UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN SEZIONE DI CATANIA

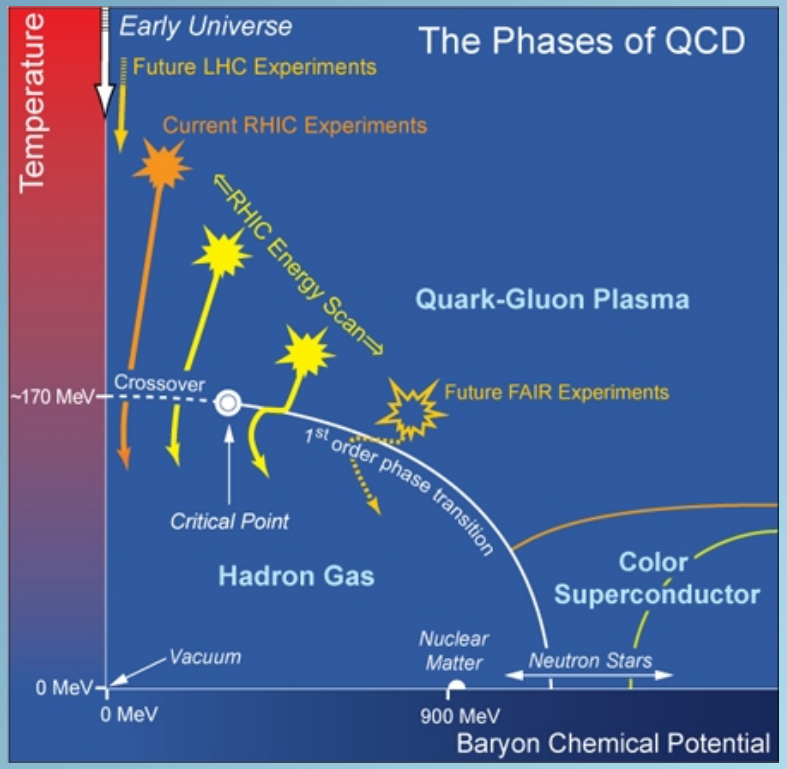
EFFETTO DELLA DINAMICA NJL SULLA GENERAZIONE DEI FLUSSI COLLETTIVI

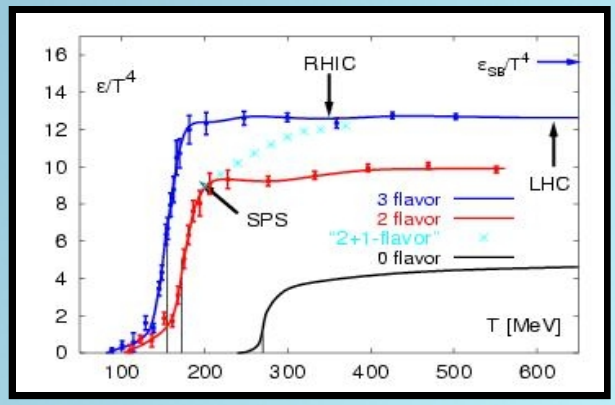
M. COLONNA, M. DI TORO,

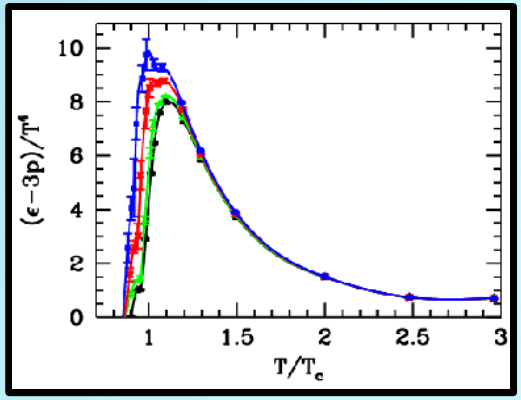
V. GRECO, S. PLUMARI

- Introduction: search of QGP.
- Effective field theory: Nambu-Jona Lasinio model (NJL).
- Non-equilibrium dynamics: Boltzmann - Vlasov transport equation for the NJL model.
- Conclusions and outlook.

Probing the phase diagram





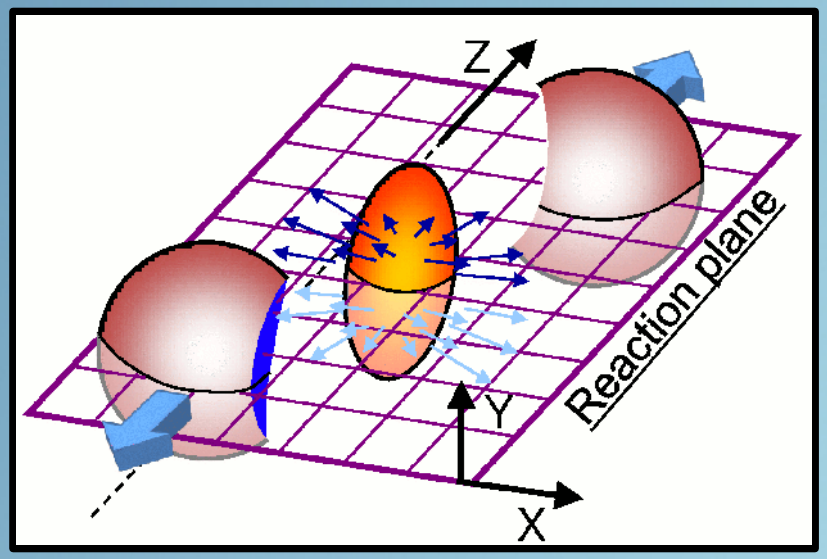


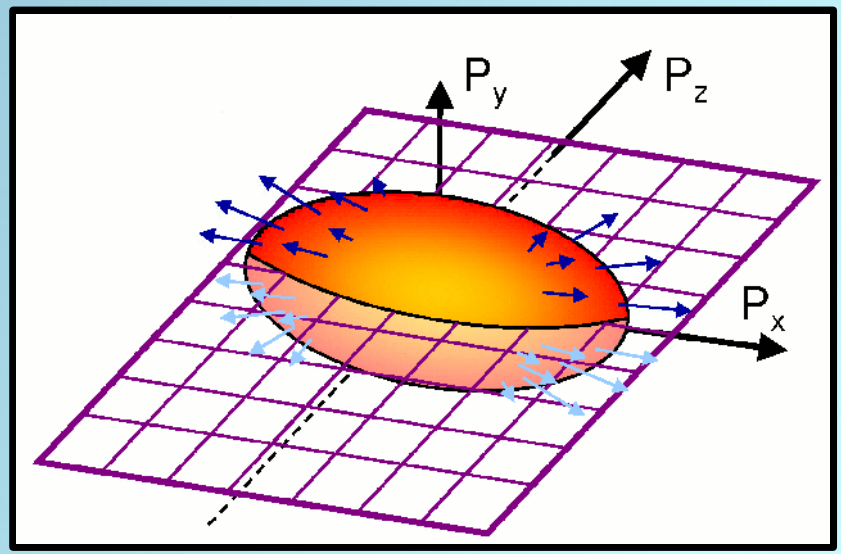
- **Enhancement of the degrees of freedom at high temperature.**

$$\varepsilon \approx 0.7\, GeV / fm^3$$

$$T_c \cong 170 \pm 20\, MeV$$

- **Stefan-Boltzmann limit not reached by 20%.**





$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

$$C_s^2 = dP/d\epsilon$$

$$v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

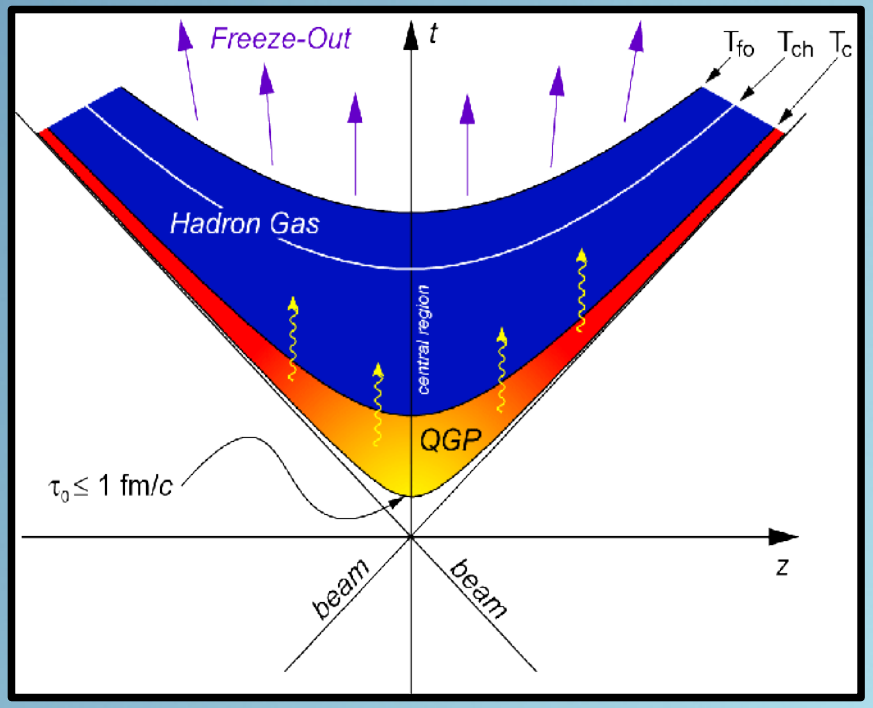
Performing a Fourier expansion of the momentum space of particle distributions

$$\frac{dN}{dp_T d\varphi} = \frac{dN}{dp_T}\left[1 + 2\sum_n \mathrm{v}_n \cos(n\varphi)\right]$$

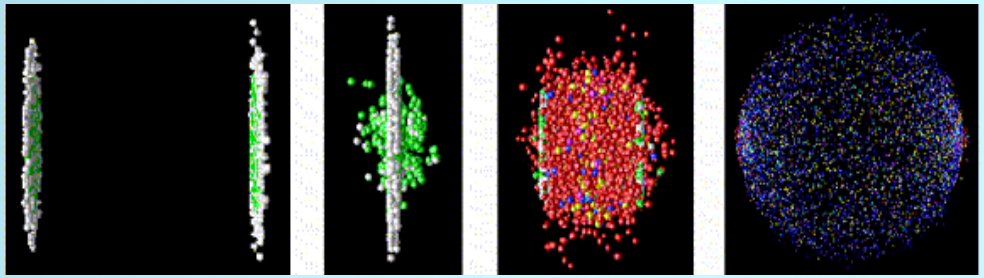
Free streaming v_2=0

v_2 is the 2nd hamonic Fourier coefficent of the particle distribution.

Probing the phase diagram



- Non-equilibrium processes of non-Abelian gauge theory are involved.
- Perturbative and non-perturbative regime of QCD.
- *Need of effective lagrangian approach.*

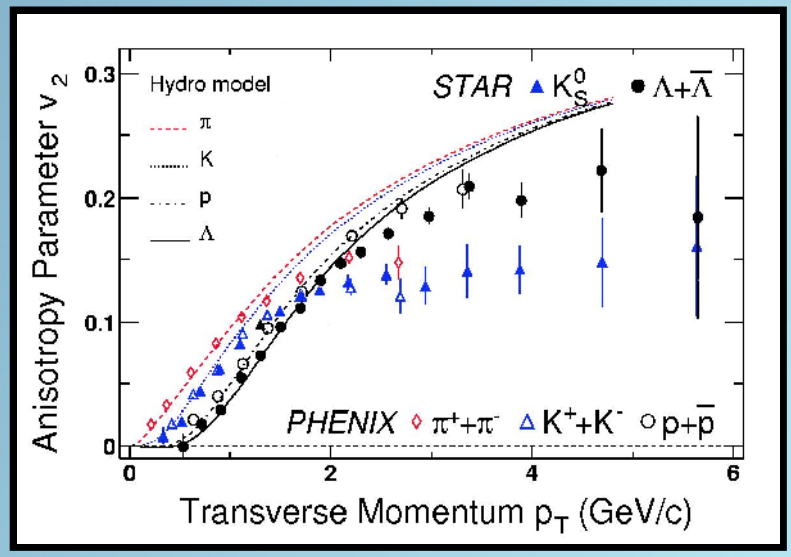


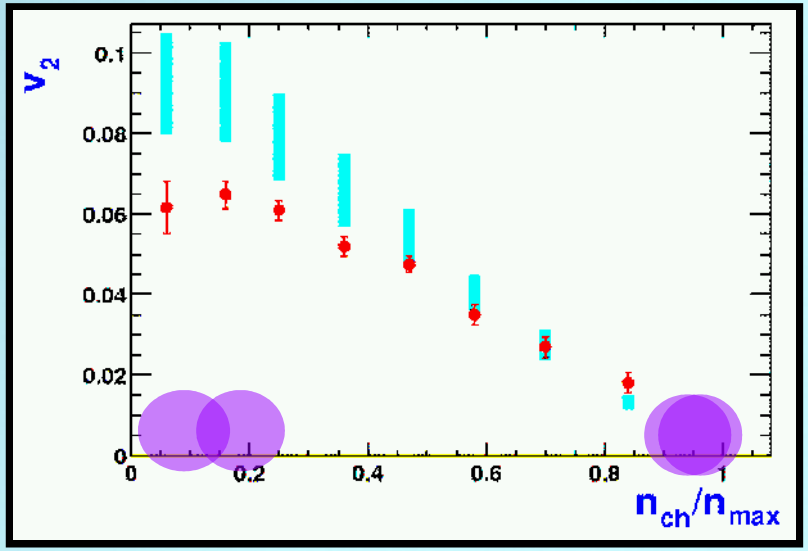
Ideal hydrodynamics

$$\begin{cases} \partial_\mu T^{\mu\nu}(x) = 0 \\ \partial_\mu j_B^\mu(x) = 0 \end{cases}$$

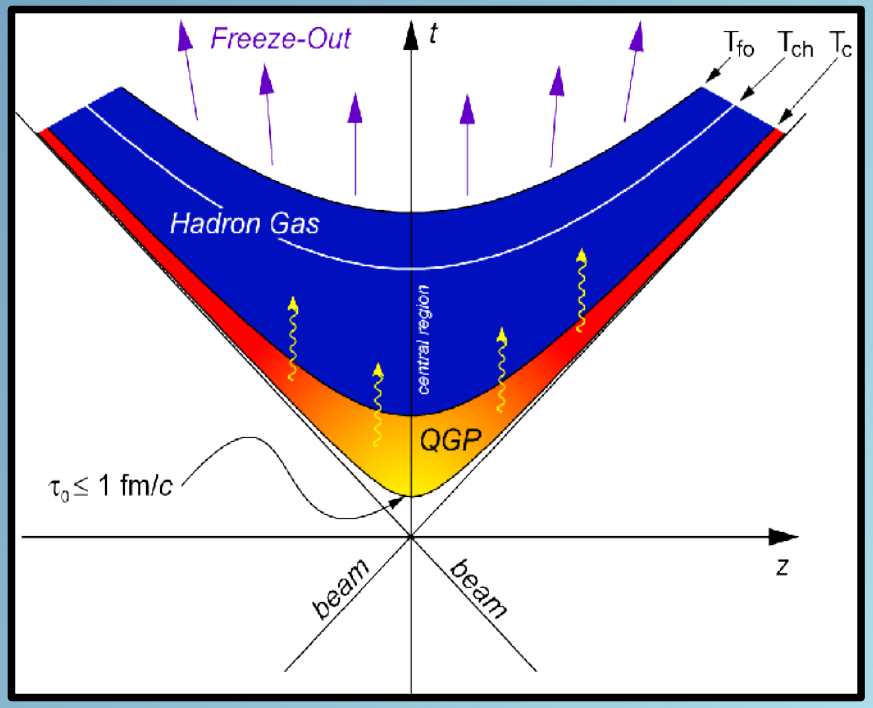
- System closed by the EoS, P(ε).
- Macroscopic desciption, no details about the dynamics.
- Mean free path vanishing.

- Good description for p_T < 1.5 GeV and b < 7 fm.
- Mass ordering of v_2 versus p_T.

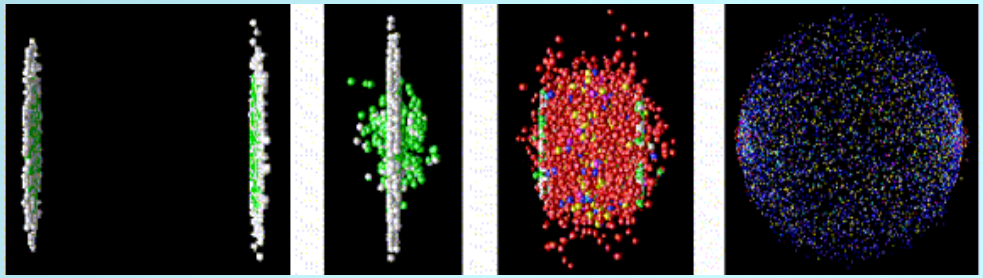




Motivation for a kinetic approach:



- Microscopic description.
- It is a 3+1D (viscous hydro is 2+1D).
- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (to study the effect of the hadronic phase).
- Extension to Bulk viscosity ζ (instabilities in hydrodynamics).



The Nambu-Jona Lasinio model

$$\mathscr{L}_{NJL} = \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + g\left[\left(\bar{\psi}\psi\right)^2 + \sum_{\alpha-1}^{N_f^2-1}\left(\bar{\psi}\tau^\alpha i\gamma_5\psi\right)^2\right]$$

- **The chiral symmetry $SU(N_f)_R \times SU(N_f)_L$ is exact in the chiral limit.**
- **The parameters m, g, Λ are fixed to reproduce m_π, f_π, $\langle\bar{\psi}\psi\rangle$ at T=0.**

Y. Nambu and G. Jona-Lasinio, Phys.Rev.**122**, 345 (1961); Phys.Rev.**124**, 246 (1961).

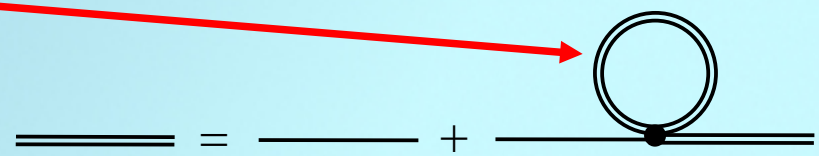
The loop expansion for the effective action,

$$\Gamma[G] = -i\,Tr\ln G^{-1} - i\,Tr(D^{-1}\cdot G) + \Gamma_2[G] + const$$

$\Gamma_2[G]$ $\xrightarrow[\text{approximation}]{\text{2-loop}}$

$$\Gamma_2[G] = g\int d^4x\{[tr\,G(x,x)]^2 - [tr\,G(x,x)G(x,x)]\} +$$
$$+ g\int d^4x\{\sum_{\alpha=1}^{N_f^2-1}([tr\,i\gamma_5\tau^\alpha G(x,x)]^2 - tr[i\gamma_5\tau^\alpha G(x,x)i\gamma_5\tau^\alpha G(x,x)])\}$$

$$0 = \frac{\delta\Gamma[G]}{\delta G} = iG^{-1}(x-y) - iD^{-1}(x-y) + \frac{\delta\Gamma_2}{\delta G}$$



$$G(x-y) = D(x-y) - 2i\,g\,tr\,G(0)\,D\cdot G$$

R. Jackiw, Phys. Rev. **D 9**, 1686 (1974).

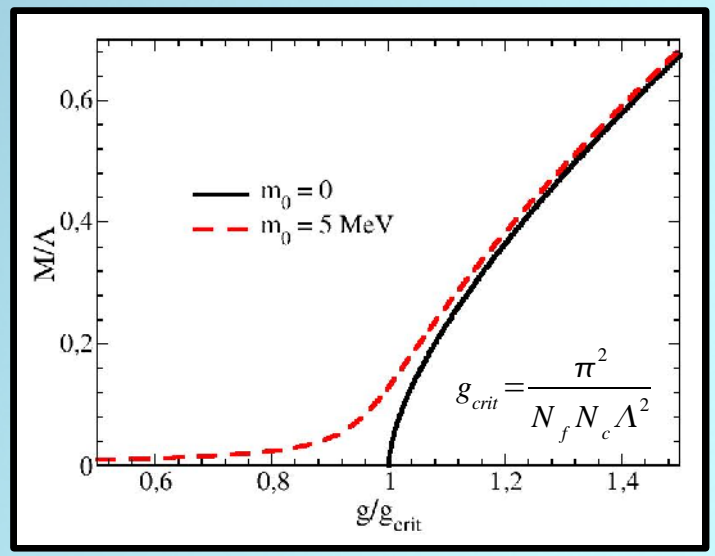
J. M. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. **D 10**, 2428 (1974).

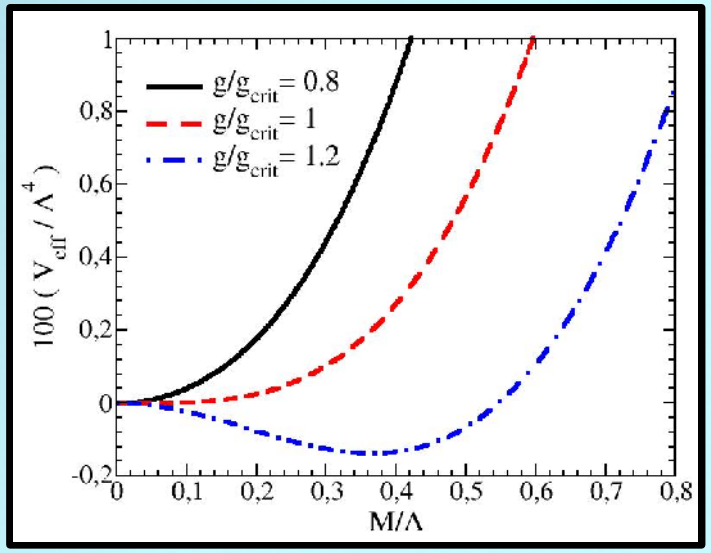
$$M = m + 4 N_f N_c g \int_{\Lambda} \frac{d^3 \vec{p}}{(2\pi)^3} \frac{M}{\sqrt{\vec{p}^2 + M^2}}$$

$$M \propto \langle \bar{\psi} \psi \rangle$$

- For $g < g_{crit}$ the solution is $M=0$.
- For $g > g_{crit}$ in addition there is a non trivial solution $M \neq 0$.

$$V_{eff} = -2 N_f N_c \int_{\Lambda} \frac{d^3 \vec{p}}{(2\pi)^3} \sqrt{\vec{p}^2 + M^2} + \frac{(M-m)^2}{4g}$$



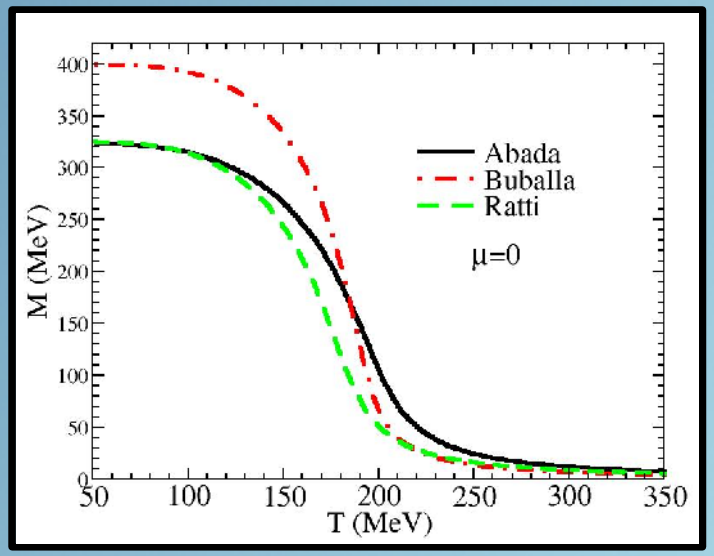


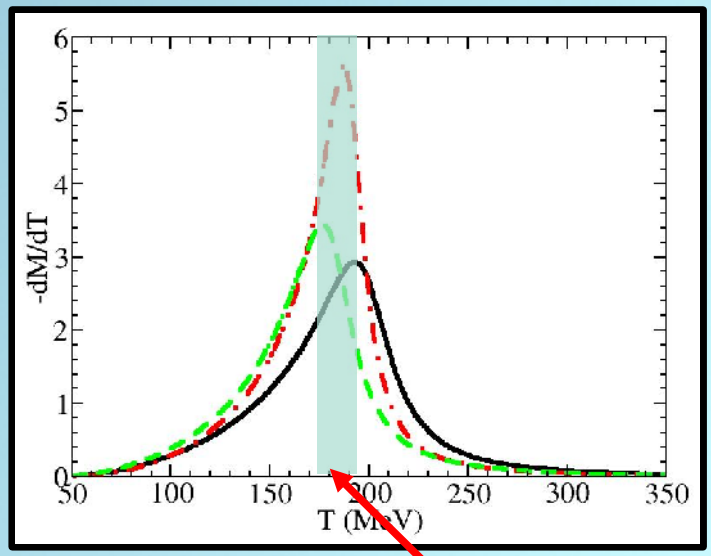
In the 2-loop approx. the effective potential is given by

$$V_{eff} = -2 N_f N_c \int_{\Lambda} \frac{d^3 \vec{p}}{(2\pi)^3} E_p - \frac{2 N_f N_c}{\beta} \int_{\Lambda} \frac{d^3 \vec{p}}{(2\pi)^3} \ln\left[\left(1 + e^{-\beta(E+\mu)}\right)\left(1 + e^{-\beta(E-\mu)}\right)\right] +$$

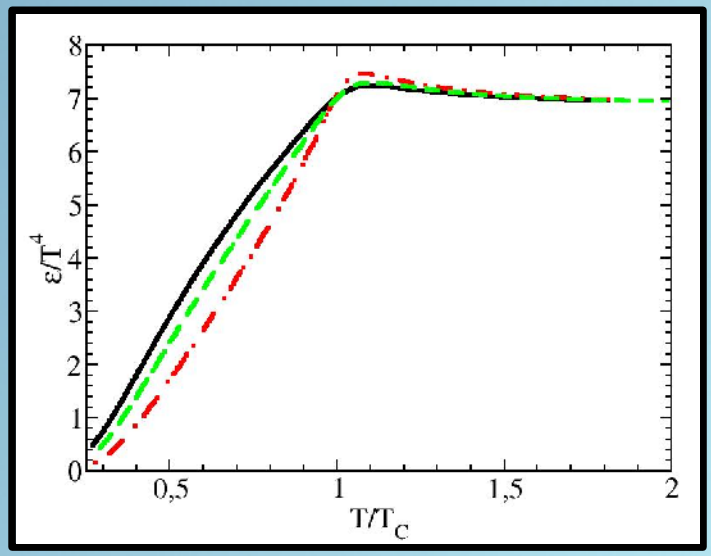
$$+ \frac{(M-m)^2}{4g} + c$$

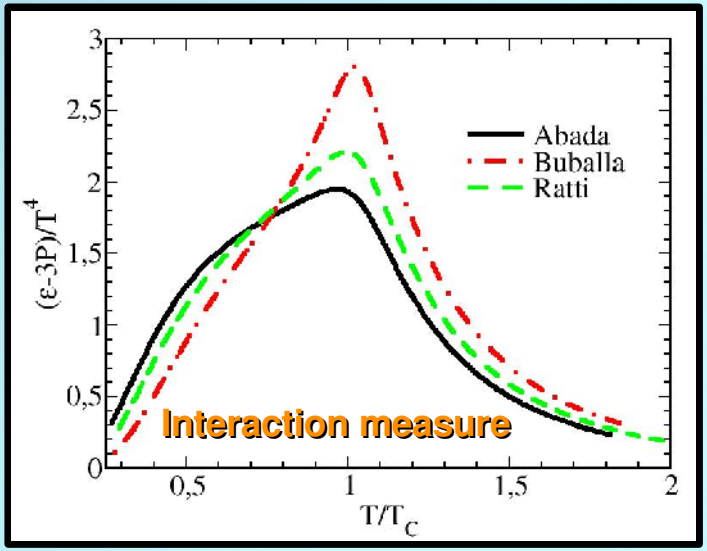
$$M(T,\mu) = m + 4 g N_f N_c M(T,\mu) \int_{\Lambda} \frac{d^3 p}{(2\pi)^3} \frac{1}{E_p} \left[1 - f^-(T,\mu) - f^+(T,\mu)\right]$$





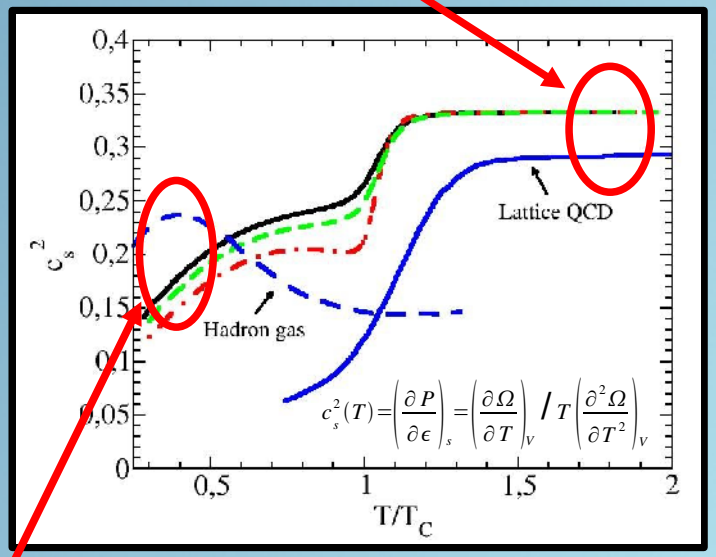
$T_c \simeq 180\ MeV$



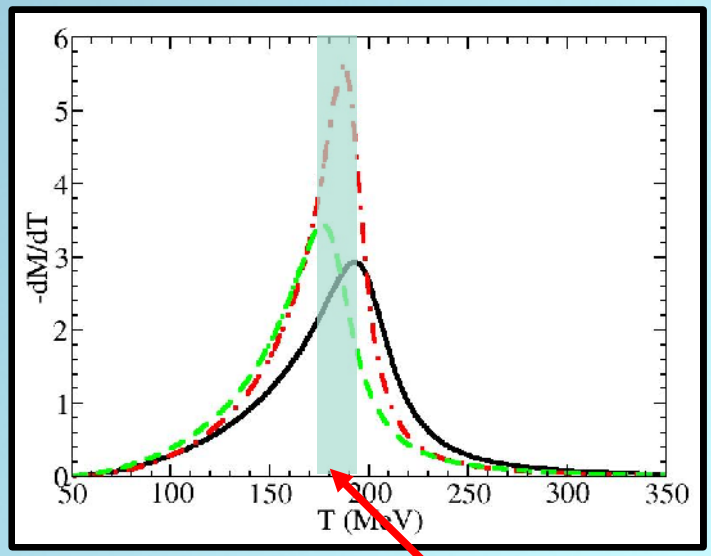


Missing of the gluon contribution

$$T^{\mu}_{\mu} \sim M^2 \sim \langle \bar{\psi}\psi \rangle^2 \xrightarrow[T \to T_c]{} 0$$



Missing of the confinement



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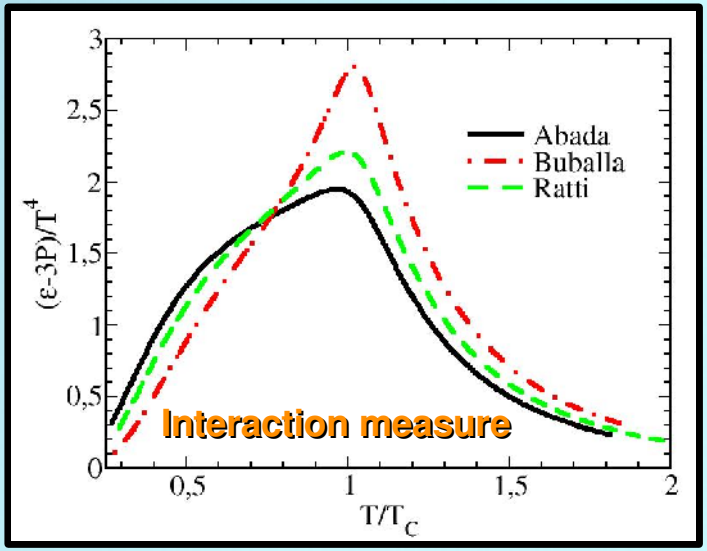
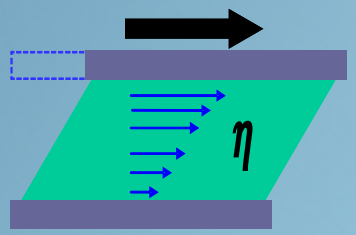


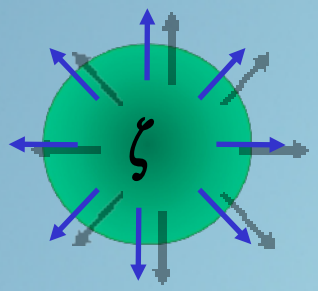
Transport coefficients: shear and bulk viscosity

Shear viscosity

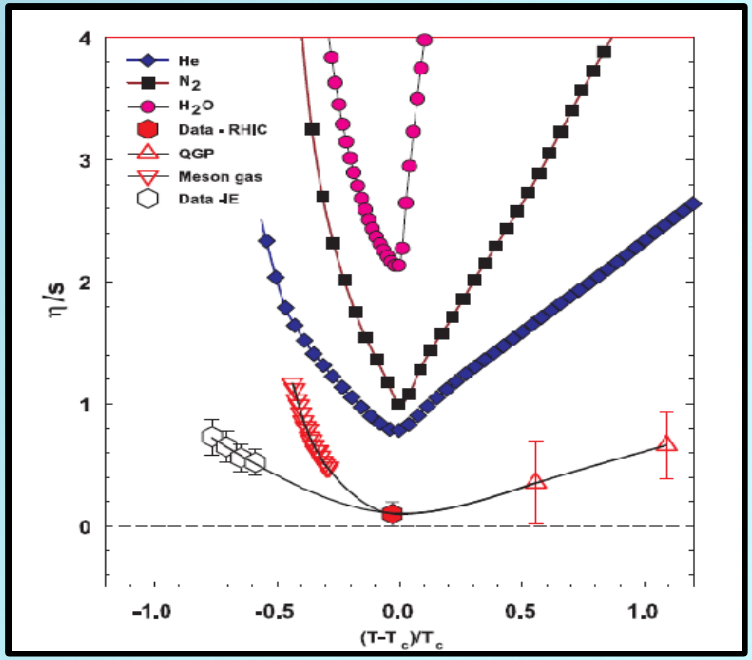


η acts as the resistance against the deformation of a fluid element.

Bulk viscosity



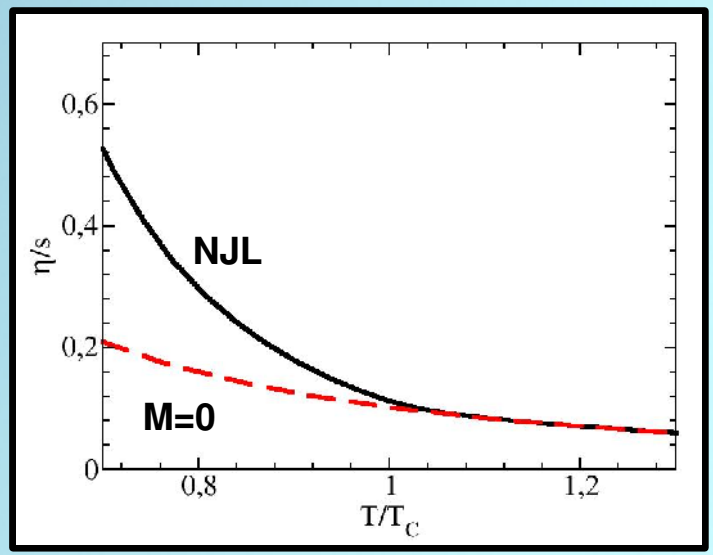
ζ acts against the expansion or compression of a fluid.



R. Lacey et al., PRL99(2006).

In the relaxation time approximation:

$$\eta = \frac{\gamma\tau}{15T}\int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^4}{E^2}\left[f_0^-(1-f_0^-)+f_0^+(1-f_0^+)\right]$$



ζ is more sensitive to the phase transition.

$$\zeta = -\frac{\gamma\tau}{3T}\int \frac{d^3p}{(2\pi)^3}\frac{M^2}{E^2}\times$$

$$\times\left\{\left(f_0^-(1-f_0^-)+f_0^+(1-f_0^+)\right)\left[\frac{\vec{p}^2}{3E}-\left(\frac{\partial p}{\partial \epsilon}\right)_n\left(E-T\frac{\partial E}{\partial T}-\mu\frac{\partial E}{\partial \mu}\right)+\left(\frac{\partial p}{\partial n}\right)_\epsilon\frac{\partial E}{\partial \mu}\right]\right\}+$$

$$+\frac{\gamma\tau}{3T}\int \frac{d^3p}{(2\pi)^3}\frac{M^2}{E^2}\left\{\left(f_0^-(1-f_0^-)-f_0^+(1-f_0^+)\right)\left(\frac{\partial p}{\partial n}\right)_\epsilon\right\}$$

C. Sasaki and K. Redlich, Phys. Rev. C 79, 055207 (2009).

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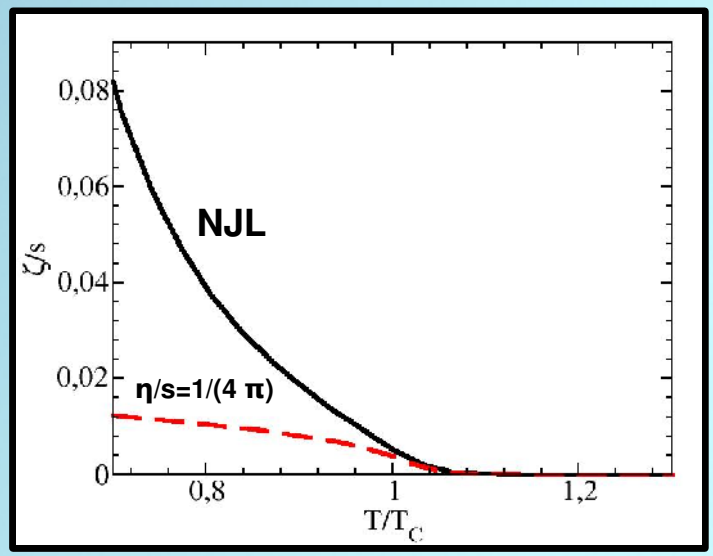

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$$+\frac{\gamma\tau}{3T}\int \frac{d^3p}{(2\pi)^3}\frac{M^2}{E^2}\left\{\left(f_0^-(1-f_0^-)-f_0^+(1-f_0^+)\right)\left(\frac{\partial p}{\partial n}\right)_\epsilon\right\}$$

C. Sasaki and K. Redlich, Phys. Rev. C 79, 055207 (2009).

Transport theory

Parton Cascade Model $\longrightarrow$ $p^{\mu}\partial_{\mu}f(X,p)=C=C_{22}+C_{23}+\ldots$ $\longrightarrow$ ε-3p=0

The Boltzmann-Vlasov equation for the NJL model

$$\left.\begin{cases} p^{\mu}\partial_{\mu}f(X,p)+M(X)\partial_{\mu}M(X)\partial_{p}^{\mu}f(X,p)=C=C_{22}+\ldots \\ M(X)=m+4gN_{c}M(X)\displaystyle\int\frac{d^{3}p}{(2\pi)^{3}}\frac{1}{E_{p}(X)}[1-f^{-}(X,p)-f^{+}(X,p)] \end{cases}\right\} \longrightarrow \varepsilon-3p\neq 0$$

The test particle method

$$f^{-}(\boldsymbol{r},\boldsymbol{p},t)=\omega\sum_{i=1}^{A}\delta^{3}(\boldsymbol{r}-\boldsymbol{r}_{i}(t))\delta^{3}(\boldsymbol{p}-\boldsymbol{p}_{i}(t))$$

The phase-space distribution function can be written as a sum of delta functions.

- **Gap-equation**

$$M_{cell}=m+2gM_{cell}\left[I(\Lambda,M_{cell})-\frac{\omega}{(2\pi)^{3}a^{3}}\left(\sum_{i=1}^{A_{cell}}\frac{1}{E_{i}}-\sum_{i=1}^{\tilde{A}_{cell}}\frac{1}{\tilde{E}_{i}}\right)\right]$$

$$I(\Lambda,M_{cell})=2N_{f}N_{c}\int_{\Lambda}\frac{d^{3}p}{(2\pi)^{3}}\frac{1}{E}$$

- **Hamilton equations for the test paticles.**

$$\begin{cases}\dot{\boldsymbol{r}}_{i}=\boldsymbol{p}_{i}/E_{i}\\ \dot{\boldsymbol{p}}_{i}=-\vec{\nabla}_{r}E_{i}+coll.=2g(M/E_{i})\vec{\nabla}_{r}\langle\bar{\psi}\psi\rangle+coll.\end{cases}\qquad i=1,\ldots,A$$

Contribution of the NJL mean field

Take into account the effects of the collision integral

C. Y. Wong, Phys. Rev. **C 25**, 1460 (1982).

For the numerical implementation of the collision integral we use the stochastic algorithm (Z. Xu and C. Greiner).

$$\frac{\Delta N_{coll}^{2\to 2}}{\Delta t(1/(2\pi)^3)\Delta^3 x\Delta^3 p_1}=\frac{1}{2E_1}\frac{\Delta^3 p_2}{(2\pi)^3 2E_2}f_1 f_2\frac{1}{\nu}\int\frac{d^3 p'_1}{(2\pi)^3 2E'_1}\frac{d^3 p'_2}{(2\pi)^3 2E'_2}\times$$
$$\times\left|\boldsymbol{M}_{12\to 1'2'}\right|^2(2\pi)^4\delta^4(p_1+p_2-p'_1-p'_2)$$

The collision rate per unit phase space volume with momenta in the range

$$p_1, p_1+\Delta^3 p_1 \quad ; \quad p_2, p_2+\Delta^3 p_2$$

$$f_i=\frac{\Delta N_i}{\frac{1}{(2\pi)^3}\Delta^3 x\Delta^3 p_i}$$

Distribution function

$$\sigma_{22}=\frac{1}{4\mathrm{F}}\frac{1}{\nu}\int\frac{d^3 p'_1}{(2\pi)^3 2E'_1}\frac{d^3 p'_2}{(2\pi)^3 2E'_2}\left|\boldsymbol{M}_{12\to 1'2'}\right|^2(2\pi)^4\delta^4(p_1+p_2-p'_1-p'_2)$$

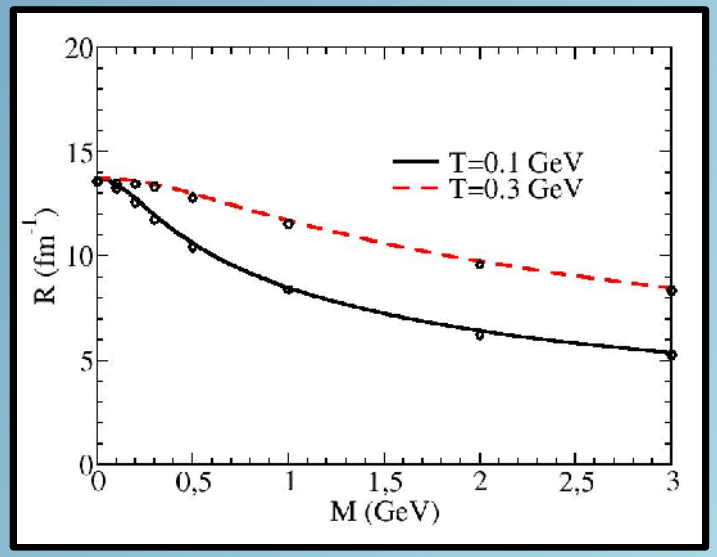
$$P_{22}=\frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1\Delta N_2}=v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

$$v_{rel}=\frac{\sqrt{\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]}}{2E_1E_2}$$

Relative velocity

Z. Xu and C. Greiner, Phys.Rev. **C 71** 064901 (2005).

Test of the collision algorithm



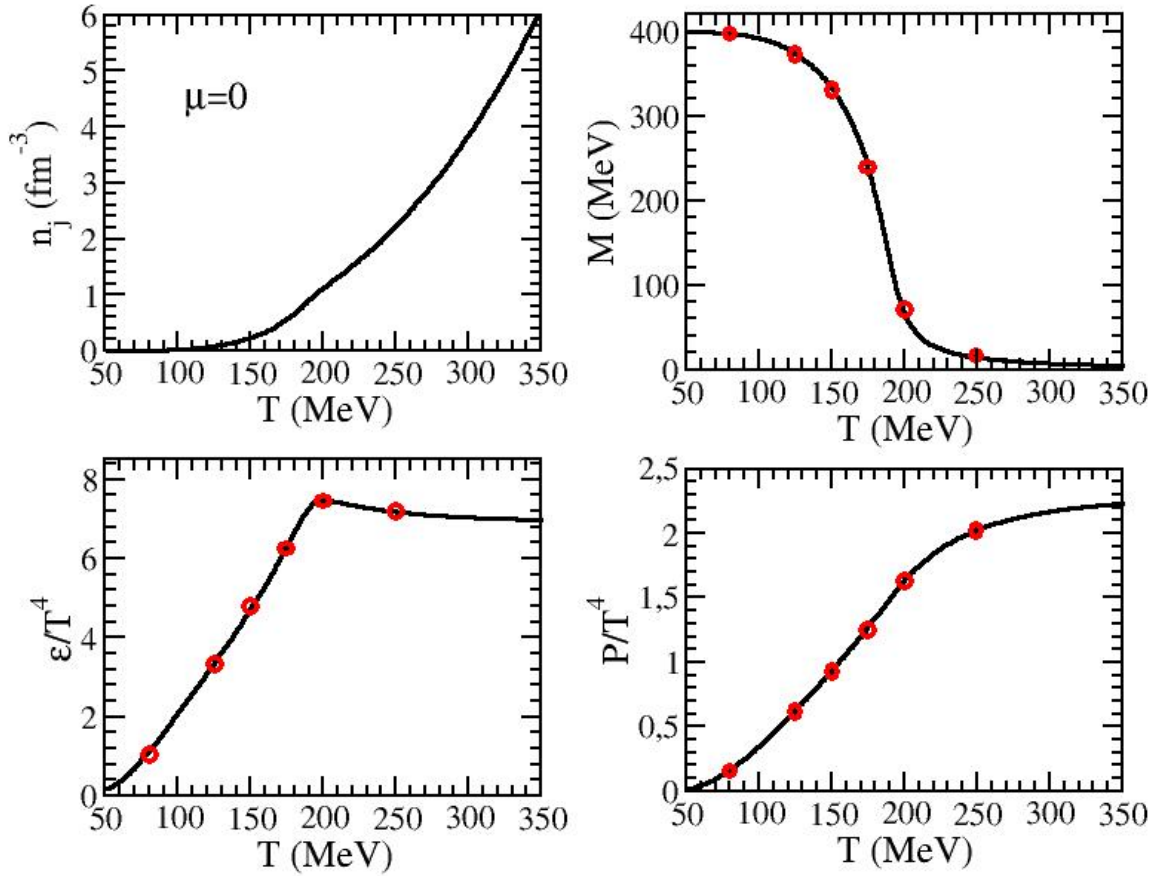
$$R = n_{tot} \langle \sigma v_{rel} \rangle = n_{tot} \frac{\beta}{4} \frac{\int_{\sqrt{s_0}}^{\infty} d\sqrt{s}\, \lambda(s) \sigma K_1(\beta \sqrt{s})}{M_a^2 M_b^2 K_2(\beta M_b)}$$

$$\lambda(s) = \left[s - (M_a + M_b)^2 \right] \left[s - (M_a - M_b)^2 \right]$$

$$K_n(z) = \frac{2^n n!}{(2n)!} z^{-n} \int_z^{\infty} d\tau\, (\tau^2 - z^2)^{n-1/2} e^{-\tau}$$

Test of the thermodynamics

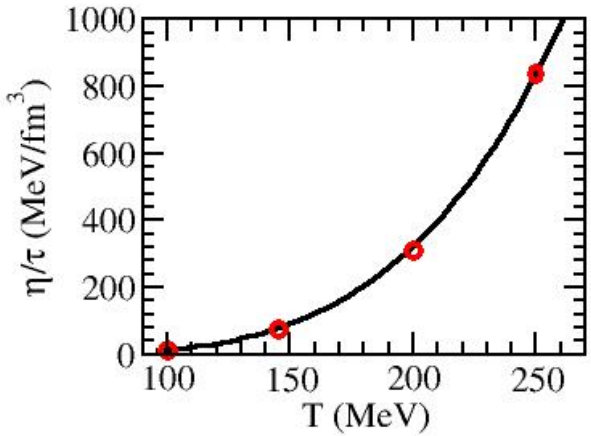
- Cubic box with periodic boundary conditions.
- At t = 0 fm/c the test particles are randomly distributed in the box.
- The momenta of the test particles are chosen according to the Fermi-Dirac distribution.



Test of the thermodynamics

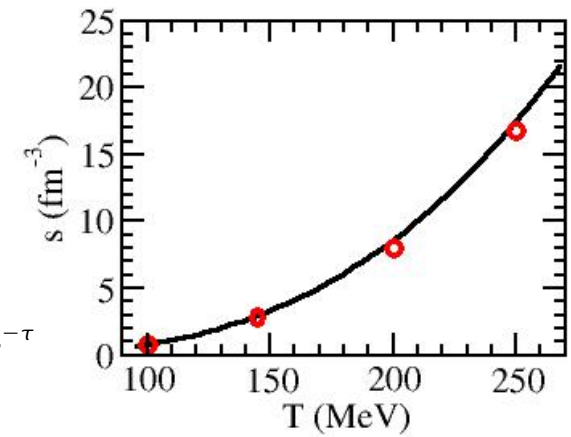
$$\eta(\vec{x},t)=\tau(\vec{x},t)\frac{n_{tot}(\vec{x},t)}{15}\left[4\left\langle\frac{p^2}{E}\right\rangle+M^2\left\langle\frac{p^2}{E^3}\right\rangle\right]$$

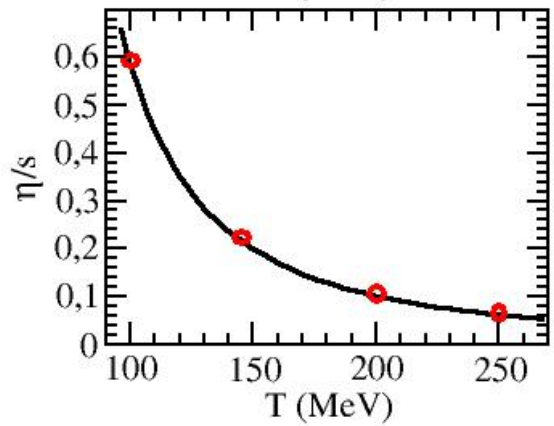
$$\xrightarrow{M\to 0}\quad \eta(\vec{x},t)=\frac{4}{15}\lambda(\vec{x},t)\,n_{tot}(\vec{x},t)\langle p\rangle$$



$$e=3T+M\frac{K_1(M/T)}{K_2(M/T)}$$

$$K_n(z)=\frac{2^n n!}{(2n)!}z^{-n}\int_z^\infty d\tau\,(\tau^2-z^2)^{n-1/2}e^{-\tau}$$



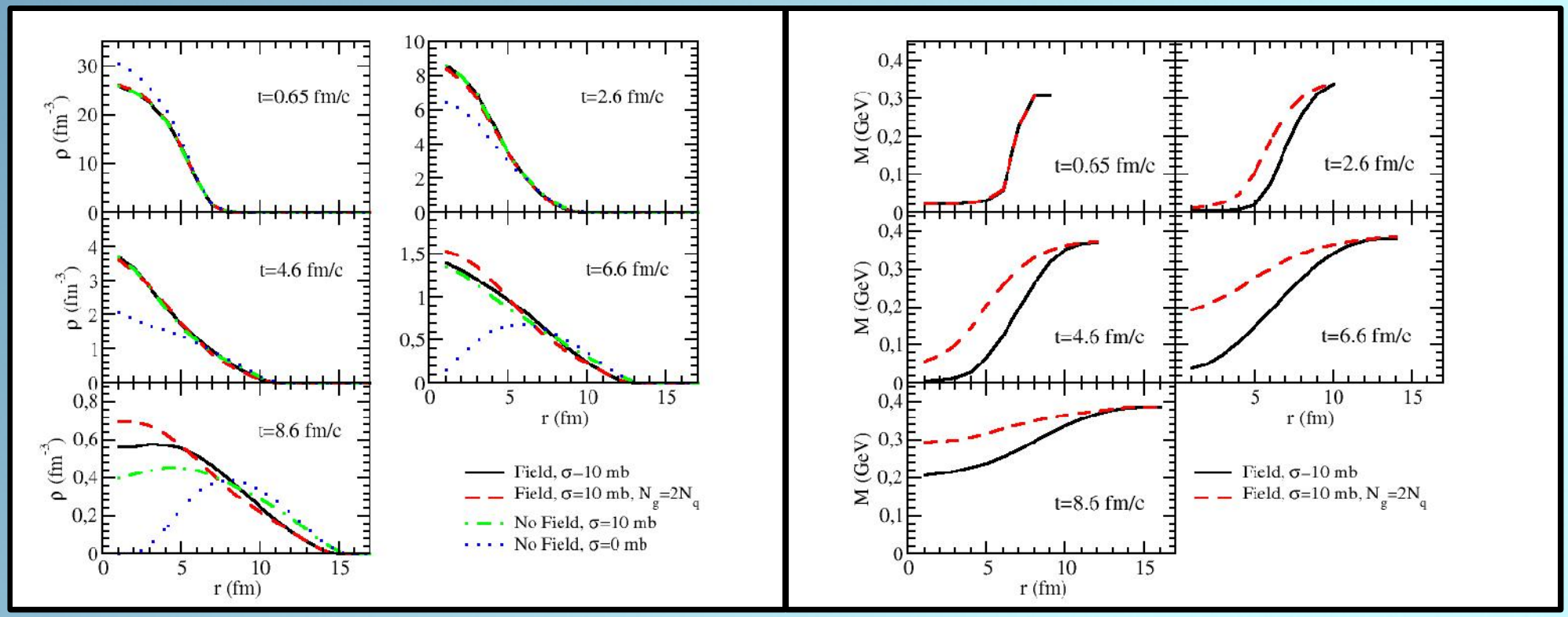


Heavy ion collisions at 200 AGeV

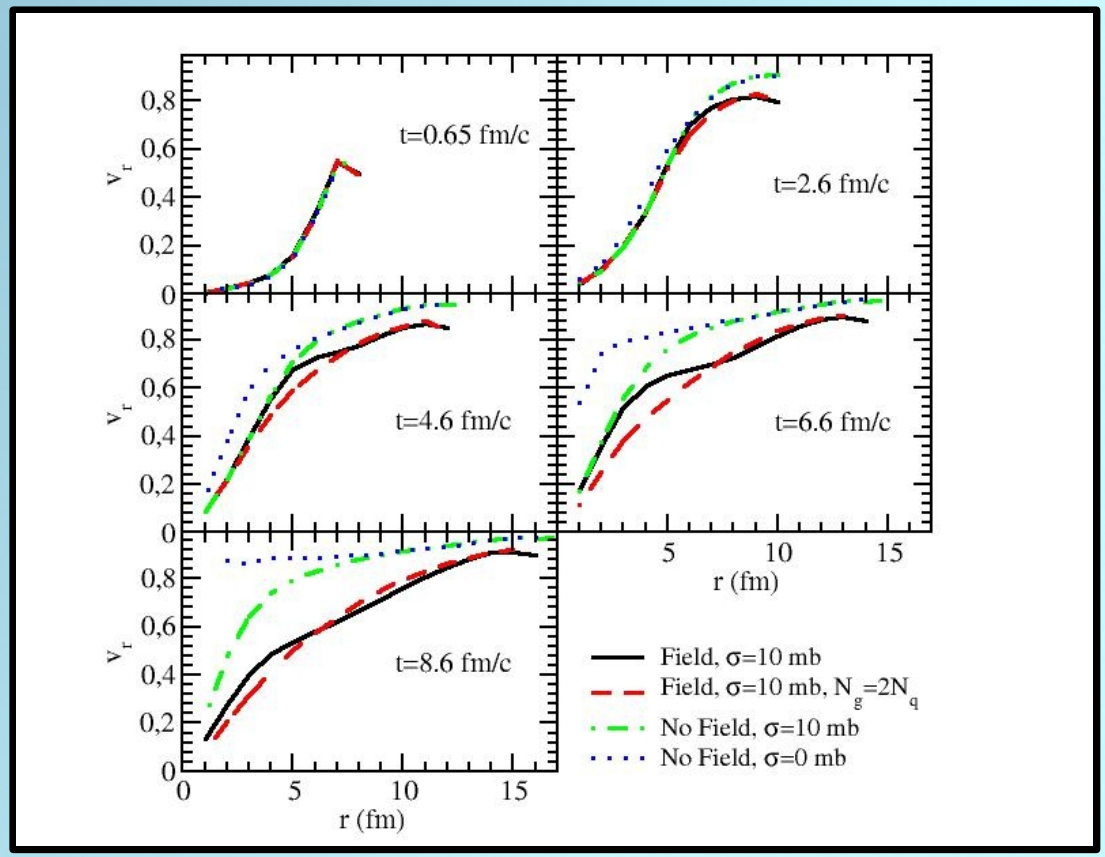
- In r Glauber model.
- In p :
  - For p_T < 2 GeV thermal distribution.
  - For p_T > 2 GeV spectra of minijets.

Heavy ion collisions

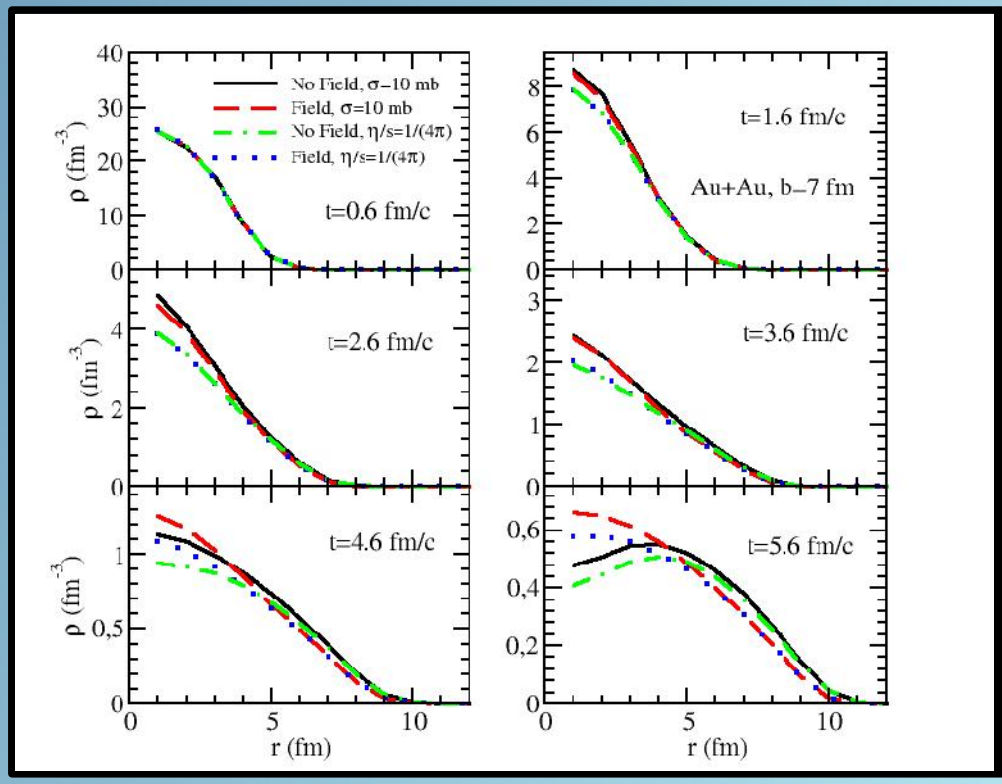
Au+Au @ 200 AGeV for central collision, b=0 fm.

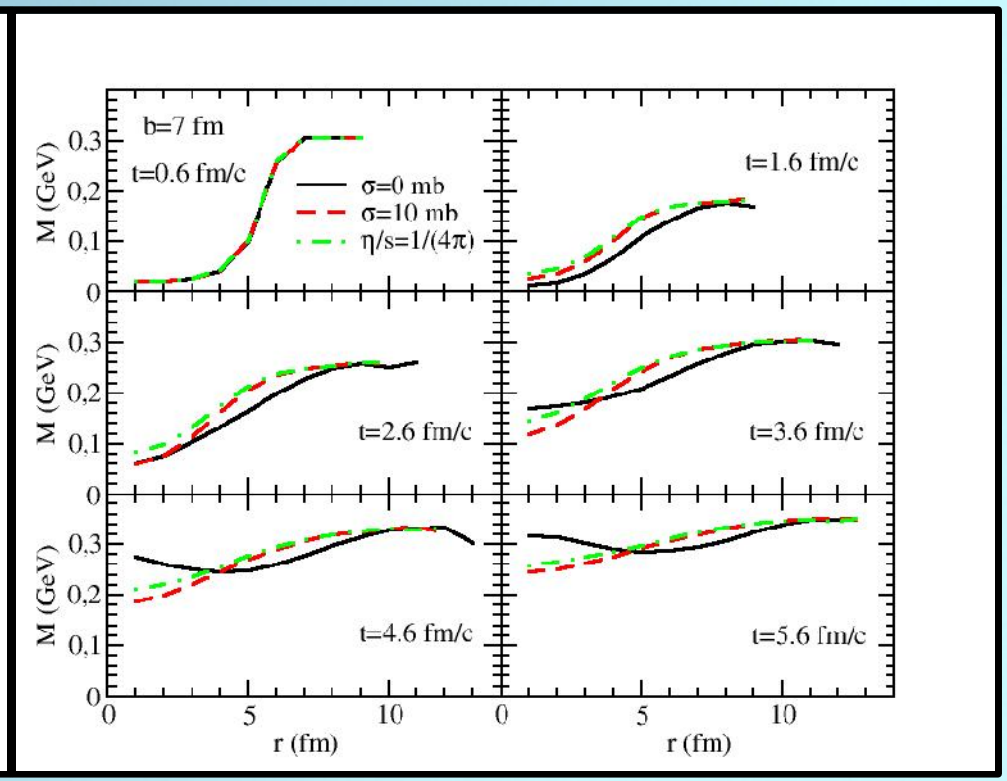


- The effect of the mean field is to reduce the radial flow v_r.
- For t < 3-4 fm/c the effect of the mean field is small because the chiral symmetry is approximatively restored.
- After the chiral phase transition the system is more massive.



Au+Au @ 200 AGeV for non central collision with b=7 fm.

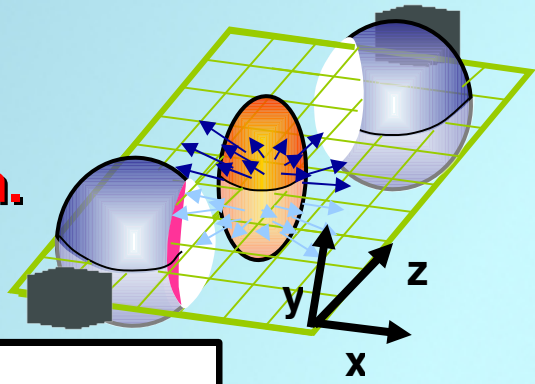




- **σ is evaluated in such way to keep the η/s of the medium costant during the dynamics.**

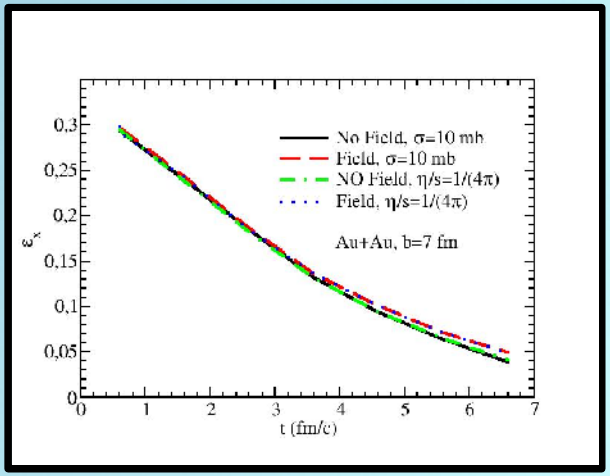
$$\sigma = \frac{1}{15}\frac{T}{\langle v_{rel}\rangle}\frac{4\langle p^2/E\rangle + M^2\langle p^2/E^3\rangle}{(\epsilon + nT)\eta/s}$$

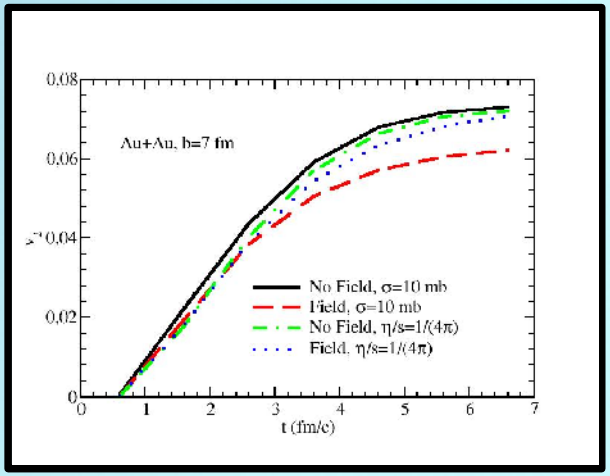
Au+Au @ 200 AGeV for non central collision with b=7 fm.

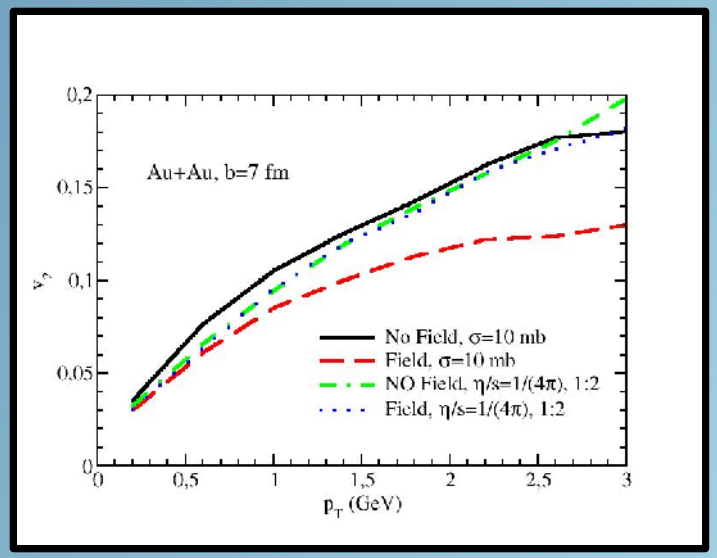


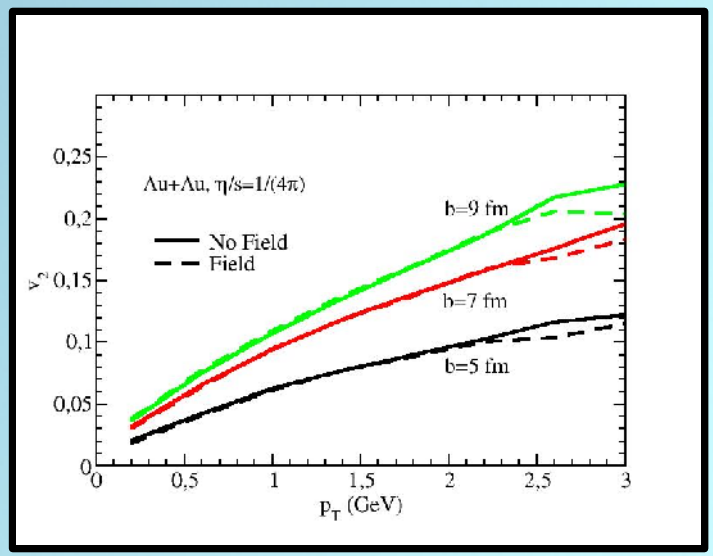
- The average elliptic flow $\langle v_2 \rangle$ is sensitive to the field dynamics.
- The effect is to reduce the $\langle v_2 \rangle$ of about 15%.
- σ is evaluated in such way to keep the η/s of the medium costant during the dynamics.

$$\sigma = \frac{1}{15}\frac{T}{\langle v_{rel} \rangle}\frac{4\langle p^2/E \rangle + M^2 \langle p^2/E^3 \rangle}{(\epsilon + nT)\eta/s}$$

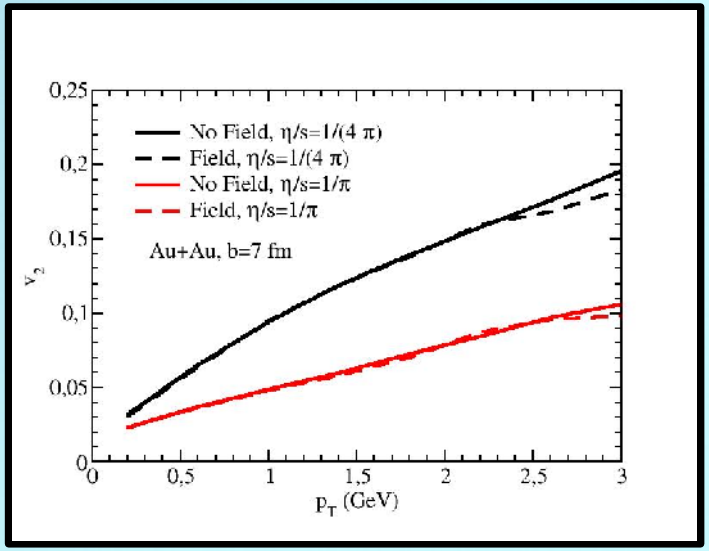




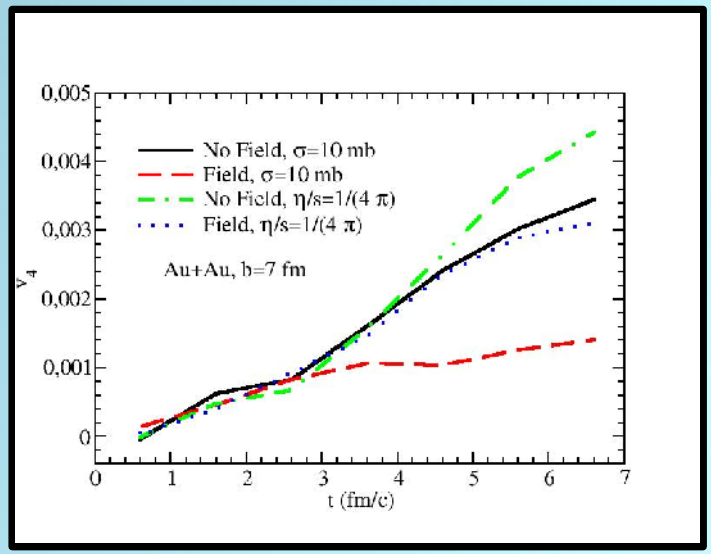


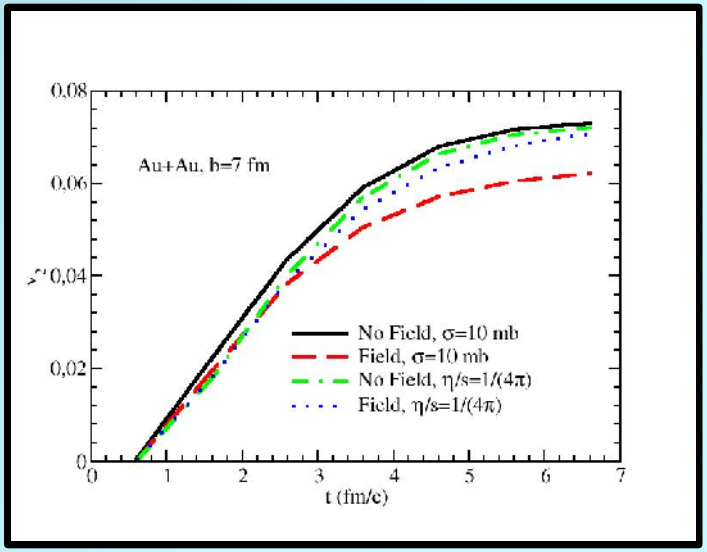


- **The role of the mean field increase with momentum.**
- **At fixed η/s there is essentially no difference with and without mean field.**
- **This effect does not depend on the centrality of the collision.**

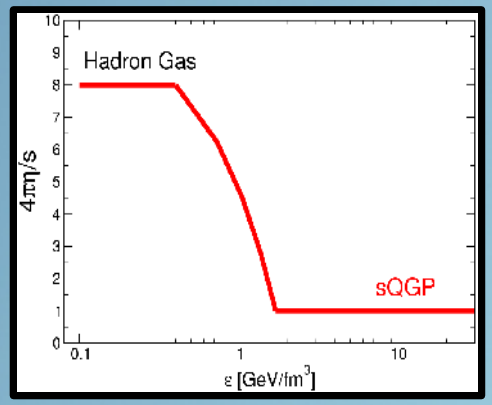


- At fixed σ the effect of the mean field is to prevent the generation of the $\langle v_4 \rangle$ with a reduction of about 50 %.
- For fixed η/s the build up of $\langle v_4 \rangle$ remain smaller for the mean field case.
- The $\langle v_4 \rangle$ is a sensitive variabe in the region of the phase transition where the EoS is different from the non-interacting one.

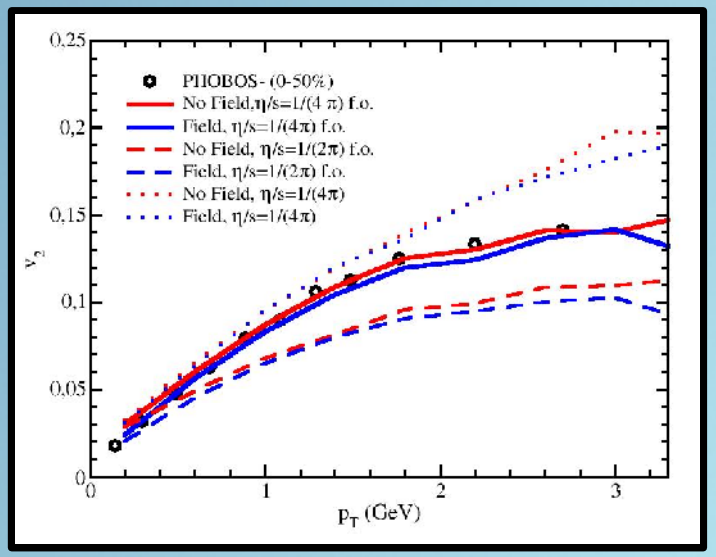


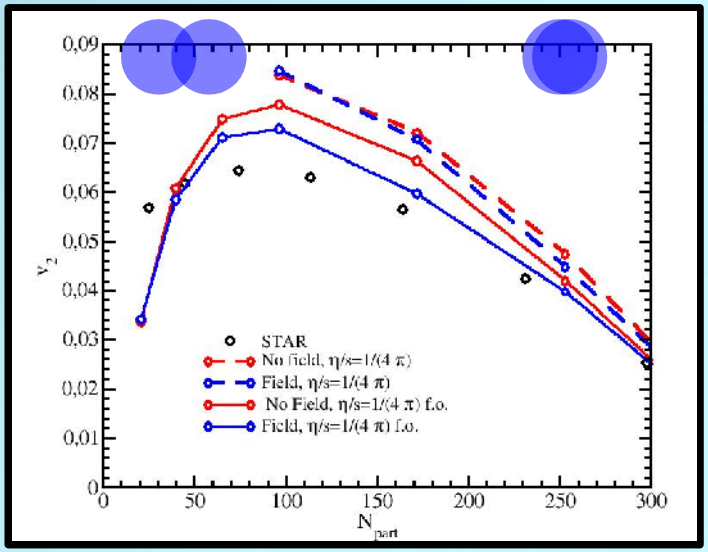


The effect of the freeze-out



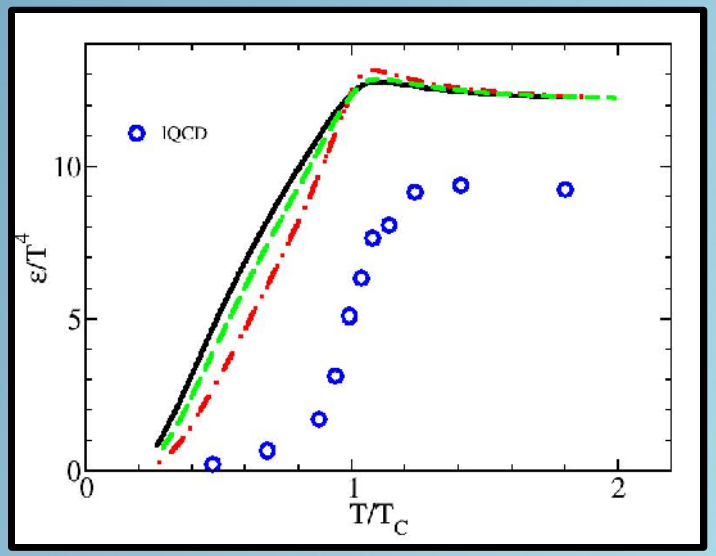
- η/s increase in the cross-over region, with a smooth transition between the QGP and the Hadronic phase.
- When the freeze-out condition is implemented a reduction of the elliptic flow is observed for peripheral collisions and at intermediate p_T.

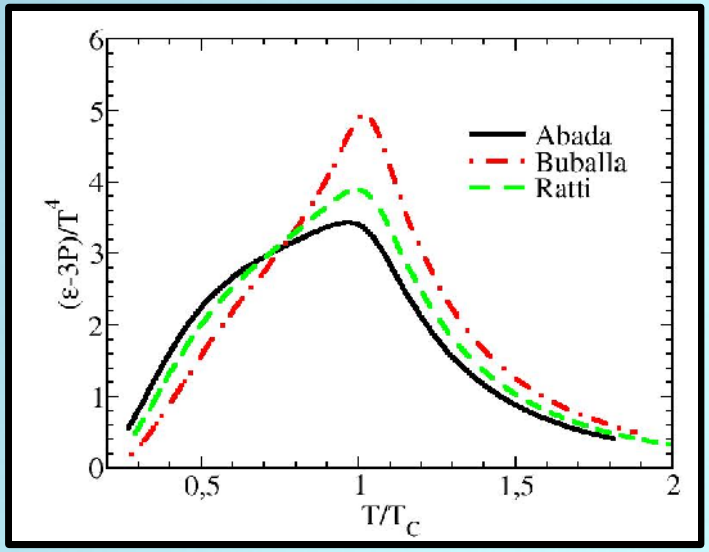


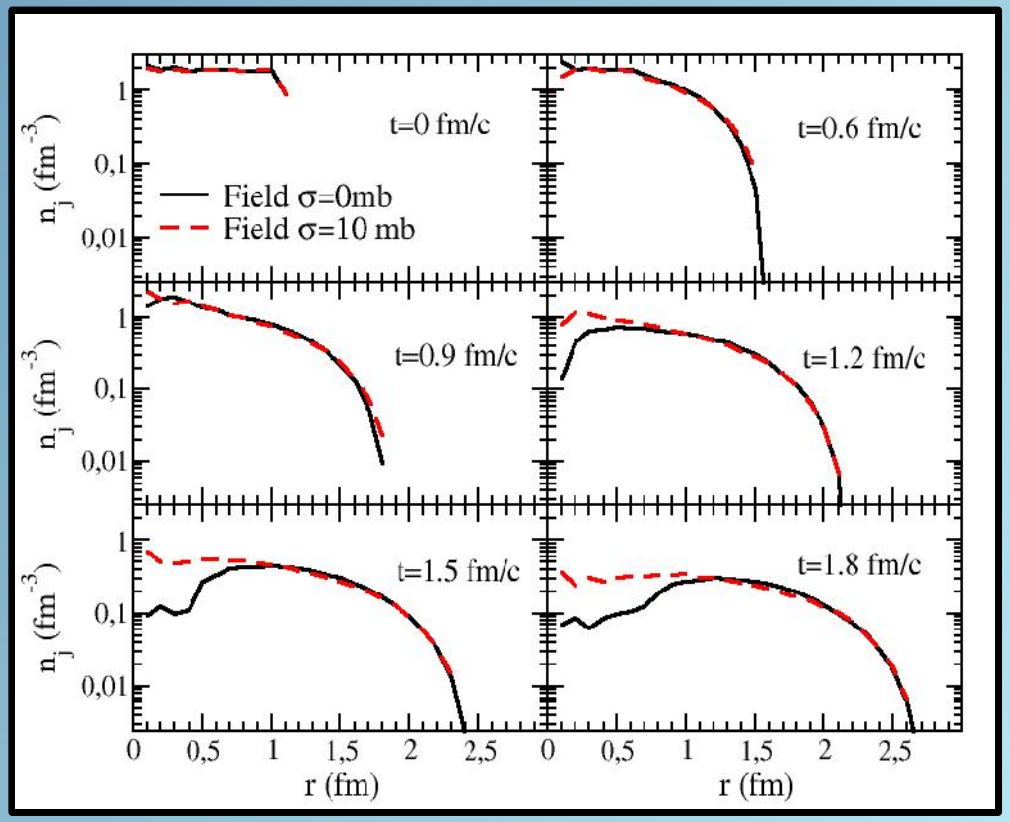


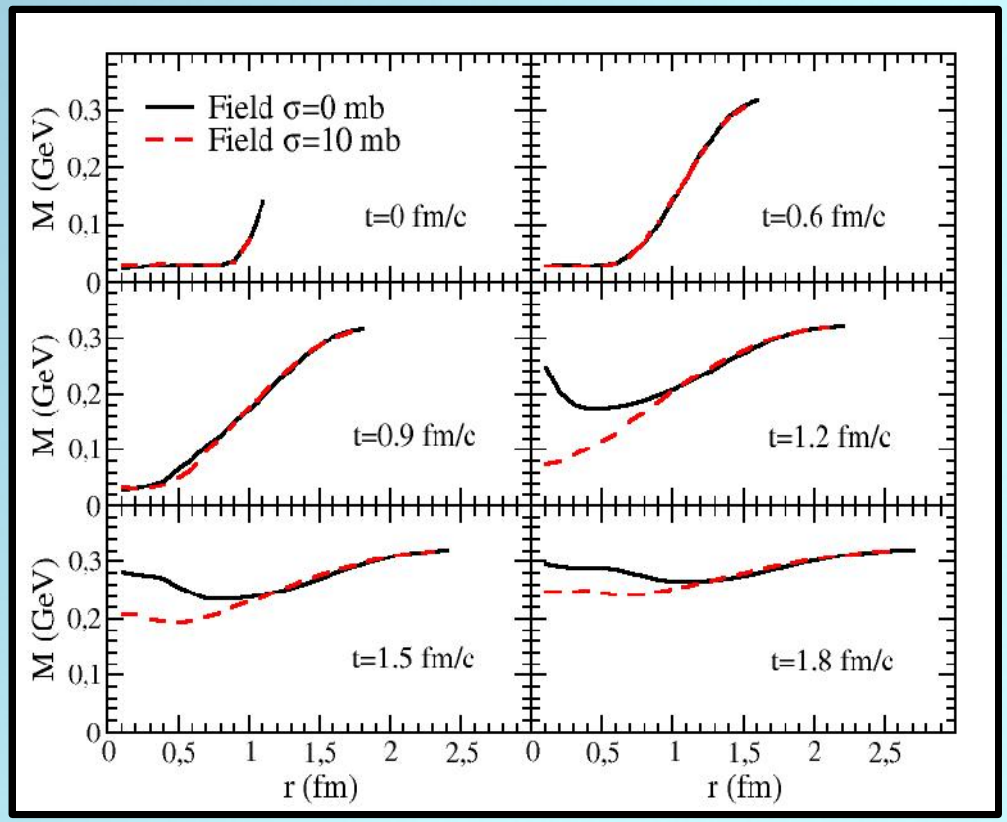
Conclusions and Outlook

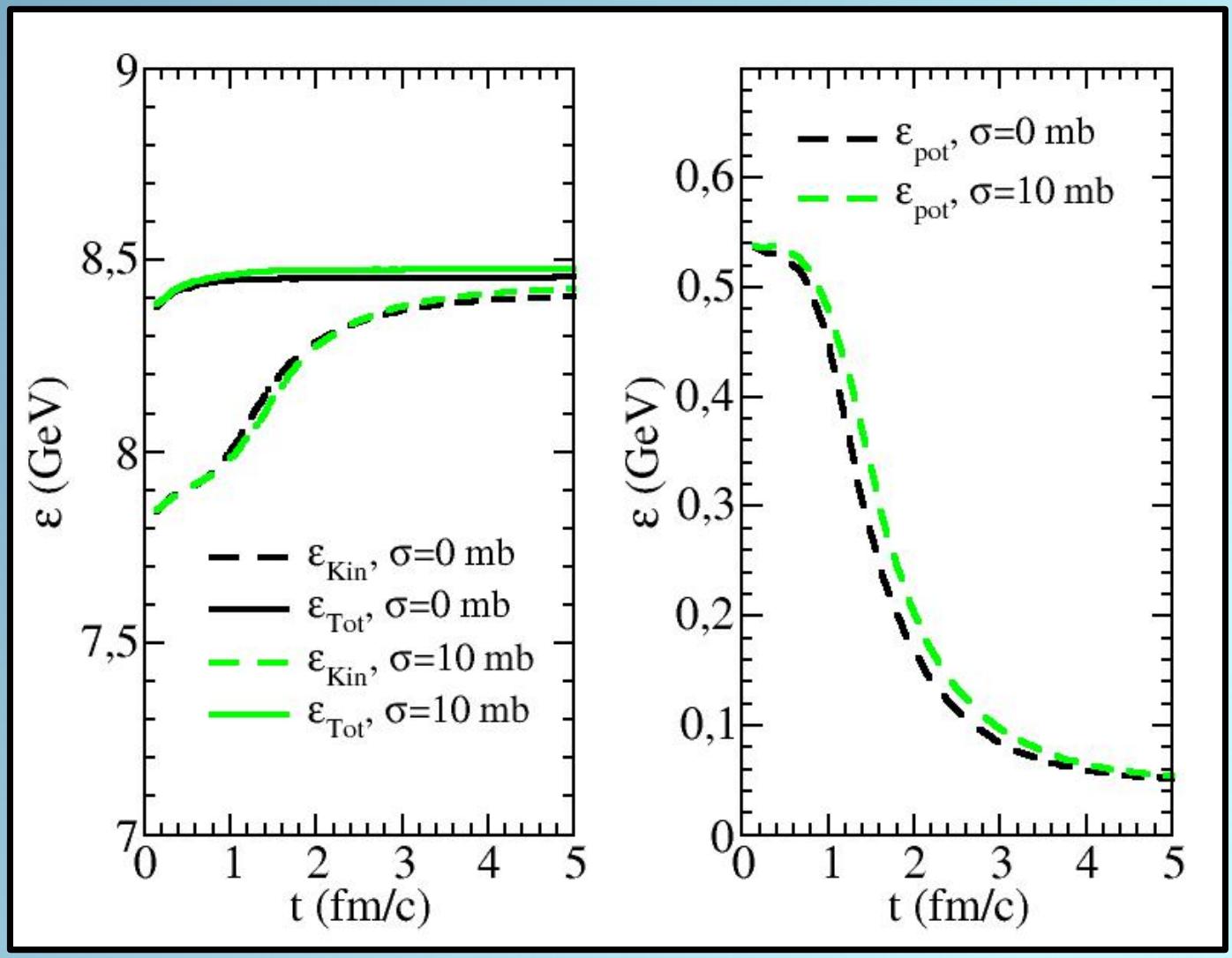
- At fixed σ the effect is to reduce the saturation value of the average elliptic flow $\langle v_2 \rangle$ and of the differential elliptic flow $v_2(p_T)$.
- The $\langle v_4 \rangle$ is particularly sensitive to the mean field in particular to the chiral phase transition.
- Agreement between the data and theory is found for $\eta/s = 1 / (4 \pi)$.
- The approach proposed can be generalized to quasi-particle models which are fitted to reproduce the energy density and pressure of lQCD.


lQCD
ε/T^4
T/T_C
0
5
10
0
1
2


Abada
Buballa
Ratti
(ε-3P)/T^4
T/T_C
0
1
2
3
4
5
6
0,5
1
1,5
2


Field σ=0mb
Field σ=10 mb
t=0 fm/c
t=0.6 fm/c
t=0.9 fm/c
t=1.2 fm/c
t=1.5 fm/c
t=1.8 fm/c
n_j (fm^-3)
r (fm)


Field σ=0 mb
Field σ=10 mb
t=0 fm/c
t=0.6 fm/c
t=0.9 fm/c
t=1.2 fm/c
t=1.5 fm/c
t=1.8 fm/c
M (GeV)
r (fm)


9
8,5
8
7,5
7
0
1
2
3
4
5
ε (GeV)
t (fm/c)
ε_Kin, σ=0 mb
ε_Tot, σ=0 mb
ε_Kin, σ=10 mb
ε_Tot, σ=10 mb
0,6
0,5
0,4
0,3
0,2
0,1
0
ε_pot, σ=0 mb
ε_pot, σ=10 mb

$$
\begin{aligned}
\mathcal{C}_{22} \;=\; & \frac{1}{2}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\,\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\,\frac{d^3p_2'}{(2\pi)^3 2E_2'} \\
& \times f_1' f_2' |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2) \\
& -\frac{1}{2}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\,\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\,\frac{d^3p_2'}{(2\pi)^3 2E_2'} \\
& \times f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2')\,.
\end{aligned}
$$

The test particle method

$$f^{-}(\boldsymbol{r},\boldsymbol{p},t)=\omega\sum_{i=1}^{A}\delta^{3}(\boldsymbol{r}-\boldsymbol{r}_{i}(t))\delta^{3}(\boldsymbol{p}-\boldsymbol{p}_{i}(t))$$

$$f^{+}(\boldsymbol{r},\boldsymbol{p},t)=\omega\sum_{i=1}^{\tilde{A}}\delta^{3}(\boldsymbol{r}-\tilde{\boldsymbol{r}}_{i}(t))\delta^{3}(\boldsymbol{p}-\tilde{\boldsymbol{p}}_{i}(t))$$

The phase-space distribution function can be written as a sum of delta functions.

ω is a normalization factor $\longrightarrow$ $$\int d^{3}r\int\frac{d^{3}p}{(2\pi)^{3}}\left[f^{-}(\boldsymbol{r},\boldsymbol{p},t)-f^{+}(\boldsymbol{r},\boldsymbol{p},t)\right]=\frac{\omega}{(2\pi)^{3}}\left(A-\tilde{A}\right)=N_{q}$$

$$M_{cell}=m+2gM_{cell}\left[I(\Lambda,M_{cell})-\frac{\omega}{(2\pi)^{3}a^{3}}\left(\sum_{i=1}^{A_{cell}}\frac{1}{E_{i}}-\sum_{i=1}^{\tilde{A}_{cell}}\frac{1}{\tilde{E}_{i}}\right)\right]$$

$$I(\Lambda,M_{cell})=2N_{f}N_{c}\int_{\Lambda}\frac{d^{3}p}{(2\pi)^{3}}\frac{1}{E}=-\frac{N_{f}N_{c}}{2\pi^{2}}M_{cell}^{2}\left\{\left(\frac{\Lambda}{M_{cell}}\right)\sqrt{\left(\frac{\Lambda}{M_{cell}}\right)^{2}+1}-\ln\left[\frac{\Lambda}{M_{cell}}+\sqrt{\left(\frac{\Lambda}{M_{cell}}\right)^{2}+1}\right]\right\}$$

C. Y. Wong, Phys. Rev. **C 25**, 1460 (1982).

Hamilton equations for the test paticles.

$$\begin{cases} \dot{\boldsymbol{r}}_i = \boldsymbol{p}_i / E_i \\ \dot{\boldsymbol{p}}_i = -\vec{\nabla}_r E_i + coll. = 2\, g\, (M/E_i)\, \vec{\nabla}_r \langle \bar{\psi}\, \psi \rangle + coll. \end{cases} \qquad i = 1, \ldots, A$$

Contribution of the NJL mean field

Take into account the effects of the collision integral

We solve the Hamilton equations and the gap-equation in a self-consistent way

$$\begin{cases} \boldsymbol{p}_i(t+\delta t) = \boldsymbol{p}_i(t-\delta t) - 2\ \delta t\ [M_{cell}(\boldsymbol{r}_i, t)/E_i(t)]\, \vec{\nabla}_r M_{cell}(\boldsymbol{r}_i, t) + coll. \\ \boldsymbol{r}_i(t+\delta t) = \boldsymbol{r}_i(t-\delta t) + 2\ \delta t\ [\boldsymbol{p}_i(t)/E_i(t)] \\ M_{cell} = m + 2\, g\, M_{cell} \left[I(\Lambda, M_{cell}) - \dfrac{\omega}{(2\pi)^3 a^3} \left(\sum_{i=1}^{A_{cell}} \dfrac{1}{E_i} - \sum_{i=1}^{\tilde{A}_{cell}} \dfrac{1}{\tilde{E}_i} \right) \right] \end{cases}$$

Transport theory

$$[W(X,p)]_{\alpha,\beta}=\int \frac{d^4u}{(2\pi)^4} e^{-ip\cdot u}\langle :\bar{\psi}_\beta(X+u/2)\psi_\alpha(X-u/2):\rangle$$

$$X=(x+y)/2$$
$$u=x-y$$

$$\langle \hat{O}\rangle=\int d^4X\int d^4p\ tr(\hat{O}\hat{W}(X,p))$$

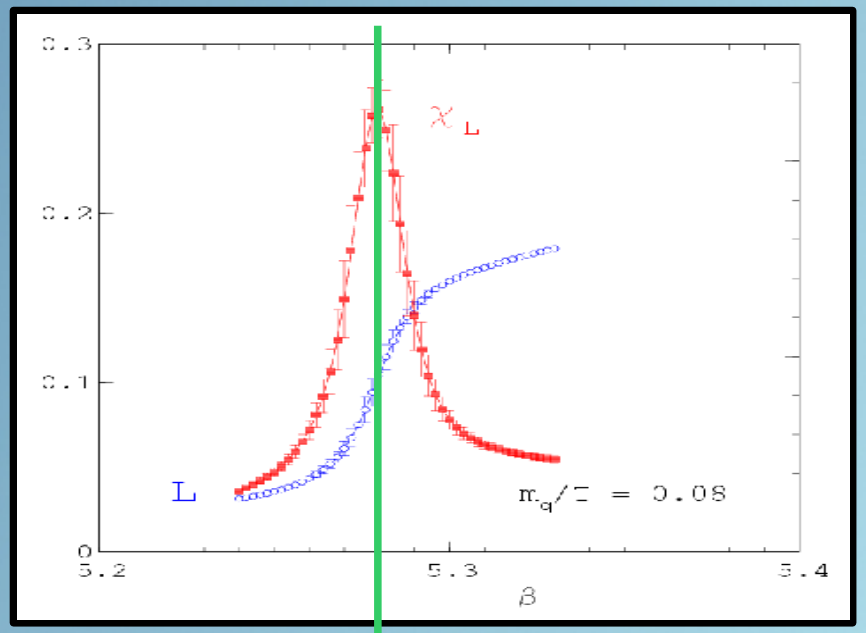
$$\begin{cases} \dfrac{\partial g(x,y)}{\partial x_\mu}\to\left[-i\,p^\mu+\dfrac{1}{2}\dfrac{\partial}{\partial X_\mu}\right]g(X,p) \\ g(x)h(x,y)\to g(X)\exp\left[-\dfrac{i}{2}\dfrac{\partial}{\partial X^\mu}\dfrac{\partial}{\partial p_\mu}\right]h(X,p)=g(X)h(X,p)-\dfrac{i}{2}\partial_\mu g(X)\partial_p^\mu h(X,p)+\cdots \end{cases}$$

$$\left[\gamma^{\mu} p_{\mu}+\frac{i}{2}\gamma^{\mu}\partial_{\mu}-m-\sigma(X)+\frac{i}{2}\partial_{\mu}\sigma(X)\partial_{p}^{\mu}-i\gamma_{5}\vec{\pi}(X)-\frac{1}{2}\gamma_{5}\partial_{\mu}\vec{\pi}(X)\partial_{p}^{\mu}\right]\hat{W}(X,p)=0$$

$$\sigma(X)=-2g\langle:\bar{\psi}(X)\psi(X):\rangle=-2g\int d^{4}p\,tr[\hat{W}(X,p)]$$

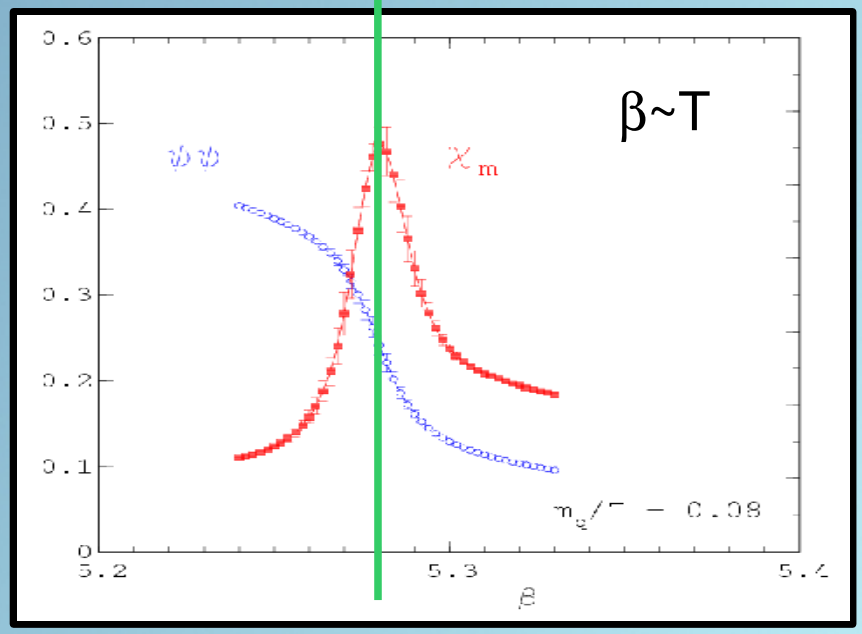
$$\vec{\pi}(X)=-2g\langle:\bar{\psi}(X)i\gamma_{5}\vec{\tau}\psi(X):\rangle=-2g\int d^{4}p\,tr[i\gamma_{5}\vec{\tau}\hat{W}(X,p)]$$

$$W=\boldsymbol{F}+i\gamma_{5}\boldsymbol{P}+\gamma^{\mu}\boldsymbol{V}_{\mu}+\gamma^{\mu}\gamma_{5}\boldsymbol{A}_{\mu}+\frac{1}{2}\sigma^{\mu\nu}\boldsymbol{S}_{\mu\nu}$$

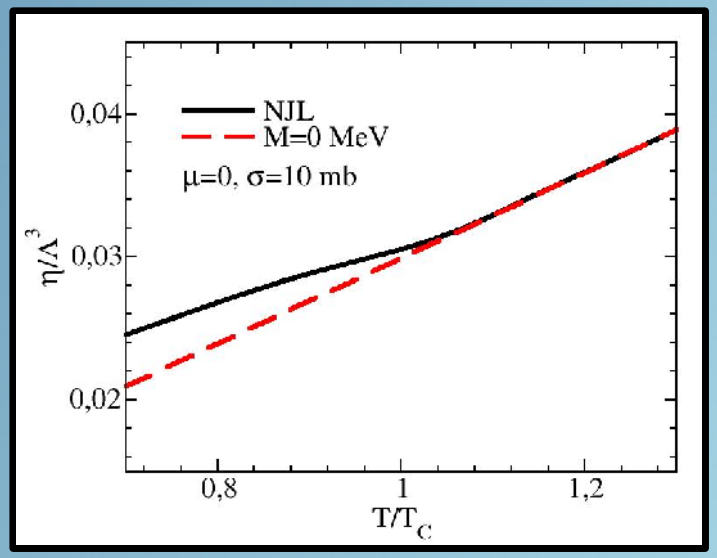


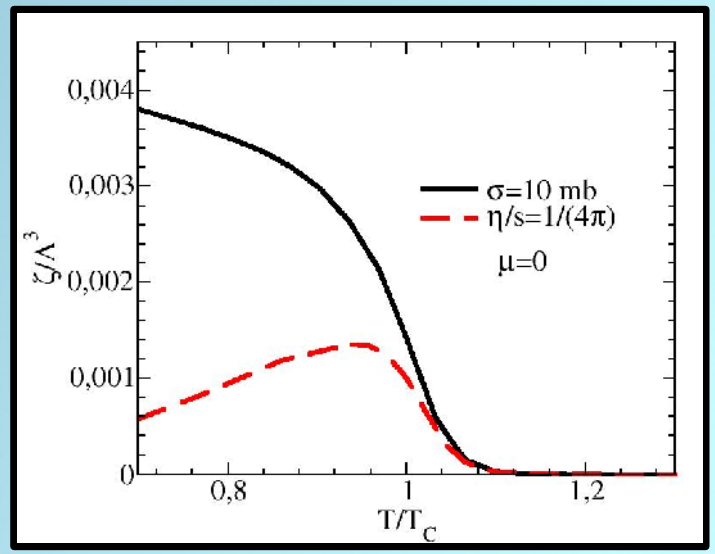
Polyakov loop

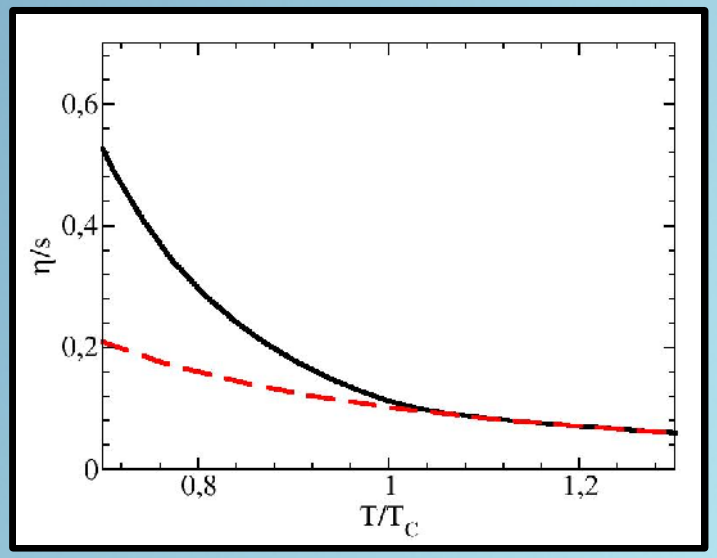
- **Coincident transitions: confinement-deconfinement transition and chiral symmetry restoration**

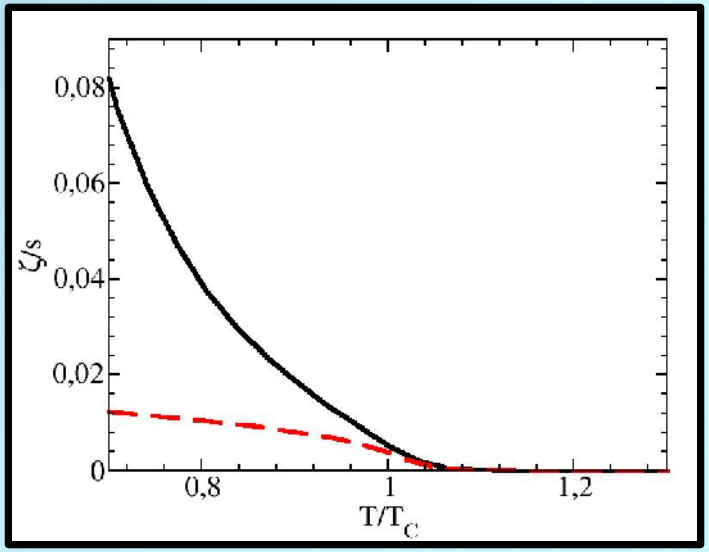


Chiral condensate


NJL
M=0 MeV
μ=0, σ=10 mb
η/Λ³
0,04
0,03
0,02
0,8
1
1,2
T/T_C


σ=10 mb
η/s=1/(4π)
μ=0
ζ/Λ³
0,004
0,003
0,002
0,001
0
0,8
1
1,2
T/T_C


η/s
0,6
0,4
0,2
0
0,8
1
1,2
T/T_C


ζ/s
0,08
0,06
0,04
0,02
0
0,8
1
1,2
T/T_C