

Astrophysics and detection of gravitational wave sources

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(Universita` di Milano Bicocca)



OUTLINE

LECTURE 1 (NOW): Setting the stage

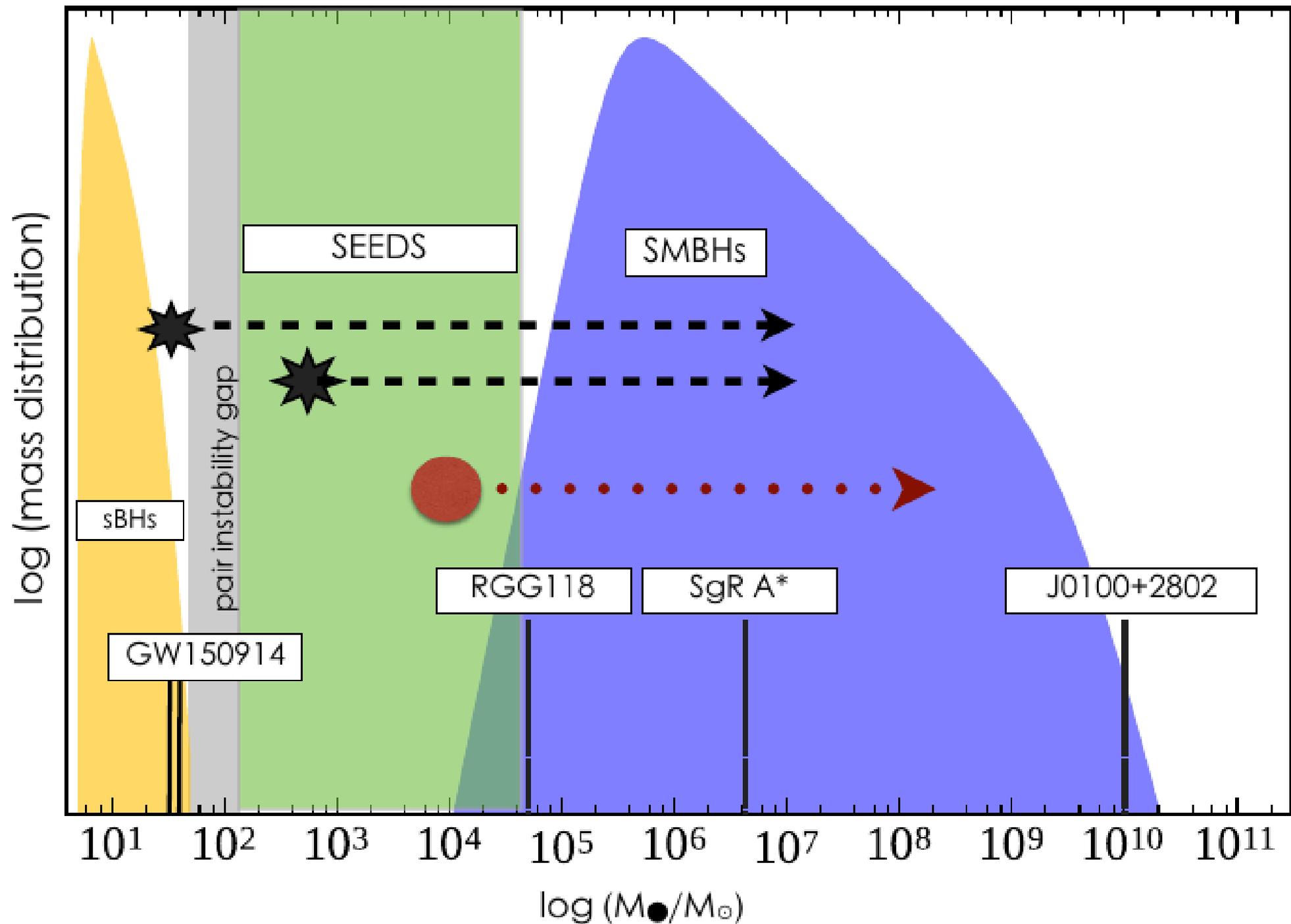
- Gravitational waves (GWs): theory and general considerations**
- GWs from binary systems, relevant scalings**

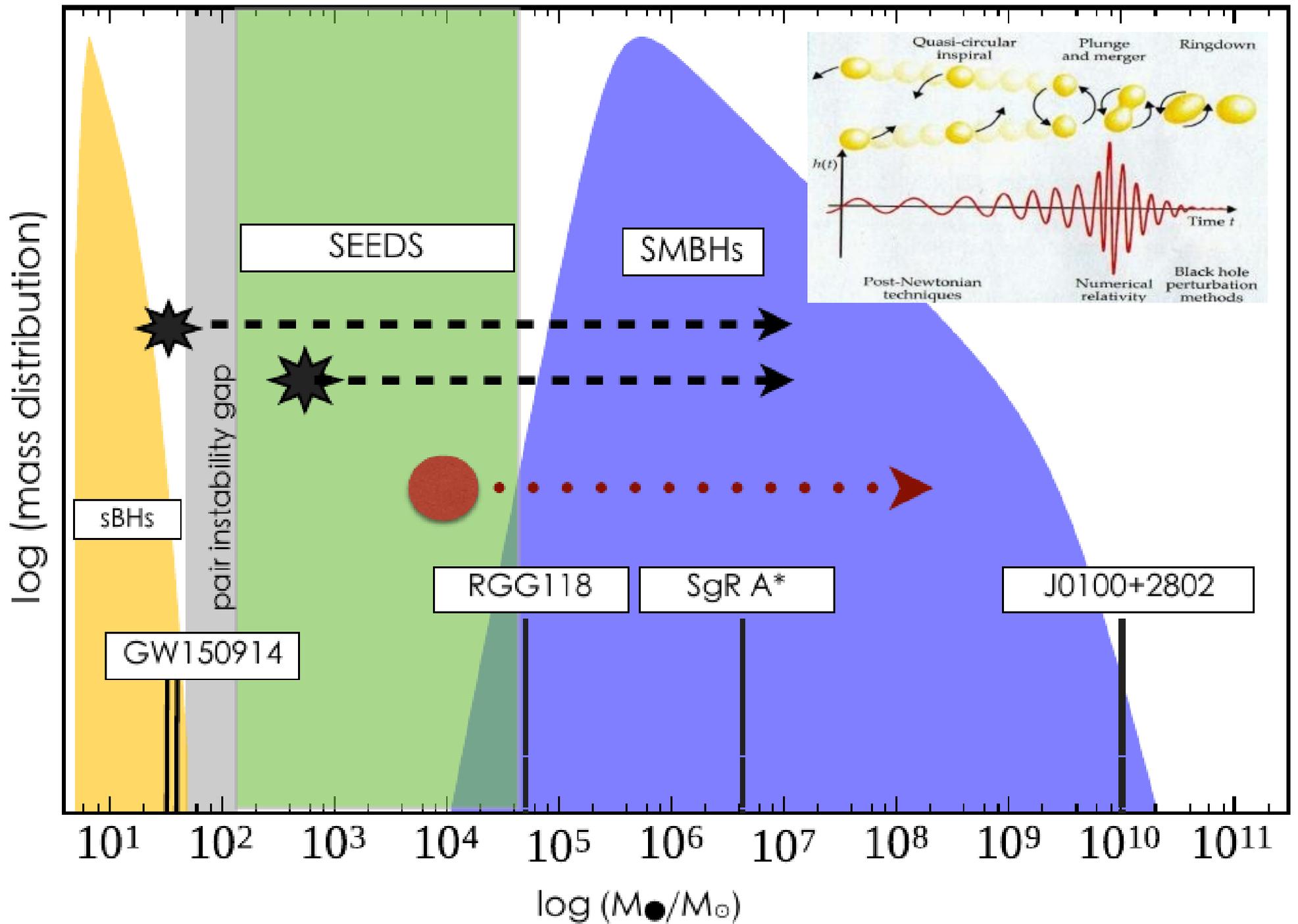
LECTURES 2/3 (Monday afternoon): ground based

- Detection of GW with ground based interferometers**
- Black hole binaries (BHBs) detected by LIGO/Virgo**
- GW170817 a neutron star binary (NSB)**
- Astrophysics of ground based GW sources: formation scenarios**
- Future from the ground: 3G detectors**

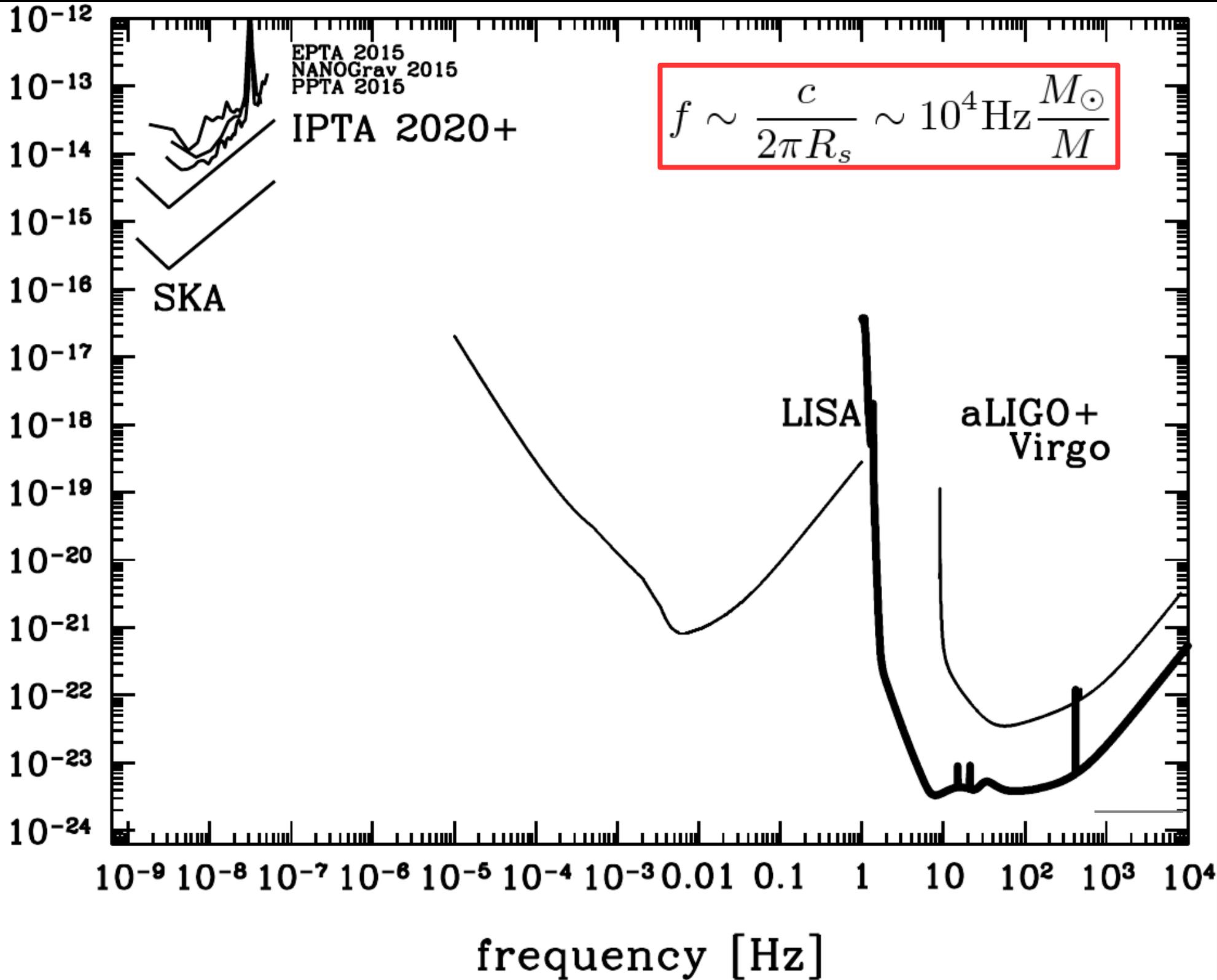
LECTURE 4/5 (Tuesday morning): space based

- Beyond the ground: GW detection from space**
- Laser interferometer space antenna and its sources**
- Galactic binaries**
- Extreme mass ratio inspirals (EMRIs)**
- Massive black hole (MBH) formation and evolution**

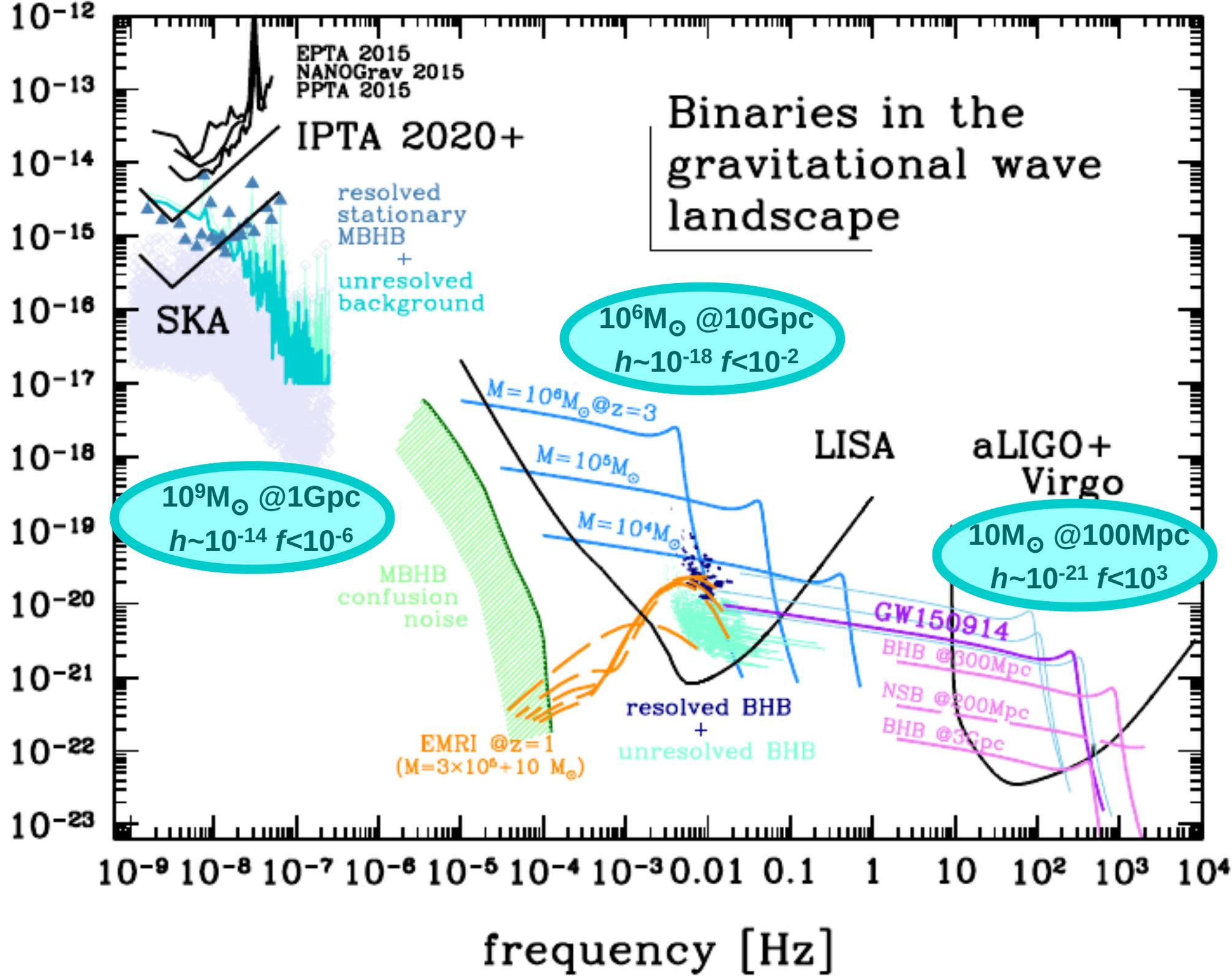




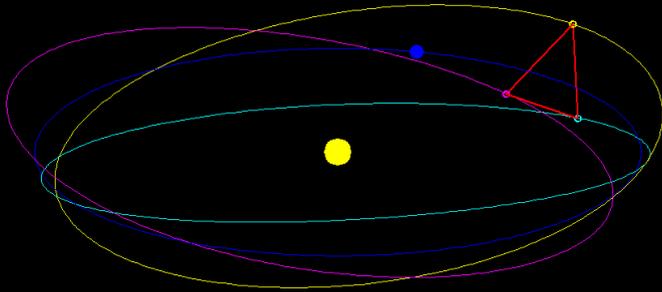
characteristic amplitude



characteristic amplitude



The Laser Interferometer Space Antenna



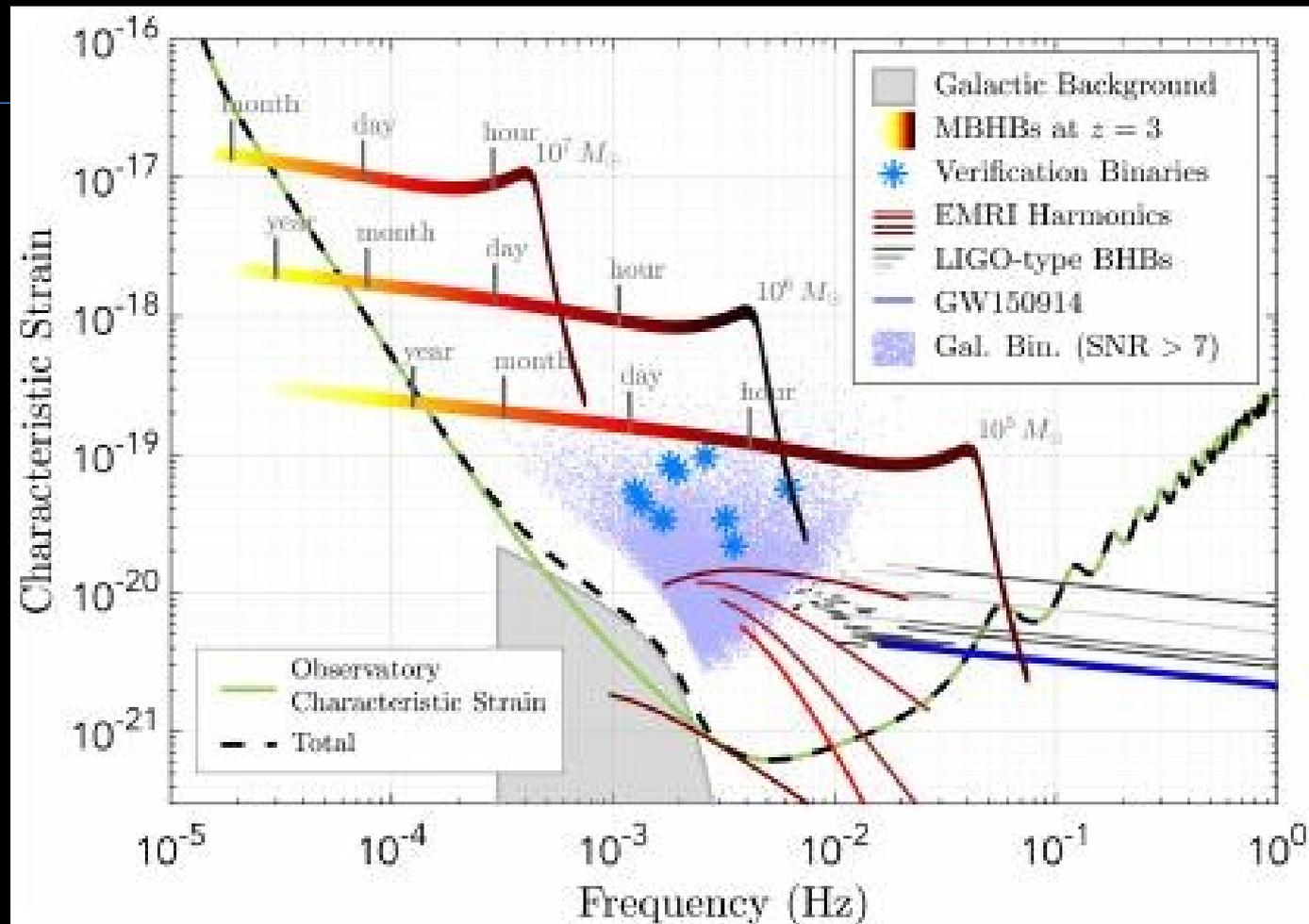
Sensitive in the mHz frequency range, arguably the richest GW-source band

- Chirping MBHBs
- Extreme mass ratio inspirals
- Galactic binaries (WDs, NSs, BHs)
- Extragalactic binaries (“LIGO-Virgo” BHs)
- Intermediate mass black holes?
- Cosmological backgrounds

Nicolas Douillet - ARTEMIS

3 satellites trailing the Earth connected through laser links

Current baseline:
2.5M km armlength
6 laser links
4 yr lifetime (10 yr goal)



The LISA Consortium

- Now a thriving community: 1300+ among full and associate members
- Several working groups connecting to the community: astrophysics, fundamental physics, cosmology, waveforms
- Several working packages defining deliverables
- 2 consortium meetings/yr, LISA symposium every 2 years, dedicated WG meetings every year

<https://www.lisamission.org/>



LISA Consortium User Guide

User guide ▾ Groups ▾ Getting help ▾ Contributing

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LISA Consortium User Guide

This User Guide goal is to gather all the information related to the LISA Consortium tools. Users are more than welcome to contribute to its improvement. To do so, see the [HowToContribute](#) page.

- LISA Consortium User Guide
- Key information
- Collaborative tools
- Development tools and guidelines
- Sharing data tools
- Computing resources

Key information

- LISA Consortium website
- Sign-up for the LISA Consortium
- Organisation
- LISA websites
- Key documents
- Next meetings (need to be logged to the wiki - see [LISA wiki](#))
- Acronyms
- Publication and Presentation Committee
- Inclusion and Diversity Committee
- Positions related to LISA

Collaborative tools

- LISA wiki
- LISA Document Management System (DMS) - Atrium
- Mailing lists
- Messaging on slack channels
- Audio / Video teleconferences

Development tools and guidelines

LISA Consortium User Guide

User guide ▾ Groups ▾ Getting help ▾ Contributing

Search Previous

Mailing lists

- Management
- Full Member Groups
 - LISA Instrument Group
 - LISA Data Processing Group
 - LISA Science Group
 - Simulation Working Groups
- Associate and Full Members Groups
 - LISA Data Challenge Working Groups
 - Astrophysics Working Groups
 - Cosmology Working Groups
 - Fundamental Physics Working Groups
 - Waveform Working Groups
 - Advocacy and Outreach Working Groups

Mailing lists

- Consortium : consortium@lisamission.org

Management

- Consortium Lead : consortiumlead@lisamission.org
- Exec Board : exec_board@lisamission.org
- Board Member : board@lisamission.org
- Coordinator : coord@lisamission.org
- Coordination Group : coordination@lisamission.org
- Publication Committee : pubcom@lisamission.org
- Publication Committee Chairs : pubcom-chairs@lisamission.org

Full Member Groups

LISA Instrument Group

- LISA Instrument Group : lig@lisamission.org
- LIG Core : lig-core@lisamission.org
- LIG Performance Modelling WG : lig-pmwg@lisamission.org
- LIG-OB : lig-ob@lisamission.org
- LIG-PMS : lig-pms@lisamission.org
- LIG-GRS : lig-grs@lisamission.org
- LIG-OMS : lig-oms@lisamission.org
- LIG-Chairs : lig-chairs@lisamission.org
- LIG SLWG Chairs : lig-slwg-chairs@lisamission.org
- LIG Performance Modelling WG Chairs : lig-pmwg-chairs@lisamission.org

Binary evolution with frequency

$$E = \frac{M_1 v_1^2}{2} + \frac{M_2 v_2^2}{2} - \frac{GM_1 M_2}{r} = \frac{\mu v^2}{2} - \frac{GM\mu}{r} = -\frac{GM\mu}{2a}$$

$$\frac{dE_{\text{rad}}}{dt} = \frac{32 G^4 \mu^2 M^3}{5 c^5 a^5} F(e) = \frac{32 G^4 M_1^2 M_2^2 (M_1 + M_2)}{5 c^5 a^5} F(e)$$

Equating $dE_{\text{rad}}/dt = -dE/dt$ and solving for a we get

$$\frac{da}{dt} = -\frac{64 G^3 M_1 M_2 (M_1 + M_2)}{5 c^5 a^3} F(e)$$

Using Kepler's law we finally get

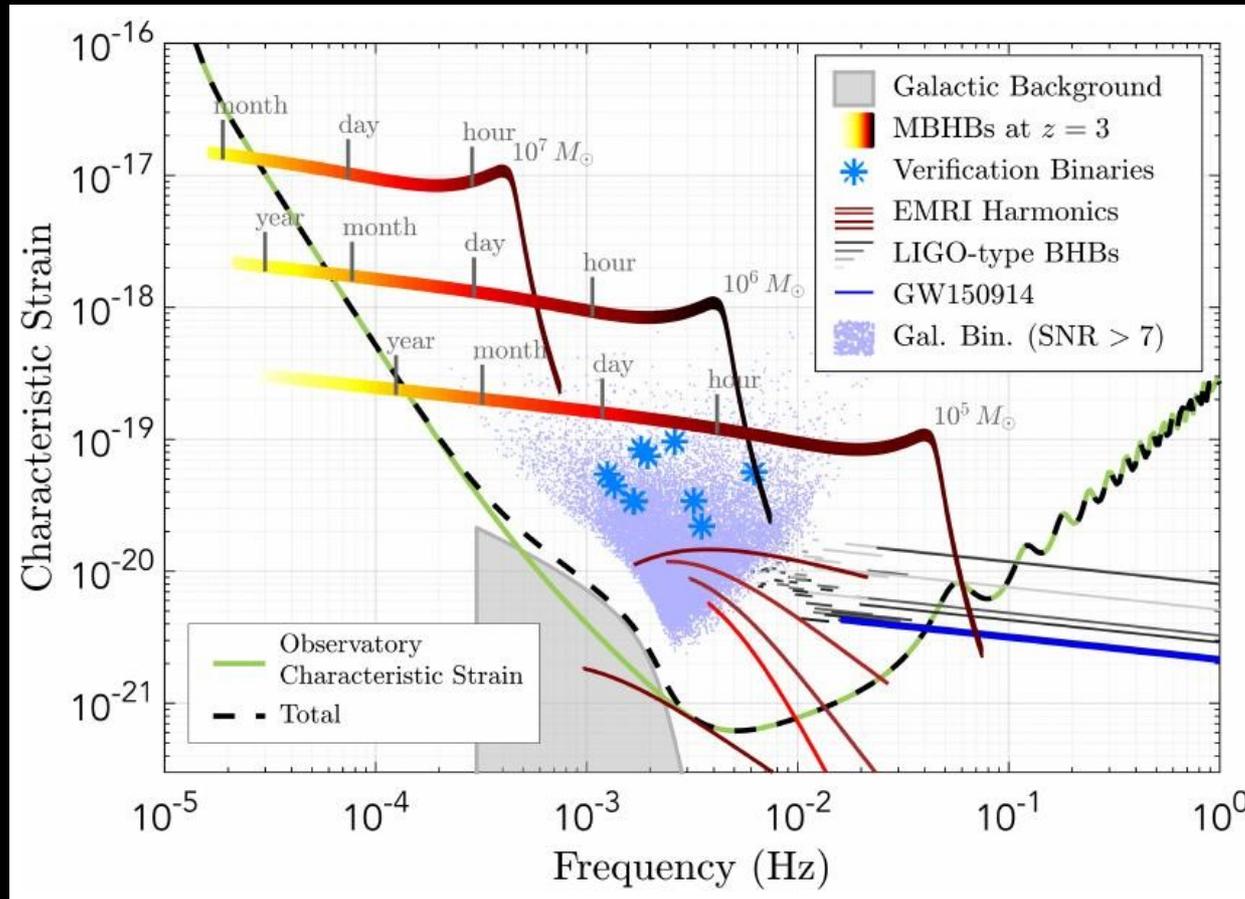
$$\frac{df_r}{dt_r} = \frac{96}{5} \pi^{8/3} \mathcal{M}^{5/3} f_r^{11/3}$$

$$\frac{dt}{d \ln f} \approx 8 \text{ s} \left(\frac{\mathcal{M}}{M_\odot} \right)^{-5/3} \left(\frac{f}{100 \text{ Hz}} \right)^{-8/3}$$

1-Solar mass binaries at mHz live many years

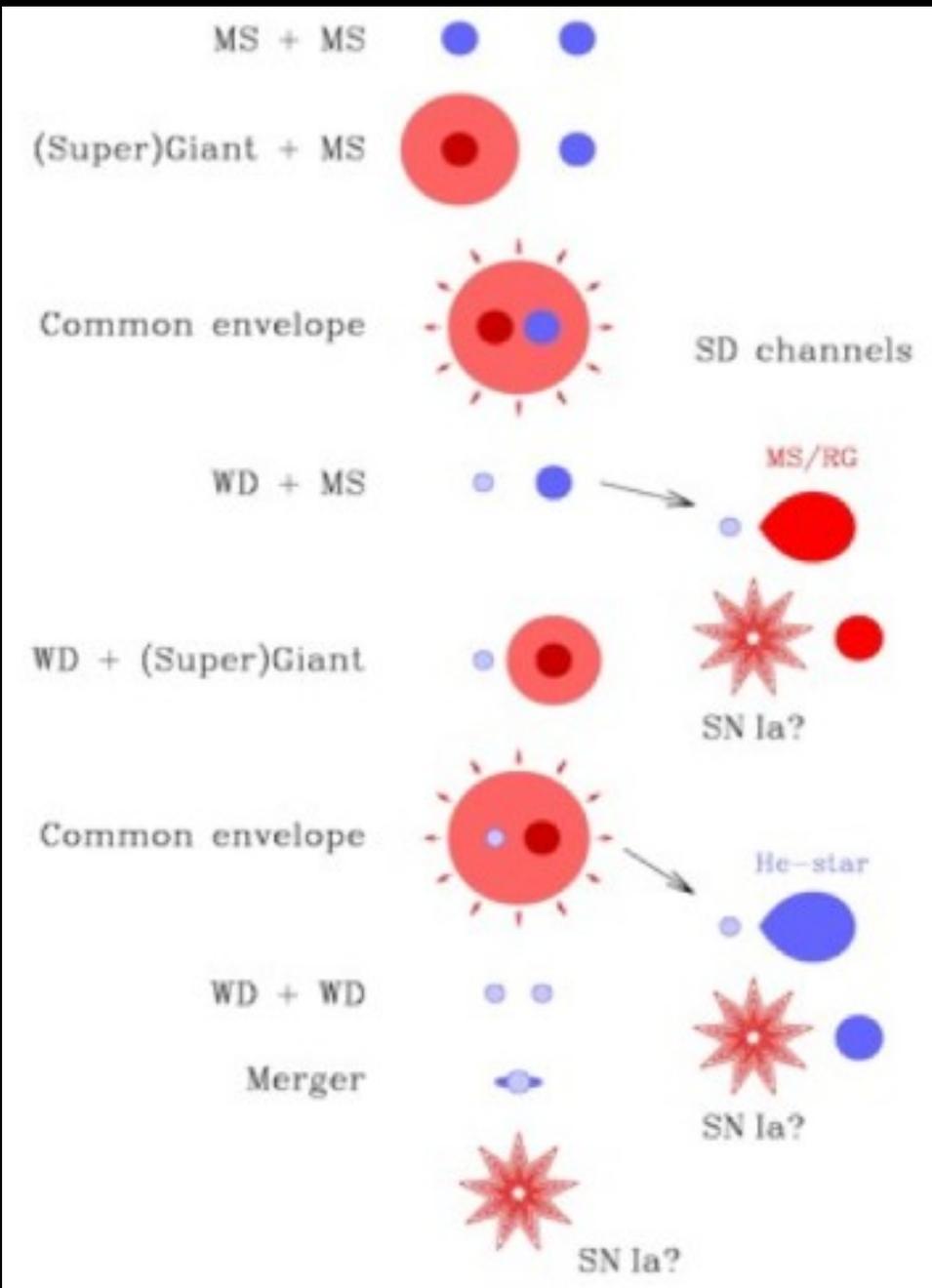
2-Massive binaries at mHz live days/hours

Massive objects inspiralling and merging:
 frequency set by the most massive object so we have:
 1-massive black hole binaries (MBHBs)
 2-extreme mass ratio inspirals (EMRIs)



Light objects far from coalescence:
 monochromatic or slowly inspiralling
 1-Galactic binaries (all flavours, most prominent WD-WD)
 2-Extragalactic stellar BHBs (multiband astronomy)

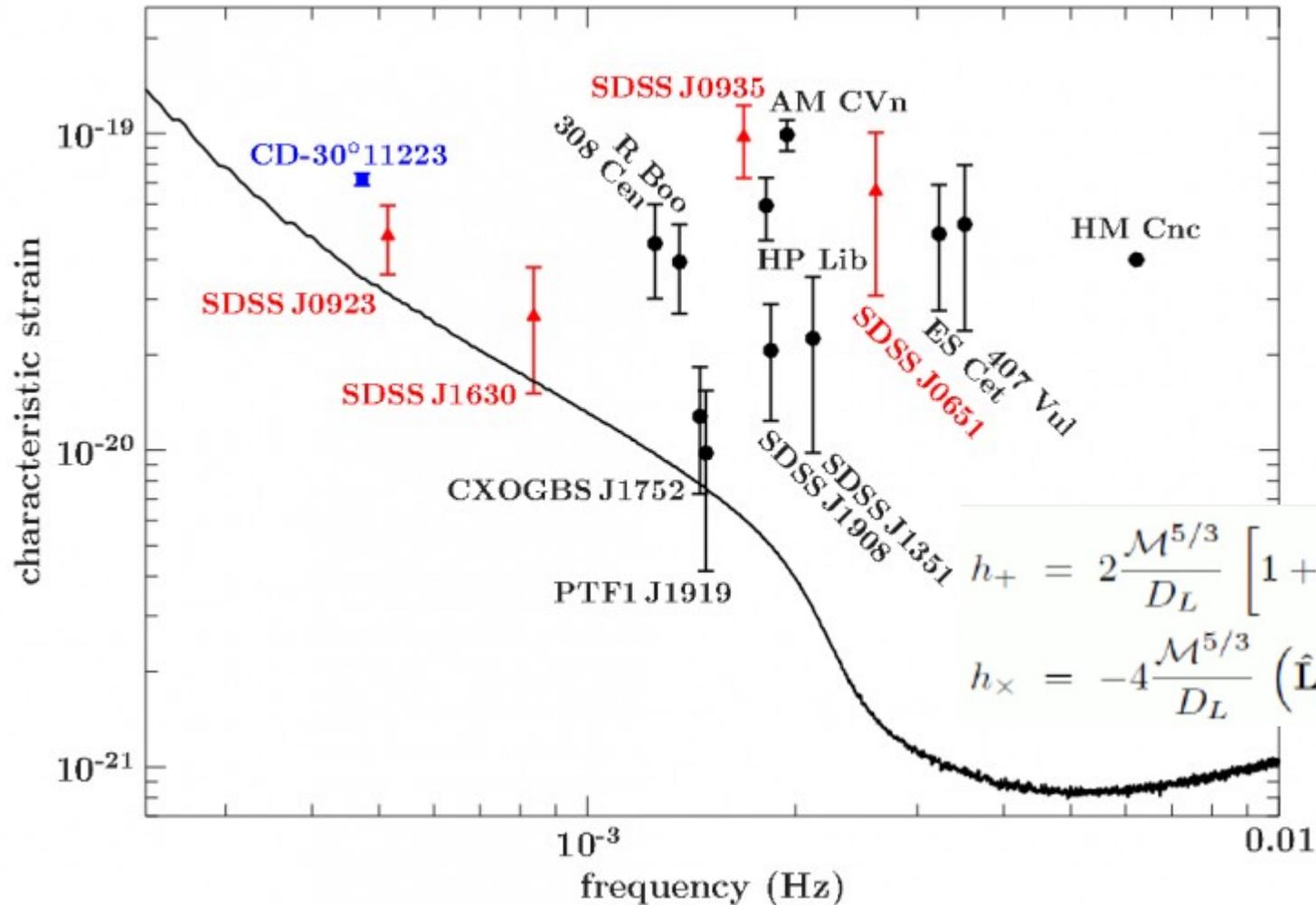
Galactic binaries



- How many ultra-compact binaries exist in the Milky Way?
- What is the merger rate of white dwarfs, neutron stars and stellar mass black holes in the Milky Way (thus better constraining the rate of the explosive events associated with these sources)?
- What does that imply for, or how does that compare to, their merger rates in the Universe?
- What happens at the moment a white dwarf starts mass exchange with another white dwarf or neutron star, and what does it tell us about the explosion mechanism of type Ia supernovae?
- What is the spatial distribution of ultra-compact binaries, and what can we learn about the structure of the Milky Way as a whole?

There are WD binaries for which we know period masses and distance (to some extent): verification binaries.

These systems are known to produce a high S/N signal in LISA.
Many more expected to come with GAIA



$$h_+ = 2 \frac{\mathcal{M}^{5/3}}{D_L} \left[1 + (\hat{\mathbf{L}} \cdot \hat{\mathbf{N}})^2 \right] (\pi f)^{2/3} \cos \phi(t)$$

$$h_\times = -4 \frac{\mathcal{M}^{5/3}}{D_L} (\hat{\mathbf{L}} \cdot \hat{\mathbf{N}}) (\pi f)^{2/3} \sin \phi(t),$$

How many binaries?

WD binary merger rate in the galaxy is estimated to be $\sim 10^{-2} \text{ } 10^{-3}/\text{yr}$

Remember:

$$\frac{df_r}{dt_r} = \frac{96}{5} \pi^{8/3} \mathcal{M}^{5/3} f_r^{11/3}$$

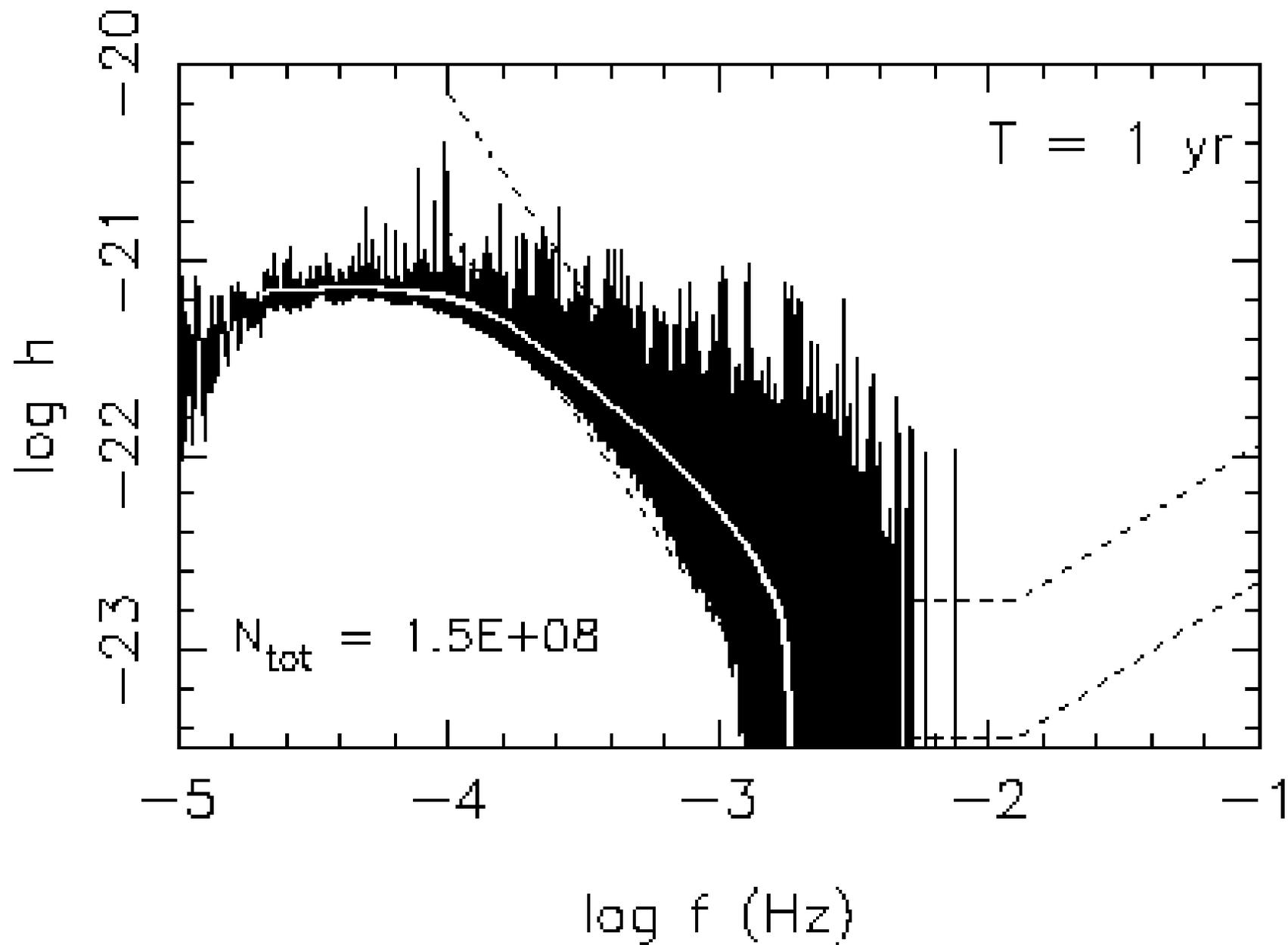
$dN/d\ln f = dN/dt \times dt/d\ln f \rightarrow$ proportional to $f^{8/3}$

$$\frac{dt}{d\ln f} \approx 8 \text{ s} \left(\frac{\mathcal{M}}{M_\odot} \right)^{-5/3} \left(\frac{f}{100 \text{ Hz}} \right)^{-8/3}$$

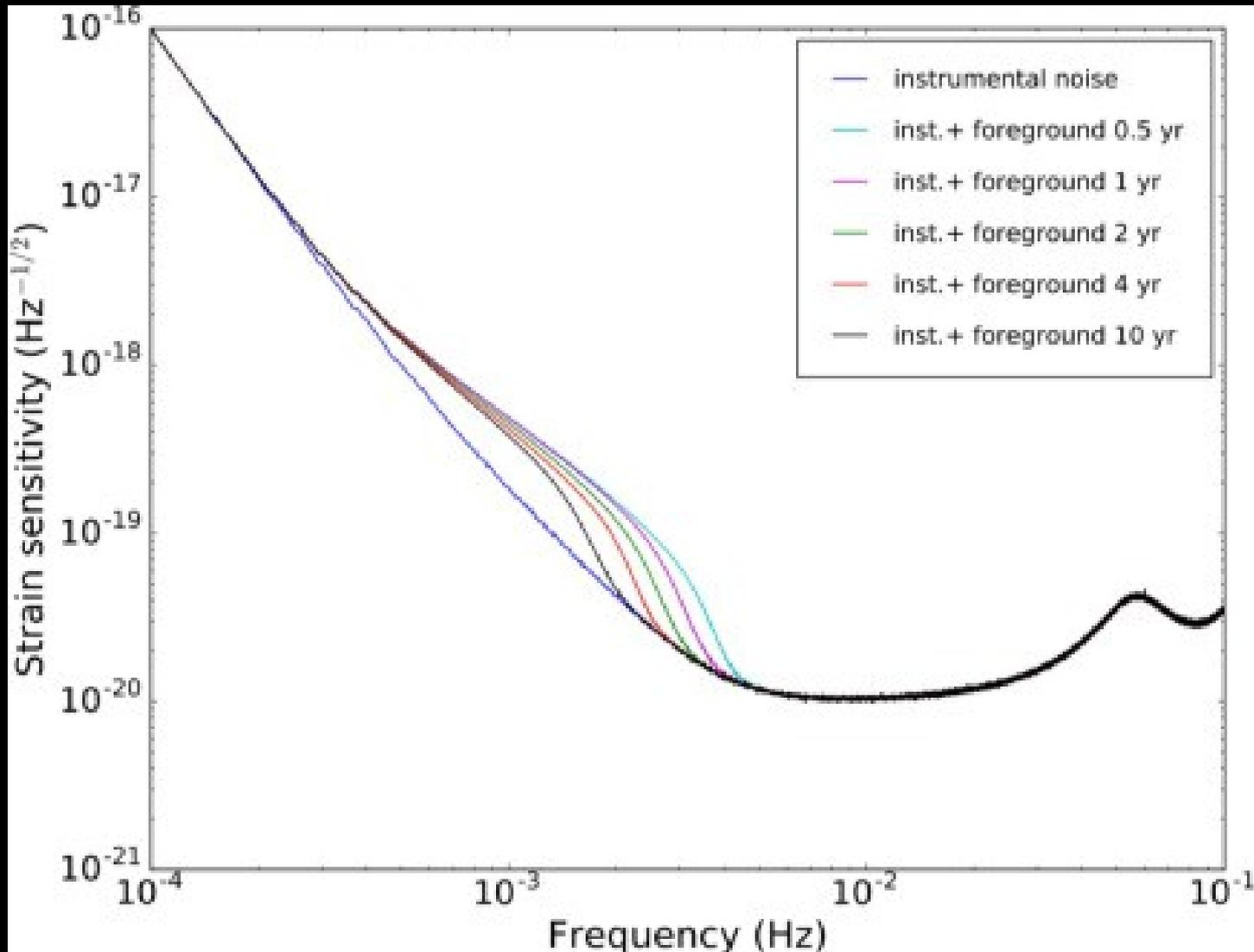
So at $f \sim 0.1 \text{ mHz}$ we have, $\sim 10^{-9} \times 10 \times 10^{16} \sim 100 \text{ M}$
There are $\sim 100 \text{ M}$ WD binaries estimated in the MW

One can make a similar calculation for NS and BH binaries to get
 $\sim 10\text{-}100$ BHBs at $f > 0.1 \text{ mHz}$ in the MW
 $\sim 10^5$ NSB at $f > 0.1 \text{ mHz}$ in the MW

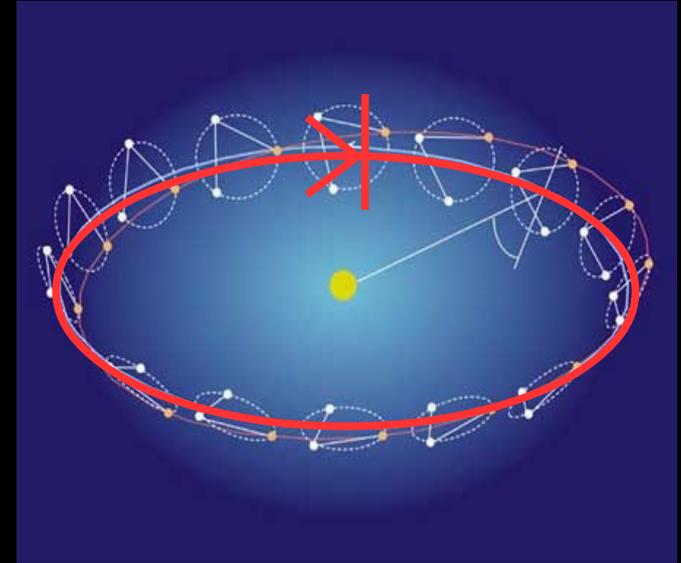
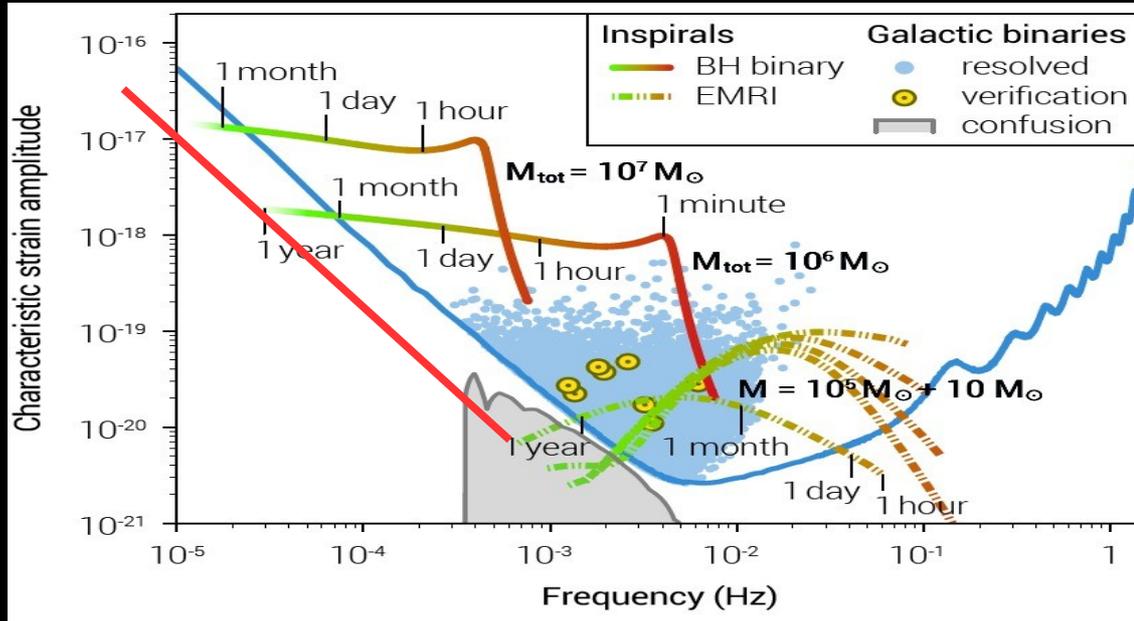
The signal looks like a 'forest' of lines piling up



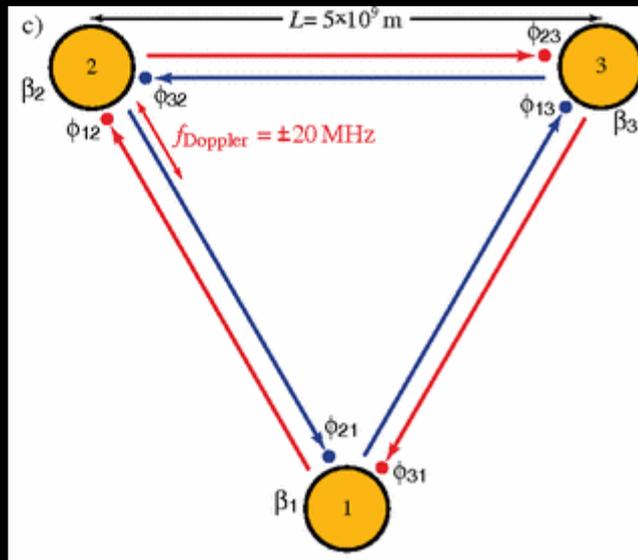
After subtraction of the brightest sources you are left as an Irreducible 'confusion noise' that will degrade the ability to detect other individual sources



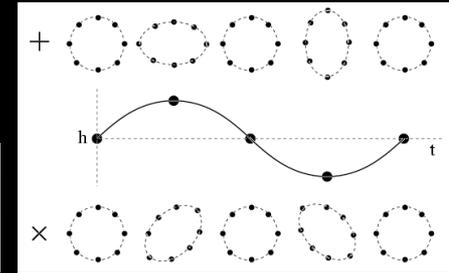
Baseline



Number of laser links



$$h(t) = \sum_A h_A(t - \hat{\Omega} \cdot x) F^A(\hat{\Omega}, \psi)$$



$$F^+(\theta, \phi, \psi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi,$$

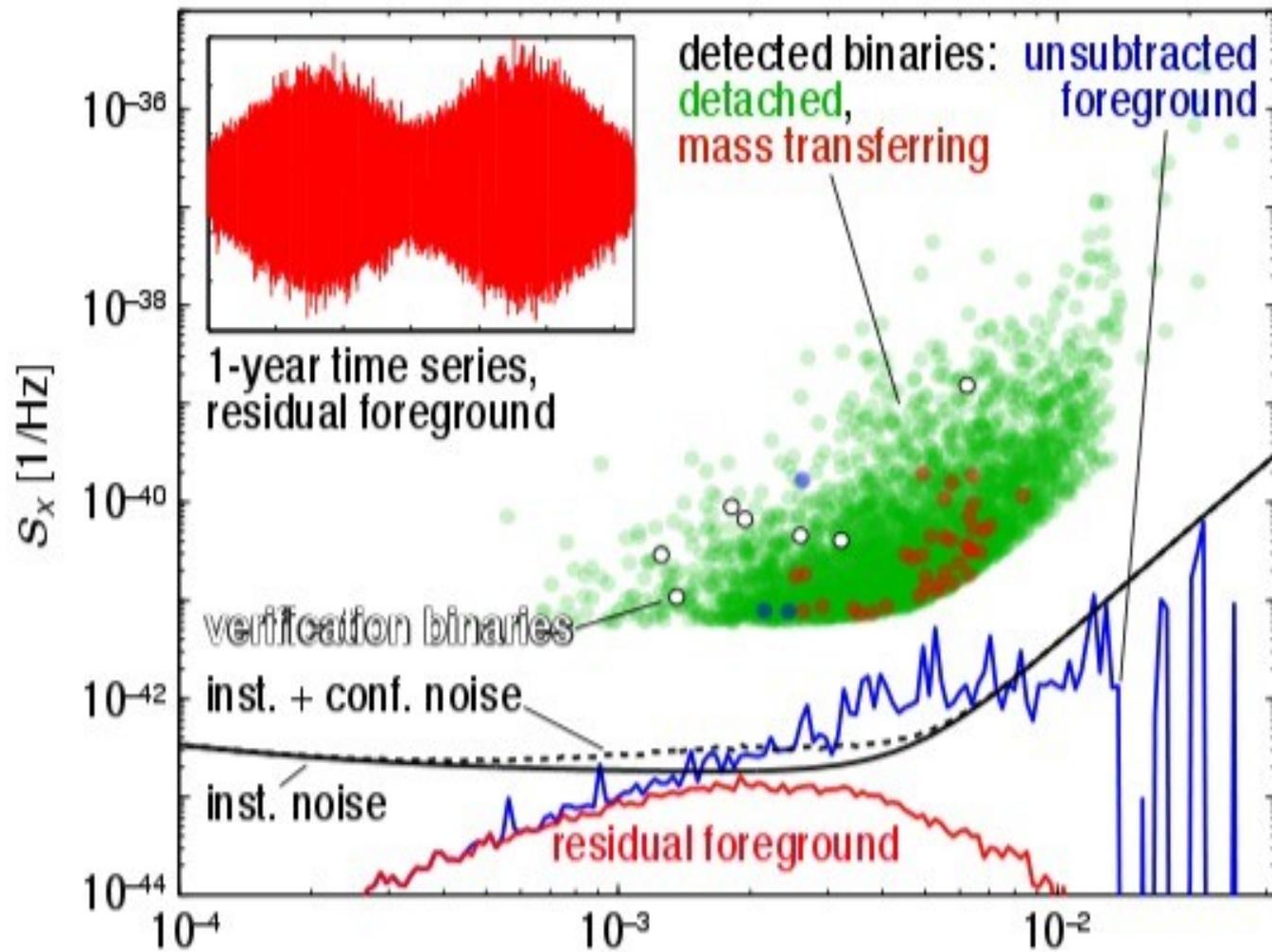
$$F^\times(\theta, \phi, \psi) = -\frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \sin 2\psi - \cos \theta \sin 2\phi \cos 2\psi,$$

$$F^x(\theta, \phi, \psi) = \sin \theta (\cos \theta \cos 2\phi \cos \psi - \sin 2\phi \sin \psi),$$

$$F^y(\theta, \phi, \psi) = -\sin \theta (\cos \theta \cos 2\phi \sin \psi + \sin 2\phi \cos \psi),$$

$$F^b(\theta, \phi) = -\frac{1}{2} \sin^2 \theta \cos 2\phi,$$

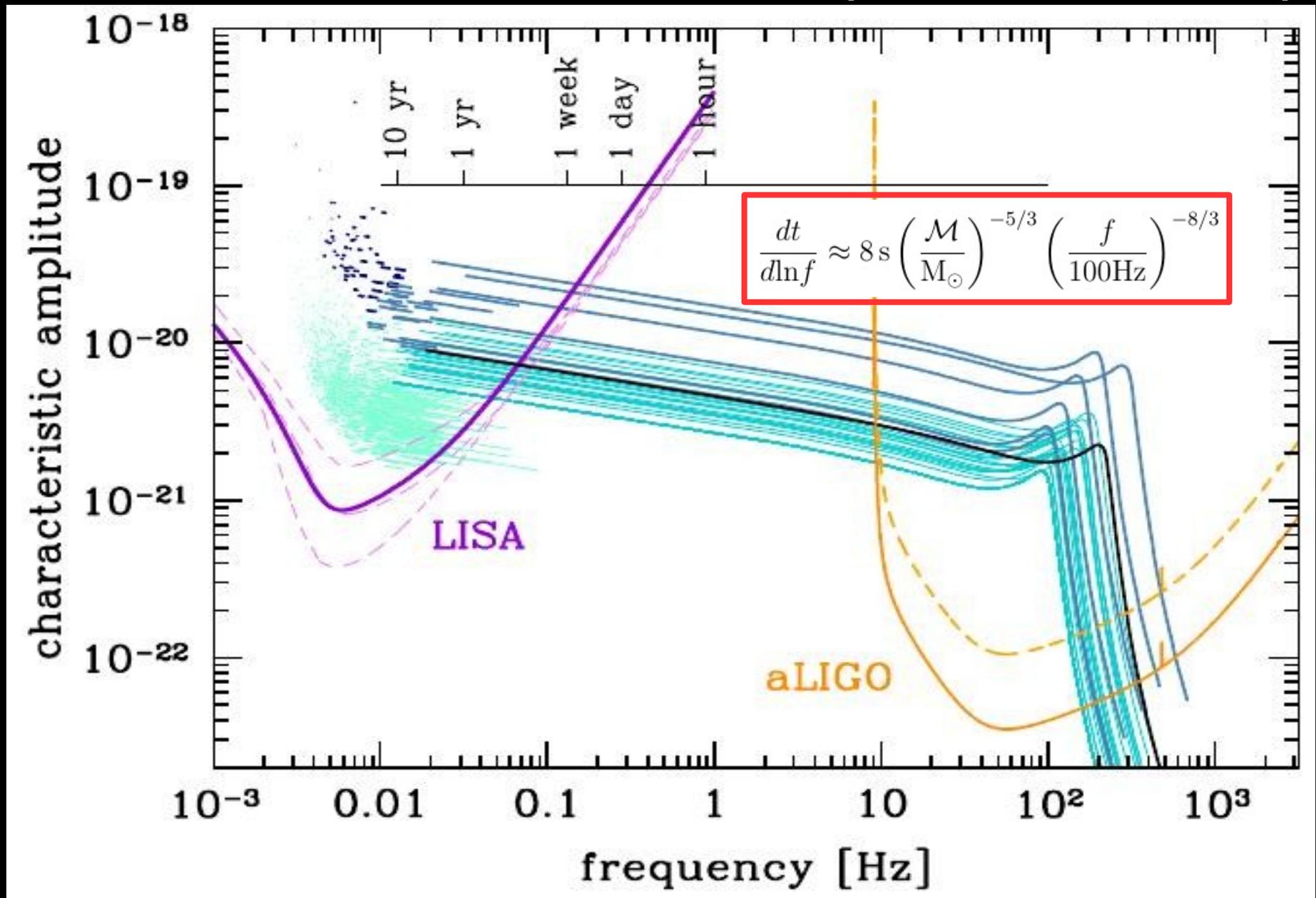
$$F^\ell(\theta, \phi) = \frac{1}{2} \sin^2 \theta \cos 2\phi.$$



	C2 - 4 links baseline	C5 - 4 links option 2Gm	C2 - 6 links option 6 links	C5 - 6 links option++
SNR > 7	4586	6827	6204	11278
$\Delta\Omega < 1 \text{ deg}^2$	1107	1821	1892	3150
$\Delta df/dt < 20\%$	1331	1814	1678	2284
3D position	370	628	457	814

An unexpected scenario: multi-band GW astronomy

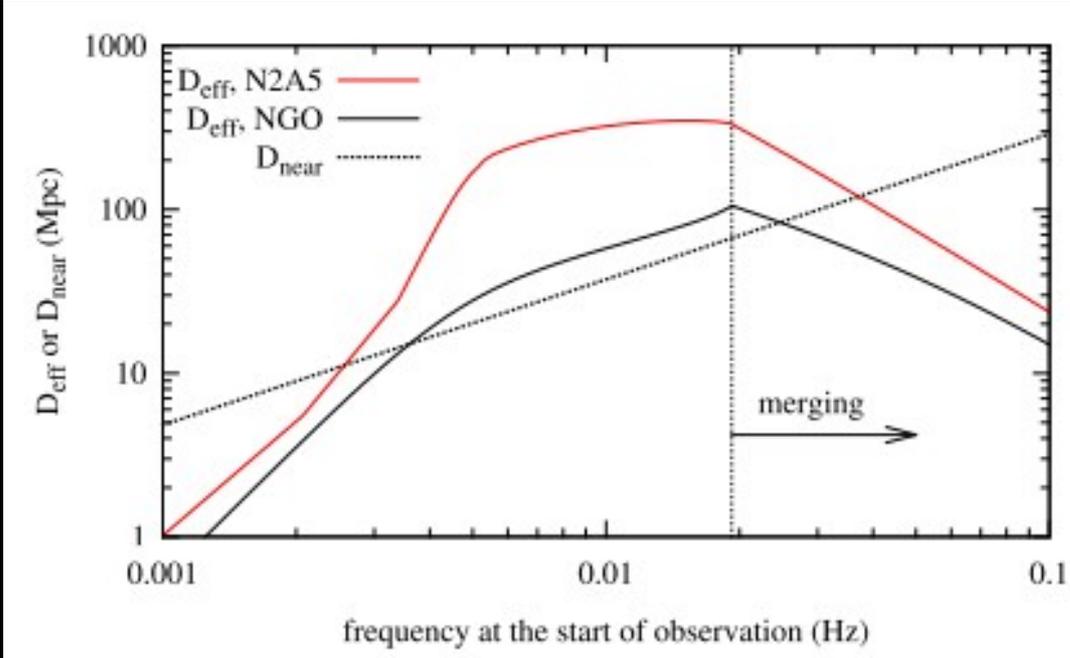
(AS 2016, PRL 116, 1102)



BHB will be detected by eLISA and cross to the LIGO band, assuming a 5 year operation of eLISA.

Distribution of sources across the band

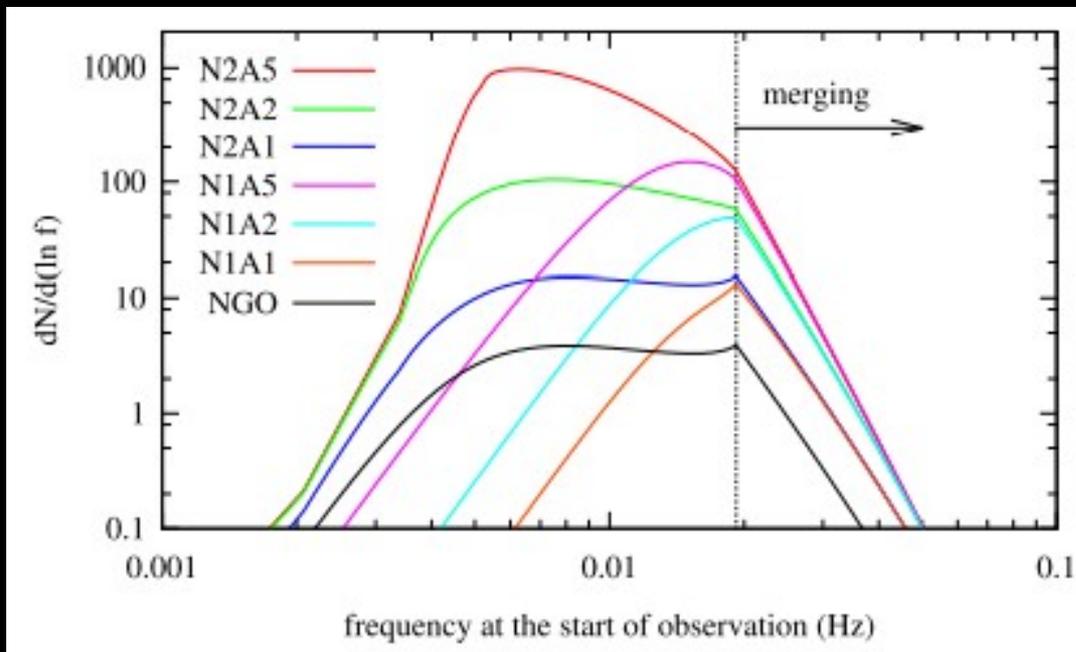
(Kyutoku & Seto 2016)



$$\frac{dt}{d \ln f} \approx 8 \text{ s} \left(\frac{\mathcal{M}}{M_{\odot}} \right)^{-5/3} \left(\frac{f}{100 \text{ Hz}} \right)^{-8/3}$$

Reach of eLISA for GW150915.
Up to $z \sim 0.1$ at $f \sim 0.01 \text{ Hz}$

-Almost stationary at $f < 0.02 \text{ Hz}$
-Evolving to the LIGO band for $f > 0.02 \text{ Hz}$

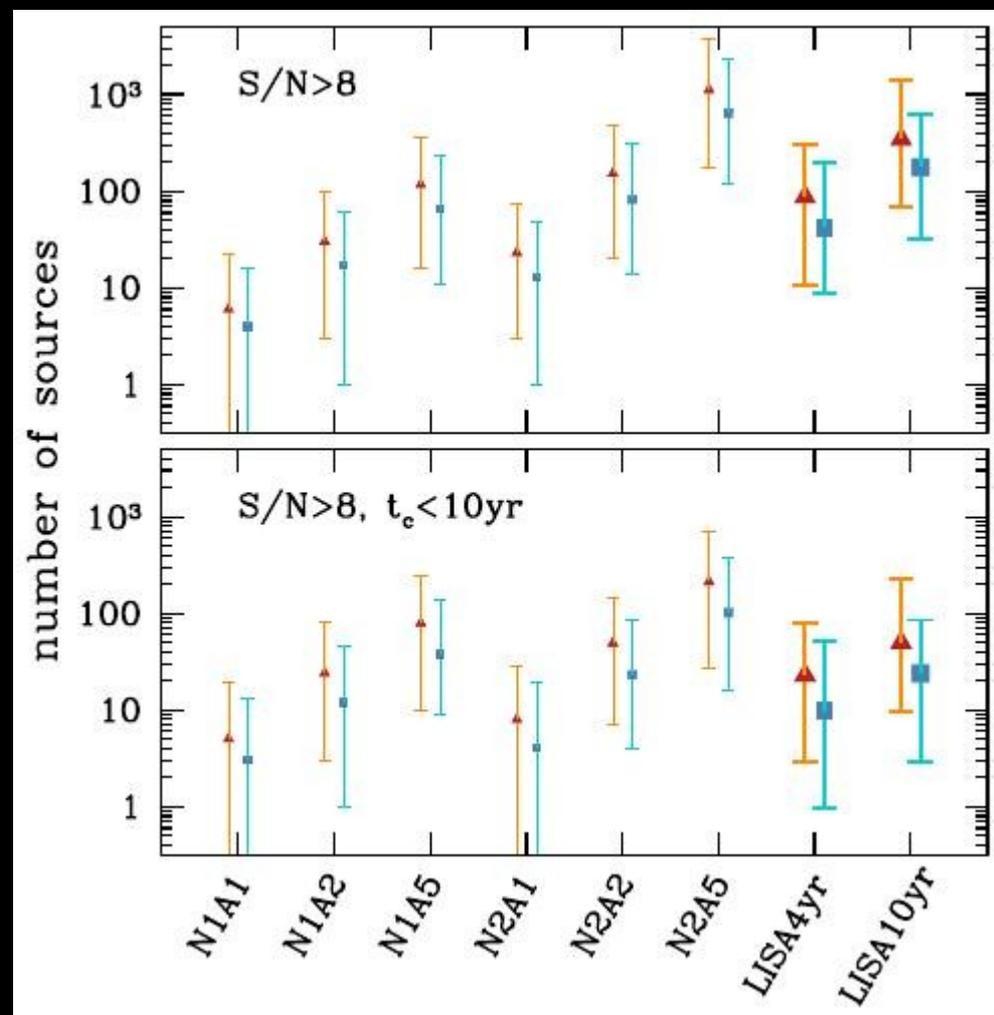
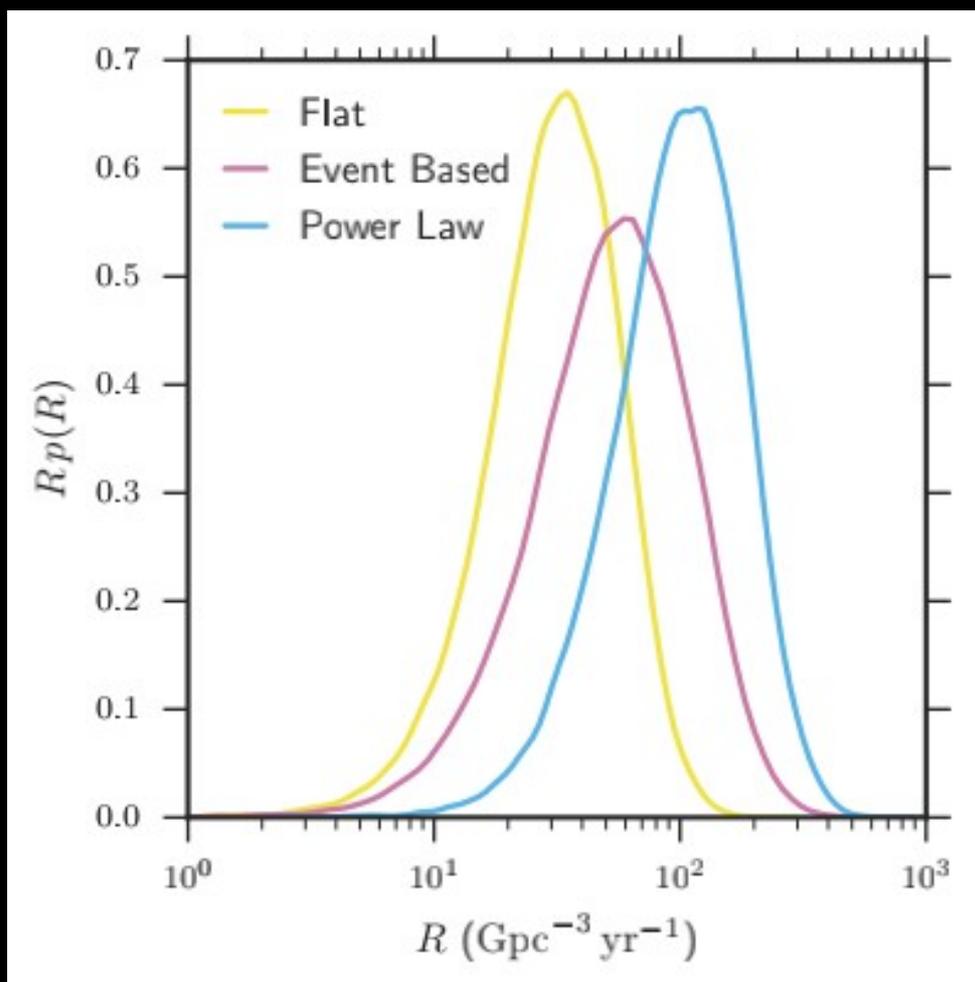


Number of observable sources ($S/N > 8$) is a strong function of frequency*.

*that is the main reason of rather pessimistic initial estimates about the observability of these sources by eLISA

How many BHBs in the eLISA band?

Implied BHB mass distributions and merger rates higher than previously thought and BHs are more massive



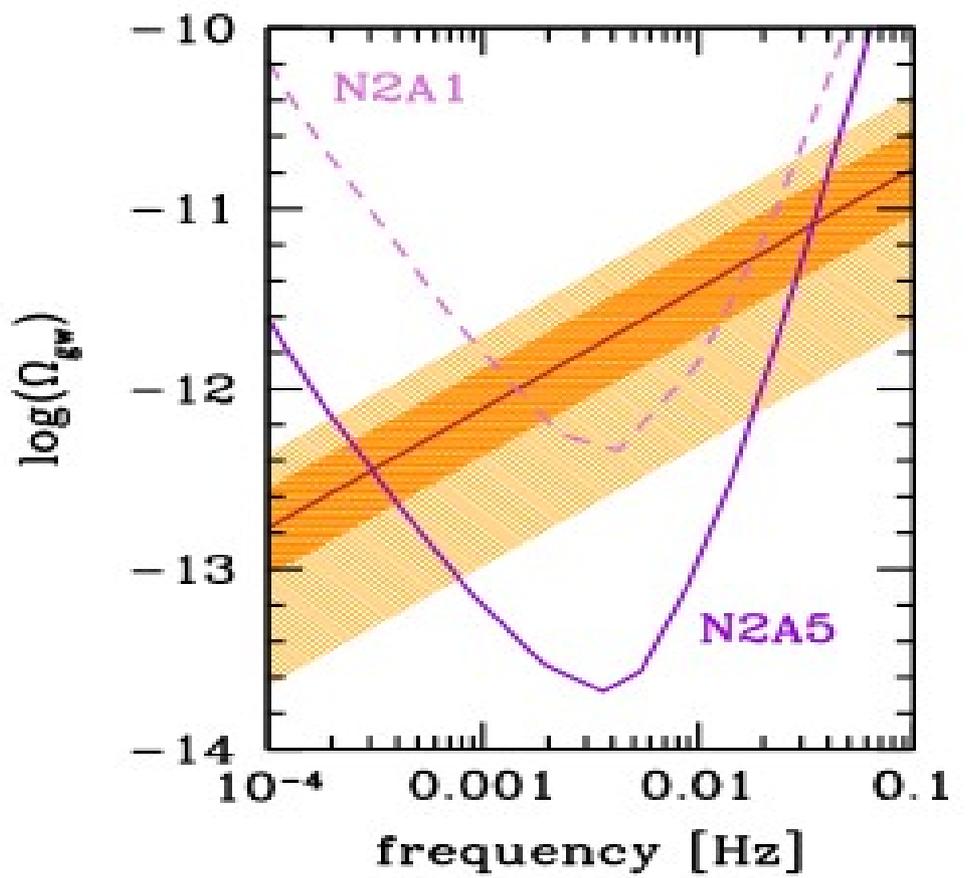
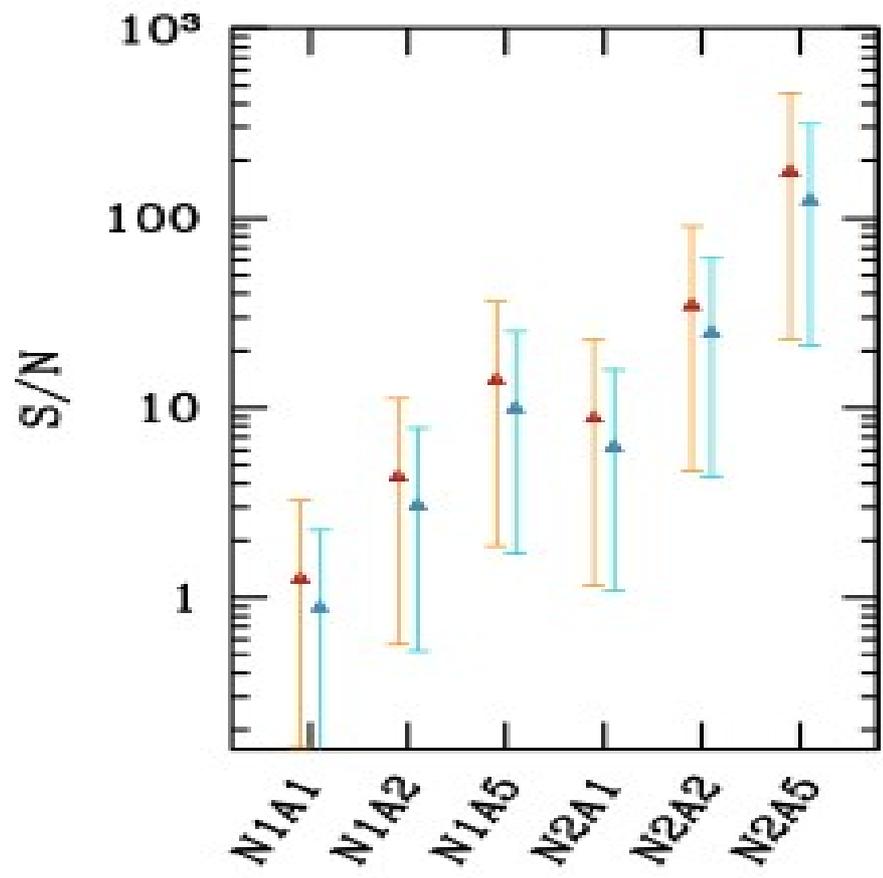
eLISA will detect up to thousands of BHBs with $S/N > 8$ up to few hundreds crossing to the aLIGO band in 5yr (Divide by 10!)

Unresolved sources will produce a stochastic GWB

$$\frac{d\rho_{\text{gw}}(f)}{d\ln f} = \frac{\pi}{4} f^2 h_c^2(f) = \int_0^\infty dz \frac{dn}{dz} \frac{1}{1+z} \left. \frac{dE_{\text{gw}}}{d\ln f_r} \right|_{f_r=f(1+z)}$$

$$h_c^2(f) = \int_0^\infty dz \int_0^\infty d\mathcal{M} \frac{d^3 N}{dz d\mathcal{M} d\ln f_r} h^2(f_r)$$

$$(S/N)_{\text{bkg}}^2 = T \int \gamma(f) \frac{h_{c,\text{bkg}}^4(f)}{4f^2 \langle S(f) \rangle^2} df$$

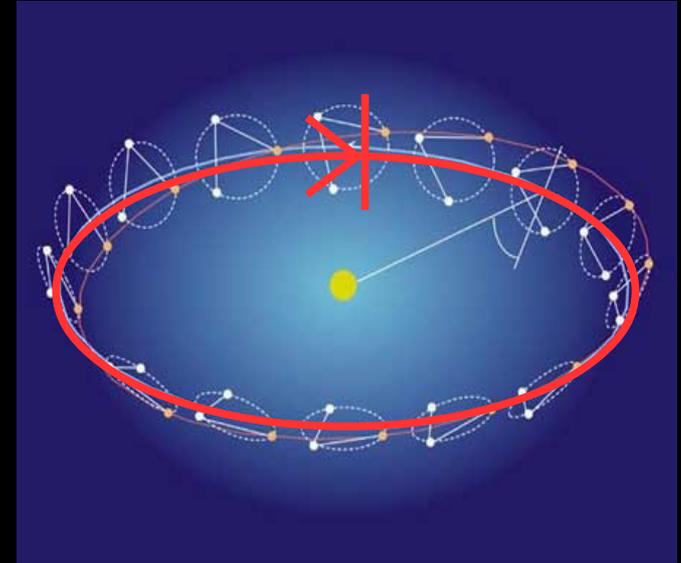
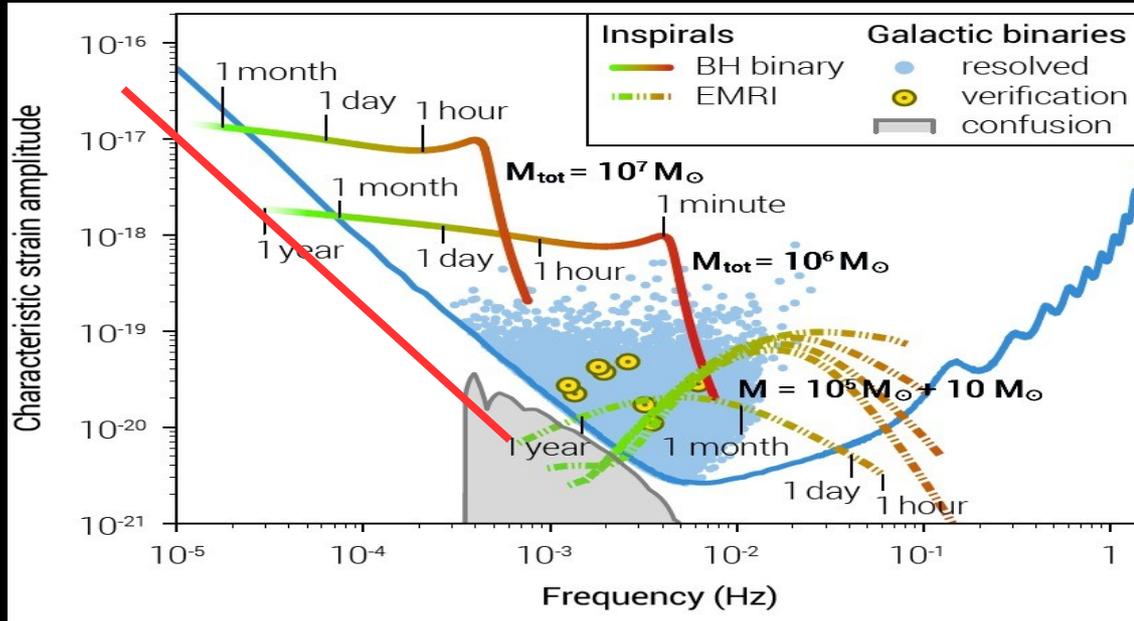


Unresolved sources will form a confusion noise detectable with high S/N

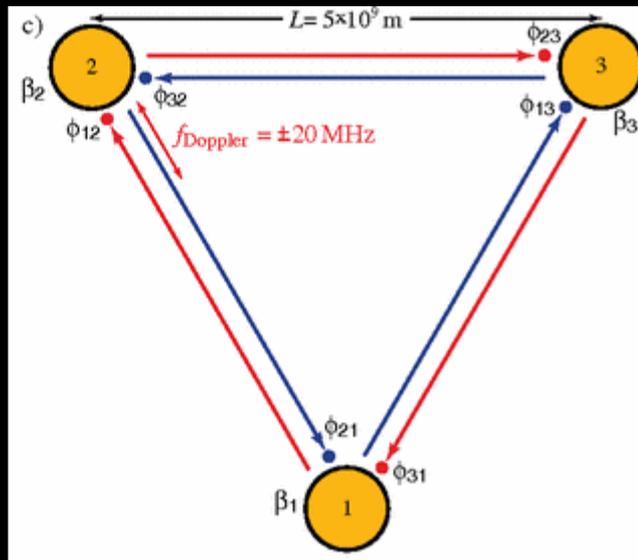
What do we do with them?

- > **Detector cross-band calibration and validation (LISA - L-V-K)**
- > **Multiband GW astronomy:**
 - LISA → L-V-K: *alert L-V-K to ensure GW detectors are on
 - *inform L-V-K with source parameters: makes detection easier
 - L-V-K → LISA: *identify sub-threshold source that can be dug out of the LISA data streams
- > **Multimessenger astronomy:**
 - point EM probes at the right location before the merger
- > **Enhanced tests of GR: e.g. strongest limits on deviations from GR**
- > **Astrophysics:**
 - independent measure of spins
 - measure of eccentricity
- > **Cosmology:**
 - new population of standard sirens?

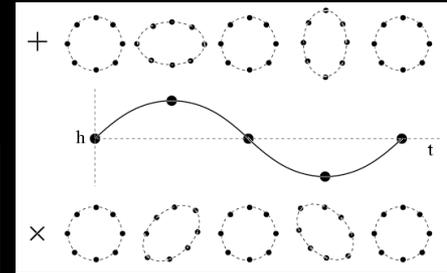
Baseline



Number of laser links



$$h(t) = \sum_A h_A(t - \hat{\Omega} \cdot x) F^A(\hat{\Omega}, \psi)$$



$$F^+(\theta, \phi, \psi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi,$$

$$F^\times(\theta, \phi, \psi) = -\frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \sin 2\psi - \cos \theta \sin 2\phi \cos 2\psi,$$

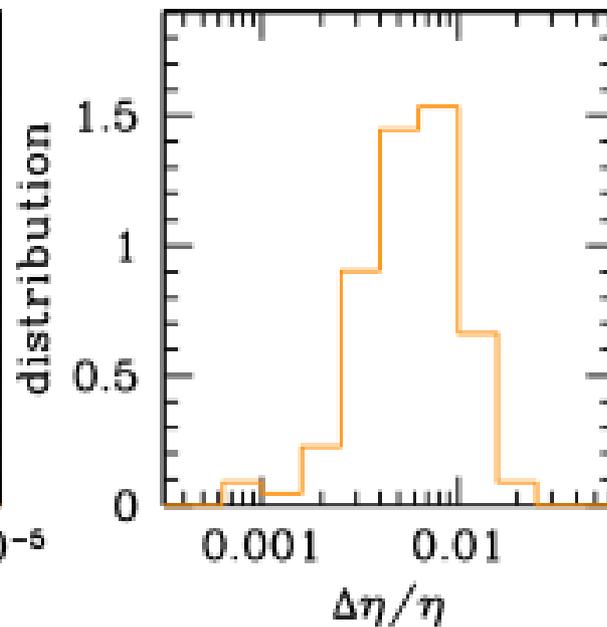
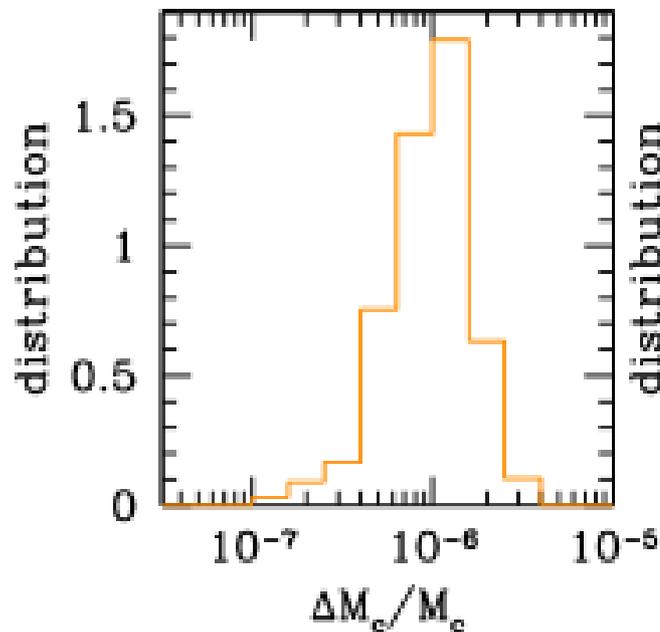
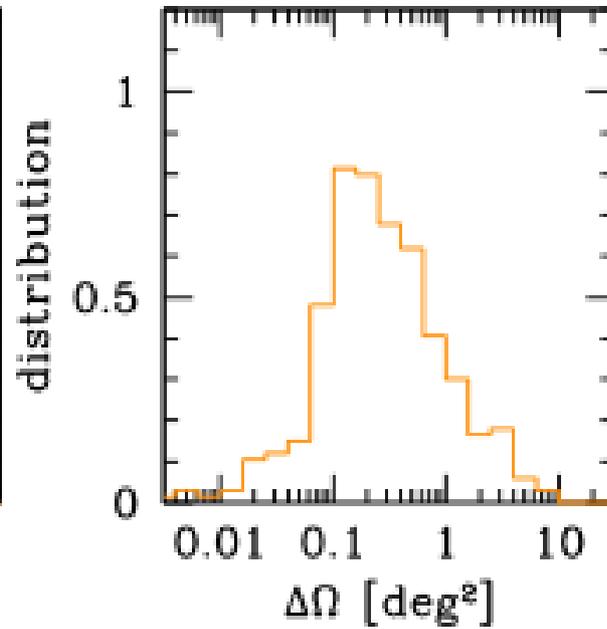
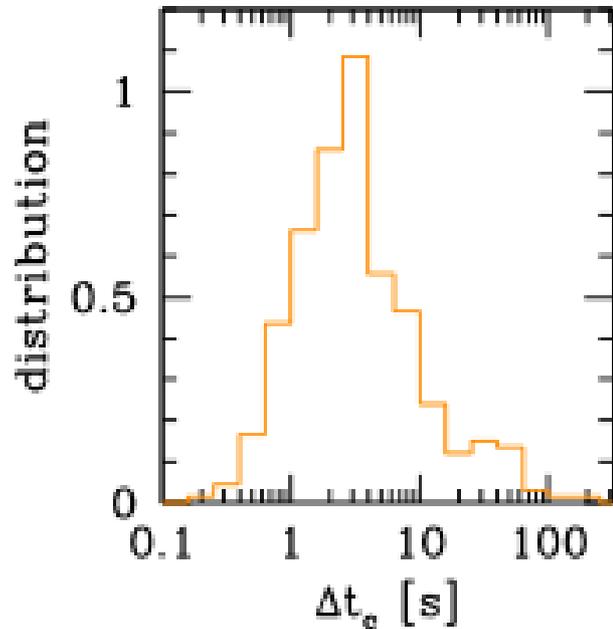
$$F^x(\theta, \phi, \psi) = \sin \theta (\cos \theta \cos 2\phi \cos \psi - \sin 2\phi \sin \psi),$$

$$F^y(\theta, \phi, \psi) = -\sin \theta (\cos \theta \cos 2\phi \sin \psi + \sin 2\phi \cos \psi),$$

$$F^b(\theta, \phi) = -\frac{1}{2} \sin^2 \theta \cos 2\phi,$$

$$F^\ell(\theta, \phi) = \frac{1}{2} \sin^2 \theta \cos 2\phi.$$

Sky pre-localization and coincident EM campaigns



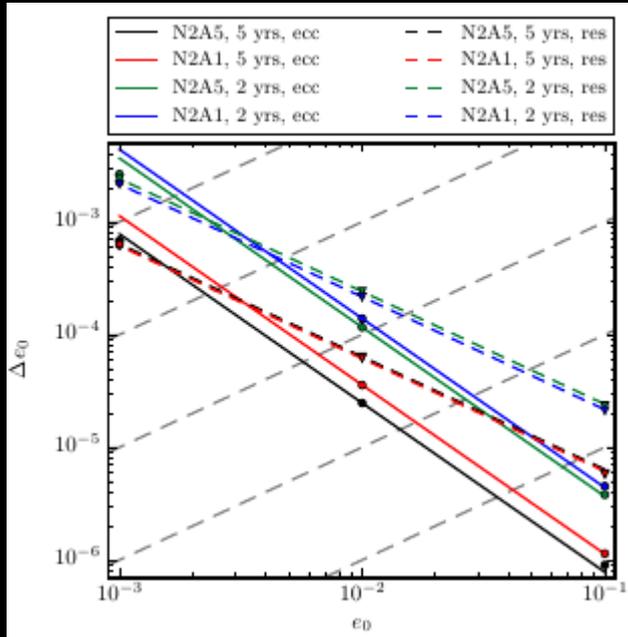
System crossing to the aLIGO band can be located with sub deg^2 precision (Klein et al. In prep.)

Merger time can be predicted within 10 seconds (but see Bonvin et al. 2016)

Make possible to pre-point all instruments: open the era of coincident GW-EM astronomy (even though a counterpart is not expected).

Measuring eccentricity with eLISA

(Nishizawa et al. 2016)



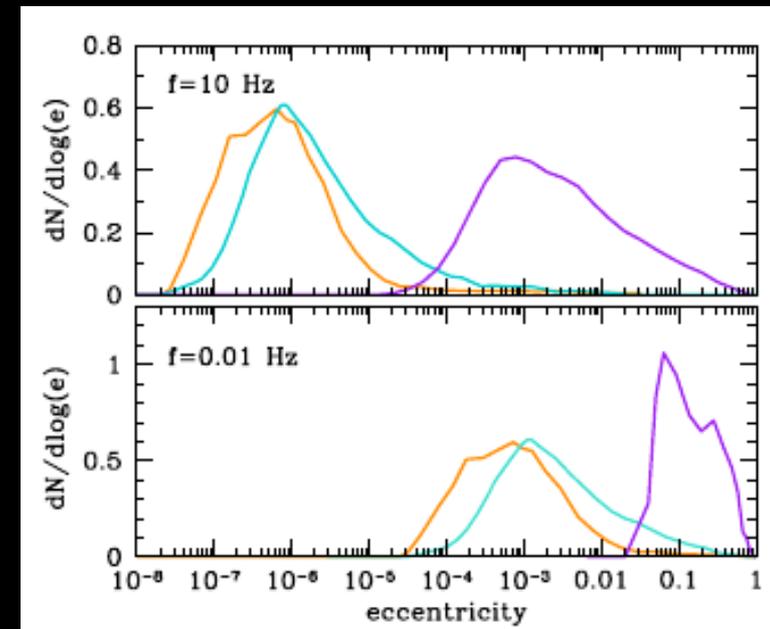
>aLIGO can only place upper bounds on e , but eLISA can measure e if $>10^{-3}$

>GW circularization implies much higher eccentricities in the eLISA band

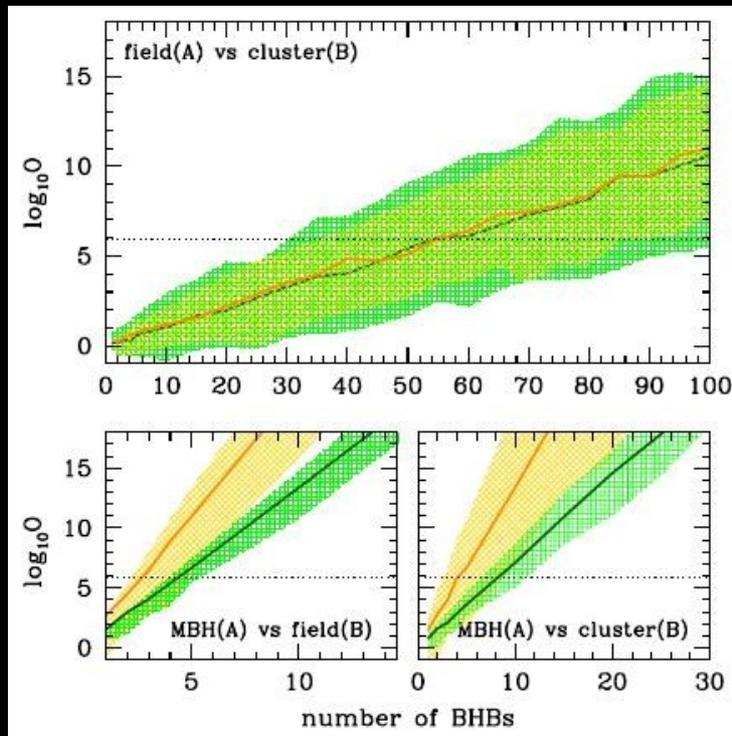
Different formation channel imply different e distributions. Too small to be measured by LIGO but accessible to LISA

Proof of concept: three BHB formation scenarios

- field binaries (Kowalska et al 2011)
- dynamical formation in Gcs (Rodriguez et al. 2016)
- Kozai resonances around a MBH (Antonini & Perets 2012)



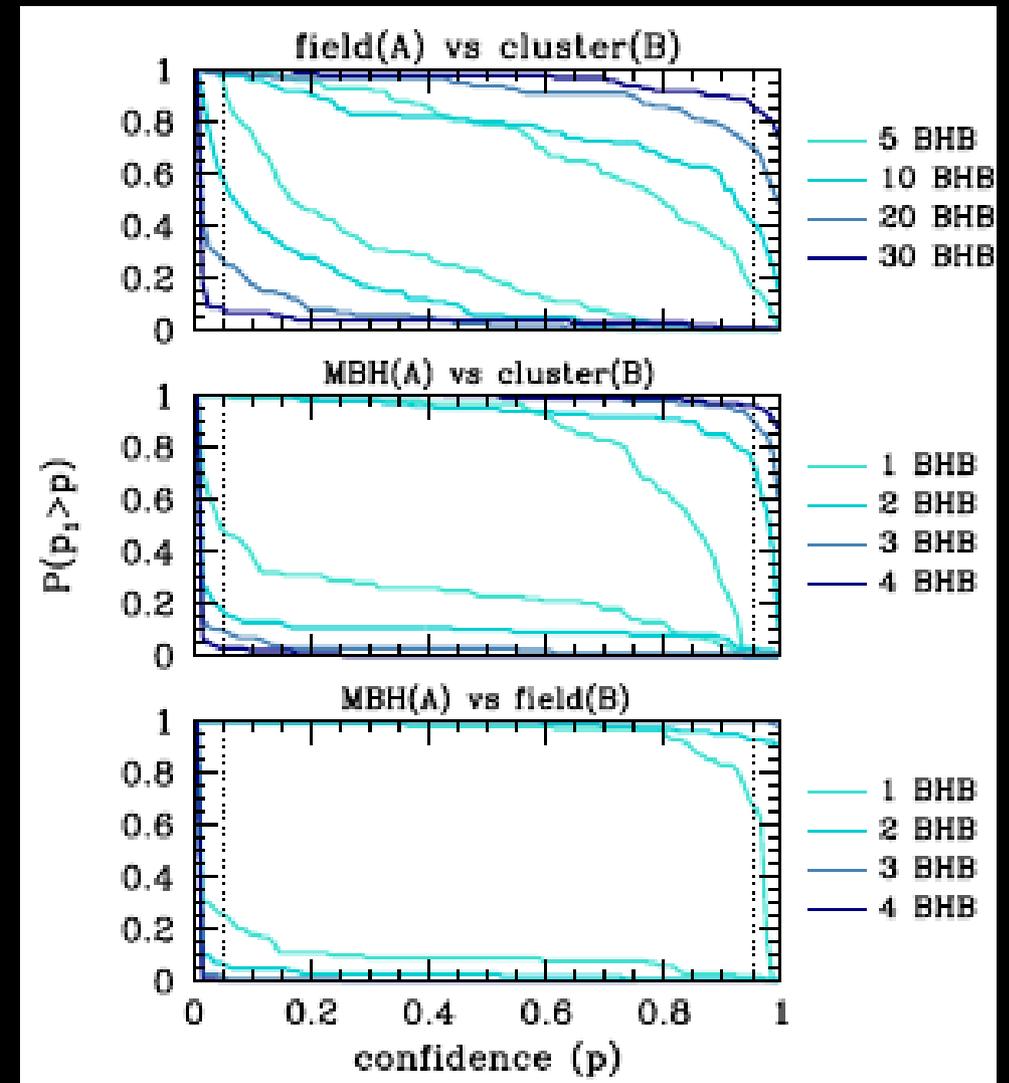
Assessing BHB origin using eccentricity



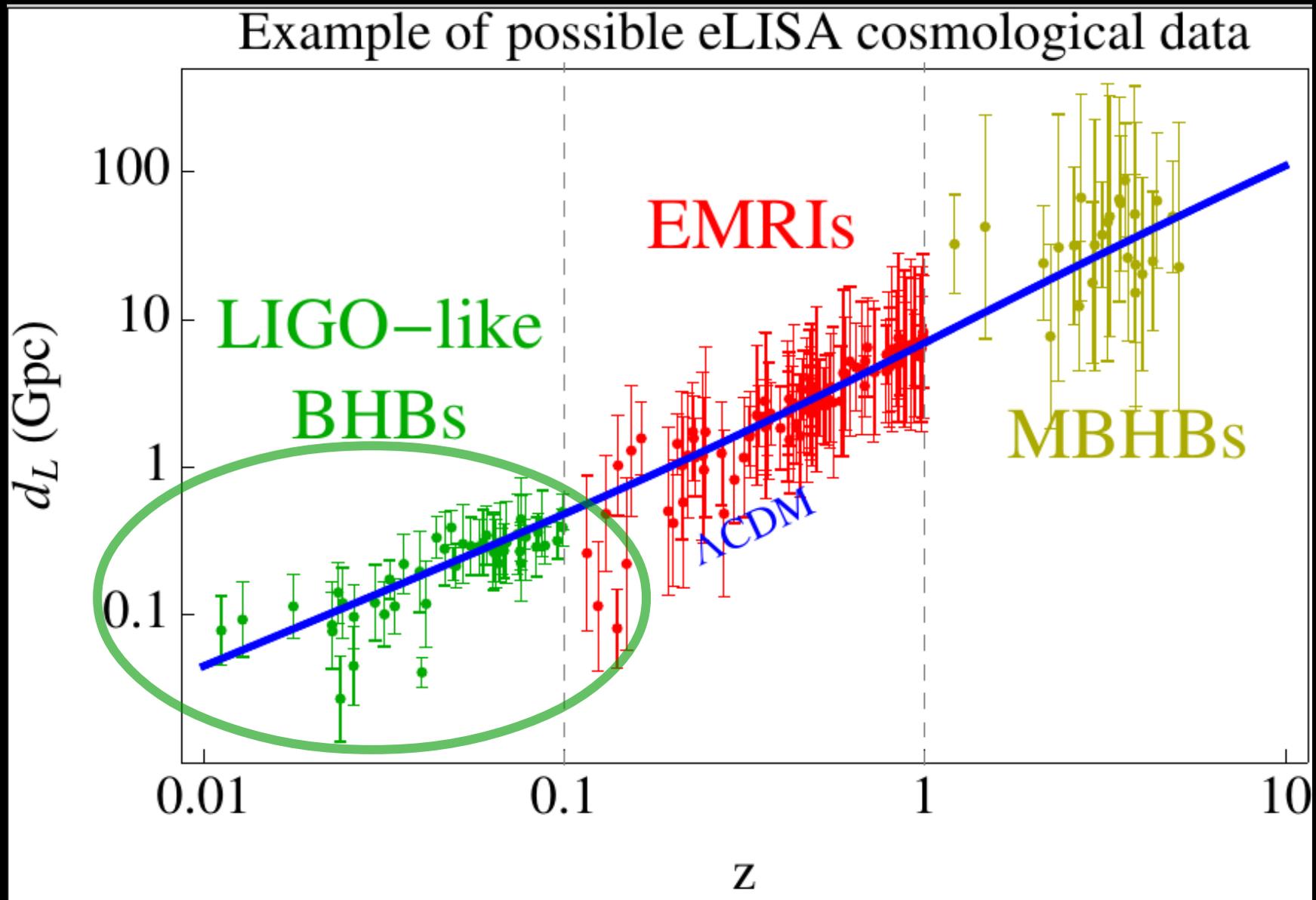
Different formation channels result in different e distributions in the eLISA band, (see also Breivik et al. 2016)

eLISA can tell formation scenarios apart with few tens of observations (Nishizawa et al. 2016)

Can be complemented to aLIGO spin measurements.



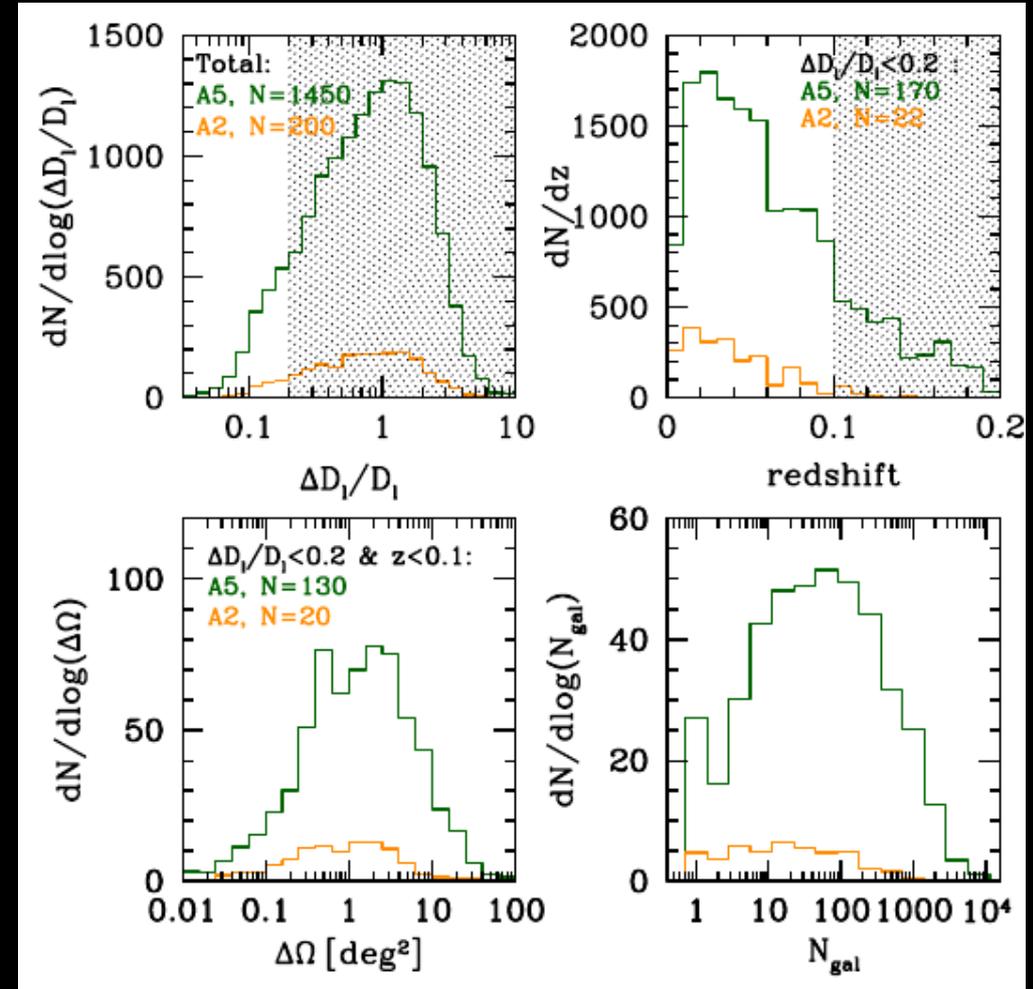
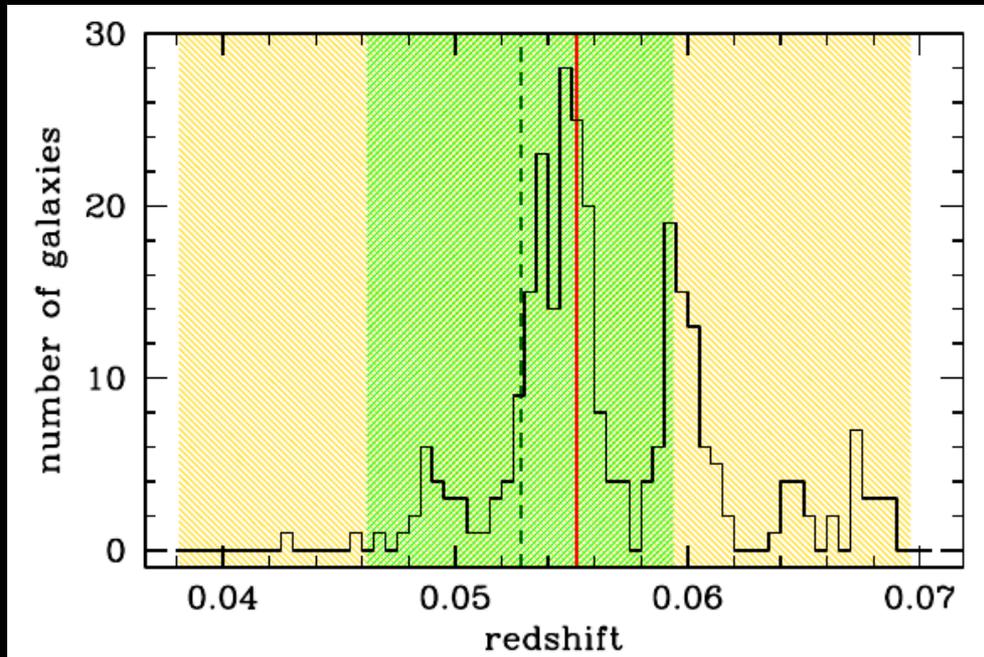
Cosmology with gravitational waves



(Courtesy of N. Tamanini)

Different GW sources will allow an independent assessment of the geometry of the Universe at all redshifts.

BHBs as standard sirens: measuring H_0

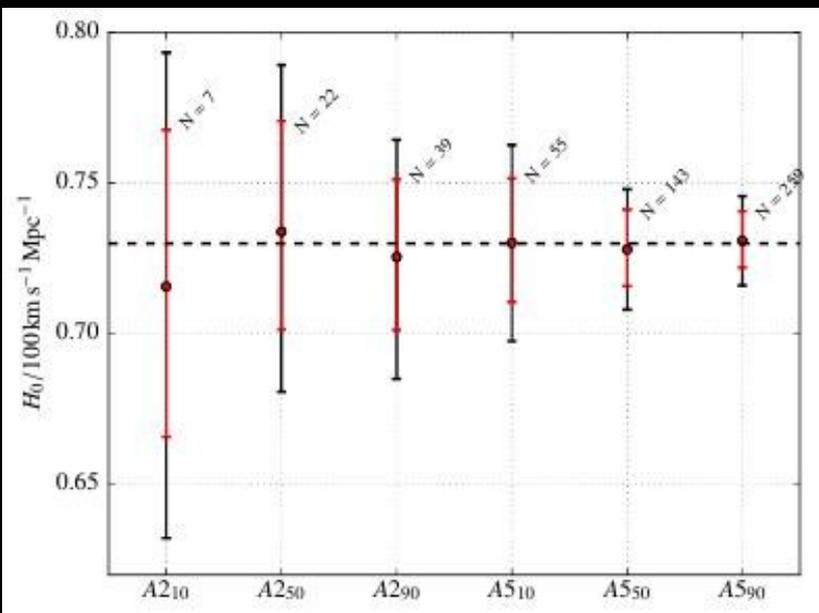


No counterpart required
(McLeod & Hogan 2008,
Petiteau et. al 2011)

- Many sources at $z < 0.1$
- small errorbox consider all possible hosts within the errorbox assuming a broad prior on h
- combine statistically the likelihood of the hosts in each errorbox to determine h

AstroBonus: few local events have 1 galaxy in the errorbox

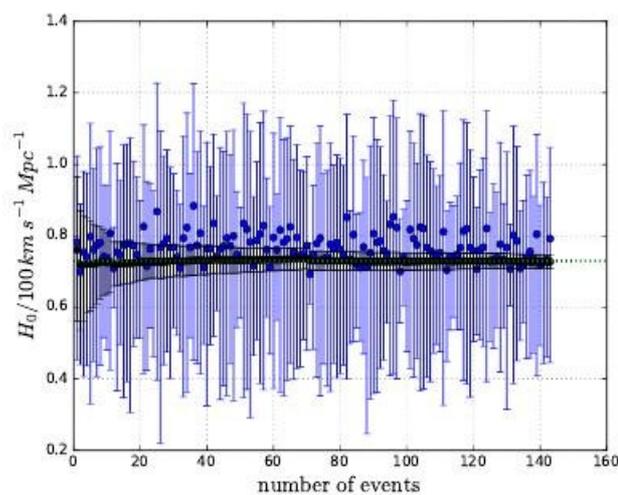
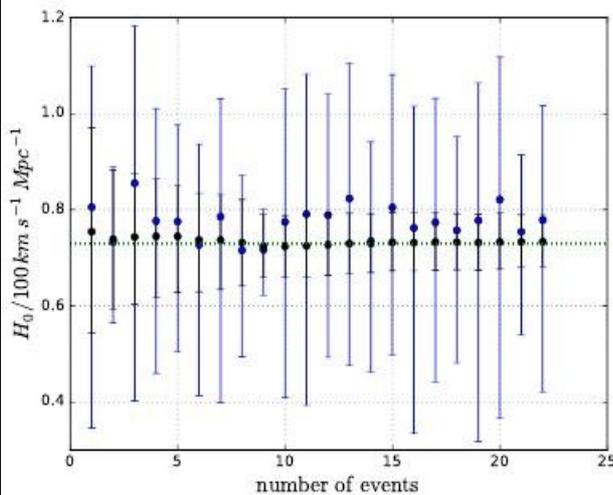
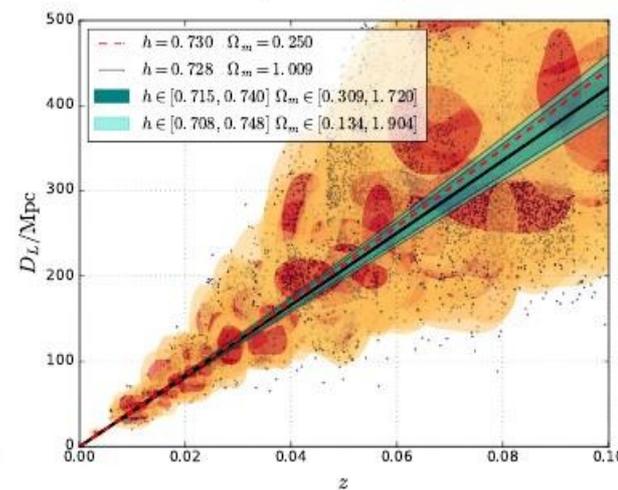
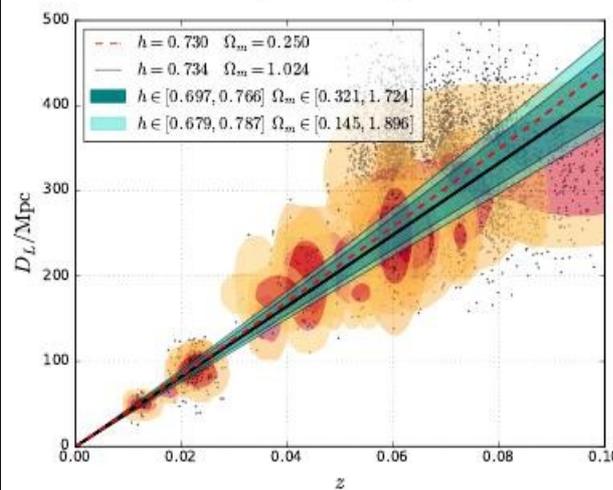
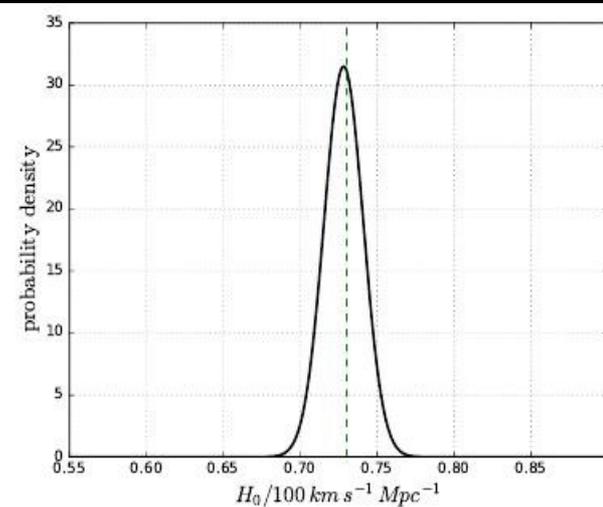
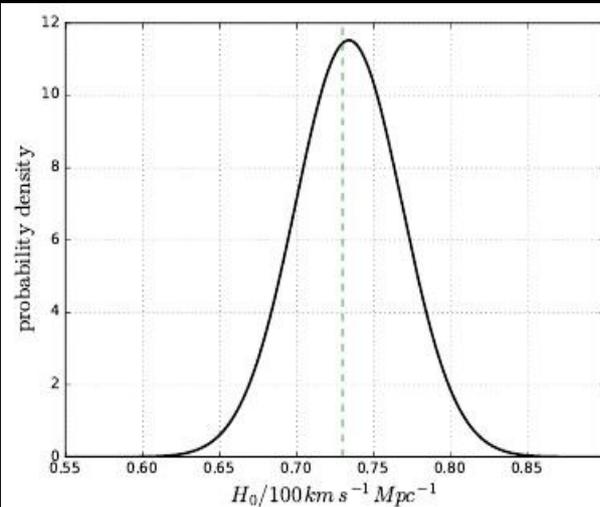
Results



The precision of the measurement scales with \sqrt{N} , regardless of the detector design.

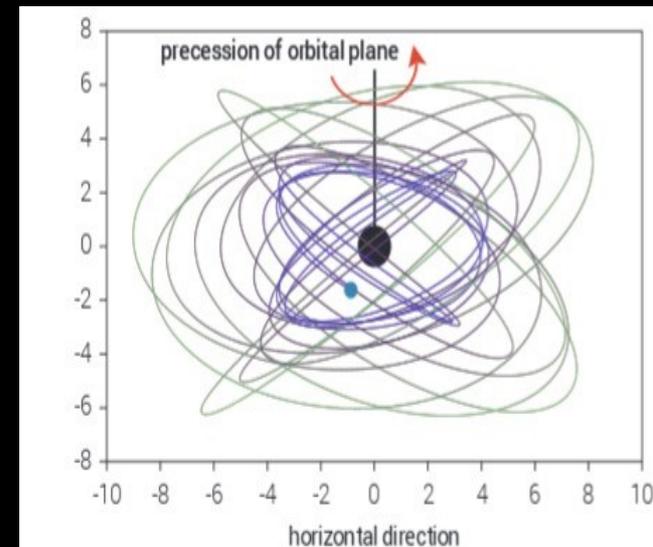
2Gm
h determined at $\sim 5\%$

5Gm
h determined at $\sim 2\%$

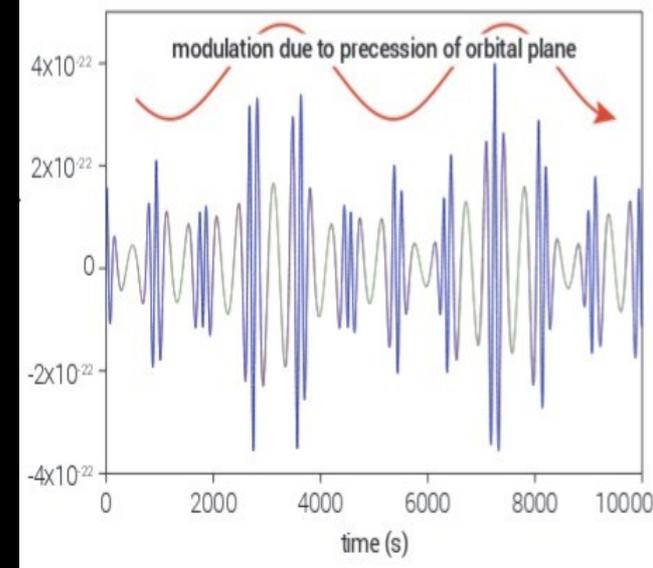


Extreme mass ratio inspirals (EMRIs)

- *What is the mass distribution of stellar remnants at the galactic centres and what is the role of mass segregation and relaxation in determining the nature of the stellar populations around the nuclear black holes in galaxies?*
- *Are massive black holes as light as $\sim 10^5 M_{\odot}$ inhabiting the cores of low mass galaxies? Are they seed black hole relics? What are their properties?*



- *Does gravity travel at the speed of light ?*
- *Does the graviton have mass?*
- *How does gravitational information propagate: Are there more than two transverse modes of propagation?*
- *Does gravity couple to other dynamical fields, such as, massless or massive scalars?*
- *What is the structure of spacetime just outside astrophysical black holes? Do their spacetimes have horizons?*
- *Are astrophysical black holes fully described by the Kerr metric, as predicted by General Relativity?*



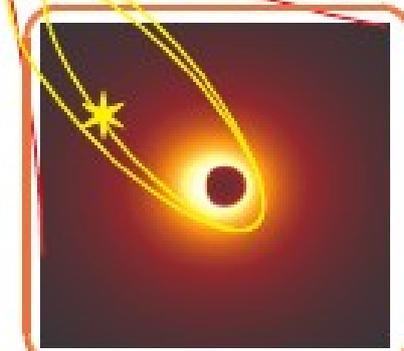
Cluster dynamics
Newtonian, collisional

$\rho_{*,cl} \sim 10^6 - 10^8 M_{\odot} \text{pc}^{-3}$
 $\sigma_{*,cl} \sim 100 - 1000 \text{ km s}^{-1}$
 $t_{\text{rlx},cl} \sim 10^8 - 10^{10} \text{ yrs}$



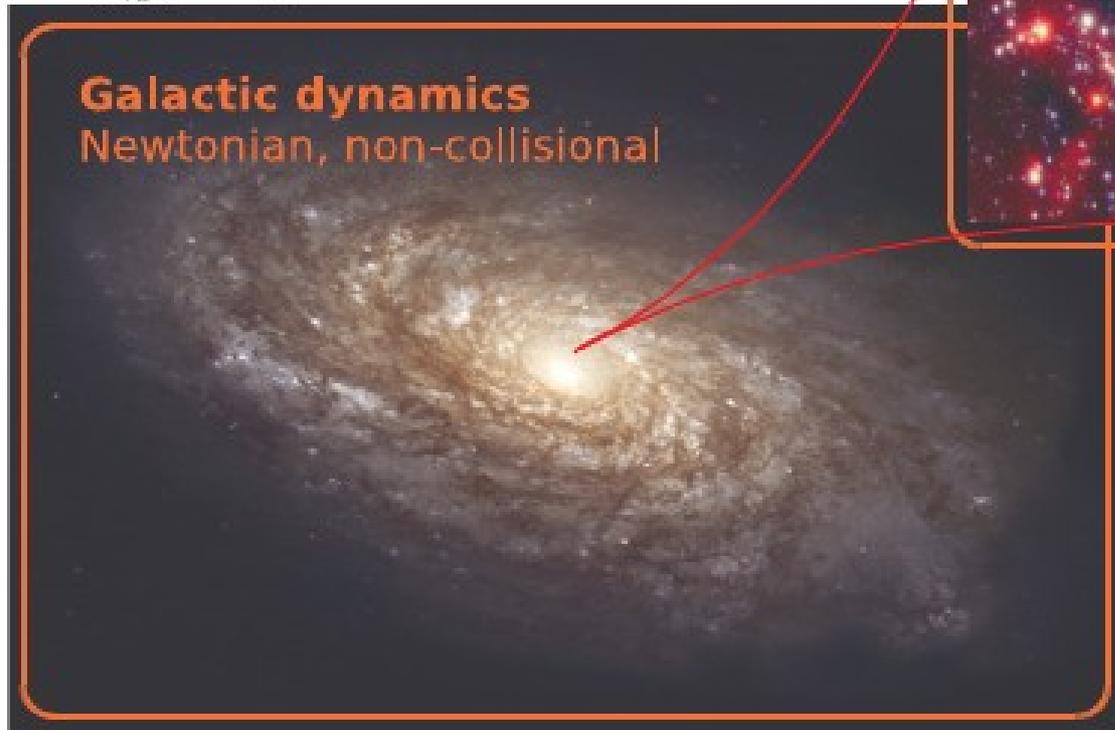
$\times 1000$

$\times 10^7$



$\rho_{*,gal} \sim 0.05 M_{\odot} \text{pc}^{-3}$
 $\sigma_{*,gal} \sim 40 \text{ km s}^{-1}$
 $t_{\text{rlx},gal} \sim 10^{15} \text{ yrs}$

Galactic dynamics
Newtonian, non-collisional



Relativistic dynamics
collisional or not (low N)

$M_{\bullet} \sim 10^6 - 10^9 M_{\odot}$
 $R_{\text{Schw}} = 10^{-7} - 10^{-4} \text{ pc}$

$$r_{\text{infl}} = \frac{GM_{\bullet}}{\sigma_0^2} \approx 1 \text{ pc} \left(\frac{M_{\bullet}}{10^6 M_{\odot}} \right) \left(\frac{60 \text{ km/s}}{\sigma_0} \right)^2$$

Two body encounters affect star orbits. The relaxation time is the time it takes for an orbit to experience a change $dv \sim v$:

$$t_{\text{rlx}}(r) = \frac{0.339}{\ln \Lambda} \frac{\sigma^3(r)}{G^2 \langle m \rangle m_{\text{CO}} n(r)} \simeq 1.8 \times 10^8 \text{ yr} \left(\frac{\sigma}{100 \text{ km s}^{-1}} \right)^3 \left(\frac{10 M_{\odot}}{m_{\text{CO}}} \right) \left(\frac{10^6 M_{\odot} \text{ pc}^{-3}}{\langle m \rangle n} \right)$$

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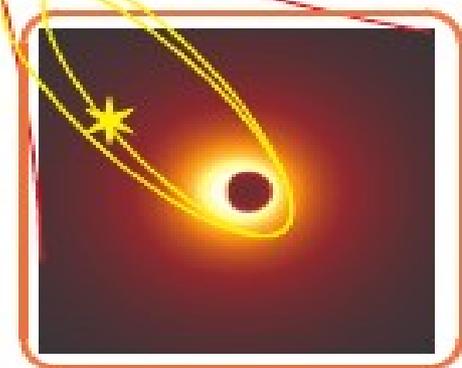
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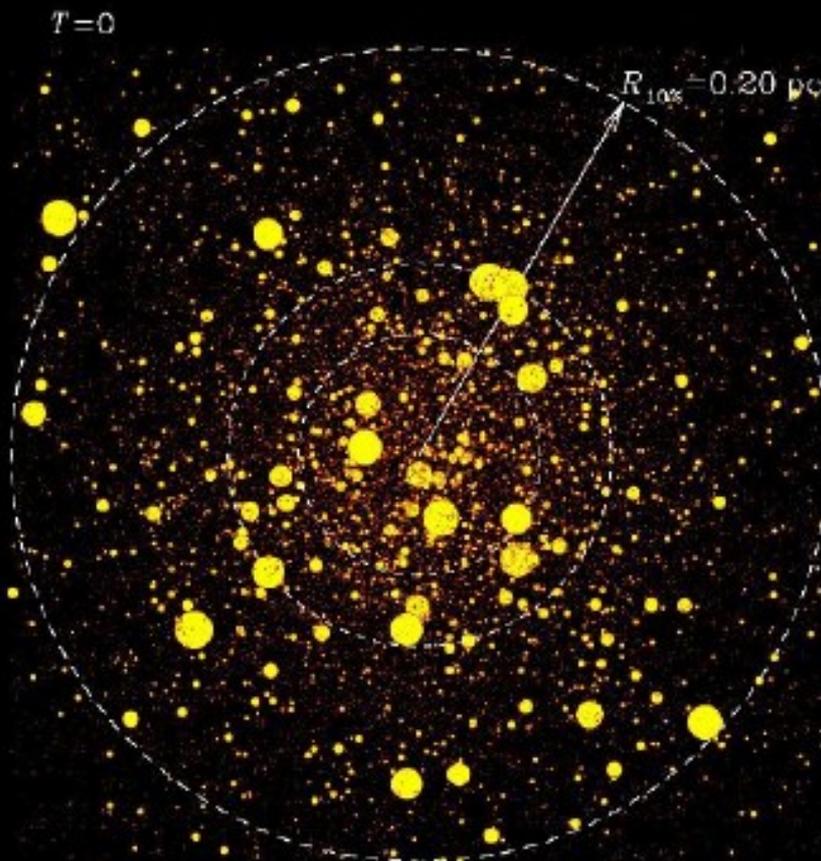
Two body encounters affect star orbits. The relaxation time is the time it takes for an orbit to experience a change $dv \sim v$:

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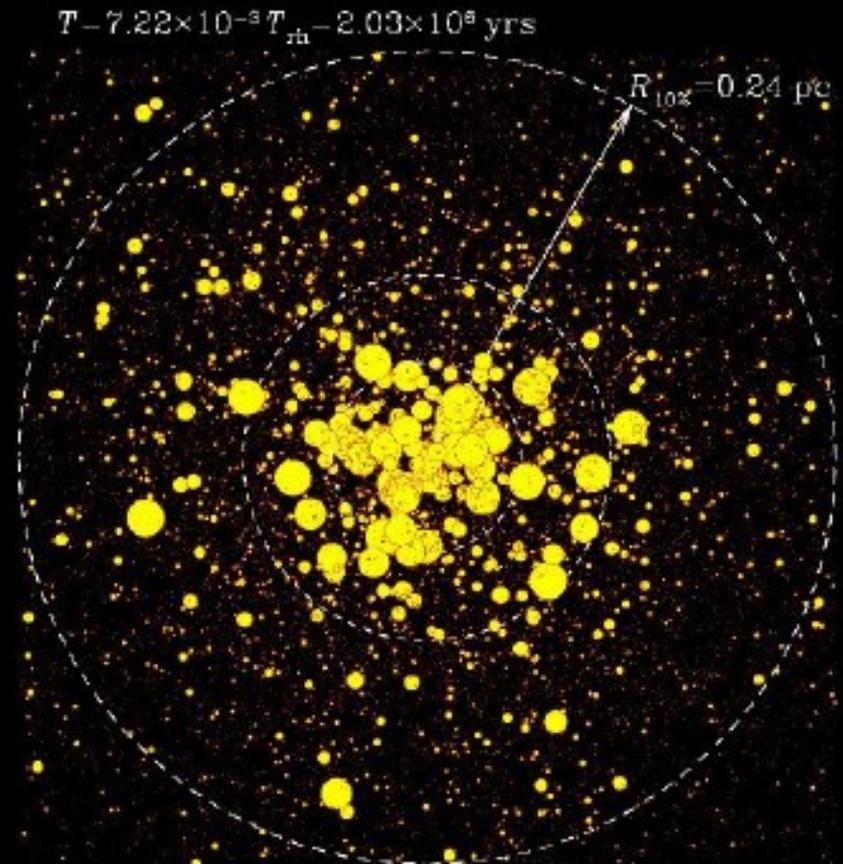
Because of segregation the density of COs goes with r^2 , within the influence radius sigma goes as $r^{1/2}$

So t_{rlx} goes with $r^{1/2}$

T
fo
TC
TI
TI
ti
ti
'c



Stellar radii magnified 1.6×10^4 times



Stellar radii magnified 2.0×10^4 times

)^{5/2}

Two body encounters affect star orbits. The relaxation time is the time it takes for an orbit to experience a change $d\mathbf{v} \sim \mathbf{v}$:

$$t_{\text{rlx}}(r) = \frac{0.339}{\ln \Lambda} \frac{\sigma^3(r)}{G^2 \langle m \rangle m_{\text{CO}} n(r)} \simeq 1.8 \times 10^8 \text{ yr} \left(\frac{\sigma}{100 \text{ km s}^{-1}} \right)^3 \left(\frac{10 M_{\odot}}{m_{\text{CO}}} \right) \left(\frac{10^6 M_{\odot} \text{ pc}^{-3}}{\langle m \rangle n} \right)$$

Because of segregation the density of COs goes with r^2 , within the influence radius sigma goes as $r^{1/2}$

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To be captured, a CO has to 'circularize'. From the quadrupole formula, the timescale for circularization is

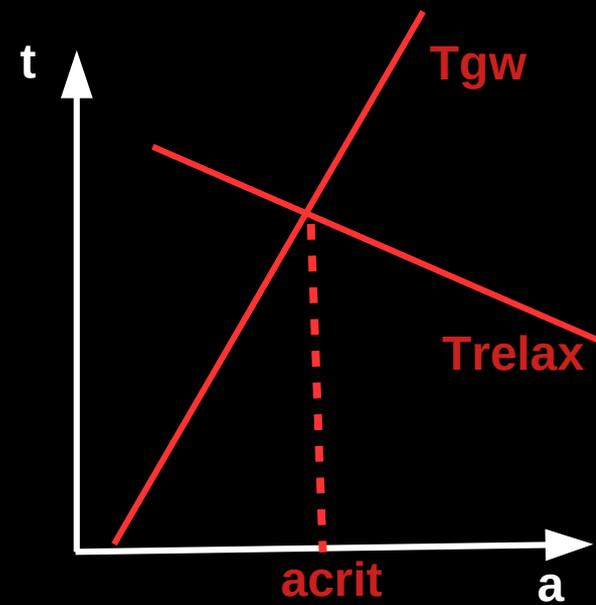
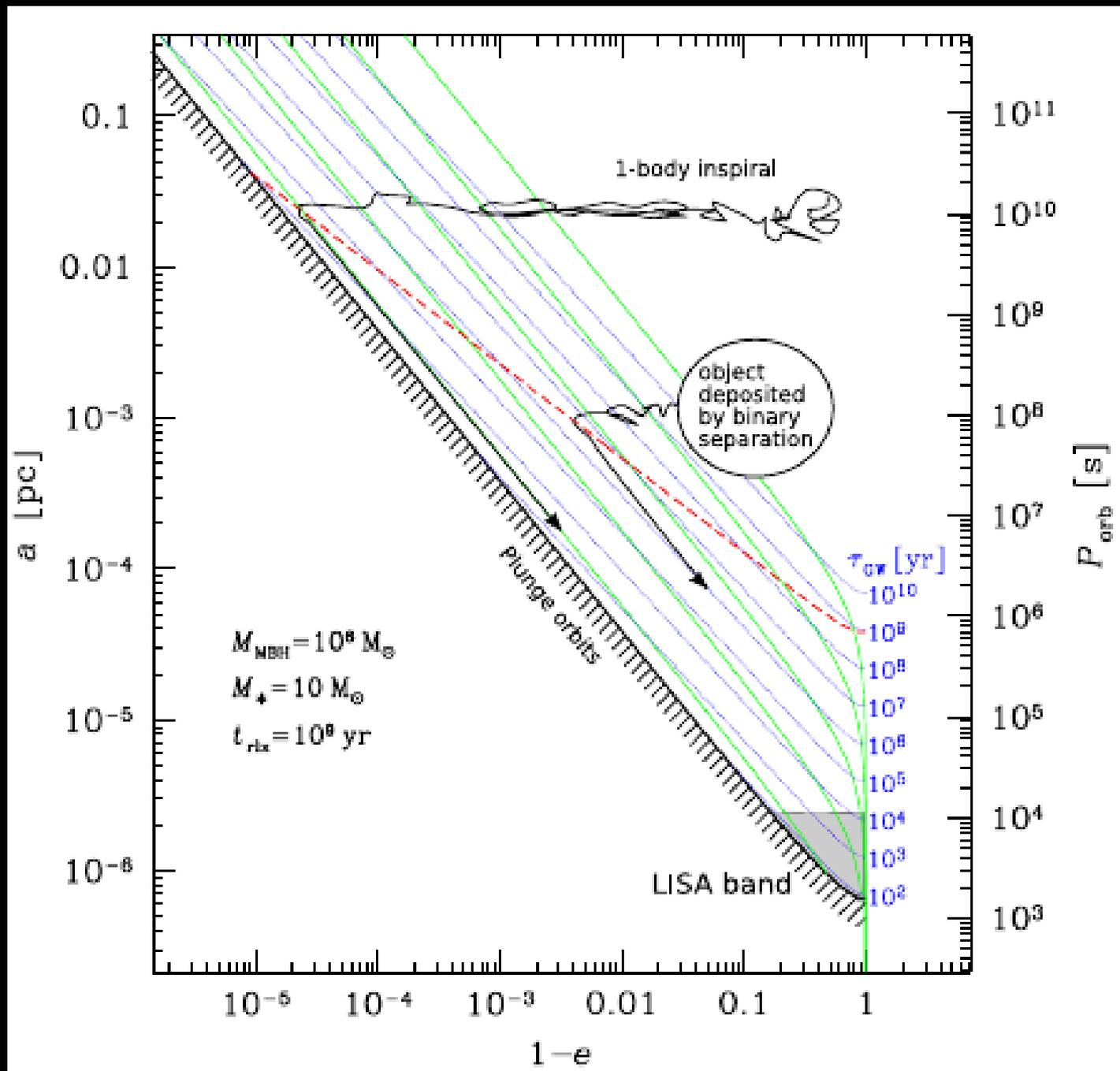
$$\tau_{\text{GW}} = \frac{e}{|de/dt|} \approx \frac{15}{304} \frac{c^5 a^4 (1 - e^2)^{5/2}}{G^3 \mu M^2} \approx 8 \times 10^{17} \text{ yr} \left(\frac{M_{\odot}}{\mu} \right) \left(\frac{M_{\odot}}{M} \right)^2 \left(\frac{a}{1 \text{ AU}} \right)^4 (1 - e^2)^{5/2}$$

The condition for circularization is

$$\tau_{\text{GW}} < C_{\text{EMRI}} (1 - e) t_{\text{rlx}}$$

The $1-e$ is because the 'circularization timescale' has to be smaller than the timescale needed to scatter the star out of the 'capture loss cone'





Equating the two timescales give

$$a_{\text{EMRI}} = 5.3 \times 10^{-2} \text{ pc } C_{\text{EMRI}}^{2/3} \times \left(\frac{t_{\text{rlx}}}{10^9 \text{ yr}} \right)^{2/3} \left(\frac{m}{10 M_{\odot}} \right)^{2/3} \left(\frac{M_{\bullet}}{10^6 M_{\odot}} \right)^{-1/3}$$
$$1 - e_{\text{EMRI}} = 7.2 \times 10^{-6} C_{\text{EMRI}}^{-2/3} \times \left(\frac{t_{\text{rlx}}}{10^9 \text{ yr}} \right)^{-2/3} \left(\frac{m}{10 M_{\odot}} \right)^{-2/3} \left(\frac{M_{\bullet}}{10^6 M_{\odot}} \right)^{4/3}$$

Note that for $\rho \propto r^2$, $dn/dr \propto \text{const}$

Moreover, the time it takes to scatter things in and out of the 'capture loss cone is $(1-e)t_{\text{rlx}}$

$(1-e)$ is proportional to $1/r$, so that

$(1-e)t_{\text{rlx}}$ is proportional to $r^{1/2}$

In practice the largest contribution to capture comes from a_{EMRI} !

What is the rate? The rate can be approximated as

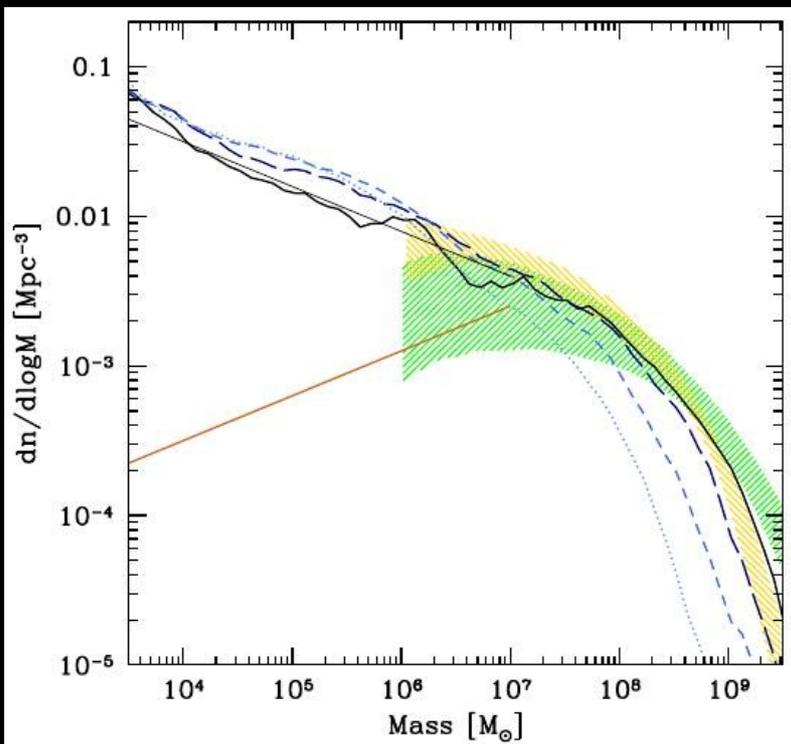
$$N_{\text{CO}}(<a_{\text{EMRI}}) / T_{\text{rlx}}(a_{\text{EMRI}})$$

For a MW cusp, $N_{\text{CO}} \sim 10^4$, $T_{\text{rlx}}(a_{\text{EMRI}}) \sim 10^{11} \text{ yr}$

So the rate is $\sim 10^{-7} / \text{yr}$ or $100 / \text{Gyr}$

$$R_0 = 300 \left(\frac{M}{10^6 M_{\odot}} \right)^{-0.19} \text{ Gyr}^{-1}$$

Astrophysical uncertainties are huge:



-MBH mass function unknown below 10^6 solar masses

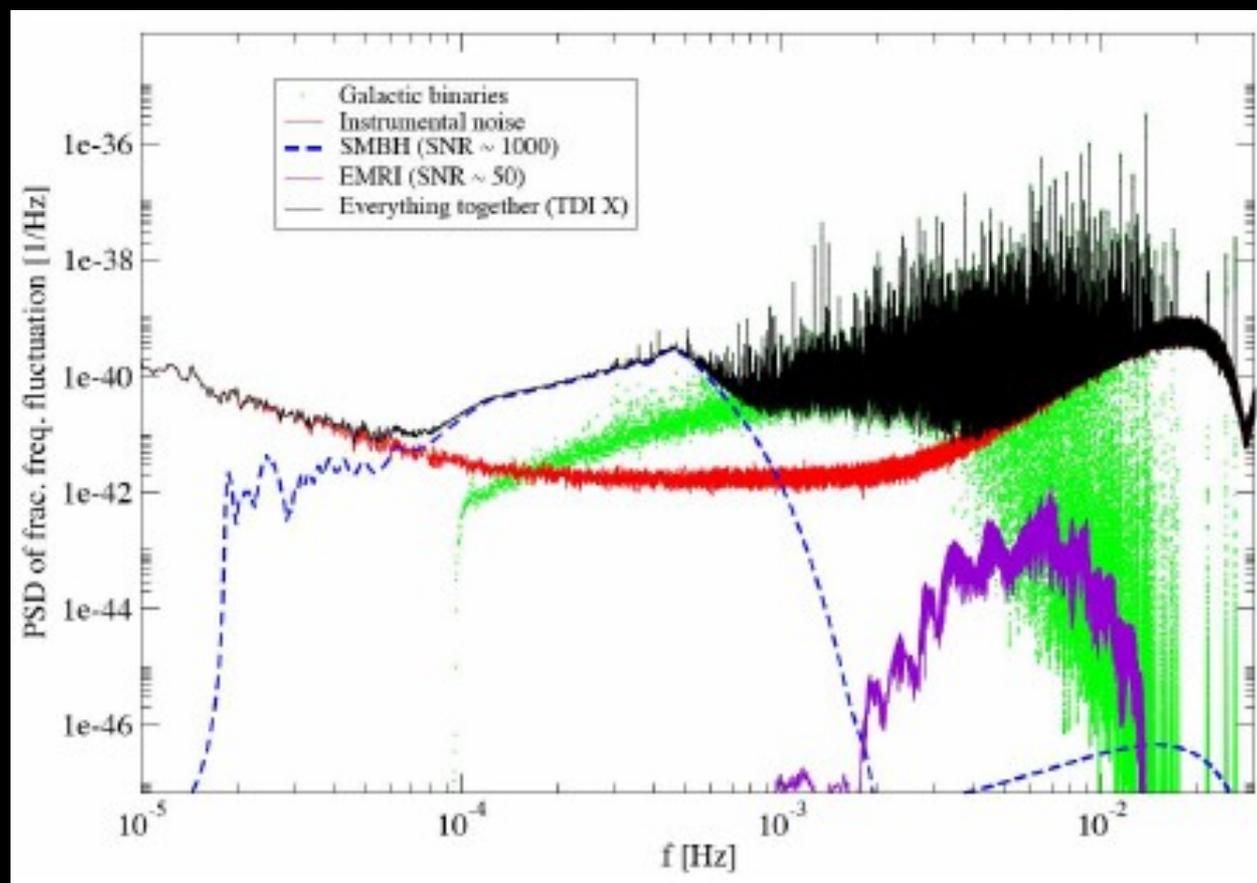
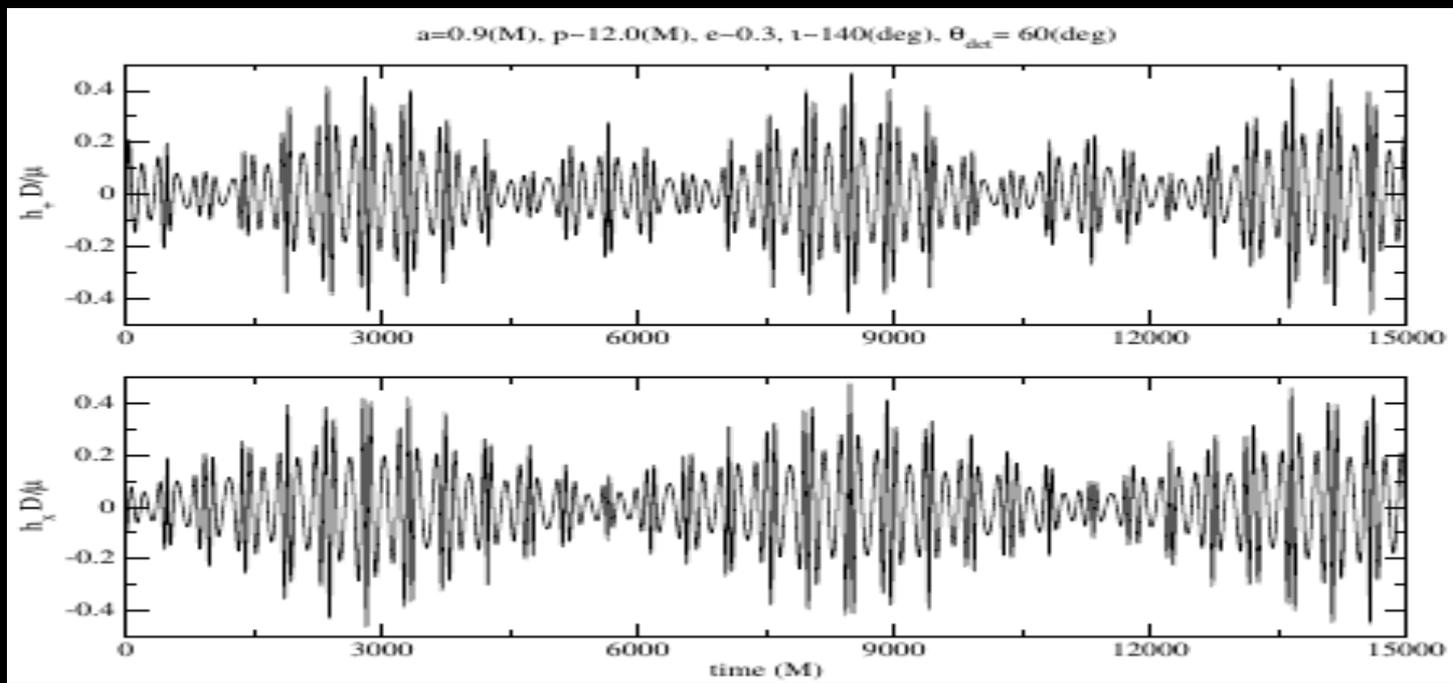
-distribution of compact objects (CO) around MBH (Preto & Amaro-Seoane 2010)?

-are COs inspiralling (thus producing EMRIs) or plunging (Merritt 2015)?

-Scaling with mass too naive! (Babak+17)

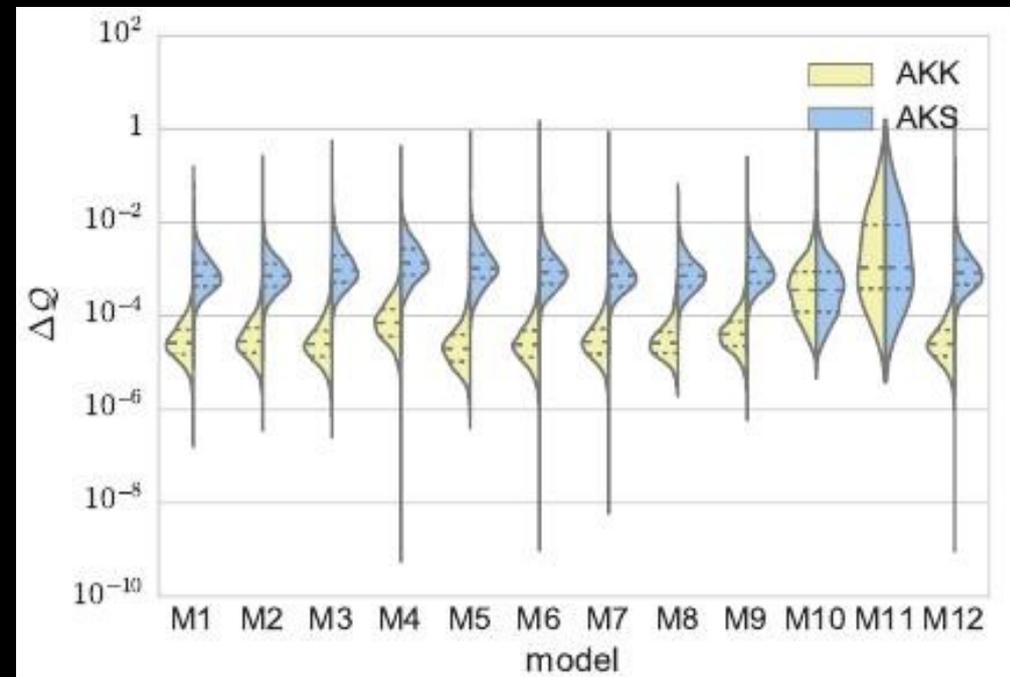
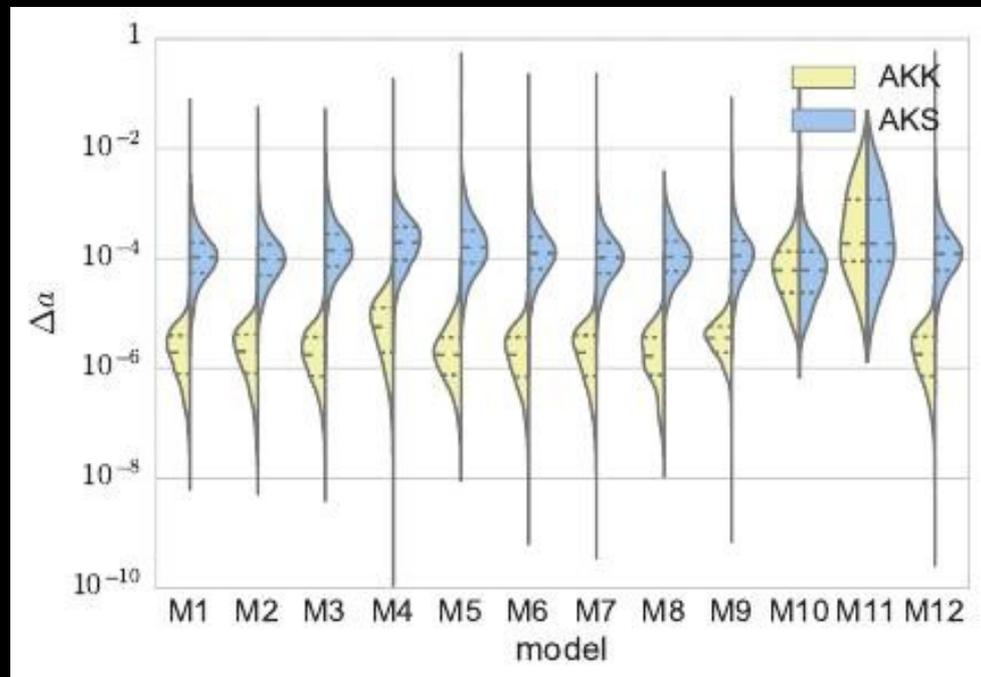
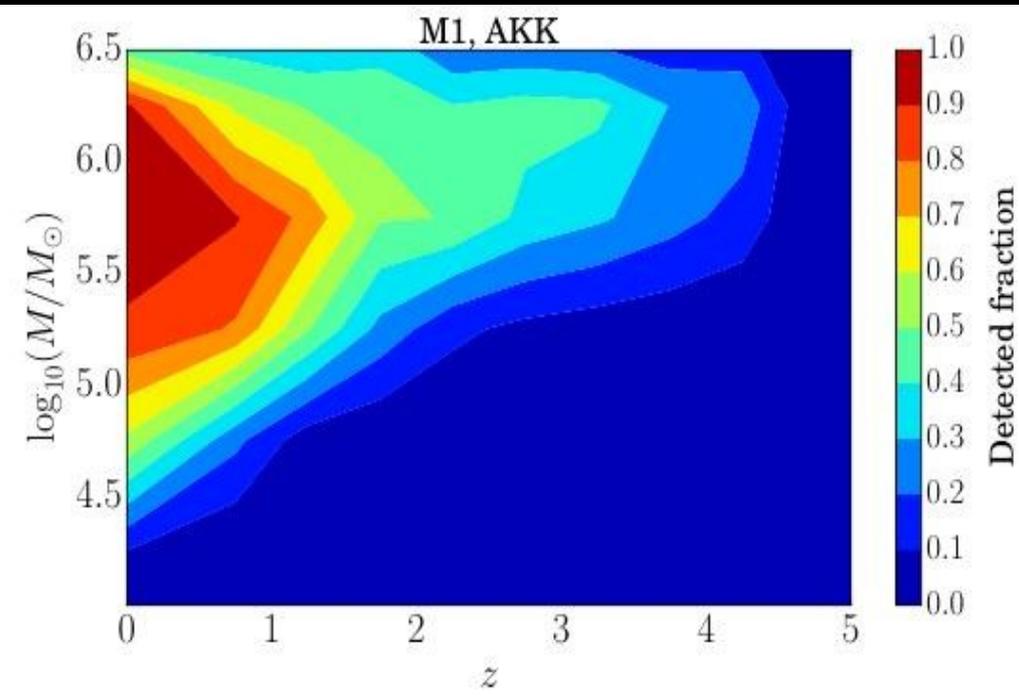
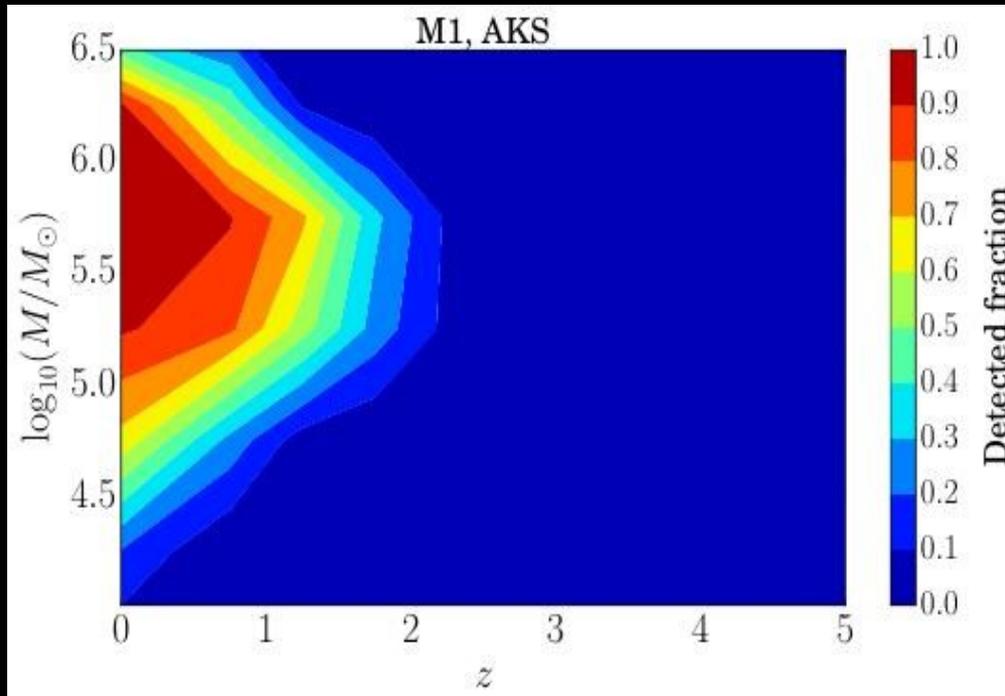
Using astrophysically motivated prescriptions we generated 12 models:

Model	Mass function	MBH spin	Cusp erosion	$M-\sigma$ relation	N_p	CO mass [M_\odot]	Total	EMRI rate [yr^{-1}] Detected (AKK)	Detected (AKS)
M1	Barausse12	a98	yes	Gultekin09	10	10	1600	294	189
M2	Barausse12	a98	yes	KormendyHo13	10	10	1400	220	146
M3	Barausse12	a98	yes	GrahamScott13	10	10	2770	809	440
M4	Barausse12	a98	yes	Gultekin09	10	30	520 (620)	260	221
M5	Gair10	a98	no	Gultekin09	10	10	140	47	15
M6	Barausse12	a98	no	Gultekin09	10	10	2080	479	261
M7	Barausse12	a98	yes	Gultekin09	0	10	15800	2712	1765
M8	Barausse12	a98	yes	Gultekin09	100	10	180	35	24
M9	Barausse12	aflat	yes	Gultekin09	10	10	1530	217	177
M10	Barausse12	a0	yes	Gultekin09	10	10	1520	188	188
M11	Gair10	a0	no	Gultekin09	100	10	13	1	1
M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279



Selected results: LISA reach and parameter estimation

(Babak et al, almost submitted...finally!)



Summary of EMRI parameter estimation

~1-1000 detections/yr

~typical sky localization better than 10 deg²

~distance to better than 10%

~MBH mass to better than 0.01%

~CO mass to better than 0.01%

~MBH spin to better than 0.001

~plunge eccentricity to better than 0.0001

~deviation from Kerr quadrupole moment to <0.001

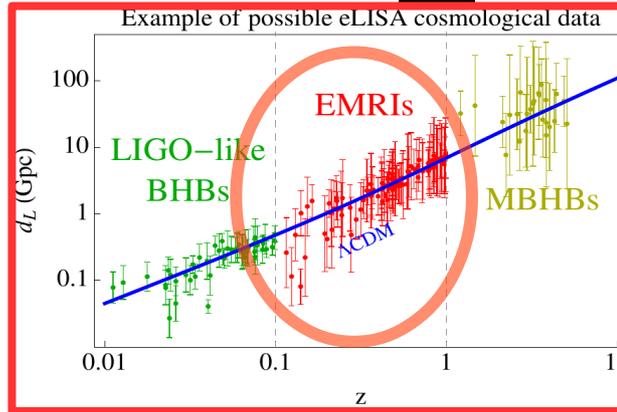
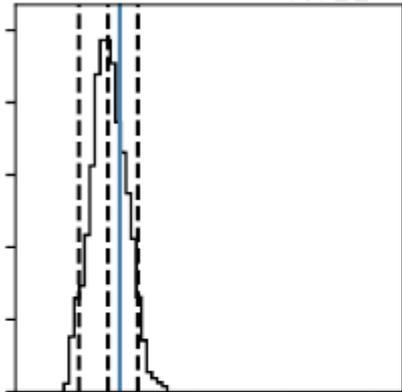
**New tool for astrophysics (Gair et al 2010) cosmology (McLeod & Hogan 2008), and fundamental physics (Gair et al 2013) ...
to be further explored**

Example: cosmology with EMRIs

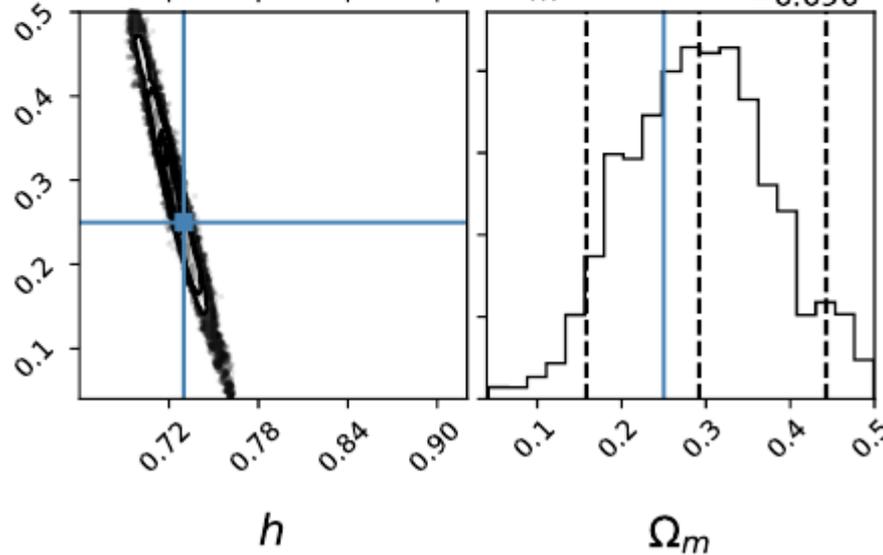
Λ CDM, 10yr

Model M1 (*fiducial*)

$$h = 0.722^{+0.013}_{-0.011}$$



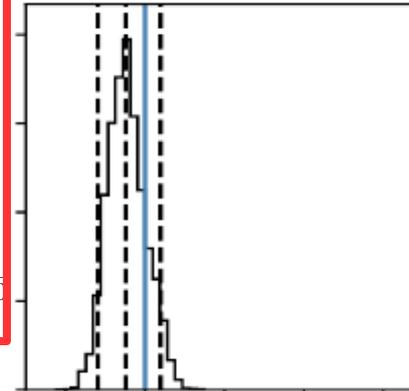
$$\Omega_m = 0.292^{+0.085}_{-0.090}$$



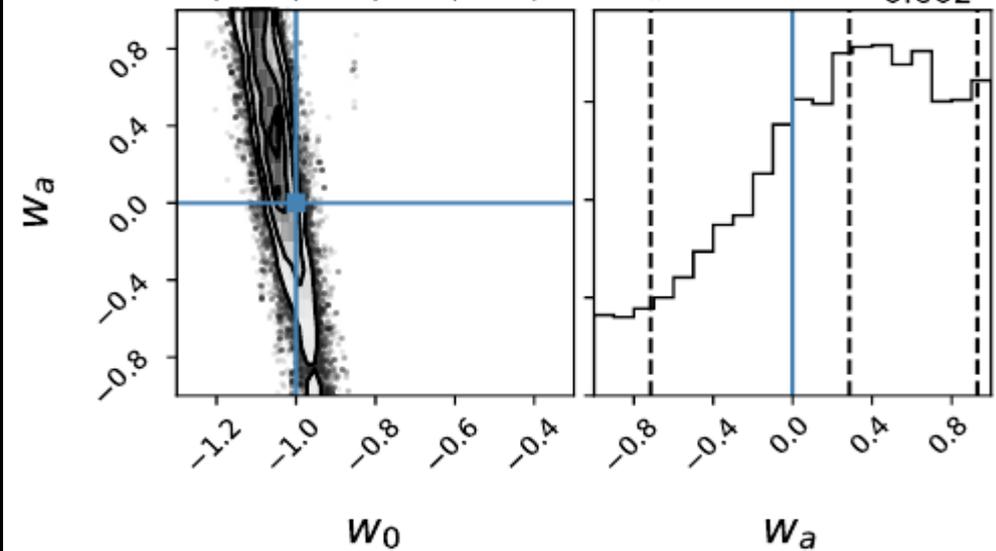
DE, 10yr

Model M1 (*fiducial*)

$$w_0 = -1.049^{+0.052}_{-0.045}$$



$$w_a = 0.286^{+0.465}_{-0.602}$$



> Independent measurement of H_0 at 1.5% level

> Independent measurement of w_0 at 5% level (assuming H_0 known)

(Laghi+, in prep.)