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Particle mixing and emergence of classicality: a spontaneous collapse model view

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Frascati (RM), Italy
September 29, 2020



Standard quantum mechanics

- linearity of Schrödinger equation allows superpositions:
 ψ_1, ψ_2 are solutions $\Rightarrow \psi = c_1\psi_1 + c_2\psi_2$ is also a solution
- evolution of quantum system due to Schrödinger equation is deterministic
- measurement destroys superposition with outcomes distributed due to Born rule:

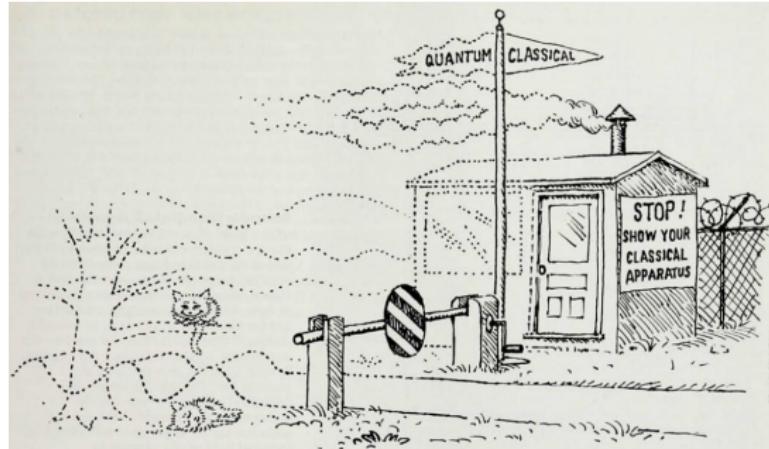
$$P_1 = |c_1|^2, P_2 = |c_2|^2 (\langle \psi_1 | \psi_2 \rangle = 0).$$

Troubles with standard QM

Standard quantum mechanics exposes two different regimes:

- ① Schrödinger evolution: linear, deterministic and reversible.
- ② Measurement: non-linear, stochastic and irreversible.

Question: Is there a border between quantum and classical worlds?



Solutions?

- Copenhagen interpretation (Bohr, 1928)
- Bohmian mechanics (Bohm, 1952)
- many-worlds interpretation (Everett, 1957)
- decoherence (Zeh, 1970)
- spontaneous collapse (Ghirardi, Rimini, and Weber, 1986)
- gravity induced collapse (Károlyházy, 1966; Diósi, 1984; Penrose, 1996)
- quantum Bayesianism (Caves, Fuchs, and Schack, 2002)
- quantum Darwinism (Zurek, 2003)
- coarse-grained measurements (Kofler and Brukner, 2007)

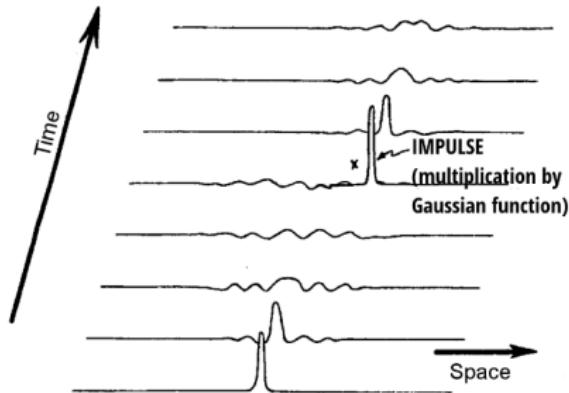
A. Bassi *et al.*, Rev. Mod. Phys. 85, 471 (2013).

Basic ideas of spontaneous collapse models

Universal dynamics should:

- ① be non-linear
- ② be stochastic
- ③ include non-unitary evolution
- ④ not allow for superluminal signaling

Proposition: *Each particle of a system of n particles experiences a sudden spontaneous localization process with defined rate, and in the time interval between two localizations system evolves due to Schrödinger equation.*



Neutral meson: Time evolution (WWA)

Time evolution of the flavour states:

$$|\psi\rangle_t = a_t |M^0\rangle + b_t |\bar{M}^0\rangle$$

$$\hat{H} = \hat{M} + \frac{i}{2}\hat{\Gamma}$$

$$\hat{H}|M_i\rangle = \left(m_i + \frac{i}{2}\Gamma_i\right)|M_i\rangle$$

$i = H \dots \text{Heavy}, L \dots \text{Light}$

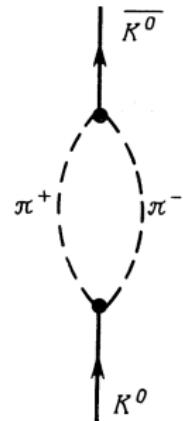
$$|M^0(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-\frac{\Gamma_H}{2}t - im_H t} |M_H\rangle + e^{-\frac{\Gamma_L}{2}t - im_L t} |M_L\rangle \right),$$

$$|\bar{M}^0(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-\frac{\Gamma_H}{2}t - im_H t} |M_H\rangle - e^{-\frac{\Gamma_L}{2}t - im_L t} |M_L\rangle \right).$$

Kaons:

$$\Gamma_H \approx 1.95 \cdot 10^7 \text{s}^{-1}$$

$$\Gamma_L \approx 1.12 \cdot 10^{10} \text{s}^{-1}$$



Transition probabilities:

$$P_{M^0 \rightarrow M^0/\bar{M}^0}(t) = \frac{1}{4} \left(e^{-\Gamma_H t} + e^{-\Gamma_L t} \underbrace{\pm 2e^{-\frac{\Gamma_H + \Gamma_L}{2}t} \cos[t\Delta m]}_{\text{interference term!}} \right).$$

Decay as an open system

$$\begin{aligned}\frac{d\hat{\rho}_t}{dt} = & -i[\hat{M}, \hat{\rho}_t] - \frac{1}{2} \sum_i [\hat{L}_i^\dagger \hat{L}_i \hat{\rho}_t + \hat{\rho}_t \hat{L}_i^\dagger \hat{L}_i - 2\hat{L}_i \hat{\rho}_t \hat{L}_i^\dagger] \\ & - \frac{1}{2}\{\hat{\Gamma}, \hat{\rho}_t\}. \end{aligned} \tag{1}$$

Not GKLS! Is the evolution still complete positive?
We can enlarge the Hilbert space!

Decay as an open system

$$\begin{aligned}\frac{d\hat{\varrho}_t}{dt} &= -i[\hat{\mathcal{H}}, \hat{\varrho}_t] - \frac{1}{2} \sum_i [\hat{\mathcal{L}}_i^\dagger \hat{\mathcal{L}}_i \hat{\varrho}_t + \hat{\varrho}_t \hat{\mathcal{L}}_i^\dagger \hat{\mathcal{L}}_i - 2 \hat{\mathcal{L}}_i \hat{\varrho}_t \hat{\mathcal{L}}_i^\dagger] \\ &\quad - \frac{1}{2} [\hat{\mathcal{L}}_D^\dagger \hat{\mathcal{L}}_D \hat{\varrho}_t + \hat{\varrho}_t \hat{\mathcal{L}}_D^\dagger \hat{\mathcal{L}}_D - 2 \hat{\mathcal{L}}_D \hat{\varrho}_t \hat{\mathcal{L}}_D^\dagger],\end{aligned}\tag{2}$$

where $\hat{\mathcal{H}} = \begin{pmatrix} \hat{\mathbf{M}} & 0 \\ 0 & 0 \end{pmatrix}$ is the Hamiltonian and $\hat{\mathcal{L}}_i = \begin{pmatrix} \hat{\mathbf{L}}_i & 0 \\ 0 & 0 \end{pmatrix}$.

The decay property is governed by $\hat{\mathcal{L}}_D = \begin{pmatrix} 0 & 0 \\ \hat{\mathbf{L}}_D & 0 \end{pmatrix}$, with

$\hat{\mathbf{L}}_D = \sum_i \sqrt{\Gamma_i} |f_i\rangle \langle M_i|$, which represents a transition between the flavor and the “decay” subspaces. In fact,

$$\hat{\Gamma} = \hat{\mathcal{L}}_D^\dagger \hat{\mathcal{L}}_D.\tag{3}$$

R. Bertlmann, W. Grimus, B. C. Hiesmayr, Phys. Rev. A 73, 054101 (2006).

Collapse models: Mathematical structure

The general stochastical differential equation of a collapse model:

$$d|\psi_t\rangle = \left[-i\hat{H}dt + \sqrt{\lambda} \sum_{i=1}^N (\hat{A}_i \underbrace{- \langle \hat{A}_i \rangle_t}_{\text{non-lin.}}) \underbrace{dW_{i,t}}_{\text{stoch.}} - \frac{\lambda}{2} \sum_{i=1}^N (\hat{A}_i \underbrace{- \langle \hat{A}_i \rangle_t}_{\text{non-lin.}})^2 dt \right] |\psi_t\rangle.$$

The corresponding master equation for $\hat{\rho}_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\frac{d\hat{\rho}_t}{dt} = -i[\hat{H}, \hat{\rho}_t] - \frac{\lambda}{2} \sum_{i=1}^N \left[\hat{A}_i^\dagger \hat{A}_i \hat{\rho}_t + \hat{\rho}_t \hat{A}_i^\dagger \hat{A}_i - 2\hat{A}_i \hat{\rho}_t \hat{A}_i^\dagger \right].$$

Example: Models QMUPL and CSL

QMUP and CSL setups:

$$\hat{A}_{QMUP} = \hat{q} \otimes \left[\frac{m_H}{m_0} |\mathbf{M}_H\rangle\langle\mathbf{M}_H| + \frac{m_L}{m_0} |\mathbf{M}_L\rangle\langle\mathbf{M}_L| \right],$$

$$\hat{A}_{CSL}(x) = \int dy g(y-x) |y\rangle\langle y| \otimes \left[\frac{m_H}{m_0} |\mathbf{M}_H\rangle\langle\mathbf{M}_H| + \frac{m_L}{m_0} |\mathbf{M}_L\rangle\langle\mathbf{M}_L| \right],$$

where $g(y-x) = \frac{1}{(\sqrt{2\pi}r_c)^d} e^{-\frac{|y-x|^2}{2r_c^2}}$.

L. Diósi, Phys. Rev. A 40, 1165 (1989).

G. C. Ghirardi, P. Pearle, A. Rimini, Phys. Rev. A 42, 78 (1990).

G. C. Ghirardi, R. Grassi, F. Benatti, Found. Phys. 25, 5 (1995).

$$\lambda_{GRW} \approx 10^{-16} s^{-1}$$

$$\lambda_{Adler} \approx 10^{-(8\pm2)} s^{-1}$$

CM & neutral mesons: State of the art

Probabilities of finding M^0 or \bar{M}^0 in beam:

$$|\langle M^0/\bar{M}^0 | M^0(t) \rangle|^2 \rightarrow \sum_{p_f} \mathbb{E} \underbrace{|\langle M^0/\bar{M}^0; p_f | U(t) | M^0; p_i, (\sqrt{\alpha}) \rangle|^2}_{\text{includes collapse now!}}$$

CSL contribution \rightarrow perturbation \Rightarrow Dyson expansion of $U(t)$.
The effect of the CSL model calculated up to the first order:

$$P_{M^0 \rightarrow M^0/\bar{M}^0}^{(1)}(t) = \frac{1}{4} \left(e^{-\Gamma_H t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_H+\Gamma_L}{2}t} (1 - \Lambda t) \cos[\Delta m t] \right),$$

where $\Lambda = \lambda_{CSL} \frac{(\Delta m)^2}{m_0^2}$, for kaons $\Lambda \propto 10^{-38} s^{-1}$.

S. Donadi et al., Found. Phys. 43, 813 (2013).

Including decay

CM in the enlarged Hilbert space:

$$\begin{aligned} d|\Psi\rangle_t &= \left\{ -i\hat{\mathcal{H}}dt + \sqrt{\lambda} \left[(\hat{\mathcal{A}} - \langle \hat{\mathcal{A}} \rangle_t) dW_t + \left(\hat{\mathcal{B}} - R_{\hat{\mathcal{B}}} \right) dW_t^D \right] \right. \\ &\quad \left. - \frac{\lambda}{2} \left[(\hat{\mathcal{A}} - \langle \hat{\mathcal{A}} \rangle_t)^2 + \hat{\mathcal{B}}^\dagger \hat{\mathcal{B}} - 2R_{\hat{\mathcal{B}}} \hat{\mathcal{B}} + R_{\hat{\mathcal{B}}}^2 \right] dt \right\} |\Psi\rangle_t, \end{aligned} \quad (4)$$

with $\hat{\mathcal{A}} = \begin{pmatrix} \hat{\mathbf{A}} & 0 \\ 0 & 0 \end{pmatrix}$, $\hat{\mathcal{B}} = \begin{pmatrix} 0 & 0 \\ \hat{\mathbf{B}} & 0 \end{pmatrix}$, $\hat{\mathbf{A}} = \sum_i \frac{m_i}{m_0} |\mathbf{M}_i\rangle\langle\mathbf{M}_i|$,
 $\hat{\mathbf{B}} = \sum_i \sqrt{\frac{\Gamma_i}{\lambda}} |\mathbf{f}_i\rangle\langle\mathbf{M}_i|$, and $R_{\hat{\mathcal{B}}} = \left\langle \frac{\hat{\mathcal{B}}^\dagger + \hat{\mathcal{B}}}{2} \right\rangle_t$.

K. Simonov, Phys. Rev. A 102, 022226 (2020).

Including decay

Back to the flavor space:

$$d|\psi\rangle_t = \left[-i\hat{M}dt + \sqrt{\lambda}(\hat{A} - \langle\hat{A}\rangle_t)dW_t - \frac{\lambda}{2}((\hat{A} - \langle\hat{A}\rangle_t)^2 + \hat{B}^\dagger\hat{B})dt \right] |\psi\rangle_t,$$

so that $\hat{\Gamma} = \lambda\hat{B}^\dagger\hat{B}$.

It is possible to simplify this equation via so-called “imaginary trick”, which is a transformation to an imaginary noise field while preserving the corresponding master equation,

$$d|\psi\rangle_t = \left[-i\hat{M}dt + i\sqrt{\lambda}\hat{A}dW_t - \frac{\lambda}{2}(\hat{A}^2 + \hat{B}^\dagger\hat{B})dt \right] |\psi\rangle_t.$$

K. Simonov, Phys. Rev. A 102, 022226 (2020).

Time asymmetry in the noise field

Noise correlation function:

$$\int_0^t dt_1 \int_0^{t_1} dt_2 \mathbb{E}[dW(t_1)dW(t_2)] = (1 - \beta)t,$$

where $\beta \in [0, 1]$ defines the time asymmetry (symmetry $\rightarrow \beta = 1/2$).
This corresponds to a family of quantum state equations

$$d|\psi\rangle_t = \left[-i\hat{M}dt + \sqrt{\lambda}(\hat{A} - \langle \hat{A} \rangle_t) dW_t - \lambda\beta\hat{A}^2dt \right] |\psi\rangle_t.$$

Hence, we can recover the decay operator and the corresponding decay rates as

$$\hat{\Gamma} = \lambda(2\beta - 1)\hat{A}^2, \quad \Gamma_i = \lambda(2\beta - 1) \frac{m_i^2}{m_0^2}.$$

Time asymmetry in the noise field

Final master equation:

$$\begin{aligned}\frac{d\hat{\rho}_t}{dt} = & i[\hat{\rho}_t, \hat{\mathbf{M}}] - \frac{\lambda}{2} [\hat{\mathbf{A}}^2 \hat{\rho}_t + \hat{\rho}_t \hat{\mathbf{A}}^2 - 2\hat{\mathbf{A}} \hat{\rho}_t \hat{\mathbf{A}}] \\ & - \frac{\lambda}{2} (2\beta - 1) \{ \hat{\mathbf{A}}^2, \hat{\rho}_t \}. \end{aligned} \quad (5)$$

CM can recover the decay dynamics of neutral mesons!

K. Simonov, Phys. Rev. A 102, 022226 (2020).

Example: QMUPL and CSL models

$$P_{M^0 \rightarrow M^0 / \bar{M}^0}^{QMUPPL}(\mathbf{t}) = \frac{1}{4} \left\{ \sum_i \left(1 + \lambda_Q \alpha (2\beta - 1) \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} \mathbf{t} \right)^{-\frac{d}{2}} \right. \\ \left. \pm \frac{2 \cos[\mathbf{t} \Delta \mathbf{m}]}{\left(1 + \frac{\lambda_Q \alpha}{2} \left((2\beta - 1) \sum_i \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} + \frac{(\Delta \mathbf{m})^2}{\mathbf{m}_0^2} \right) \mathbf{t} \right)^{\frac{d}{2}}} \right\}, \quad (6)$$

$$P_{M^0 \rightarrow M^0 / \bar{M}^0}^{CSL}(\mathbf{t}) = \frac{1}{4} \left\{ \sum_i e^{-\lambda_{CSL} (2\beta - 1) \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} \mathbf{t}} \right. \\ \left. \pm 2 e^{-\frac{1}{2} \lambda_{CSL} \left((2\beta - 1) \sum_i \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} + \frac{(\Delta \mathbf{m})^2}{\mathbf{m}_0^2} \right) \mathbf{t}} \cos[\mathbf{t} \Delta \mathbf{m}] \right\}. \quad (7)$$

These results can be found by solving master equation exactly or doing perturbation theory in quantum state equation.

K. Simonov, B. C. Hiesmayr, Phys. Rev. A 94, 052128 (2016).

K. Simonov, Phys. Rev. A 102, 022226 (2020).

Example: QMUPL and CSL models

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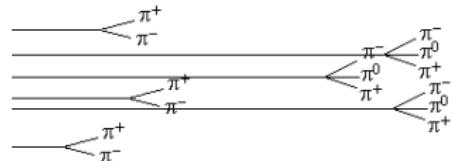
$$P_{M^0 \rightarrow M^0/\bar{M}^0}^{CSL}(\mathbf{t}) = \frac{1}{4} \left\{ \sum_i e^{-\lambda_{CSL} (2\beta - 1) \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} \mathbf{t}} \right. \\ \left. \pm 2 e^{-\frac{1}{2} \lambda_{CSL} \left((2\beta - 1) \sum_i \frac{\mathbf{m}_i^2}{\mathbf{m}_0^2} + \frac{(\Delta \mathbf{m})^2}{\mathbf{m}_0^2} \right) \mathbf{t}} \cos[\mathbf{t} \Delta \mathbf{m}] \right\}. \quad (9)$$

For a neutral meson system, QMUPL and CSL are not equivalent!

Impact of the CSL model prediction: Masses



$$\frac{m_\mu}{m_0} \longleftrightarrow \frac{m_0}{m_\mu}$$



$$\frac{2\Delta\Gamma}{\Delta\Gamma\pm 2\Gamma} m_L^2 + 2(\Delta m)m_L + (\Delta m)^2 = 0,$$

with $\Gamma = \frac{\Gamma_H + \Gamma_L}{2}$.

	$\Gamma_L^{\text{exp}} [\text{s}^{-1}]$	$\Gamma_H^{\text{exp}} [\text{s}^{-1}]$	$\Delta m^{\text{exp}} [\hbar s^{-1}]$	$m_L [\hbar s^{-1}]$	$m_H [\hbar s^{-1}]$
K-mesons	$1.117 \cdot 10^{10}$	$1.955 \cdot 10^7$	$0.529 \cdot 10^{10}$	$2.311 \cdot 10^8$	$5.524 \cdot 10^9$
D-mesons	$2.454 \cdot 10^{12}$	$2.423 \cdot 10^{12}$	$0.950 \cdot 10^{10}$	$1.468 \cdot 10^{12}$	$1.477 \cdot 10^{12}$
B_d-mesons	$6.582 \cdot 10^{11}$	$6.576 \cdot 10^{11}$	$0.510 \cdot 10^{12}$	$1.020 \cdot 10^{15}$	$1.020 \cdot 10^{15}$
B_s-mesons	$7.072 \cdot 10^{11}$	$6.158 \cdot 10^{11}$	$1.776 \cdot 10^{13}$	$2.477 \cdot 10^{14}$	$2.655 \cdot 10^{14}$

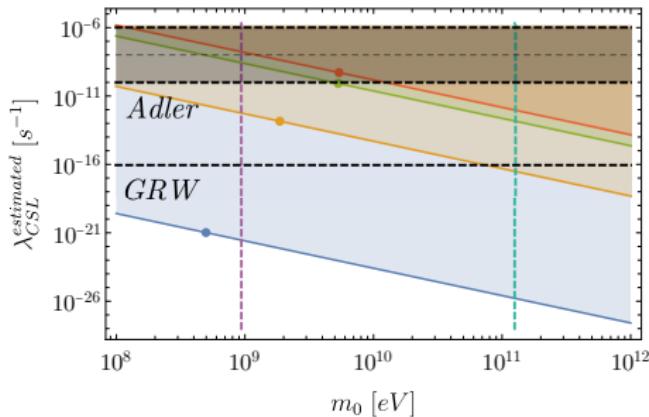
K. Simonov, B. C. Hiesmayr, Phys. Rev. A 94, 052128 (2016).

K. Simonov, Phys. Rev. A 102, 022226 (2020).

CSL model prediction: Collapse rate

In terms of experimentally measured quantities:

$$\lambda_{CSL}^{\text{estimated}} \geq \left(\frac{\Delta m}{m_0 (\sqrt{\Gamma_L^{\pm 1}} - \sqrt{\Gamma_H^{\pm 1}})} \right)^{\mp 2}, \quad (10)$$



K. Simonov, B. C. Hiesmayr, Phys. Rev. A 94, 052128 (2016).

K. Simonov, Phys. Rev. A 102, 022226 (2020).

Conclusions

- The effect of a collapse model on a neutral meson system crucially depends on the nature of the noise, namely, its time (a)symmetry β .
- Any asymmetric value $\beta \neq \frac{1}{2}$ leads to a dependence on the absolute masses which does not show up in the standard quantum mechanics and, moreover, recovers decay dynamics in both master and quantum state equations.
- In turn, it is possible to deduce the absolute masses m_H , m_L of the neutral mesons and predict the value of the collapse rate λ .

THANK YOU FOR YOUR ATTENTION!