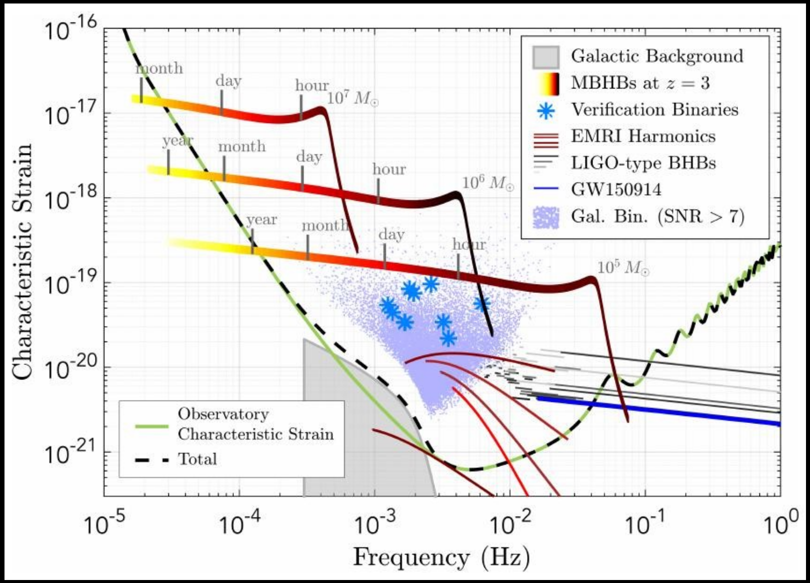
Astrophysics and detection of gravitational wave sources

Alberto Sesana
(Universita` di Milano Bicocca)

Alberto Sesana (Relativistic Astrophysics Gravitational Wave astrophyisics)

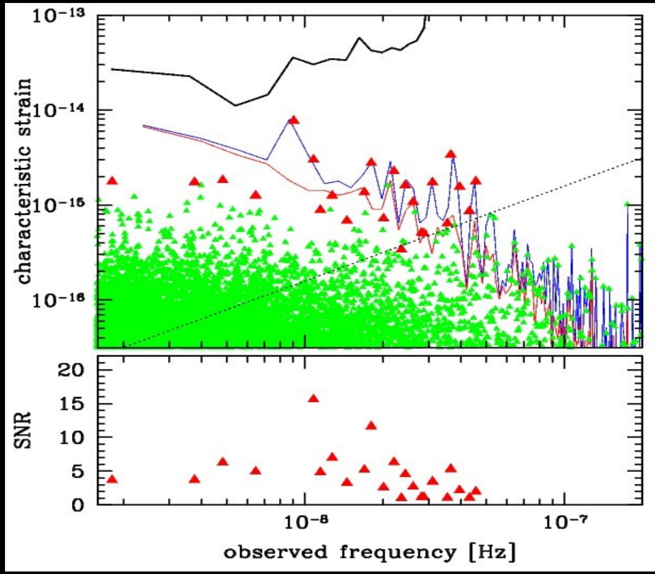
Research interests:
- formation and dynamics of massive black holes
- emission of gravitational waves with pulsar timing and LISA



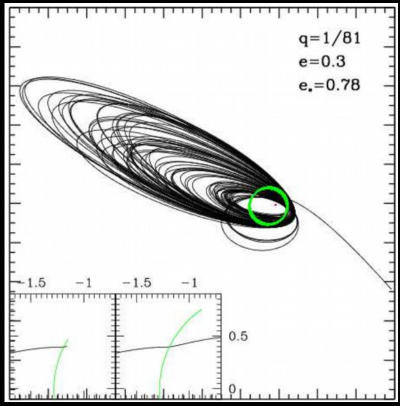
Curva di sensibilita` di LISA con esempi di segnali provenienti da varie sorgenti

Esempi di tesine:

- calcolare osservabilita` di sorgenti specifiche di onde gravitazionali con diversi detectors
- analizzare simulazioni di interazioni tra buchi neri binari e stelle
- modellizzare l'emissione elettromagnetica e osservabilita` di buchi neri massicci binari



Esempio di segnali di onde gravitazionali osservabili da pulsar timing array con relativo calcolo del segnale su rumore



Esempio di interazione dinamica a tre corpi tra una binaria di buchi neri supermassicci (verde) e una stella (nero)

OUTLINE

LECTURE 1 (NOW): Setting the stage

- -Gravitational waves (GWs): theory and general considerations
- -GWs from binary systems, relevant scalings

LECTURES 2/3 (Monday afternoon): ground based

- -Detection of GW with ground based interferometers
- -Black hole binaries (BHBs) detected by LIGO/Virgo
- -GW170817 a neutron star binary (NSB)
- -Astrophysics of ground based GW sources: formation scenarios
- -Future from the ground: 3G detectors

LECTURE 4/5 (Tuesday morning): space based

- -Beyond the ground: GW detection from space
- -Laser interferometer space antenna and its sources
- -Galactic binaries
- -Extreme mass ratio inspirals (EMRIs)
- -Massive black hole (MBH) formation and evolution

OUTLINE

LECTURE 6/7 (Tuesday afternoon): space based/PTA

-Massive black hole binaries (MBHBs): formation and dynamics
-LISA science with MBHBs
-Pulsar Timing Arrays (PTA): principles

LECTURE 8 (Wednesday morning): PTA

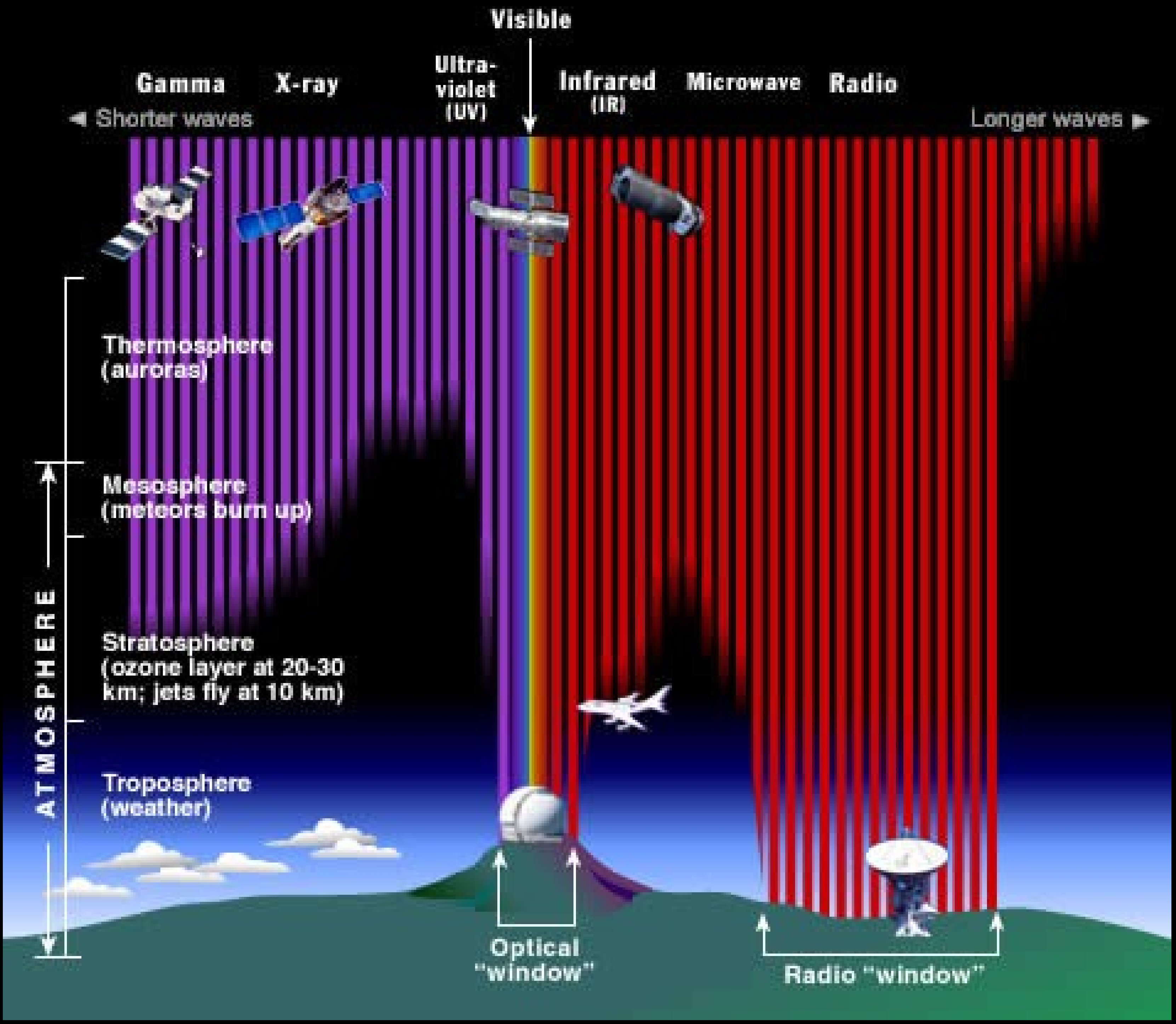
-MBHB detecion PTAs: status and prospects

*Physics of compact objects in GR and beyond
(Prof. Gualtieri)

*Data analysis and GR tests
(Prof. Del Pozzo)

*Multimessenger astronomy with GW and EM signals
(Prof. Branchesi)

Electromagnetic radiation spectrum



Pan-cromatica vision of the Universe

Different wavelengths are key to access different information

- **-Optical: thermal phenomena, dust absorption, stars**
- **-infrared: reprocessing from dust, cold gas**
- **-X ray: violent phenomena, shocks, hot gas, accretion**

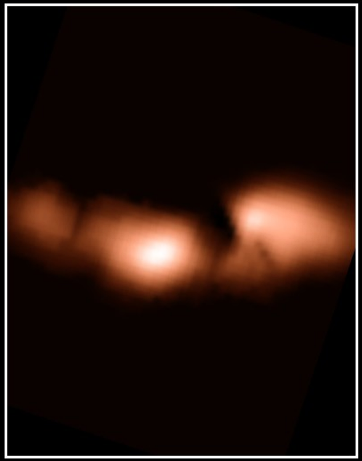
X ray

UV

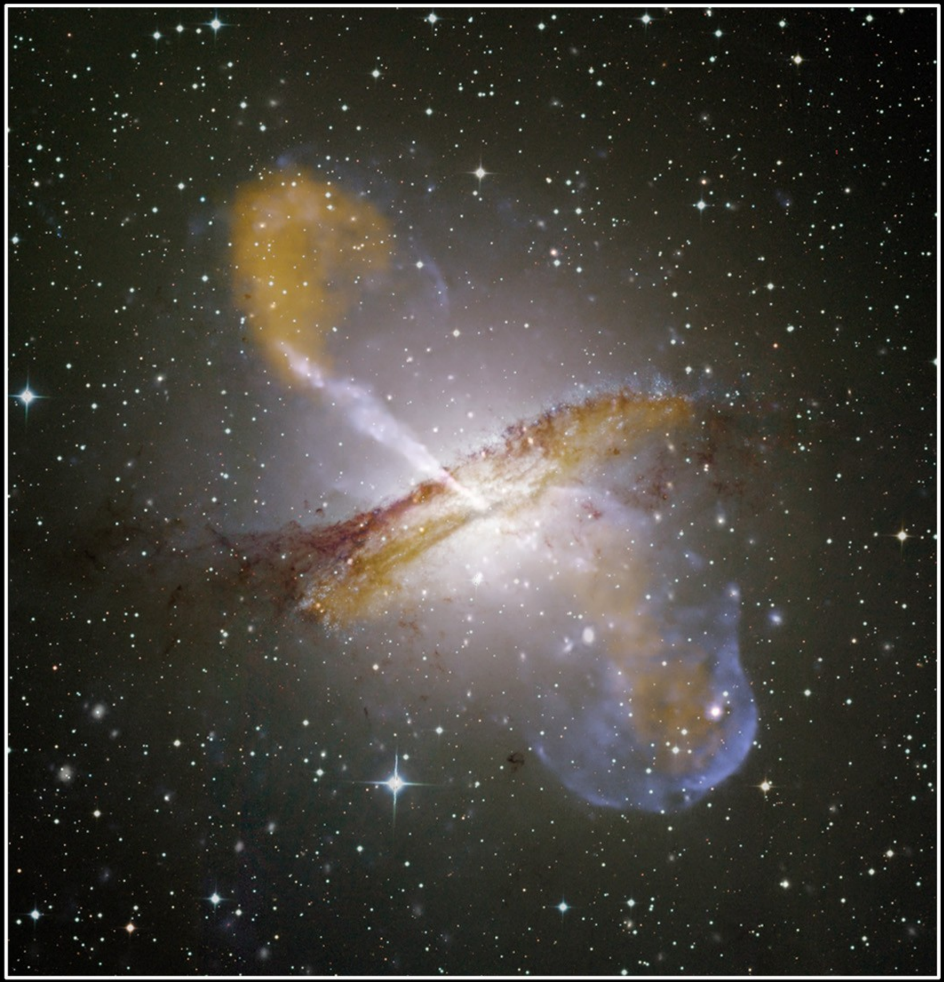
Optical

MIR

Radiocontinuum



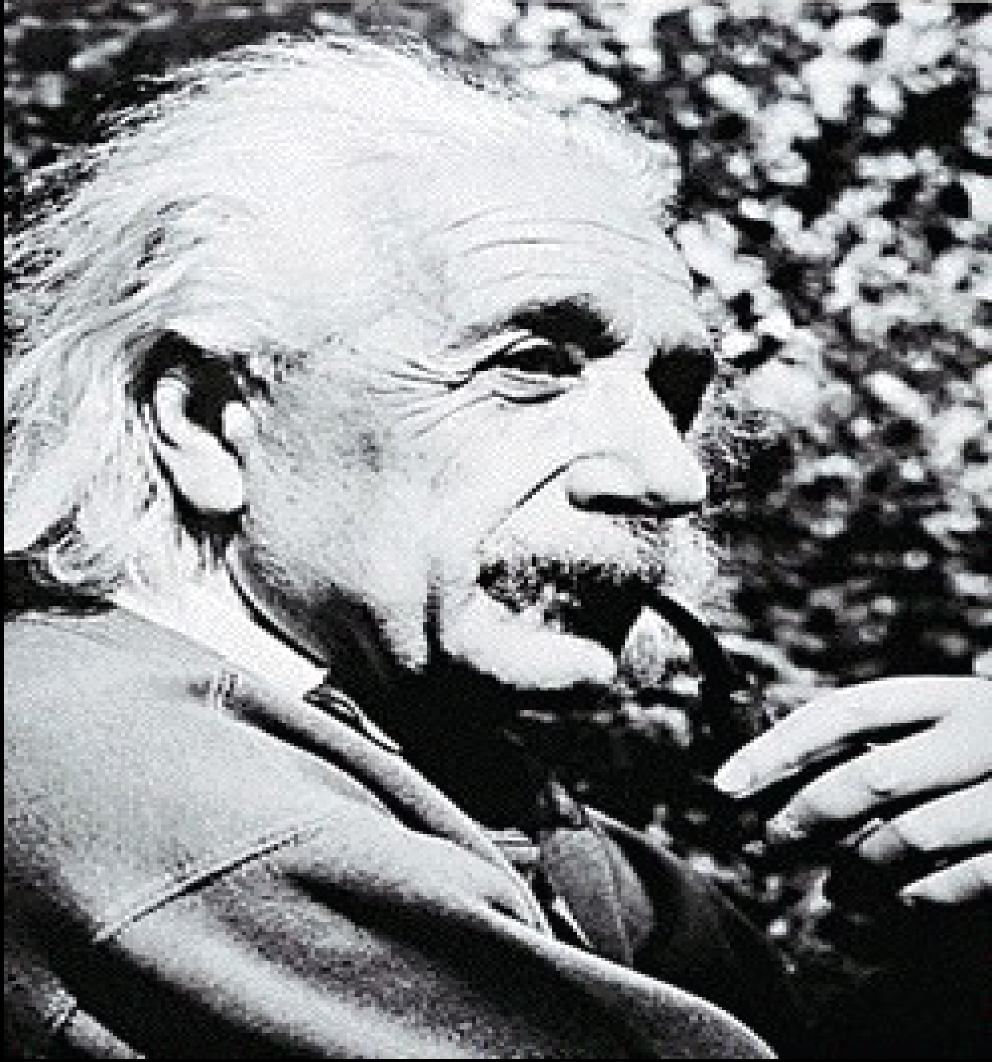
H I



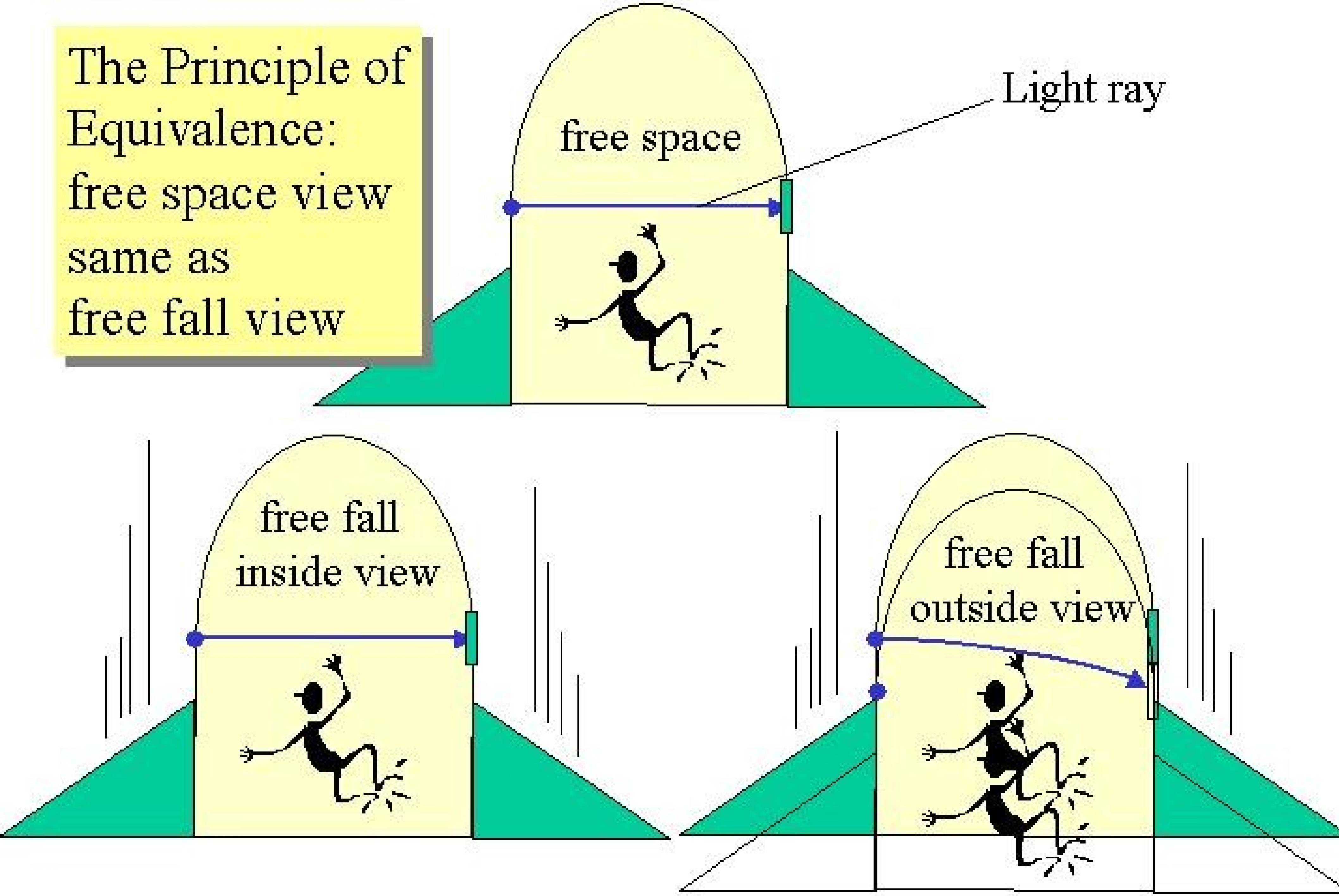
X ray + Optical + Submillimetre + Radio Composition

What if we can listen to the Universe?

In 1916 Einstein publishes the theory of general relativity, based on a simple principle:
"the gravitational field is locally equivalent to a non inertial reference frame"

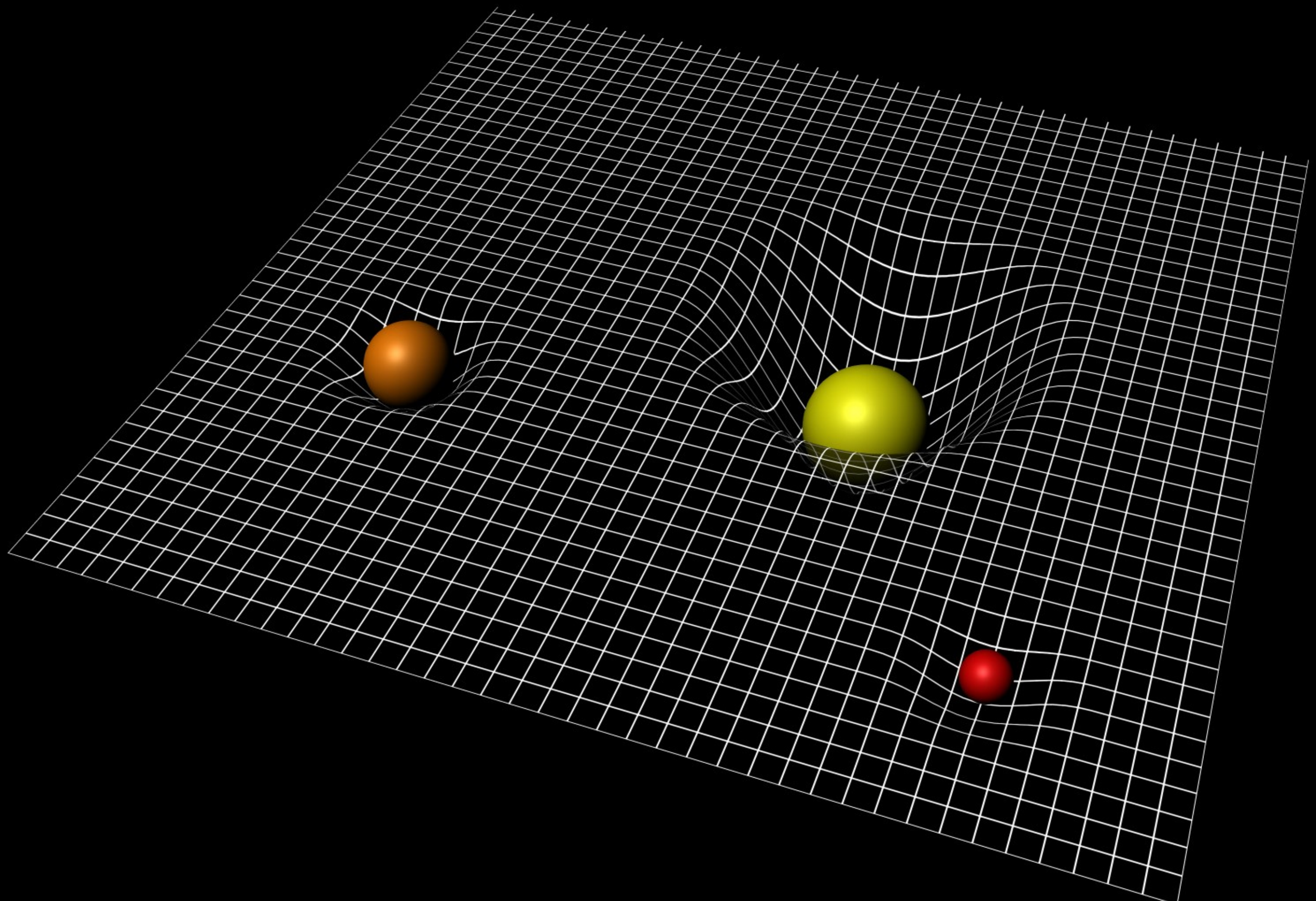


The consequences of this principle are astonishing *the presence of mass cuves space*, as it was an elastic material.


The Principle of Equivalence:
free space view
same as
free fall view
free space
Light ray
free fall
inside view
free fall
outside view

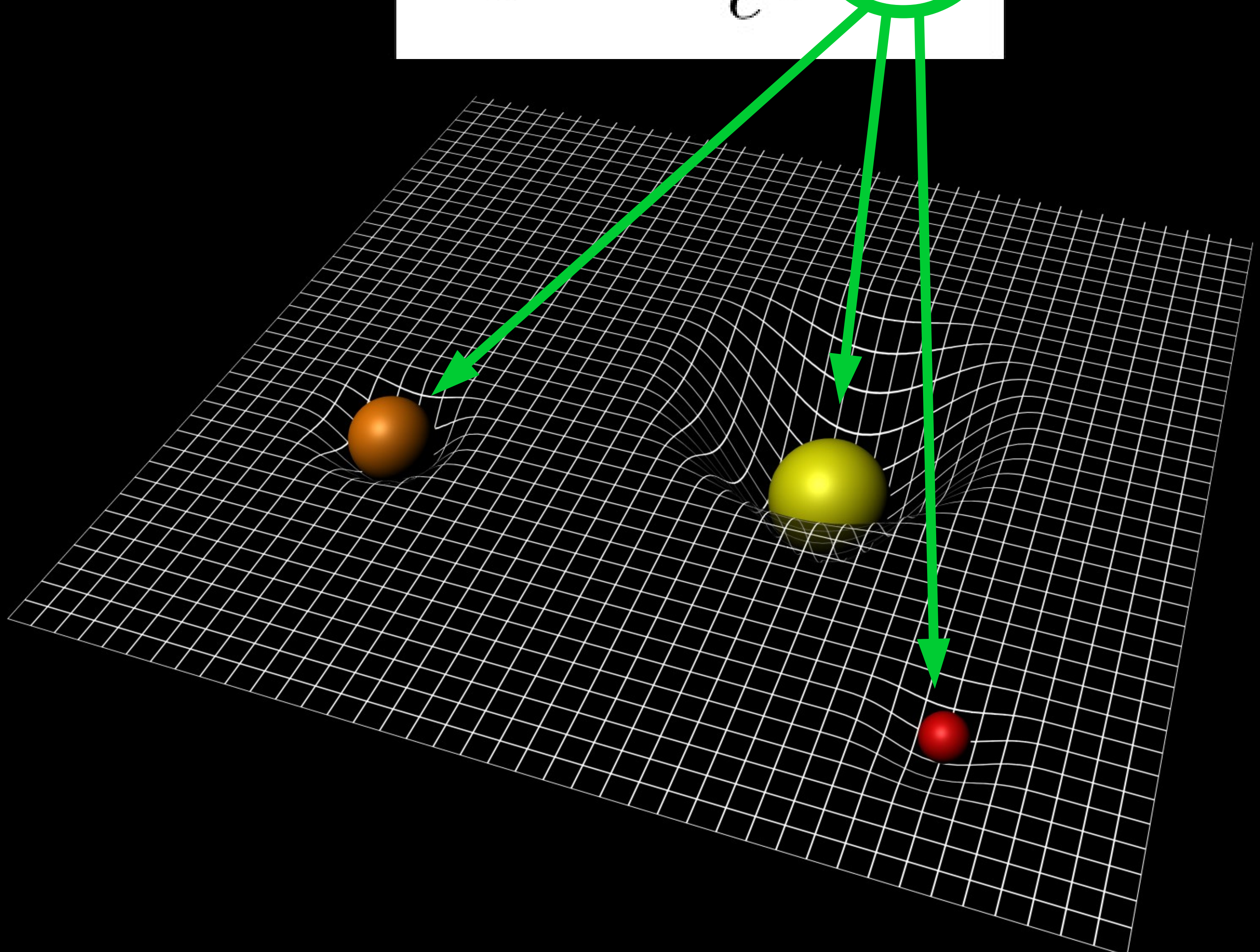
1915: Einstein publishes the theory of general relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



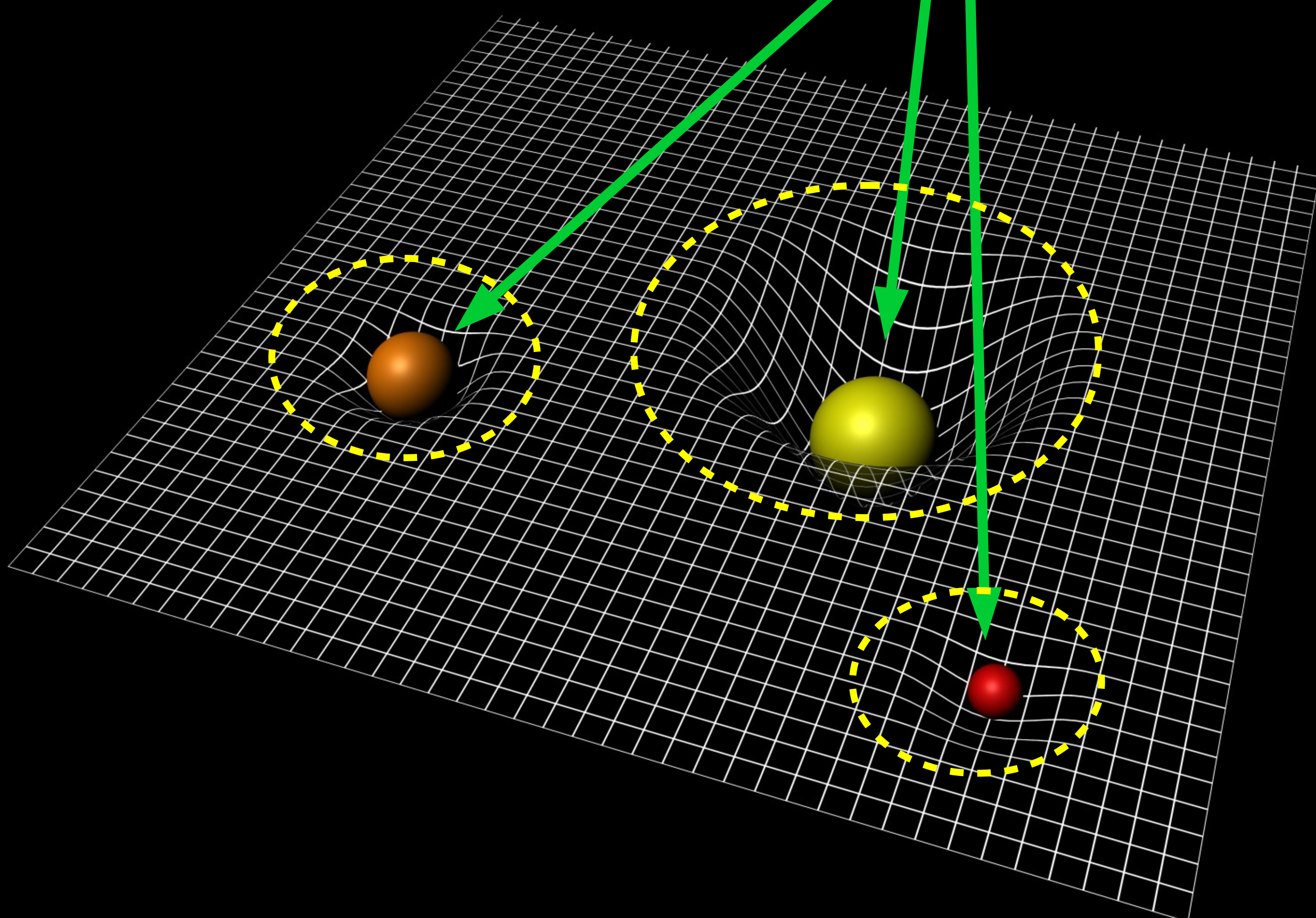
1915: Einstein publishes the theory of general relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

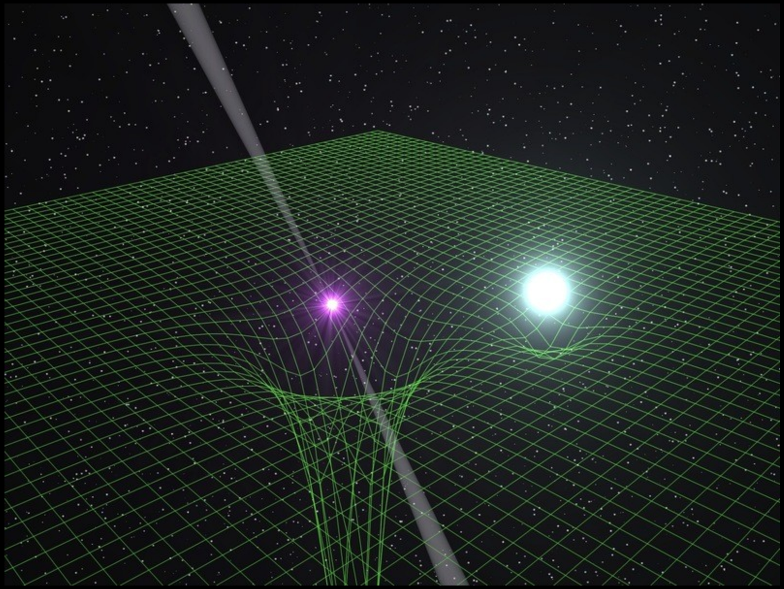


1915: Einstein publishes the theory of general relativity

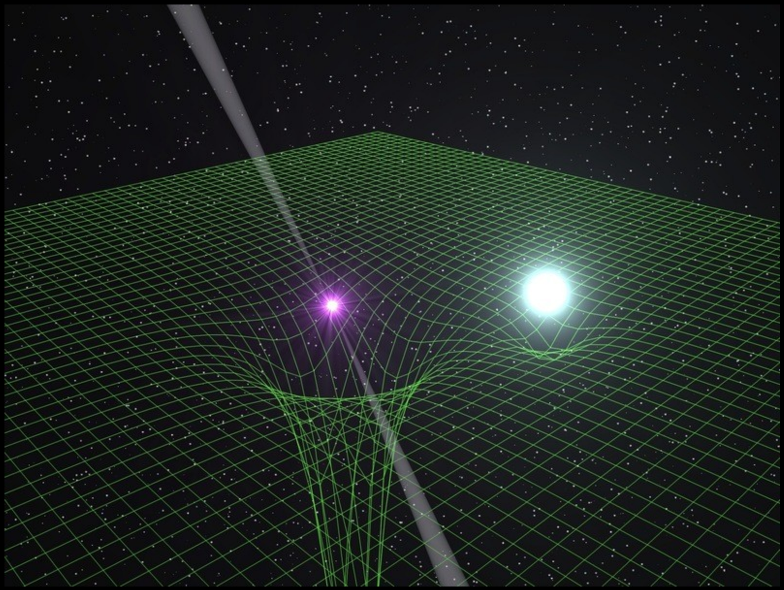
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

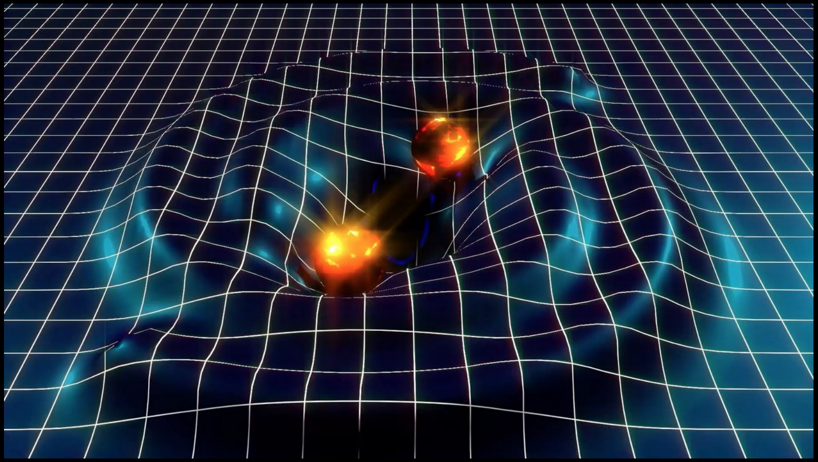


What happens if two compact objects (e.g. black holes) orbit each other?



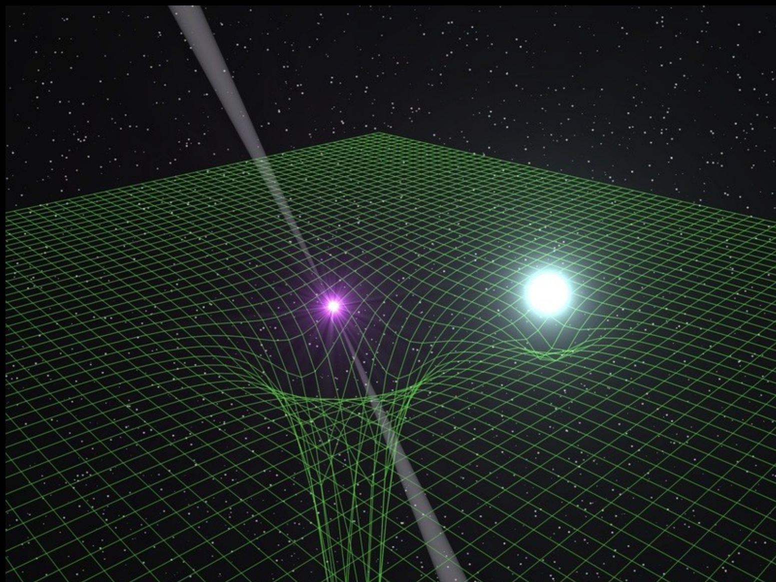
What happens if two compact objects (e.g. black holes) orbit each other?

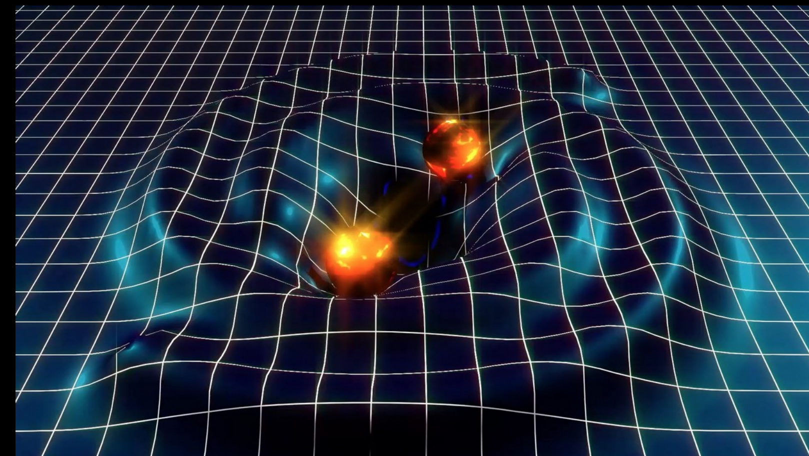




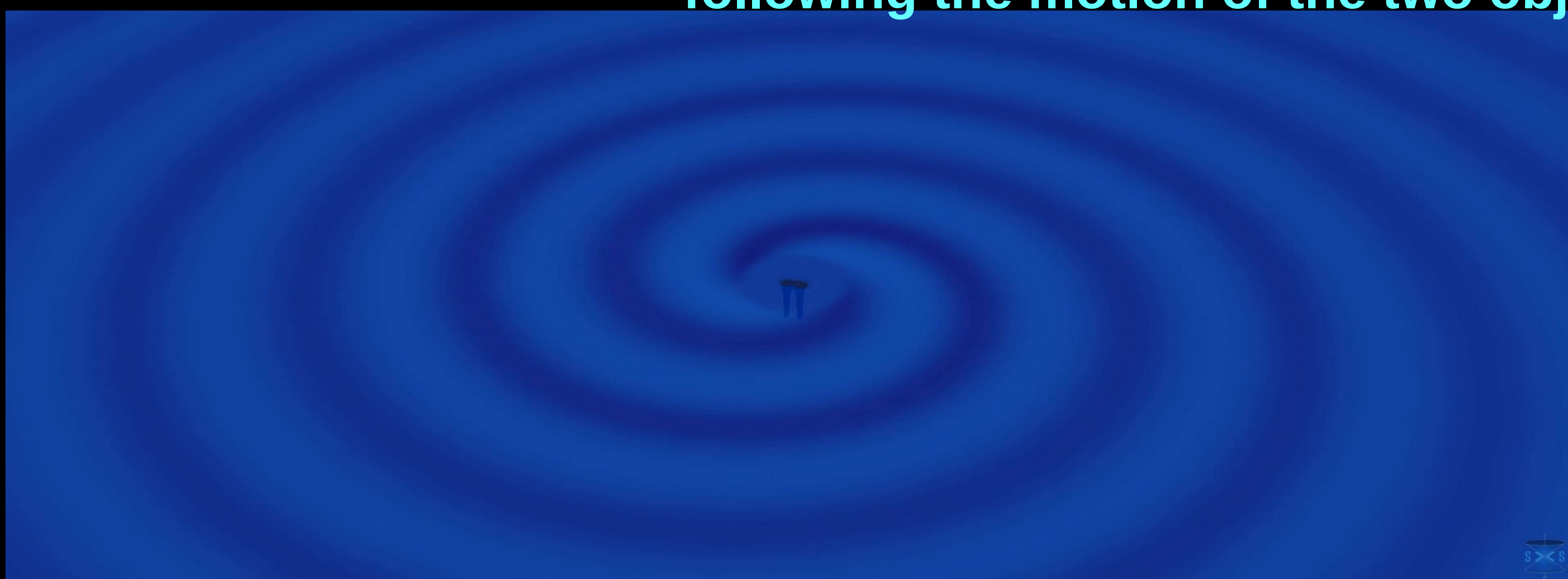
The space distortion changes in time, following the motion of the two objects

What happens if two compact objects (e.g. black holes) orbit each other?



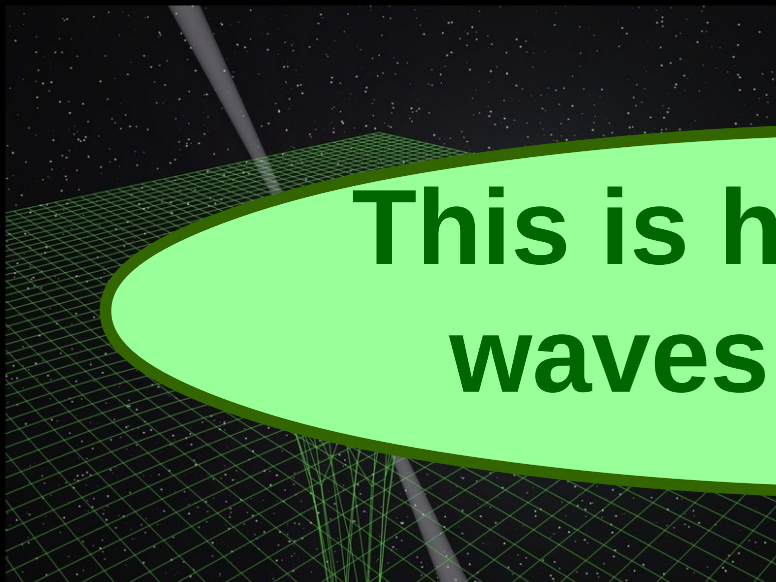


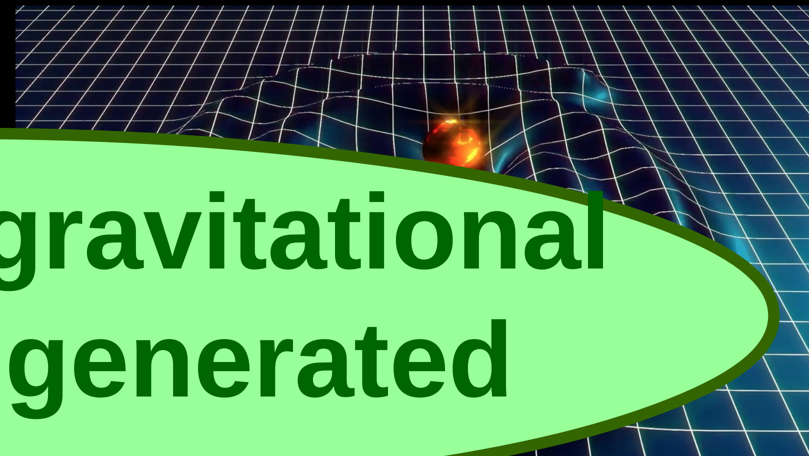
The space distortion **changes in time,** following the motion of the two objects



The distortion propagates outwards at the speed of light

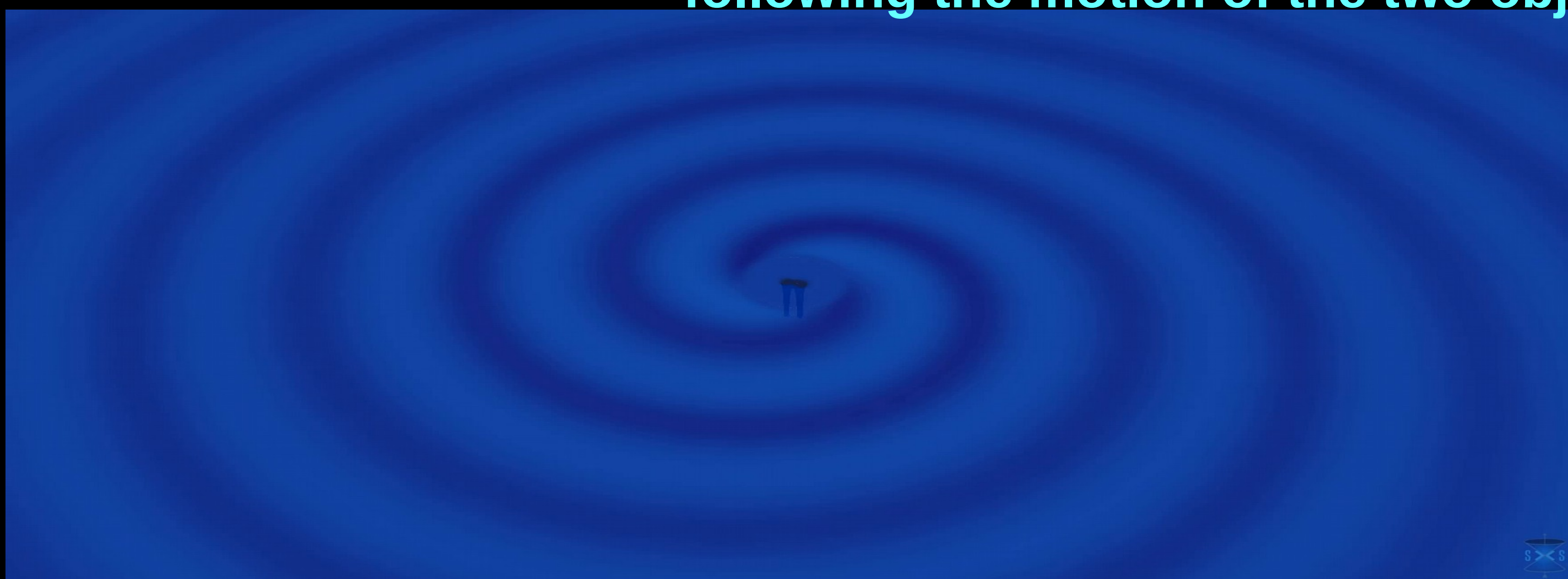
What happens if two compact objects (e.g. black holes) orbit each other?





This is how gravitational waves are generated

The space distortion **changes in time,** following the motion of the two objects



The distortion propagates outwards at the speed of light

Gravitational wave basics

Gravitational waves are natural solutions of linearized Einstein equation. Take a flat metric plus a small perturbation

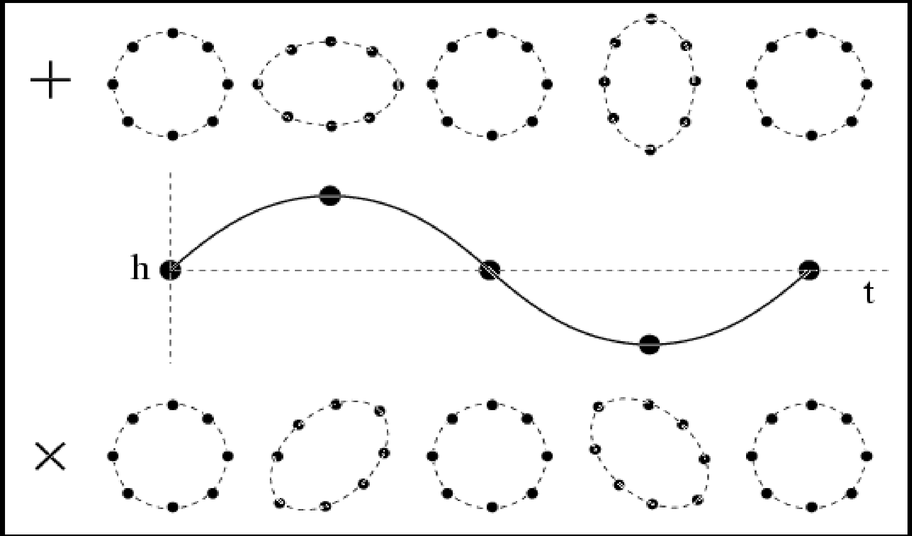
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \ll 1$$

The metric satisfies a wave equation with the wave sourced by *T*

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Vacuum solution (*T*=0) is a tranverse wave with two independent polarizations traveling at the speed of light

$$h_{ij}^{TT}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos\left[\omega\left(t - \frac{z}{c}\right)\right]$$



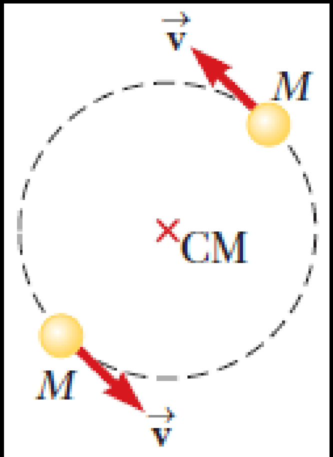
$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_+^{TT} & h_\times^{TT} \\ 0 & 0 & h_\times^{TT} & 1 - h_+^{TT} \end{pmatrix}$$

Solving with the source term provides an expression of the GW strain as a function of the source mass-quadrupole moment

$$h_{ij}^{TT}(t,r) = \frac{2G}{c^4 r}\left[\Lambda_{ij,kl}\ddot{q}^{\,kl}\left(t-\frac{r}{c}\right)\right]$$

Every accelerating mass with non-zero quadrupole mass moment emits gravitational waves

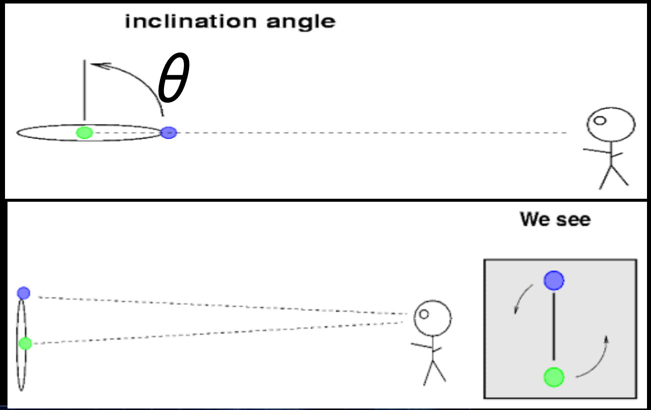
Binary systems are primary sources of gravitational waves!



$$h_+(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\pi f_{gw}t+2\phi)$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\cos\theta\sin(2\pi f_{gw}t+2\phi)$$

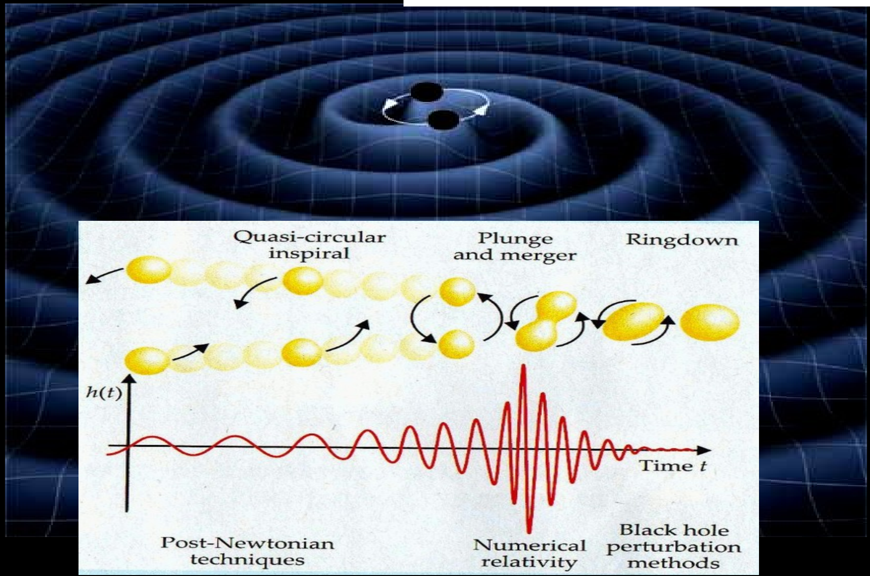
$$M_C = \mu^{3/5}M^{2/5} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$



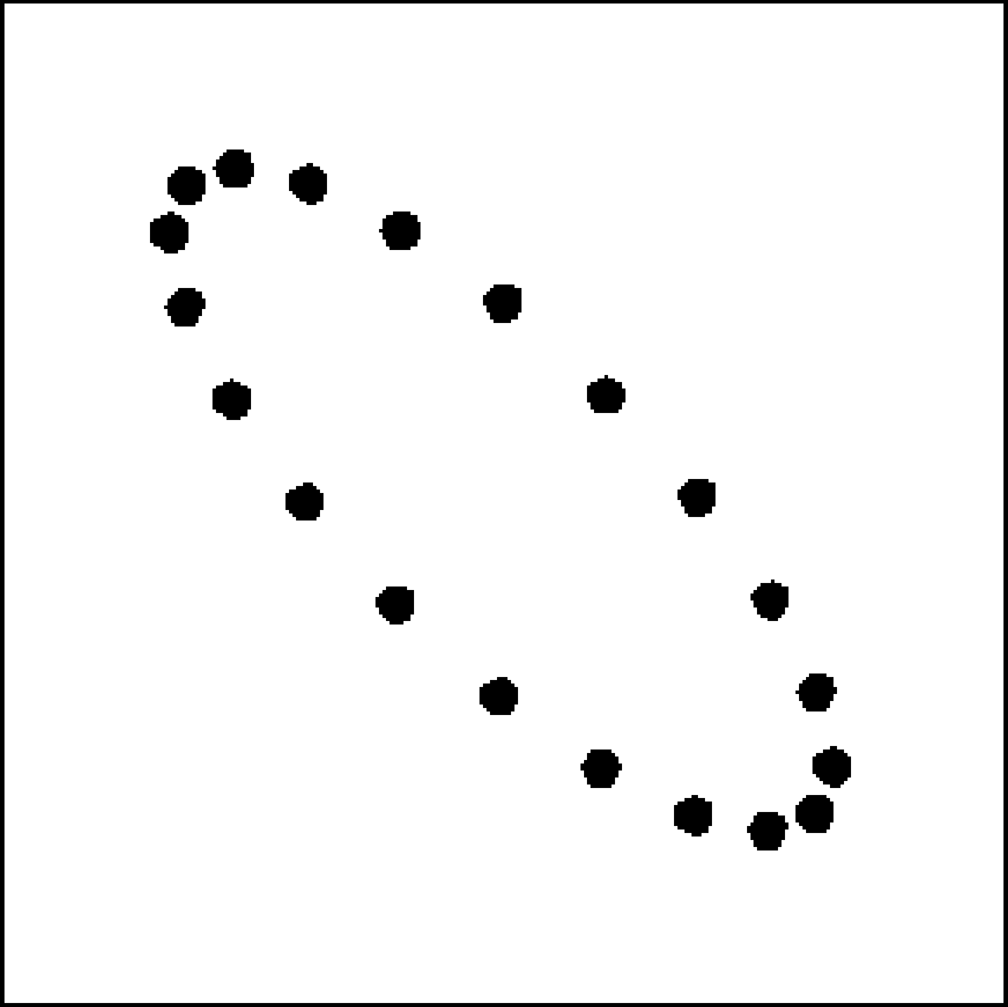
GWs take away energy from the binary system, the luminosity is

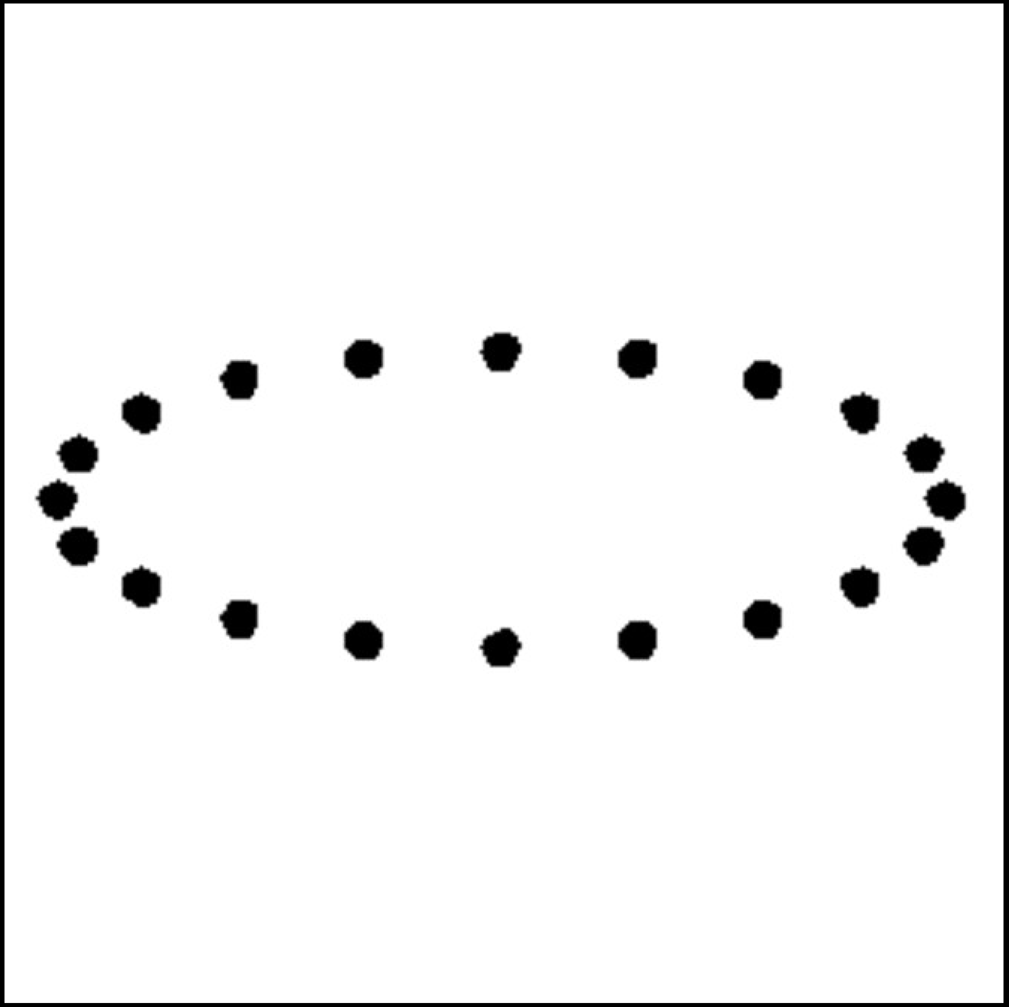
$$\frac{dE_{gw}}{dt} = \frac{32c^5}{5G}\left(\frac{GM_C 2\pi f_{gw}}{2c^3}\right)^{\frac{10}{3}} = P$$

$\sim 10^{-4} c^5/G \sim 10^{56}$ erg/s



-gravitational waves are distortions of the space itself, propagating at the speed of light
-Distortions are perpendicular to the propagation direction
-They have two polarizations





Houston we have a problem: Space is extraordinarily rigid!

Solving Einstein equations we can get the amplitude of the waves as a function of the source parameters:

$$h_+(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\pi f_{gw}t + 2\phi)$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\cos\theta\sin(2\pi f_{gw}t + 2\phi)$$

Mmmm...let's take a 1Ton bar of 1m and spin it a thousand times per second (1000Hz).

Let's measure h at the distance of one meter from the bar…..

$$h = 10^{-35}$$

This is a variation of a billionth of a billionth of a centimetre over a distance of a billion of a billion of centimetres (which is roughly the distance to Proxima Centauri)

We need something more massive!

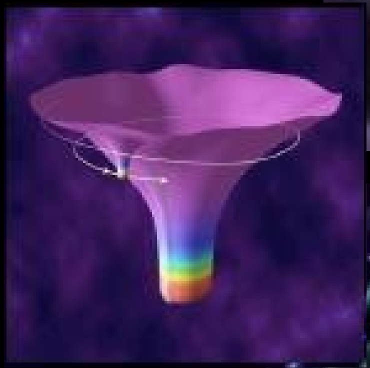
$$h_+(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\pi f_{gw}t + 2\phi)$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_C}{c^2}\right)^{\frac{5}{3}}\left(\frac{\pi f_{gw}}{c}\right)^{\frac{2}{3}}\cos\theta\sin(2\pi f_{gw}t + 2\phi)$$

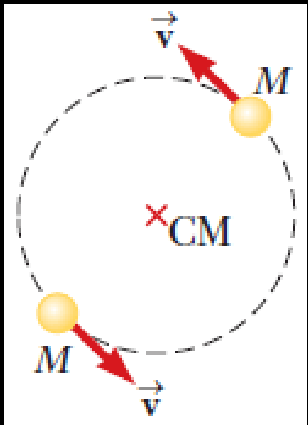
The only possibility is to look at astronomical sources, for example binary systems of compact objects:

1) binaries of black holes or neutron stars

2) compact objects orbiting a supermassive black hole



3) supermassive black hole binaries



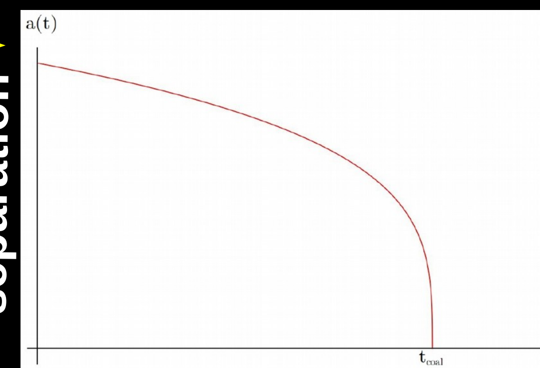
Binary evolution with frequency

$$E = \frac{M_1 v_1^2}{2} + \frac{M_2 v_2^2}{2} - \frac{G M_1 M_2}{r} = \frac{\mu v^2}{2} - \frac{G M \mu}{r} = -\frac{G M \mu}{2a}$$

$$\frac{dE_{\rm rad}}{dt} = \frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{a^5} F(e) = \frac{32}{5}\frac{G^4}{c^5}\frac{M_1^2 M_2^2 (M_1 + M_2)}{a^5} F(e)$$

Equating $dE_{rad}/dt = -dE/dt$ and solving for a we get

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{M_1 M_2 (M_1 + M_2)}{a^3} F(e)$$



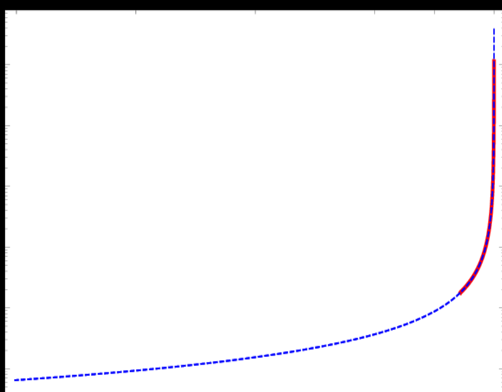
separation

time

Using Kepler's law we finally get

$$\frac{\mathrm{d} f_{\rm r}}{\mathrm{d} t_{\rm r}} = \frac{96}{5} \pi^{8/3} \mathcal{M}^{5/3} f_{\rm r}^{11/3}$$

frequency



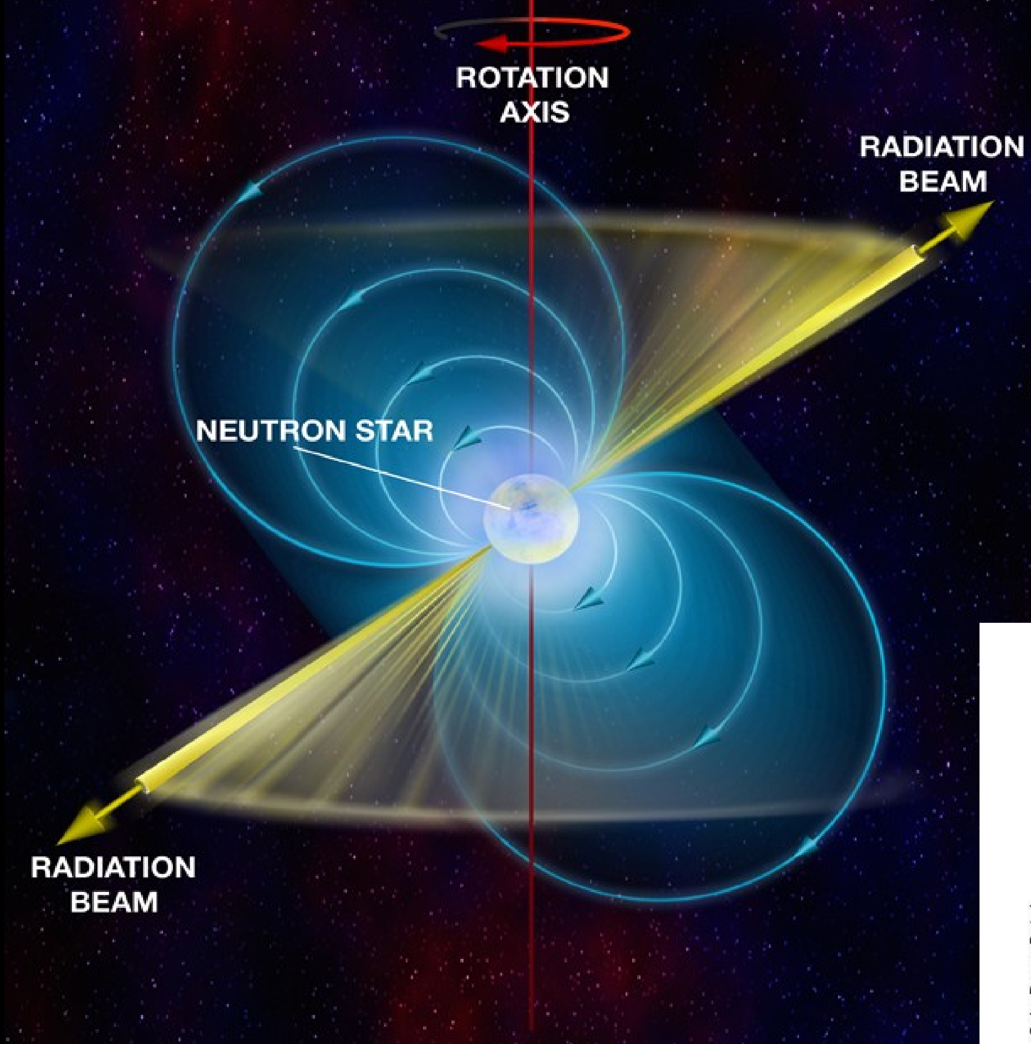
time

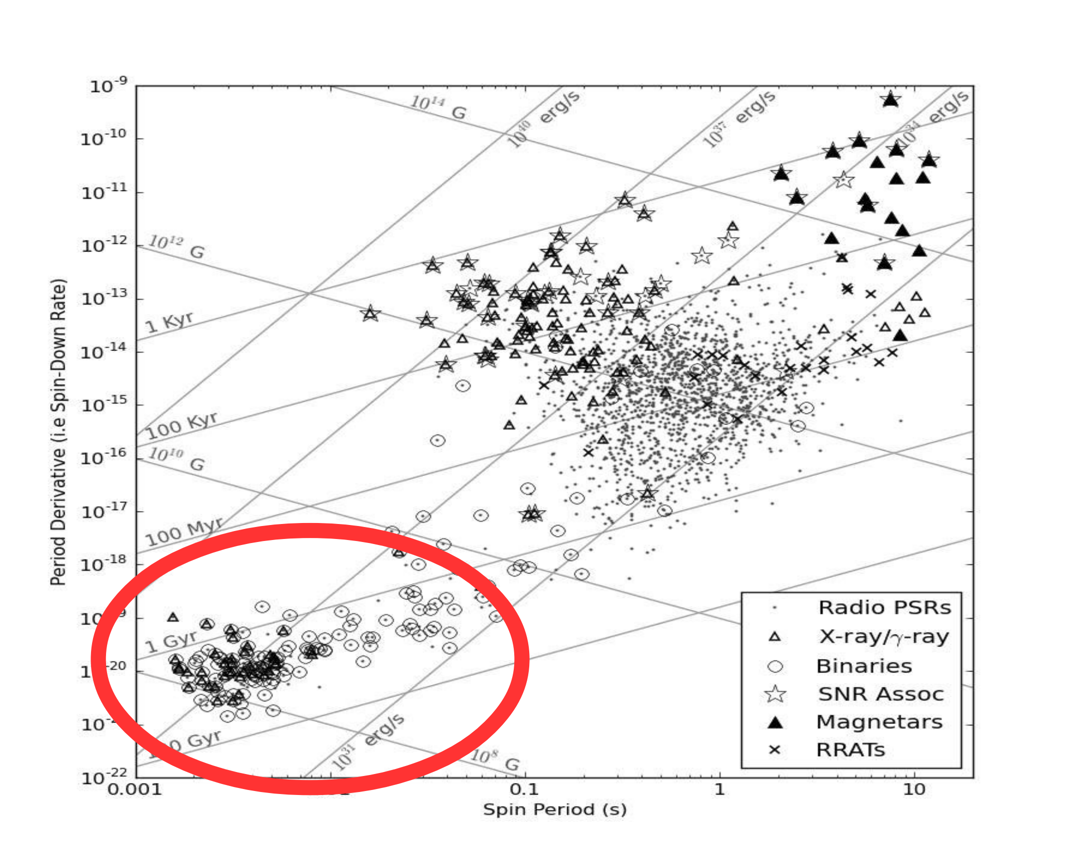
1-Mass proportionality: at a given f, low mass binaries live longer

2-Frequency proportionality: at low f binaries live muuuuch longer

3-Redshift: $f_{obs} = f_r/(1+z)$: high z binaries are observed at lower f

Before GW detection: pulsars

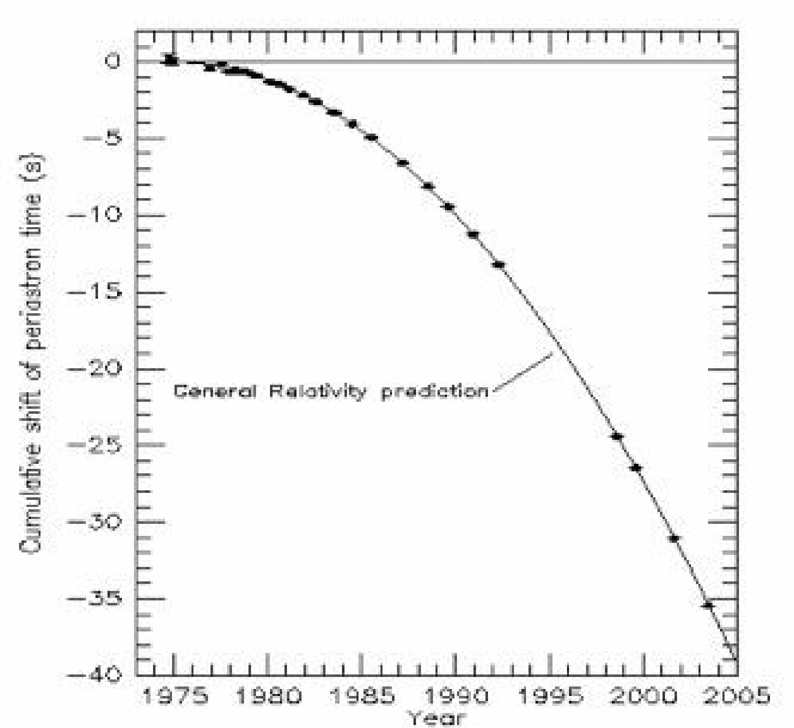




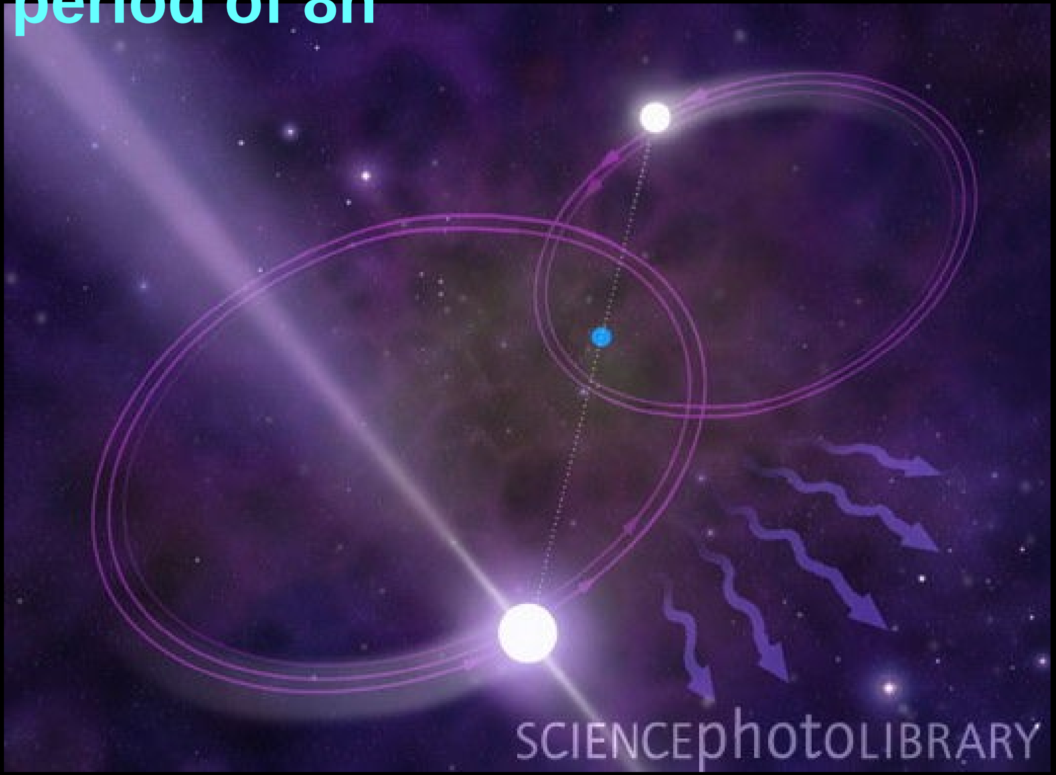
- -M ~1.4 masse solari
- -R~10 km
- -P~0.0014-10 s
- -B~10^8 -10^{15} G

Hulse & Taylor double pulsar

Observed in 1974: two neutron stars orbiting each other (one observed as a pulsar) with a period of 8h



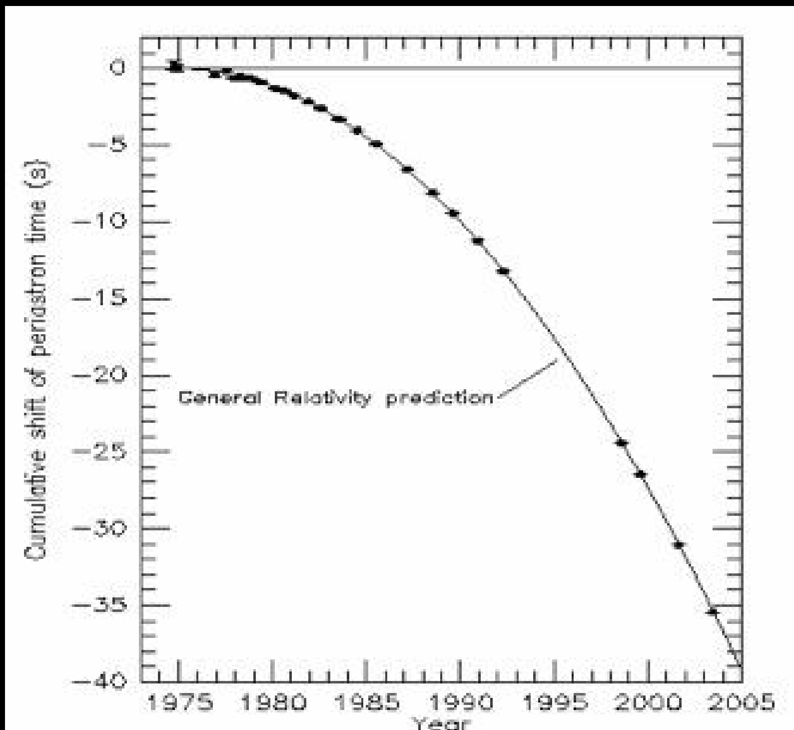
$$L_{gw} = \frac{G}{5c^5}\left\langle \sum_{ij} \frac{d^3}{dt^3}Q_{ij}\left(t-\frac{x}{c}\right)\frac{d^3}{dt^3}Q^{ij}\left(t-\frac{x}{c}\right)\right\rangle$$



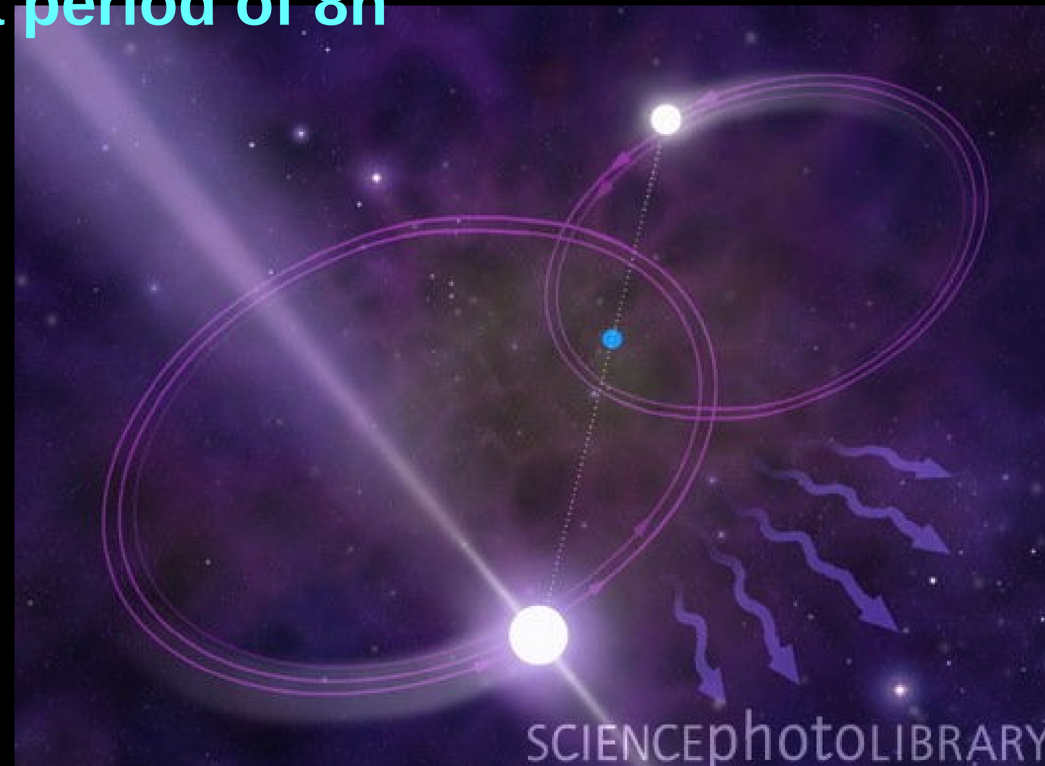
The system is so compact that the variation of the orbital period due to GW emission is observable in the pulsar time of arrivals (doppler shift due to binary orbit).

Hulse & Taylor double pulsar

Observed in 1974: two neutron stars orbiting each other (one observed as a pulsar) with a period of 8h

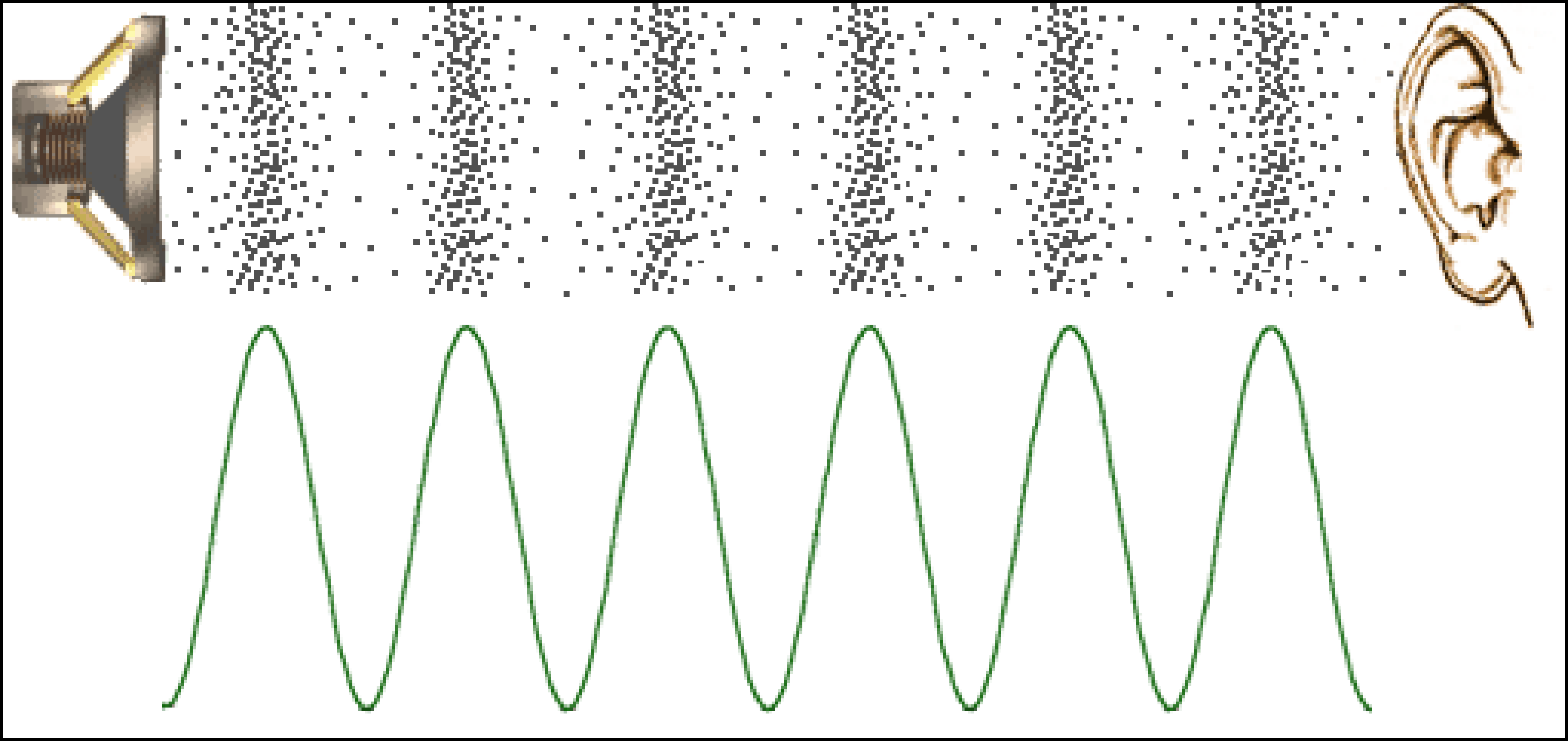


$$L_{gw} = \frac{G}{5c^5}\left\langle \sum_{ij} \frac{d^3}{dt^3}Q_{ij}\left(t-\frac{x}{c}\right)\frac{d^3}{dt^3}Q^{ij}\left(t-\frac{x}{c}\right)\right\rangle$$



The system is so compact that the variation of the orbital period due to GW emission is observable in the pulsar time of arrivals (doppler shift due to binary orbit).

Remember the analogy with sound waves



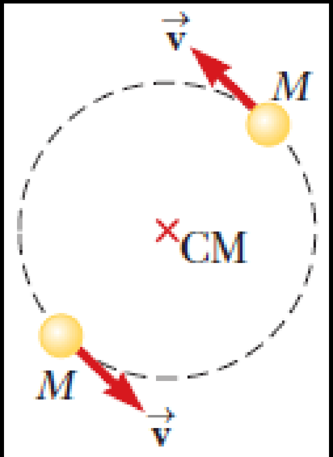
Sound is a compression wave that propagates in a medium (air for example)

A gravitational wave is a distortion that does not need a medium to propagate! It is a distortion of space itself

Heuristic scalings

We need accelerating compact objects
We consider a binary of mass *M* and separation *a*

$$h \sim \frac{R_S}{a}\frac{R_S}{r} \sim \frac{(GM)^{5/3}(\pi f)^{2/3}}{c^4 r}$$

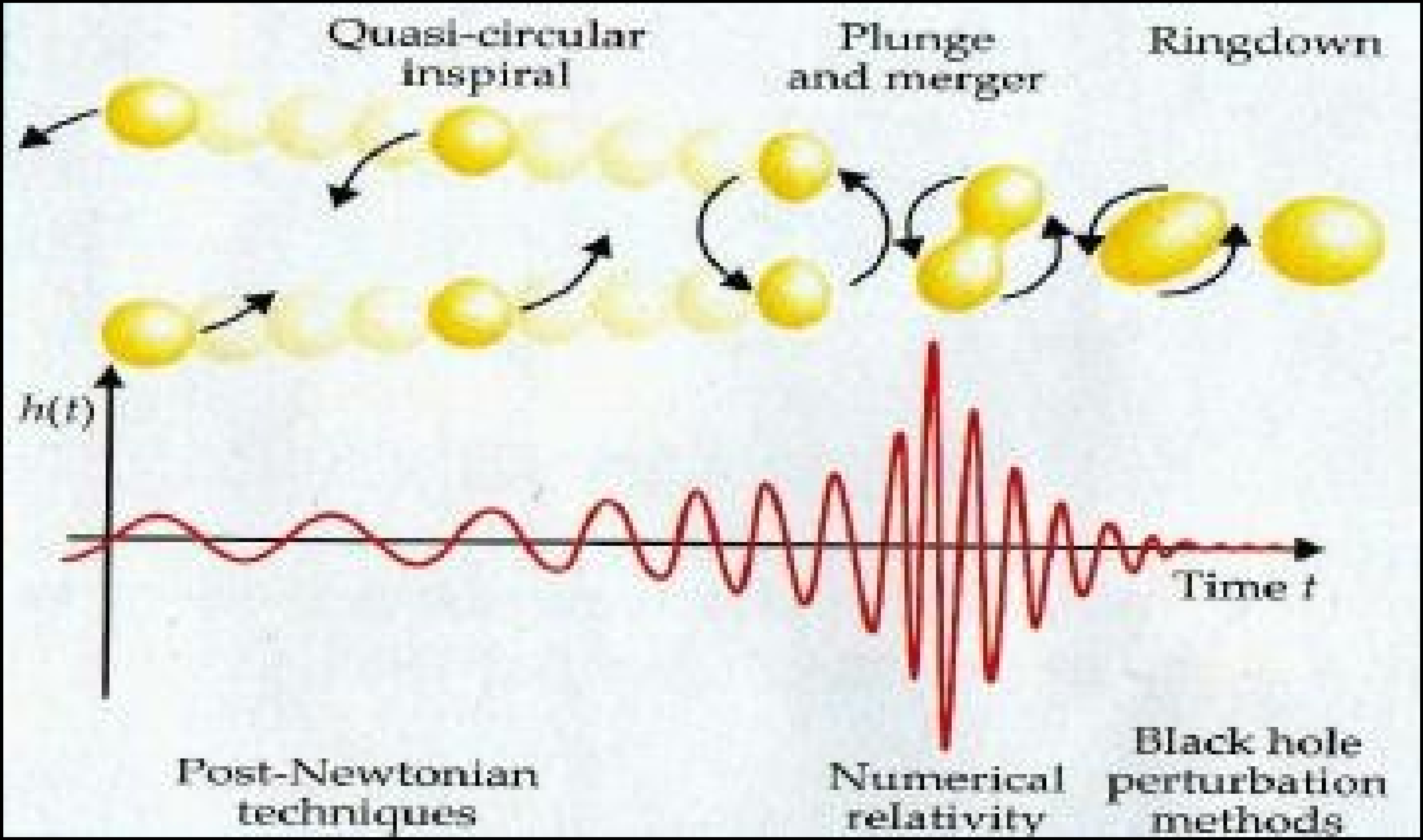


In astrophysical scales

$$h \sim 10^{-20}\,\frac{M}{M_\odot}\,\frac{\mathrm{Mpc}}{D}$$

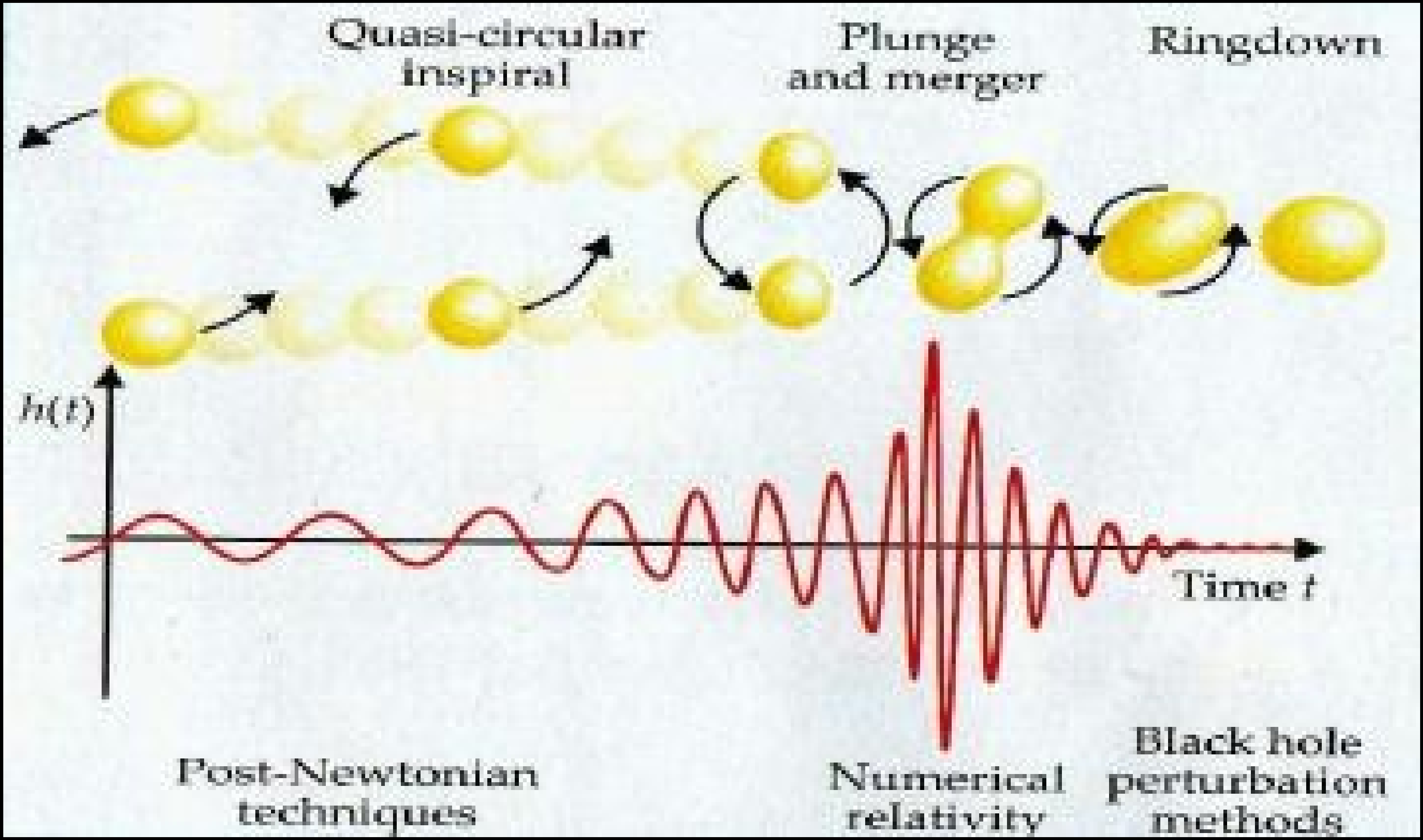
$$f \sim \frac{c}{2\pi R_s} \sim 10^4\,\mathrm{Hz}\,\frac{M_\odot}{M}$$

Let’s take two black holes as an example:



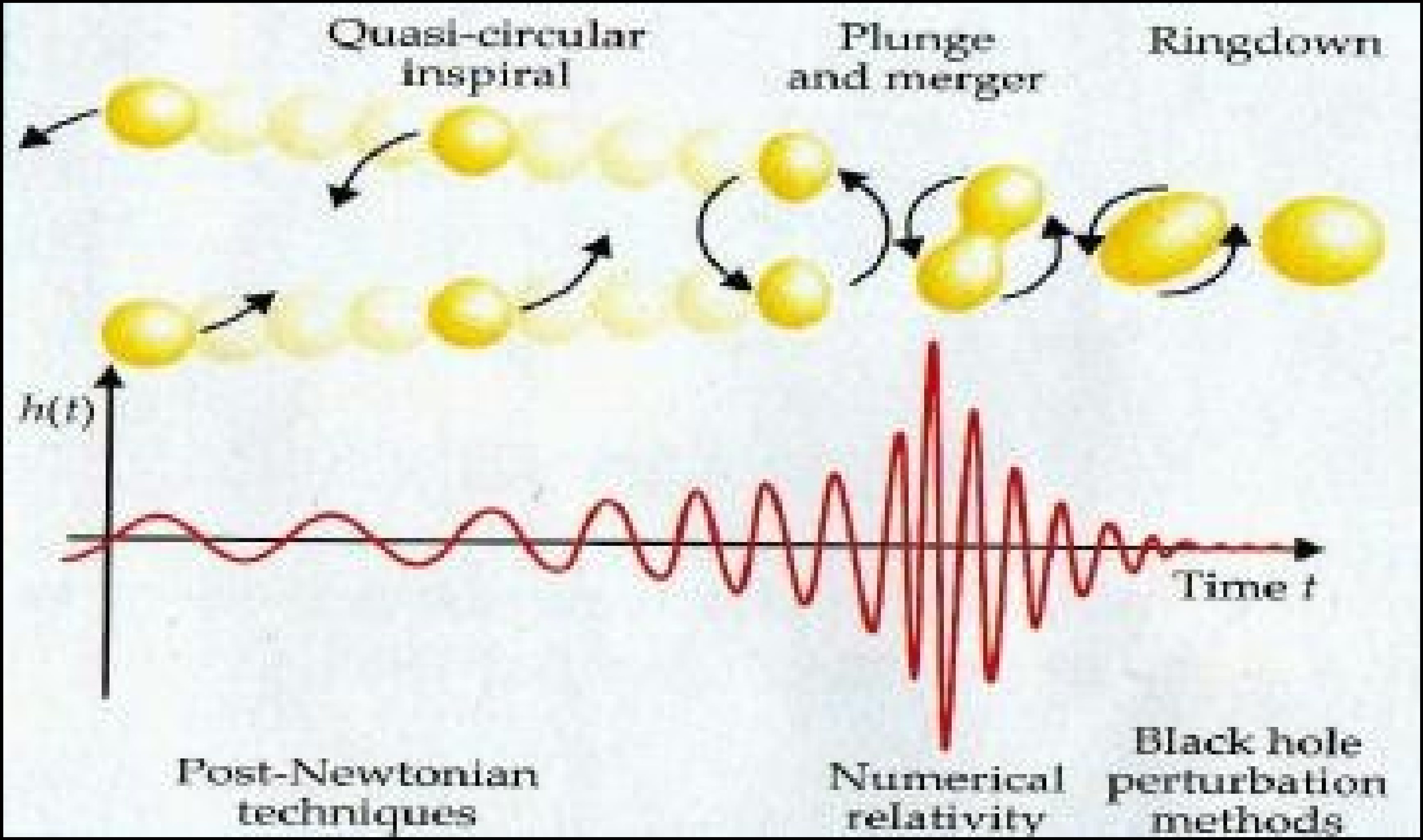
Say two black hole with the mass of 5 Suns

Let's take two black holes as an example:



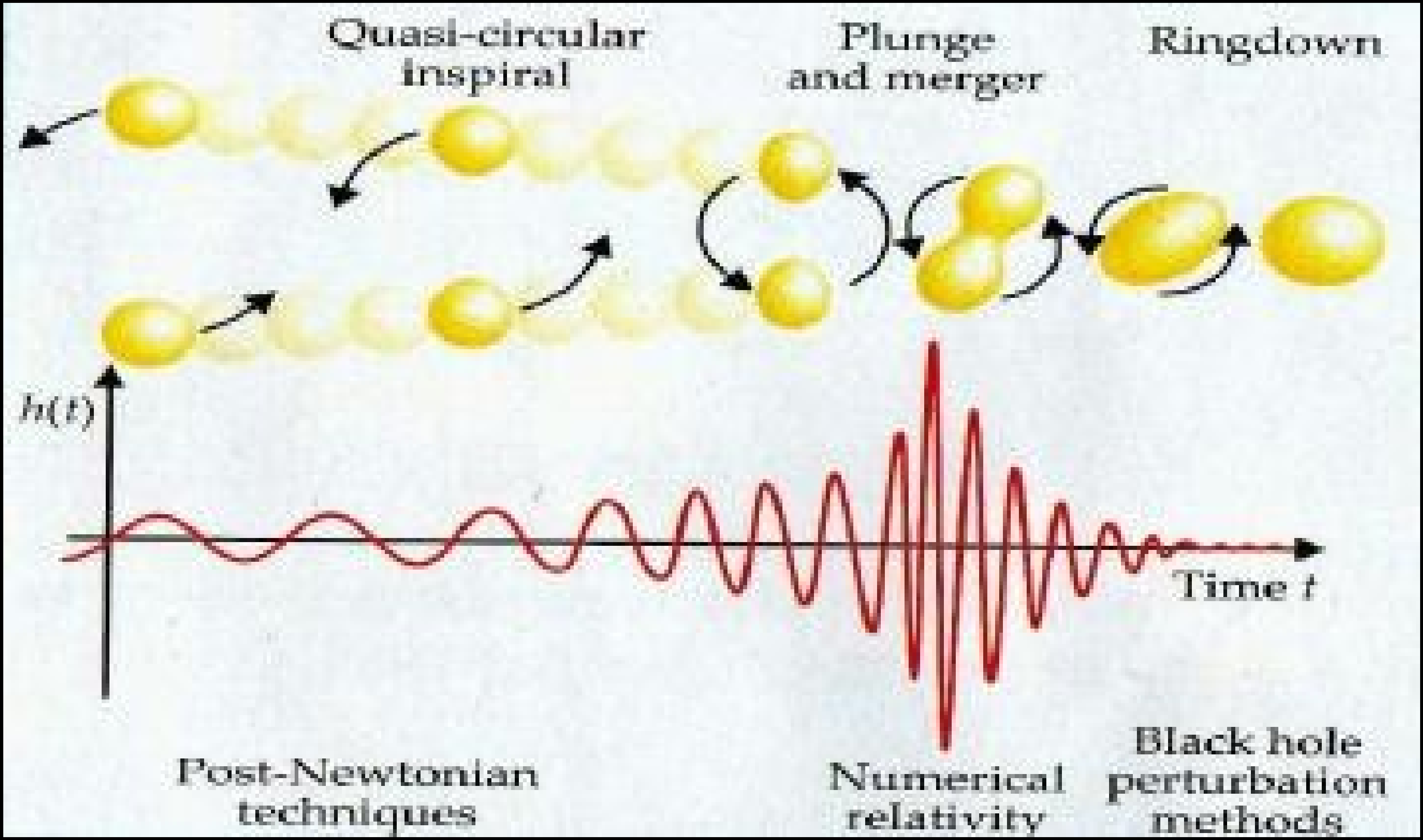
Say two black hole with the mass of 5 Suns

Let's take two black holes as an example:



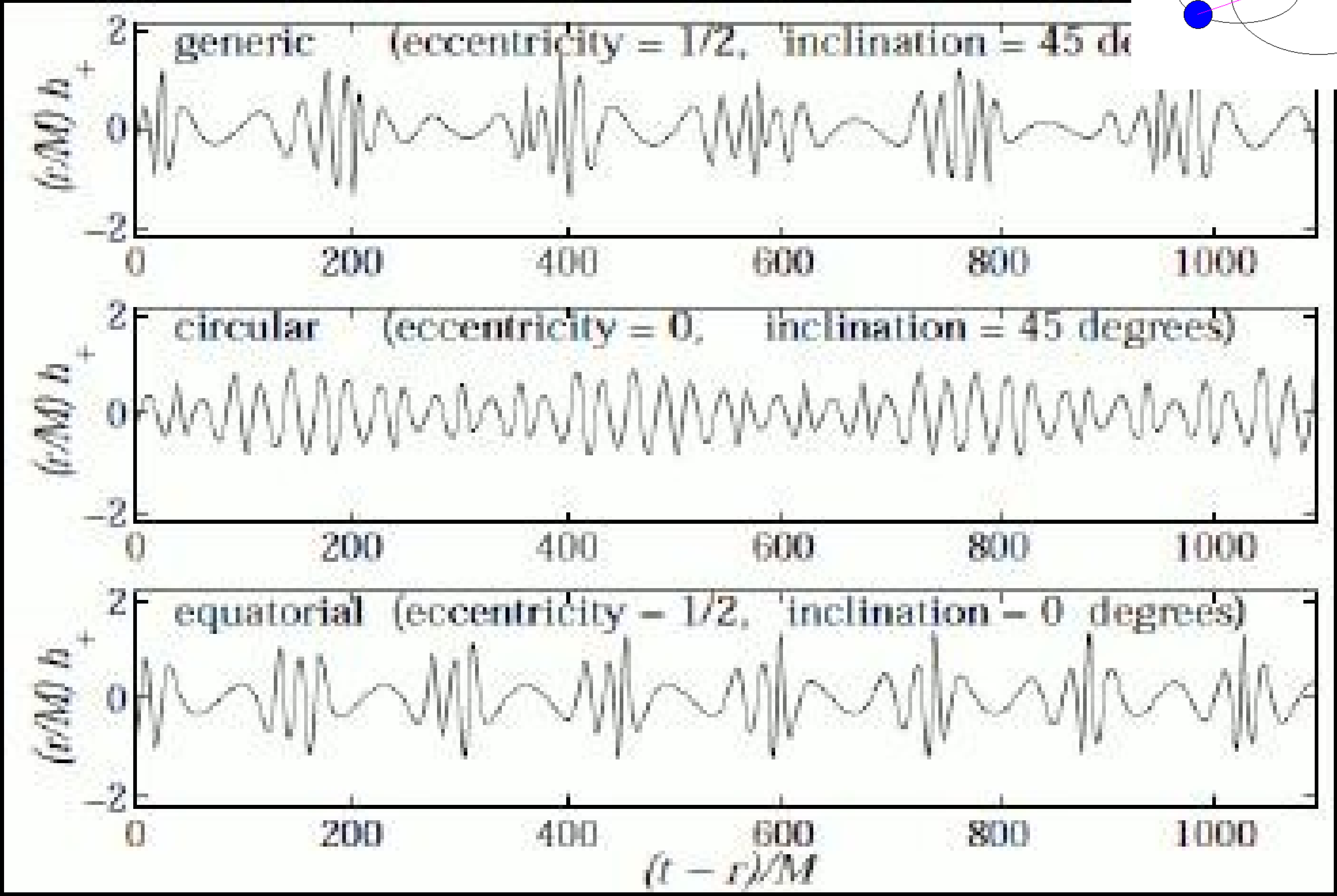
Now a bit fatter, say the mass of 100 Suns

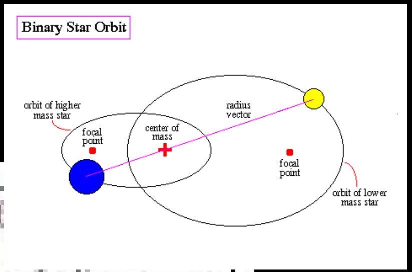
Let's take two black holes as an example:



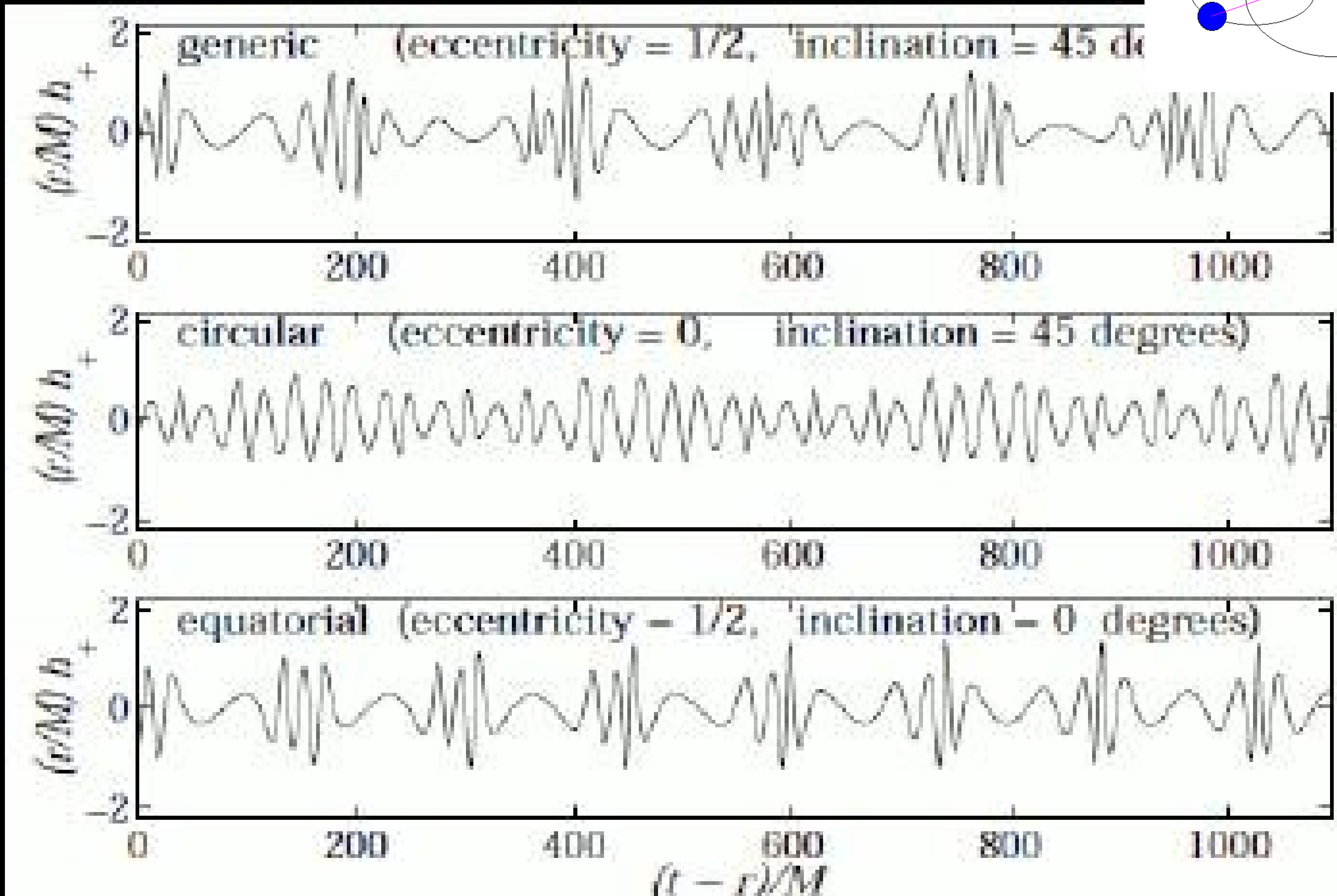
Now a bit fatter, say the mass of 100 Suns

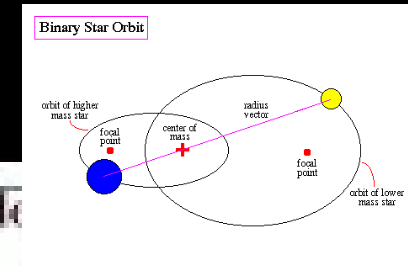
Now on an eccentric orbit





Now on an eccentric orbit





Heuristic scalings

We want compact accelerating systems
Consider a BH binary of mass M, and semimajor axis a

$$h \sim \frac{R_S}{a}\frac{R_S}{r} \sim \frac{(GM)^{5/3}(\pi f)^{2/3}}{c^4 r}$$

In astrophysical scales

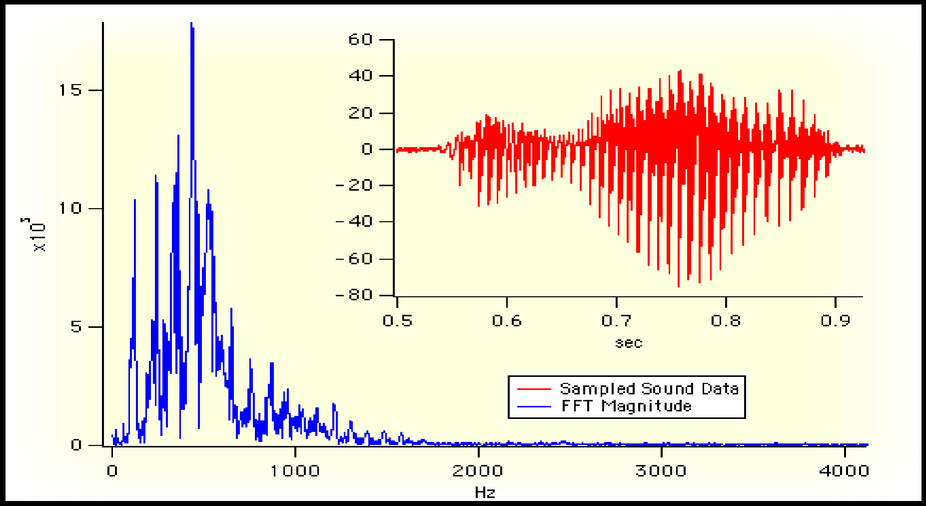
$$h \sim 10^{-20}\frac{M}{M_\odot}\frac{\mathrm{Mpc}}{D}$$

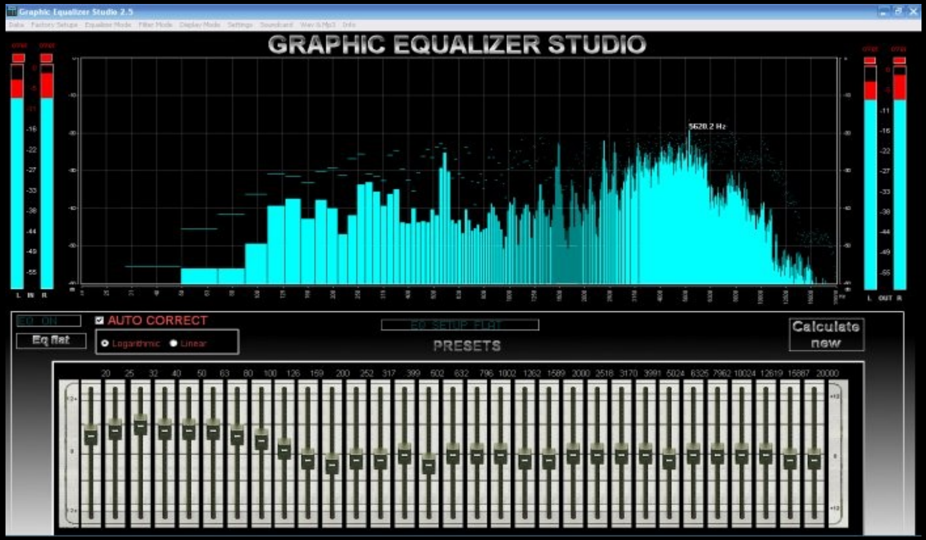
$$f \sim \frac{c}{2\pi R_s} \sim 10^4\,\mathrm{Hz}\frac{M_\odot}{M}$$

10 $M_\odot$ binary at 100 Mpc: $h \sim 10^{-21}$, $f < 10^3$
10^6 $M_\odot$ binary at 10 Gpc: $h \sim 10^{-18}$, $f < 10^{-2}$
10^9 $M_\odot$ binary at 1Gpc: $h \sim 10^{-14}$, $f < 10^{-5}$

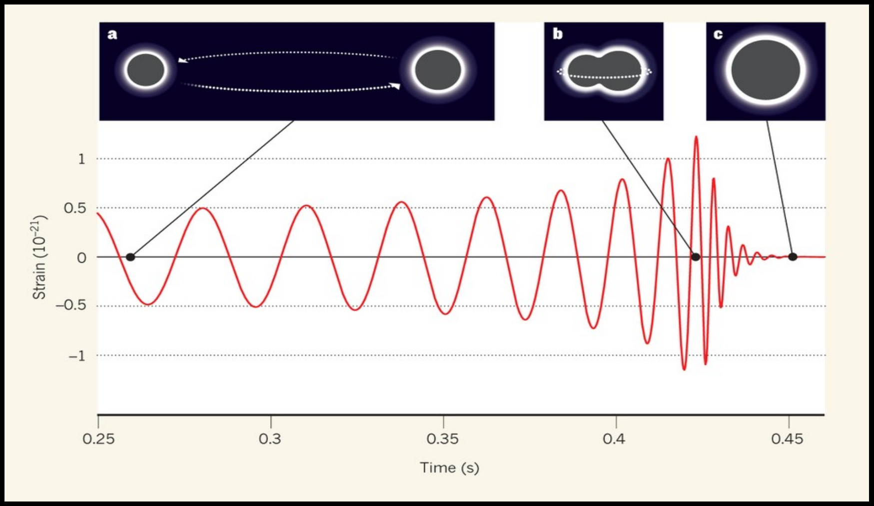
Fourier representation

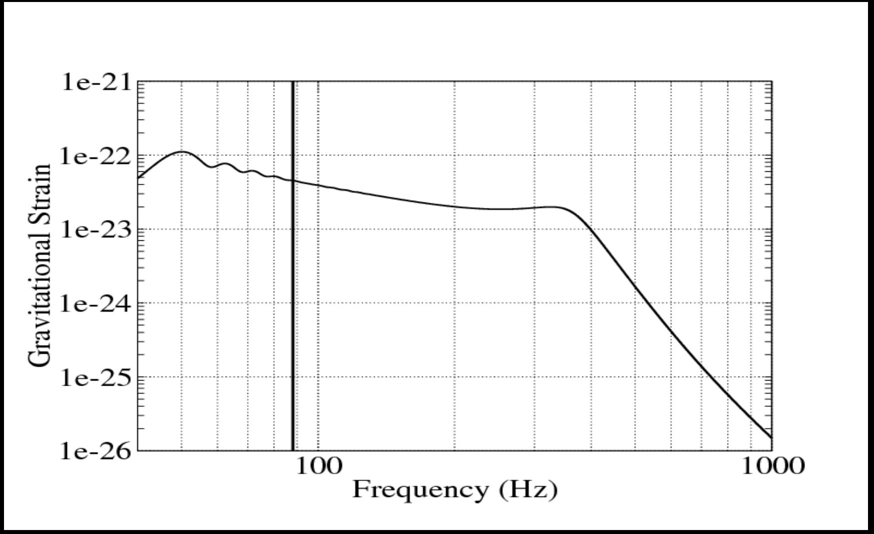
Sound waves



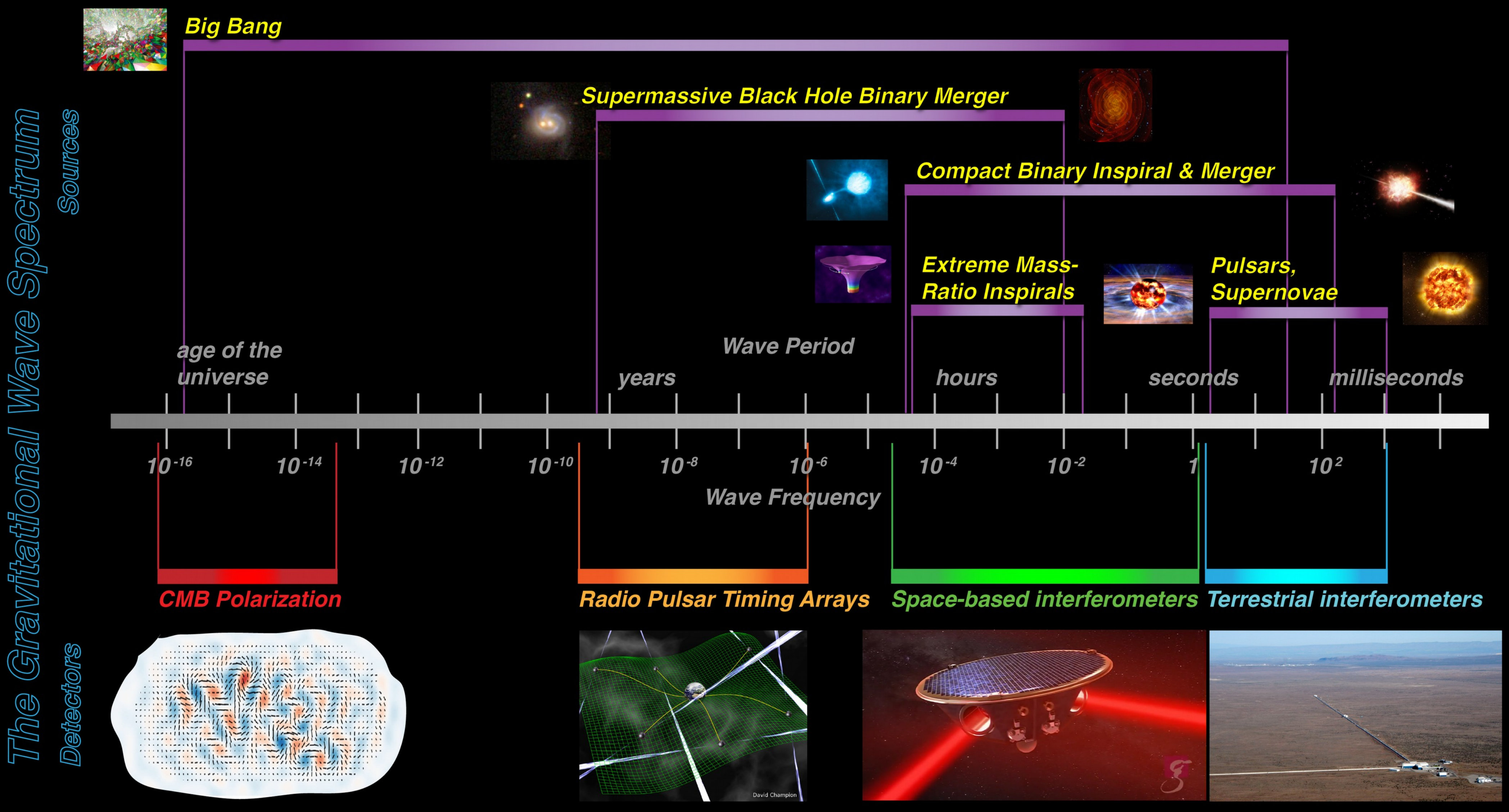


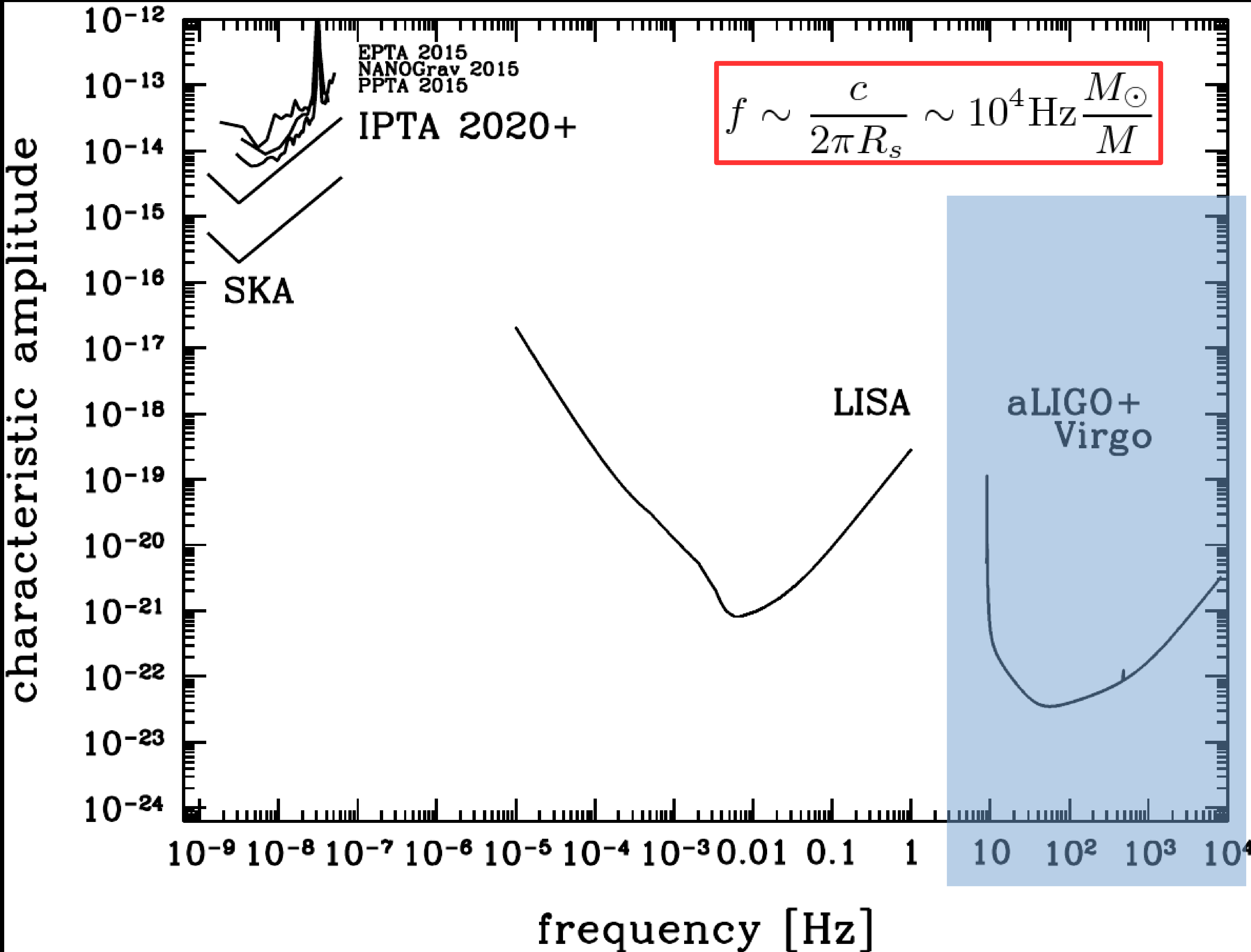
Gravitational waves



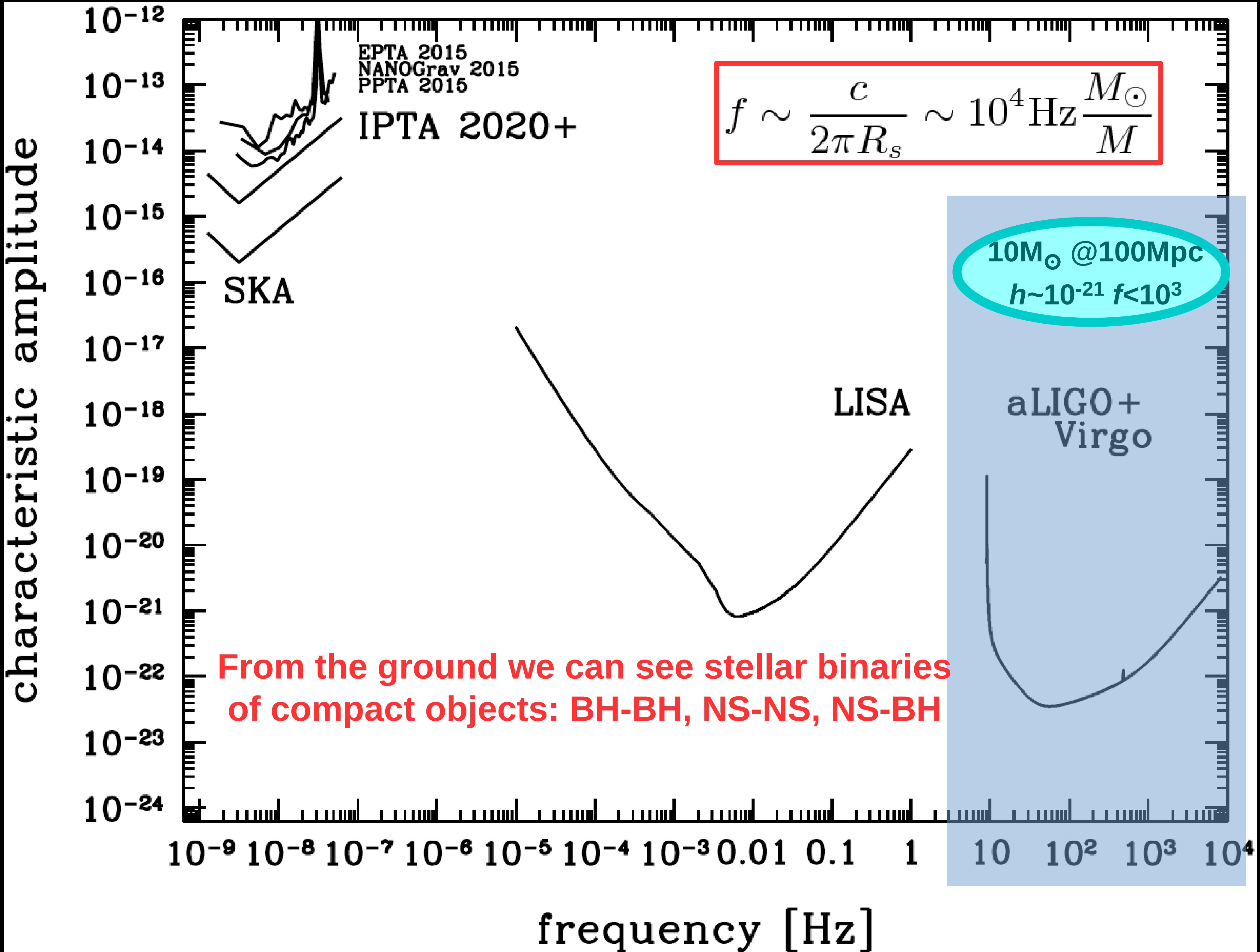


In the frequency domain it is useful to represent the GW with its characteristic strain (strain times the number of cycles per unit log frequency bin). In the inspiral $h_c \propto f^{-1/6}$

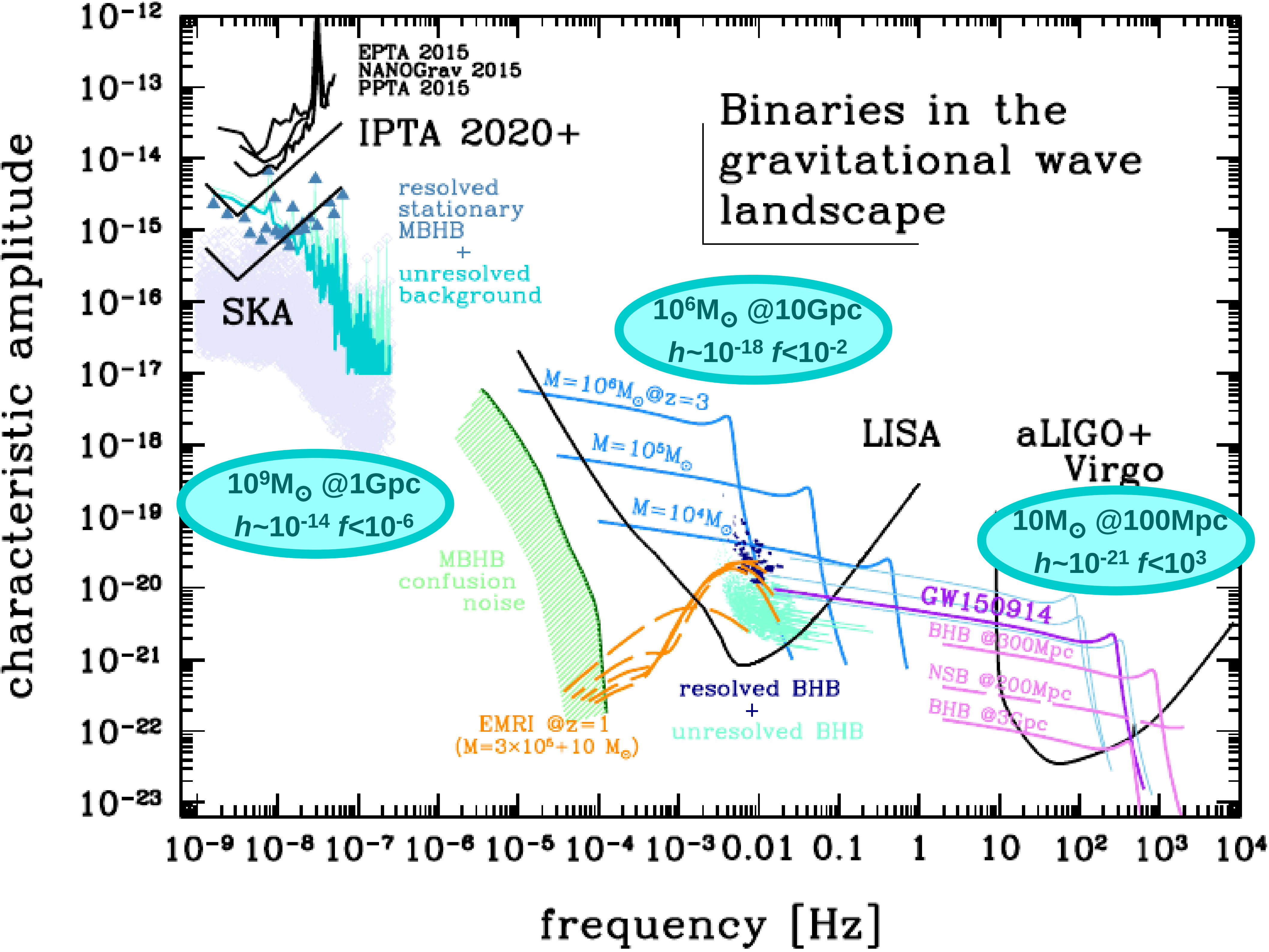

The Gravitational Wave Spectrum
Sources
Big Bang
Supermassive Black Hole Binary Merger
Compact Binary Inspiral & Merger
Extreme Mass-Ratio Inspirals
Pulsars, Supernovae
age of the universe
Wave Period
years
hours
seconds
milliseconds
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
Wave Frequency
CMB Polarization
Radio Pulsar Timing Arrays
Space-based interferometers
Terrestrial interferometers
Detectors
David Champion



EPTA 2015
NANOGrav 2015
PPTA 2015
IPTA 2020+
SKA
LISA
aLIGO+
Virgo
$f \sim \frac{c}{2\pi R_s} \sim 10^4 \mathrm{Hz} \frac{M_\odot}{M}$
characteristic amplitude
frequency [Hz]
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 0.01 0.1 1 10 10^2 10^3 10^4



characteristic amplitude
frequency [Hz]
EPTA 2015
NANOGrav 2015
PPTA 2015
IPTA 2020+
SKA
LISA
aLIGO+ Virgo
$f \sim \frac{c}{2\pi R_s} \sim 10^4 \mathrm{Hz} \frac{M_\odot}{M}$
$10M_\odot$ @100Mpc
$h \sim 10^{-21}$ $f < 10^3$
From the ground we can see stellar binaries of compact objects: BH-BH, NS-NS, NS-BH



EPTA 2015
NANOGrav 2015
PPTA 2015
IPTA 2020+
SKA
resolved stationary MBHB + unresolved background
Binaries in the gravitational wave landscape
10⁶M☉ @10Gpc h~10⁻¹⁸ f<10⁻²
10⁹M☉ @1Gpc h~10⁻¹⁴ f<10⁻⁶
10M☉ @100Mpc h~10⁻²¹ f<10³
M=10⁶M☉@z=3
M=10⁵M☉
M=10⁴M☉
LISA
aLIGO+ Virgo
MBHB confusion noise
GW150914
BHB @300Mpc
NSB @200Mpc
BHB @3Gpc
resolved BHB + unresolved BHB
EMRI @z=1 (M=3×10⁵+10 M☉)
characteristic amplitude
frequency [Hz]