

# *Astrophysics and detection of gravitational wave sources*

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# **OUTLINE**

## **LECTURE 1 (NOW): Setting the stage**

- Gravitational waves (GWs): theory and general considerations**
- GWs from binary systems, relevant scalings**

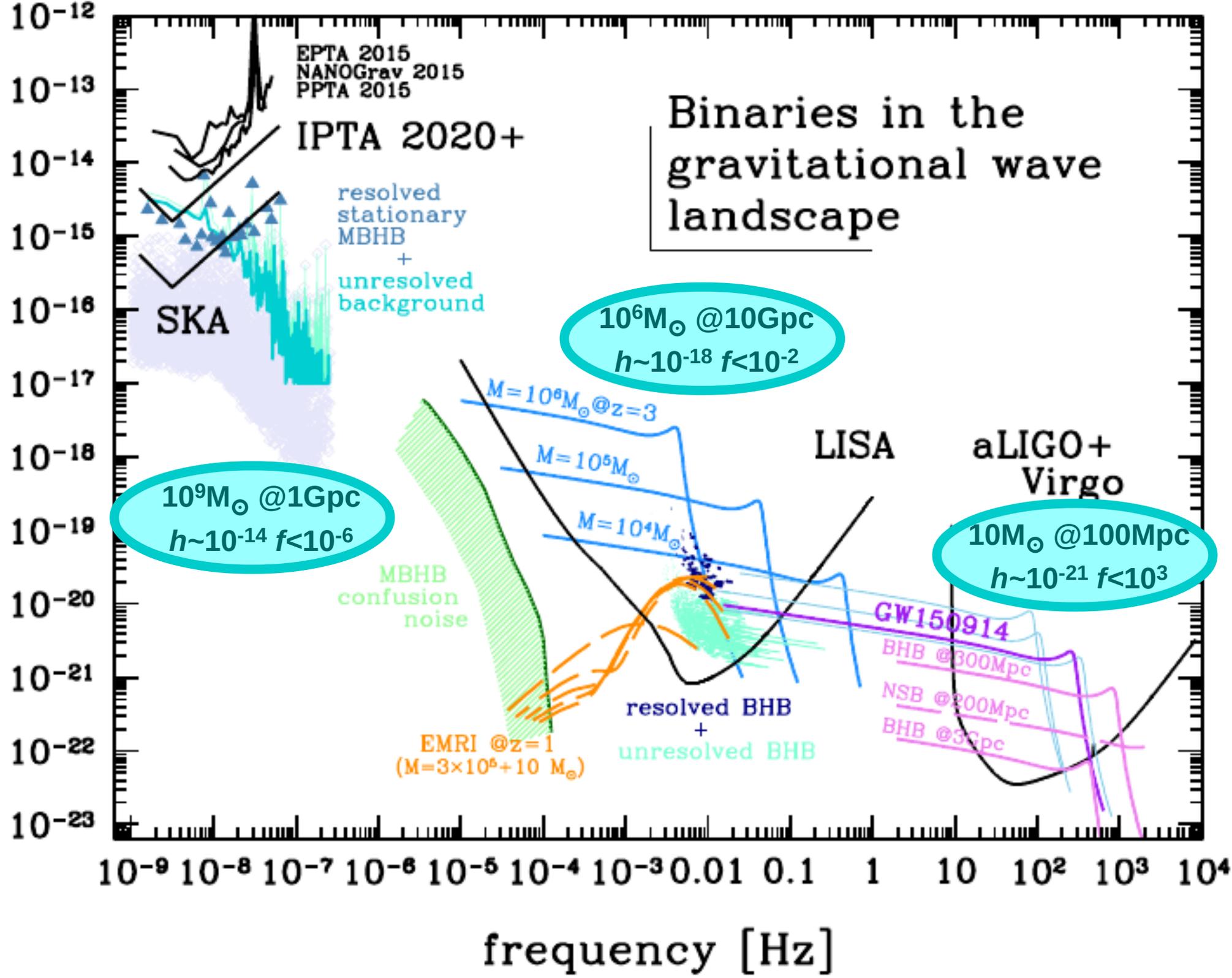
## **LECTURES 2/3 (Monday afternoon): ground based**

- Detection of GW with ground based interferometers**
- Black hole binaries (BHBs) detected by LIGO/Virgo**
- GW170817 a neutron star binary (NSB)**
- Astrophysics of ground based GW sources: formation scenarios**
- Future from the ground: 3G detectors**

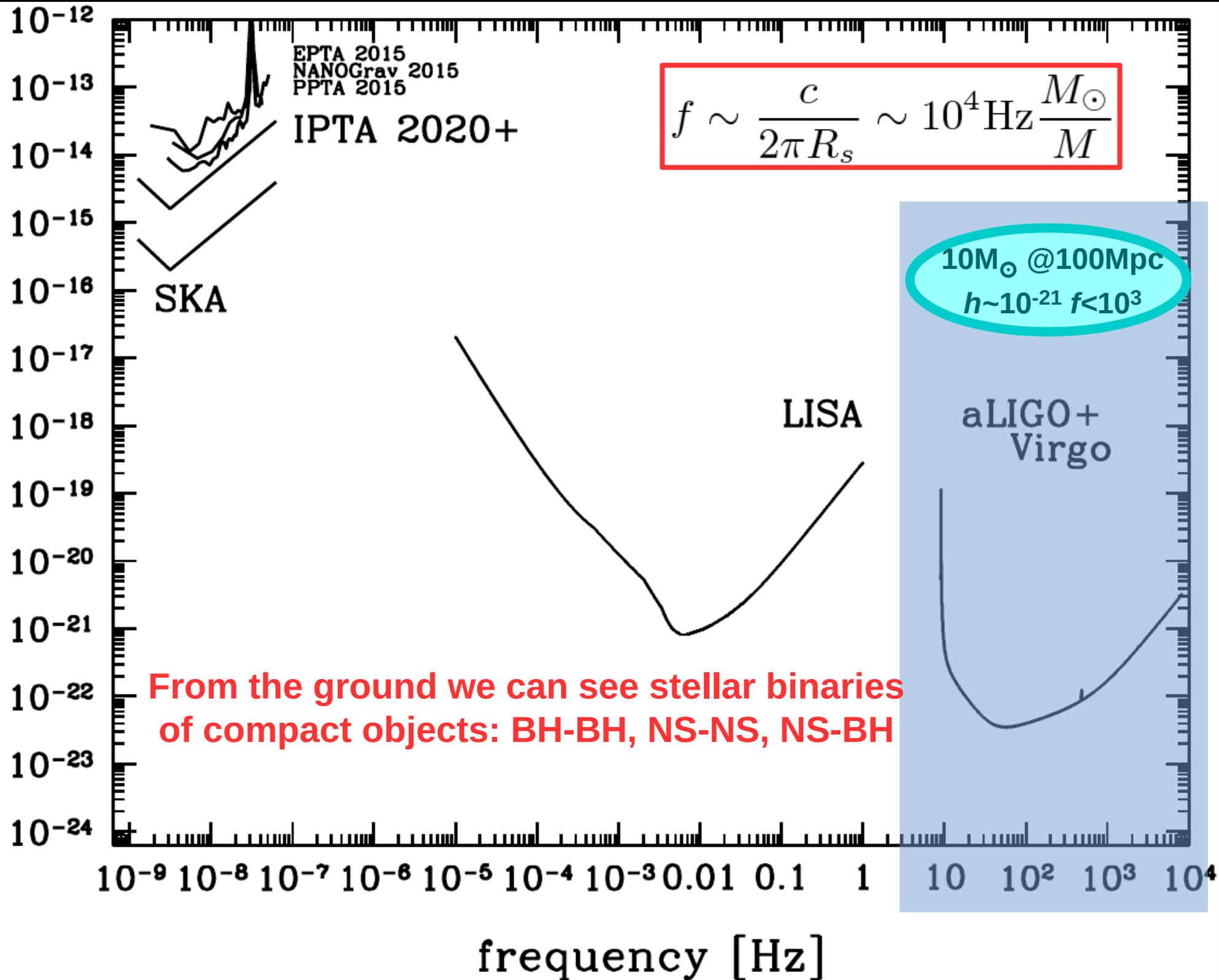
## **LECTURE 4/5 (Tuesday morning): space based**

- Beyond the ground: GW detection from space**
- Laser interferometer space antenna and its sources**
- Galactic binaries**
- Extreme mass ratio inspirals (EMRIs)**
- Massive black hole (MBH) formation and evolution**

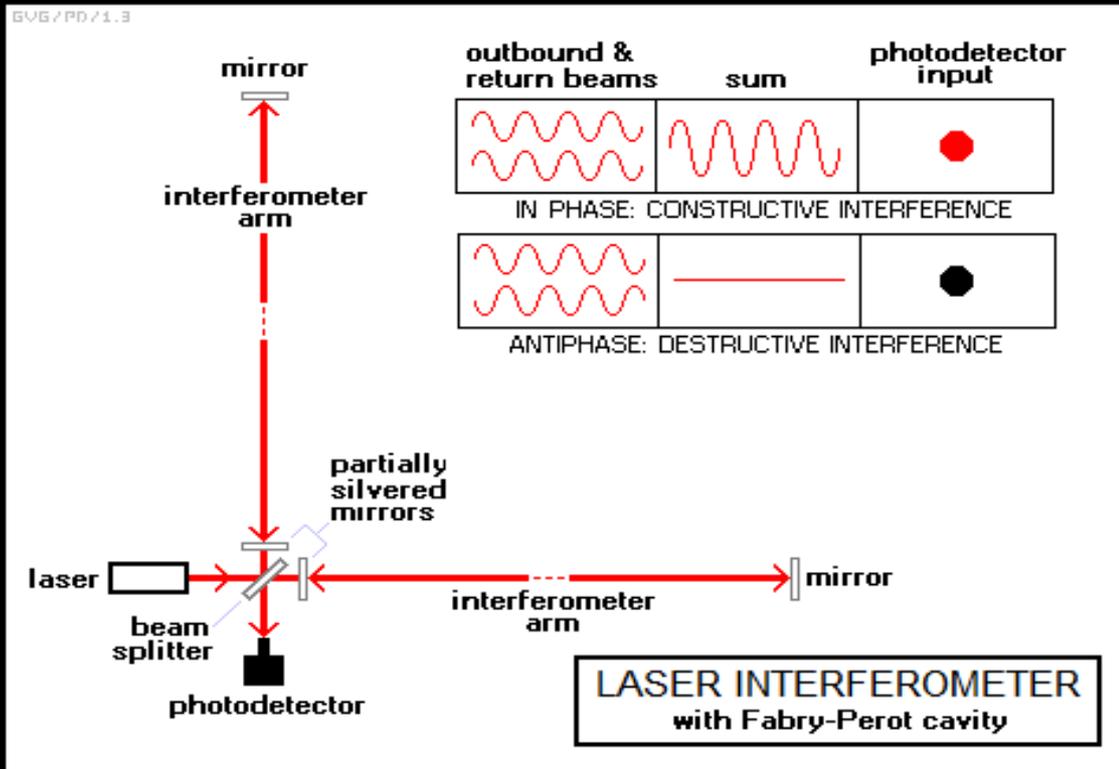
characteristic amplitude



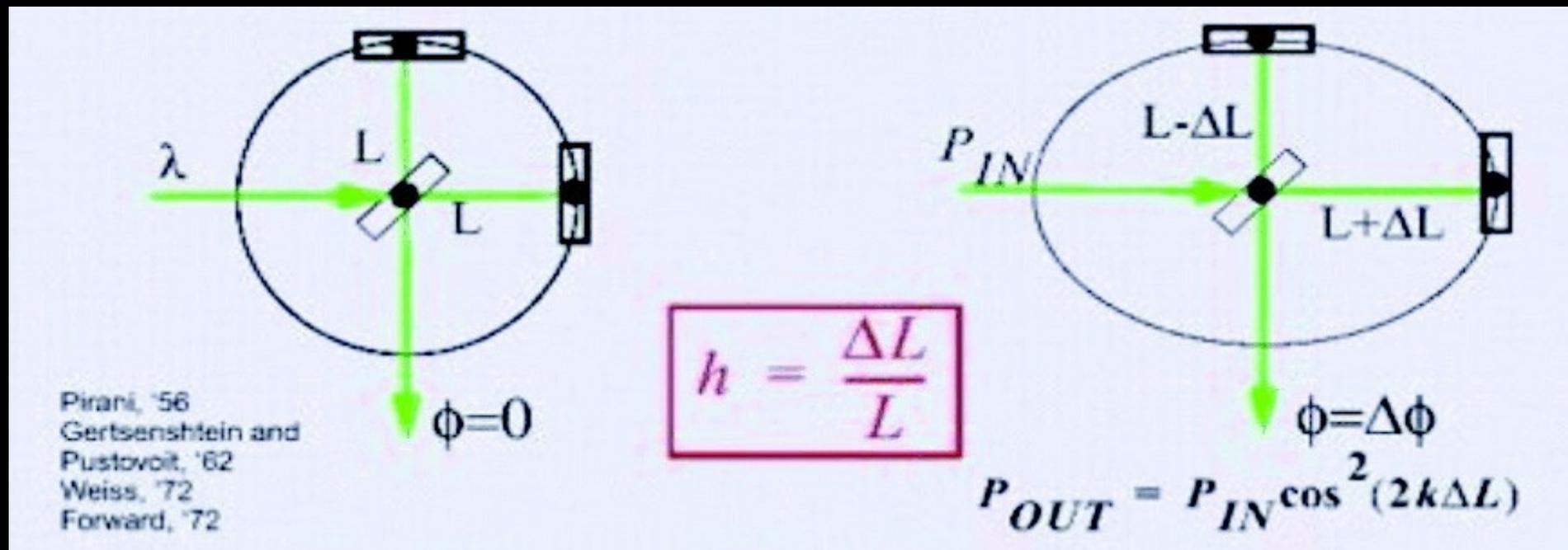
characteristic amplitude



# Observation technique: laser interferometry

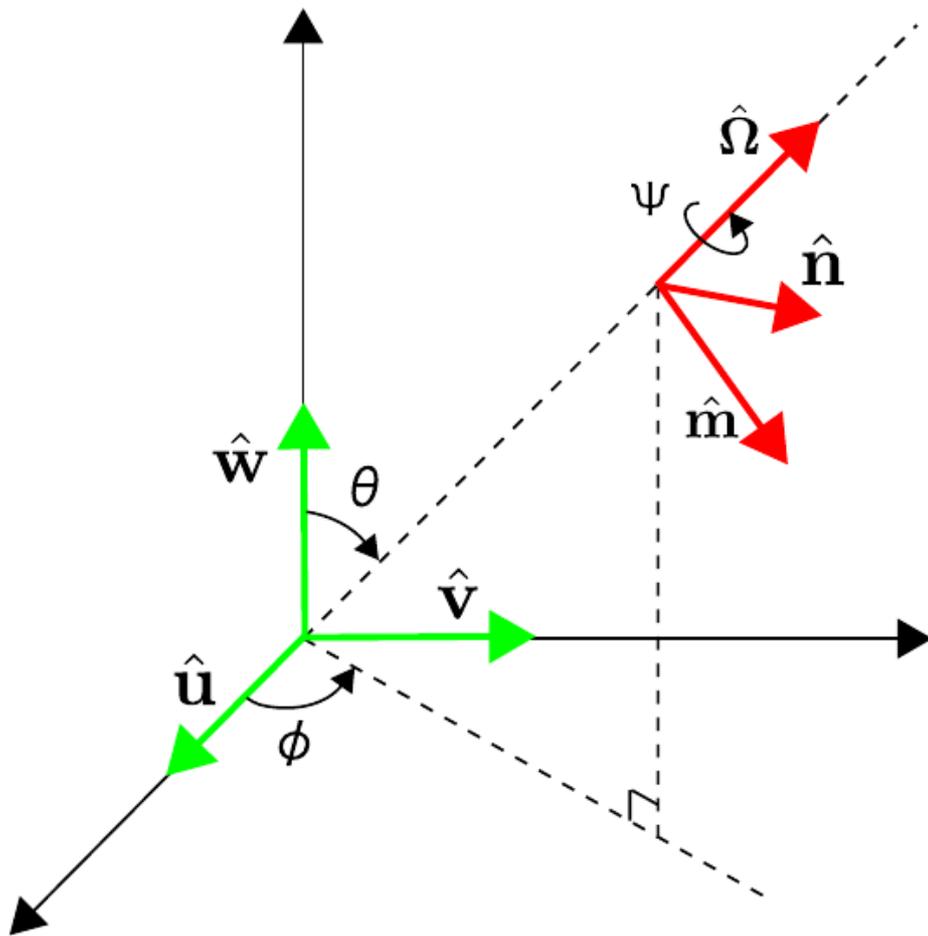


The wave passage changes the length of the path covered by the light in the two arms. The result is a de-phasing in the recombination of the two laser beams.



$$h_+(t) = \frac{4}{r} \left( \frac{GM_C}{c^2} \right)^{\frac{5}{3}} \left( \frac{\pi f_{gw}}{c} \right)^{\frac{2}{3}} \left( \frac{1 + \cos^2 \theta}{2} \right) \cos(2\pi f_{gw}t + 2\phi)$$

$$h_\times(t) = \frac{4}{r} \left( \frac{GM_C}{c^2} \right)^{\frac{5}{3}} \left( \frac{\pi f_{gw}}{c} \right)^{\frac{2}{3}} \cos \theta \sin(2\pi f_{gw}t + 2\phi)$$



**The response of a detector to the two polarization waves depends on relative orientation of the detector and the incoming wave.**

$$h_{ij}(t - \hat{\Omega} \cdot \vec{x}) = \sum_A h^A(t - \hat{\Omega} \cdot \vec{x}) \epsilon_{ij}^A(\hat{\Omega})$$

$$\epsilon_{ij}^+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \epsilon_{ij}^\times = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\epsilon_{ij}^x = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \epsilon_{ij}^y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\epsilon_{ij}^b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \epsilon_{ij}^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\hat{x} = (1, 0, 0), \hat{y} = (0, 1, 0), \hat{z} = (0, 0, 1)$$

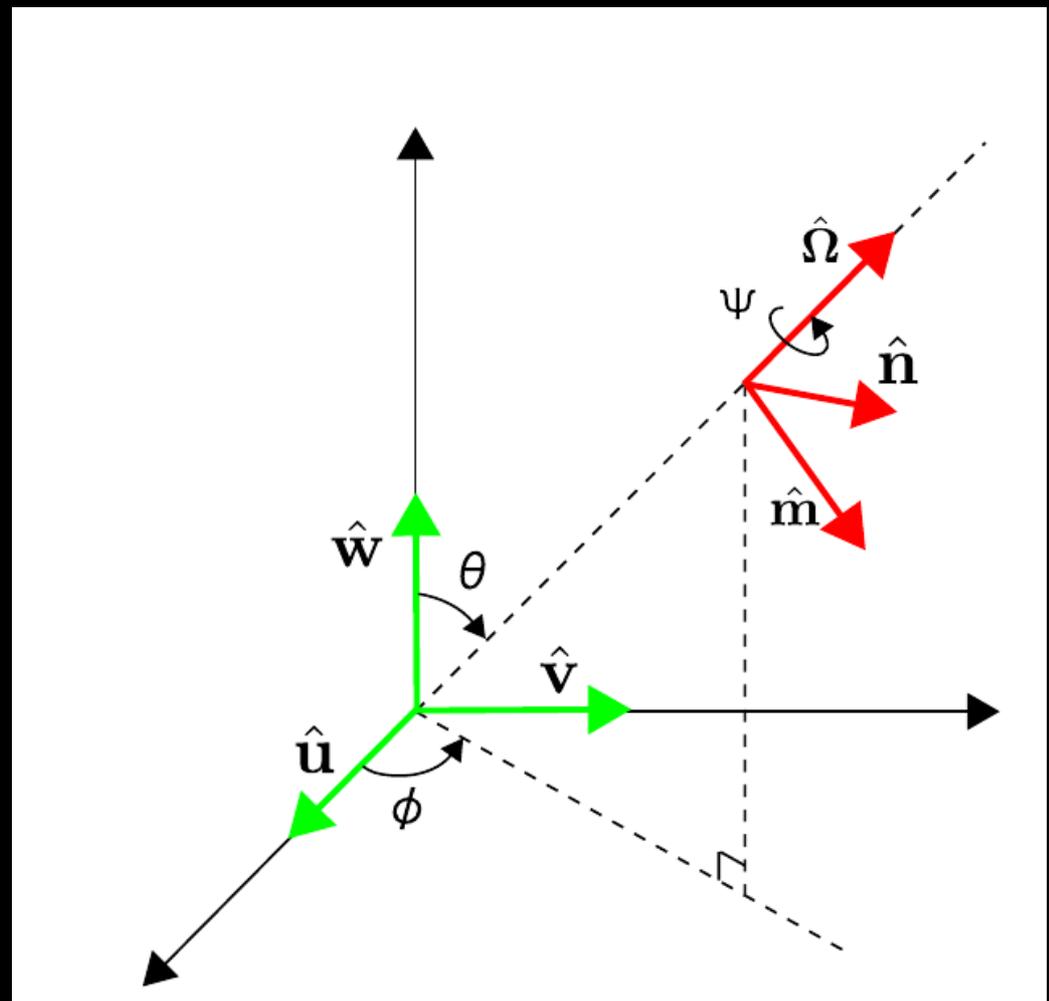
$$\hat{x}' = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta)$$

$$\hat{y}' = (-\sin \phi, \cos \phi, 0)$$

$$\hat{z}' = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

$$\begin{aligned} \hat{m} &= \hat{x}' \cos \psi + \hat{y}' \sin \psi, \\ \hat{n} &= -\hat{x}' \sin \psi + \hat{y}' \cos \psi, \\ \hat{\Omega} &= \hat{z}'. \end{aligned}$$

**$\Psi$  is the polarization angle. Defining the relative orientation of the polarization axis with respect to the tirection of the interferometer**



$$\begin{aligned} \epsilon^+ &= \hat{m} \otimes \hat{m} - \hat{n} \otimes \hat{n}, \\ \epsilon^\times &= \hat{m} \otimes \hat{n} + \hat{n} \otimes \hat{m}, \\ \epsilon^x &= \hat{m} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{m}, \\ \epsilon^y &= \hat{n} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{n}, \\ \epsilon^b &= \hat{m} \otimes \hat{m} + \hat{n} \otimes \hat{n}, \\ \epsilon^\ell &= \hat{\Omega} \otimes \hat{\Omega}. \end{aligned}$$

$$h(t) = \sum_A h_A(t - \hat{\Omega} \cdot x) F^A(\hat{\Omega}, \psi)$$

$$F^A(\hat{\Omega}, \psi) = \frac{1}{2} (\hat{x}^i \hat{x}^j - \hat{y}^i \hat{y}^j) \epsilon_{ij}^A(\hat{\Omega}, \psi)$$

$$F^+(\theta, \phi, \psi) = \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi,$$

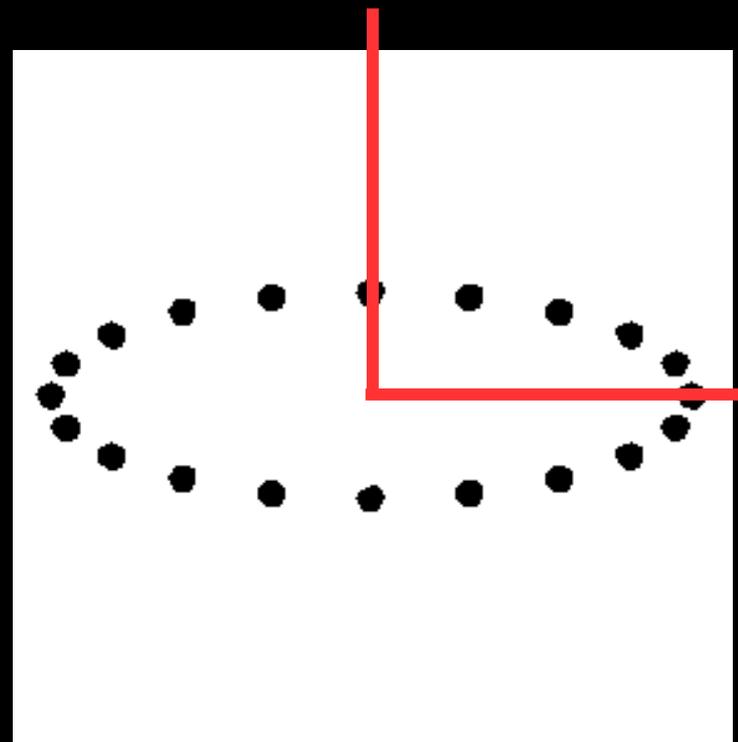
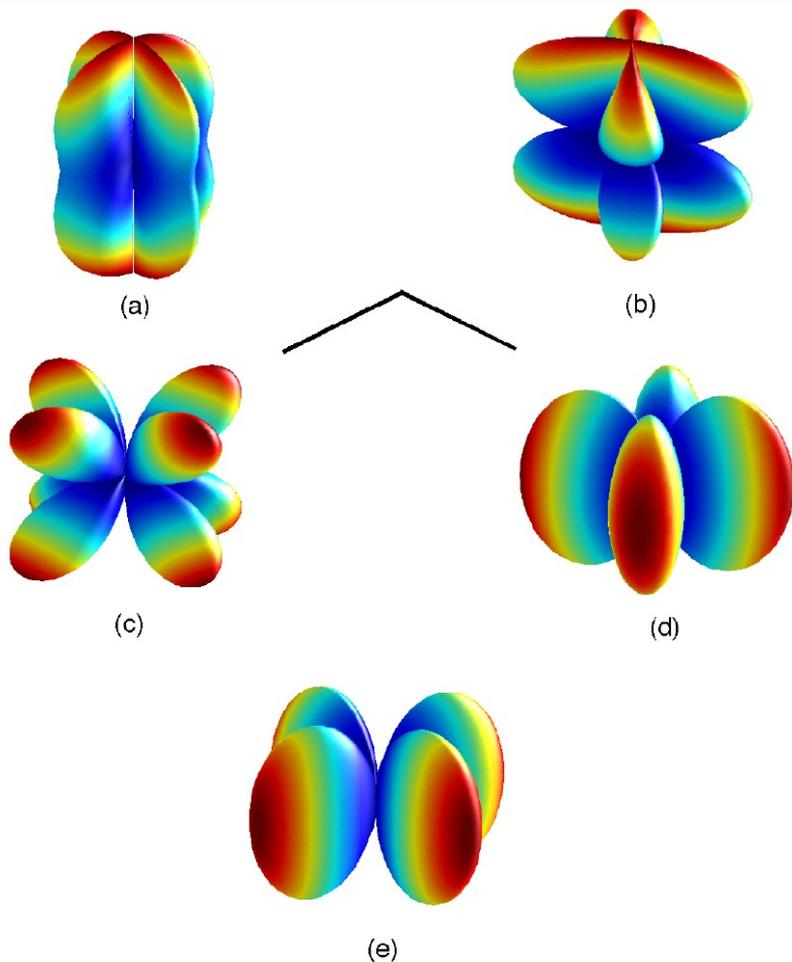
$$F^\times(\theta, \phi, \psi) = -\frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi - \cos \theta \sin 2\phi \cos 2\psi,$$

$$F^x(\theta, \phi, \psi) = \sin \theta (\cos \theta \cos 2\phi \cos \psi - \sin 2\phi \sin \psi),$$

$$F^y(\theta, \phi, \psi) = -\sin \theta (\cos \theta \cos 2\phi \sin \psi + \sin 2\phi \cos \psi),$$

$$F^b(\theta, \phi) = -\frac{1}{2} \sin^2 \theta \cos 2\phi,$$

$$F^\ell(\theta, \phi) = \frac{1}{2} \sin^2 \theta \cos 2\phi.$$

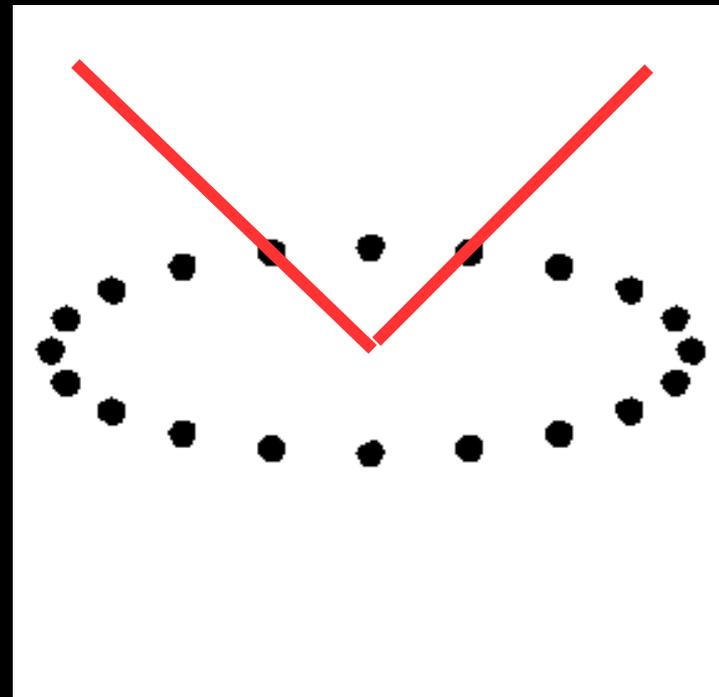
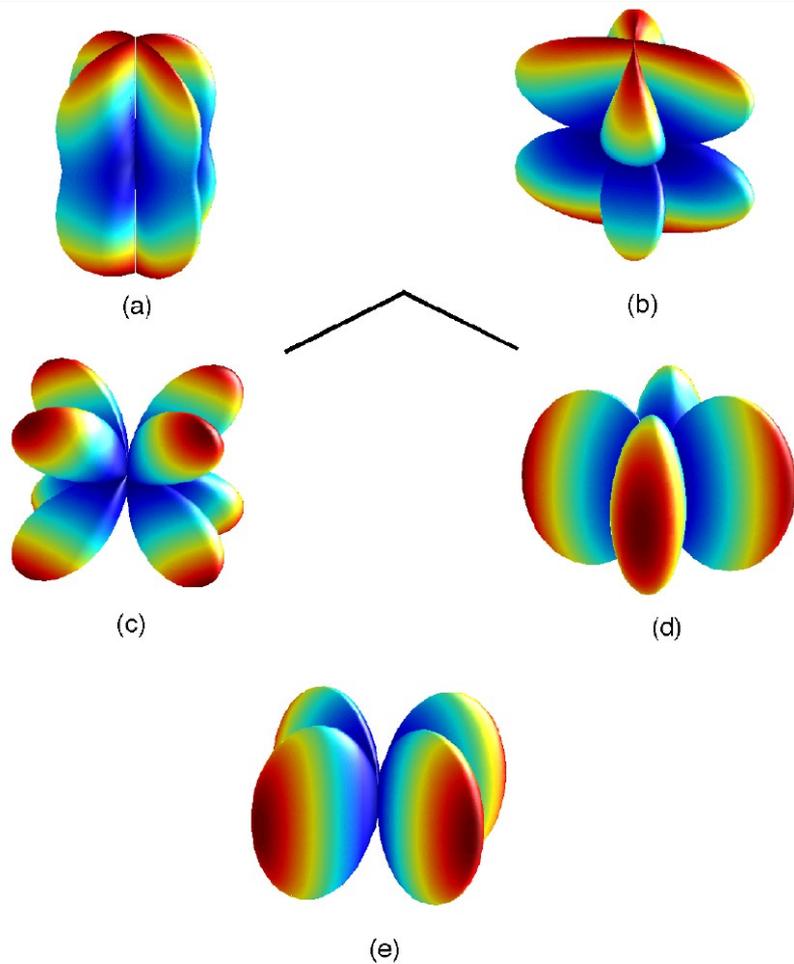


**$\Delta L$  is maximum: the response of the detector to the wave is maximum**

$$h(t) = \sum_A h_A(t - \hat{\Omega} \cdot x) F^A(\hat{\Omega}, \psi)$$

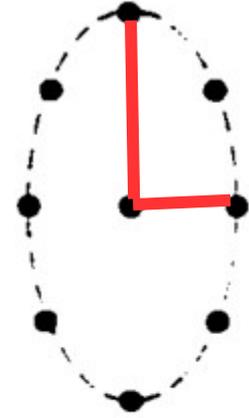
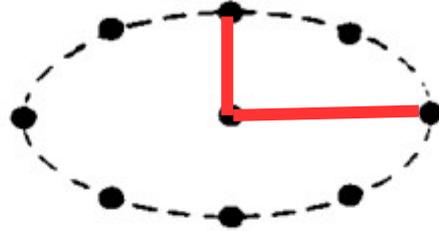
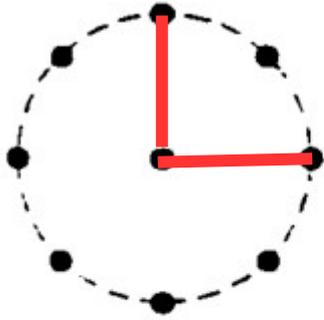
$$F^A(\hat{\Omega}, \psi) = \frac{1}{2} (\hat{x}^i \hat{x}^j - \hat{y}^i \hat{y}^j) \epsilon_{ij}^A(\hat{\Omega}, \psi)$$

$$\begin{aligned} F^+ (\theta, \phi, \psi) &= \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi, \\ F^\times (\theta, \phi, \psi) &= -\frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi - \cos \theta \sin 2\phi \cos 2\psi, \\ F^x (\theta, \phi, \psi) &= \sin \theta (\cos \theta \cos 2\phi \cos \psi - \sin 2\phi \sin \psi), \\ F^y (\theta, \phi, \psi) &= -\sin \theta (\cos \theta \cos 2\phi \sin \psi + \sin 2\phi \cos \psi), \\ F^b (\theta, \phi) &= -\frac{1}{2} \sin^2 \theta \cos 2\phi, \\ F^\ell (\theta, \phi) &= \frac{1}{2} \sin^2 \theta \cos 2\phi. \end{aligned}$$

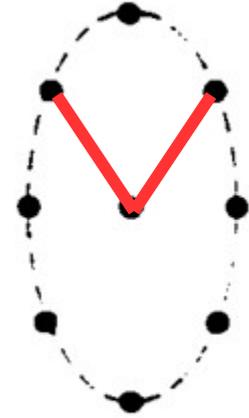
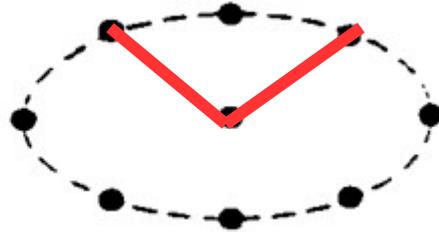
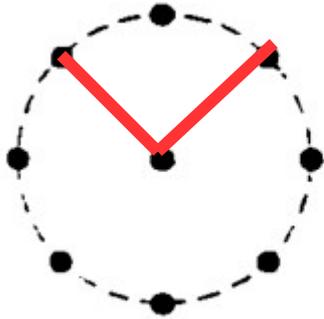


**$\Delta L$  is null, so is the response of the detector to the wave**

"plus"  
polarization



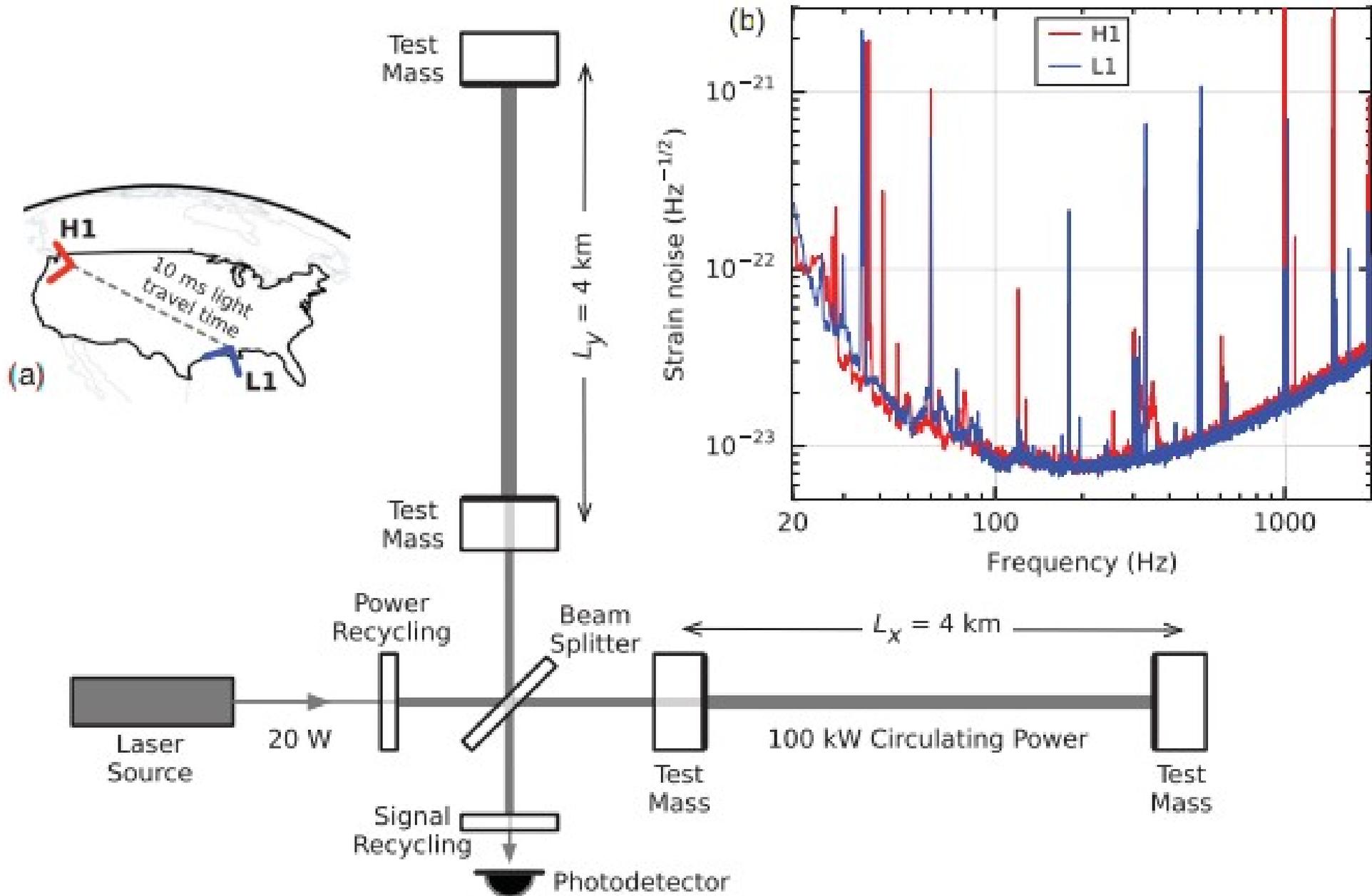
"plus"  
polarization





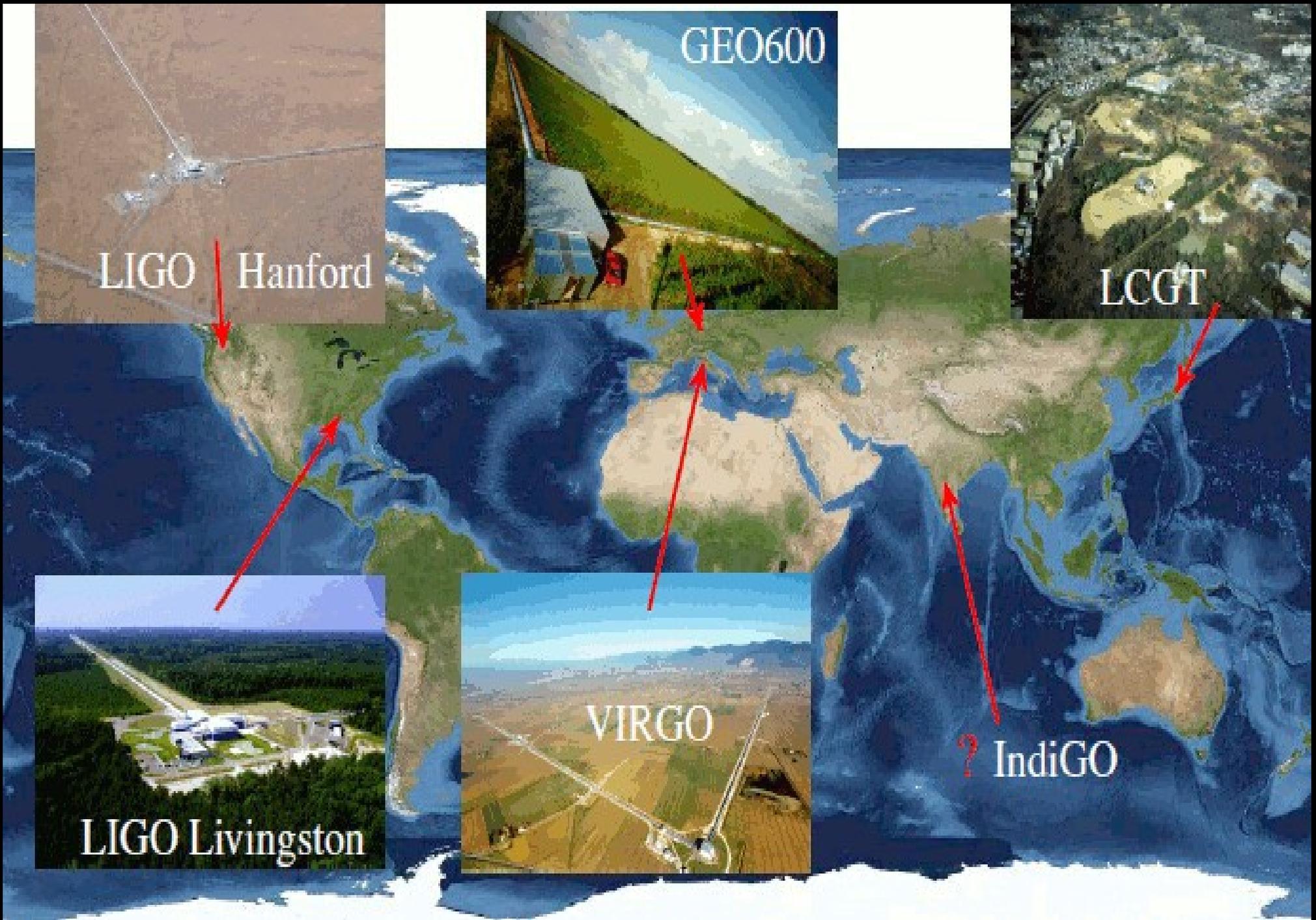
**To see relative variations of  $10^{-22}$ , we need a long interferometer...  
a really long one!**





**Interferometers are huge microphones. Contrary to telescopes, that are the equivalent to eyes and are pointed in the desired direction, interferometers are huge ears, sensitive in all directions.**

# Network of ground based interferometers



# ***What did LIGO see on September 14 2015?***

Long time ago (~1 billion years) in a galaxy far away (~1 billion light years)

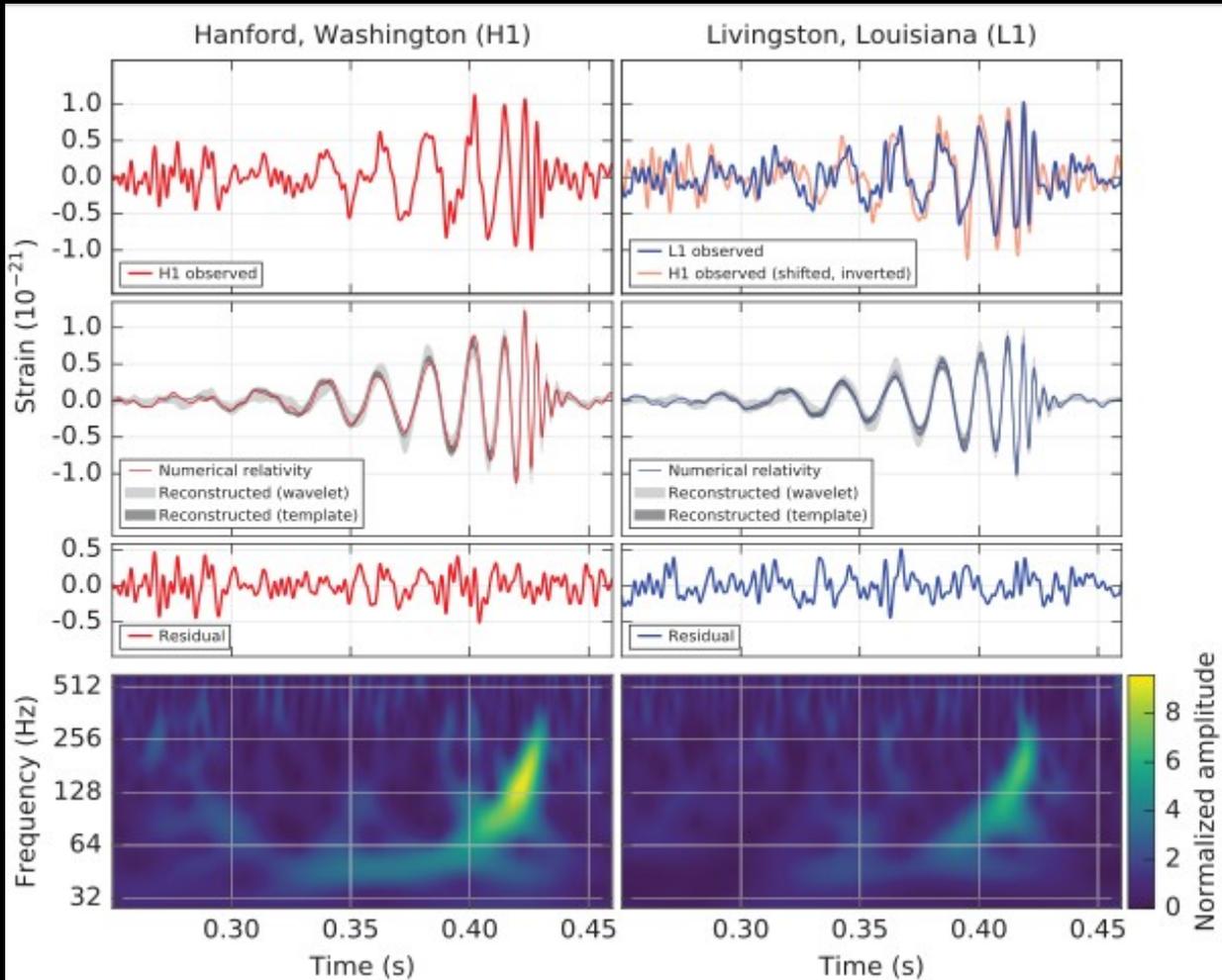
...

**One billion years later on Earth....**

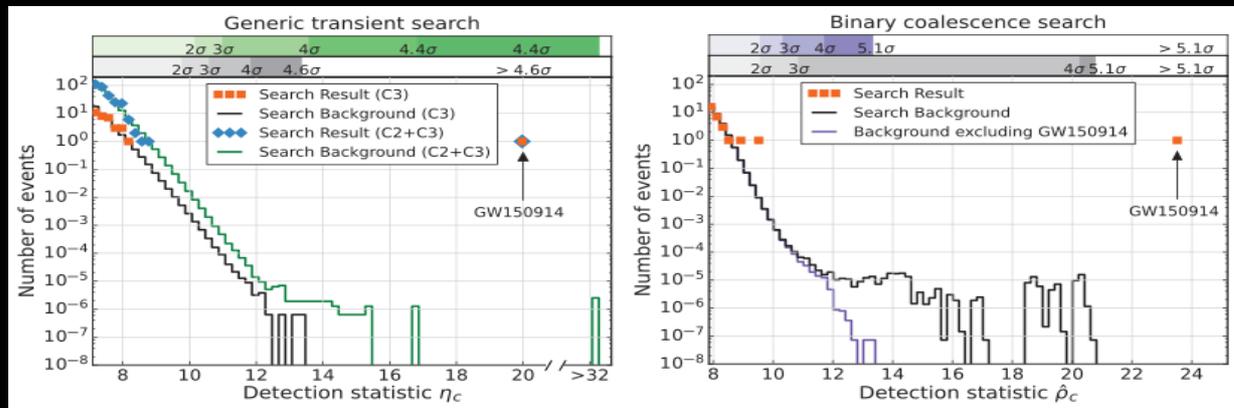
**As the signal passes through the LIGO arms...**

**This is what LIGO saw, or I should say heard**

# Making history: 14 September 2015



On September 14 2015, the two LIGO detectors observed a coincident signal of 0.2 seconds



The signal was so strong that it was immediately recognized by the 'detection pipelines'

Nominal S/N: 24  
Significance:  $5.1\sigma$



## Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*\*

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of  $1.0 \times 10^{-21}$ . It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than  $5.1\sigma$ . The source lies at a luminosity distance of  $410_{-180}^{+160}$  Mpc corresponding to a redshift  $z = 0.09_{-0.04}^{+0.03}$ . In the source frame, the initial black hole masses are  $36_{-4}^{+5} M_{\odot}$  and  $29_{-4}^{+4} M_{\odot}$ , and the final black hole mass is  $62_{-4}^{+4} M_{\odot}$ , with  $3.0_{-0.5}^{+0.5} M_{\odot} c^2$  radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: [10.1103/PhysRevLett.116.061102](https://doi.org/10.1103/PhysRevLett.116.061102)

The Physical Review Letters server registered more than 100000 accesses following the announcement.

Server crashed!



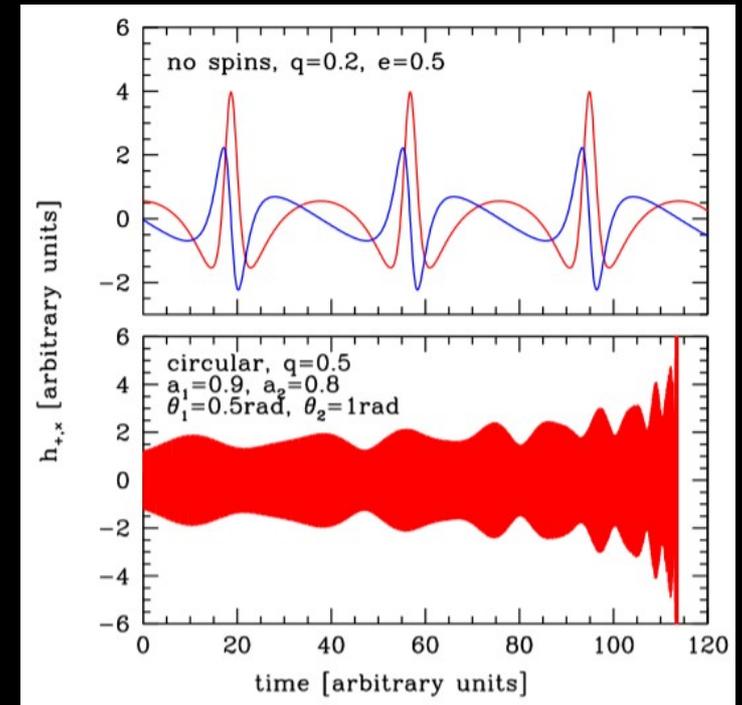
# What information do we extract from the signal?

>Masses have great impact and the phase modulation and amplitude of the signal

>Eccentricity has a great impact on the waveform shape and phase modulation

>Spins have an impact on the waveform amplitude (precession) and phase

The precision of the measurement depends on the S/N and on the number of observed cycles



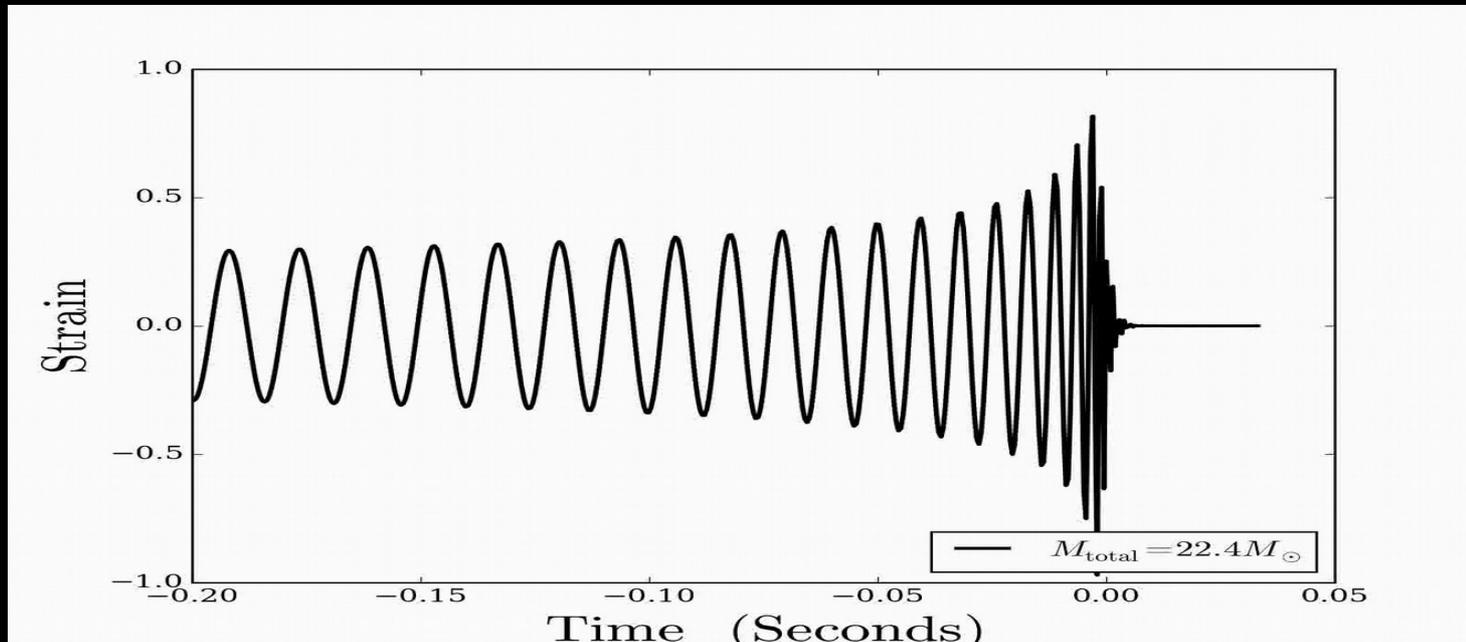
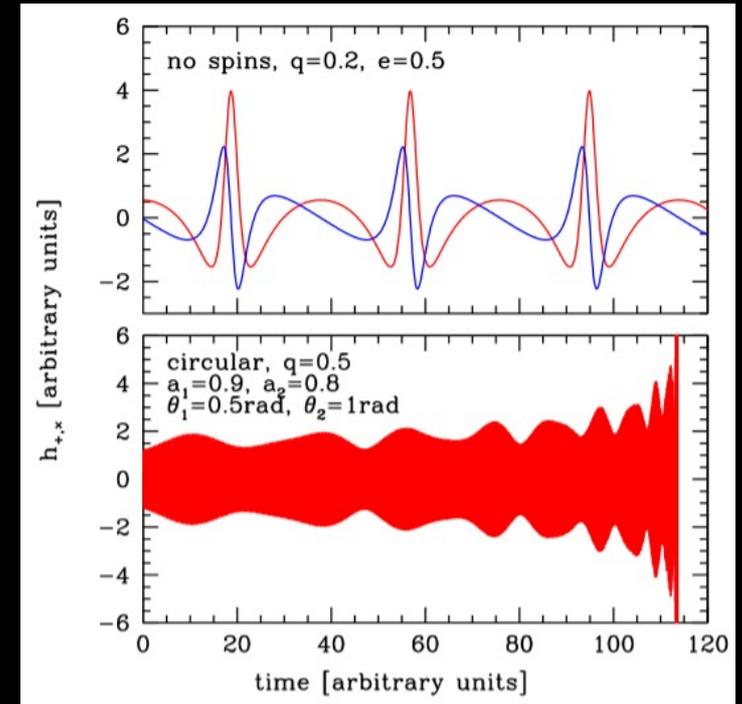
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(Courtesy W. del Pozzo)

# What information do we extract from the signal?

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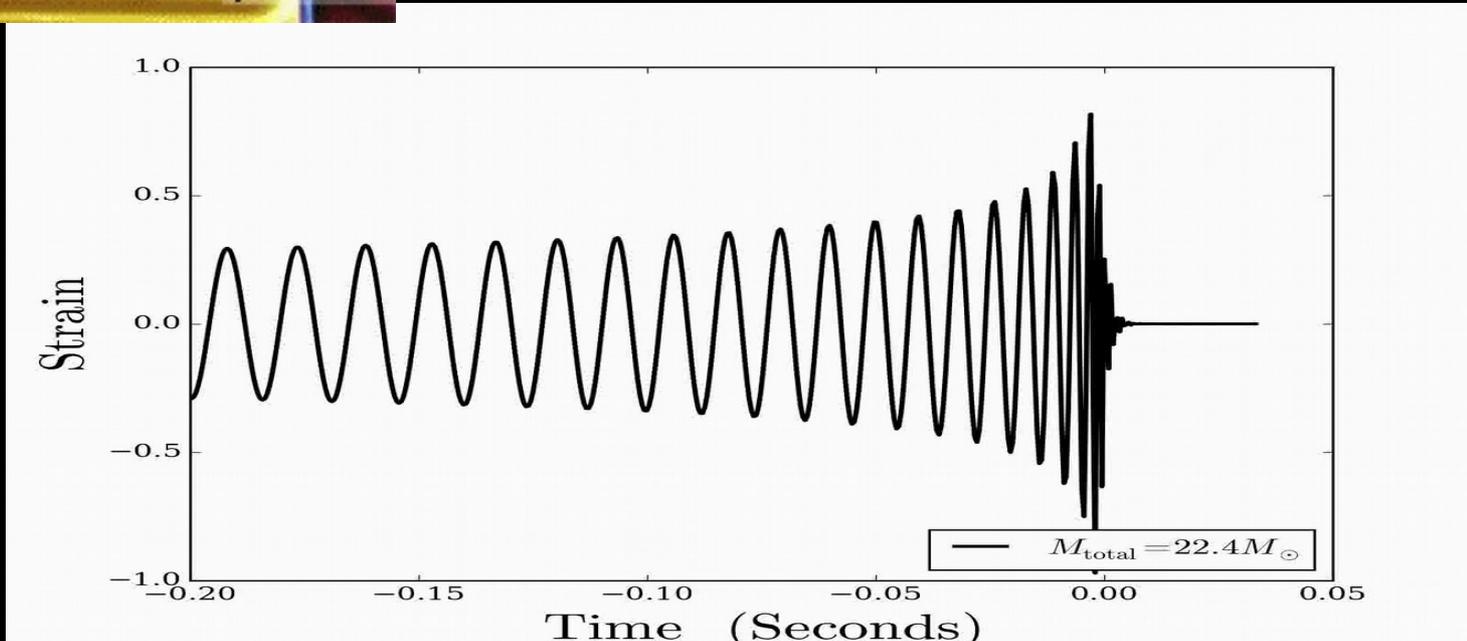
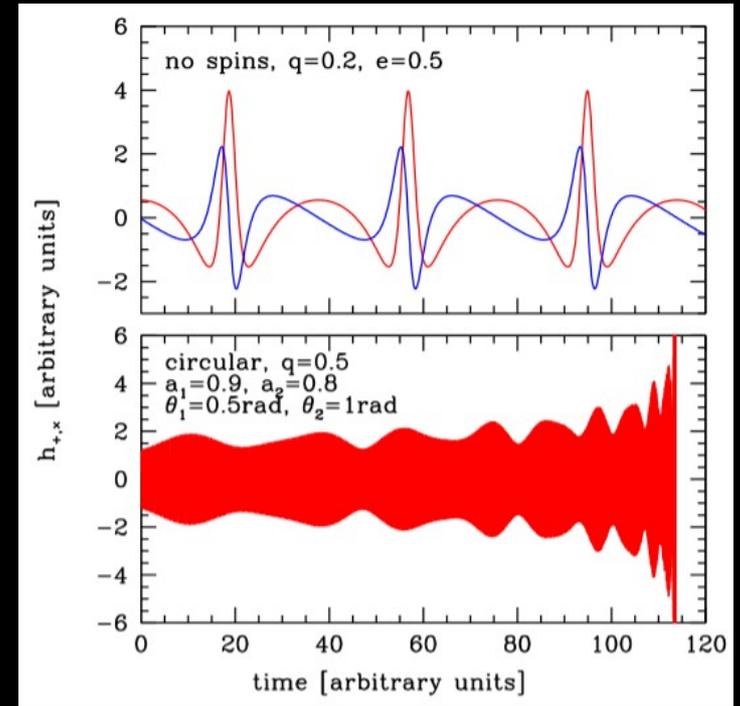


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(Courtesy W. del Pozzo)

# Source parameter extraction

$$h^{(t)}(t) = \frac{\sqrt{3}}{2} \left[ F_+^{(t)}(t) h_+(t) + F_\times^{(t)}(t) h_\times(t) \right]$$

Detected signal: combination of the two wave **polarization amplitude** and the **antenna beam pattern**

$$h_+ = 2 \frac{\mathcal{M}^{5/3}}{D_L} \left[ 1 + (\hat{\mathbf{L}} \cdot \hat{\mathbf{N}})^2 \right] (\pi f)^{2/3} \cos \phi(t)$$

$$h_\times = -4 \frac{\mathcal{M}^{5/3}}{D_L} (\hat{\mathbf{L}} \cdot \hat{\mathbf{N}}) (\pi f)^{2/3} \sin \phi(t),$$

**polarization amplitude:**

function of the source intrinsic parameters ( $M, f$ ), of the source distance  $D_L$ , and of the source inclination  $i = \mathbf{L} \cdot \mathbf{N}$

$$F_+(\theta'_N, \phi'_N, \psi'_N) = \frac{1}{4} (1 + \cos \theta_N'^2) \cos 2\phi'_N \cos 2\psi'_N - \cos \theta'_N \sin 2\phi'_N \sin 2\psi'_N$$

$$F_\times(\theta'_N, \phi'_N, \psi'_N) = \frac{1}{2} (1 + \cos \theta_N'^2) \cos 2\phi'_N \sin 2\psi'_N + \cos \theta'_N \sin 2\phi'_N \cos 2\psi'_N$$

**Antenna pattern:**

function of the relative source-detector orientation. Depends on: source sky location and polarization ( $\theta \square \varphi \square \psi$ )

$$\phi(f) = \phi_c - \frac{1}{16} (\pi f \mathcal{M})^{-5/3} \left[ 1 + \frac{5}{3} \left( \frac{743}{336} + \frac{11}{4} \eta \right) (\pi M f)^{2/3} - \frac{5}{2} (4\pi - \beta) (\pi M f) \right. \\ \left. + 5 \left( \frac{3058673}{1016064} + \frac{5429}{1008} \eta + \frac{617}{144} \eta^2 - \sigma \right) (\pi M f)^{4/3} \right];$$

**Phase evolution:**

depends on the system masses and spins and eccentricity ( $M_1, M_2, \mathbf{a}_1, \mathbf{a}_2, \mathbf{e}$ )

The full waveform for an eccentric spinning binary depends on **17 parameters**. Each of them leave a peculiar imprint in the waveform amplitude and phase.

>The position in the sky is essentially measured via triangulation  
two interferometers  poor performance

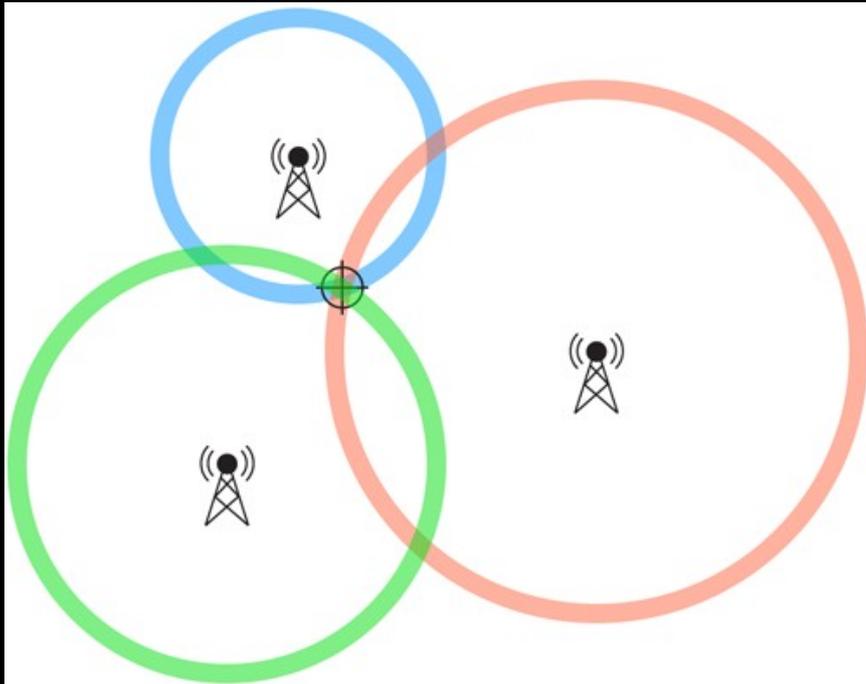
>Distance is measured from the waveform amplitude (but is degenerate with  
sky position and orbit inclination)

The accuracy of the measurement depends on the number of  
interferometers and on the possibility of  
disentangle the two wave polarizations

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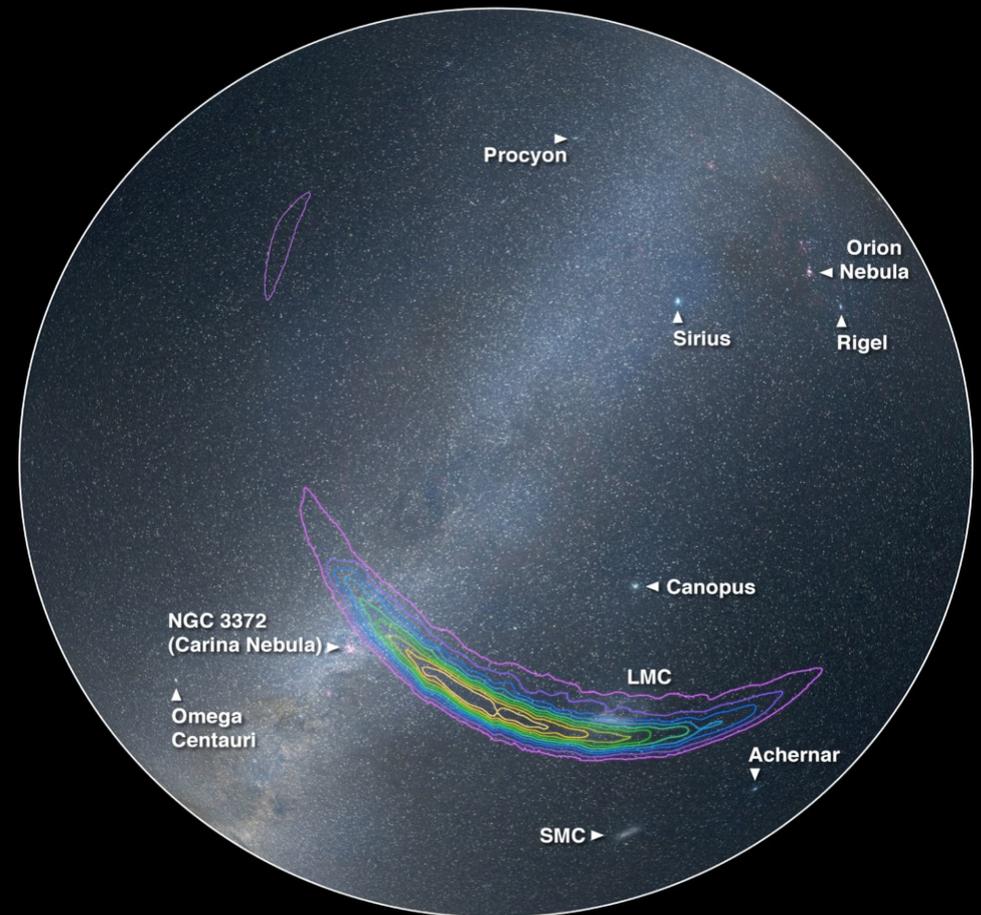
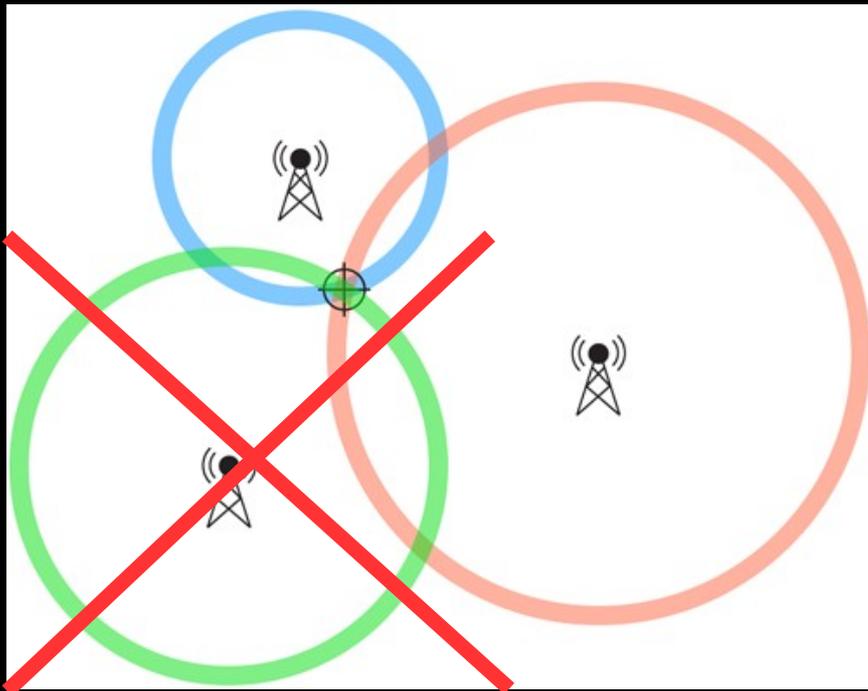
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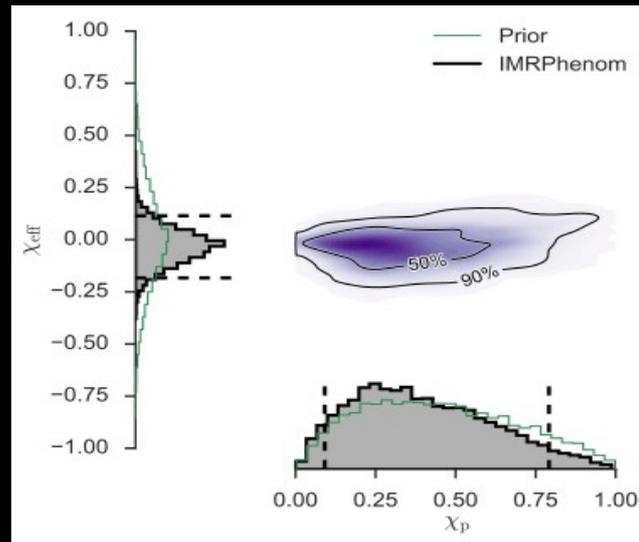
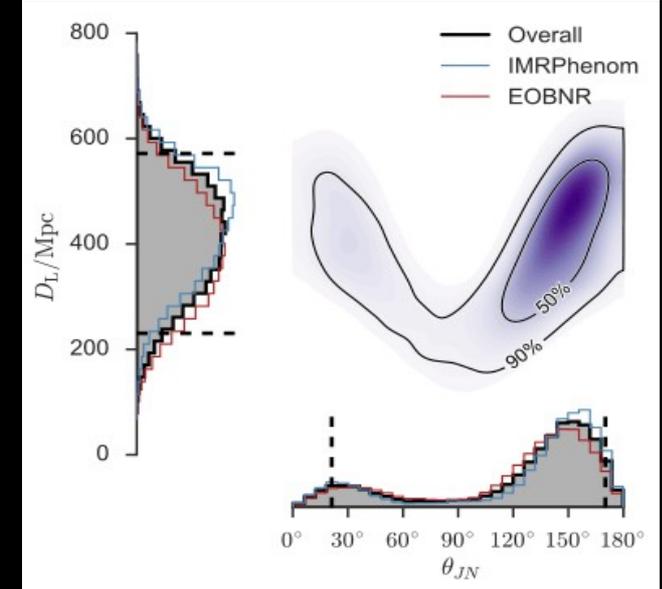
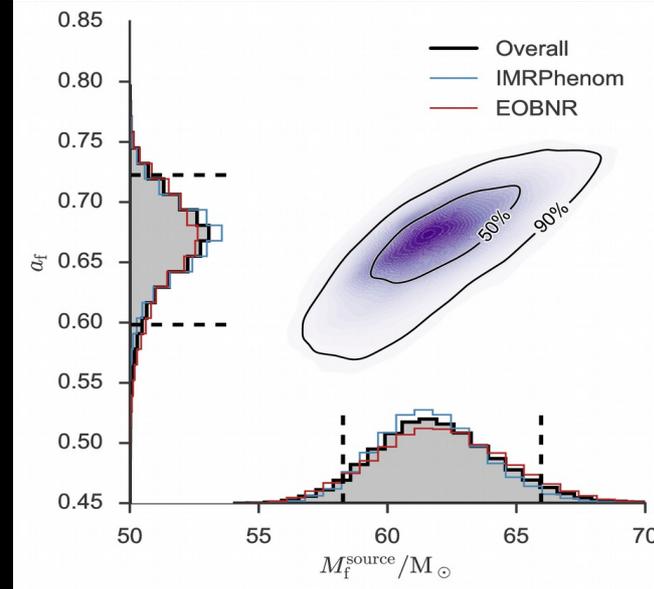
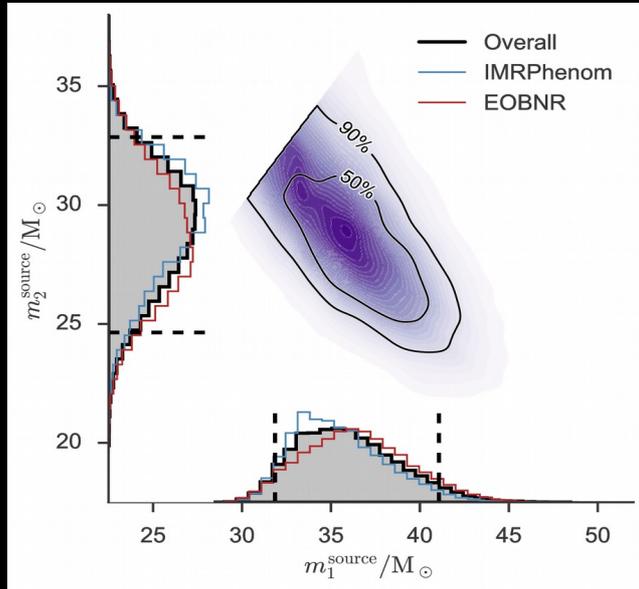
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# GW150914 (astro)physical properties



The signal comes from the coalescence of two black holes

-Masses  $M_1=36M_\odot$   $M_2=29M_\odot$   $M_f=62M_\odot$

-Distances  $D=400\text{Mpc}$ ,  $z=0.09$

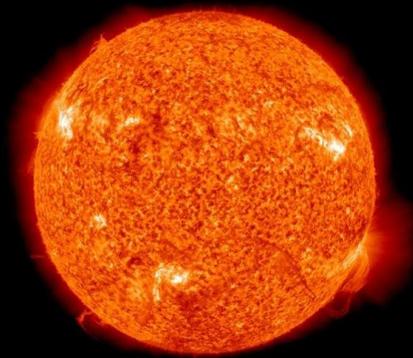
-Small spins

-Small eccentricity

$$\chi_p = \frac{c}{B_1 G m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp}) > 0$$

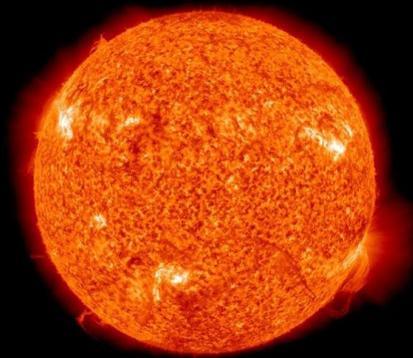
$$\chi_{\text{eff}} = \frac{c}{G} \left( \frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \frac{\hat{\mathbf{L}}}{M}$$

The system irradiated 3 solar masses worth of energy in gravitational waves with a peak luminosity of  $L \sim 3 \times 10^{56} \text{ erg/s}$



**x30**

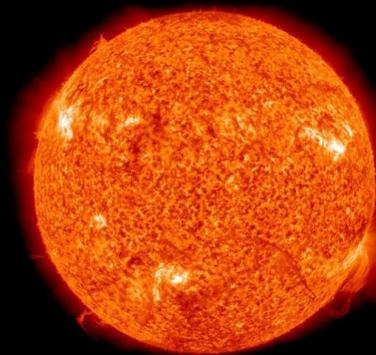
SDO/AIA 304 2010-11-17 18:05:33 UT



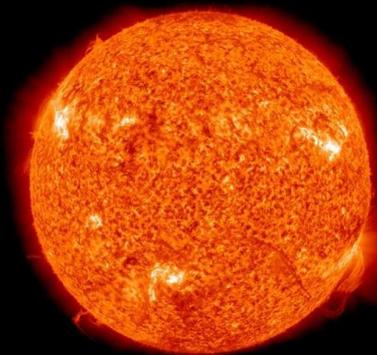
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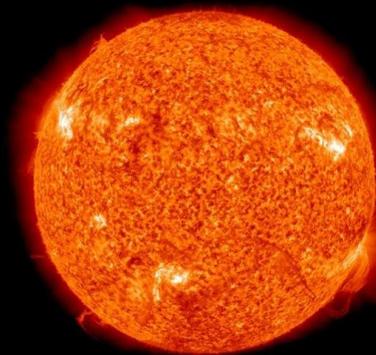


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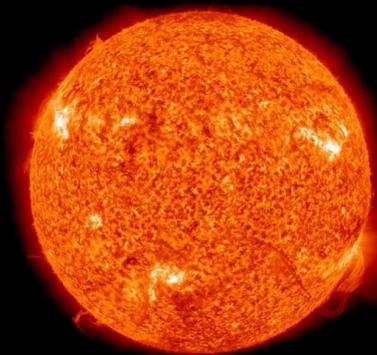
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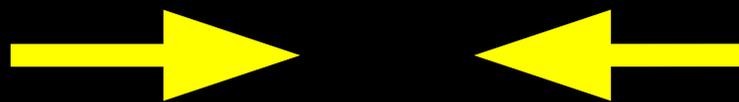
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**x30**

S00/AA 304 2010-11-17 18:05:33 UT



**$v \sim 200000 \text{ km/s}$**



# ***GW150914***

**First direct observation of gravitational waves**

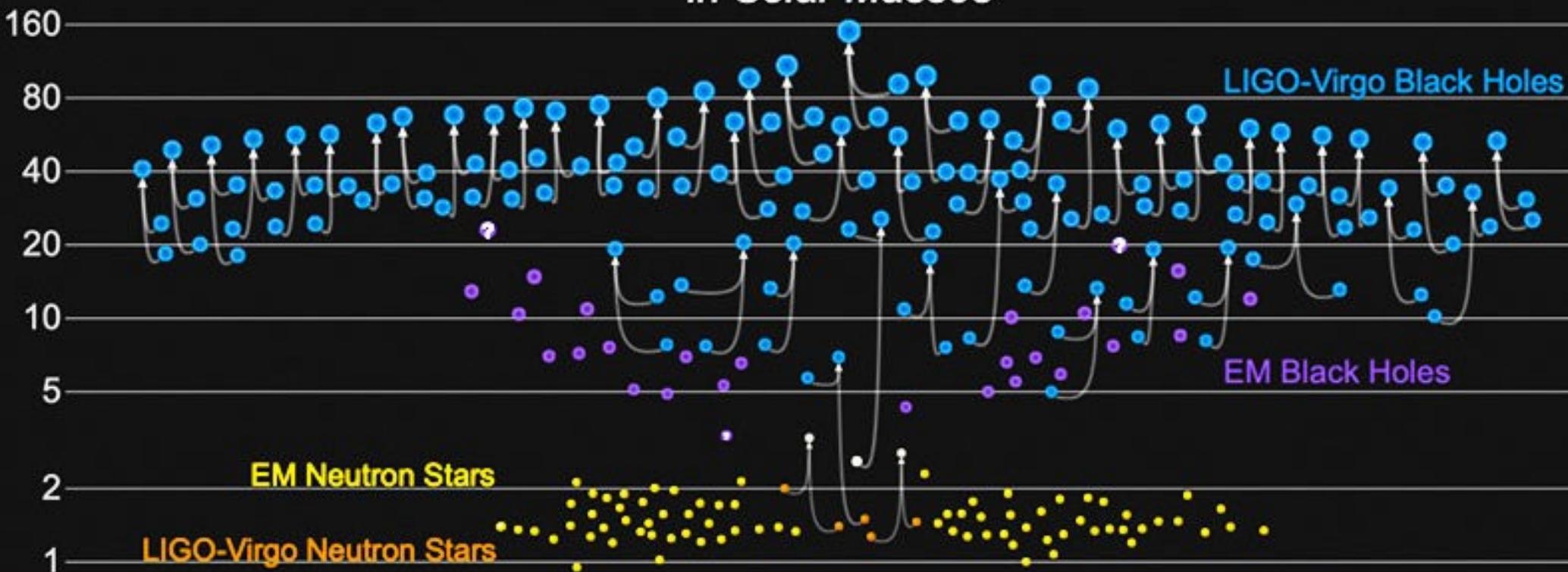
**First direct observation of black holes**

**First direct observation of a black hole binary**

**First test of General Relativity in the strong field regime**

# Masses in the Stellar Graveyard

*in Solar Masses*

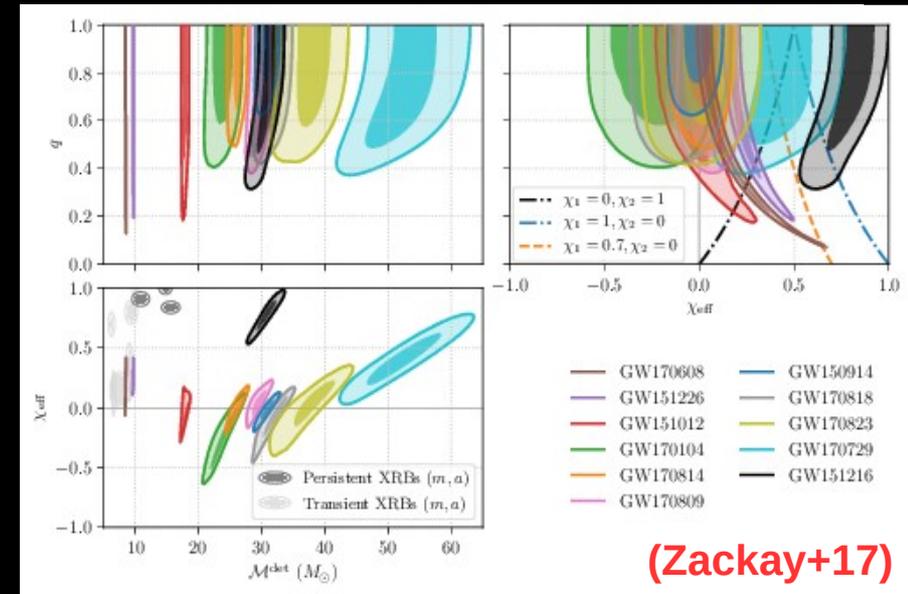
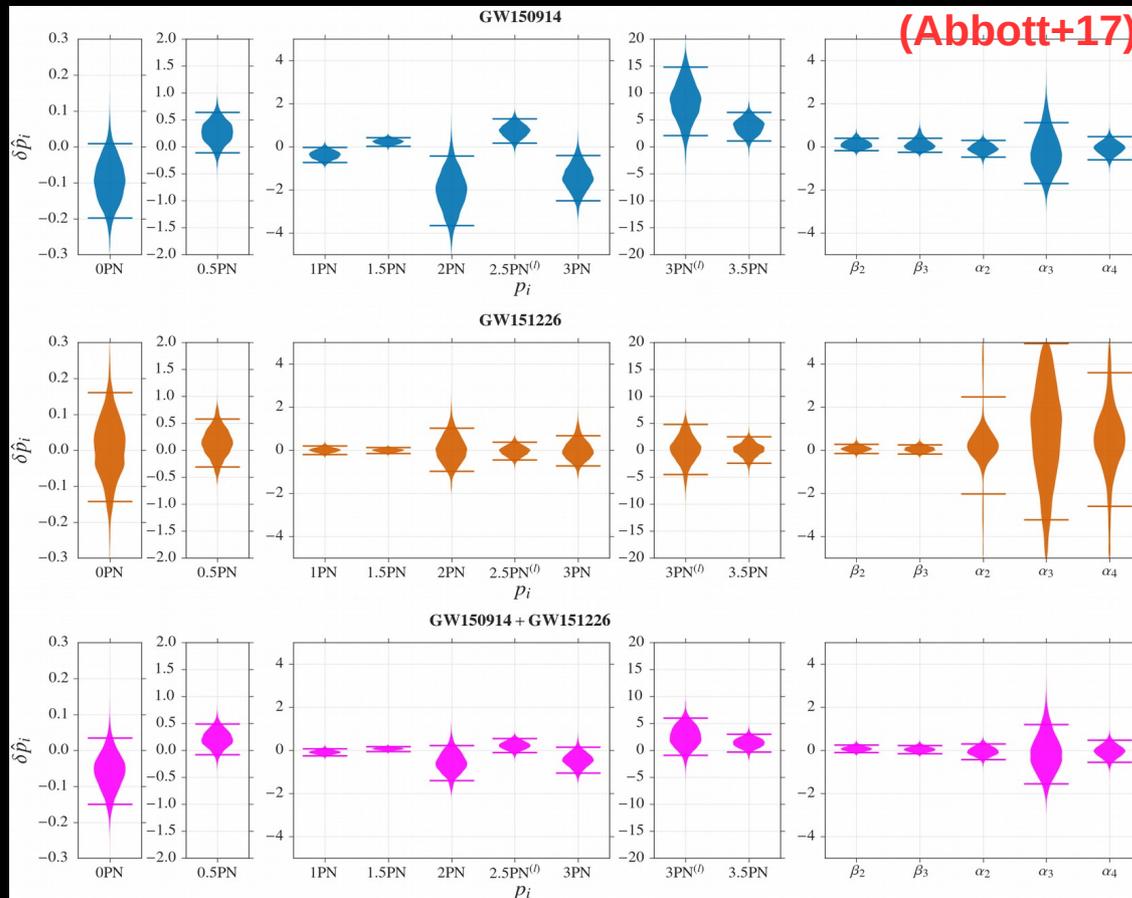
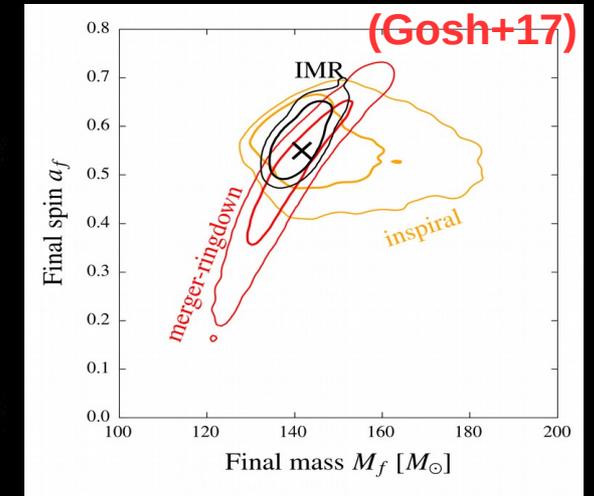
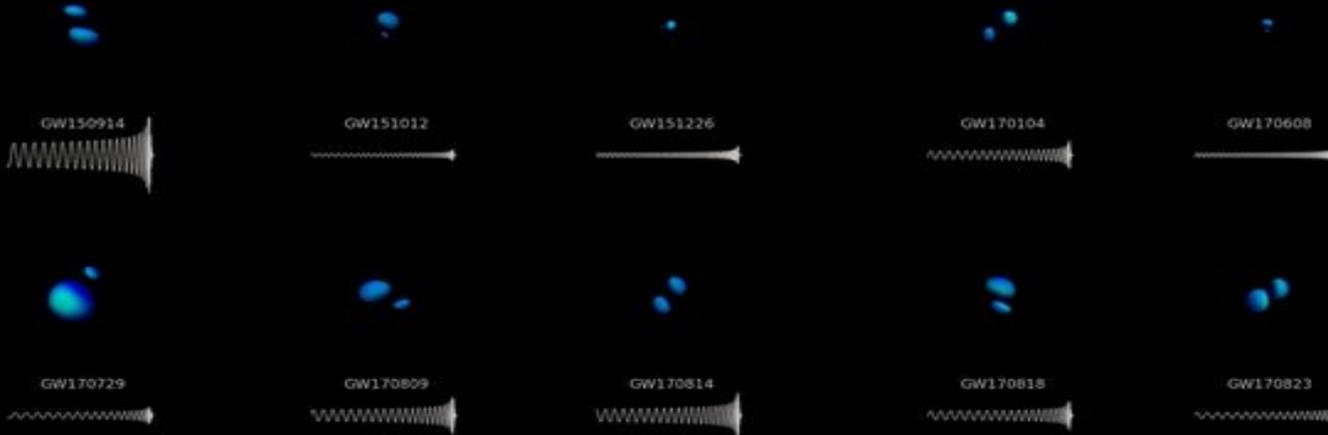


GWTC-2 plot v1.0

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

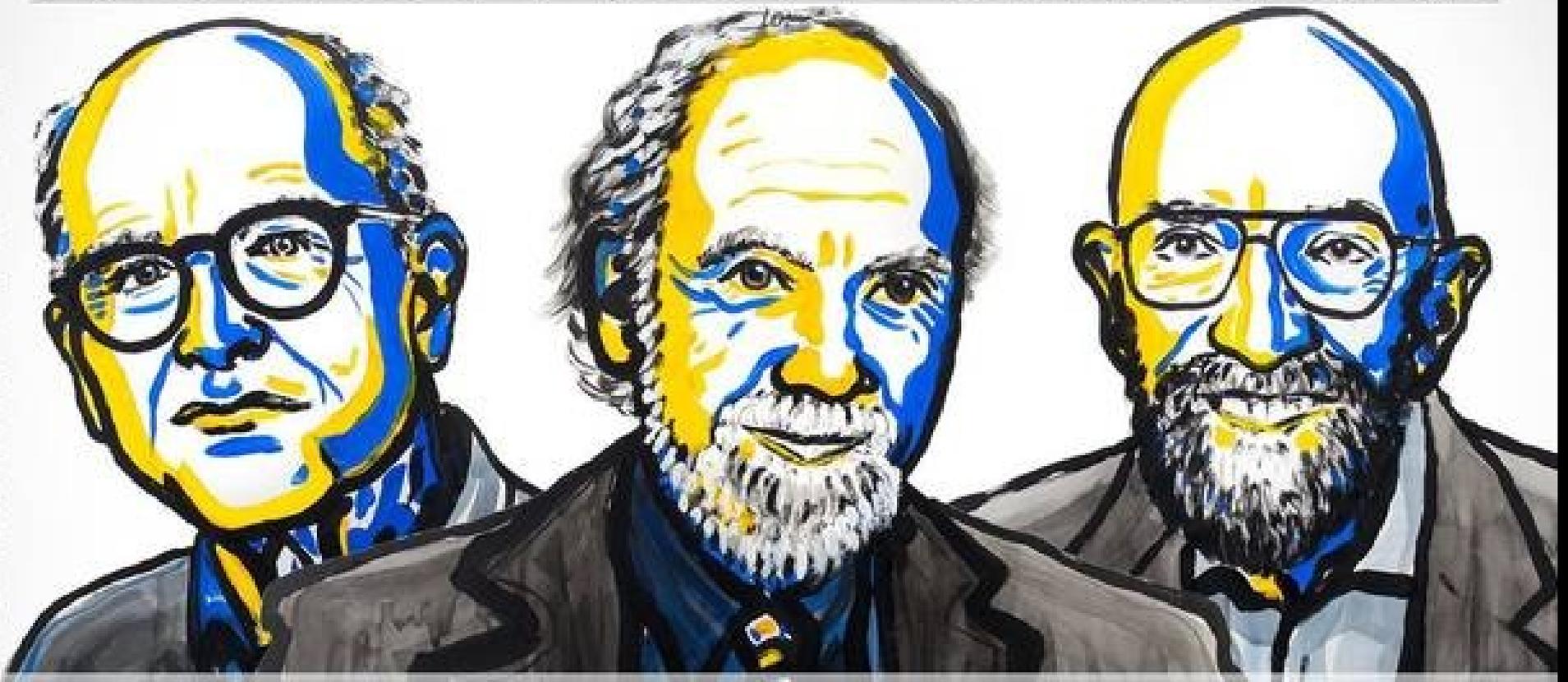
# Learning from BHBs

We've seen black hole binaries (BHBs) coalescing for the first time (Abbott+ 2016 2017...)



-First tests of GR in the strong field regime  
 -Interesting astrophysical information (masses, spins)  
 → Formation scenario?

# 2017 NOBEL PRIZE IN PHYSICS



Rainer Weiss  
Barry C. Barish  
Kip S. Thorne

*“for decisive contributions to the LIGO detector and the observation of gravitational waves”*

*August 17 2017....*

Both LIGO detectors observed a clear signal

The Gamma-ray detector Fermi, independently observed a burst 1.7 seconds after the end of the LIGO signal

Fermi



Gamma rays, 50 to 300 keV

GRB 170817A

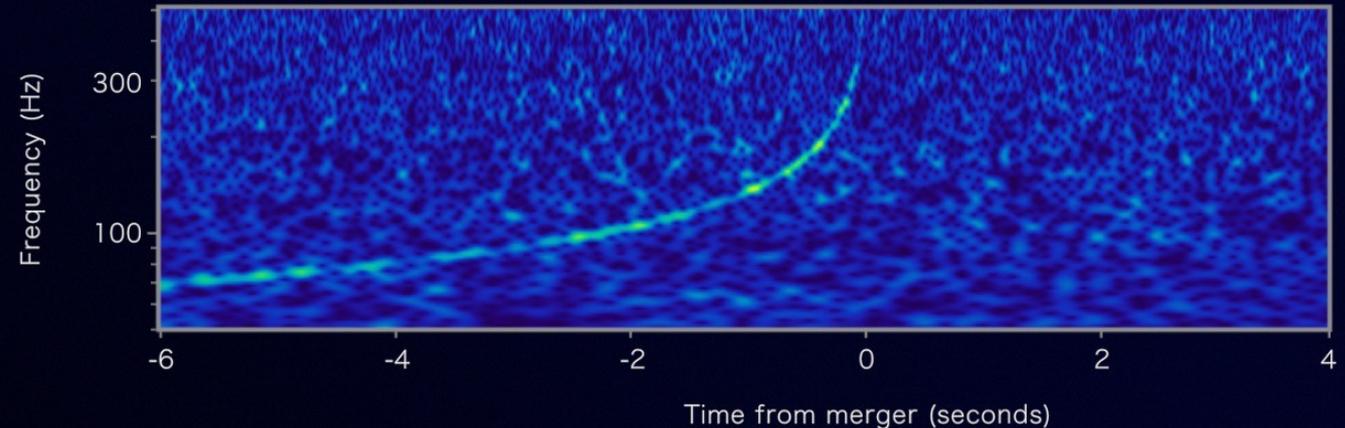


LIGO

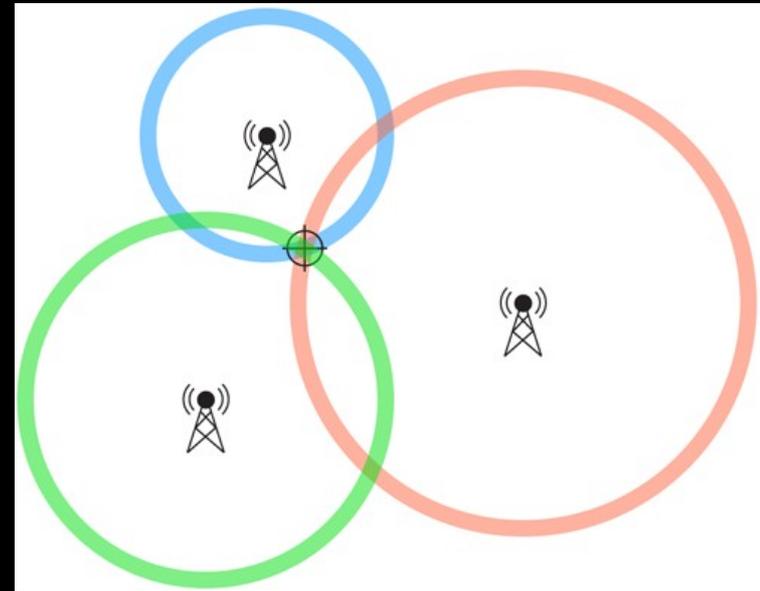
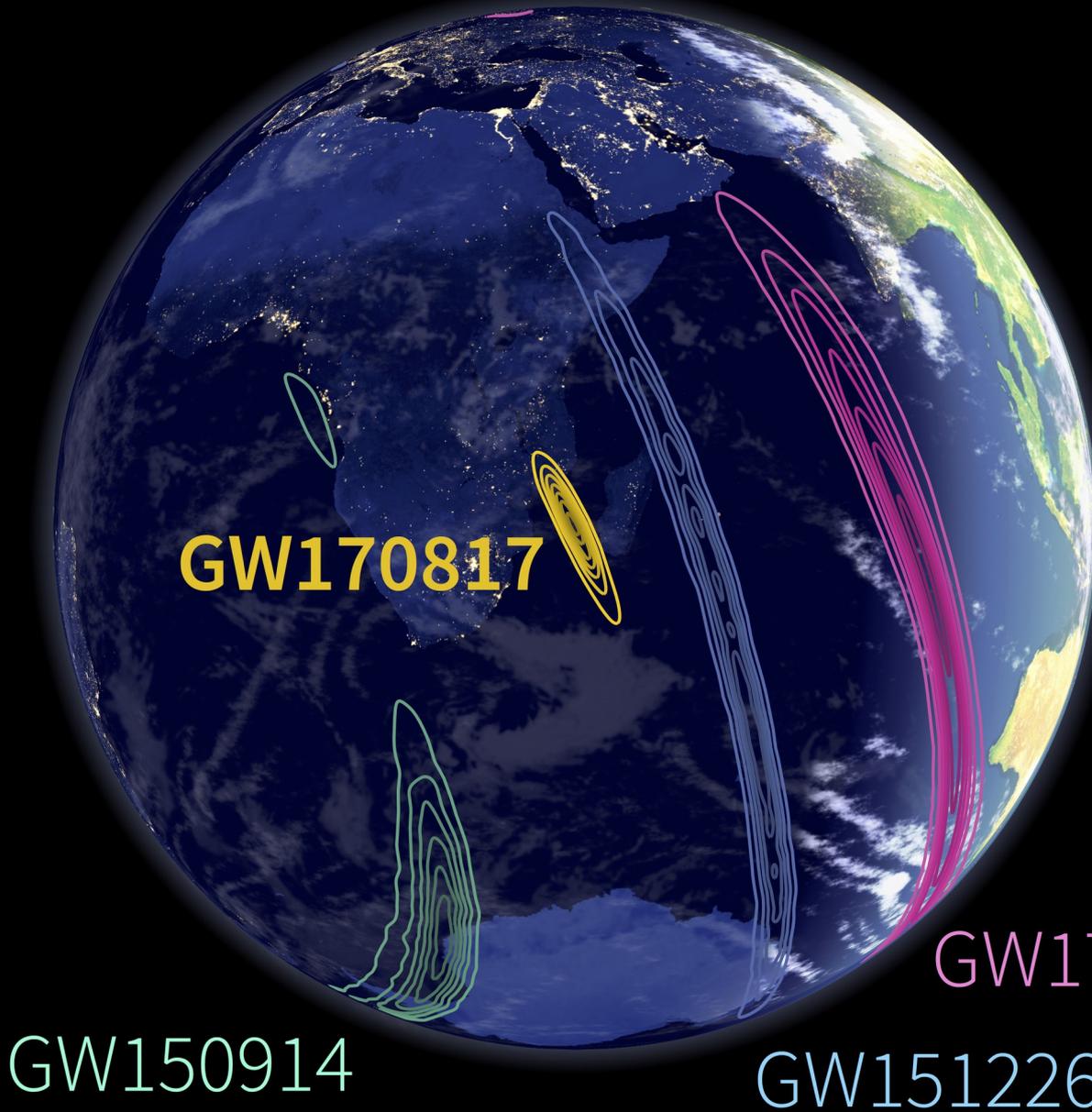


Gravitational-wave strain

GW170817



Triangulation using the LIGO-Virgo detectors allowed a decent sky localization of the signal

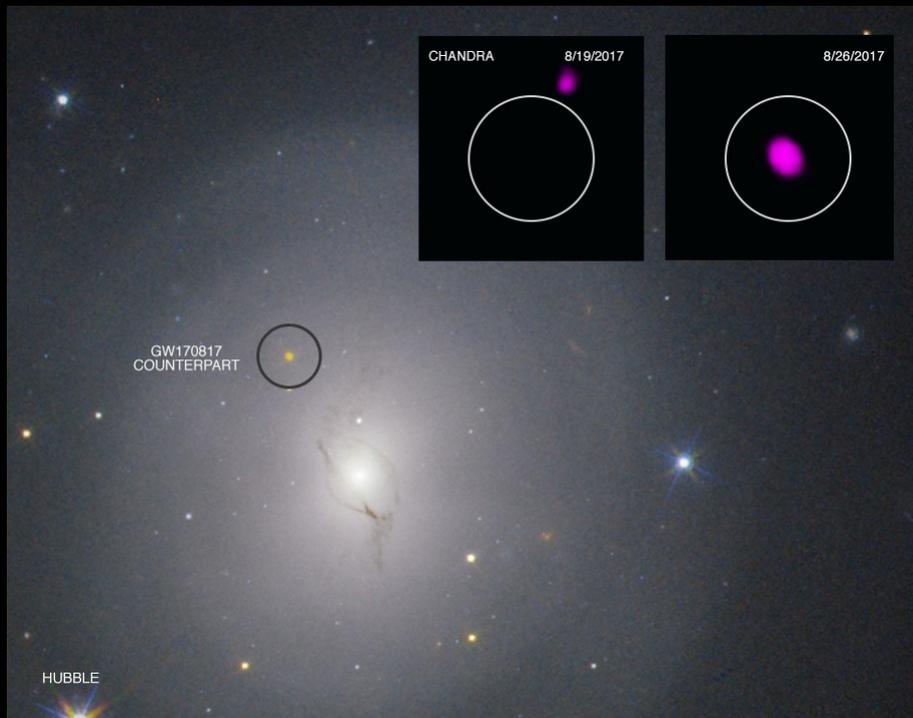
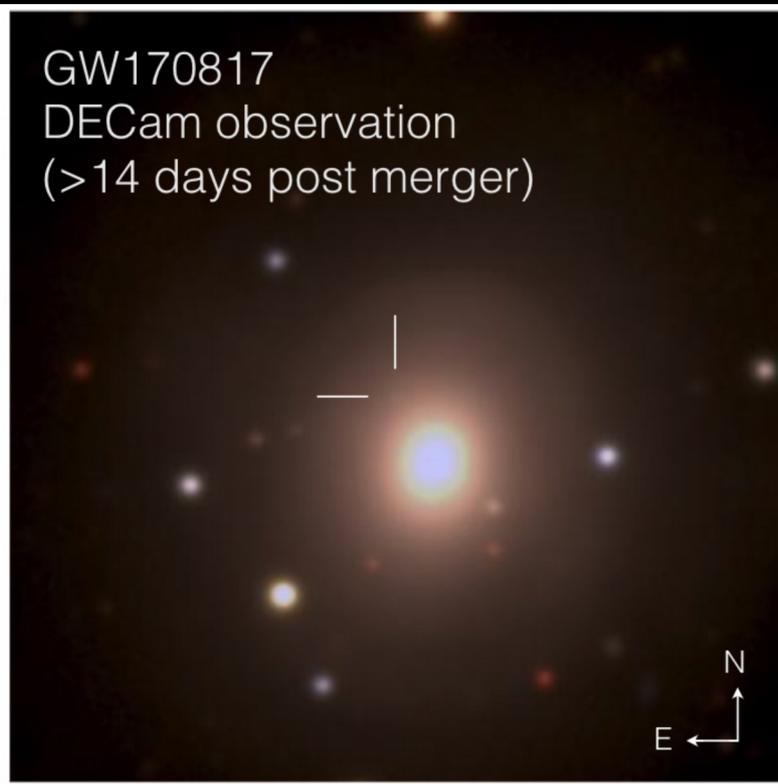
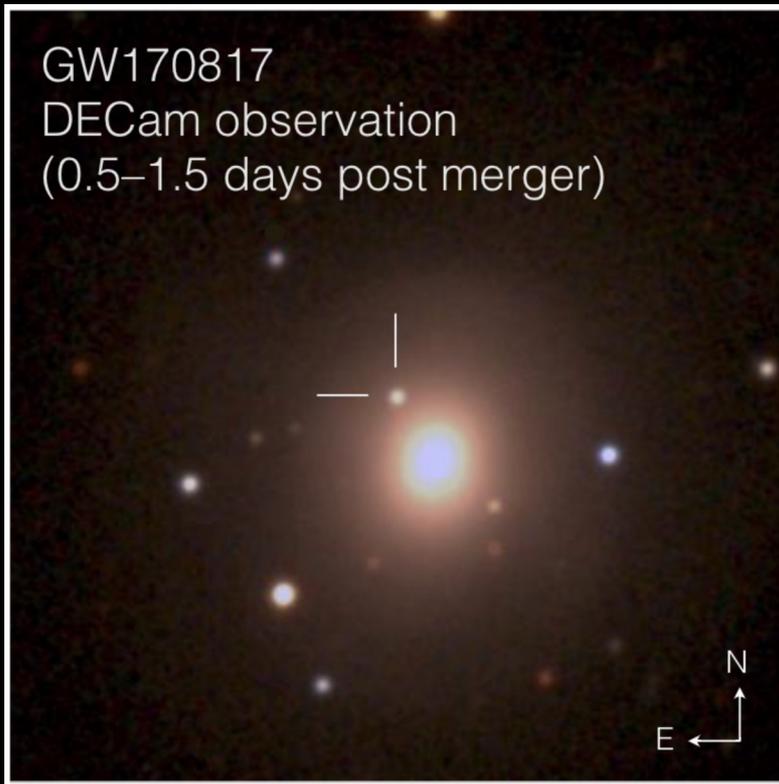


GW150914

GW151226

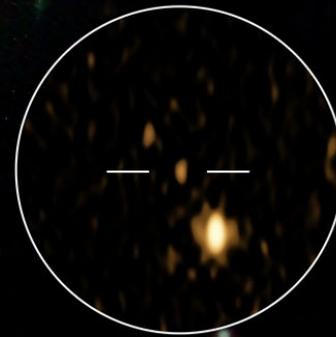
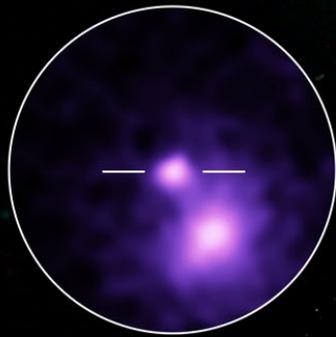
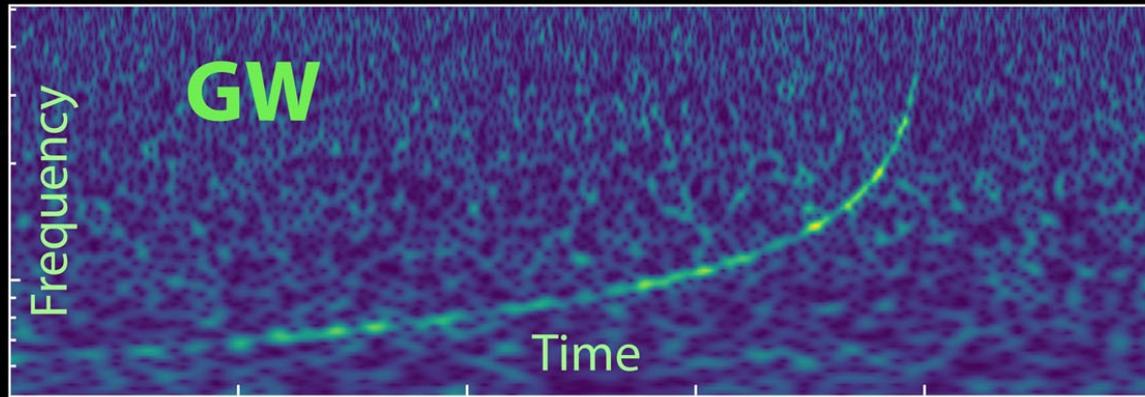
GW170104

Alerts are sent out to observatories around the world: the hunt begins!



**12 h later a counterpart was found! First in optical, then at all wavelengths.**

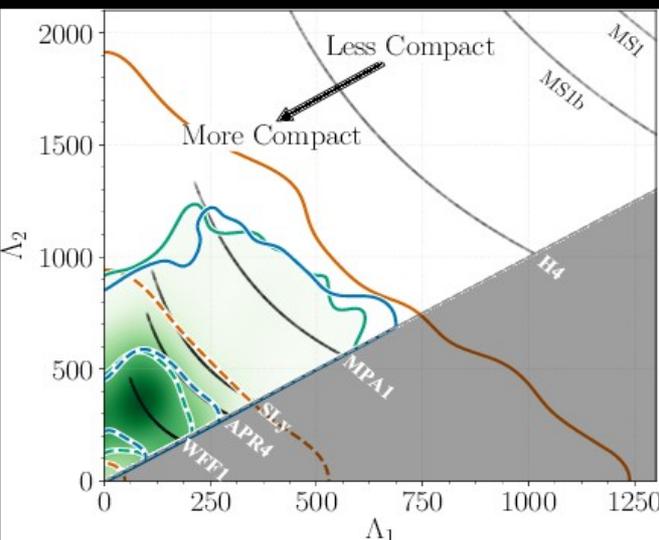
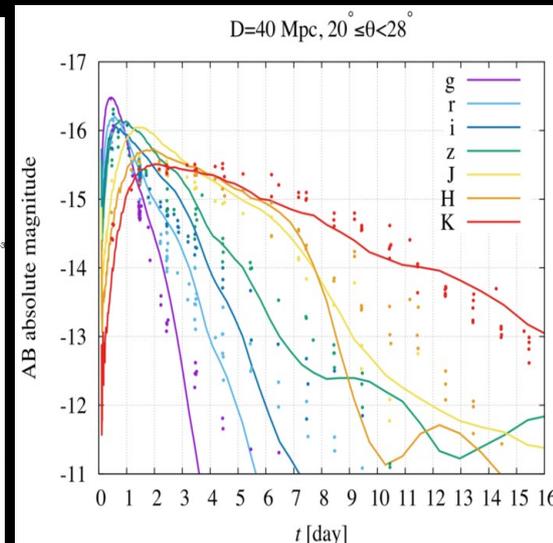
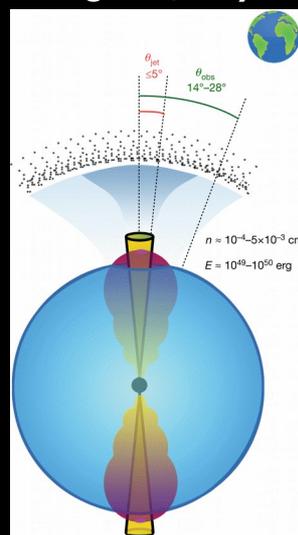
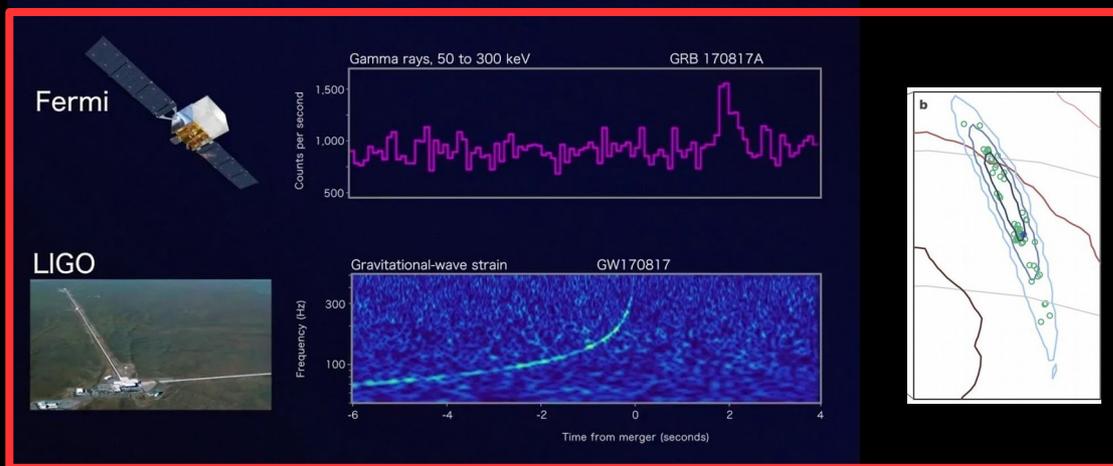
**The event occurred in the outskirts of NGC4993, an otherwise boring galaxy at 40 megaparsecs from us.**



# A unique event

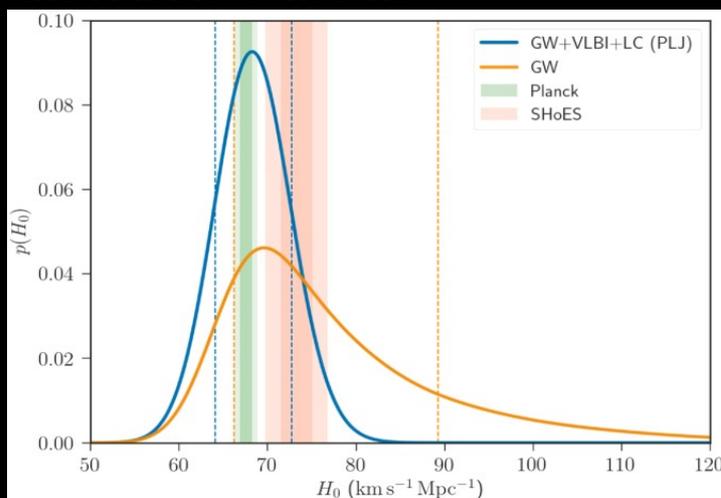
We've seen a merging neutron star (NS) binary GW170817 (Abbott+ 2017 2018)

- Confirm GRB – BNS merger connection
  - Kilonova from radioactive decay
  - Heavy element production
  - Structured jet launching and emergence
- (Kawaguchi+18, Mooley+19, Ghirlanda+19, Fong+19, ....)

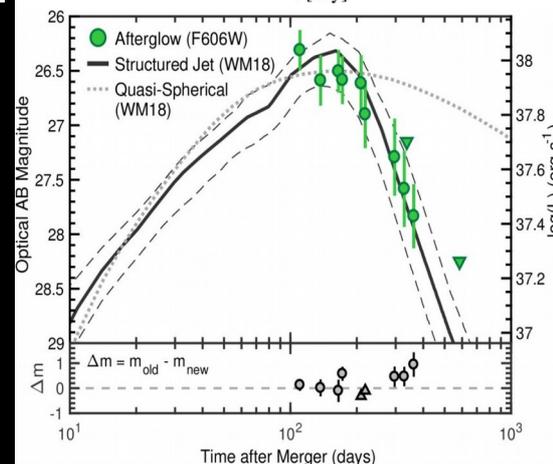


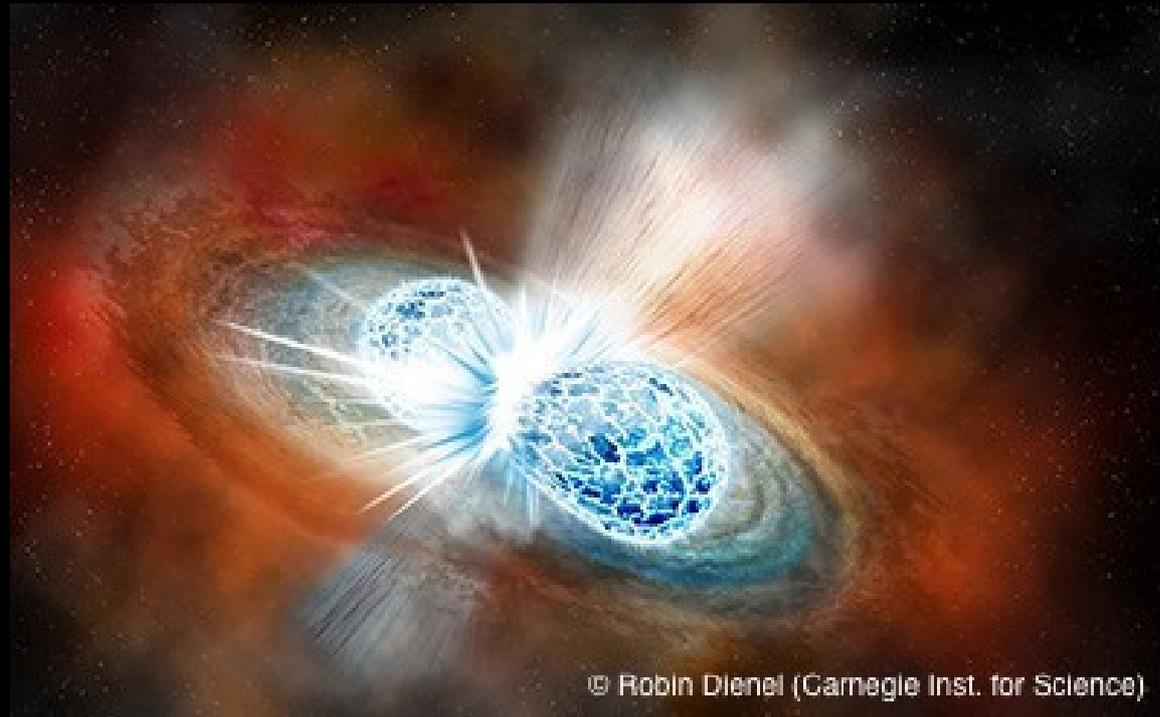
Constraints on the NS EoS from tidal deformability (Abbott+18, ...)

First GW measurement of the Hubble constant



(Abbott+18, Hotokezaka+18...)





**For the first time we observed an astrophysical event both in gravitational and electromagnetic waves**

**The event was a merger of two neutron stars**

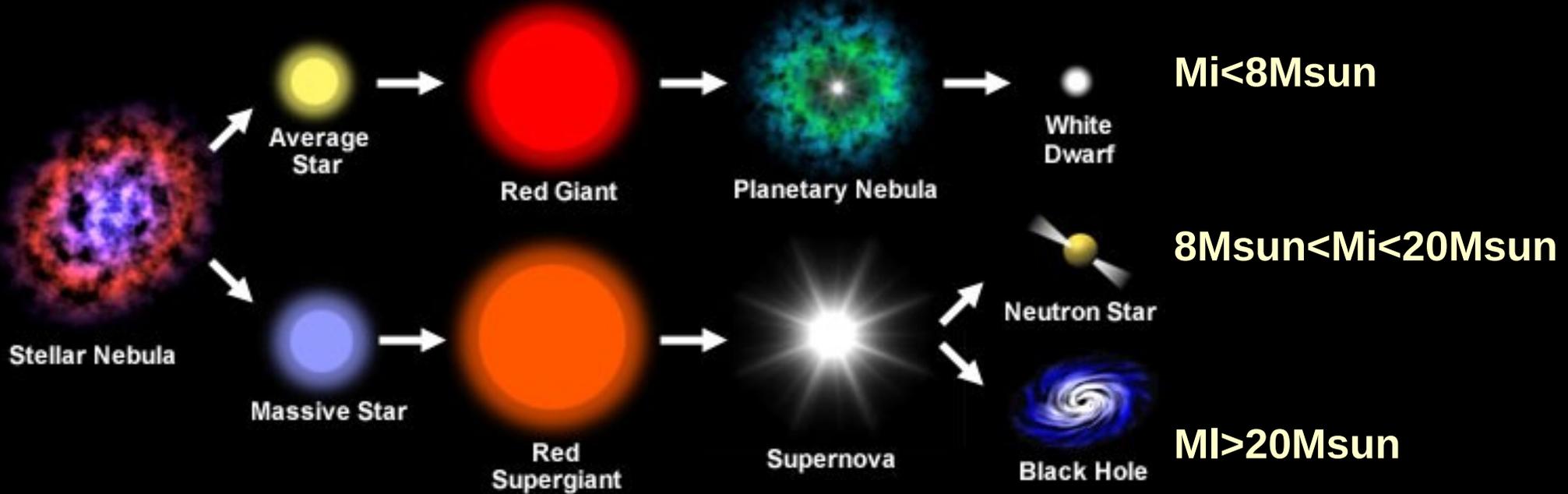
**The merger prompt the formation of relativistic jet, thus generating a short gamma ray burst**

**Neutron rich material congregates in nuclei via r-processes**

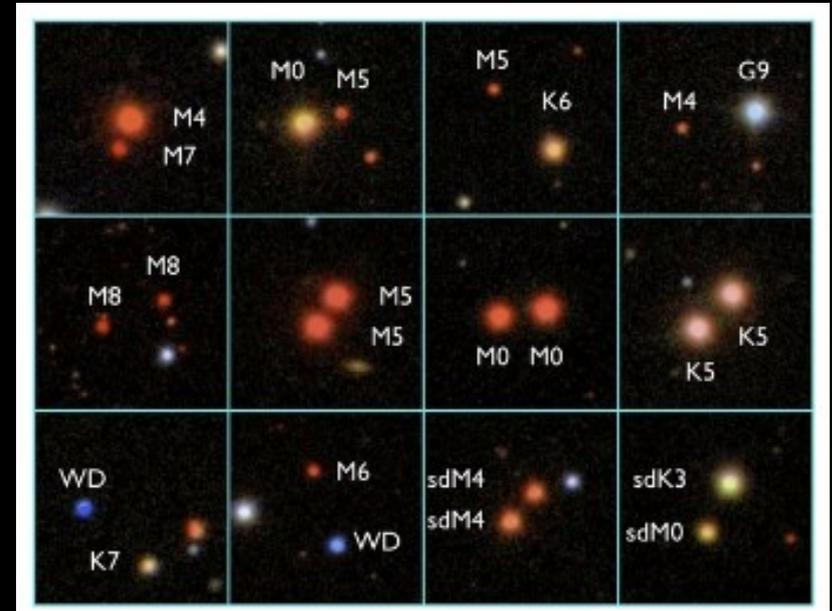
**It is estimated that ~10 Earth masses of gold were produced in the event.**

# Formation of compact binaries

## Life Cycle of a Star



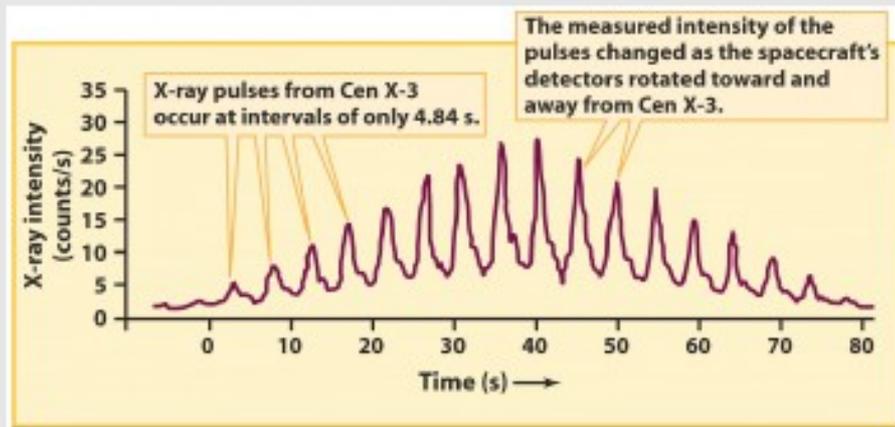
Mass (solar masses)	Time (years)	Spectral type
60	3 million	O3
30	11 million	O7
10	32 million	B4
3	370 million	A5
1.5	3 billion	F5
1	10 billion	G2 (Sun)
0.1	1000s billions	M7



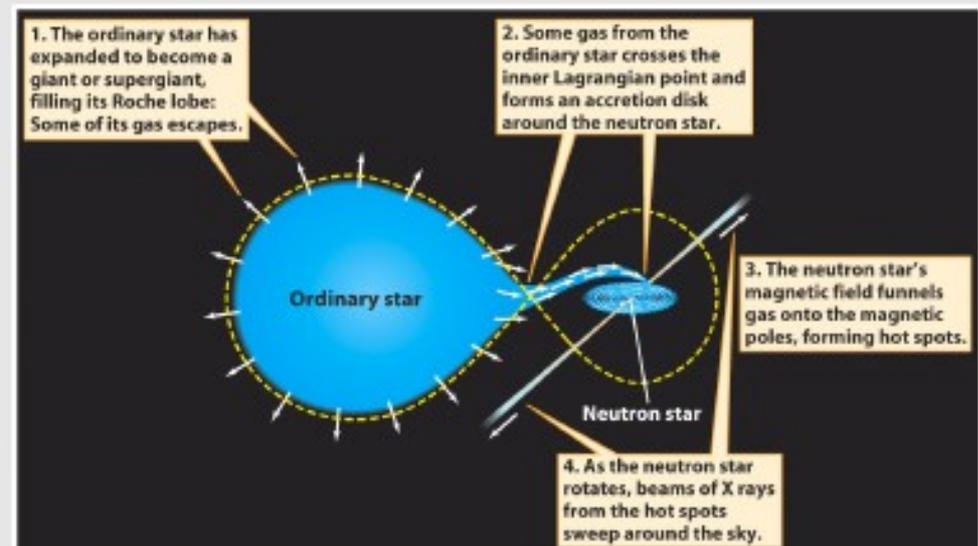
- Massive stars: burn fast and die young
- Most massive stars (70%) are in binaries (e.g. Sana et al 2012)

# X-ray binaries: some pulsating X-ray sources are neutron stars in close binaries

- Cen X-3 is an example of a bright pulsating X-ray source which also varies substantially on longer timescales.
- Magnetic forces funnel gas accreting from a companion star onto the neutron star's magnetic poles, producing hot ( $\sim 10^8$  K) spots.
- These hot spots radiate intense beams of X-rays.
- As the neutron star rotates, the X-ray beams appear to flash on and off.
- Periodic variations at the orbital period (typically days) may also be seen.

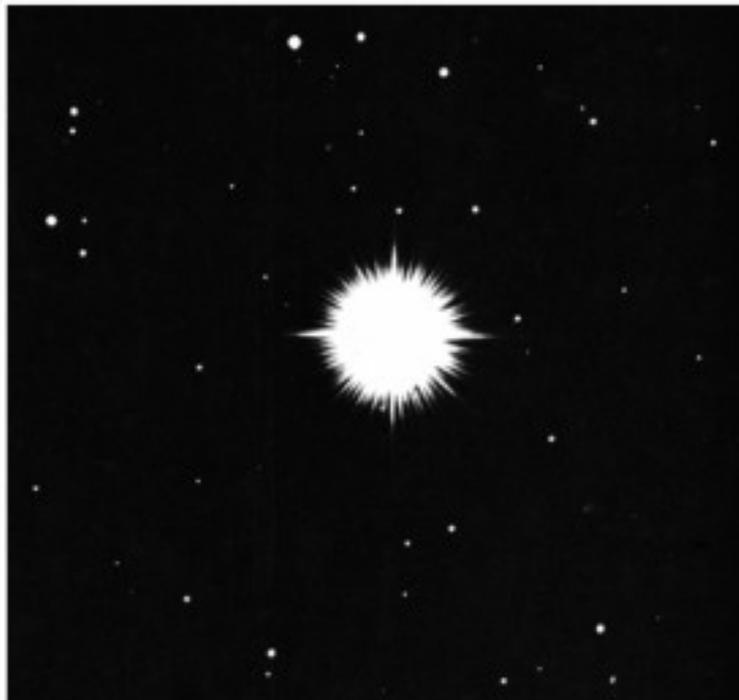


Longer term variations can arise due to the orbital motion of the binary

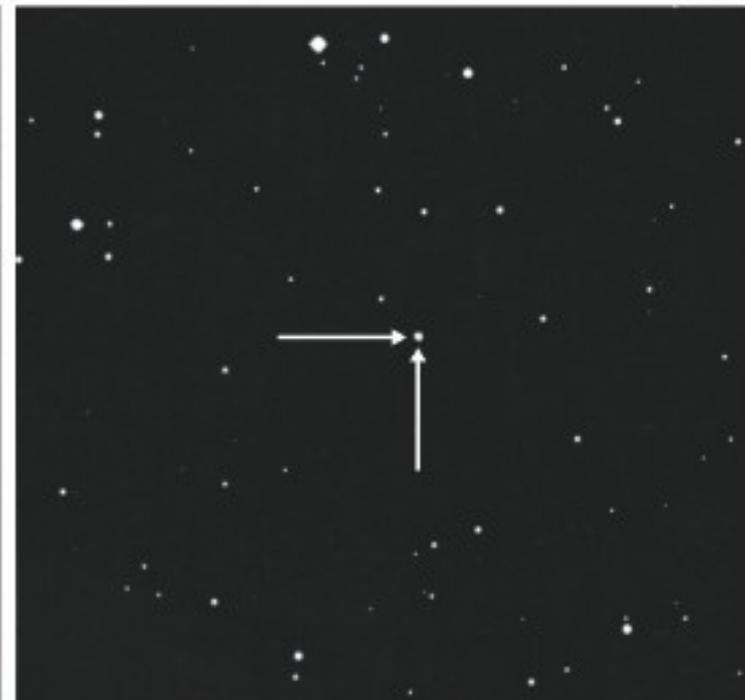


# More binaries: novae and X-ray bursters

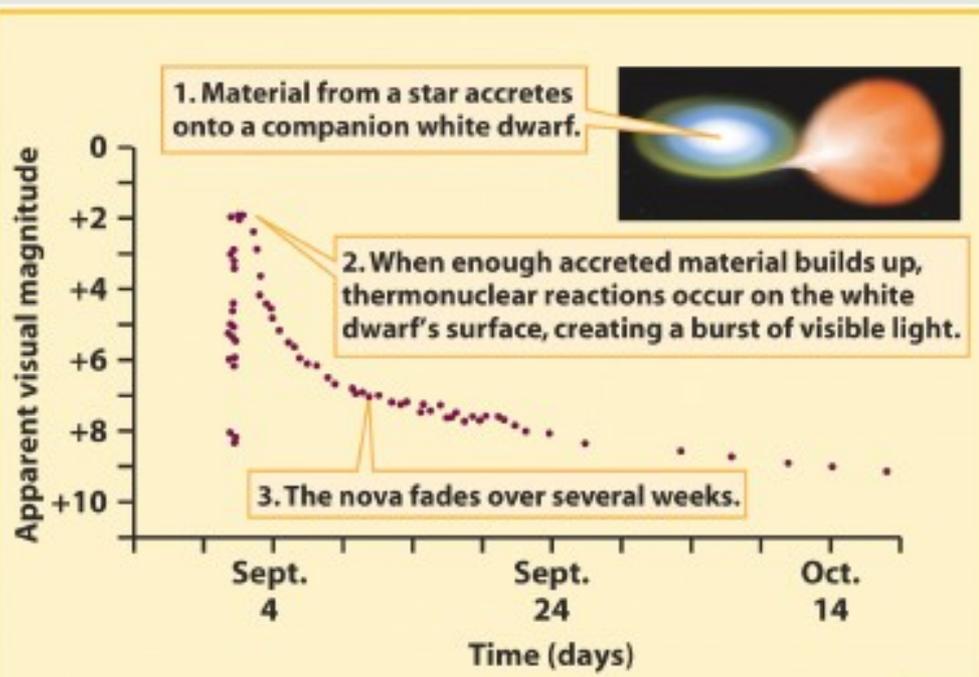
- Some stars which have high energy radiation (e.g. X-rays) also show remarkable outbursts.
- For example, novae exhibit a rise in optical brightness by a factor of 1000 or more, which then decays away on a timescale of weeks.
- The peak optical luminosity is  $\sim 10^{-4}$  of that observed in a supernova.
- Detailed studies have shown that a nova results from a thermonuclear explosion on the surface of a white dwarf, which is accreting material from a companion star in a close binary.



(a) Nova Herculis 1934 shortly after peak brightness

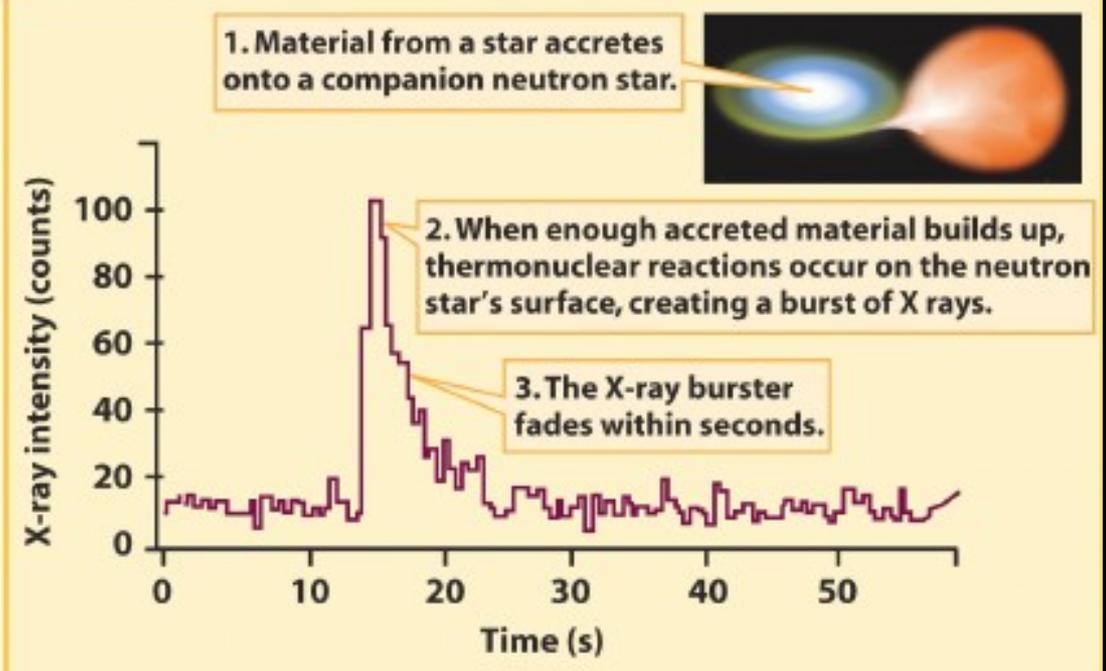


(b) Two months later



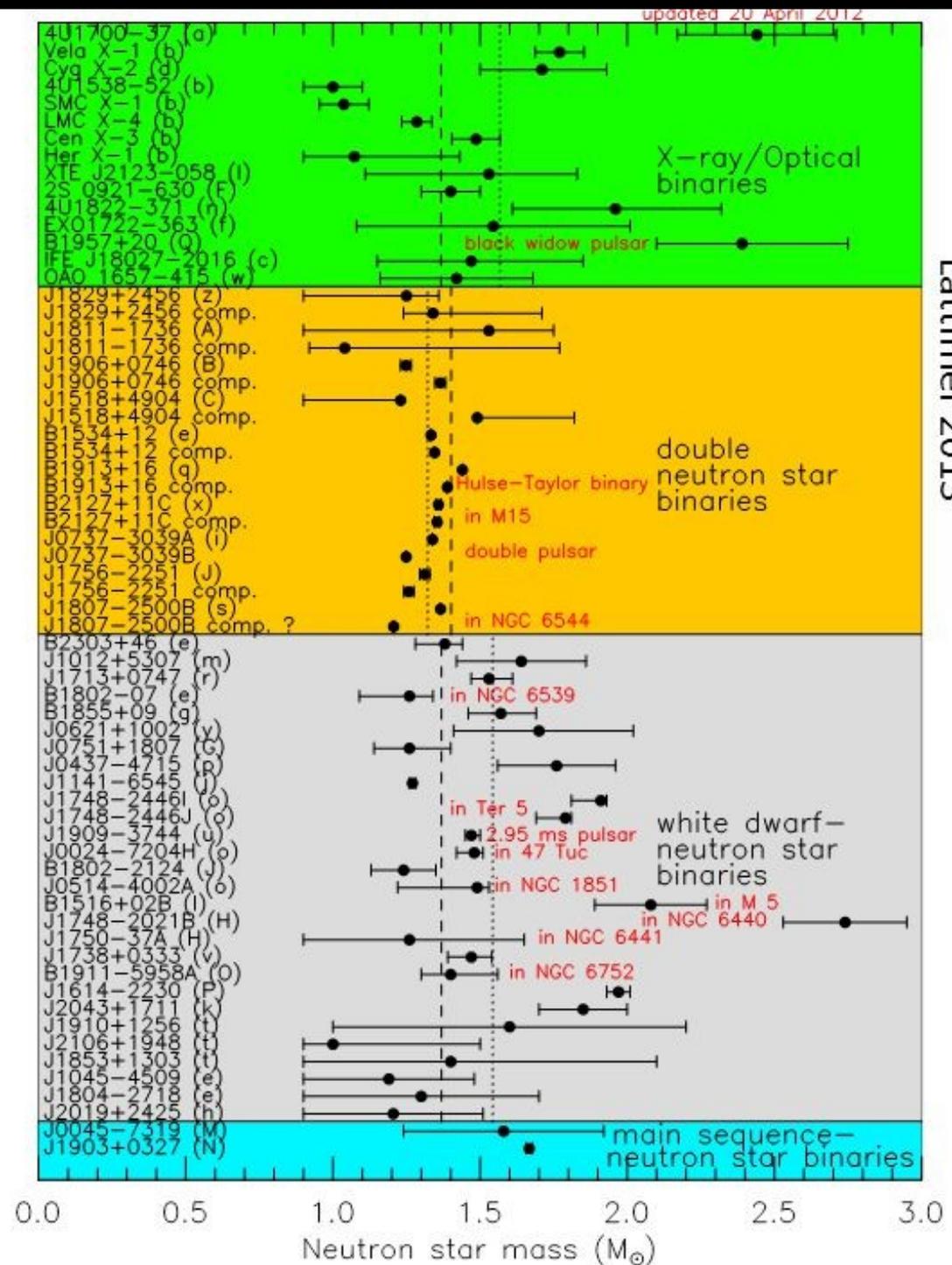
- Some stars which have high energy radiation (e.g. X-rays) also show remarkable outbursts.
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- A similar phenomenon, happening at higher energies and on shorter timescales, is seen in the X-ray burst sources.
- These emit flashes of bright X-ray emission lasting less than 1 minute.
- This is believed to occur due to a thermonuclear flash on the surface of an accreting neutron star.



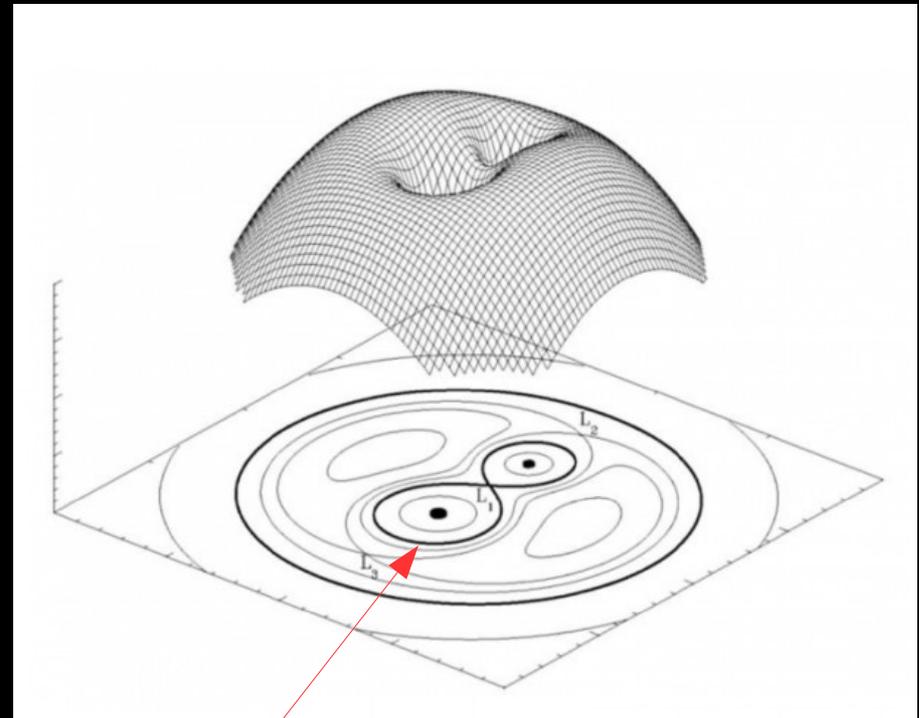
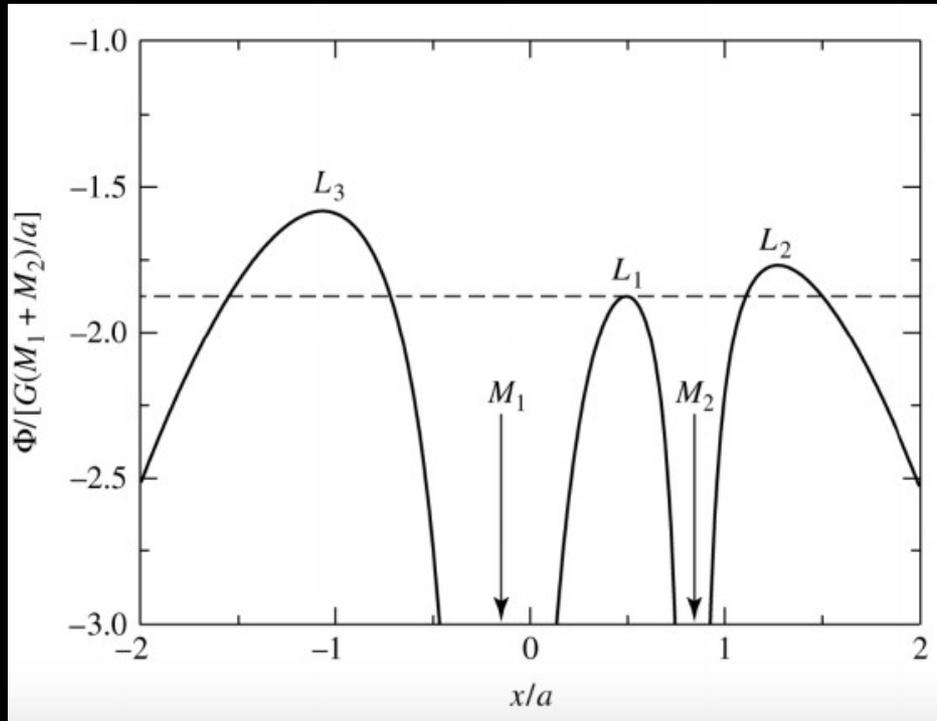
# Neutron Star Masses

- Observations of neutron star binaries provide a growing list of neutron star mass estimates.
- Current observations predict a range of NS masses from 1.0 to  $>2$  solar masses.
- Can we explain these masses?



# Evolution of binaries: effective potential

$$U = -\frac{Gm(M_1 + M_2)}{r_{\text{CM}}} - \frac{1}{2}m\omega^2 r_{\text{CM}}^2 = -Gm \left( \frac{M_1}{s_1} + \frac{M_2}{s_2} \right) - \frac{1}{2}m\omega^2 r_{\text{CM}}^2$$



The region over which each component of the binary exerts its influence is called **Roche Lobe**.

The RL are connected by the unstable Lagrangian point **L1**.

$$\frac{R_1}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

# Mass transfer

Assuming a circular binary, the angular momentum can be written as

$$J = \mu\sqrt{GMa}$$

Suppose that the secondary loses mass that is partly accreted by the primary, passing through L1

$$\dot{M}_2 < 0$$

$$\dot{M}_1 = -(1 - \beta)\dot{M}_2$$

$$\beta \in [0, 1]$$

By differentiating the J equation one can easily show that

$$\begin{aligned}\frac{\dot{a}}{a} &= 2\frac{\dot{J}}{J} - 2\frac{\dot{\mu}}{\mu} - \frac{\beta\dot{M}_2}{M} \\ &= 2\frac{\dot{J}}{J} - 2\left[(\beta - 1)\frac{M_2}{M_1} + 1\right]\frac{\dot{M}_2}{M_2}\end{aligned}$$

If the mass transfer is conservative then

$$\frac{\dot{a}}{a} = -2\left(1 - \frac{M_2}{M_1}\right)\frac{\dot{M}_2}{M_2}$$

So the binary shrinks if  $M_2 > M_1$ . This is important because the most massive star in the binary it is the first to evolve

# Supernova kick

The binary can be described as the motion of  $\mu$  in the potential of  $M$

$$v_i = \sqrt{\frac{G(M_2 + M_1)}{a_i}}$$

M1 goes boom (SN)  $M_c < M_1$ , and  $\Delta M = M_1 - M_c$

The explosion generates a kick

$$\mu_f = M_c M_2 / (M_c + M_2) \quad \vec{v}_f = \vec{v}_i + \vec{v}_{\text{kick}}$$

$$E_f = \frac{1}{2} \mu_f v_f^2 - \frac{G M_c M_2}{a_i}$$

If the final velocity is larger than the escape velocity (calculated from  $E_f$ ) then the binary gets unbound

$$v_f \leq v_e = \sqrt{\frac{2G(M_2 + M_c)}{a_i}}$$

If  $v_{\text{kick}}$  is null the binary stays bound unbound if

$$\Delta M \leq \frac{M_1 + M_2}{2}$$

The kick velocity depends on the nature of the object

**Neutron stars:** bounce of the imploding layers onto the hard NS core can produce large kick if the overall process is asymmetric

The NS kick distribution is estimated to follow

$$f(v_{\text{kick}}) = \sqrt{\frac{1}{\pi}} \frac{v_{\text{kick}}^2}{\sigma_v^3} e^{-\frac{v_{\text{kick}}^2}{2\sigma_v^2}}$$

with  $\sigma \sim 190$  km/s (estimated from the distribution of peculiar velocities of PSR wrt their local ref frame).

A binary with masses of  $\sim 10 M_{\text{sun}}$  @ 5AU separation has an escape speed of  $< 100$  km/s.

So the first SN kick is expected to destroy many NS binaries (but obviously a fraction does survive since we do see NS binaries (e.g. Hulse & Taylor binary, Double Pulsar and many more...)

**Black holes:** estimates are way more uncertain:

-theoretically since the horizon forms, we do not expect any bounce, so the kick velocity should be small

-observationally we cannot estimate  $v_{\text{kick}}$  from lone BHs in the MW

# Common envelope evolution

Compact object (CO) inspiralling into an envelope of a giant star. The inspiral of the CO 'heats up' the envelope eventually unbinding it:

$$\Delta E_{\text{env}} = -E_{\text{env},i} = \frac{GM_{\text{gs}}M_{\text{env}}}{\lambda R_L}$$

By equating this to the change in energy of the compact object, one finds a relation between the initial and final separation of the binary:

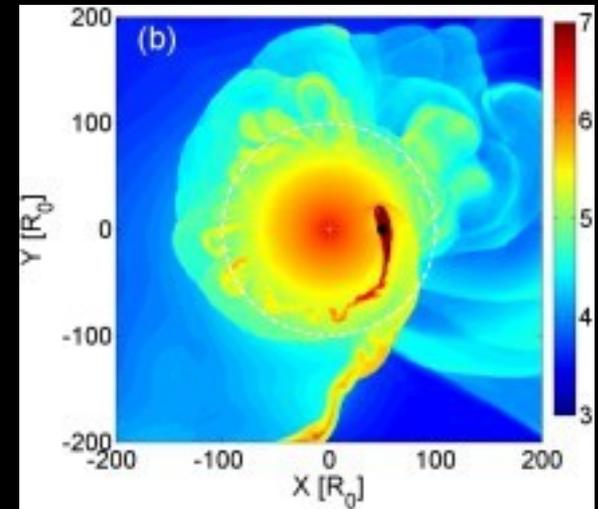
$$\frac{a_f}{a_i} = \frac{M_{\text{core}}}{M_{\text{gs}}} \left( 1 + \frac{2}{\lambda \alpha_{\text{ce}}} \frac{a_i}{R_L} \frac{M_{\text{env}}}{M_{\text{BH}}} \right)^{-1} = \frac{M_{\text{core}}}{M_{\text{gs}}} \left( \frac{M_{\text{BH}}}{M_{\text{BH}} + \frac{2M_{\text{env}}}{\lambda \alpha_{\text{ce}}} \frac{a_i}{R_L}} \right)$$

For typical massive star values:

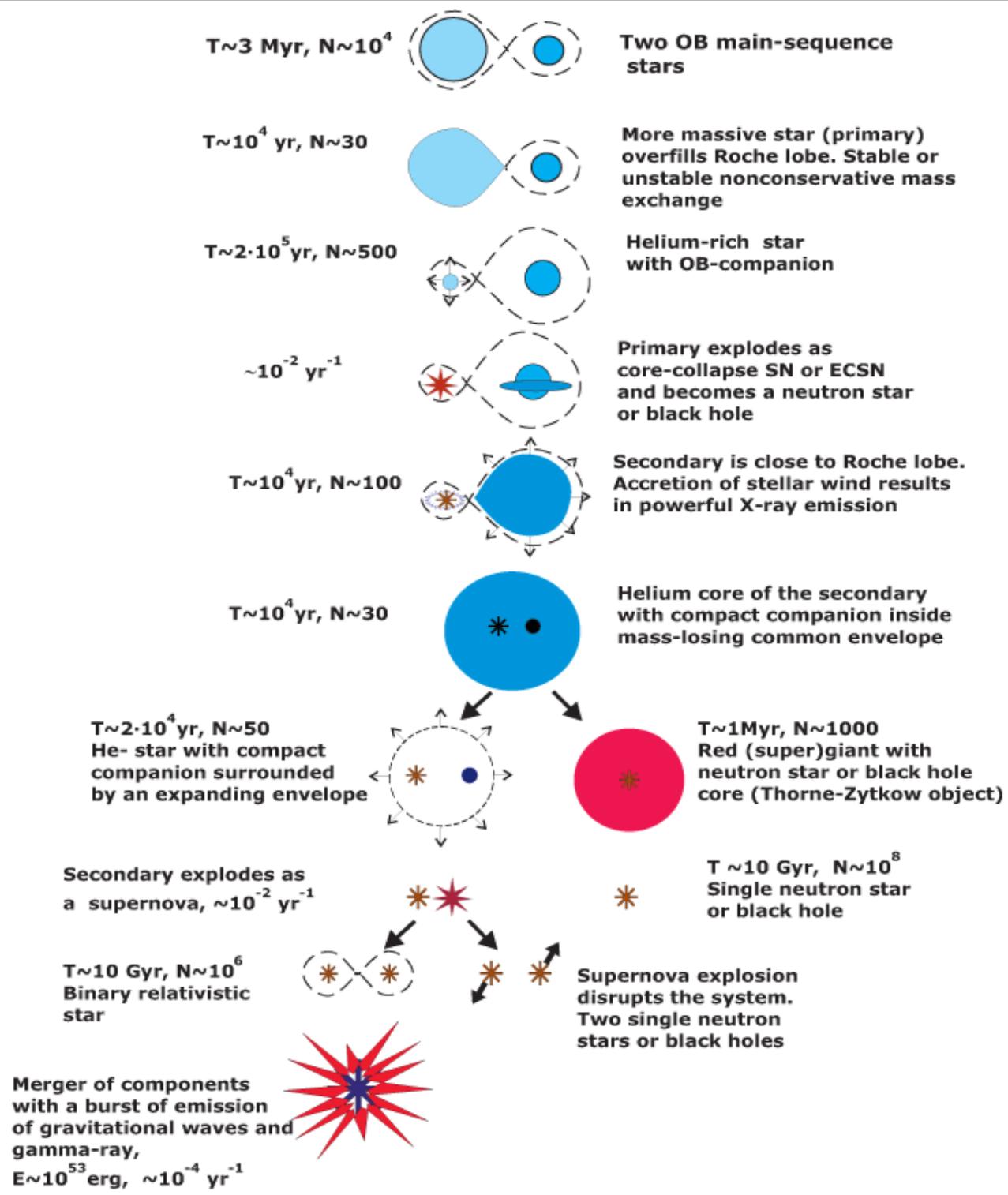
$$\frac{M_{\text{core}}}{M_{\text{gs}}} \approx 0.2 - 0.3, \quad \frac{M_{\text{BH}}}{M_{\text{BH}} + \frac{2M_{\text{env}}}{\lambda \alpha_{\text{ce}}} \frac{a_i}{R_L}} \approx 0.05 - 0.01$$

One gets  $a_f/a_i \sim 0.01-0.001$ .

So, in principle the system can shrink from  $\sim \text{AU}$  to  $\sim \text{solar radii}$ .



# Evolution channels of massive binaries



# Evolution channels of massive binaries

## 'Standard' Channel

Initial binary:  $M_1 = 14 M_\odot$ ,  
 $M_2 = 9 M_\odot$ ,  $P_{\text{orb}} = 190 \text{ d}$

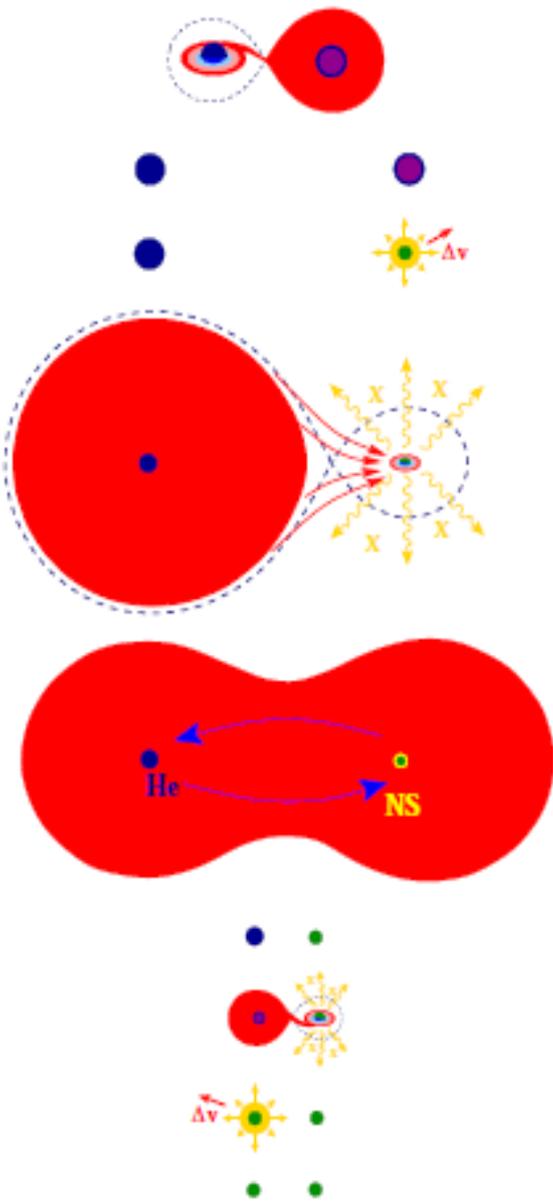
Stable non-conservative Case  
B mass transfer leaving a  
helium star with  $M_{\text{He}}^A = 4 M_\odot$   
and  $M_2' = 11 M_\odot$ ,  $P_{\text{orb}} = 350 \text{ d}$

After first supernova (with  
kick  $v_{\text{kick}} = 50 \text{ km s}^{-1}$ ):  
 $M_A' = 1.337 M_\odot$ ,  $M_2' = 11 M_\odot$ ,  
 $P_{\text{orb}} = 8.8 \text{ yr}$ ,  $e = 0.82$ ,  
 $\Delta v_{\text{qs}}^A = 13 \text{ km s}^{-1}$

High-mass X-ray binary phase  
leading to unstable mass  
transfer and a  
common-envelope and  
spiral-in phase and leaving  
 $M_A'' = 1.337 M_\odot$ ,  
 $M_{\text{He}}^B = 2.4 M_\odot$ ,  $P_{\text{orb}} = 2.8 \text{ hr}$

Helium star mass transfer  
phase (+ spin-up of neutron  
star) leaving  $M_A = 1.338 M_\odot$ ,  
 $M_{\text{He}} = 1.559 M_\odot$ ,  $P_{\text{orb}} = 2.6 \text{ hr}$

Immediately after second  
supernova:  $M_A = 1.338 M_\odot$ ,  
 $M_{\text{He}} = 1.249 M_\odot$ ,  $P_{\text{orb}} = 3.3 \text{ hr}$ ,  
 $e = 0.12$ ,  $\Delta v_{\text{qs}}^B = 35 \text{ km s}^{-1}$



## Double-Core Channel

Initial binary:  $M_1 = 11.5 M_\odot$ ,  
 $M_2 = 11 M_\odot$ ,  $P_{\text{orb}} = 3.1 \text{ yr}$

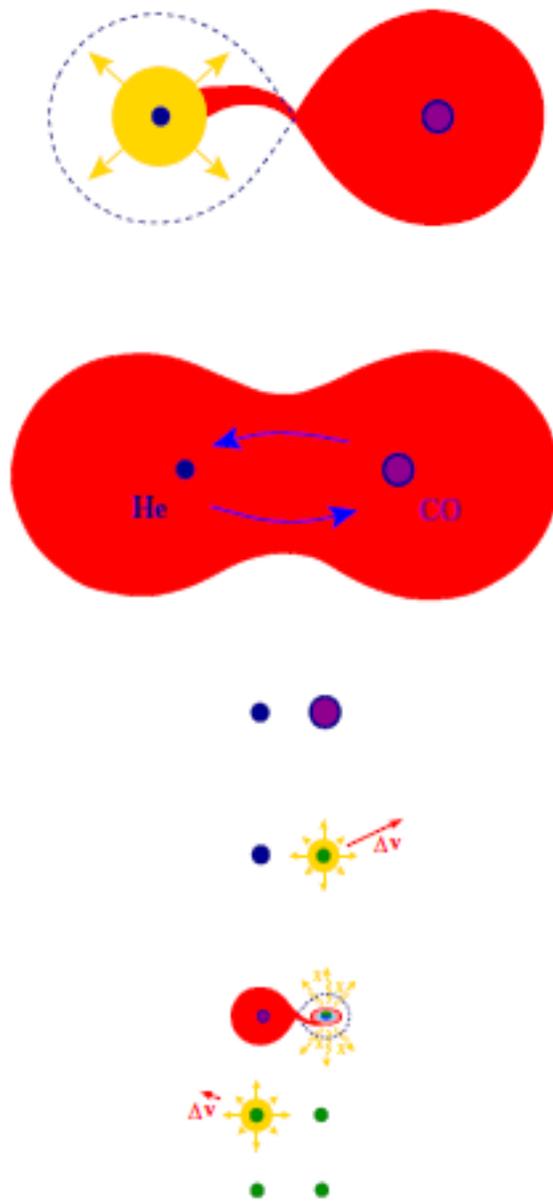
Unstable Case C mass  
transfer: secondary expands  
to fill its Roche lobe

Double-core common-envelope  
and spiral-in phase leaving a  
CO star with  $M_{\text{CO}} = 3.0 M_\odot$   
and a He star with  
 $M_{\text{He}} = 2.4 M_\odot$ ,  $P_{\text{orb}} = 3.8 \text{ hr}$

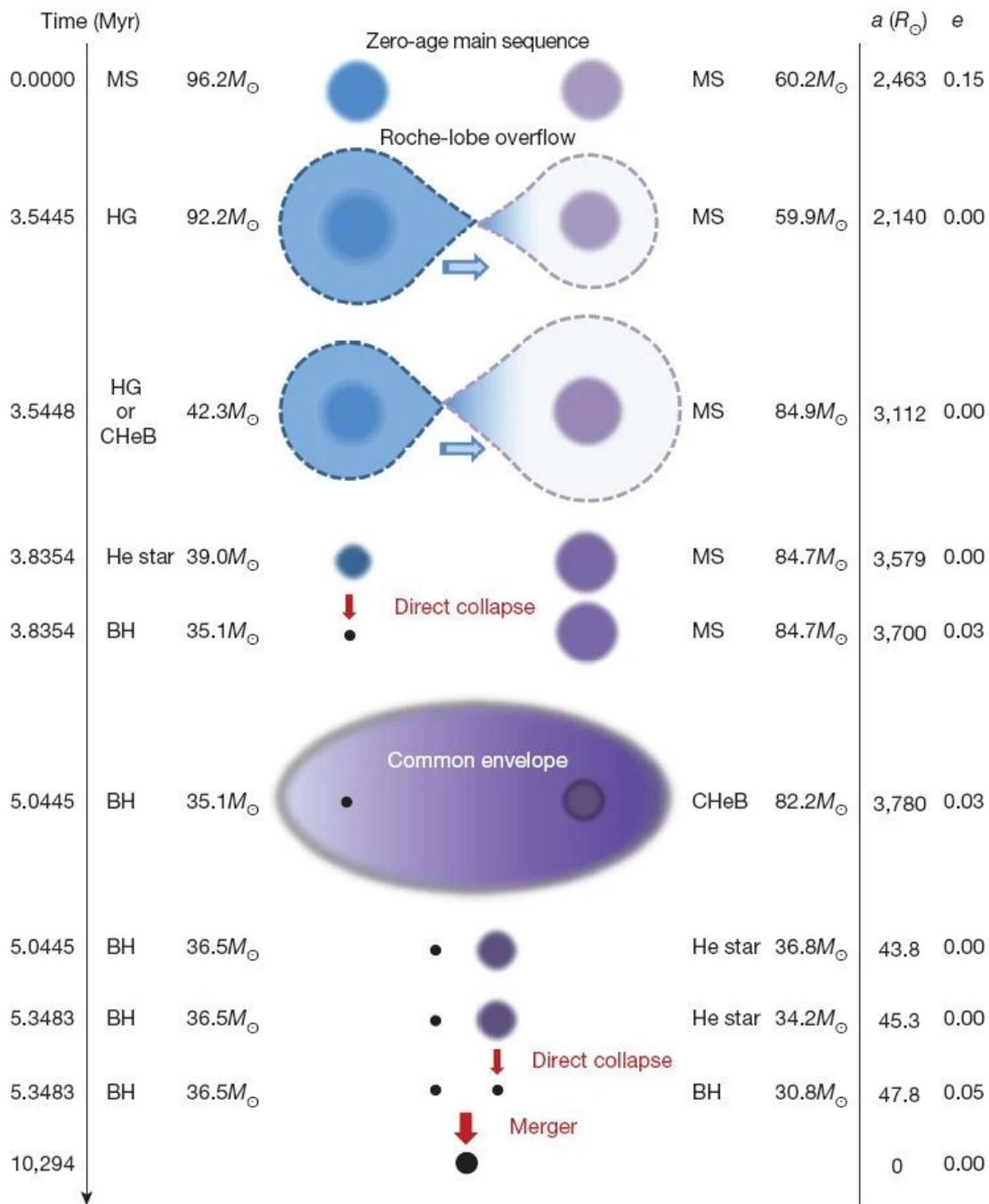
After first supernova (with  
kick  $v_{\text{kick}} = 300 \text{ km s}^{-1}$ ):  
 $M_A' = 1.337 M_\odot$ ,  
 $M_{\text{He}}^B = 2.4 M_\odot$ ,  $P_{\text{orb}} = 3.3 \text{ hr}$ ,  
 $e = 0.33$ ,  $\Delta v_{\text{qs}}^A = 230 \text{ km s}^{-1}$

Helium star mass transfer  
phase (+ spin-up of neutron  
star) leaving  $M_A = 1.338 M_\odot$ ,  
 $M_{\text{He}} = 1.559 M_\odot$ ,  $P_{\text{orb}} = 2.6 \text{ hr}$

Immediately after second  
supernova:  $M_A = 1.338 M_\odot$ ,  
 $M_{\text{He}} = 1.249 M_\odot$ ,  $P_{\text{orb}} = 3.3 \text{ hr}$ ,  
 $e = 0.12$ ,  $\Delta v_{\text{qs}}^B = 35 \text{ km s}^{-1}$



# GW150914: an example scenario



(Belczynski et al. 2016)

$$a_0 = 1.6 R_\odot \left( \frac{M_1}{M_\odot} \right)^{3/4} \left[ q(1+q)F(e) \left( \frac{t_{\text{coal}}}{1\text{Gyr}} \right) \right]^{1/4}$$

## Evolution of massive Binaries

### Complications

- common envelope
- kicks
- metallicity
- rotation

### Features:

- Preferentially high, aligned spins?
- small formation eccentricity

**A back of the envelope estimate of the rate:**

-stellar density in the universe  $\rho=3 \times 10^8 \text{ Msun/Mpc}^3$

-assuming  $1 \text{ Msun}$  average mass this means  $3 \times 10^8 \text{ stars/Mpc}^3$

-for a Salpeter IMF,  $N(m) \propto m^{-2.35}$   $\sim 0.3\%$  have  $m > 30 \text{ Msun}$  (thus leading to a BH)

-70% of those stars are in binaries

-so we can estimate  $\sim 3 \times 10^5 \text{ binaries/Mpc}^3$

-if they form steadily over an Hubble time we get a formation rate of  $\sim 2 \times 10^4 \text{ binaries/Gpc}^3/\text{yr}$

-if those binaries all merge in  $t \ll t_{\text{Hubble}}$  then this is also the merger rate.

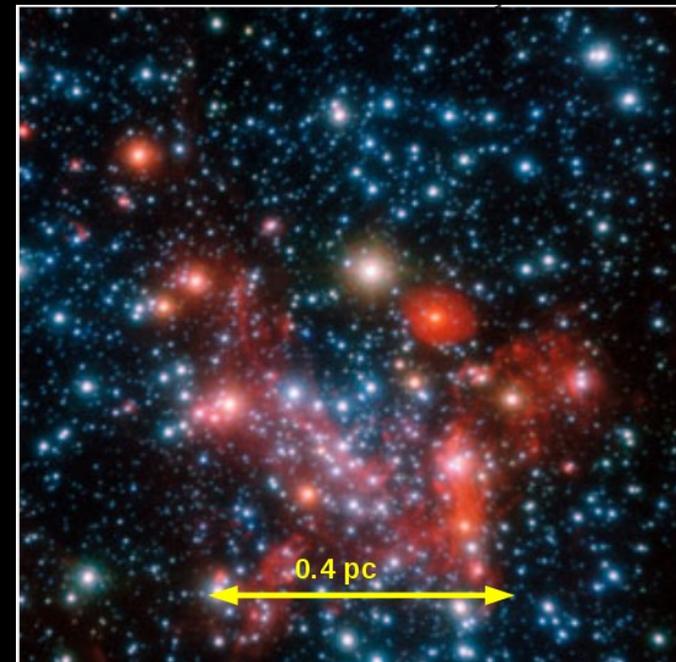
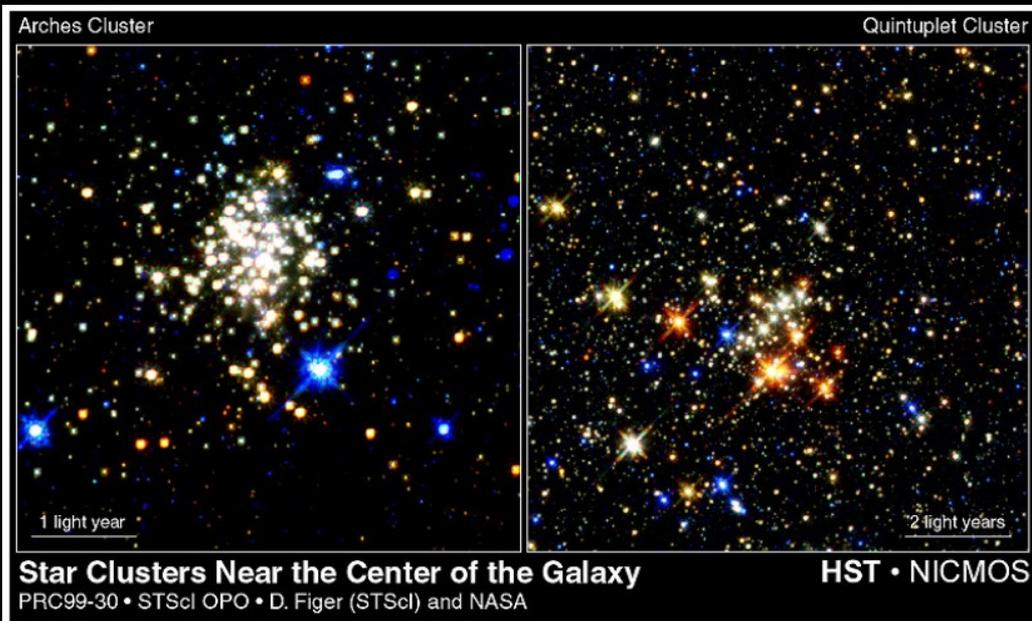
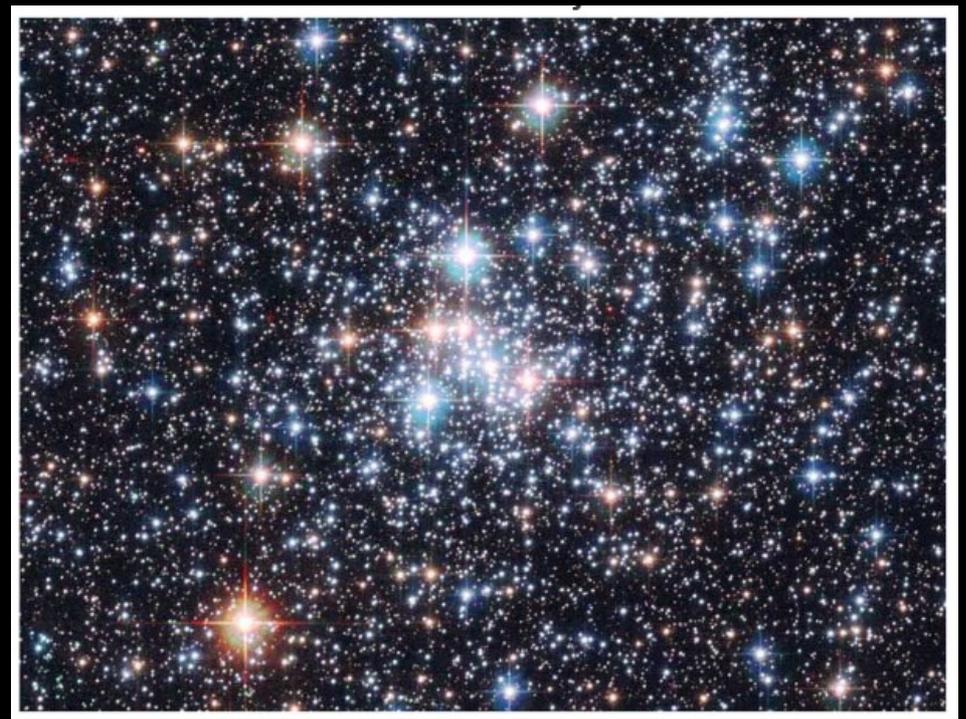
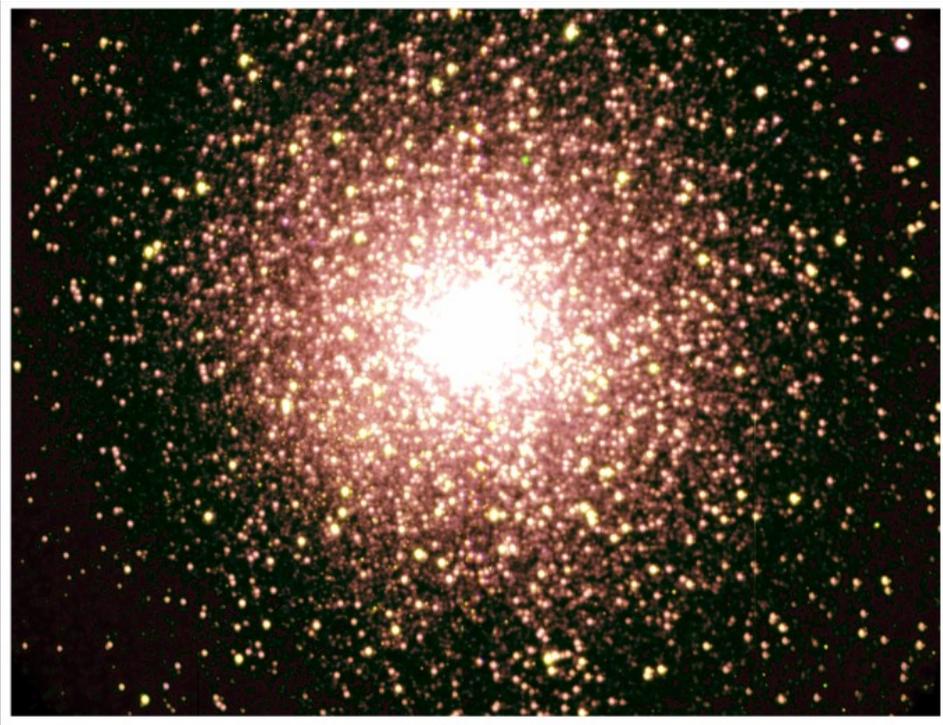
**This is an upper limit and is 2-3 orders of magnitude larger than the measured LIGO/Virgo rate**

-SN kicks can easily disrupt the vast majority of those binaries

-Efficiency of CE unknown, it might be not efficient enough or way too efficient

(one can do a similar calculation with NS binaries)

# *BHB origin: dynamical formation*

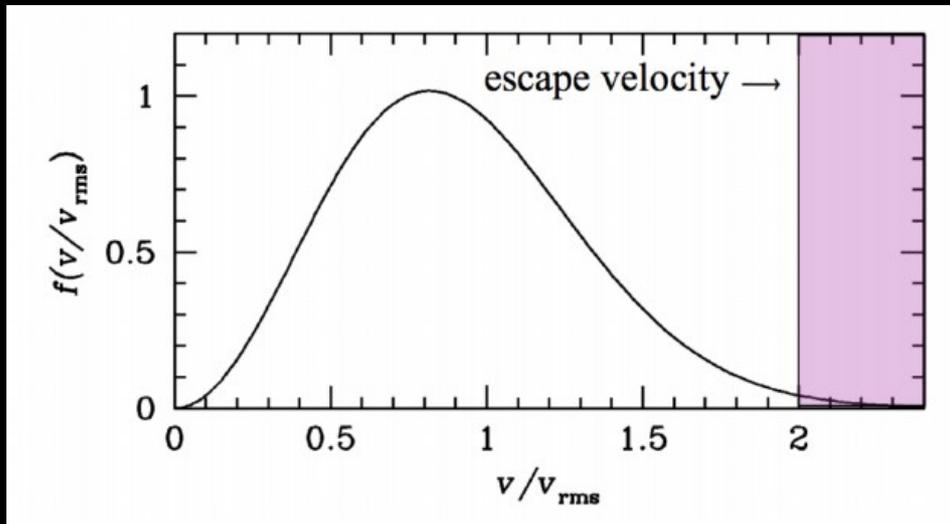


# Dynamics of clusters (Celoria+ 2018)

## Two body relaxation

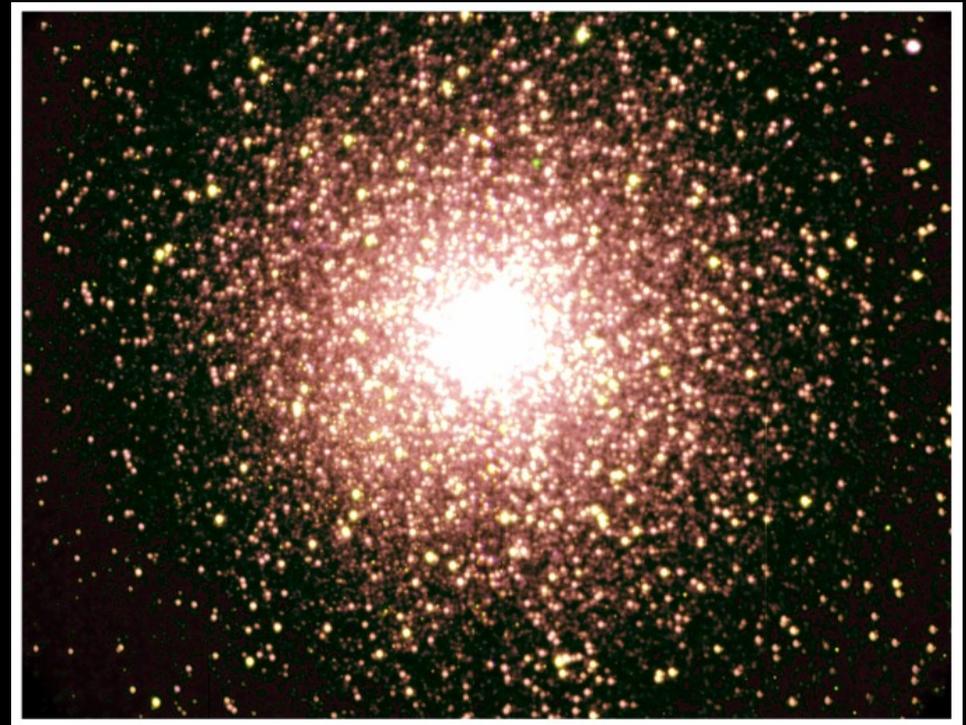
$$t_{\text{rlx}} \approx \frac{10^8 \text{ yr}}{\log N} \left( \frac{M}{10^5 M_\odot} \right) \left( \frac{R}{1 \text{ pc}} \right)^{3/2} \left( \frac{1 M_\odot}{m} \right)$$

## Leads to evaporation



## Leads to core collapse

$$\frac{\rho(t)}{\rho(0)} \propto \left( \frac{M(t)}{M(0)} \right)^{3\zeta-5}$$



## Because of equipartition

$$\frac{1}{2} m_i \langle v_i^2 \rangle = \frac{1}{2} m_j \langle v_j^2 \rangle$$

## Massive stars tend to segregate to the center of the cluster

$$t_{\text{segr}}(M) = \frac{\langle m \rangle}{M} t_{\text{rlx}}$$

**For typical clusters this migration/evolution timescale is  $>3\text{Myr}$  which is the lifetime of the most massive stars.**

**Therefore BHs tend to migrate through the centre where they form binaries either via direct GW capture or via triple interactions.**

**A variety of mechanisms then can lead to the merger of the binary:**

**-exchanges**

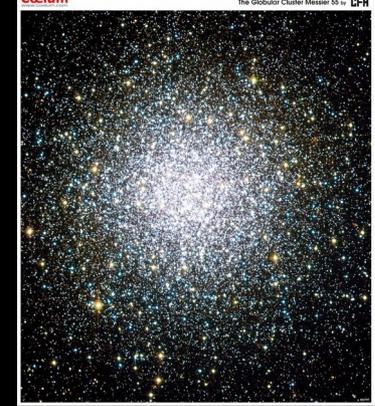
**-Kozai-Lidov oscillations**

**-Hardening**

**.....**

**Estimated rates through this channel are  $\sim 10/\text{Gpc}^3/\text{yr}$  (Rodriguez+2016...)**

# BHB origin: dynamical formation



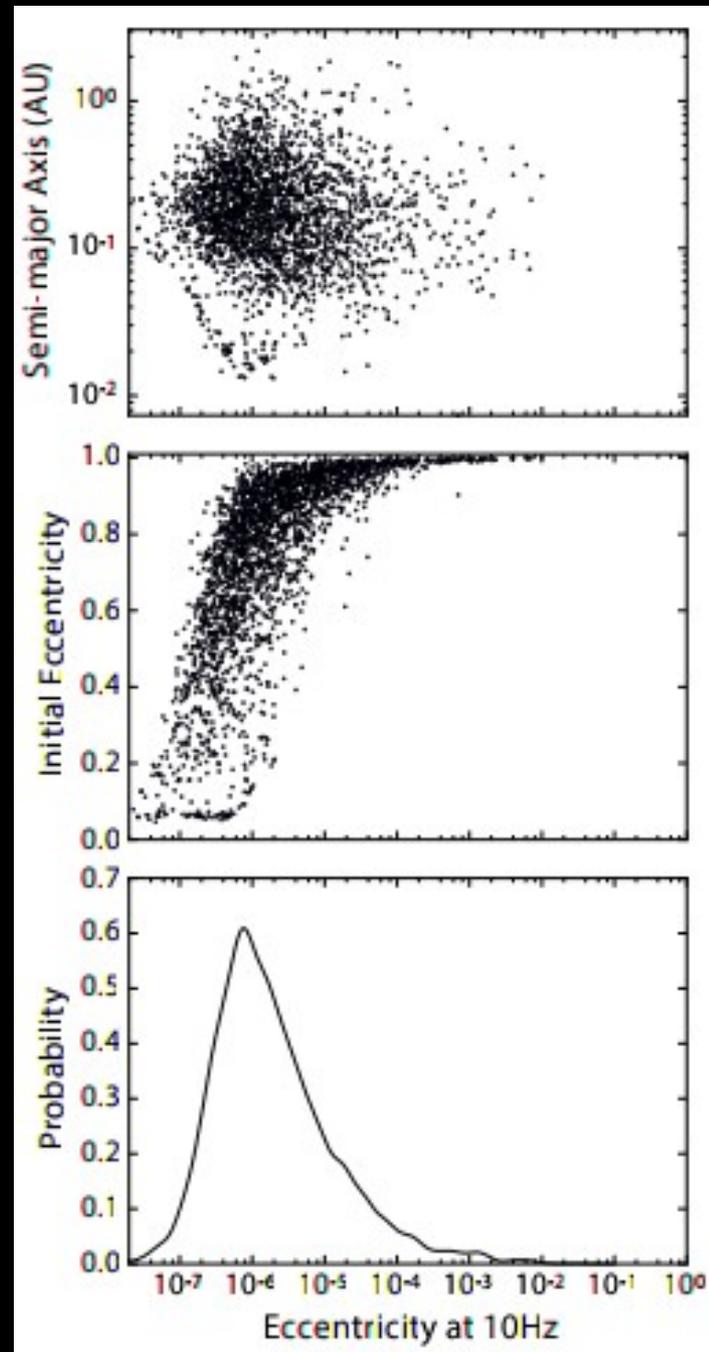
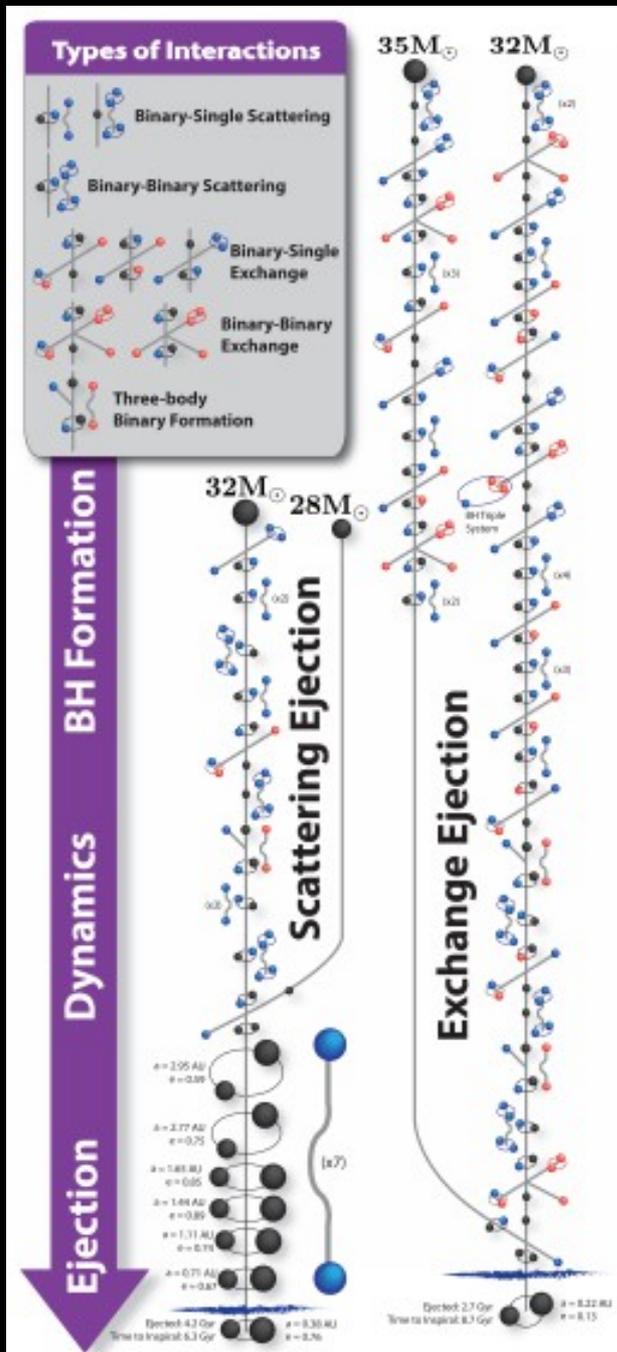
## Dynamical capture

## Complications

- mass segregation
- winds
- ejections
- multiple interactions
- resonant dynamics (Kozai-Lidov)

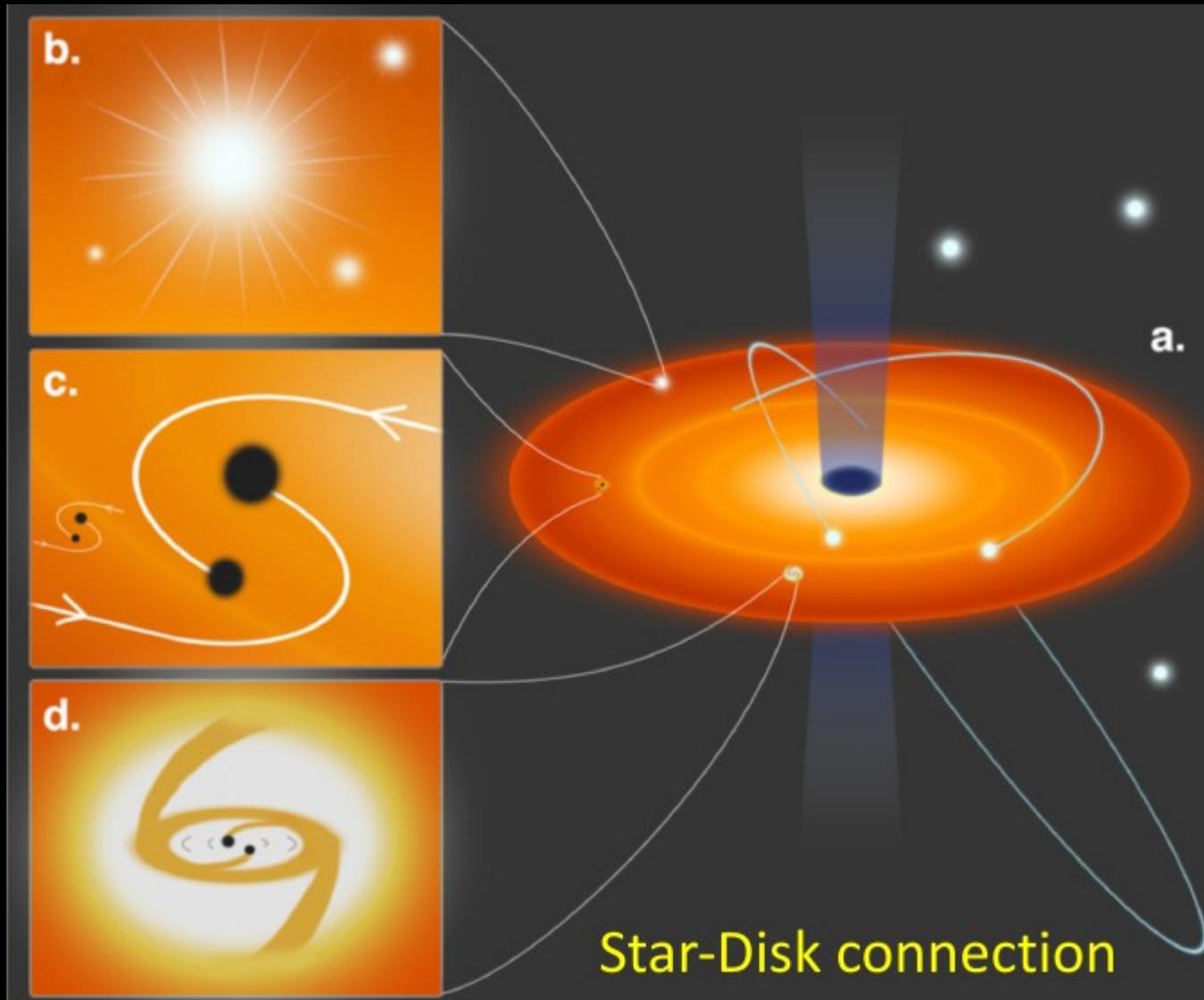
## Features:

- randomly oriented spins?
- high formation eccentricities



(Rodriguez et al. 2016)

# BHB origin: AGN disks



**GALACTIC NUCLEI**

**AGN disks**

**Complications**

- capture
- evolution
- migration traps
- multiple interactions

**Features:**

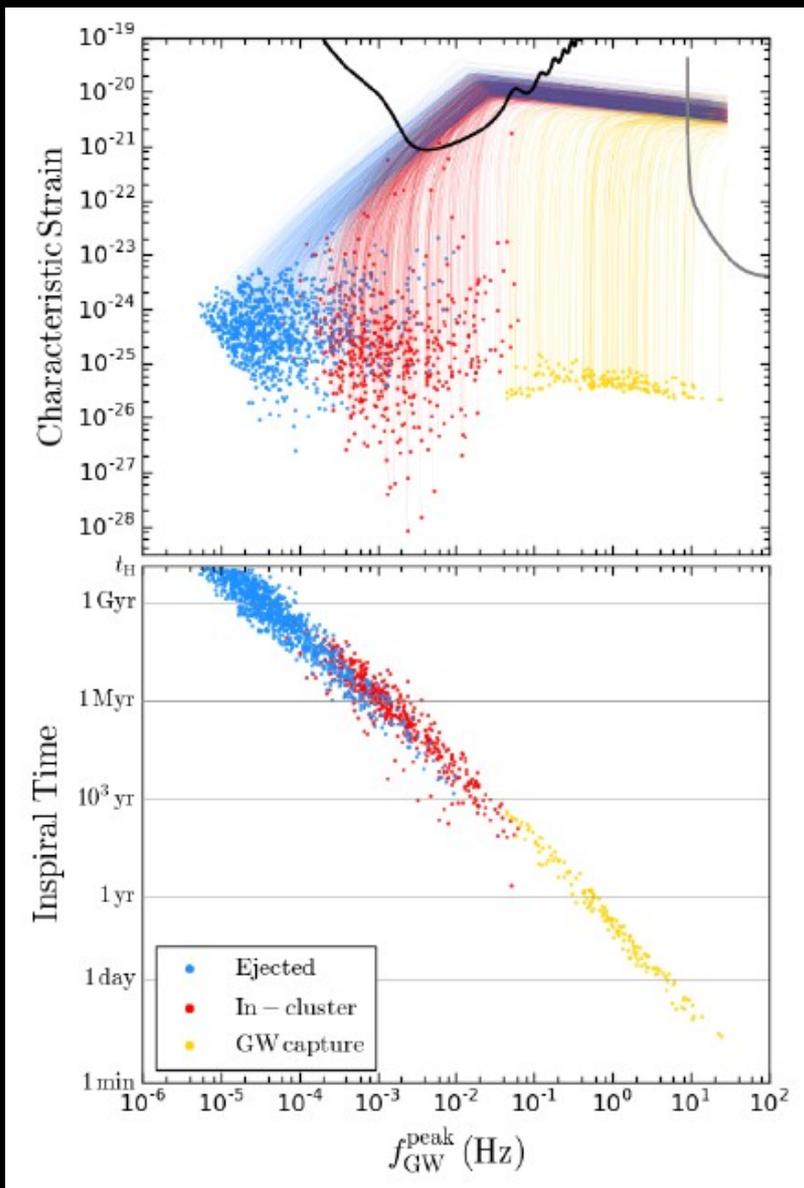
- accelerated by large MBH potential
- EM counterparts?
- associated to galactic centers

**Kozai by SMBH**

**-very eccentric**

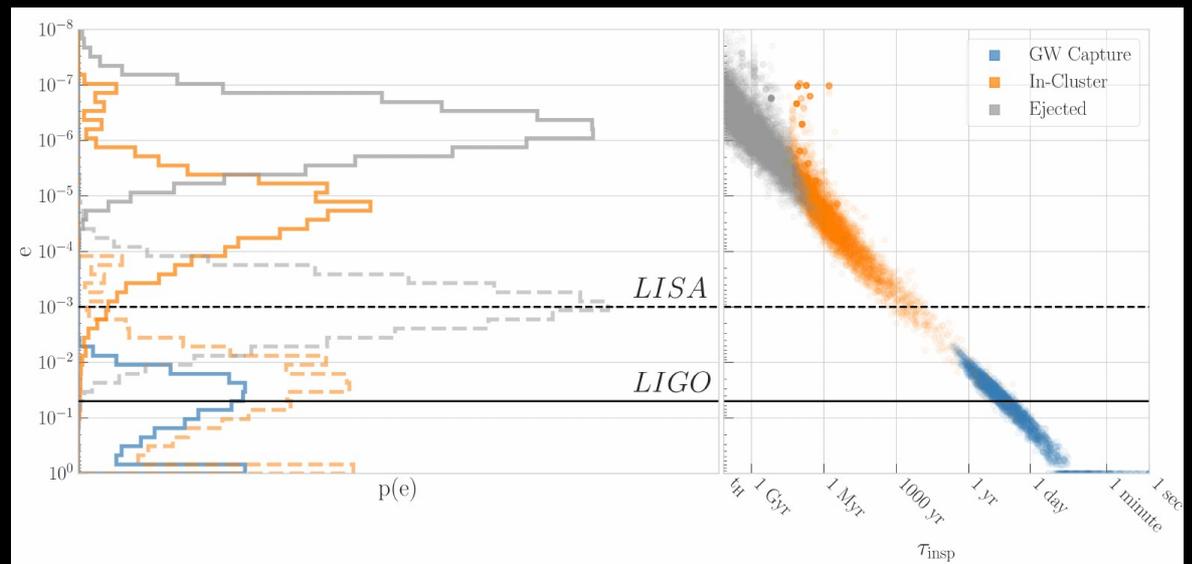
(Ford, McKernan, Haiman, Lin, Antonini....)

# BHB formation triplets quadruplets...

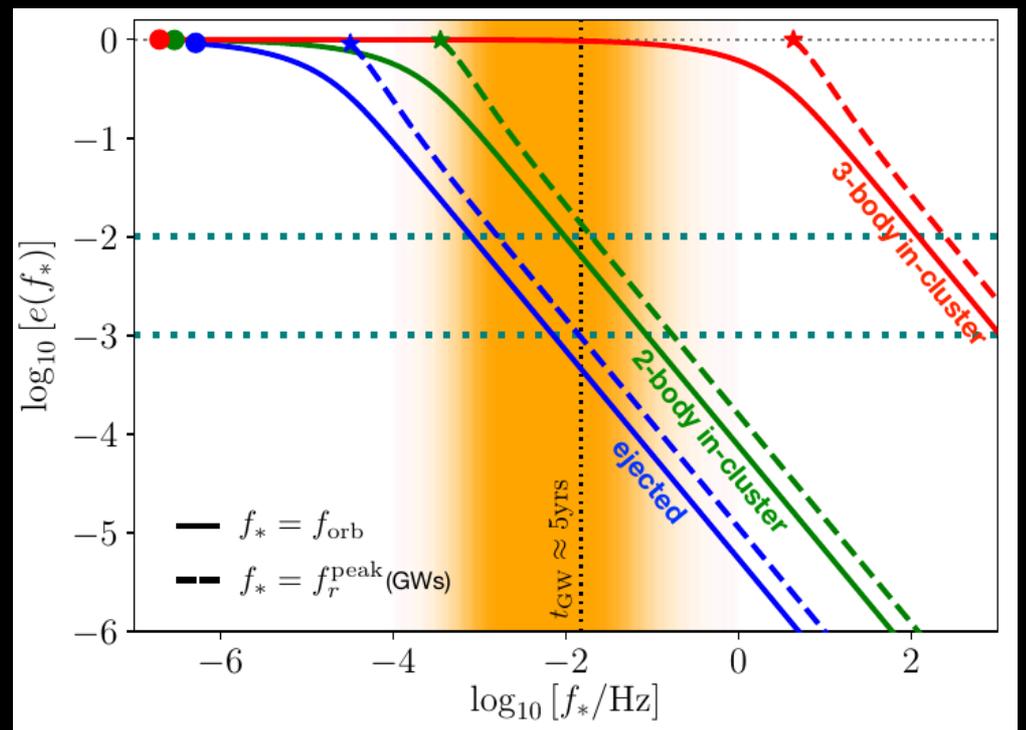


(Kramer+)

Those channels might be relatively rare!



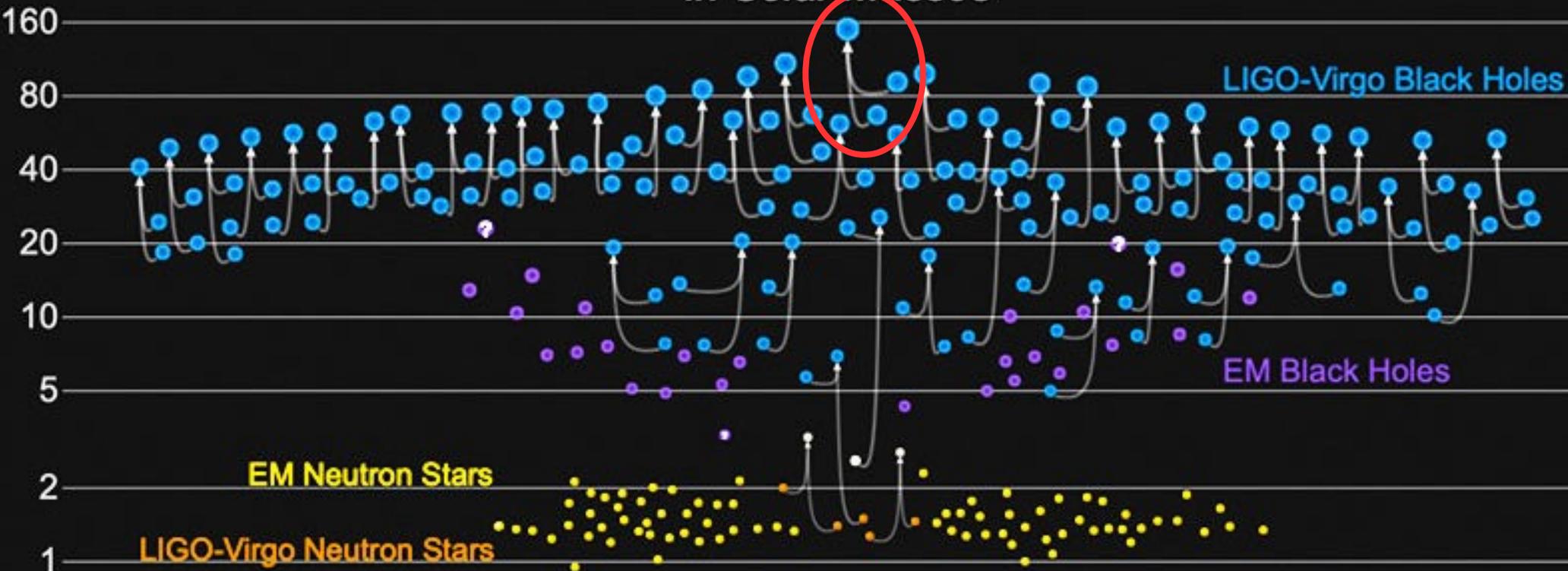
(Zevin+)



(D'Orazio & Samsing)

# Masses in the Stellar Graveyard

*in Solar Masses*

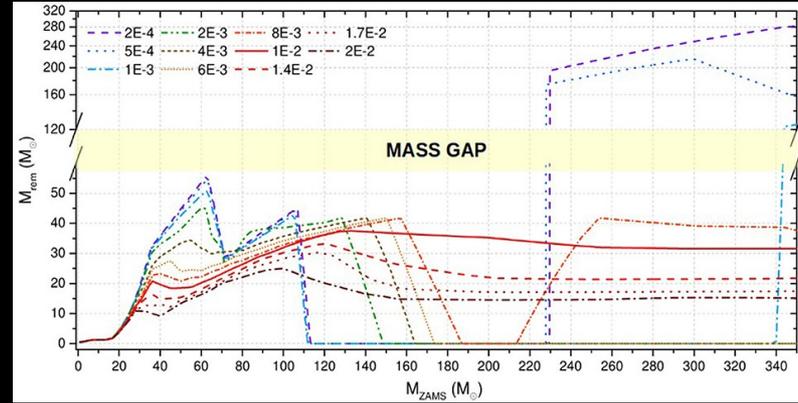


GWTC-2 plot v1.0  
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

# GW190521

~70+90 solar masses → 160Msun remnant: IMBH!!!!

The two progenitors are in the pair instability supernova gap



Route to formation:

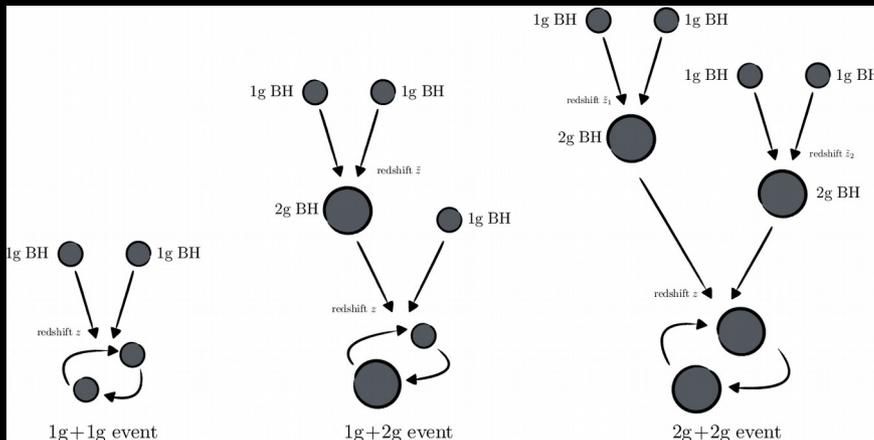
-very low metallicity popIII progenitors (Kinugawa+20, Farrell+20, Liu+20)

-primordial BHs

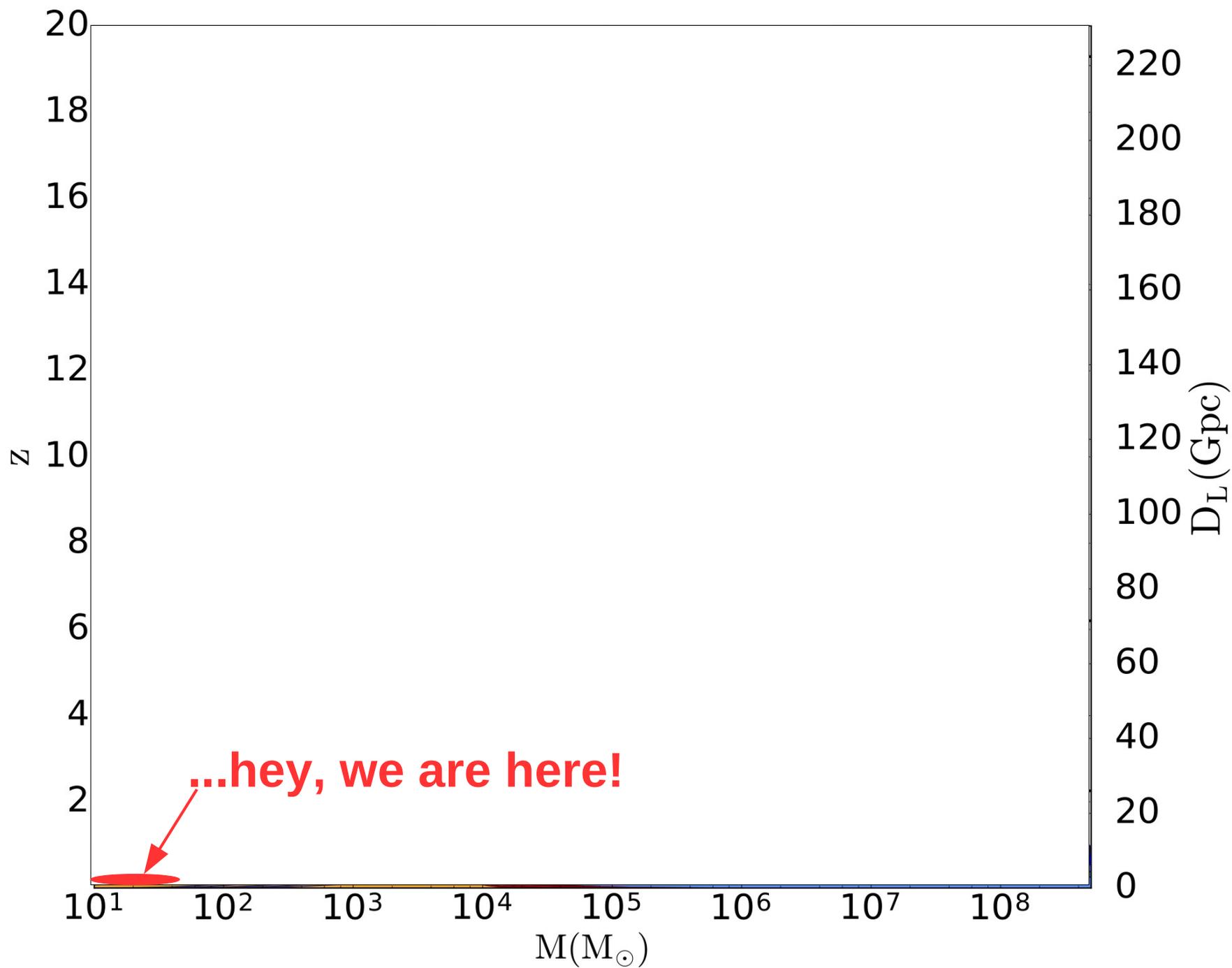
-'across gap binary' (Mangiagli+2019, Fishback+20)

-Accretion in AGN disks (Graham+20, also proposing a counterpart)

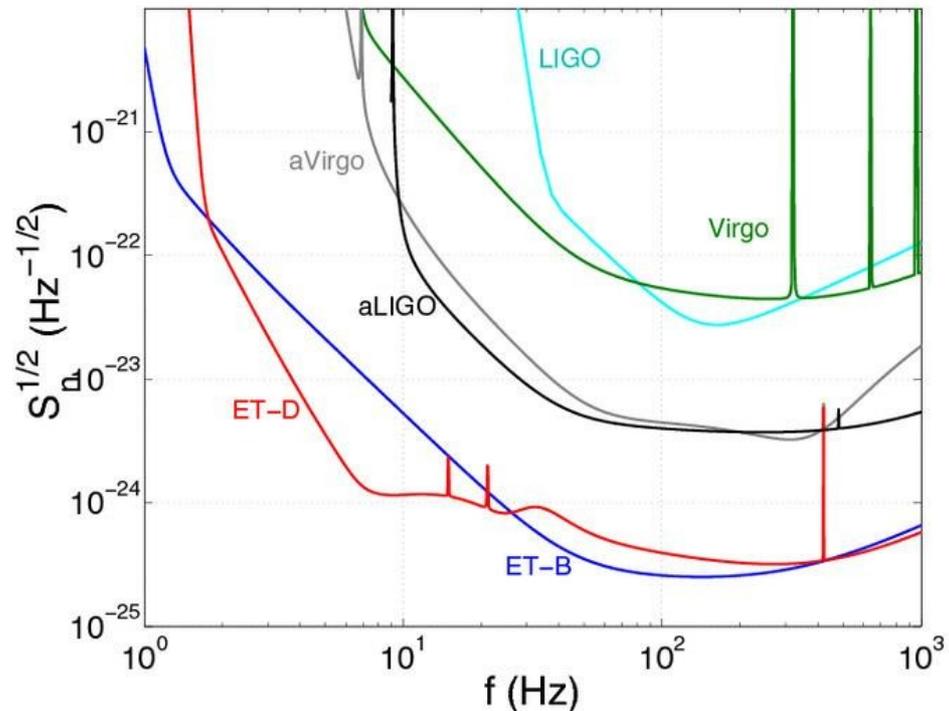
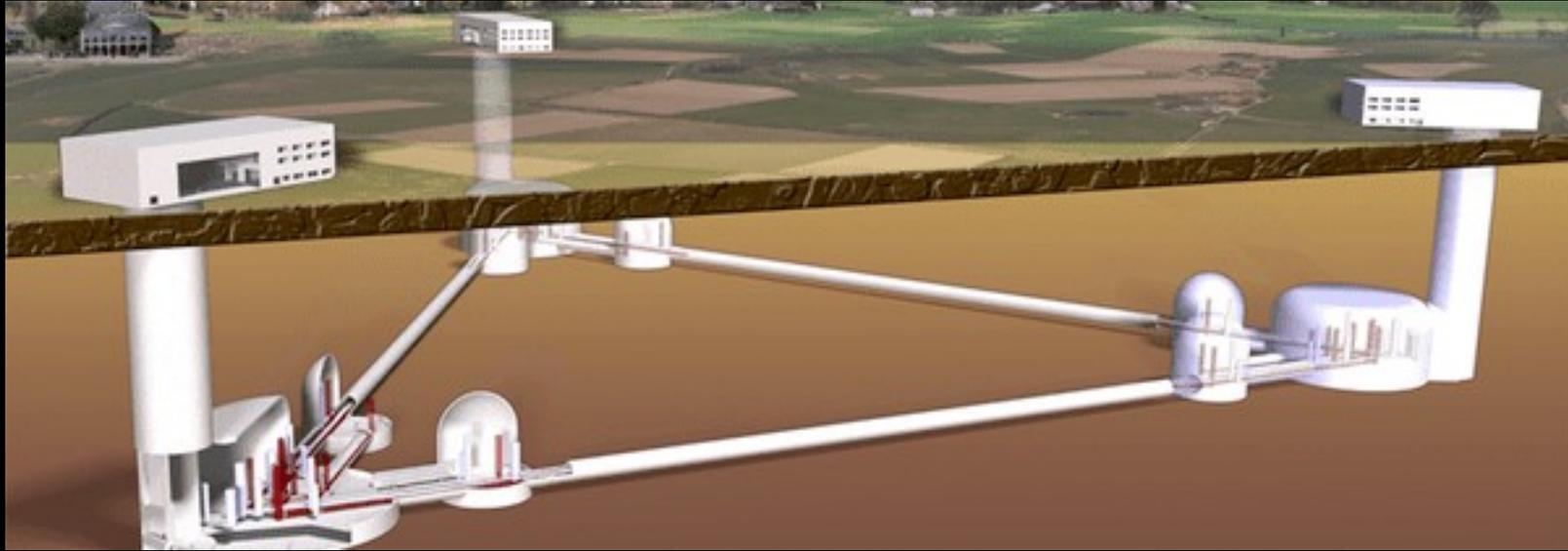
-2<sup>nd</sup> Generation (Gerosa+2017, Rordiguez+2020, Bin+20)



# ***Binaries in the z-mass parameter space***



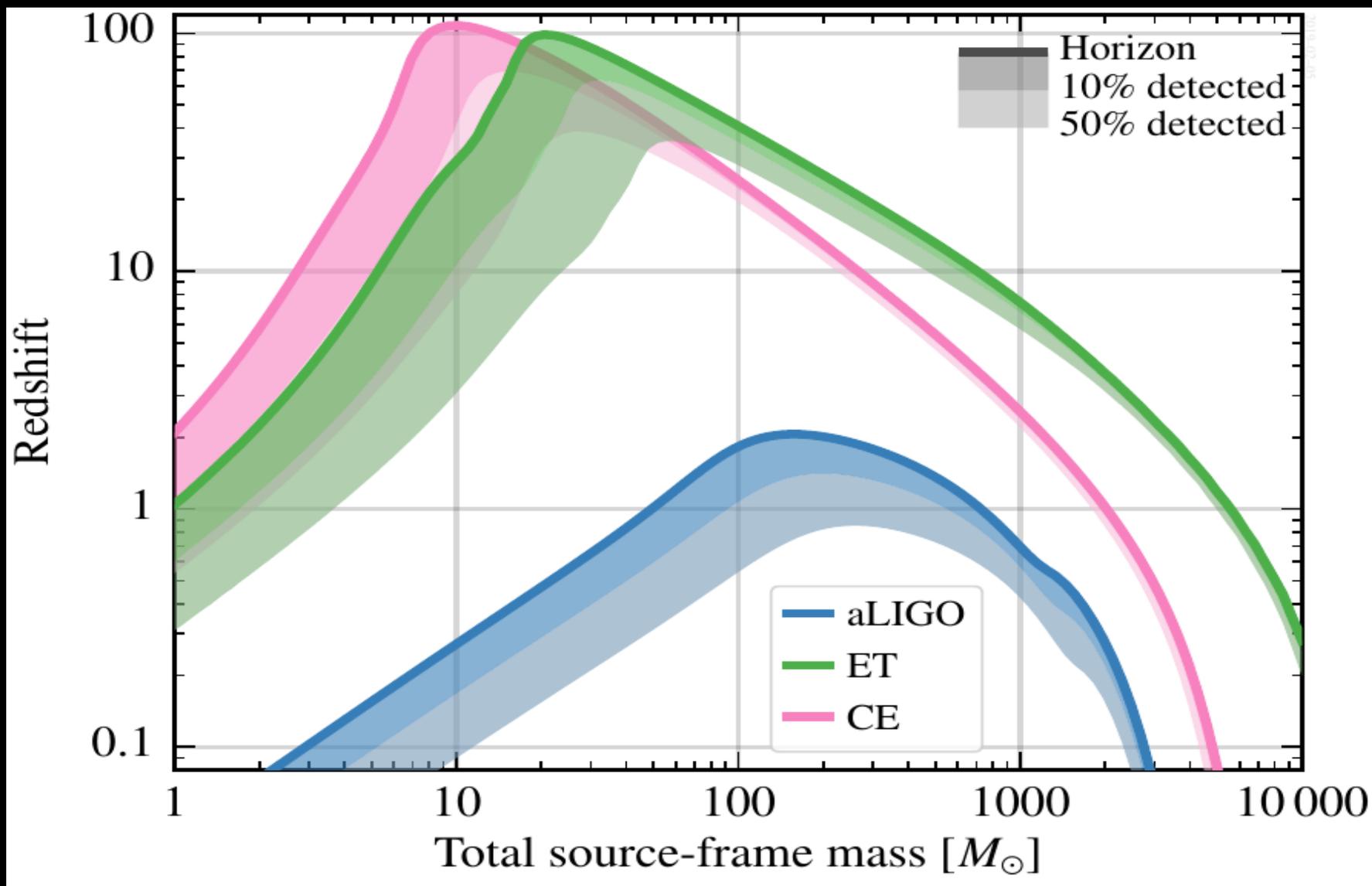
# 3G detectors: ET and CE



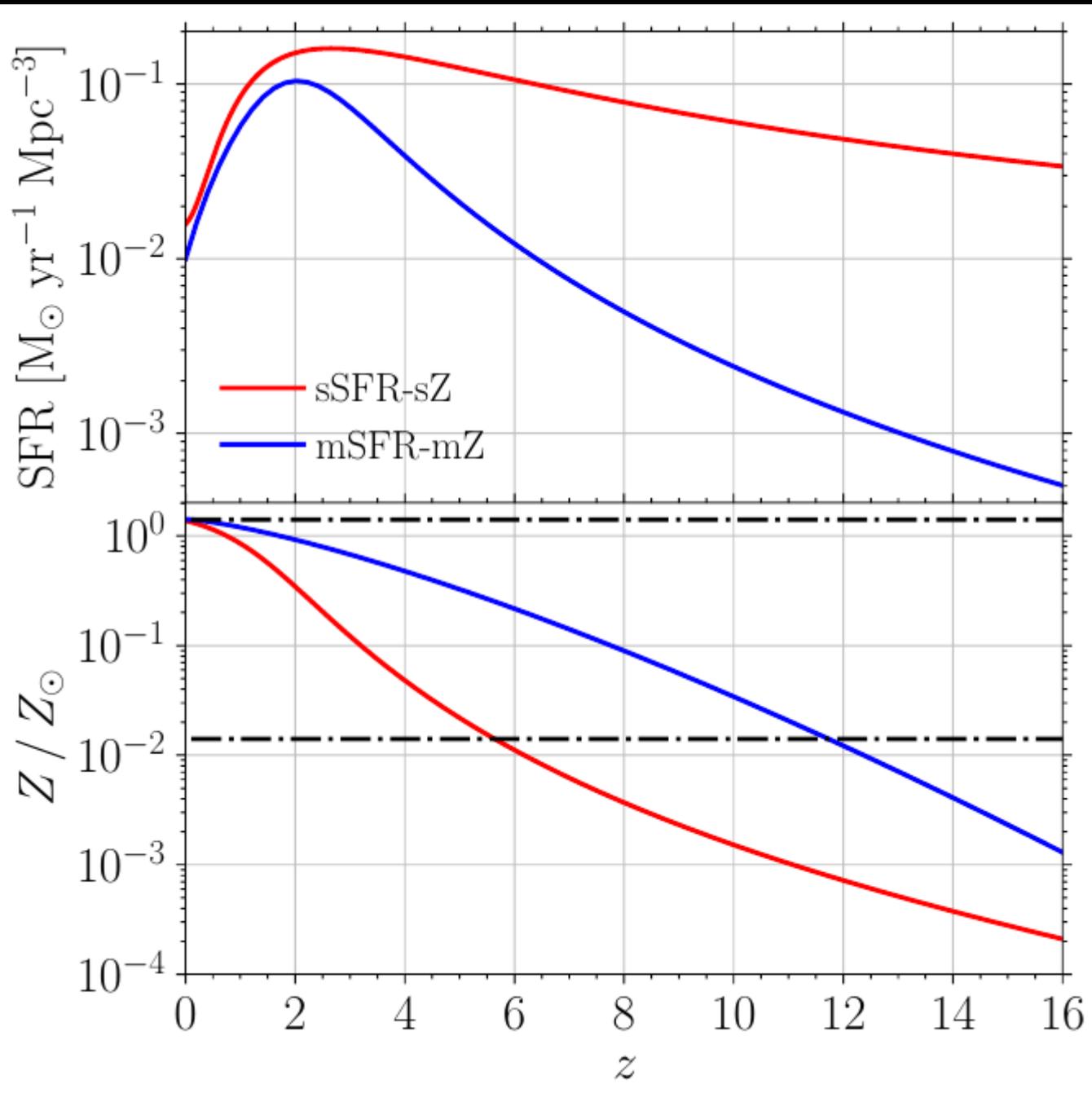
## Einstein Telescope (ET)

- underground facility (suppress noise)
- triangular configuration
- 10 km long armlength
- 2030s'?
- Netherlands? Sardinia?

# 3G detectors: ET and CE



- All LIGO/Virgo-like BHBs in the Universe up to  $z \sim 20$  ( $\sim 10^5/\text{yr}$ )
- All neutron star binaries (NSBs) to  $z \sim 2-3$  ( $\sim 10^4/\text{yr}$ )
- intermediate mass BHs (IMBHs) up to  $z \sim 2$  (???)
- ~100 solar mass seed BHs to  $z \sim 20$



The cosmic merger rate depends on many things:

- mass function
- cosmic star formation rate
- metallicity evolution
- detailed binary evolution
- time delay distribution (from formation to merger)

(Vitale 2018, Mangiagli+ 2019)

**3G will truly probe the cosmic history of star formation and evolution, possibly beyond the epoch of reionization**

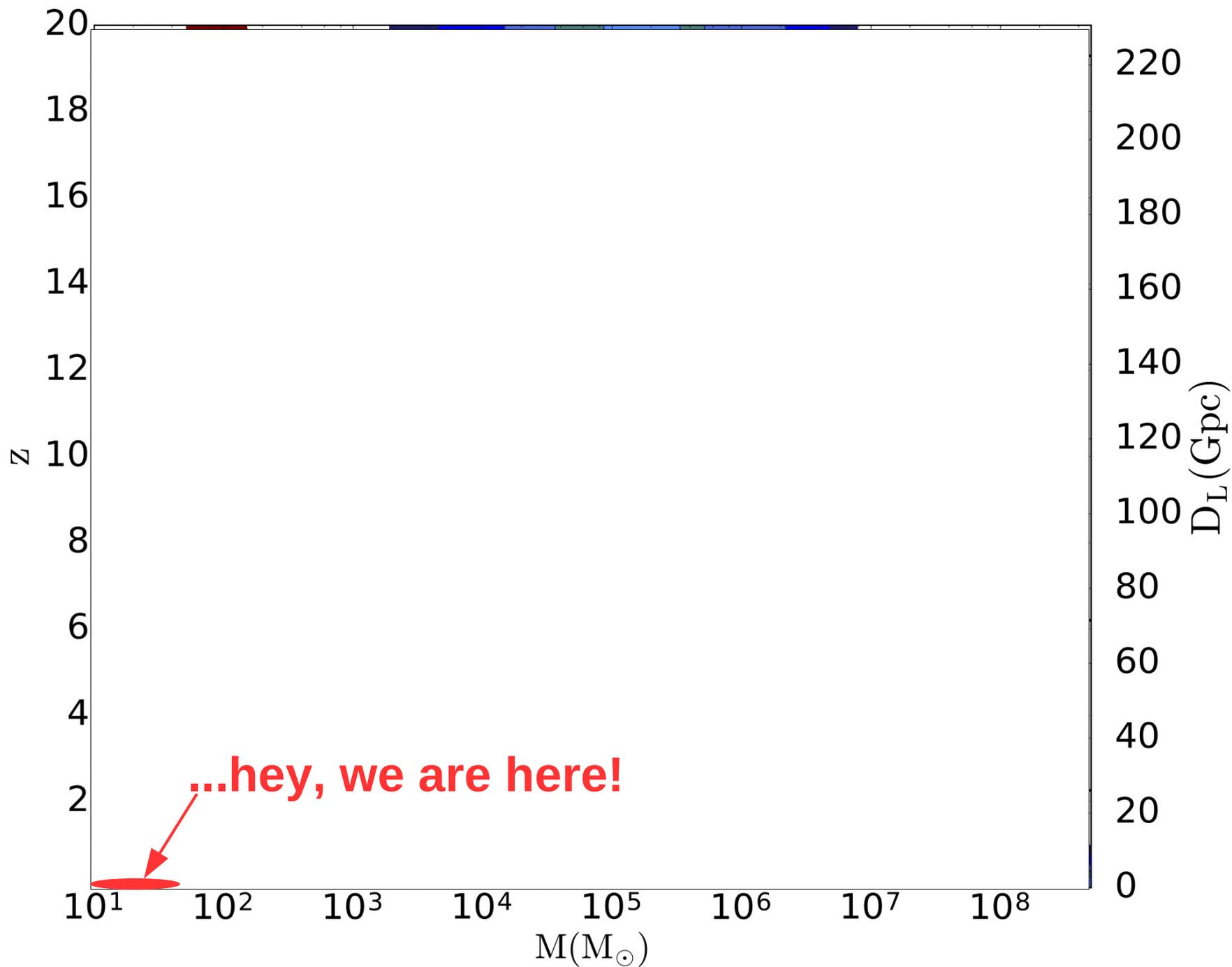
**2G detectors are detecting many BHBs  
(and many more will come) however:**

- 3G will increase the sample by at least 2 orders of magnitudes**
- Detailed distribution, can easily separate subpop**
- AGN channel: cross correlation with AGN catalogues?**
- exotic channels: might be too rare for 2G, identifiable in 3G via eccentricity measurement?**

**MOST IMPORTANTLY:**

- 3G have higher redshift reach**
- 3G have higher mass reach**

# ***Binaries in the z-mass parameter space***



# 3G detectors reach

