Investigating light polarization with mathematical and experimental tools

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Project AIM

What is our goal?

Facilitate the process of **teaching contemporary quantum physics** in **regular** high school **classroom activities**

Project strategy

How to do it?

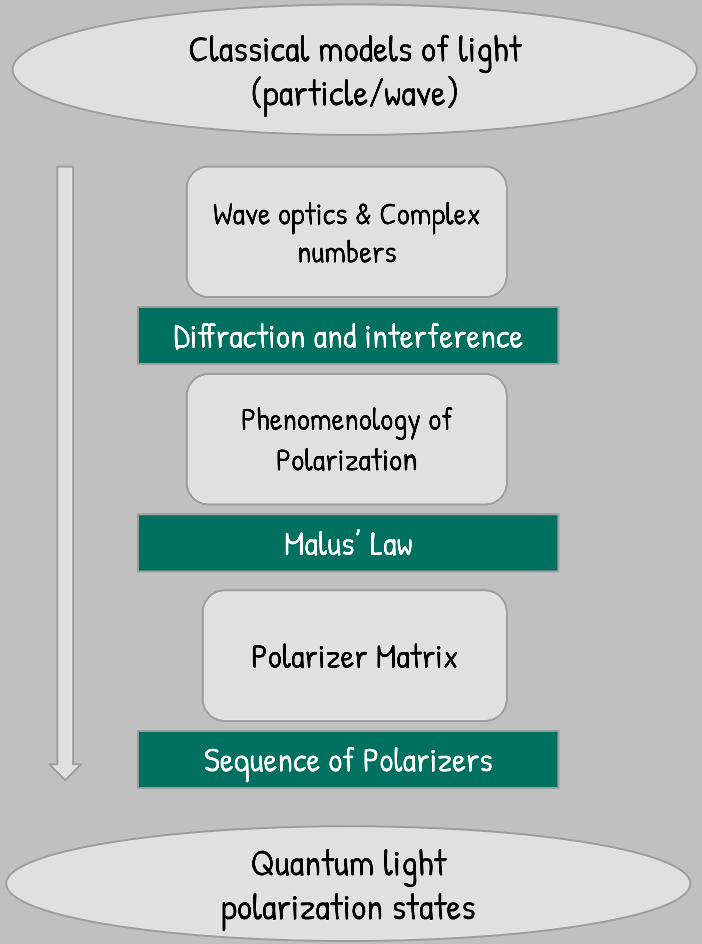
Promoting the **use of mathematical concepts and tools**, such as complex numbers, vectors and matrices to generate significative **connections** between physical phenomena and experiments.

the Maths of Light Polarization

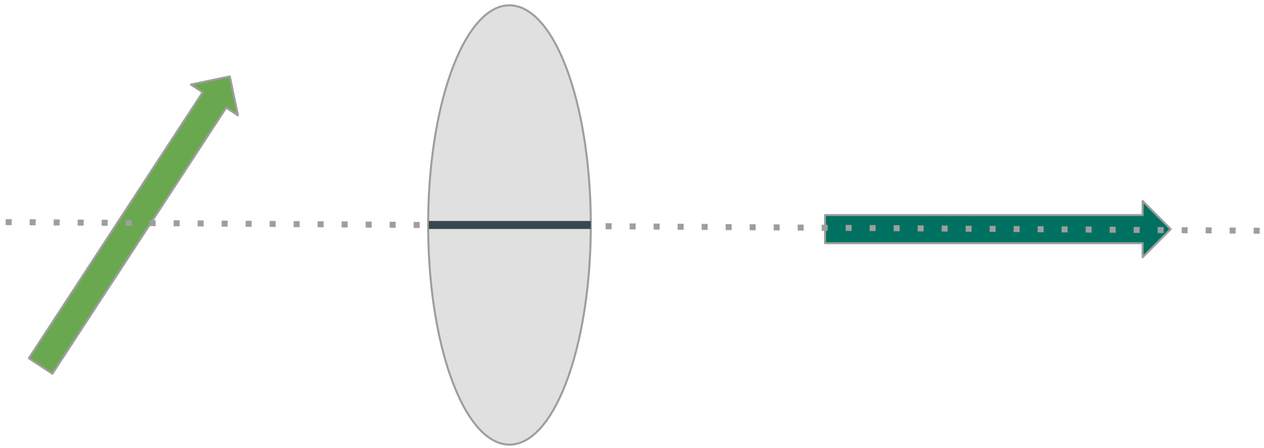
Teaching - Learning Sequence in a nutshell

"[...] to promote the comprehension of the fundamental role of linear algebra (vectors and matrices) in physics"
[**Italian Maths Curriculum learning outcomes**]

- **Significative context** for complex numbers.
- Introduce an **effective formalism** to analyze the **state** of **quantum objects** and their **evolution**.
- Experimental activities to support conceptual understanding.



Polarizers and their mathematical representation



Task 1
Find the matrix of the H polarizer

$$\begin{bmatrix} E_{0X} \\ E_{0Y} \end{bmatrix}$$

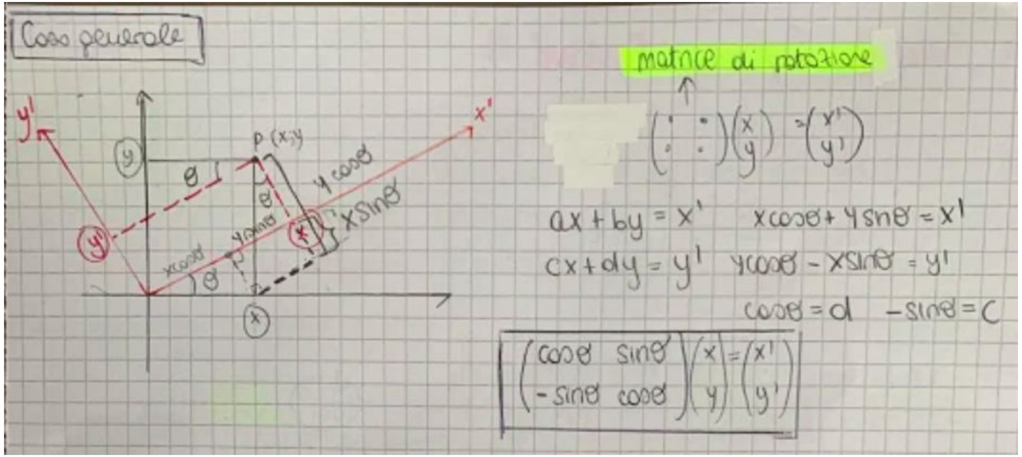
$$P_H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} E_{0X} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_{0X} \\ E_{0Y} \end{bmatrix}$$

Polarizers and their mathematical representation

Generalization:

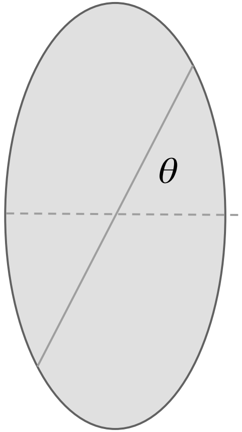
the polarizer action could be represented as a “rotation” of the “light vector”



Task 2
Find the rotation matrix

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Polarizers and their mathematical representation



$$P_\theta = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$$

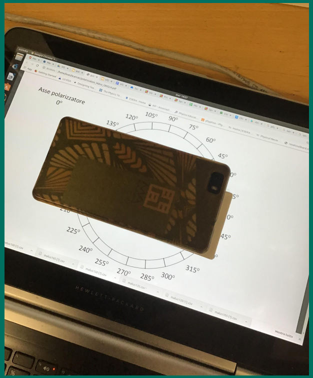
$$P_\theta := R(-\theta)\, P_H\, R(\theta)$$

$$P_H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Task 3
Find the polarizer matrix

Malus’ Law

Experimental activity

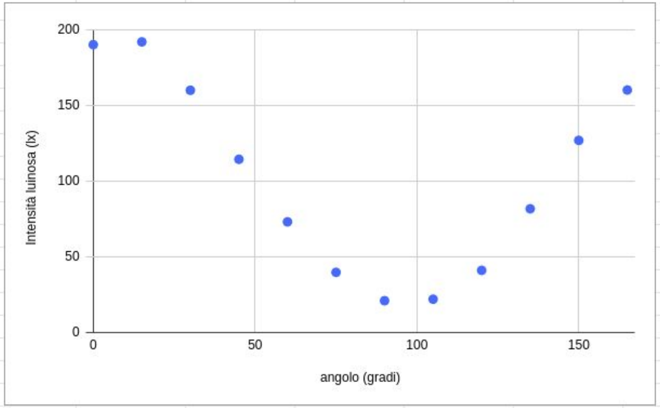


$$E_0 = A\begin{bmatrix}\cos\theta_0\\ \sin\theta_0\end{bmatrix} \qquad I_0 = |A|^2$$

$$E_1 = \begin{bmatrix}\cos^2\theta_1 & \sin\theta_1\cos\theta_1\\ \sin\theta_1\cos\theta_1 & \sin^2\theta_1\end{bmatrix}E_0 = A\begin{bmatrix}\cos^2\theta_1 & \sin\theta_1\cos\theta_1\\ \sin\theta_1\cos\theta_1 & \sin^2\theta_1\end{bmatrix}\begin{bmatrix}\cos\theta_0\\ \sin\theta_0\end{bmatrix}$$

$$= A\begin{bmatrix}\cos\theta_0\cos^2\theta_1 + \sin\theta_0\sin\theta_1\cos\theta_1\\ \cos\theta_0\sin\theta_1\cos\theta_1 + \sin\theta_0\sin^2\theta_1\end{bmatrix} = A\cos(\theta_1-\theta_0)\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix}$$

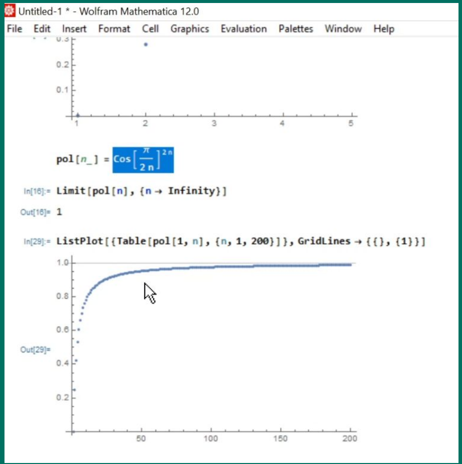
		Intensità (lx)	
angolo(°)	angolo (rad)	media	devst
0	0	190,25	4,652188425
15	0,2617993878	192,1	3,842742077
30	0,5235987756	160,1	2,726414006
45	0,7853981634	114,4615385	3,230463966
60	1,047197551	73,14285714	2,672612419
75	1,308996939	39,75	5,970163913
90	1,570796327	21	4,816637832
105	1,832595715	22	3,082207001
120	2,094395102	41,07142857	5,676730989
135	2,35619449	81,76923077	3,419401657
150	2,617993878	127	2,449489743
165	2,879793266	160,2857143	2,870208222

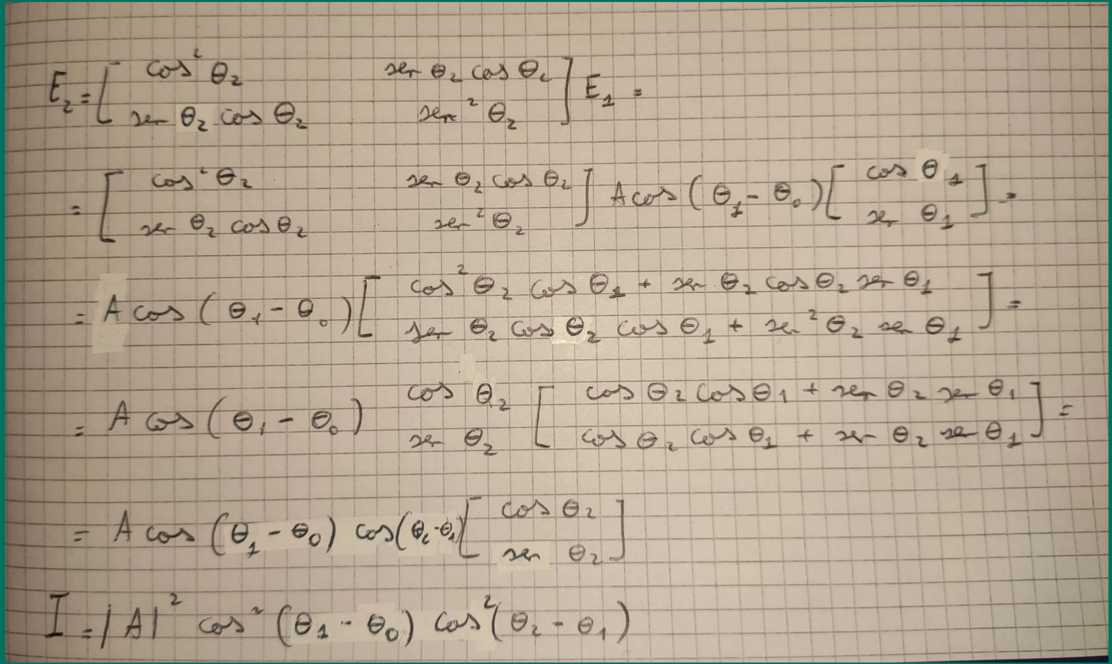


Sequence of polarizers

Mathematical exploration

In this activity students **analyze** how light intensity changes in a sequence of polarizers.





$$E_2 = \begin{bmatrix} \cos^2\theta_2 & \operatorname{sen}\theta_2\cos\theta_2 \\ \operatorname{sen}\theta_2\cos\theta_2 & \operatorname{sen}^2\theta_2 \end{bmatrix} E_1 =$$

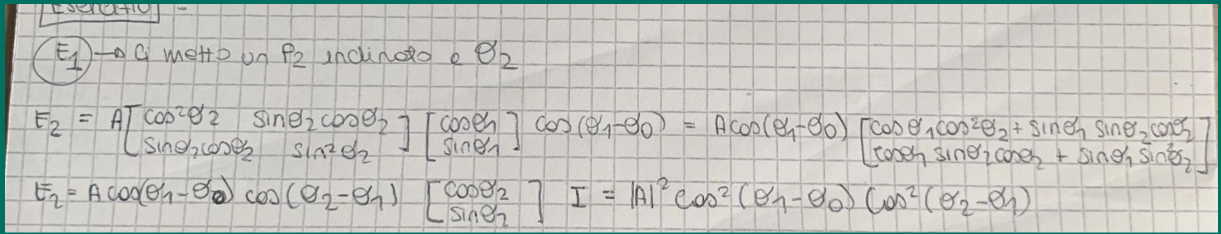
$$= \begin{bmatrix} \cos^2\theta_2 & \operatorname{sen}\theta_2\cos\theta_2 \\ \operatorname{sen}\theta_2\cos\theta_2 & \operatorname{sen}^2\theta_2 \end{bmatrix} A\cos(\theta_1-\theta_0)\begin{bmatrix} \cos\theta_1 \\ \operatorname{sen}\theta_1 \end{bmatrix} =$$

$$= A\cos(\theta_1-\theta_0)\begin{bmatrix} \cos^2\theta_2\cos\theta_1 + \operatorname{sen}\theta_2\cos\theta_2\operatorname{sen}\theta_1 \\ \operatorname{sen}\theta_2\cos\theta_2\cos\theta_1 + \operatorname{sen}^2\theta_2\operatorname{sen}\theta_1 \end{bmatrix} =$$

$$= A\cos(\theta_1-\theta_0)\begin{matrix} \cos\theta_2 \\ \operatorname{sen}\theta_2 \end{matrix}\begin{bmatrix} \cos\theta_2\cos\theta_1 + \operatorname{sen}\theta_2\operatorname{sen}\theta_1 \\ \cos\theta_2\cos\theta_1 + \operatorname{sen}\theta_2\operatorname{sen}\theta_1 \end{bmatrix} =$$

$$= A\cos(\theta_1-\theta_0)\cos(\theta_2-\theta_1)\begin{bmatrix} \cos\theta_2 \\ \operatorname{sen}\theta_2 \end{bmatrix}$$

$$I = |A|^2\cos^2(\theta_1-\theta_0)\cos^2(\theta_2-\theta_1)$$



E_1 → Ci metto un P_2 inclinato a θ_2

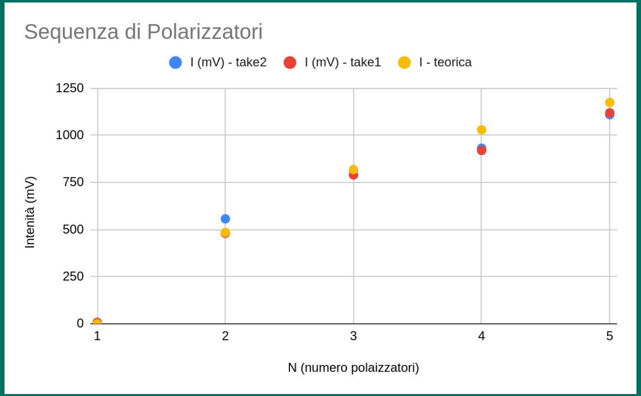
$$E_2 = A\begin{bmatrix} \cos^2\theta_2 & \sin\theta_2\cos\theta_2 \\ \sin\theta_2\cos\theta_2 & \sin^2\theta_2 \end{bmatrix}\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}\cos(\theta_1-\theta_0) = A\cos(\theta_1-\theta_0)\begin{bmatrix} \cos\theta_1\cos^2\theta_2 + \sin\theta_1\sin\theta_2\cos\theta_2 \\ \cos\theta_1\sin\theta_2\cos\theta_2 + \sin\theta_1\sin^2\theta_2 \end{bmatrix}$$

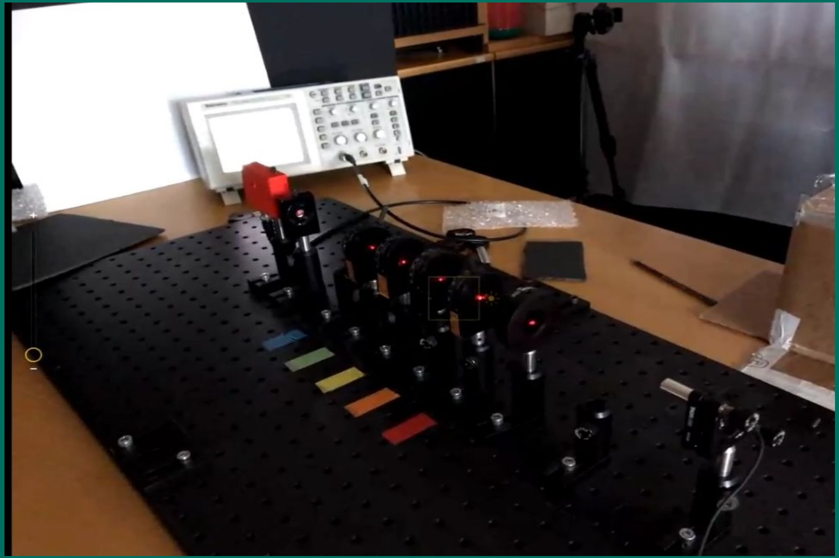
$$E_2 = A\cos(\theta_1-\theta_0)\cos(\theta_2-\theta_1)\begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix} \quad I = |A|^2\cos^2(\theta_1-\theta_0)\cos^2(\theta_2-\theta_1)$$

Sequence of polarizers

Experimental activity

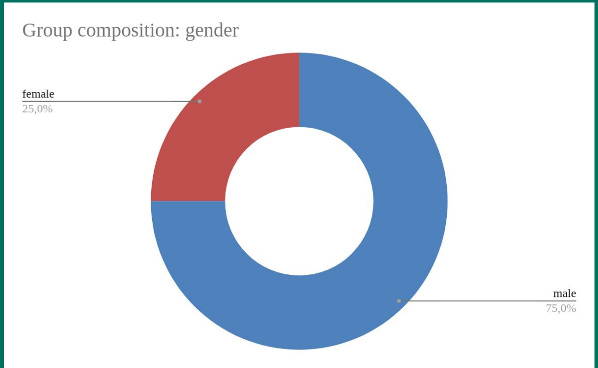
In this activity students **measure** how light intensity changes in a sequence of polarizers.

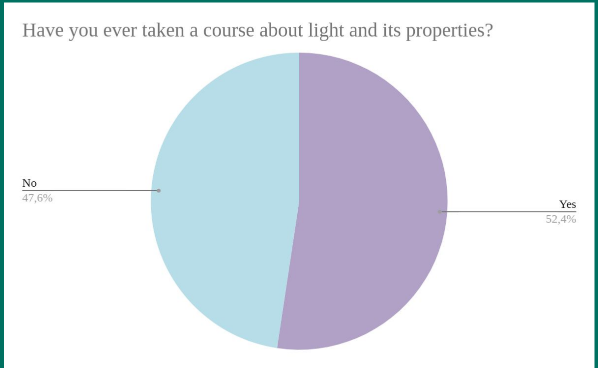


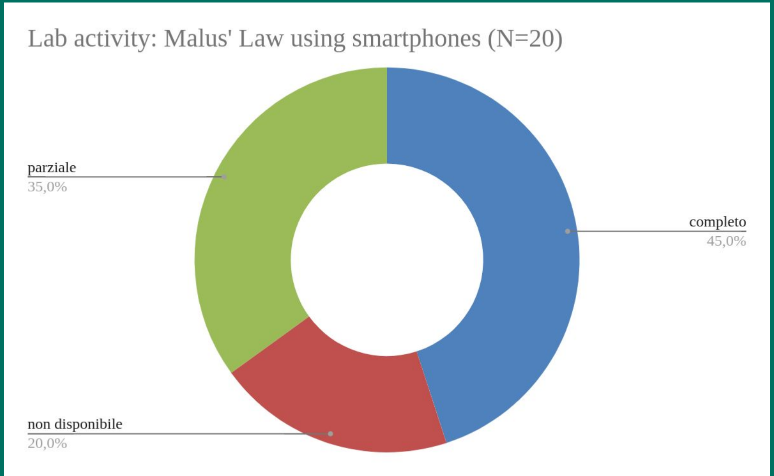


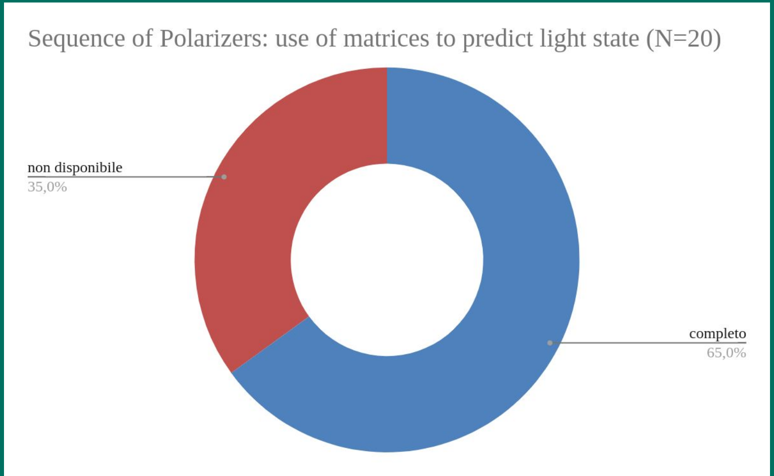
Results

N = 20 students
IV Liceo Scientifico (17 yo)
context: Virtual Optics Summer School (Online course)





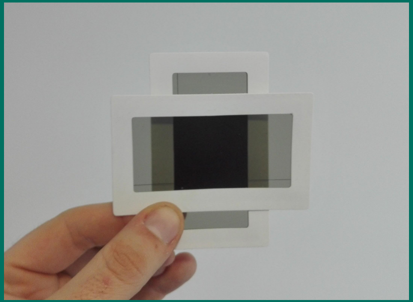


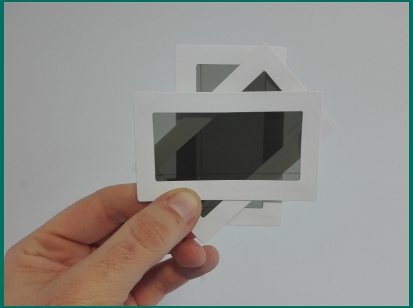


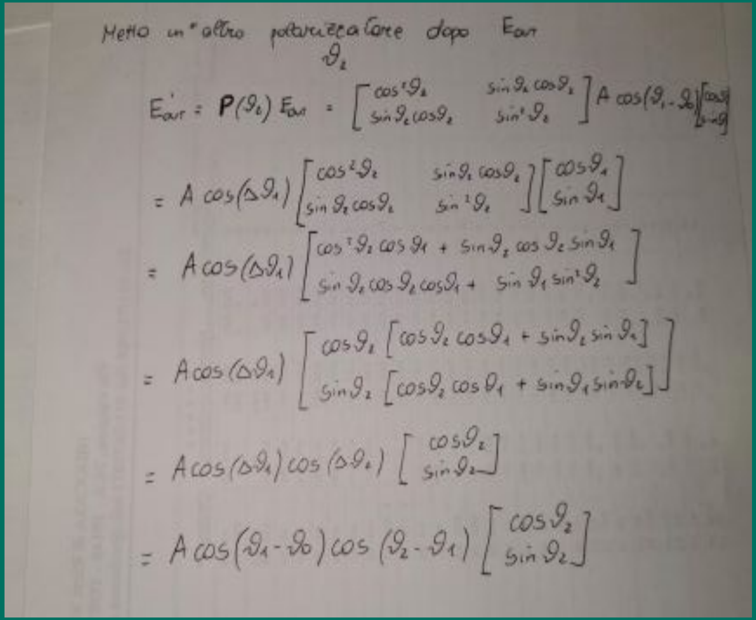
Conclusions

How to facilitate the process of teaching contemporary quantum physics in regular high school classroom activities

In a **significative context** (*light polarization*) students learn how to use **useful mathematical concepts** (*complex numbers, vectors, matrices*, all part of Maths curriculum) that **support the understanding of physical phenomena**.







Conclusions

How to facilitate the process of teaching contemporary quantum physics in regular high school classroom activities

The use of a **powerful mathematical language** can support students understanding of contemporary quantum experiments, such as the **single photon interference** in a Mach-Zehnder interferometer using polarized beam splitters (PBS)