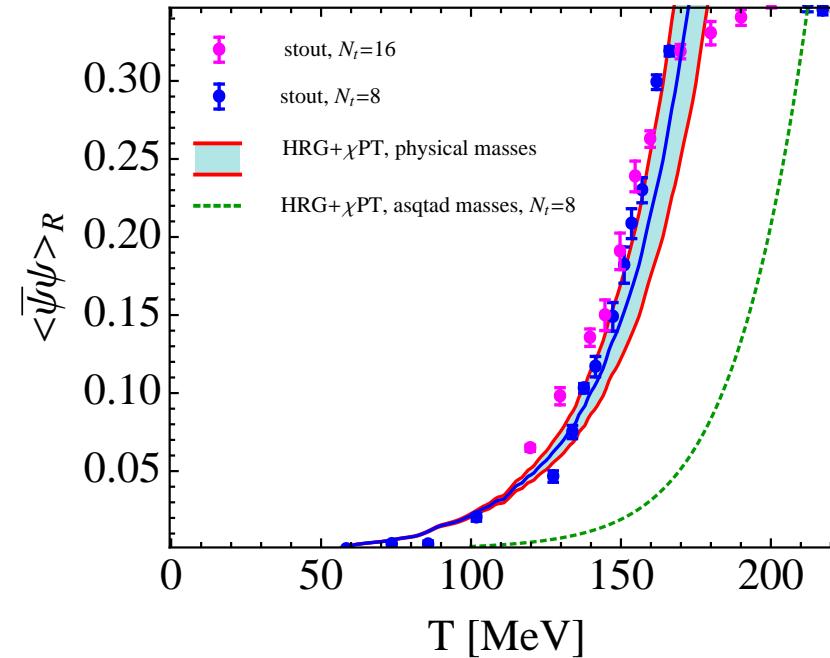
Calculation of the chiral condensate

$$\langle \bar{\psi} \psi \rangle = -\frac{\partial p}{\partial m_q}, \quad \langle \bar{\psi} \psi \rangle_R = [\langle \bar{\psi} \psi \rangle - \langle \bar{\psi} \psi \rangle_0] \frac{m_q}{m_\pi^4}$$

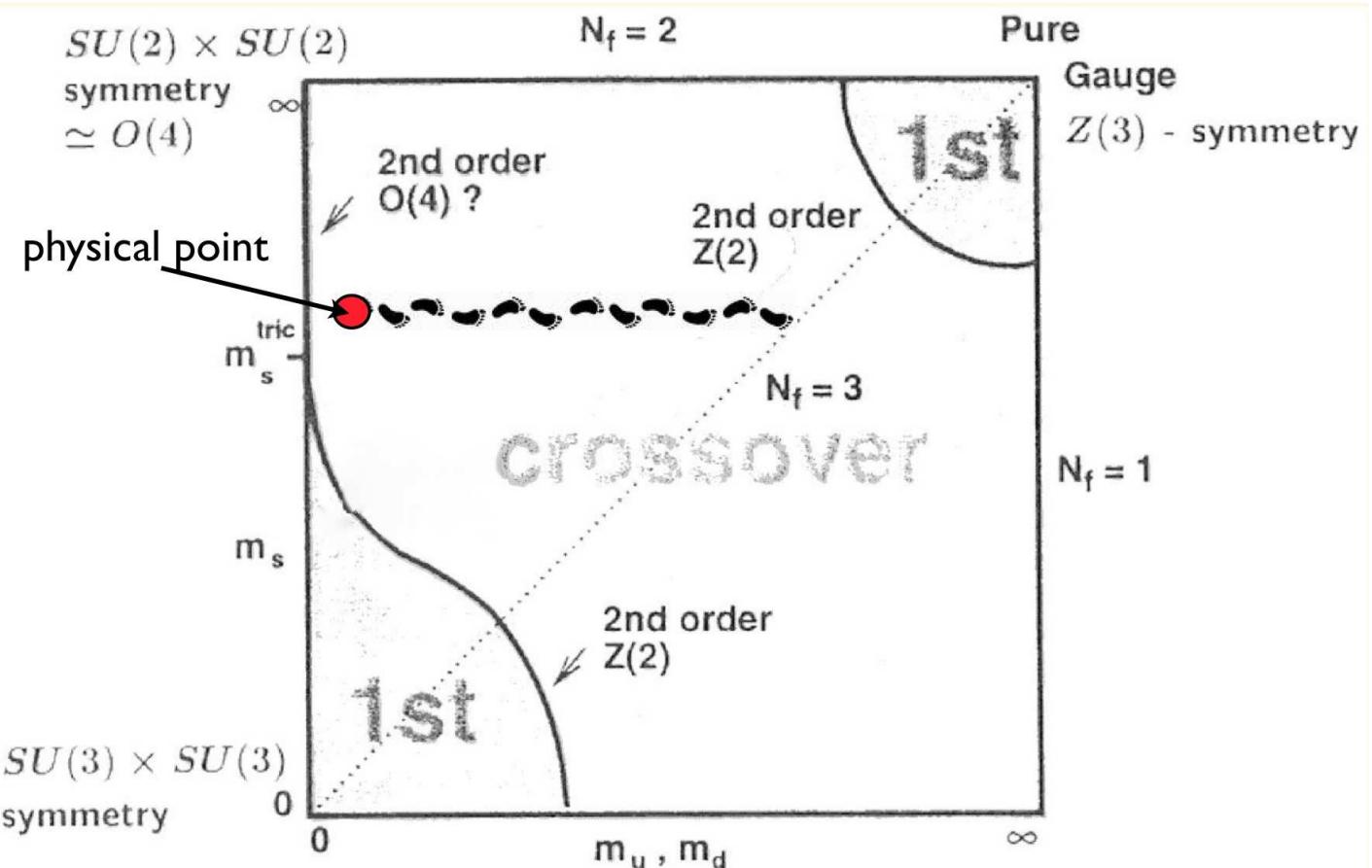
$$\langle \bar{\psi} \psi \rangle = \langle \bar{\psi} \psi \rangle_0 + \langle \bar{\psi} \psi \rangle_\pi + \sum_{i \in mesons} \frac{\partial \ln Z_{m_i}^M}{\partial m_i} \frac{\partial m_i}{\partial m_\pi^2} \frac{\partial m_\pi^2}{\partial m_q} + \sum_{i \in baryons} \frac{\partial \ln Z_{m_i}^B}{\partial m_i} \frac{\partial m_i}{\partial m_\pi^2} \frac{\partial m_\pi^2}{\partial m_q}.$$

◆ Contribution of pions from Chiral Perturbation Theory

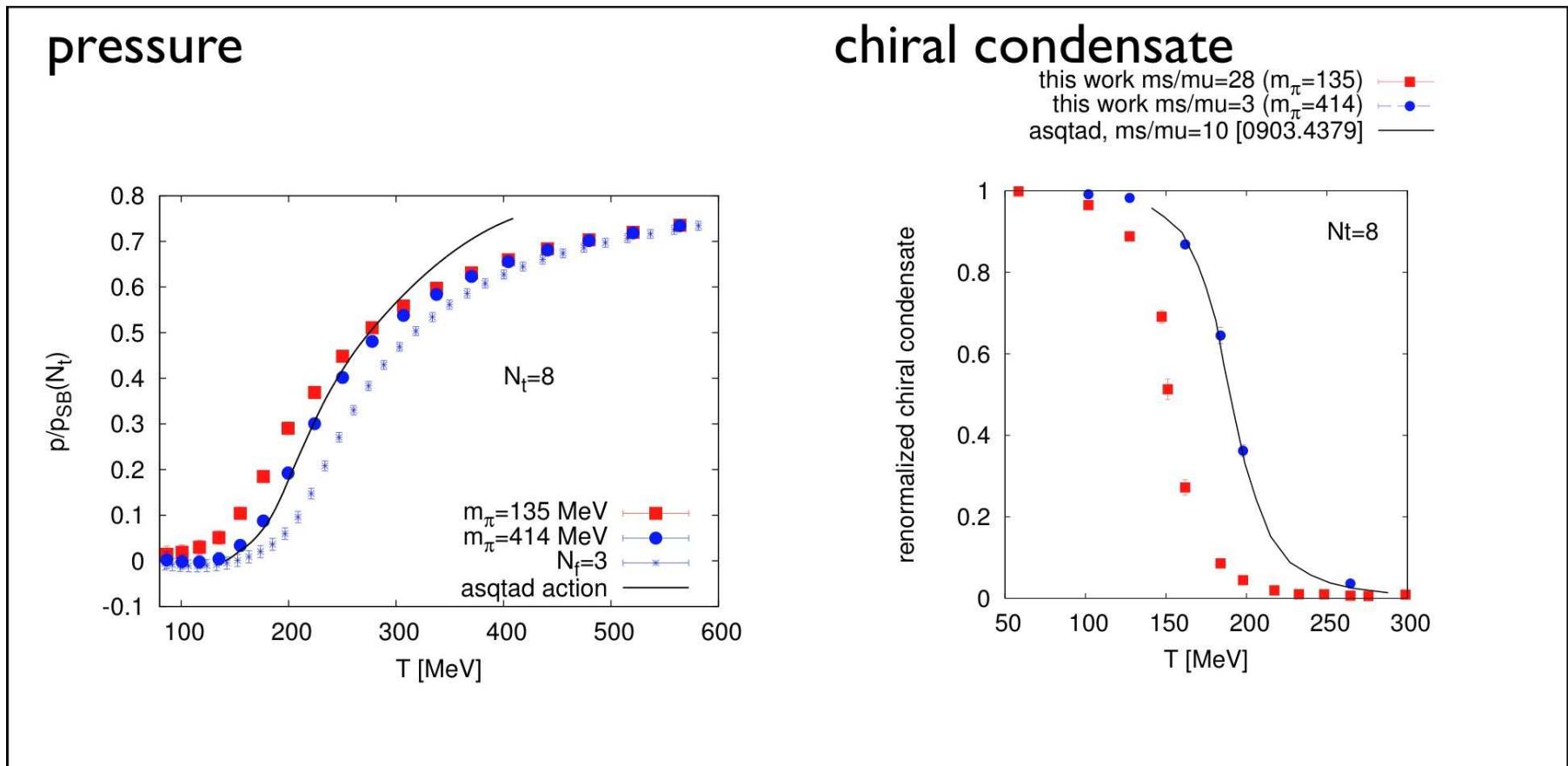
Gerber and Leutwyler (1989)



Pion mass dependence



Transition temperature and m_π



- ❖ We reproduce the **HotQCD transition temperature** by using a heavier pion mass
- ❖ We see coincidence of **chiral** and **deconfinement** temperatures at this pion mass

Conclusions

- ◆ Hadron Resonance Model reliable for $T \leq 150 - 160$ MeV
- ◆ Low temperature thermodynamics dominated by pions
- ◆ HRG results very sensitive to pion mass
- ◆ pseudoscalar meson splitting in stout action not too large
 - ⇒ good agreement with HRG results with physical masses
- ◆ pseudoscalar meson splitting in asqtad action more severe (even more with p4 action)
 - ⇒ inclusion of mass splitting and lattice spacing dependence in HRG model explains temperature shift